Light nuclei production and flow in relativistic HIC

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- Introduction
- Experiments on light nuclei production
- Coalescence model
  - AMPT+ Coalescence
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  - Comparison with kinetic approach
- Summary

Based on work [PRC 92, 064911 (2015); 95, 054613 (2017); arXiv.1710.05139 [nucl-th]] in collaboration Yifeng Sun, Xuejiao Yin & Lilin Zhu; and [PRC 76, 054910 (2007)] with Yongseok Oh & Zie-wei Lin

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Introduction

- Light nuclei production can provide information on entropy production in HIC [J. Kapusta & P. J. Siemens, Phys. Rev. Lett. 43, 1486 (1979).

$$\text{Entropy/nucleon}: S_N = \frac{5}{2} - \ln(2^{-2/3}\langle d_N \rangle) = 3.95 - \ln R_{dp}$$

- Light nuclei production also provides a complementary way to HBT for studying the emission source in HIC [S. Mrowczynski, Phys. Lett. B277, 43 (1992), Scheibl & Heinz, PRC 59, 1585 (1999)].

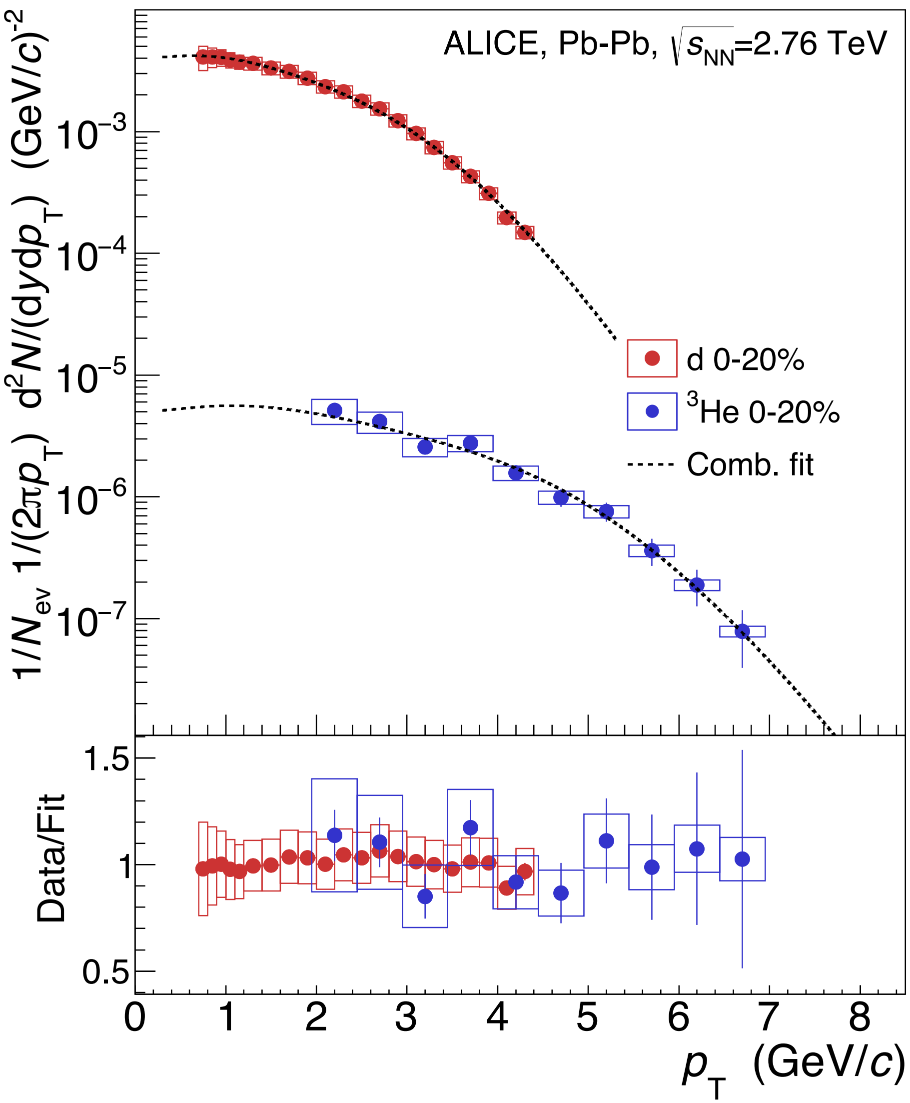
- Coalescence vs statistical production of light nuclei.

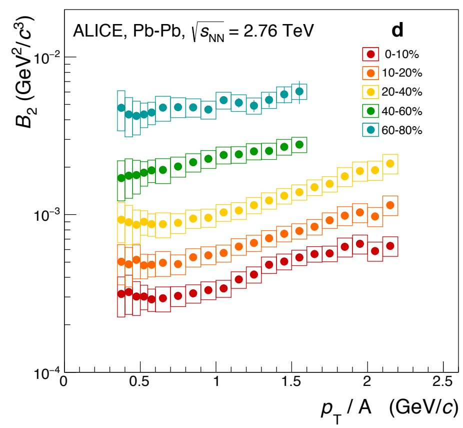
- Coalescence vs kinetic production of light nuclei.

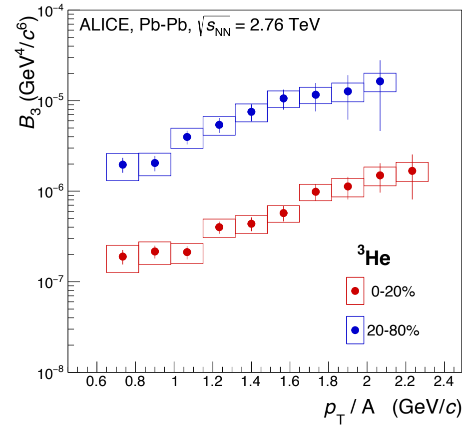
Deuteron and helium spectra and yields from ALICE

ALICE Collaboration: arXiv:1505.08951 [nucl-ex]

$$E_A \frac{d^3 N_A}{dp_A^3} = B_A \left(E_p \frac{d^3 N_p}{dp_p^3} \right)^A$$
$$p_A = A p_p$$



