Theory view on Hyperon-Hyperon Interaction from Correlations

Kenji Morita
(University of Wroclaw / RIKEN)

Ref: KM, T.Furumoto, A.Ohnishi, PRC91, 024916 ('15). ΛΛ

KM, A.Ohnishi, F.Etminan, T.Hatsuda, PRC94, 031901(R) ('16). pΩ

A. Ohnishi, KM, K.Miyahara, T.Hyodo, NPA954, 294 ('16). ΛΛ, Kbar N

EXHIC Collaboration, Prog. Part. Nucl. Phys.95, 279 ('17). Review

T.Hatsuda, KM, A.Ohnishi, K.Sasaki, NPA967, 856 ('17). pΞ

Outline

1. Introduction
2. Correlation from Final State Interaction in Heavy-Ion Collisions
3. Applications : Dibaryon Candidate

   From S=-6 to -2

   combined w/ Lattice QCD@(almost) phys.point

   1. $\Omega\Omega$
   2. $p\Omega$
   3. $p\Xi$
4. Concluding Remarks

Hadron-Hadron Interaction

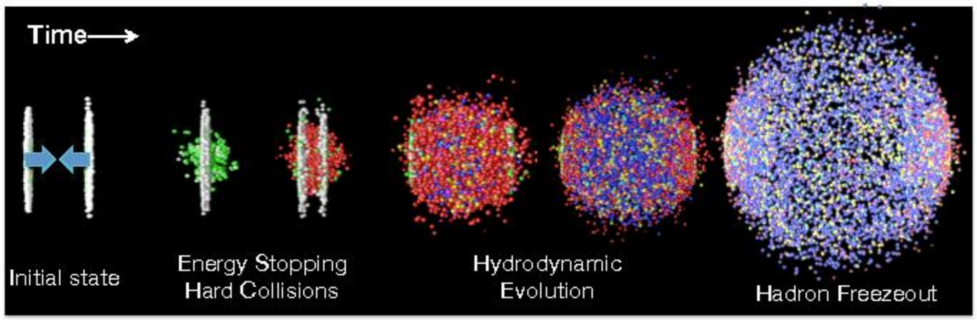
- **Testing Ground for QCD at Low Energy**
  - **Foundation of Nuclear Physics**
  - **Baseline for many-body physics**
    - Exotic hadrons / finite T, ρ

Chiral Symmetry

Confinement

H-Dibaryon	Heavy Ion Collisions
*N*Ω Dibaryon	Compact Stars
Λ(1405)etc	Hypernuclei

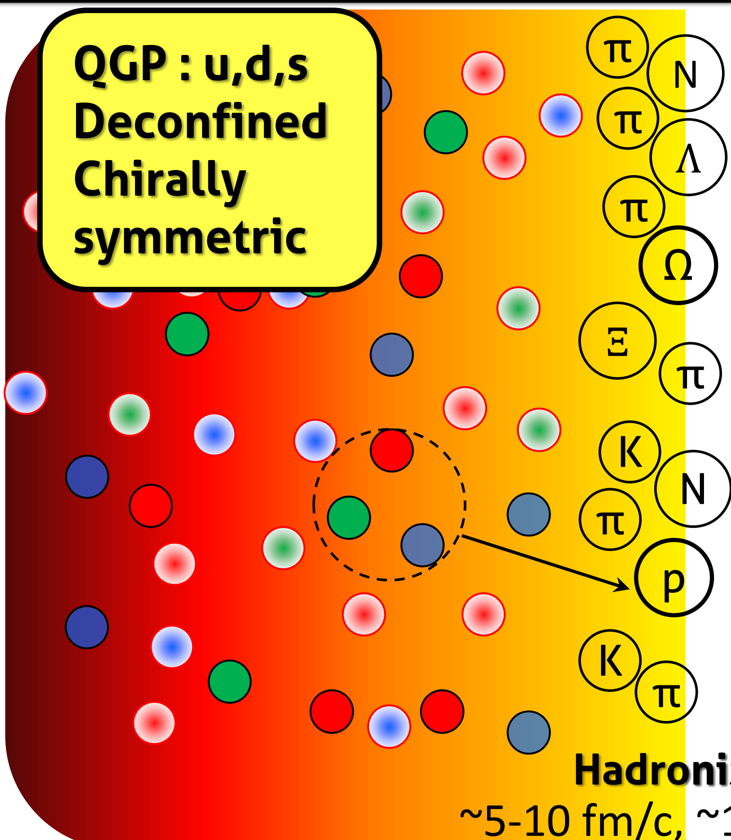
Heavy Ion Collisions as Hyperon Factory



Pb+Pb **2.76ATeV@LHC** $\times 10^7$ events

$$\left.\frac{dN_Y}{dy}\right|_{y=0} \simeq \begin{cases} 1-26, & \Lambda\,(S=-1) \\ 0.12-3.3 & \Xi\,(S=-2) \\ 0.015-0.6 & \Omega\,(S=-3) \end{cases}$$

ALICE, PLB'15

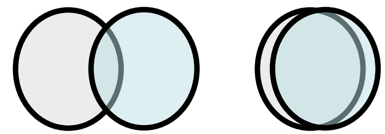


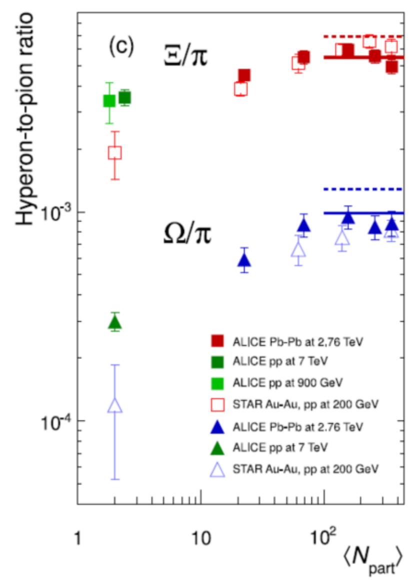
Hadronization
~5-10 fm/c, ~160MeV

time

Enhanced pair creation of s-quarks

(60-80%) (0-5%)





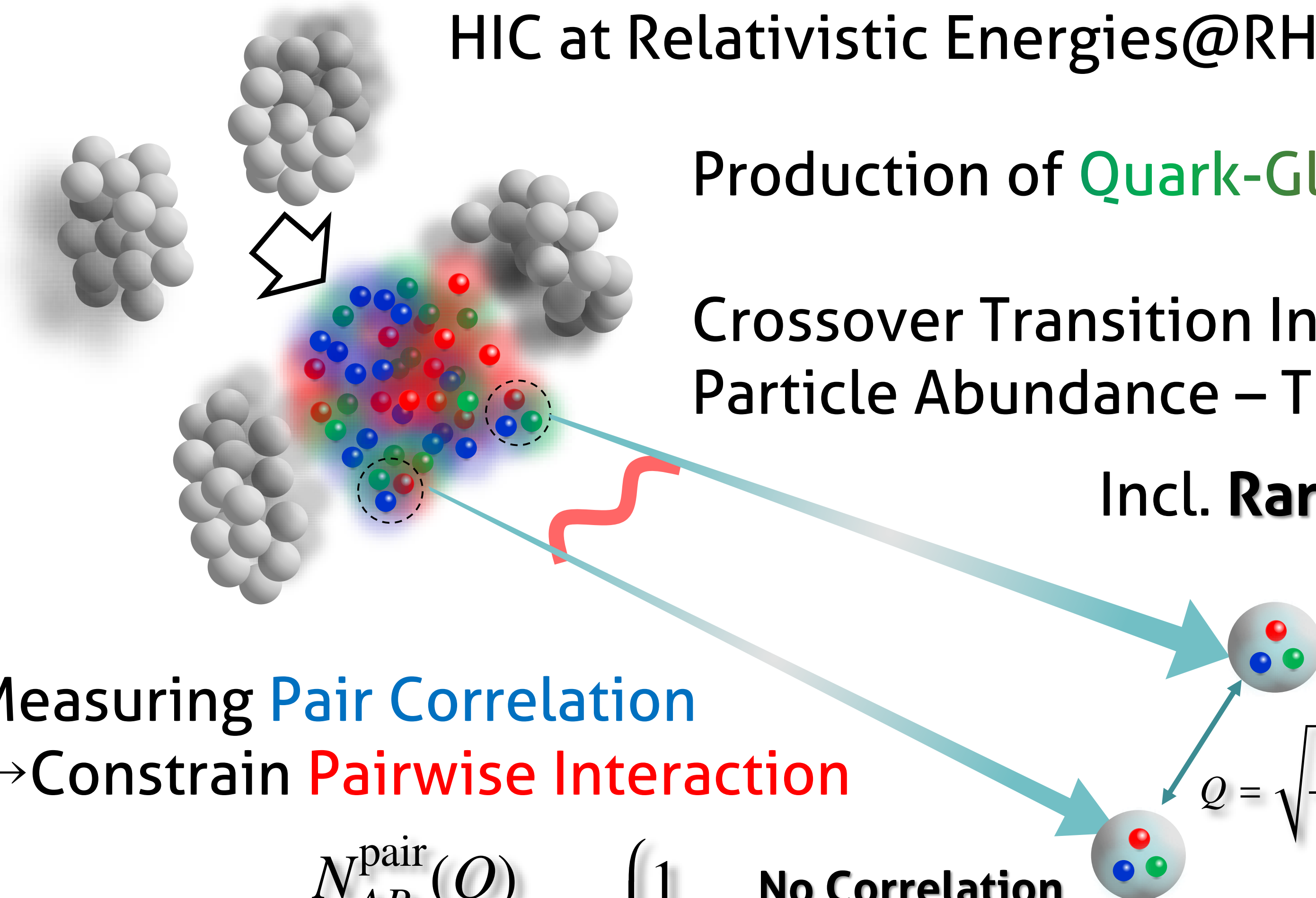
How HIC Can Tell Us Interaction?

HIC at Relativistic Energies@RHIC, LHC

Production of Quark-Gluon Plasma

Crossover Transition Into Hadron
Particle Abundance – Thermal Eq.

Incl. **Rare Particles**



Measuring Pair Correlation
→Constrain Pairwise Interaction

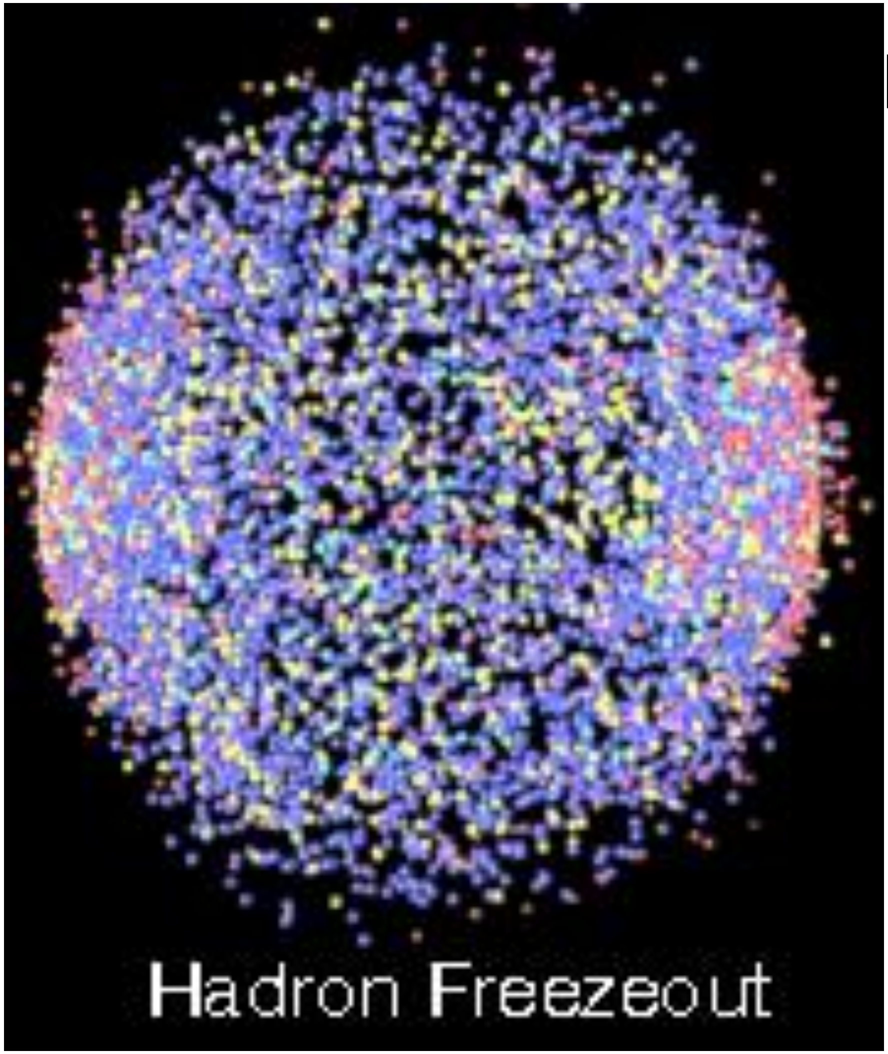
$$Q=\sqrt{-\left(\frac{p_1-p_2}{2}-\frac{(p_1-p_2)\cdot P}{P^2}P\right)^2}$$

$$C_{AB}(Q)=\frac{N_{AB}^{\mathrm{pair}}(Q)}{N_A N_B(Q)}=\begin{cases}1 & \text{No Correlation}\\ \text{others} & \text{Interaction, Interference etc}\end{cases}$$

No Correlation

Interaction
Interference etc

Counting Correlated Pairs



Freeze-out:

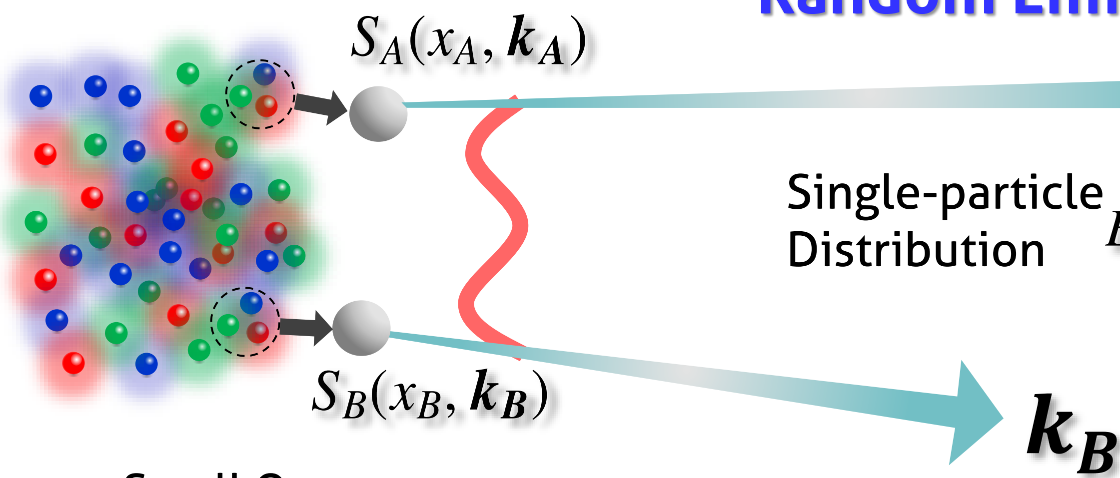
- Independent (chaotic) production from thermal source : $S_A(x_1, \boldsymbol{k_1}) S_B(x_2, \boldsymbol{k_2})$
- dilute after FO - pairwise interaction only
- State of pairs – evaluated in the pair-rest frame * $\boldsymbol{k_1} + \boldsymbol{k_2} = \boldsymbol{0}$

⇩

Scattering w.f. $\psi(Q^*, r^*)$

Theoretical Description

Random Emission + Final State Int.