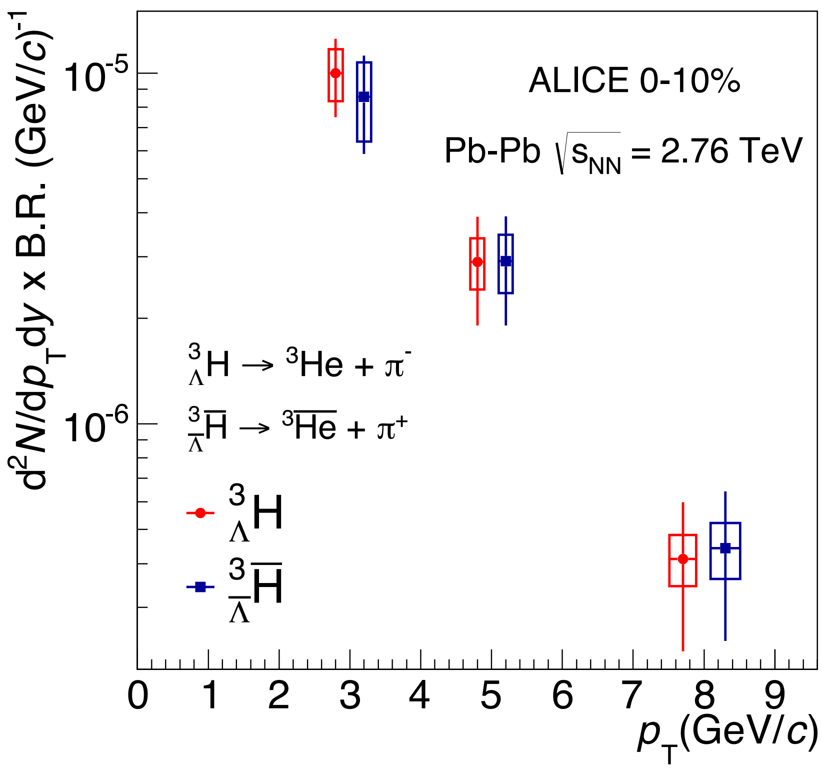


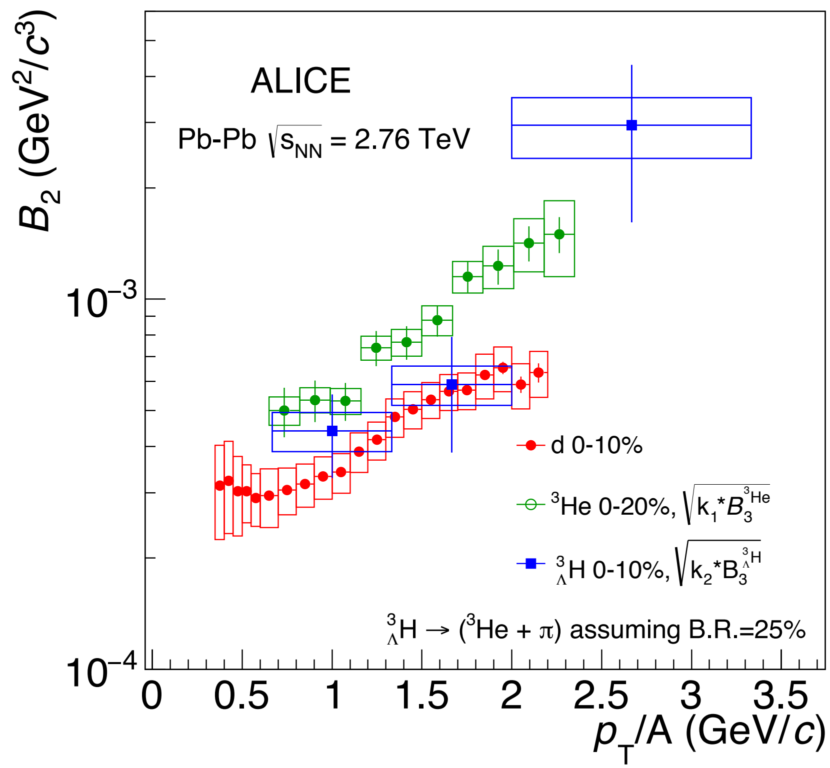


- Spectrum can be fitted by coalescence model with coalescence parameters B_2 and B_3 increasing with p_t as well as with centrality.

Light hypernuclei production from ALICE

ALICE, arXiv:1506.08453 [nucl-ex]





- Similar yields and spectra for hyperhelium and anti-hyperhelium.
- Fitting with coalescence model requires coalescence parameter B_3 also increasing with $p_{T\ as}$ well as with centrality.

Coalescence model in the sudden approximation

Wave functions for initial |i>=|1,2> and final |f>=|3> states

$$\langle \mathbf{r}_1, \mathbf{r}_2 | i \rangle = \phi_1(\mathbf{r}_1)\phi_2(\mathbf{r}_2)$$

$$\langle \mathbf{r}_1, \mathbf{r}_2 | f \rangle = \frac{1}{\sqrt{V}} e^{i\mathbf{K}\cdot(\mathbf{r}_1+\mathbf{r}_2)/2}\Phi(\mathbf{r}_1 - \mathbf{r}_2)$$

Probability for 1+2 -> 3

$$\mathcal{P} = |\langle f | i \rangle|^2$$

Probability for particle 1 of momentum $\mathbf{k}_1$ and particle 2 of momentum $\mathbf{k}_2$ to coalescence to cluster 3 with momentum $\mathbf{K}$

$$\frac{dN}{d^3\mathbf{K}} = g \int d^3\mathbf{x}_1 d^3\mathbf{k}_1 d^3\mathbf{x}_2 d^3\mathbf{k}_2 W_1(\mathbf{x}_1, \mathbf{k}_1) W_2(\mathbf{x}_2, \mathbf{k}_2)$$

$$\times W(\mathbf{y}, \mathbf{k})\delta^{(3)}(\mathbf{K} - \mathbf{k}_1 - \mathbf{k}_2), \qquad \mathbf{y} = \mathbf{x}_1 - \mathbf{x}_2, \quad \mathbf{k} = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2}$$

Wigner functions

$$W(\mathbf{x}, \mathbf{k}) = \int d^3\mathbf{y}\phi^*\left(\mathbf{x} - \frac{\mathbf{y}}{2}\right)\phi\left(\mathbf{x} + \frac{\mathbf{y}}{2}\right)e^{-i\mathbf{k}\cdot\mathbf{y}}$$

For a system of particles 1 and 2 with phase-space distributions $f_i(\mathbf{x}_i,\mathbf{k}_i)$ normalized to $\int d^3\mathbf{x_i} d^3\mathbf{k}_i f_i(\mathbf{x}_i,\mathbf{k}_i) = N_i$, number of particle 3 produced from coalescence of N_1 of particle 1 and N_2 of particle 2

$$\frac{dN}{d^3\mathbf{K}} \approx g\int d^3\mathbf{x}_1 d^3\mathbf{k}_1 d^3\mathbf{x}_2 d^3\mathbf{k}_2 f_1(\mathbf{x}_1,\mathbf{k}_1) f_2(\mathbf{x}_2,\mathbf{k}_2) \times \overline{W}(\mathbf{y},\mathbf{k})\delta^{(3)}(\mathbf{K}-\mathbf{k}_1-\mathbf{k}_2)$$

$$\overline{W}(\mathbf{y},\mathbf{k}) = \int \frac{d^3\mathbf{x}'_1 d^3\mathbf{k}'_1}{(2\pi)^3}\frac{d^3\mathbf{x}'_2 d^3\mathbf{k}'_2}{(2\pi)^3} W_1(\mathbf{x}'_1,\mathbf{k}'_1) W_2(\mathbf{x}'_2,\mathbf{k}'_2) W(\mathbf{y}',\mathbf{k}')$$

Wigner function $W_i(\mathbf{x}_i',\mathbf{k}_i')$ centers around $\mathbf{x}_i$ and $\mathbf{k}_i$

$g = \frac{2J+1}{(2J_1+1)(2J_2+1)}$ Statistical factor for two particles of spin J_1 and J_2 to form a particle of spin J

The above formula can be straightforwardly generalized to multi-particle coalescence, but is usually used by taking particle Wigner functions as delta functions in space and momentum.

Coalescence model for deuteron production

$$\frac{d^3N_d}{dp_d^3} = \frac{3}{4}\int d^3x_1 d^3p_1 d^3x_2 d^3p_2 f_p(x_1,p_1) f_n(x_2,p_2)$$

$$\times f_d^W(x_1^{'},x_2^{'};p_1^{'},p_2^{'})\delta^{(3)}(p_d - p_1 - p_2)$$

$f_{p/n}(x,p)$: proton/neutron phase-space distribution function at freeze out

f_d^W: deuteron Wigner distribution based on harmonic oscillator wave function

$$f_d^W(x_1^{'},x_2^{'};p_1^{'},p_2^{'}) = 8\exp\left[-\left(p_1^{'} - p_2^{'}\right)^2\sigma^2/4 - \left(x_1^{'} - x_2^{'}\right)^2/\sigma^2\right]$$

with $\sigma = 1/\sqrt{\mu\omega}, \quad \mu = m_N/2$

$$\sqrt{\left\langle r_d^2\right\rangle} = 1.96\,\text{fm} \rightarrow \omega = 8.06\times 10^{-3}\ \text{GeV}$$

Wigner phase-space distribution function for triton and helium3

Gaussian wave function $$\psi(\vec{r}_1,\vec{r}_2,\vec{r}_3) = (3\pi^2 b^4)^{-3/4} e^{-\frac{\rho^2+\lambda^2}{2b^2}}$$

Jacobi coordinates

$$\vec{R} = \frac{1}{3}(\vec{r}_1+\vec{r}_2+\vec{r}_3), \quad \vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1-\vec{r}_2), \quad \vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1+\vec{r}_2-2\vec{r}_3)$$

$$\vec{K} = \vec{k}_1+\vec{k}_2+\vec{k}_3, \quad \vec{k}_\rho = \frac{1}{\sqrt{2}}(\vec{k}_1-\vec{k}_2), \quad \vec{k}_\lambda = \frac{1}{\sqrt{6}}(\vec{k}_1+\vec{k}_2-2\vec{k}_3)$$

$$\Rightarrow \quad \rho^W_{t(He)}(\rho,k_\rho;\lambda,k_\lambda) = 8^2 e^{-\frac{\rho^2+\lambda^2}{b^2}} e^{-(k_\rho^2+k_\lambda^2)b^2}$$

b=1.61 fm for triton and 1.74 for He $\longrightarrow$ correct radii

A multiphase transport (AMPT) model

Lin, Ko, Li, Zhang & Pal, PRC 72, 064901 (05);
http://www-cunuke.phys.columbia.edu/OSCAR

Default: Zhang, Ko, Li & Lin, PRC 61, 067901 (00); Lin, Pal, Ko, Li & Zhang, PRC 64, 041901 (01);

- Initial conditions: HIJING (soft strings and hard minijets)
- Parton evolution: ZPC
- Hadronization: Lund string model for default AMPT
- Hadronic scattering: ART

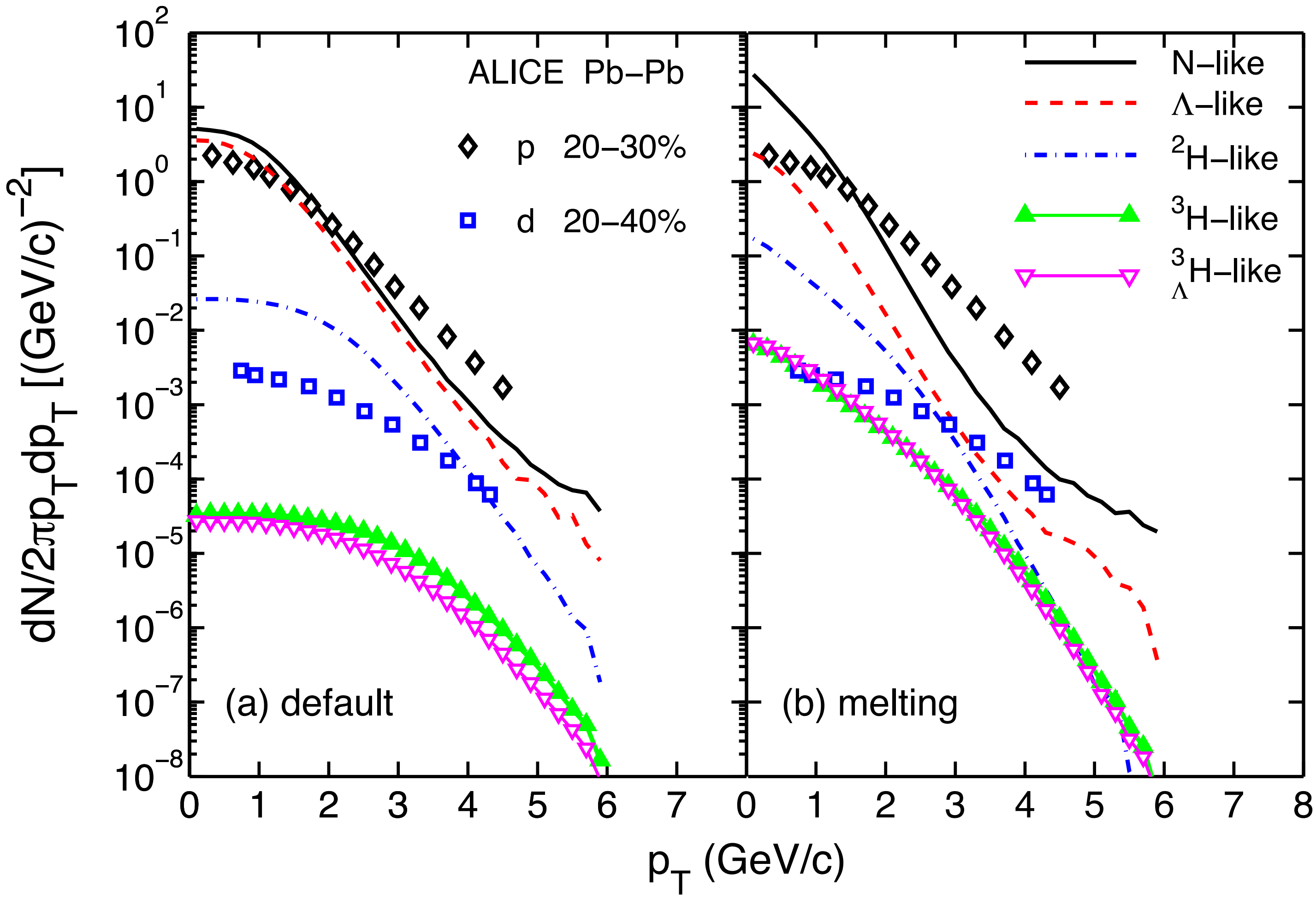
String melting: Lin & Ko, PRC 65, 034904 (02); Li, Ko & Pal, PRL 89, 152301 (02)

- Convert hadrons from string fragmentation into quarks and antiquarks
- Evolve quarks and antiquarks with ZPC
- When partons stop interacting, combine nearest quark and antiquark to meson, and nearest three quarks to baryon (coordinate-space coalescence)
- Hadron flavors are determined by the invariant mass of quarks

Pb+Pb @ 2.76 GeV from AMPT

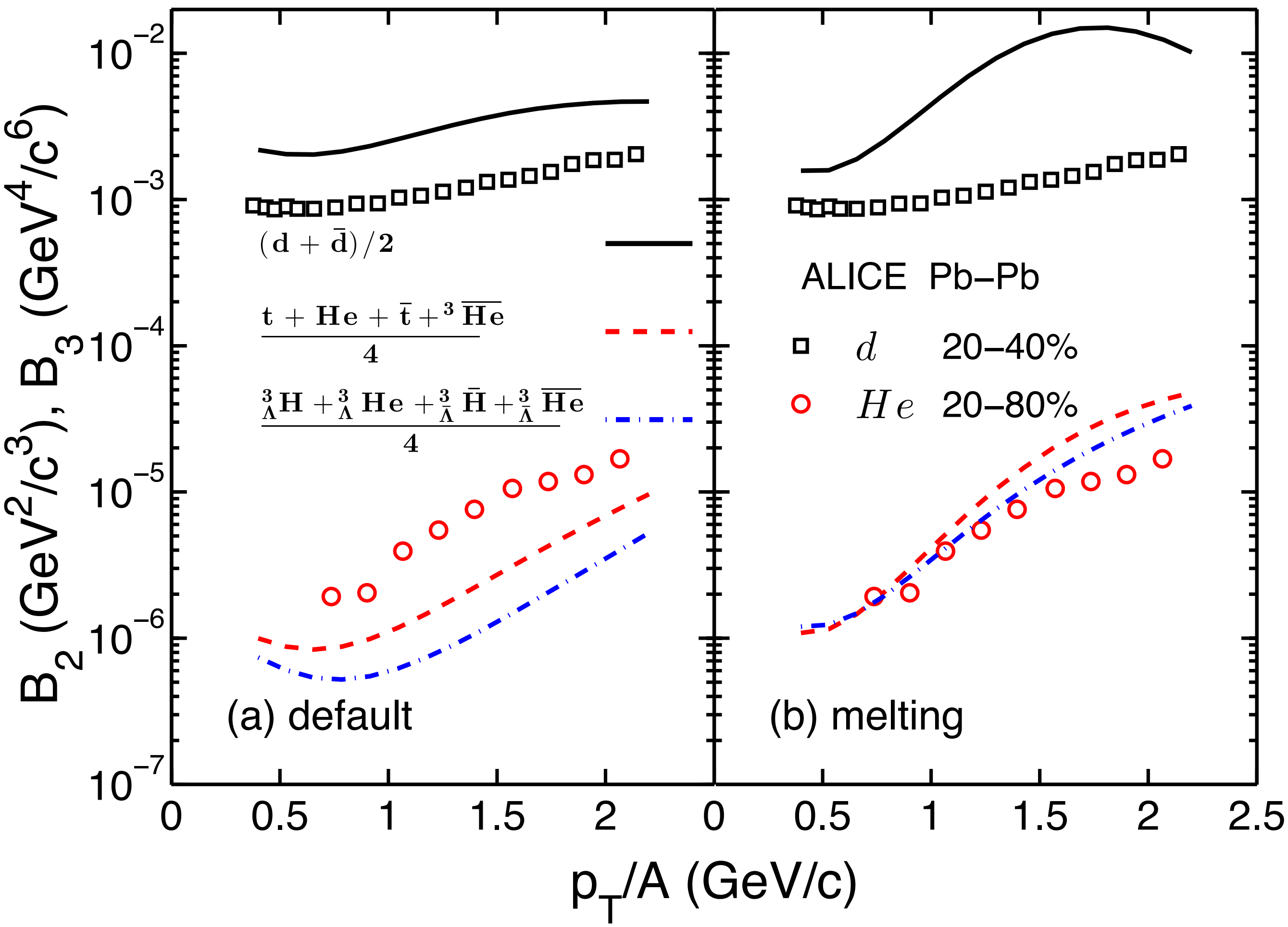
Transverse momentum spectra

Zhu, Ko & Yin, PRC 92, 064911 (2015)



- Default AMPT works better than string melting due to baryon problem in latter, but both not perfect.