Single-particle Distribution $E_k \dfrac{dN_i}{d^3\boldsymbol{k}} = \int d^4x S_i(x, \boldsymbol{k})$

↓ Constraints

Small Q

$$N^{\text{pair}}(Q) \simeq \int_{\Delta \boldsymbol{k}} \int_{x_A} \int_{x_B} S_A(x_A, \boldsymbol{k}_A) S_B(x_B, \boldsymbol{k}_B) |\psi_{AB}^{(-)}(\boldsymbol{r}^*, \boldsymbol{Q}^*)|^2$$

(# of pair) = integration of (emission probability x weight factor)

Random emission : no correlation btw. emission probability of A and B

Scattering wave function (← Schrödinger eq.)
FSI and symmetrization (for identical pairs)

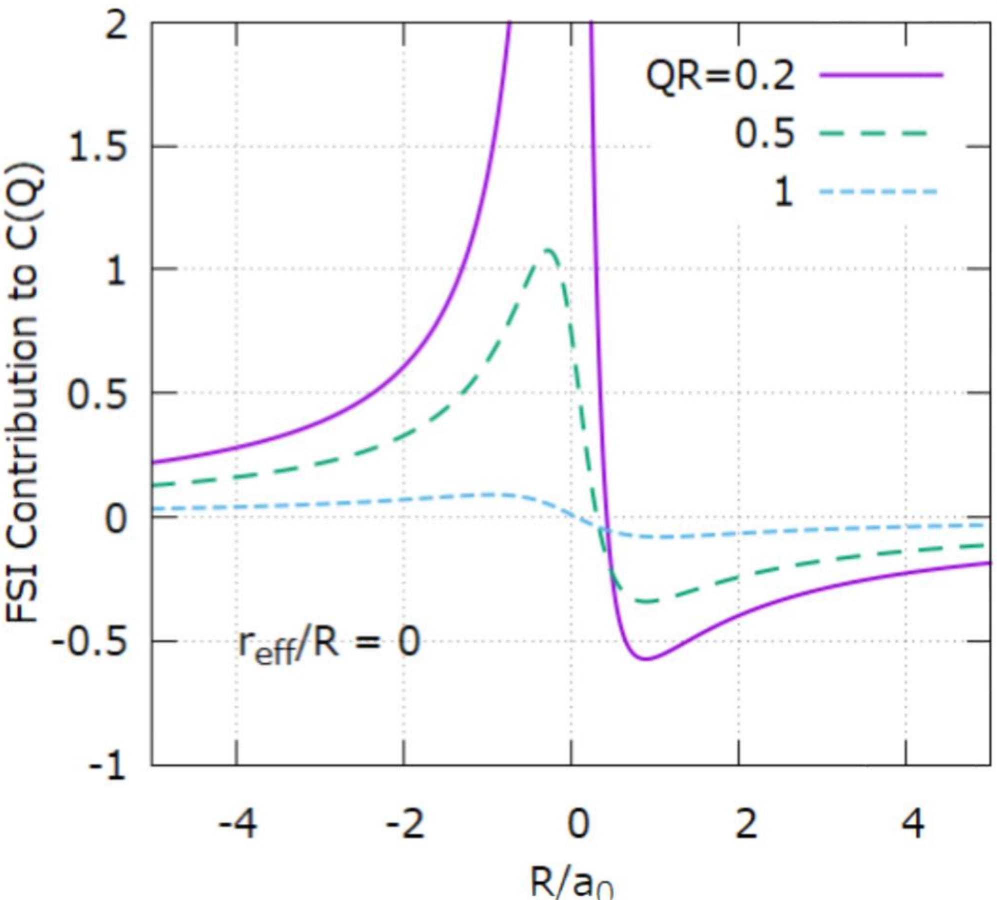
More rigorous formula found in Anchishkin, Heinz, Renk, PRC57 ('98)

Correlation from FSI

$$(\pi R^2)^{3/2} \exp\left(-\frac{r^2}{4R^2}\right)$$ Static/Spherical Source

$$C_{AB}(Q) - 1 = \frac{4\pi}{(2\pi R^2)^3} \int dr r^2 S^{\rm rel}(r)[|\chi_Q(r)|^2 - |j_0(Qr)|^2]$$

Lednicky+ '82



Asymptotic form $\sin(Qr + \delta)/(Qr)$

Shape-independent approx.

$$Q \cot \delta = -\frac{1}{a_0} + \frac{1}{2} r_{\rm eff} Q^2$$

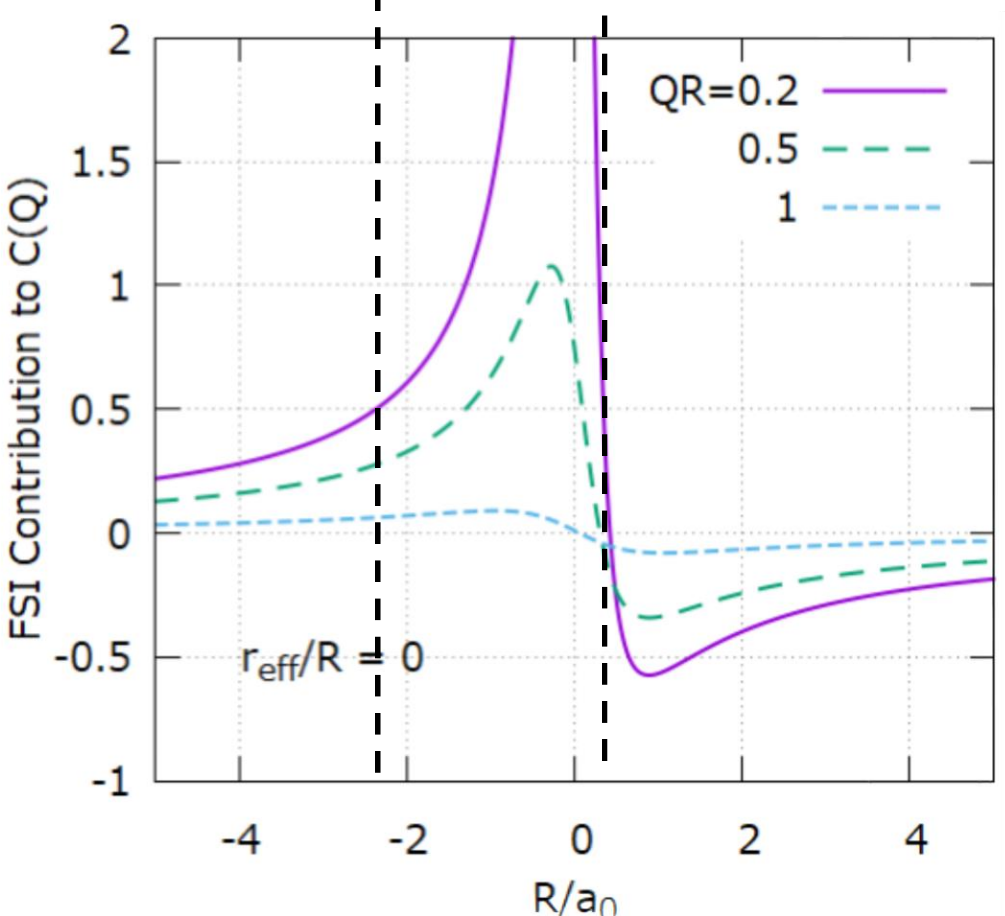
Small Q: Sensitive to S-wave scattering length a_0 and less sensitive to effective range $r_{\rm eff}$

Approximately scale with R/a_0 and QR

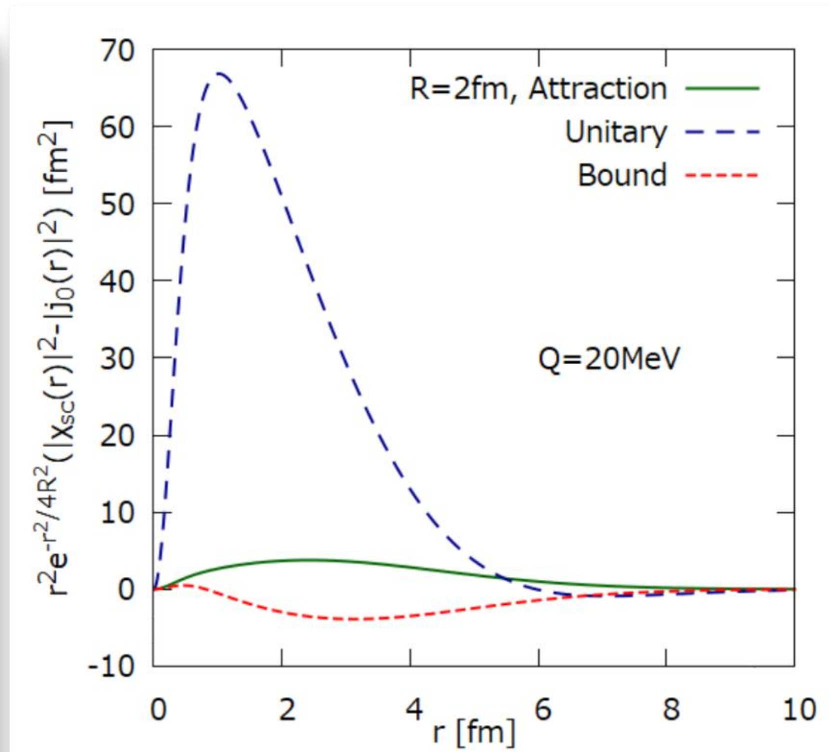
Holds in the presence of Coulomb

Correlation from FSI

Attraction Regime | Unitary Regime | Bound (or repulsive) Regime

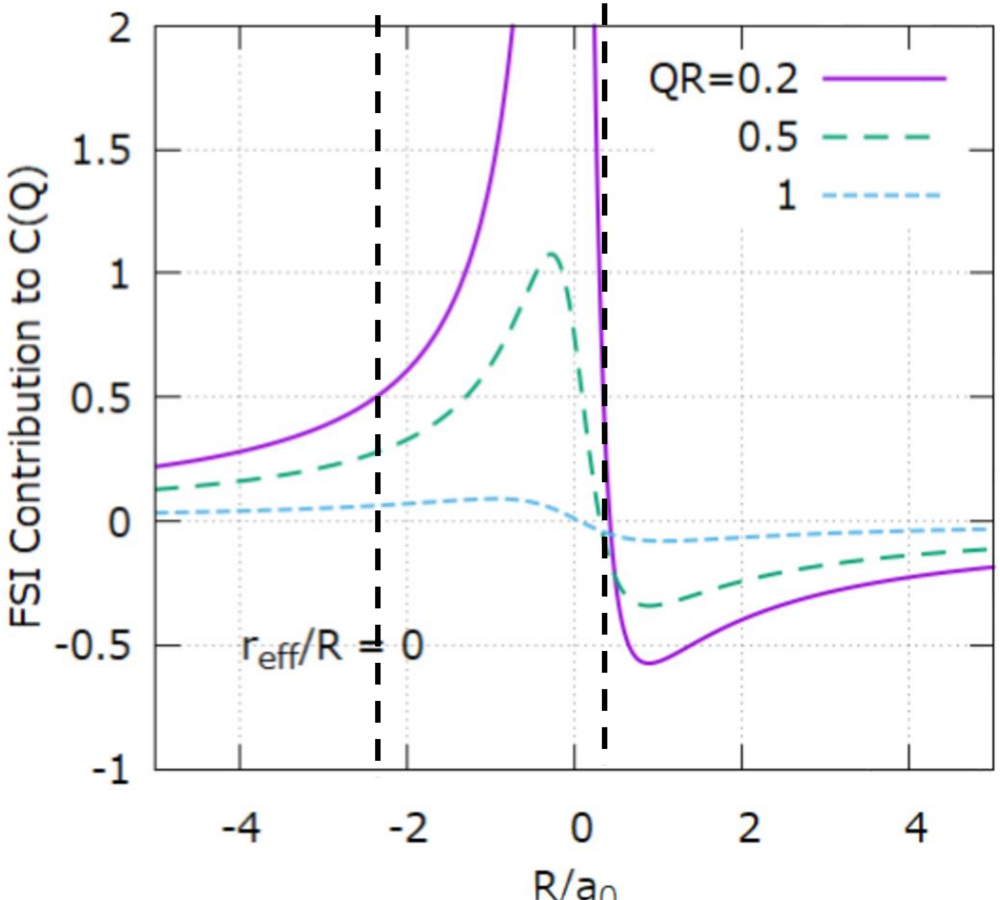


Source func × Wave func diff.

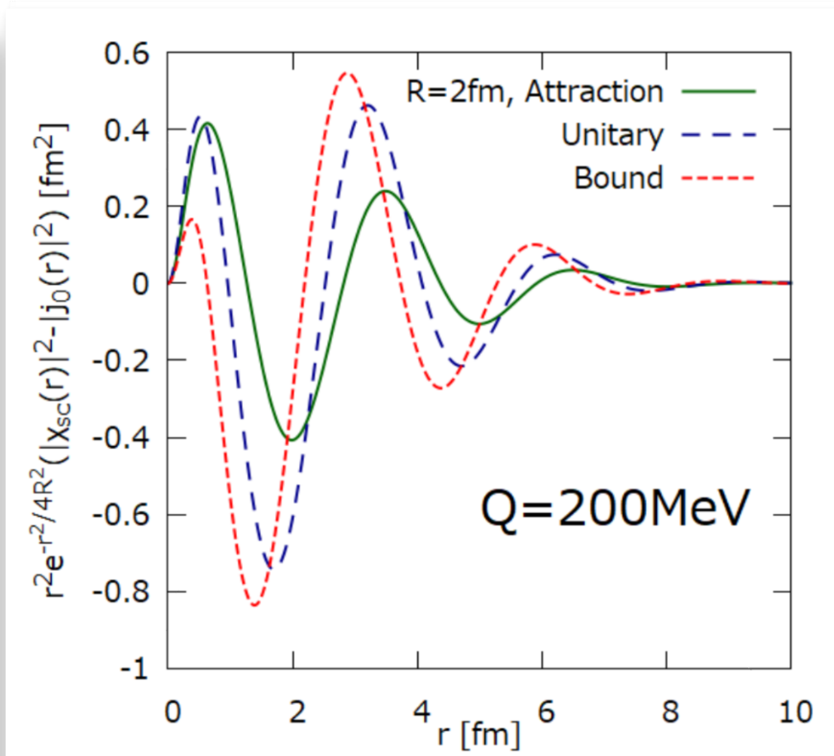


Correlation from FSI

Attraction Regime | Unitary Regime | Bound (or repulsive) Regime

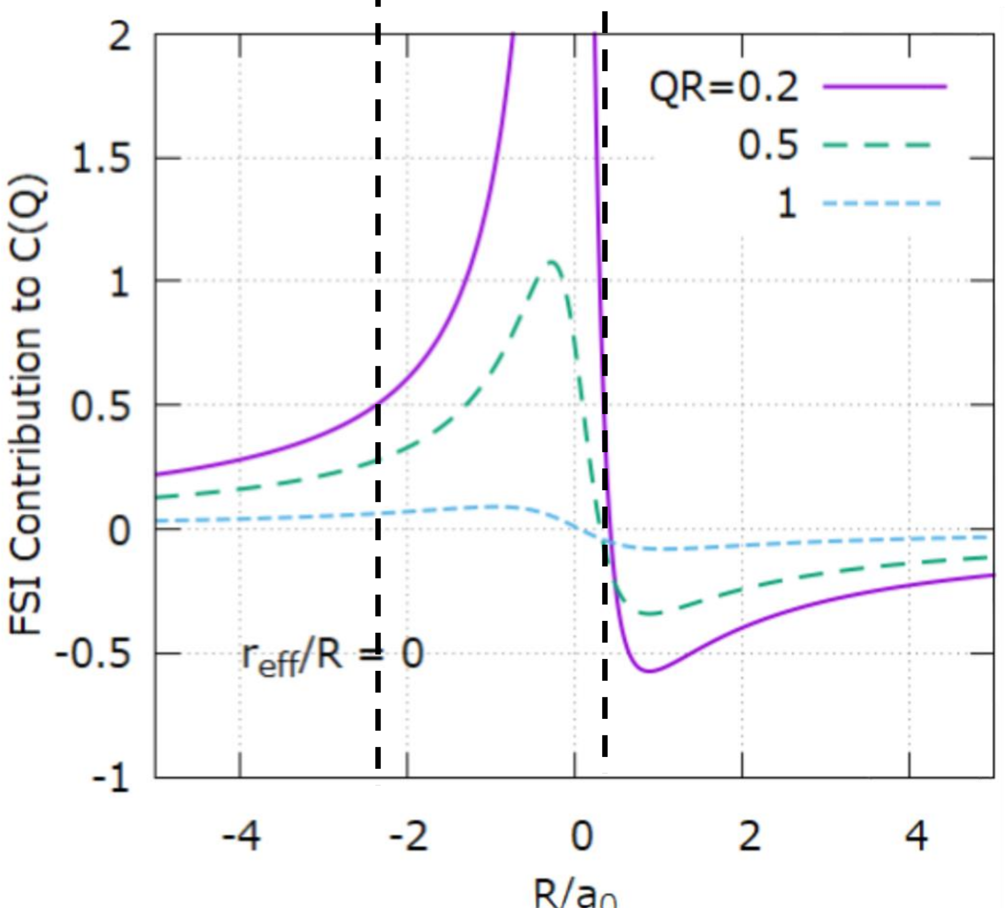


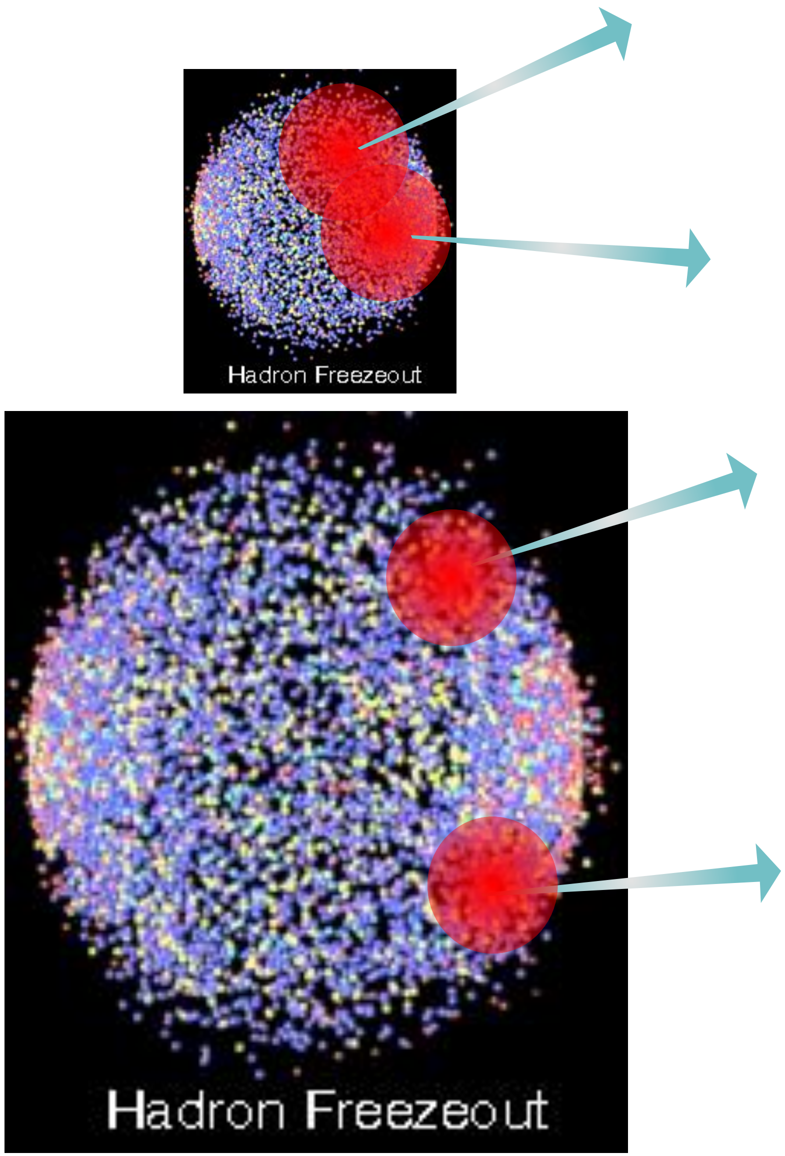
Source func × Wave func diff.