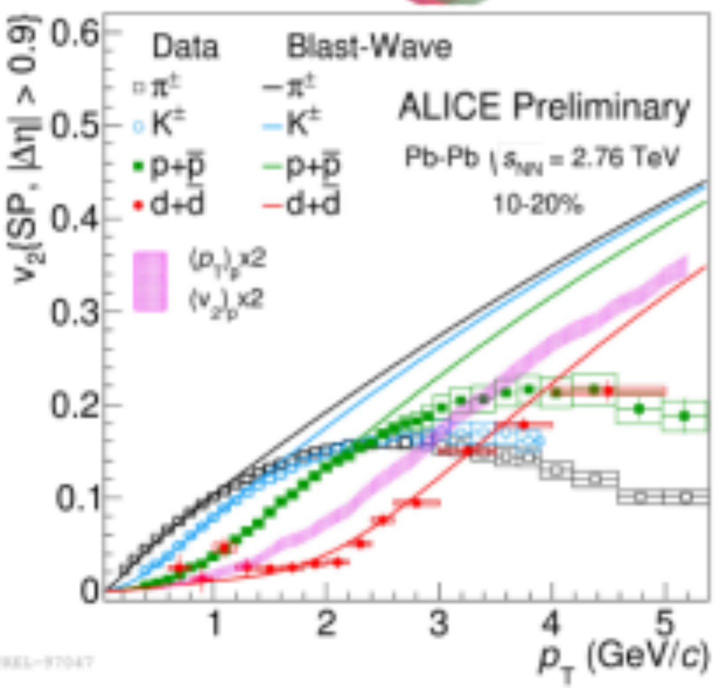
Coalescence factor



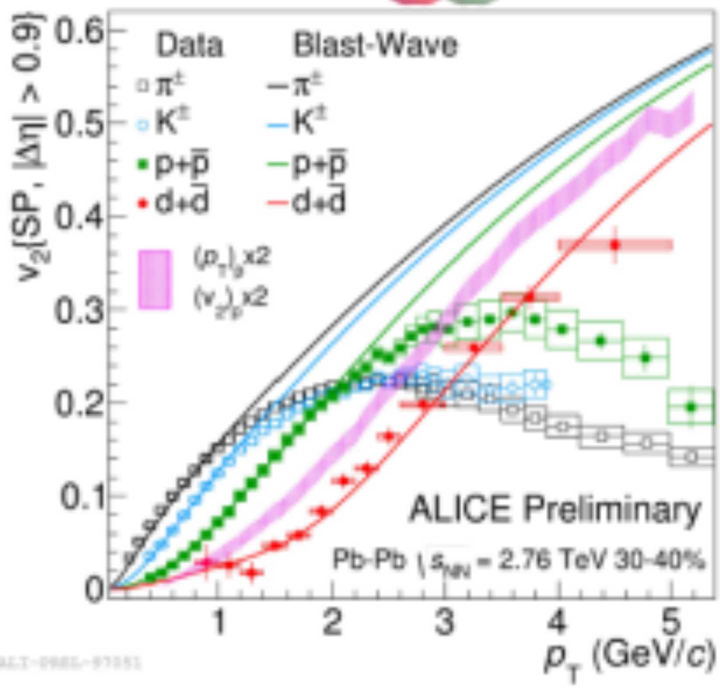
- Coalescence factor is overestimated for deuteron and underestimated (default) and overestimated (melting) for triton and helium-3. Need to improve AMPT for nucleon production

Blast-wave vs Coalescence

10-20%



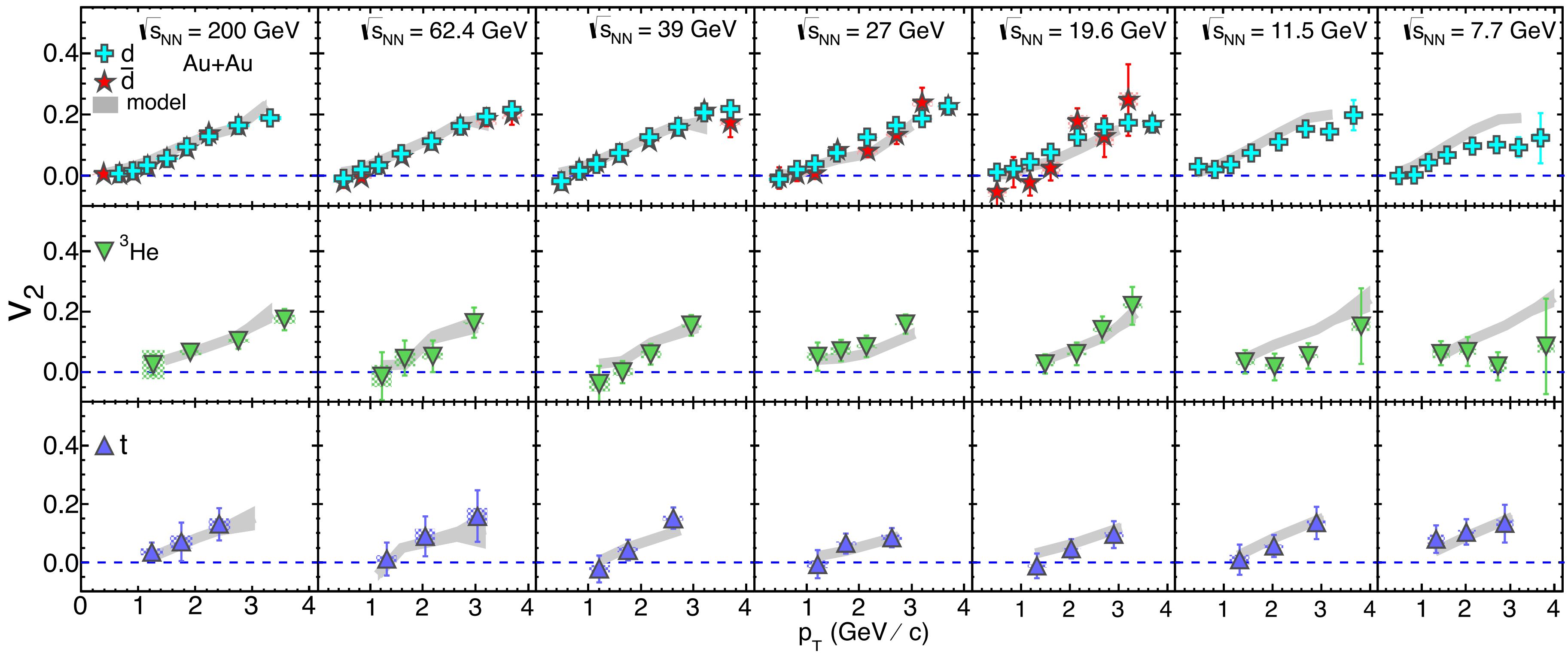
30-40%



- How does the Blast-Wave prediction differ from expectations from coalescence?
  - The measured v_2 of protons was used to compute the expected v_2 of deuterons (reverse n_q scaling = both p_T and v_2 of measured p was multiplied by 2) and the results were compared to data and Blast-Wave curves:

Courtesy of Jurgen Schukraft

Ellitpic flow from STAR Beam Energy Scan arXiv:1601.0705



- AMPT + Coalescence reproduces data reasonably well.
- Blast wave mode fails.

The blast-wave model

Momentum spectrum $$E\frac{d^3N}{d^3\mathbf{p}} = \int_{\Sigma^\mu} d^3\sigma_\mu p^\mu f(x,p)$$

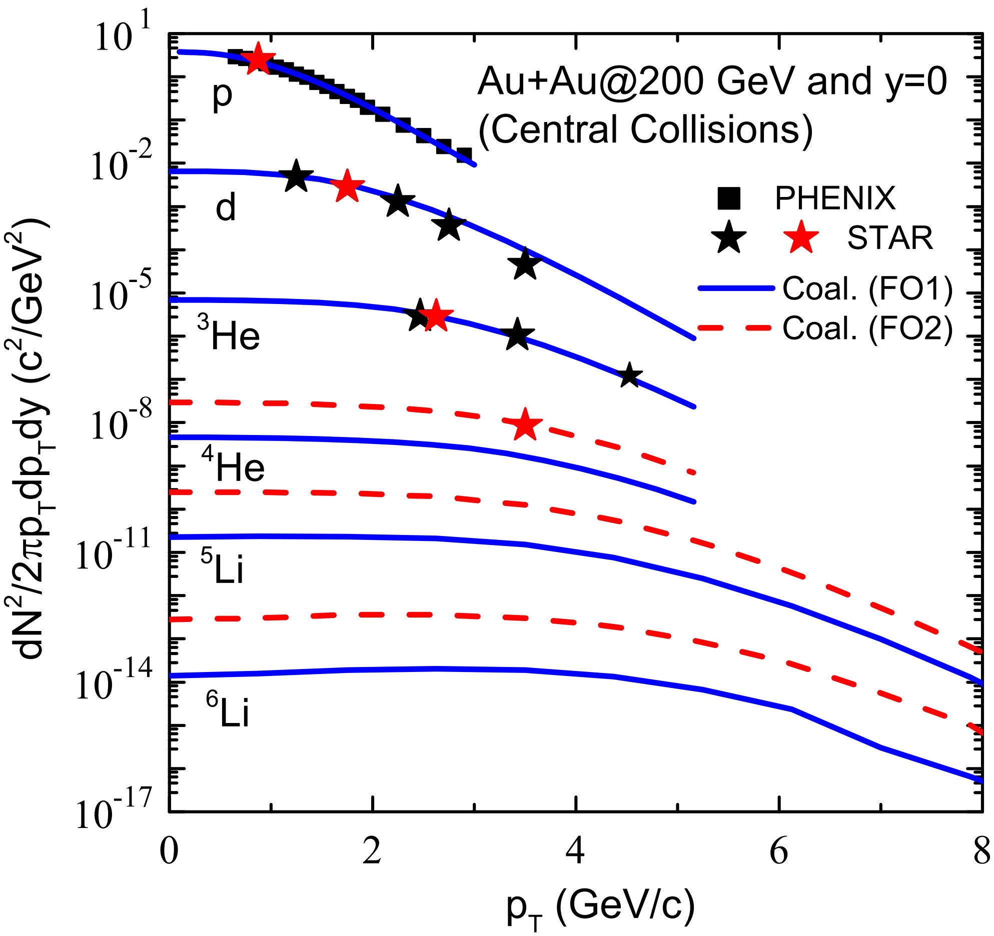
$$\frac{d^3N}{p_T dp_T dy d\phi_p} = \frac{2\xi\tau_0}{(2\pi)^3}\int_{\Sigma^\mu} d\eta r dr d\phi m_T \cosh(\eta - y) \times \exp\left[-\frac{m_T\cosh\rho\cosh(\eta-y) - p_T\sinh\rho\cos(\phi_p-\phi_b)}{T_K}\right]$$

$$\eta = \frac{1}{2}\ln\frac{t+z}{t-z}, \quad y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z}, \quad \rho = \frac{1}{2}\ln\frac{1+|\beta|}{1-|\beta|}$$

Flow velocity : $$\beta = \beta(r)\left[1+\varepsilon(p_T)\cos(2\phi_b)\right]$$
$$\beta(r) = \beta_0 r/R, \quad \varepsilon(p_T) = c_1\exp(-p_T/c_2)$$

Transverse radius : $$r \le R_0\left[1+s_2\cos(2\phi)\right]$$

Binding energy effect on antimatter production



Sun and Chen, PLB 751, 272 (2015)

- ^{4}He is formed earlier because its larger binding energy.
- Assuming a similar effect for ^{5}Li and ^{6}Li leads to their enhanced production.

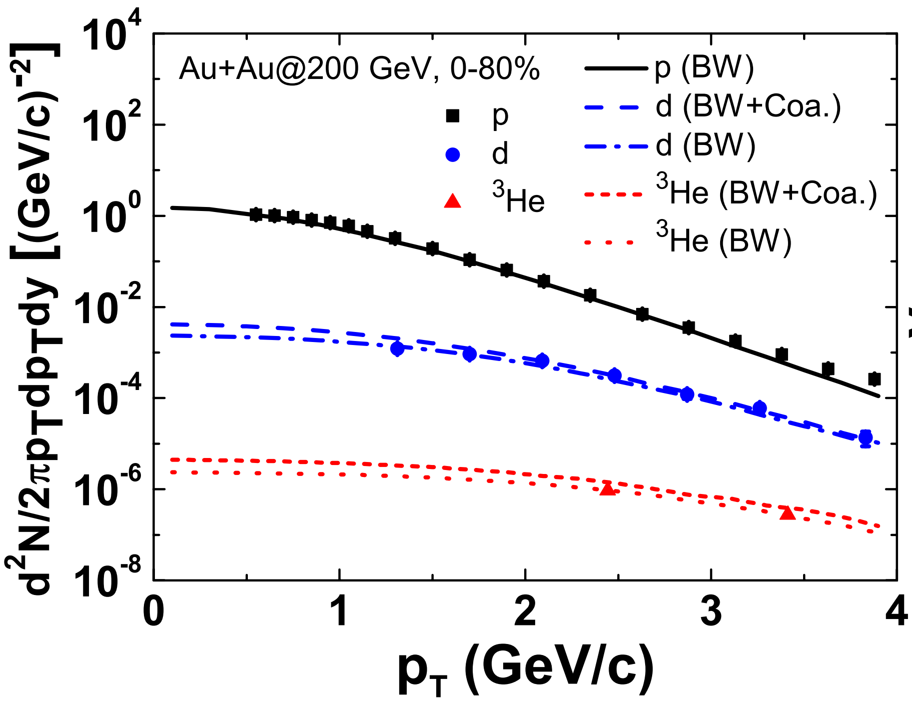
Table 1
Parameters of the blast-wave-like analytical parametrization for (anti-)nucleon phase–space freezeout configuration.

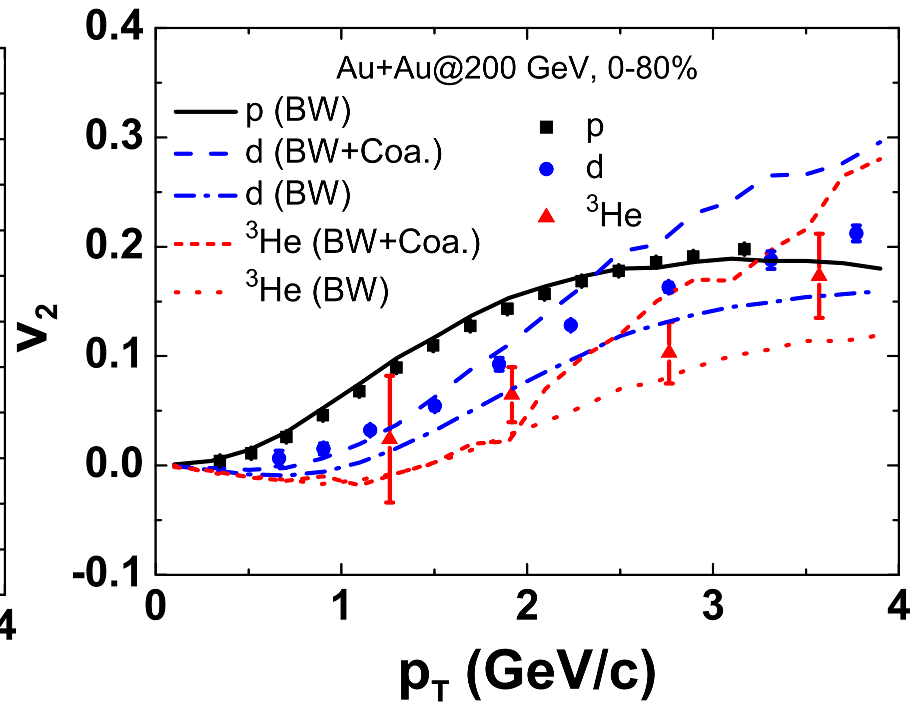
	T (MeV)	ρ_0	R_0 (fm)	τ_0 (fm/c)	$\Delta\tau$ (fm/c)	ξ_p	$\xi_{\overline{p}}$
FO1	111.6	0.98	15.6	10.55	3.5	10.45	7.84
FO2	111.6	0.98	12.3	8.3	3.5	21.4	16.04

Transverse momentum spectrum and elliptic flow at RHIC

Yin, Ko, Sun & Zhu, PRC 95, 054913 (2017)

Centr. (%)	ξ	τ_0 (fm/c)	T_K (MeV)	β_0	R_0 (fm)	c_1	c_2 (GeV/c)	s_2	a (GeV/c)$^{-1}$
0-80	1.76	9.0	130	0.67	10.0	0.148	2.12	-0.04	0.25