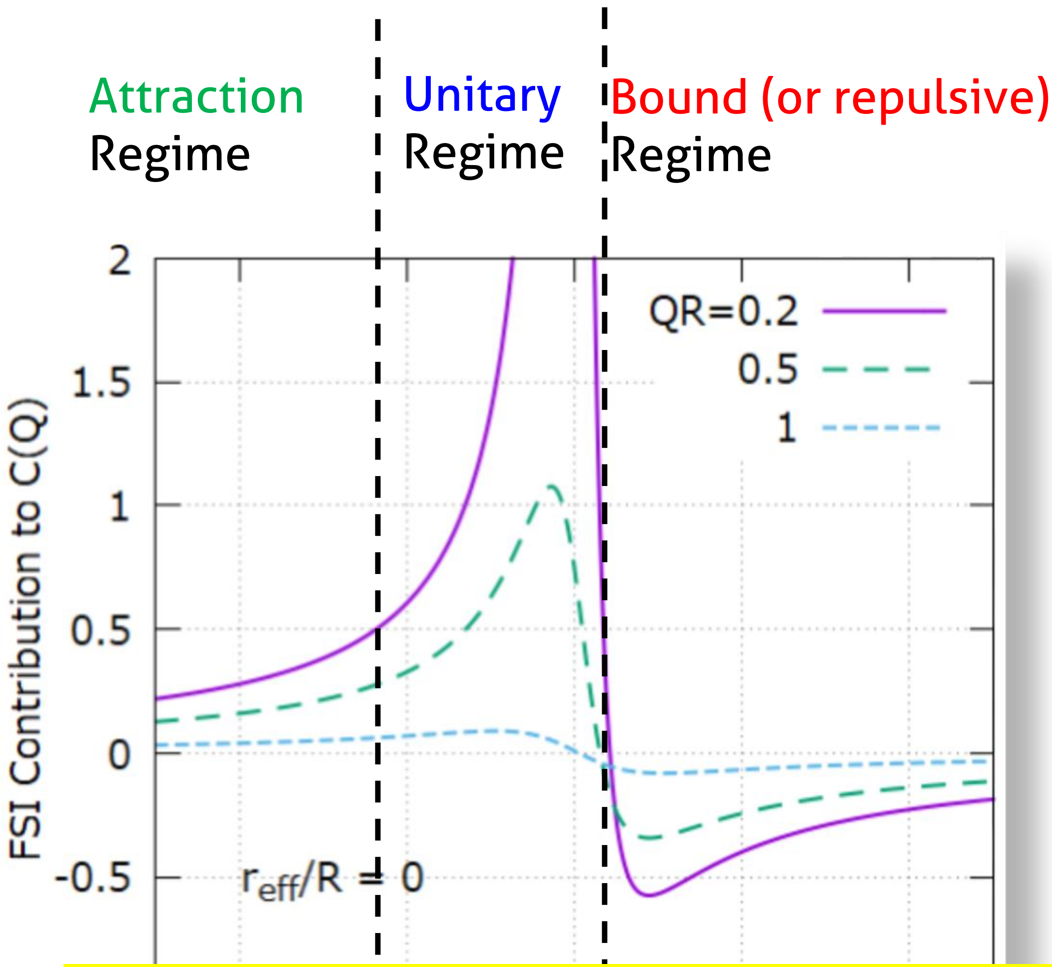
Correlation from FSI

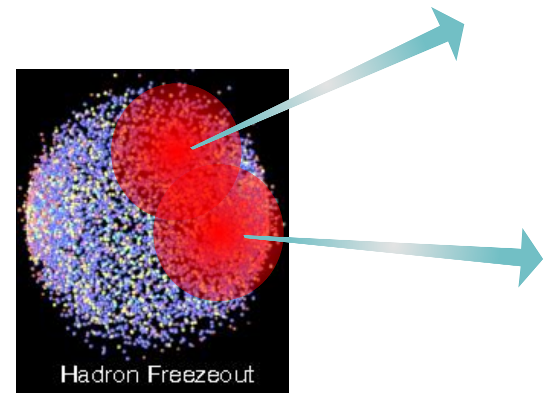
Attraction Regime | Unitary Regime | Bound (or repulsive) Regime

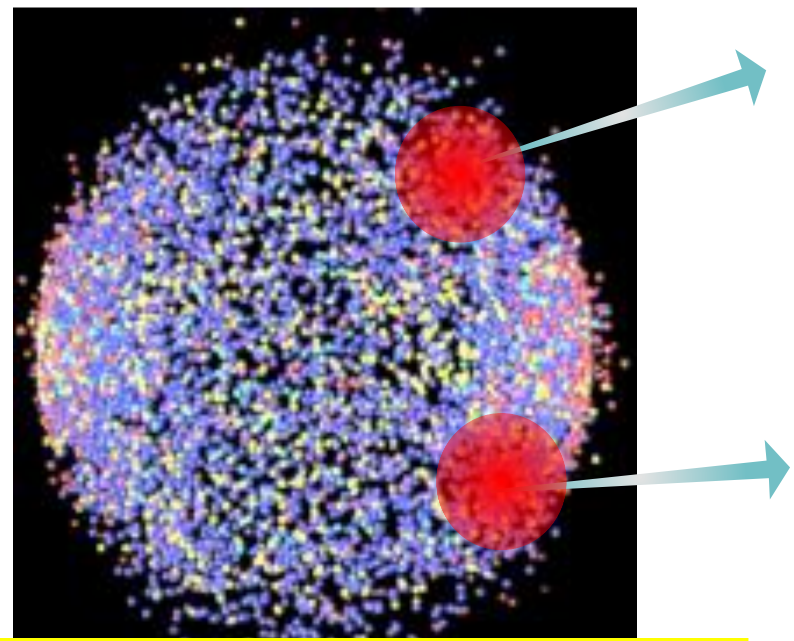




Correlation from FSI







Measuring C(Q) for different system size helps to disentangle the FSI-induced correlation from others

Application to Dibaryon Candidates

w/ inputs from Lattice QCD at (almost) physical point

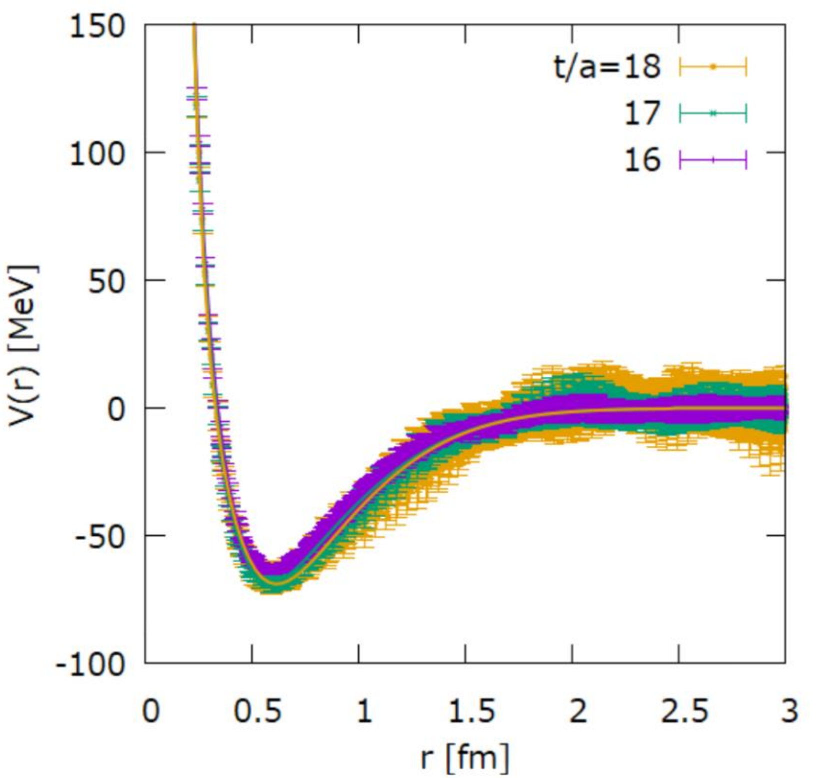
Details will be given by T.Inoue and K.Sasaki on Thursday

The Most Strange System: ΩΩ (S=-6)

$^1S_0$ bound state from Lattice QCD

S.Gongyo et al., (HAL QCD), 1709.00654

m_π=146MeV, m_Ω=1713MeV



+Coulomb repulsion

t/a	a_0 [fm]	r_{eff} [fm]	E_B [MeV]
16	65.3	1.29	0.1
17	17.6	1.23	0.5
18	11.7	1.21	1.0

⇓

Unitary regime in typical source size for HIC

ΩΩ Correlation : elements

Wave function

$$|\varphi_{\Omega\Omega}^{\text{spin-averaged}}(\boldsymbol{q}^*, \boldsymbol{r}^*)|^2 = \frac{1}{16}|\varphi(J=0)|^2 + \sum_{J=1}^{3} \frac{2J+1}{16}|\varphi(J)|^2$$

FSI+Coulomb+symmetrization Coulomb+(a)symmetrization

Source function

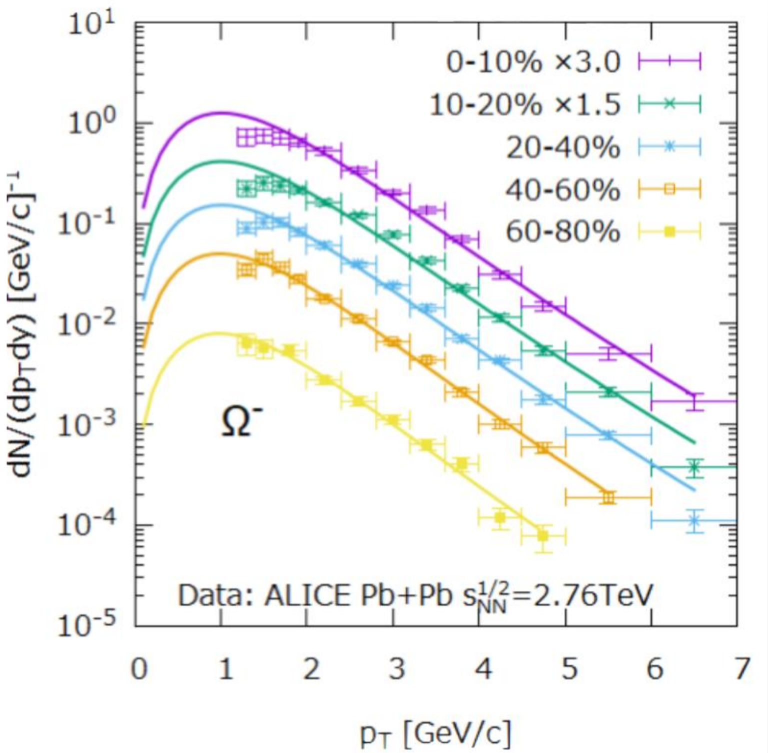
$$S(x, \boldsymbol{k}) = \frac{d}{(2\pi)^3} m_T \cosh(y - \eta_s) n_f(u \cdot k, T)$$