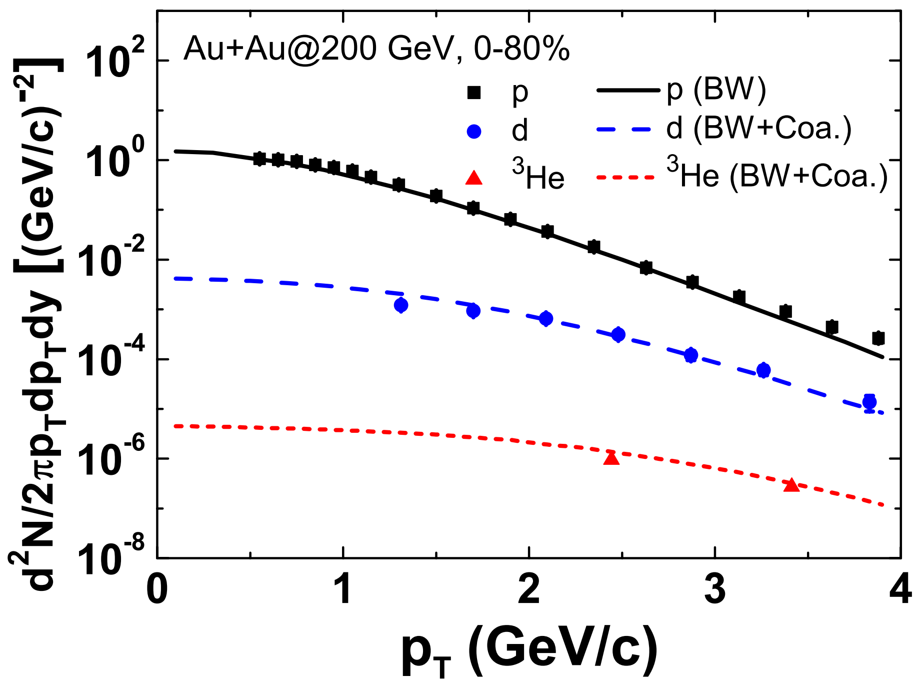


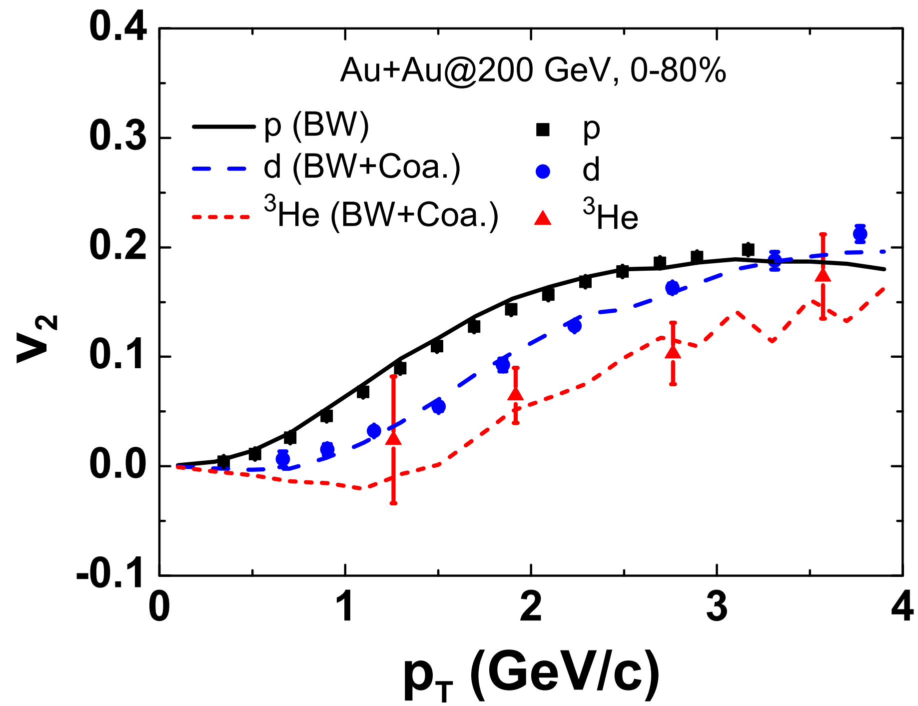


- Transverse momentum spectra are well reproduced.
- Deuteron elliptic flow is too large, while ^{3}He elliptic flow is small.

Extended blast-wave model

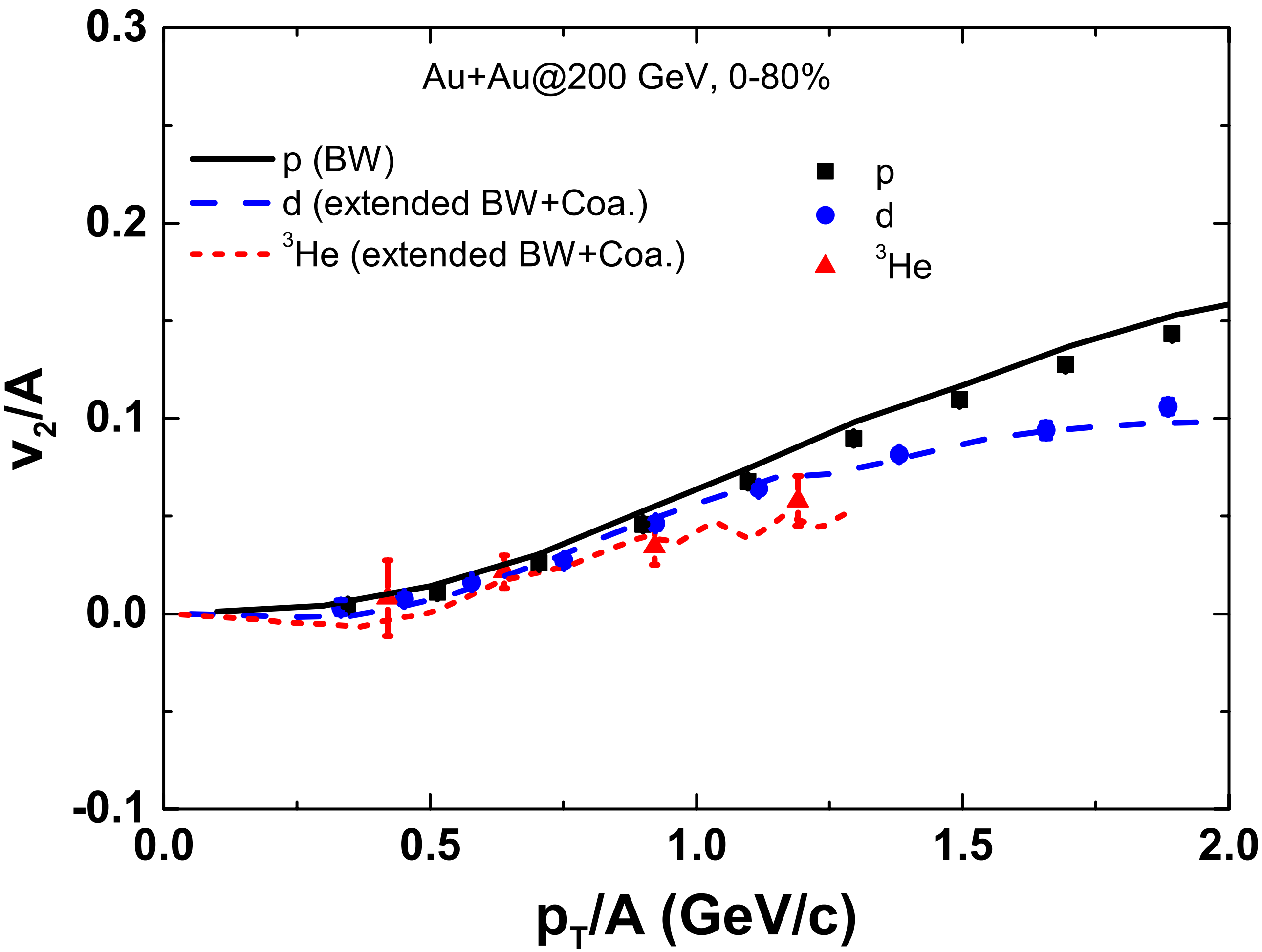
$$\mathrm{Momentum-space\ correlation}: R = R_0 e^{a(p_T - p_0)},\ (|p_x| > |p_y|)$$





- Both deuteron and ^{3}He elliptic flows are better described after allowing nucleons with momenta larger than p_0=0.9 GeV more spread in space when their momenta are more aligned along the reaction plane.

Nucleon number scaled elliptic flow

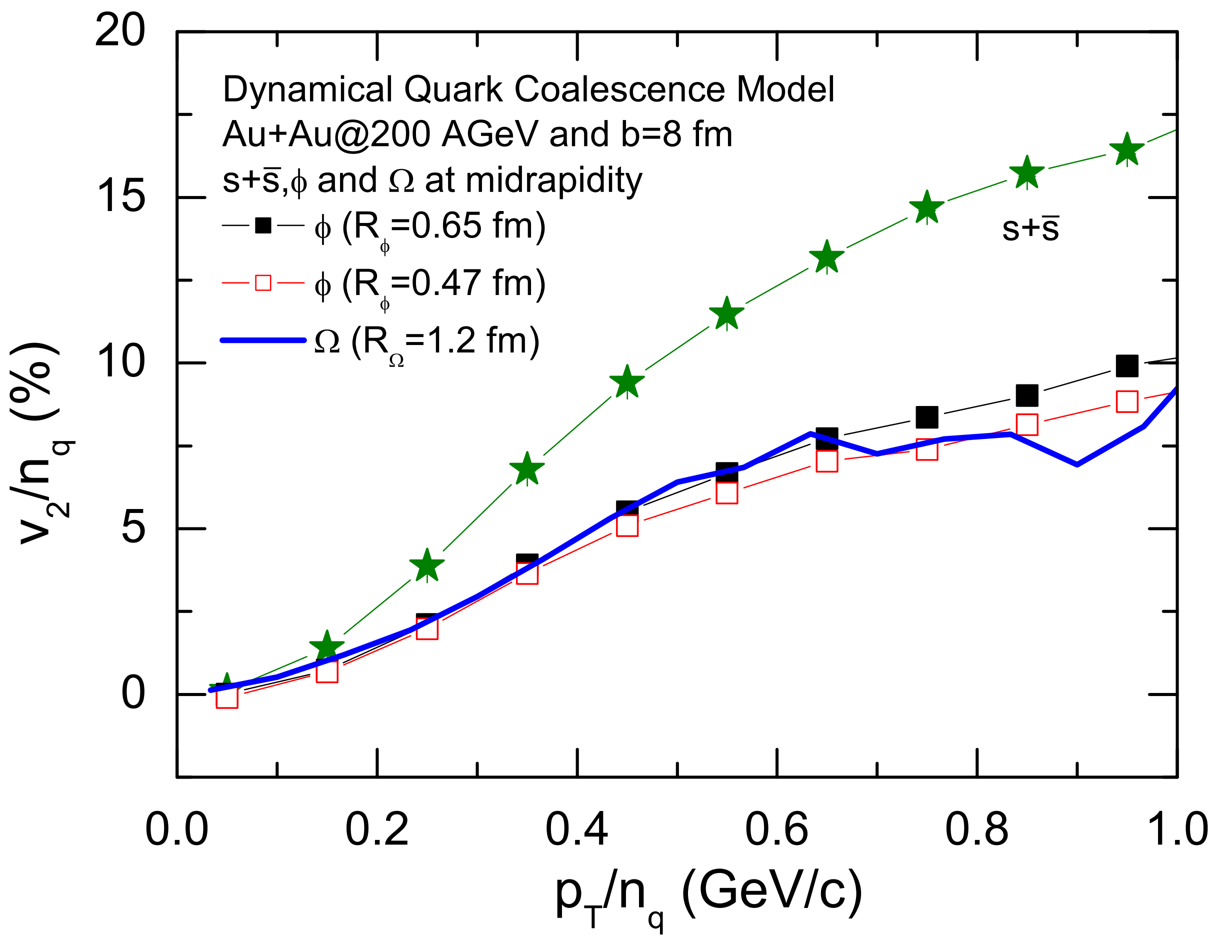


- Approximate nucleon number scaling of deuteron and helim-3 elliptic flow at low p_T/A from coalescence model.

Dynamical quark coalescence model

Chen & Ko, PRC 73, 044903 (06)

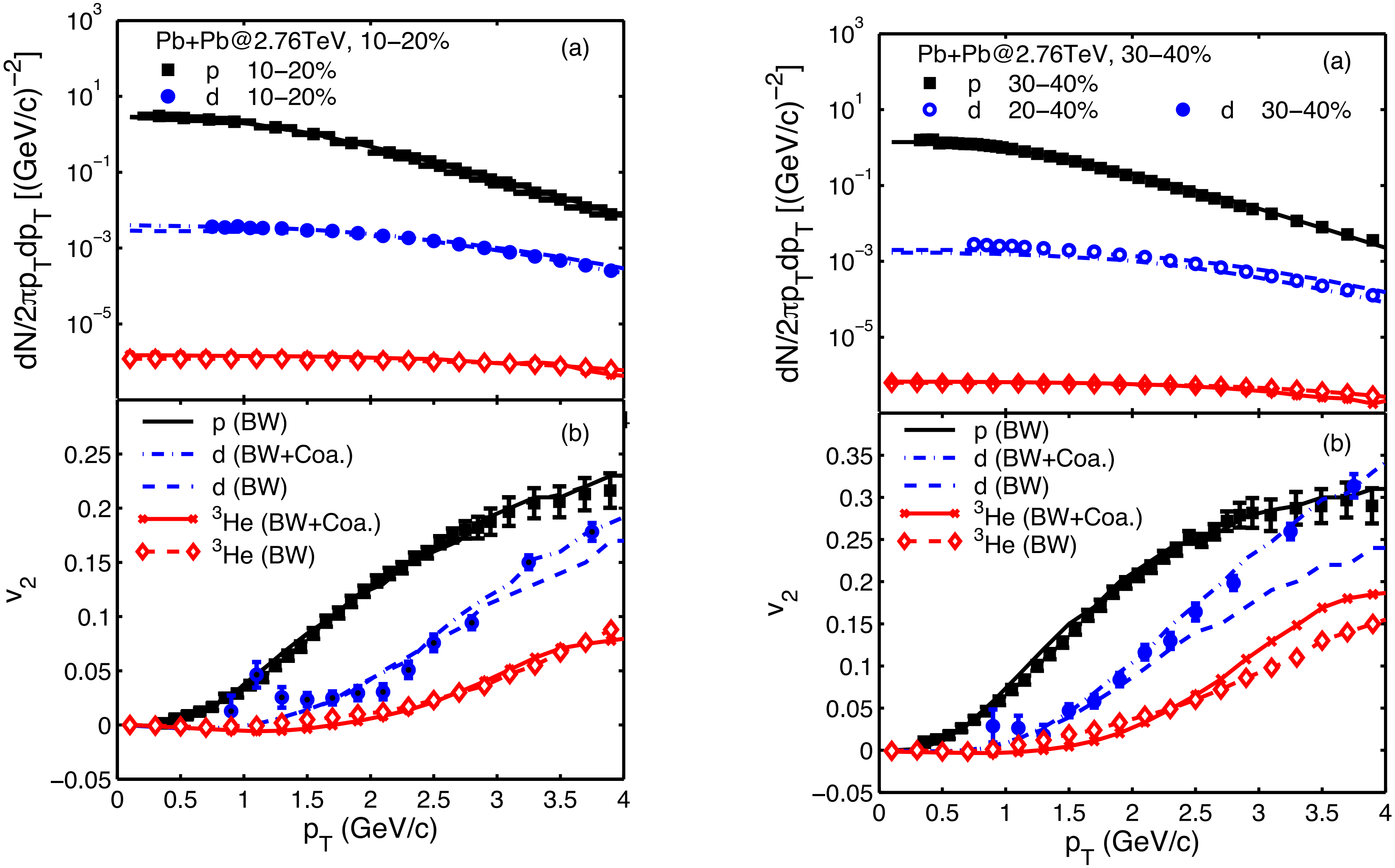
Based on the phase-space distribution of strange quarks from AMPT and including quark spatial and momentum distributions in hadrons



Although scaled phi and Omega satisfy constituent quark number scaling, they are smaller than the strange quark elliptic flow.

Transverse momentum spectra and elliptic flow at LHC

Zhu, Zheng, Ko & Sun, arXiv:1710.05139 [nucl-th]



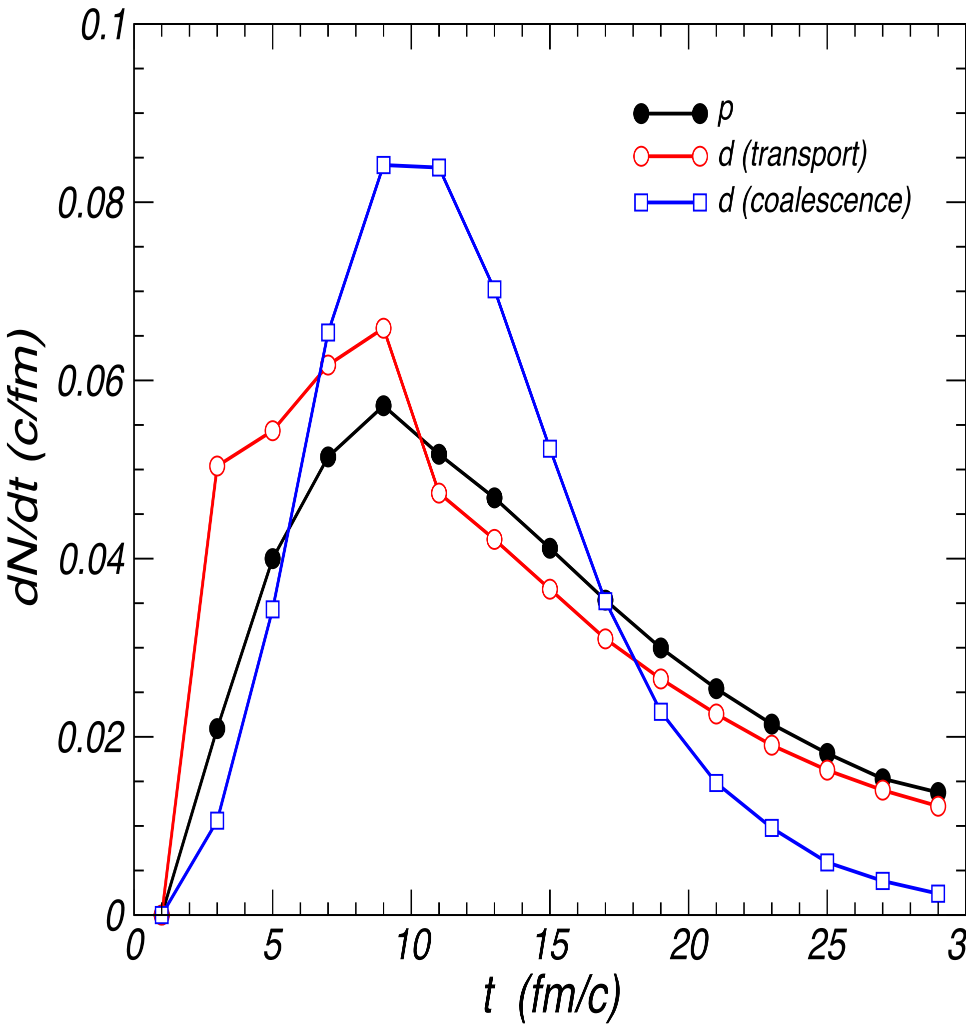
Centr. (%)	ξ	τ_0 (fm/c)	T_K (MeV)	β_0	R_0 (fm)	c_1	c_2 (GeV/c)	s_2	a (GeV/c)$^{-1}$
10-20	5.5	13.5	120	0.84	17.0	0.09	4.6	-0.07	0.05
30-40	5.0	10.5	120	0.825	13.0	0.15	3.3	-0.12	0.02