$$\times \exp\left(-\frac{x^2+y^2}{2R^2}\right) \delta(\tau - \tau_0)$$

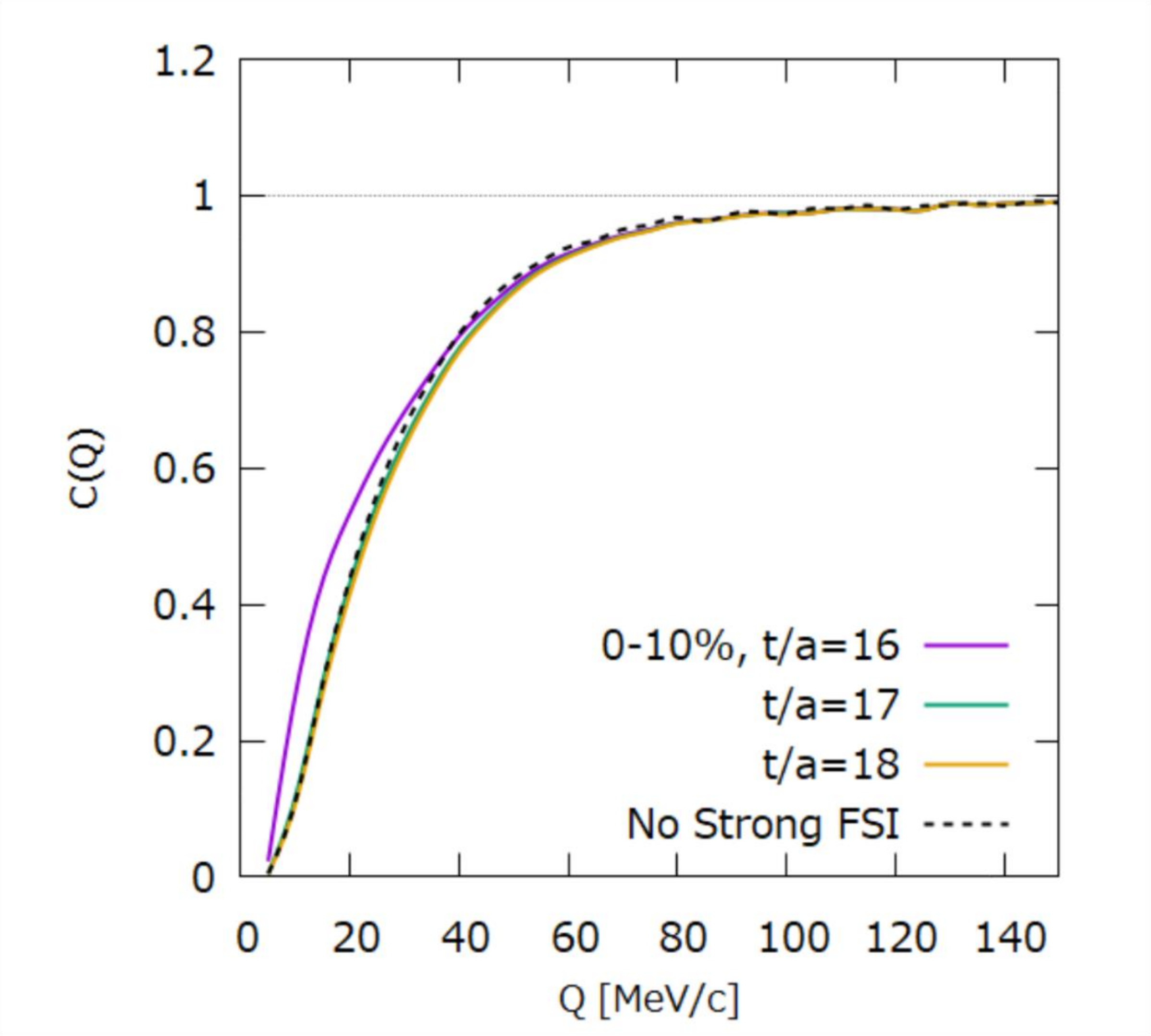
$$y_T = \alpha r / R$$

Boost-invariant, azimuthal symmetric transverse flow

Fit to p_T spectrum with T=164 MeV

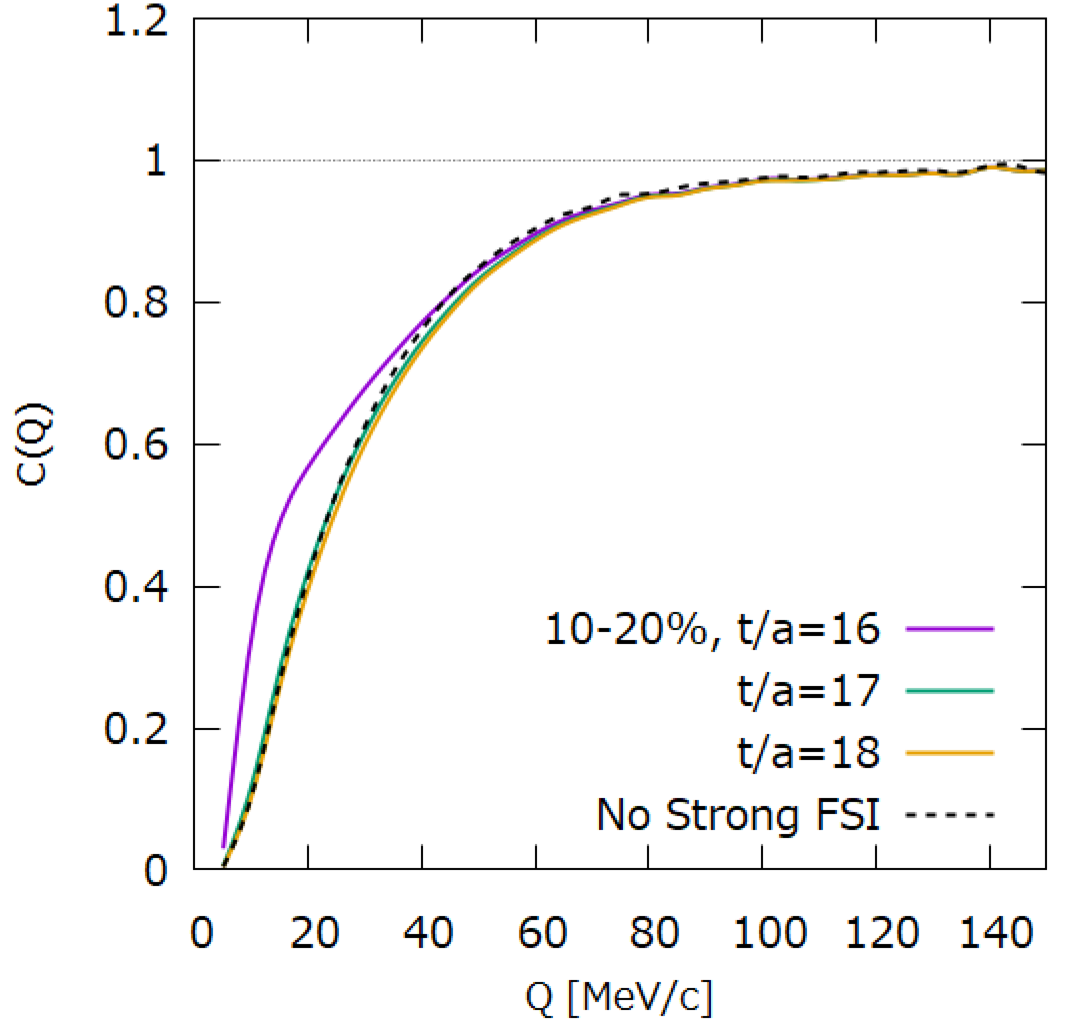


ΩΩ Correlation@LHC



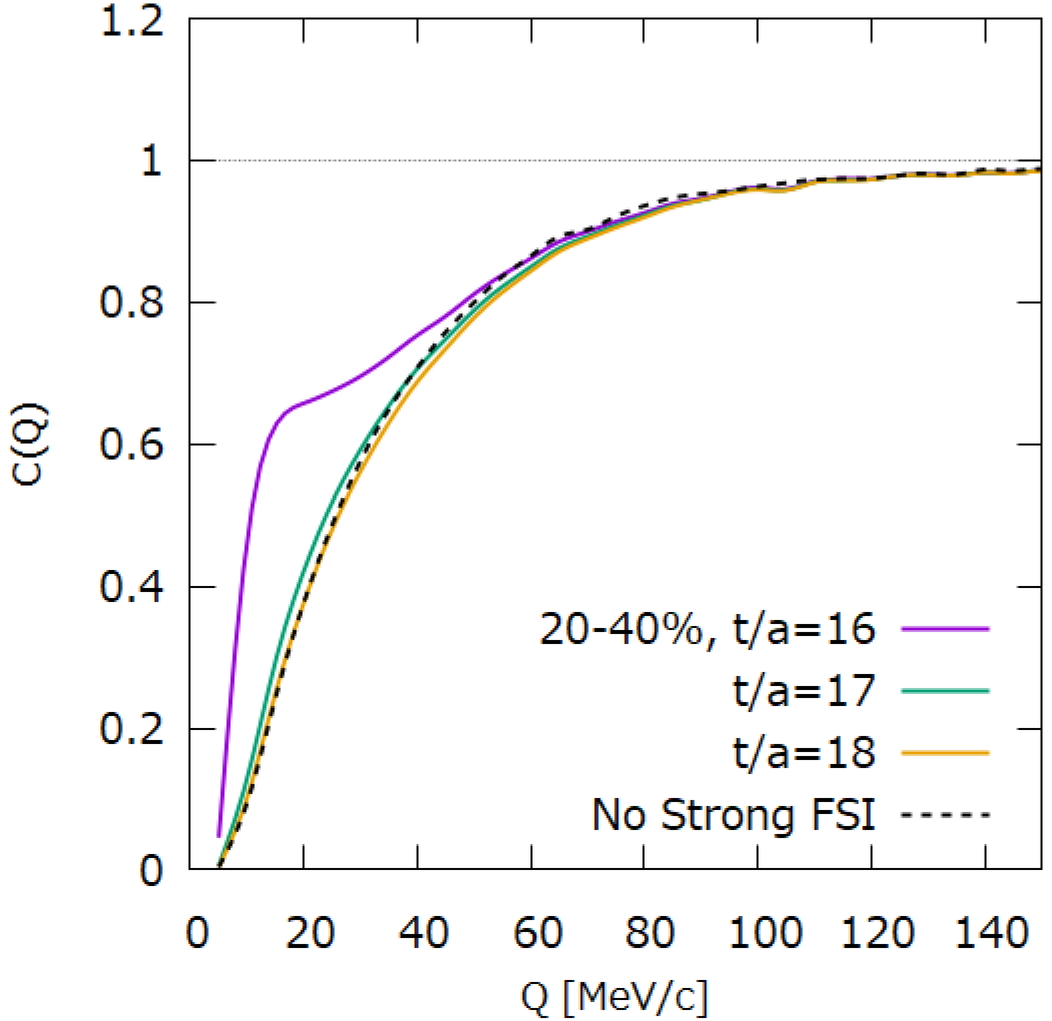
System is too large
Further suppressed by the spin degeneracy factor 1/16

ΩΩ Correlation@LHC



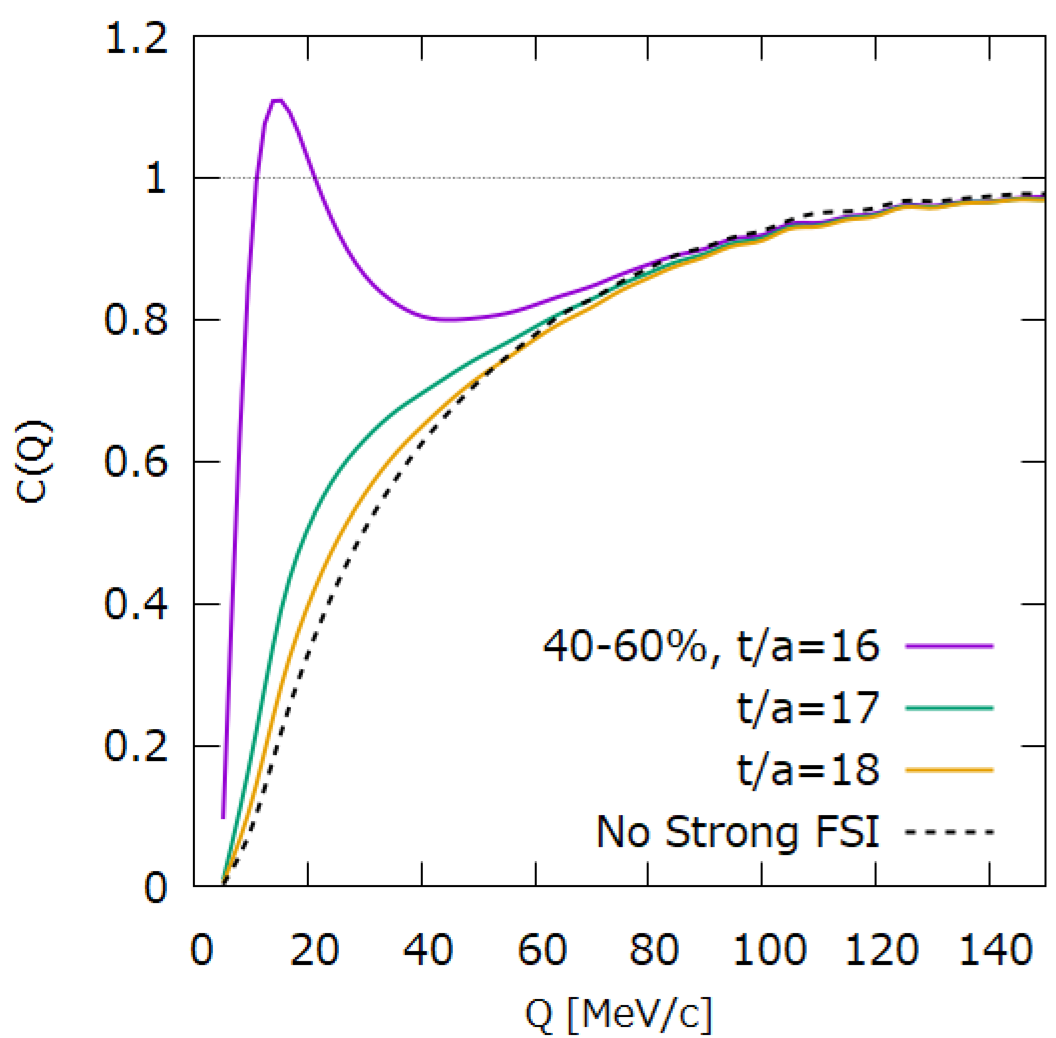
System is too large
Further suppressed by the spin degeneracy factor 1/16

ΩΩ Correlation@LHC



System is too large
Further suppressed by the spin degeneracy factor 1/16

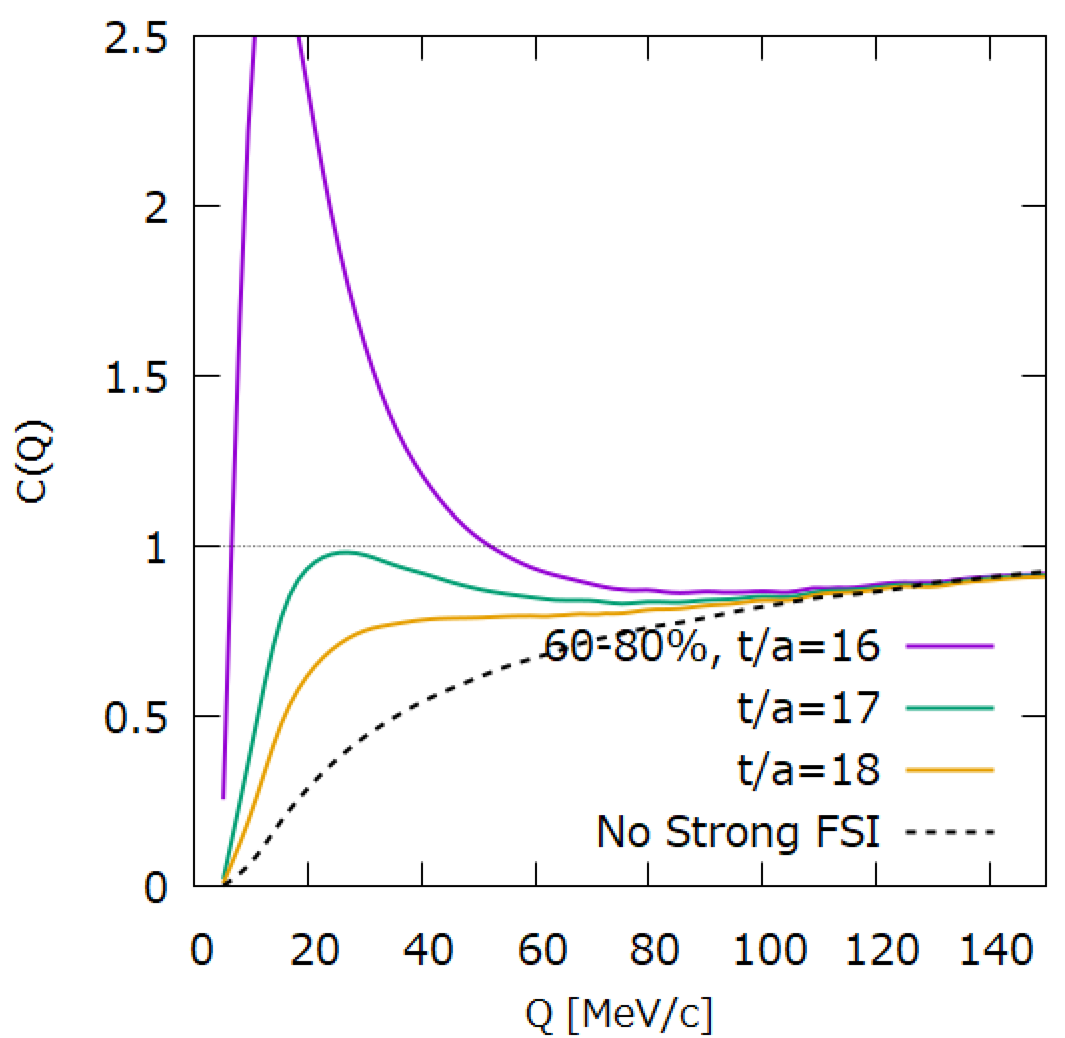
ΩΩ Correlation@LHC



System is too large
Further suppressed by the spin degeneracy factor 1/16

Moderate enhancement from Coulomb+HBT case

ΩΩ Correlation@LHC



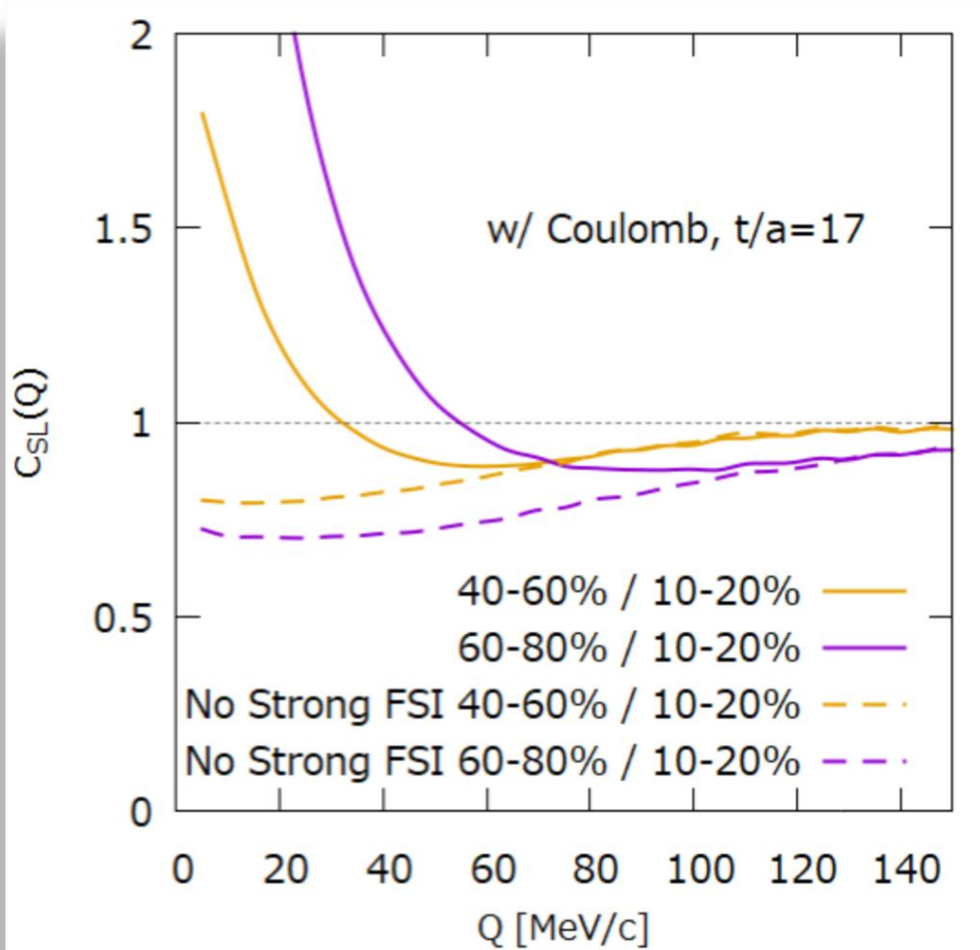
System is too large
Further suppressed by the spin degeneracy factor 1/16

Moderate enhancement from Coulomb+HBT case

Strong enhancement from Coulomb+HBT case

ΩΩ Correlation@LHC

The Small-Large Ratio $C_{SL}(Q)$



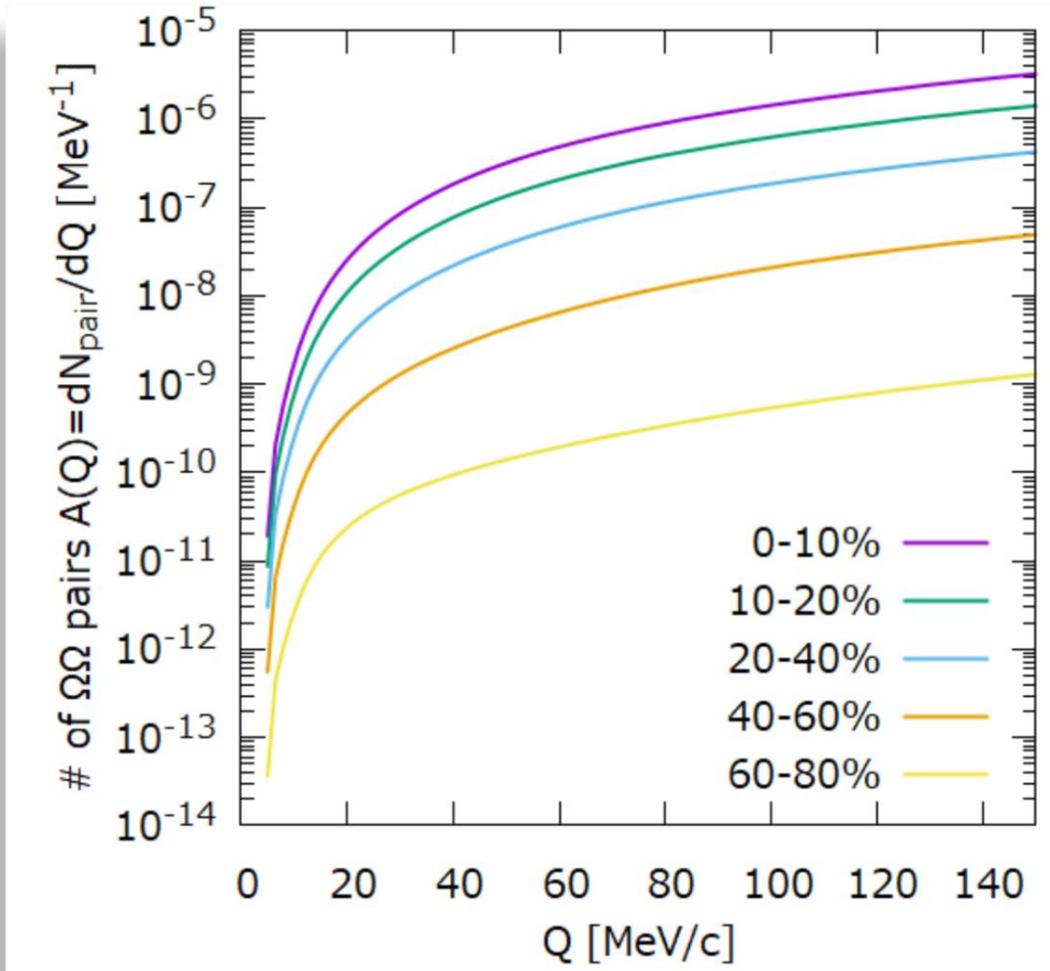
Response to system size change

QS (HBT) Correlation suppresses the ratio

Nevertheless FSI dominates at low Q

ΩΩ Correlation: Statistics?