A relativistic transport (ART) model for HIC

Li & Ko, PRC 52, 2037 (1995)

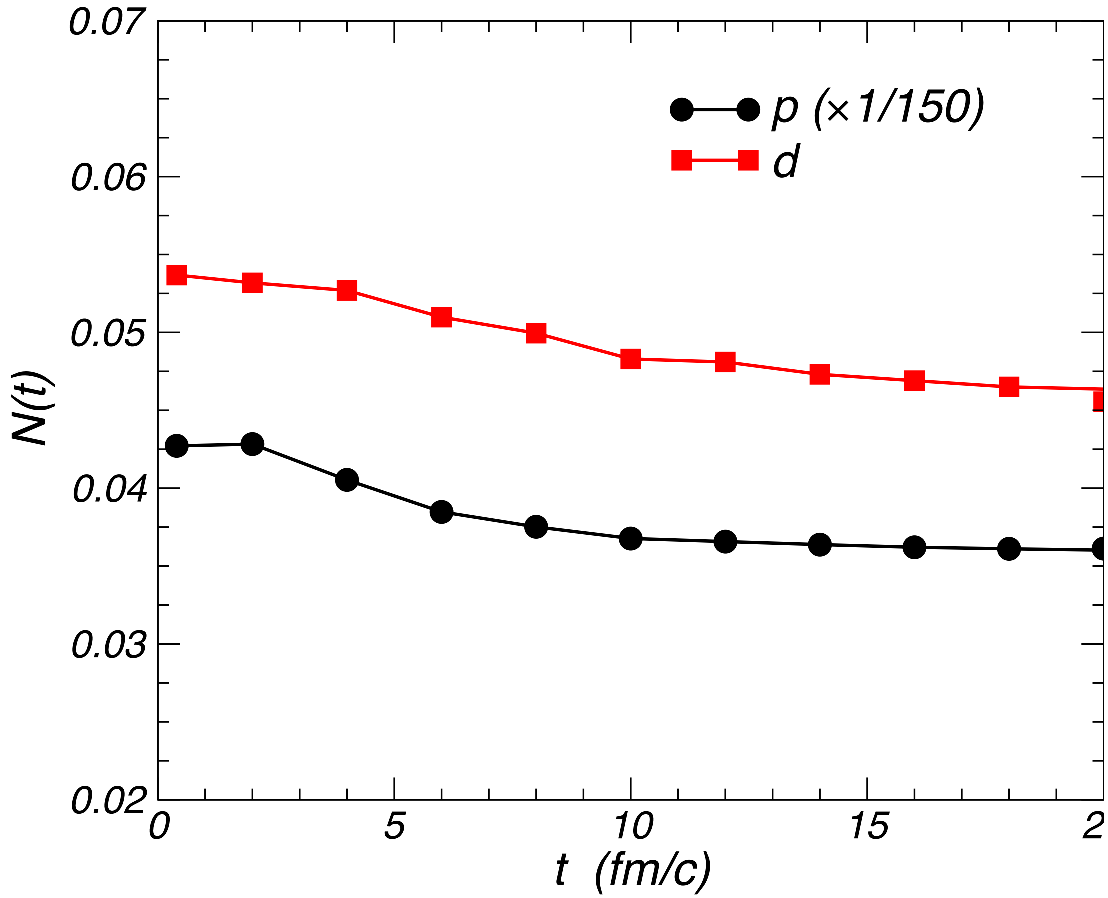
- Based on BUU model with explicit isospin dependence
- Including baryons N, Δ(1232), N*(1440), N*(1535), Λ, Σ and mesons π, ρ, ω, η, K
- Including baryon-baryon, meson-baryon and meson-meson elastic and inelastic scattering with empirical cross sections if available, otherwise from theoretical models
- Effects of higher nucleon and delta resonances up to 2 GeV are included as intermediate states in meson-baryon scattering
- Very successful in describing many experimental results at AGS
- Used as a hadronic afterburner in the AMPT model
- Extended to include deuteron production ($n+p \to d+\pi$) and annihilation ($d+\pi \to n+p$) as well as its elastic scattering [Oh, Ko & Lin, PRC 76, 054910 (2007)]

Deuteron emission time distributions



- Similar emission time distributions for protons and deuterons in coalescence model
- Slight different deuteron early emission time distribution in transport and coalescence models

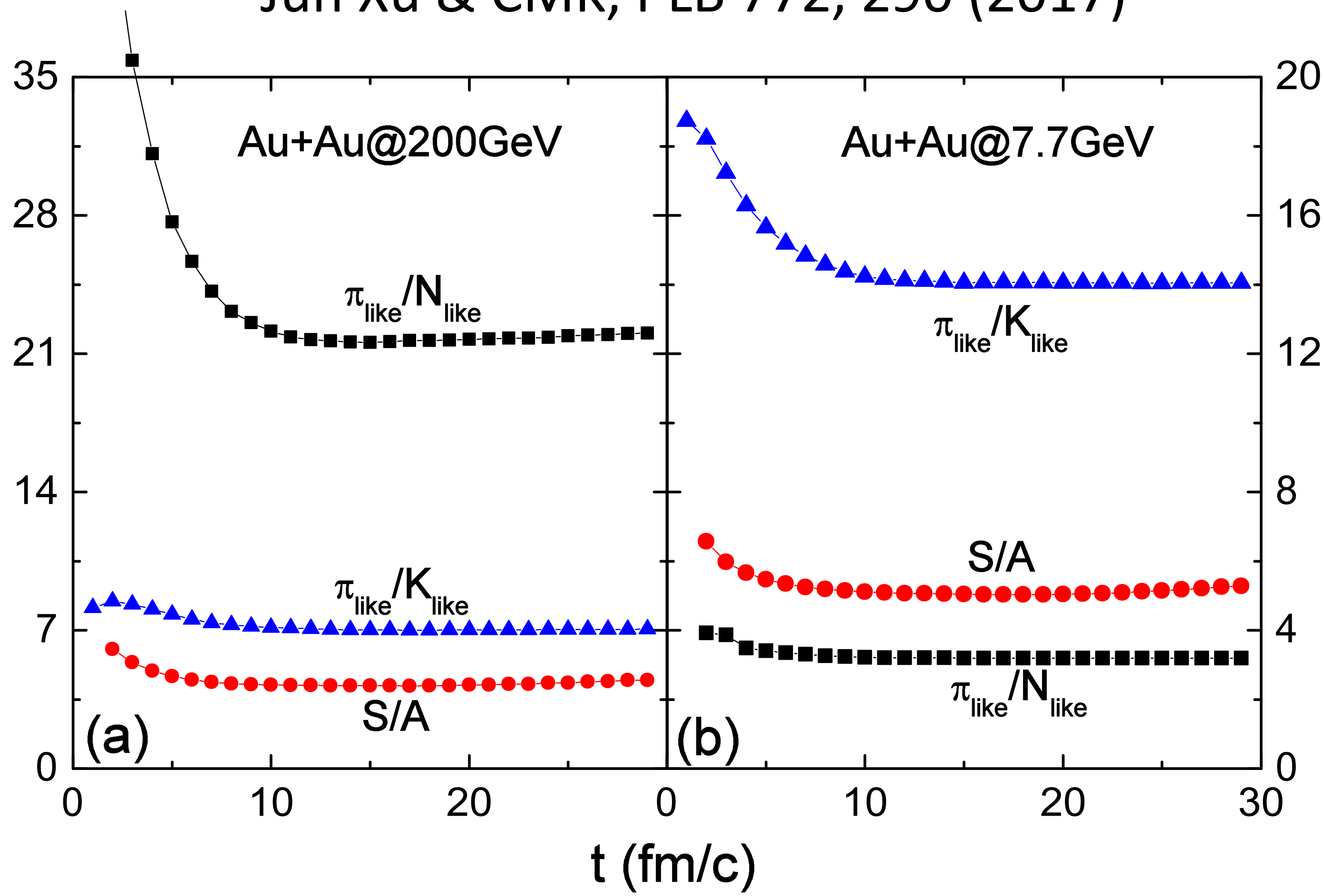
Time evolution of proton and deuteron numbers



- Both proton and deuteron numbers decrease only slightly with time → early chemical equilibration

Chemical freeze-out in relativistic heavy ion collisions

Jun Xu & CMK, PLB 772, 290 (2017)



- Both ratio of effective particle numbers and entropy per particle remain essentially constant from chemical to kinetic freeze-out.

Summary

- Coalescence model using kinetic freeze-out nucleons from blast-wave model → light nuclei transverse momentum spectra consistent with data from RHIC and LHC.
- To describe experimental data for light nuclei elliptic flow requires high momentum nucleons more spread in space when their momenta are more aligned along the reaction plane.
- Deuteron yield in kinetic approach are essentially fixed at chemical freeze out (higher temperature) as in statistical model. Like other hadrons, they acquire non-zero chemical potential at kinetic freeze out (lower temperature). Their constancy during hadronic evolution is accompanied by the constancy of entropy/particle.