of pair A(Q)



To have 100 pairs at low Q:

Acceptance×Efficiency : 0.01

Probability of events with more than 2Ω (assuming Poisson)

0.12 for 0-10%

10^{-4} for 60-80%

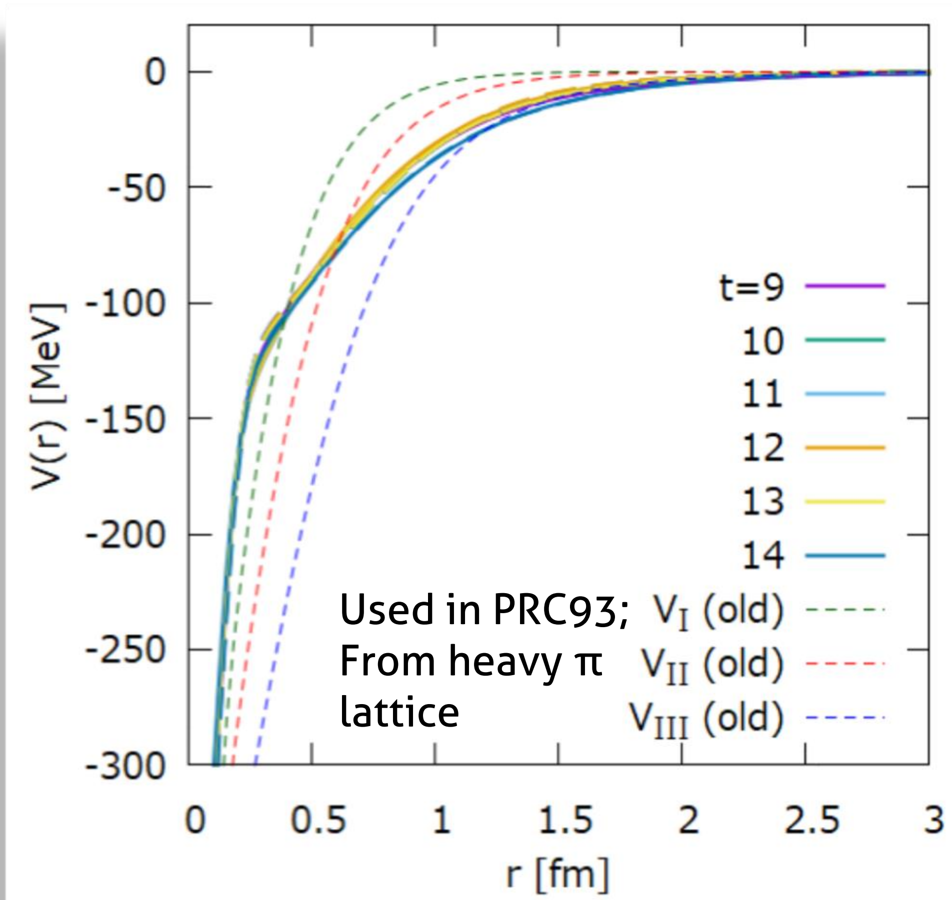
$10^{12} – 10^{15}$ events : unreachable at LHC

Not impossible at Future J-PARC ?(int. rate 10^8 Hz)

S=-3:
*p*Ω Correlation
@almost phys.point

*p*Ω Interaction ($^5S_2$)

NΩ potential (fitted to Lattice data) : bound state exists



T.Iritani et al. (HAL QCD)

+Coulomb attraction

t/a	a_0 [fm]	r_{eff} [fm]	E_B [MeV]
11	3.77	1.37	1.6
12	3.89	1.38	1.5
13	3.47	1.37	2.0

Bound state regime for Heavy Ion Collisions
Close to unitary for smaller system

pΩ Correlation

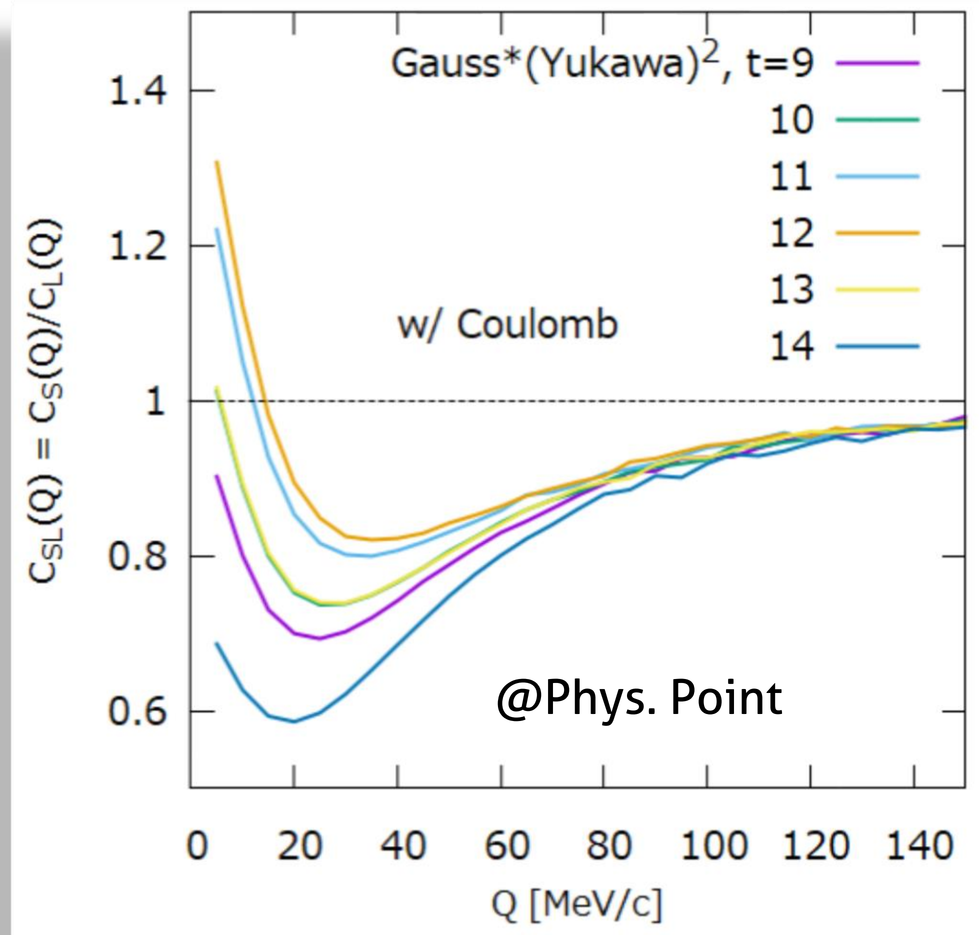
$$|\varphi_{p\Omega}^{\text{spin-averaged}}(\boldsymbol{q}^*, \boldsymbol{r}^*)|^2 = \frac{3}{8}|\varphi(^3S_1)|^2 + \frac{5}{8}|\varphi(^5S_2)|^2$$

⇓

Coupled to ΛΞ (2430) and ΣΞ(2507)

Absorption of S-wave component

$$V_{J=1}(r) = -i\theta(r_0 - r)V_0$$



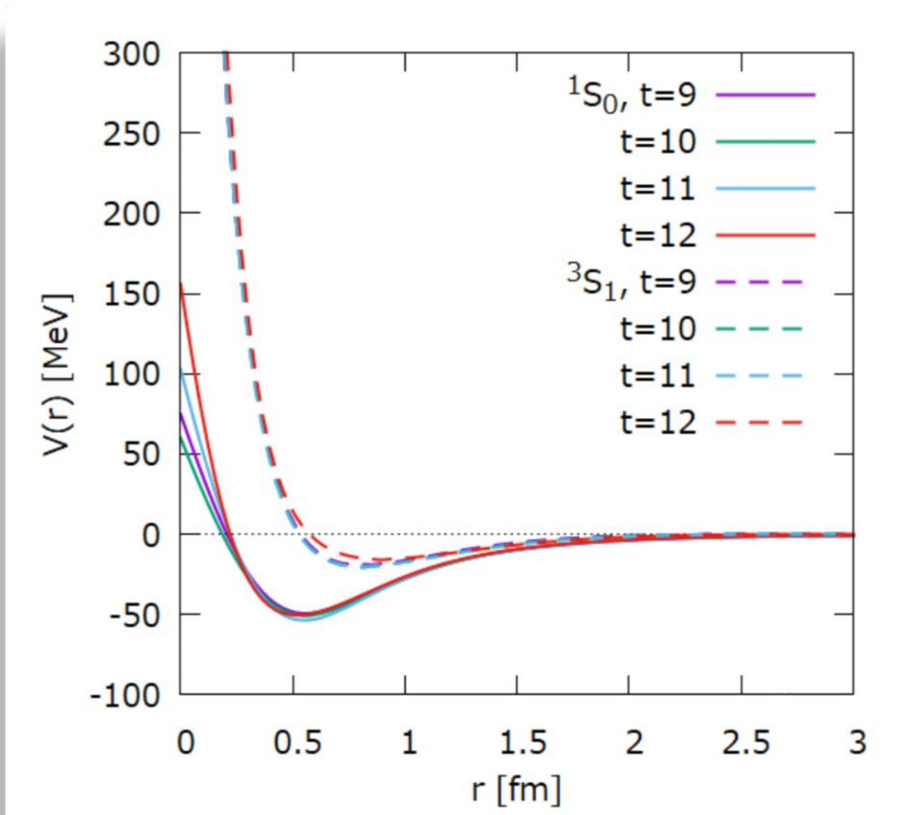
Bound state regime:
Suppression of $C_{SL}(Q)$
Below unity
At low Q

See Talk by J.Chen on Monday

S=-2:
*p*Ξ Correlation
@(almost)Phys. Point

$p\Xi$ Interaction ($^1S_0$, $^3S_1$)

$N\Xi$ potential (fitted to Lattice data)



+Coulomb attraction

	Effective $^1S_0$		$^3S_1$	
t/a	a_0 [fm]	r_{eff} [fm]	a_0 [fm]	r_{eff} [fm]
9	-22.66	2.46	-0.60	4.53
10	-19.86	2.30	-0.73	4.17
11	-23.95	2.30	-0.80	4.17
12	-12.39	2.40	-0.61	5.30

$^1S_0$ channel (coupling to $\Sigma\Sigma$ incorpolated) dominates
Close to unitary for HIC source

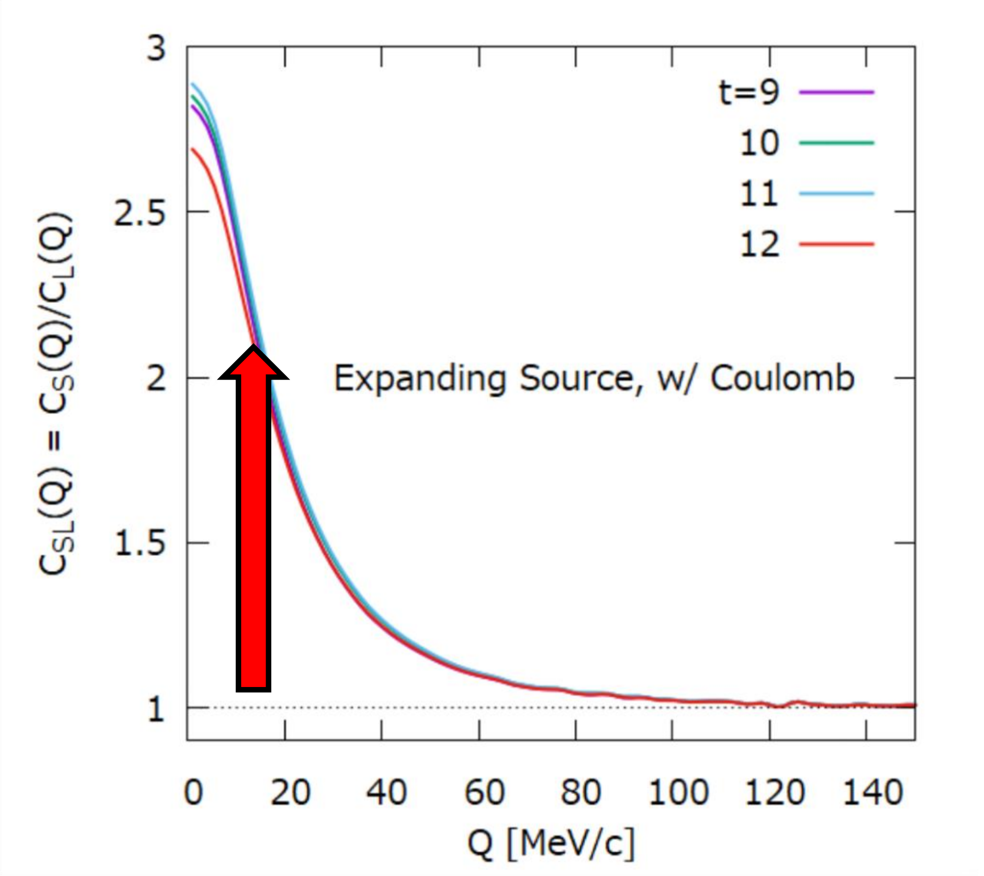
K.Sasaki et al. (HAL QCD)

$p\Xi^-$ Correlation at Physical Point

$$|\psi_{p\Xi^-}|^2 = \frac{1}{2}|\psi_{p\Xi^-}^{I=0}|^2 + \frac{1}{2}|\psi_{p\Xi^-}^{I=1}|^2$$

$$= \frac{1}{8}|\psi_{p\Xi^-}^{I=0}({}^1S_0)|^2 + \frac{3}{8}|\psi_{p\Xi^-}^{I=0}({}^3S_1)|^2 + \frac{1}{2}|\psi_{p\Xi^-}^{I=1}|^2$$

Unitary regime: Notable enhancement by FSI



Concluding Remarks

- **Correlation measurement in HIC can constrain low energy scattering param.**
  - New opportunity for multistrange systems
  - FSI contribution is sensitive to system size : Comparing small and large systems via $C_{SL}(Q)$
  - Different systems useful for disentangle other correlation origins
- **Indirect search for dibaryon states**
  - $\Omega\Omega$: Unitary regime, but statistically difficult
  - $p\Omega$: Bound regime - suppression of $C_{SL}(Q)$
  - $p\Xi$: Unitary regime – enhancement of $C_{SL}(Q)$

Backup

More on Ω Source Function

Fix τ

- $\tau \sim R_{long} \sim \langle N_{ch} \rangle^{1/3}$
- Ω freeze-out from phase boundary due to small cross section

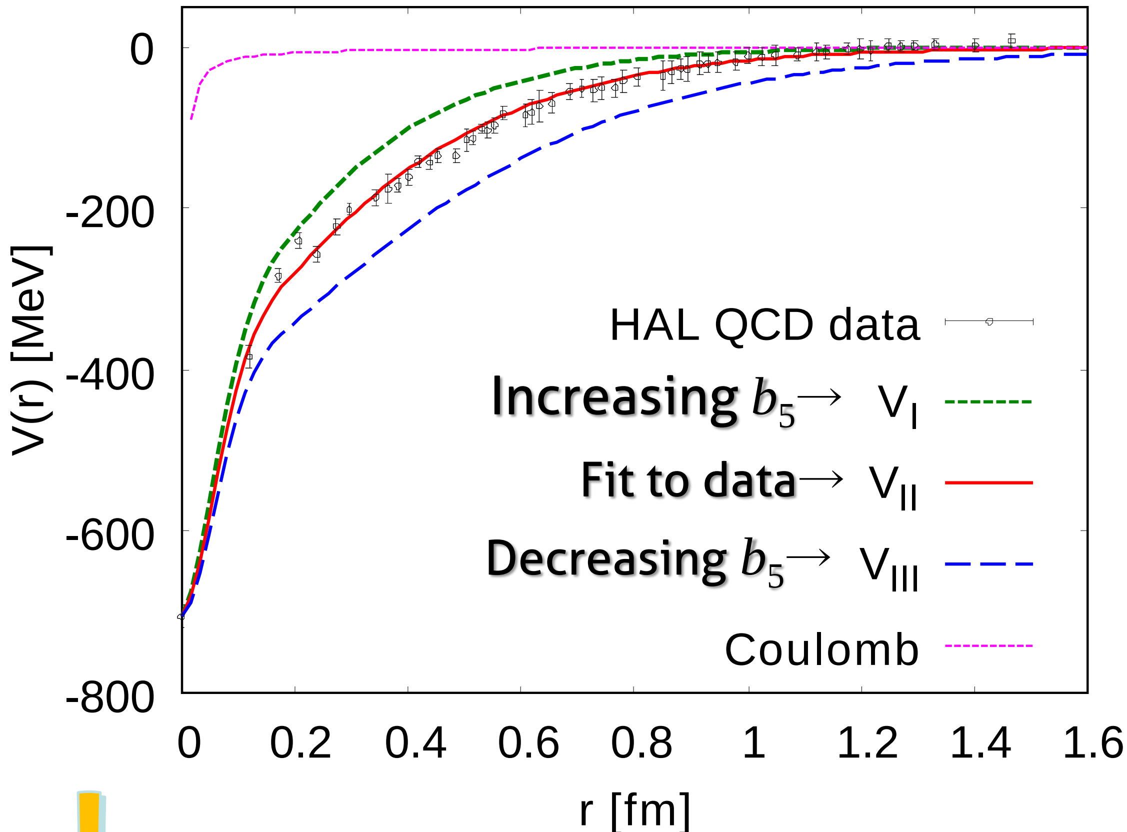
Hybrid model: Zhu et al., PRC'15, Takeuchi et al., PRC'15

Parameters

Centrality	0-10%	10-20%	20-40%	40-60%	60-80%
τ_0 [fm]	10.0	7.9	6.75	4.89	2.0
R [fm]	5.18	4.74	3.8	2.55	1.6
α	0.38	0.38	0.38	0.38	0.37

NΩ potential ($^5S_2$) : motivated by LQCD

$$V(r) = b_1 e^{-b_2 r^2} + b_3 (1 - e^{-b_4 r^2})^n (e^{-b_5 r}/r)^2$$



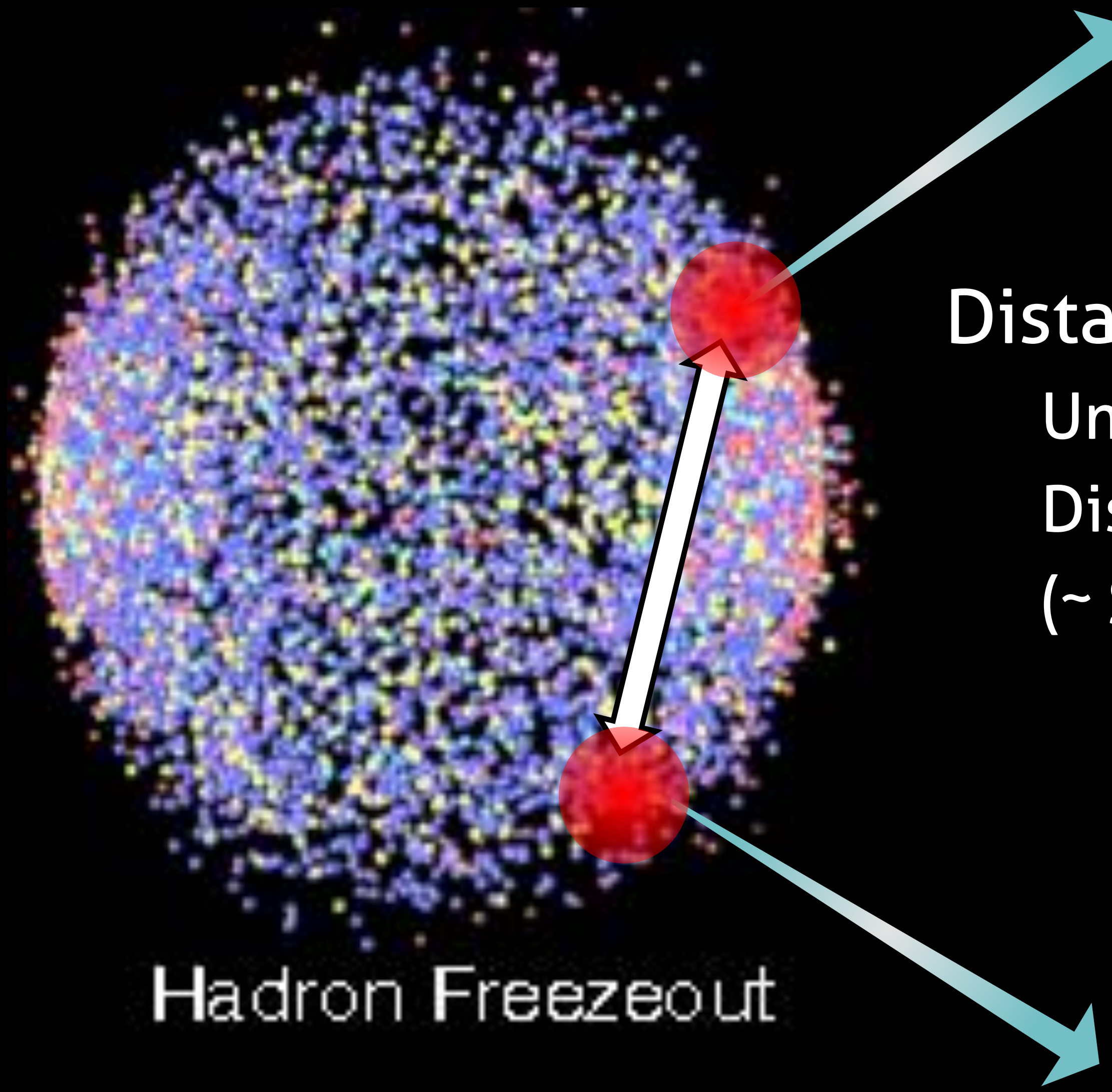
Pot	a_0 [fm]	r_{eff} [fm]	E_B [MeV]
V_I	-1.0	1.15	N/A
V_{II}	23.1	0.95	0.05
V_{III}	1.6	0.65	24.80

w/Coulomb

V_I+V_c	-1.12	1.16	N/A
$V_{II}+V_c$	5.79	0.96	6.3
$V_{III}+V_c$	1.29	0.65	26.9

! Caveat : data obtained from heavy quark mass (Etminan+ ‘14)
Following calculations use the physical baryon masses

Counting Correlated Pairs



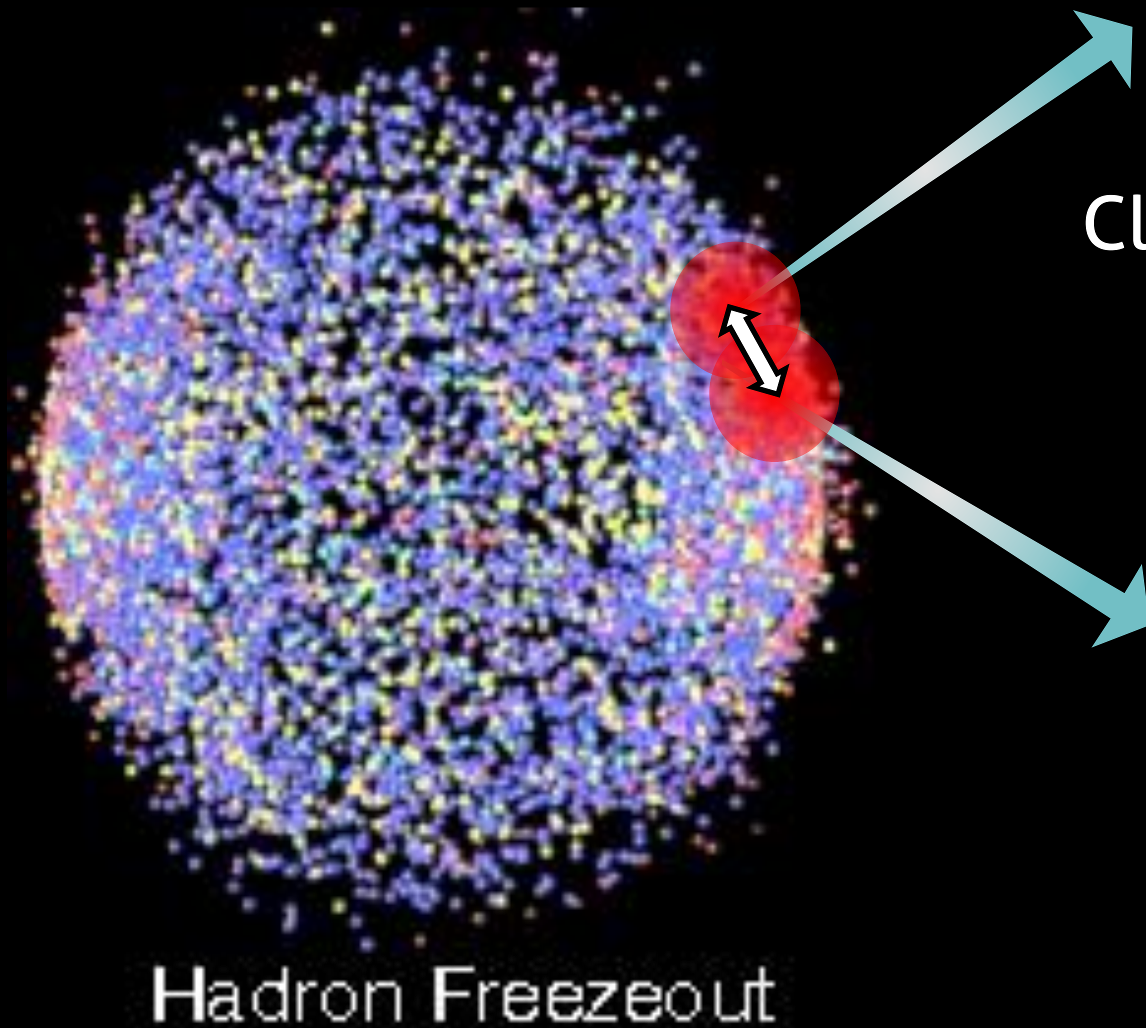
Distant pairs:

Uncorrelated because

Distance < int. range

(~ System size)

Counting Correlated Pairs



Close pairs:

Correlated through FSI

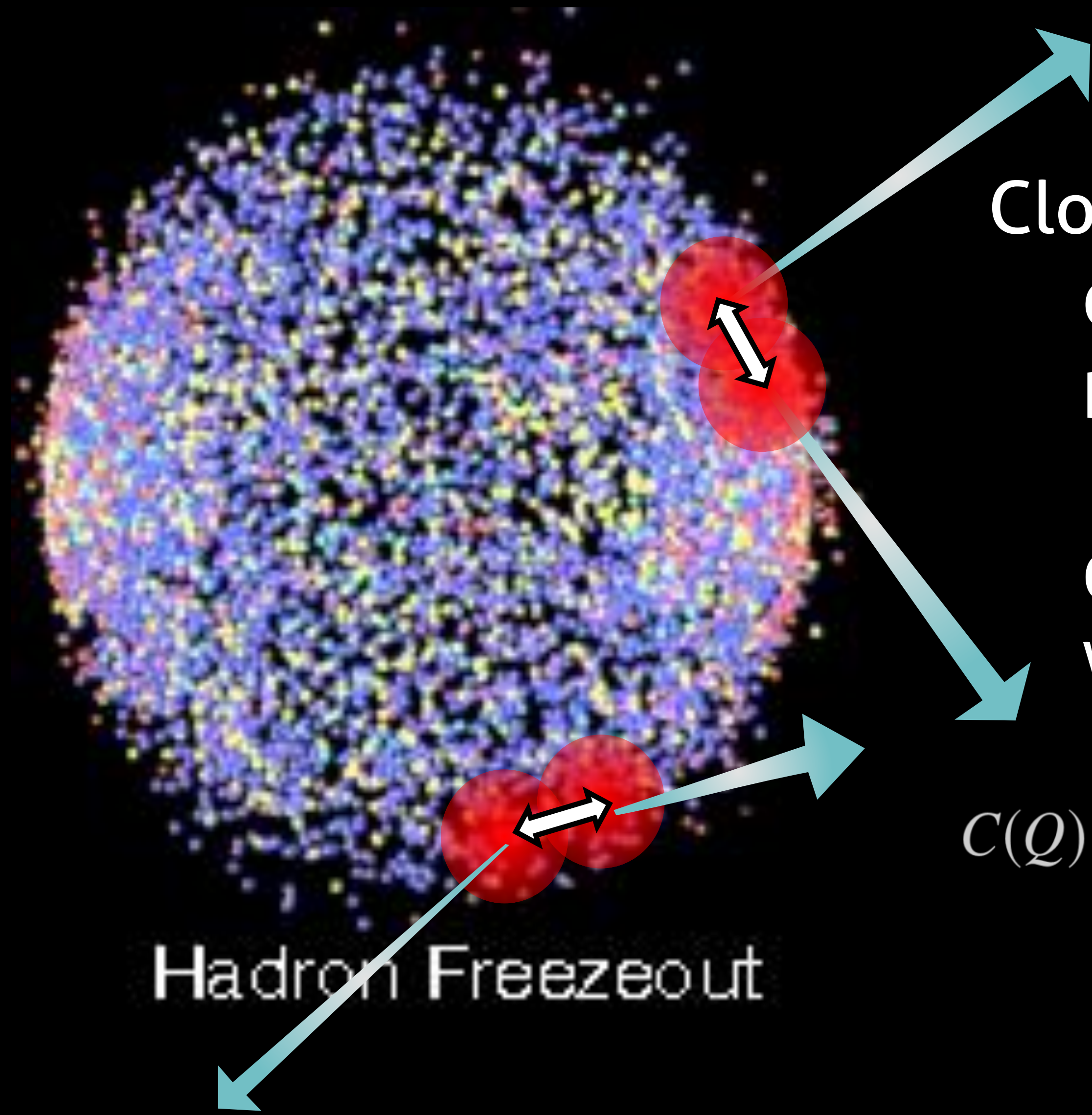
Distance < int. range

FSI : $\psi(Q^*, r^*)$

Input from V(r)

(phenomenological / Lattice)

Counting Correlated Pairs



Close but large Q pairs:

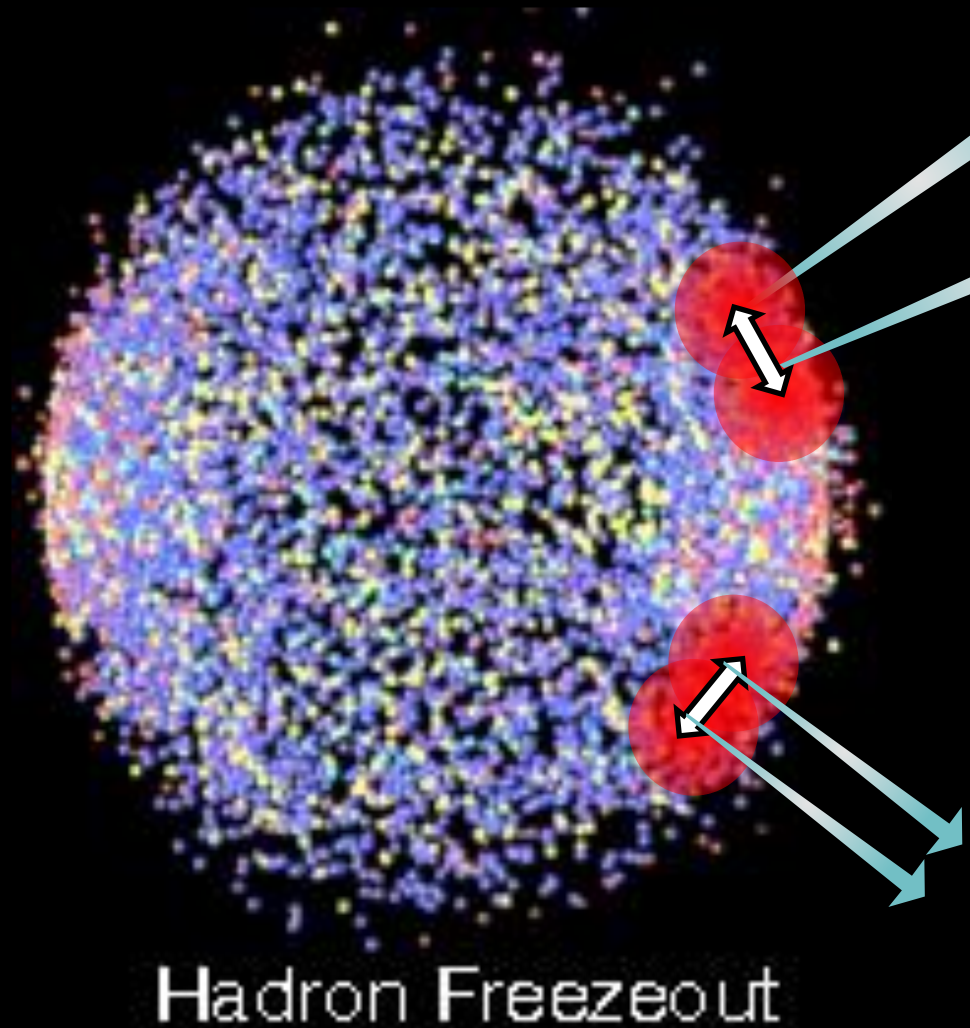
Correlated through FSI

Distance < int. range

Oscillating $|\psi(Q^*, r^*)|^2$ washes out correlation

$$C(Q) \propto \int_r S(r)|\chi_Q(r)|^2 - |j_0(Qr)|^2$$

Counting Correlated Pairs



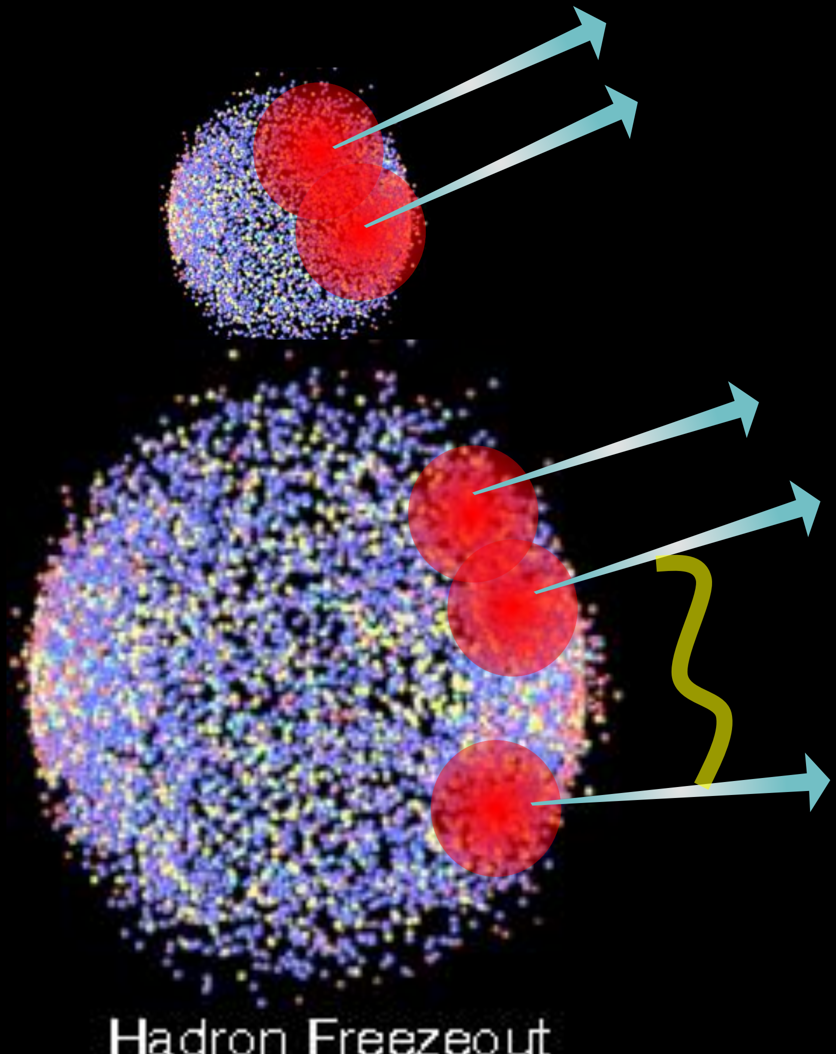
Close and small Q pairs:

Correlated through FSI

Distance < int. range

$|\psi(Q^*, r^*)|^2$ leads enhance or reduction of # of pairs

System Size?



Small System:

Most of observed pairs with small Q correlated

Large System:

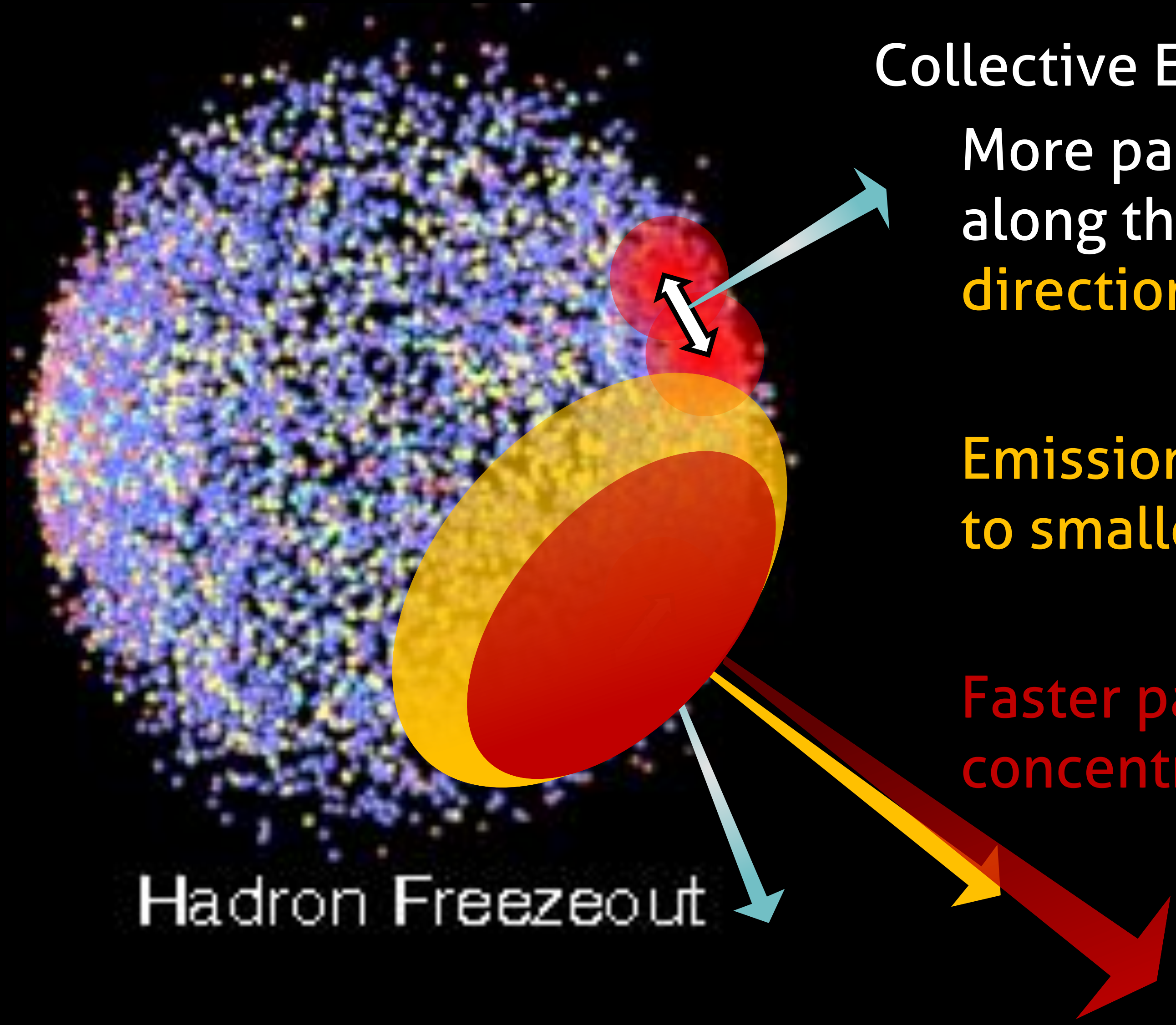
Less pairs coming from close distance

Important Remark:

Coulomb FSI for charged pairs!

Conclusion : measure small Q pairs coming from small region!

Effect of Collectivity



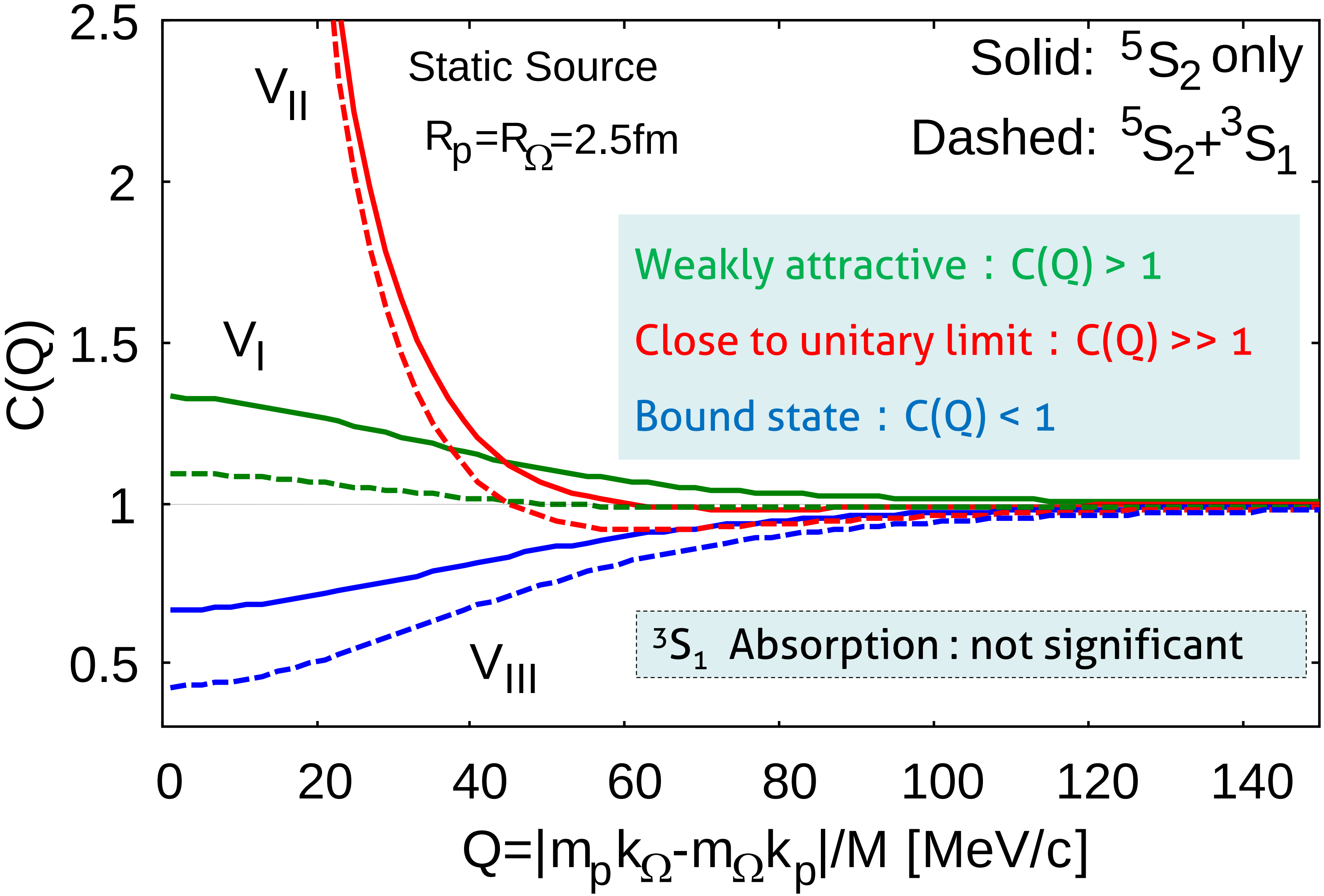
Collective Expansion:

More particles produced along the expanding direction

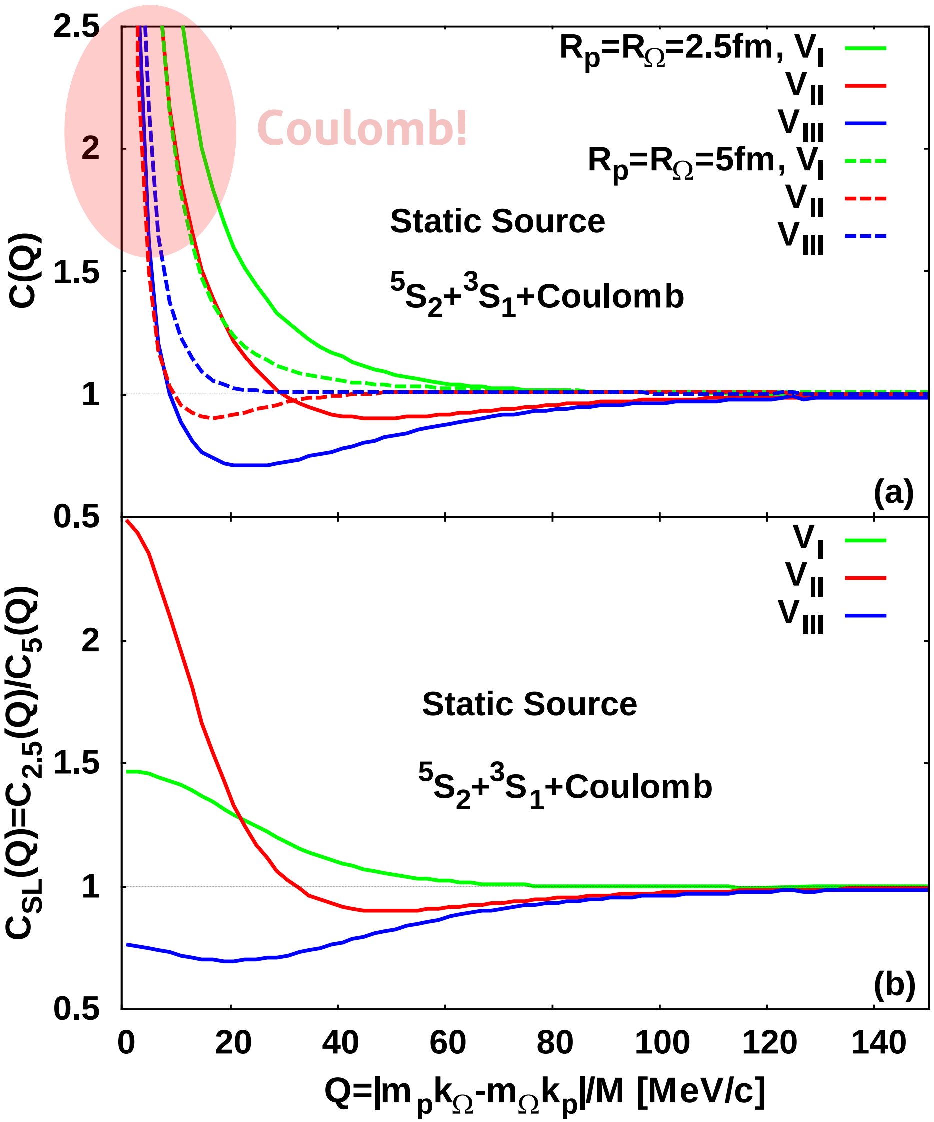
Emission region deformed to smaller one

Faster pairs more concentrated

$p\Omega$ Correlation : Strong int. Only



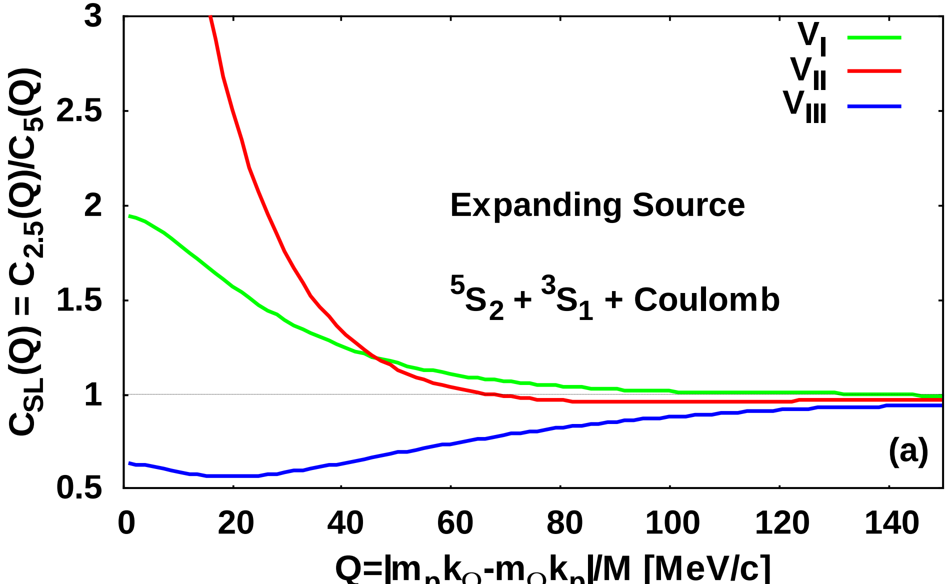
Small-Large Ratio $C_{SL}(Q)$



Taking ratio of C(Q) for small and large systems

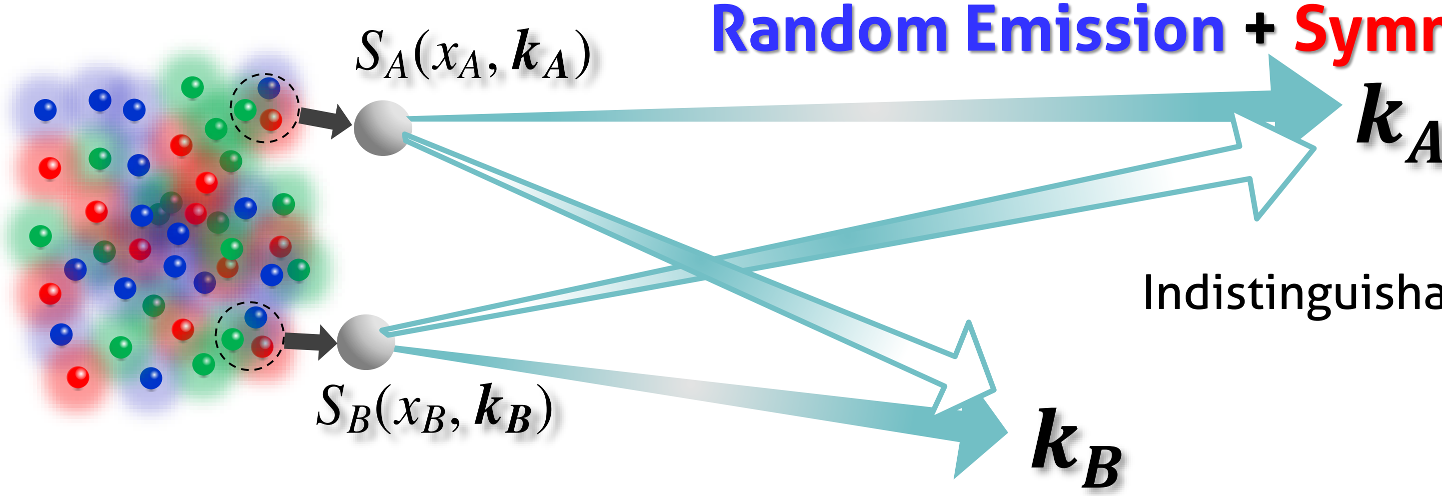
Resemble the ideal case

Even with expansion



Quantum Statistics (HBT/GGLP)

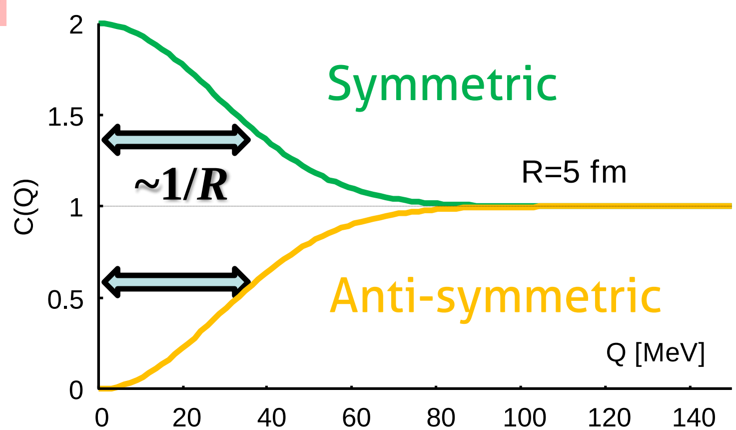
Random Emission + Symmetrization



$$\psi_{AB} = \frac{1}{\sqrt{2}}\left(e^{i\boldsymbol{k}_A\cdot\boldsymbol{x}_A}e^{i\boldsymbol{k}_B\cdot\boldsymbol{x}_B} \pm e^{i\boldsymbol{k}_A\cdot\boldsymbol{x}_B}e^{i\boldsymbol{k}_B\cdot\boldsymbol{x}_A}\right)$$

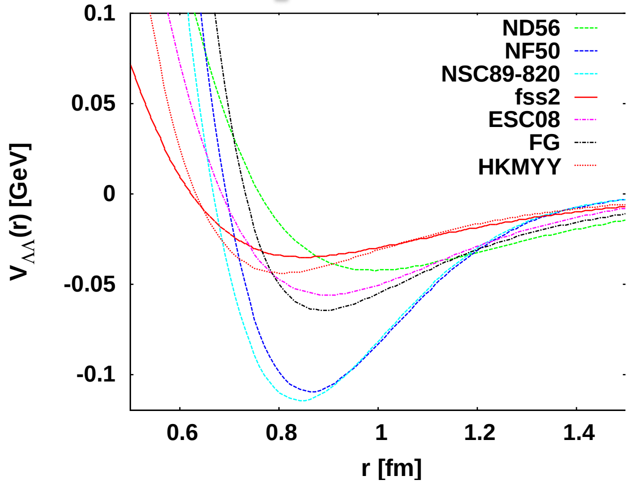
$$= \begin{cases} e^{i\boldsymbol{K}\cdot\boldsymbol{X}}\sqrt{2}\cos(\boldsymbol{Q}\cdot\boldsymbol{r}) \\ e^{i\boldsymbol{K}\cdot\boldsymbol{X}}\sqrt{2}i\sin(\boldsymbol{Q}\cdot\boldsymbol{r}) \end{cases}$$

$$C_{id}(\boldsymbol{Q}) = 1 \pm \frac{1}{\mathcal{N}}\int d^3\boldsymbol{r}\cos(2\boldsymbol{Q}\cdot\boldsymbol{r})S^{\mathrm{rel}}_{\boldsymbol{K}}(\boldsymbol{r})$$



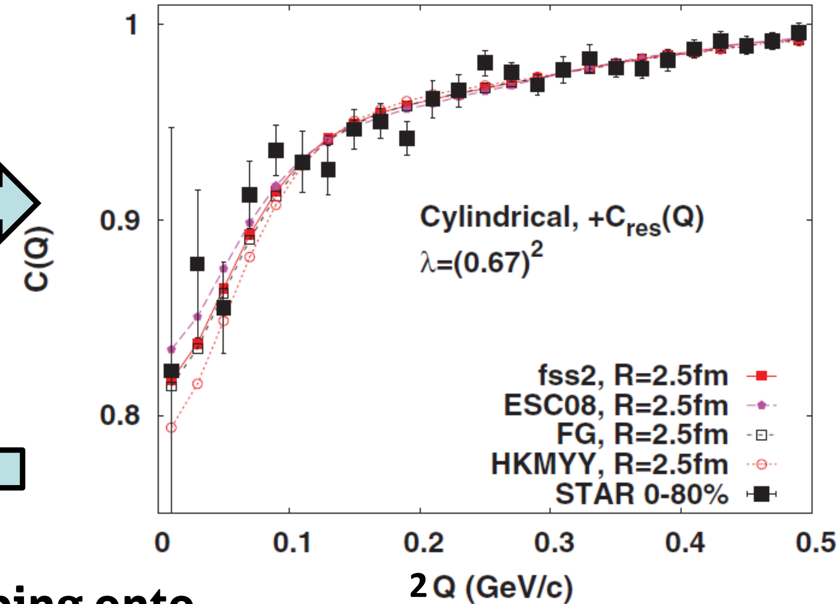
Fourier tr. of the emission func.

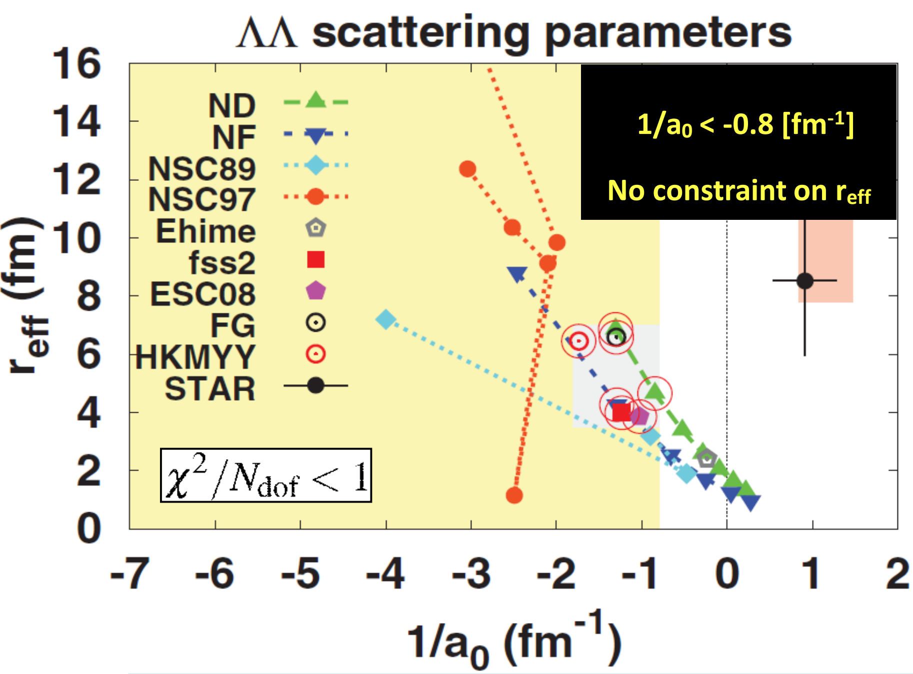
Comparison with STAR data (RHIC)



Expansion
Σ^0 decay

$$C(Q) \to C(Q) + a_{\rm res} e^{-r_{\rm res}^2 Q^2}$$





Mapping onto scattering param.

Long tail in data – 2 components structure?
Weakly attracting potential can fit data
(Consistent w/ Nagara event)

Koonin-Pratt Formula

In pair-rest frame $P = (M, \mathbf{0})$ $x = x_A - x_B = (t, \boldsymbol{r})$

Relative emission function : emission probability for pairs with distance $\boldsymbol{r}$

$$S^{\mathrm{rel}}(\boldsymbol{r}) = \frac{\int dt \int d^4X S_A(X + E_B^* x/M, \boldsymbol{k_A}) S_B(X - E_A^* x/M, \boldsymbol{k_B})}{\int d^4x_1 S_A(x_A, \boldsymbol{k_A}) \int d^4x_B S_B(x_B, \boldsymbol{k_B})}$$

$$C_{AB}(\boldsymbol{Q}, \boldsymbol{P}) = \int d^3\boldsymbol{r} S^{\mathrm{rel}}(\boldsymbol{r}) |\psi_{AB}^{(-)}(\boldsymbol{r}, \boldsymbol{Q})|^2$$

Coulomb Interaction

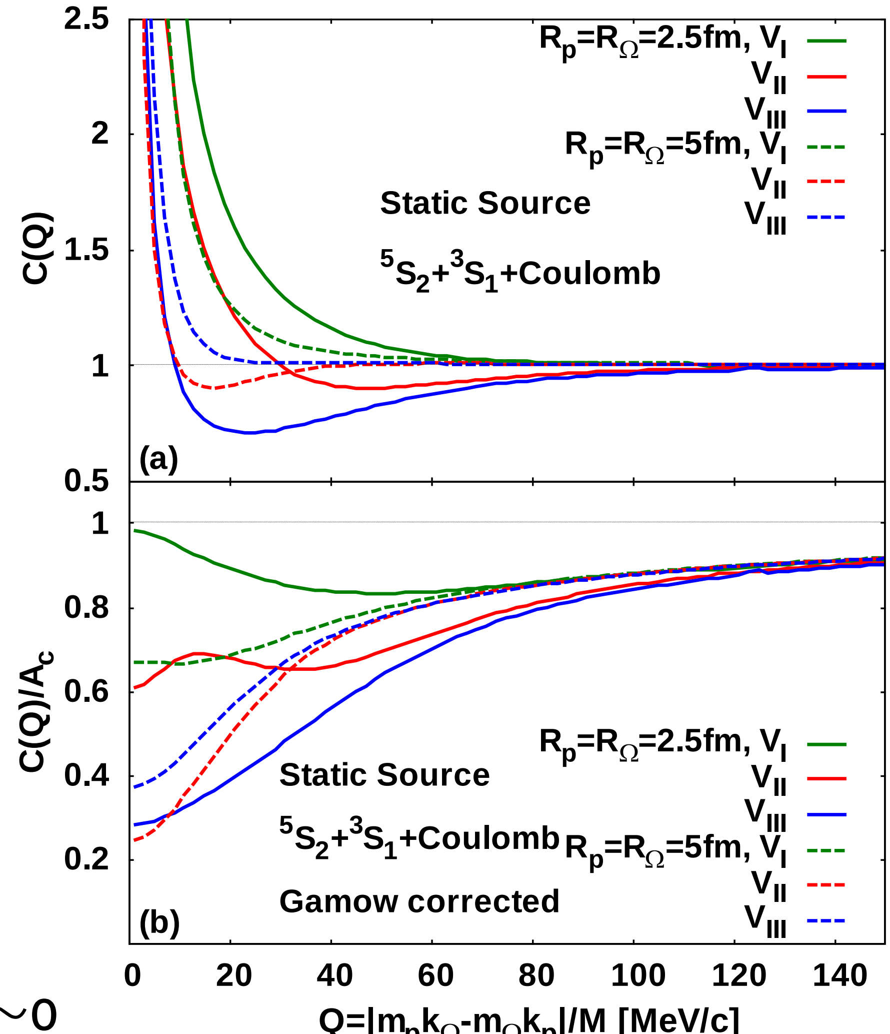
$$\eta = -\frac{1}{Qa_{\mathrm{Bohr}}}$$ =45fm

$$e^{i\boldsymbol{Q}\cdot\boldsymbol{r}} \to \psi^C_{\boldsymbol{Q}}(\boldsymbol{r}) = e^{\frac{\pi\eta}{2}}\Gamma(1+i\eta)e^{i\boldsymbol{Q}\cdot\boldsymbol{r}}{}_1F_1(-i\eta, 1, -i(Qr+\boldsymbol{Q}\cdot\boldsymbol{r}))$$

$$j_0(Qr) \to \chi^C_0 = \sqrt{\frac{\pi\eta}{\sinh\pi\eta}}e^{\frac{\pi\eta}{2}}e^{-iQr}{}_1F_1(1+i\eta, 2, 2iQr)$$

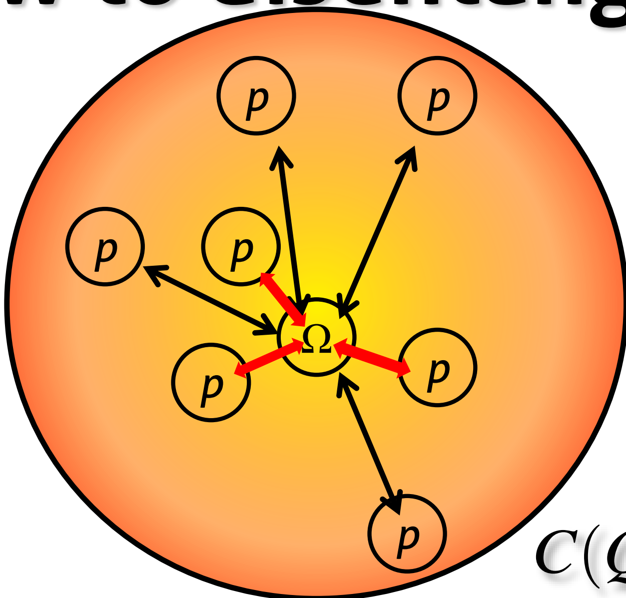
$$\chi_{\mathrm{abs}}(r) = \theta(r-r_0)\frac{1}{2iQr}\left[H_0^+(Qr) - \frac{H_0^+(Qr_0)}{H_0^-(Qr_0)}H_0^-(Qr)\right]$$

$$A_c = \frac{2\pi\eta}{e^{2\pi\eta}-1}$$



Coulomb int. dominates C(Q) near Q～0

How to disentangle Coulomb / Strong?

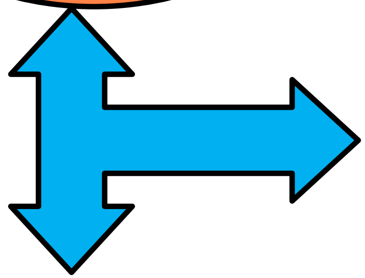


⟷ **Coulomb dominated**

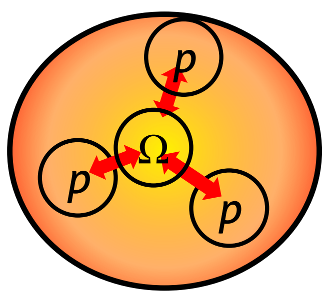
⟷ (red) **Strong int. dominated**

Large R : many pairs are from long-distance - Coulomb-dominated

$$C(Q) - 1 \propto \int dr r^2 e^{-r^2/(2\rho^2)} [\chi_Q^2 - j_0^2(Qr)]$$

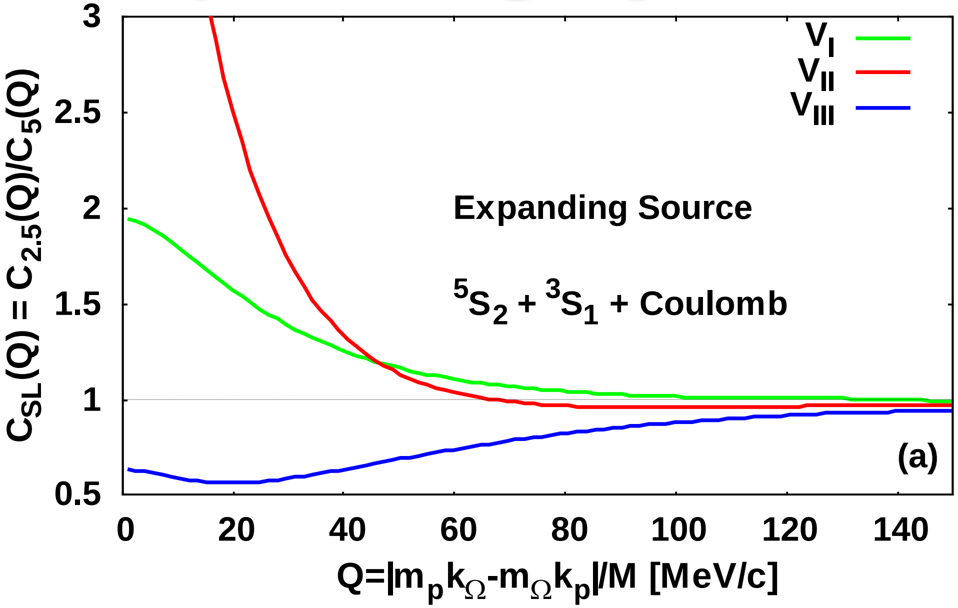


Difference informs about Strong int.



Small R : most of pairs are from short-distance - Strong-int. dominated

Expanding System : no difference



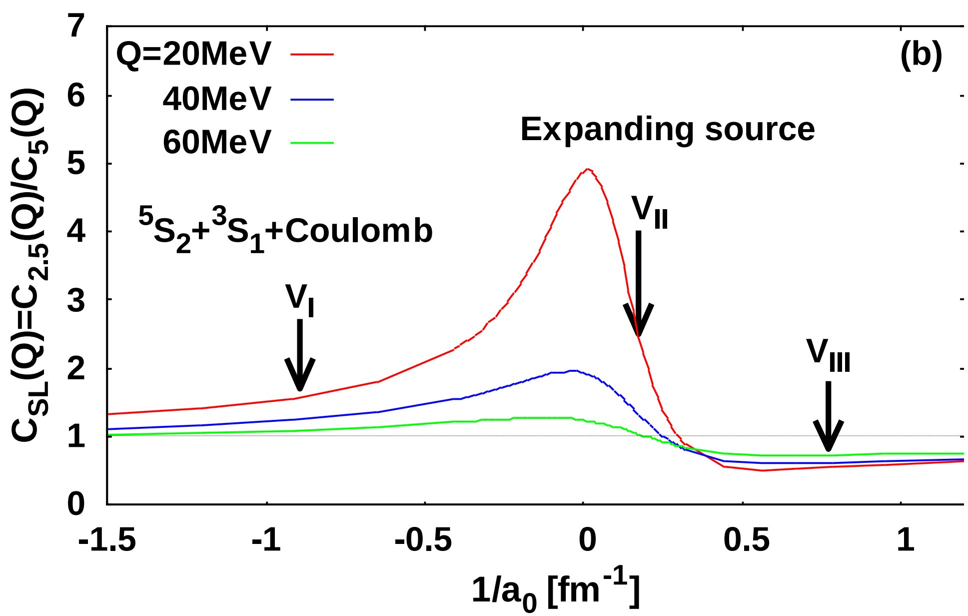
1-dim Boost-invariant expansion

$$S(x_i,k_i)=\frac{d_i}{(2\pi)^3}\frac{M_{T,i}\cosh(y_i-\eta_s)}{e^{M_{T,i}\cosh(y_i-\eta_s)/T}+1}e^{-\frac{x^2+y^2}{2R_i^2}}\delta(\tau-\tau_i)$$

LHC Pb+Pb 2.76AGeV

Central : τ_p = 20 fm/c, τ_Ω = 10 fm/c
T_p = 120 MeV, T_Ω = 164 MeV
$R_p = R_\Omega$ = 5fm
k_t = 1.25 GeV (= $\langle k_t \rangle$)

Peripheral : τ_p = 3 fm/c, τ_Ω = 2 fm/c
$T_p = T_\Omega$ = 164 MeV
$R_p = R_\Omega$ = 2.5fm
k_t = 1 GeV (= $\langle k_t \rangle$)



Feed-Down Contribution

- **Short-lived (Σ*, N* etc) : absorbed into R**
- **$\Xi \rightarrow \Lambda + \pi$: (partly) subtracted**
- **$\Sigma^0 \rightarrow \Lambda + \gamma$**

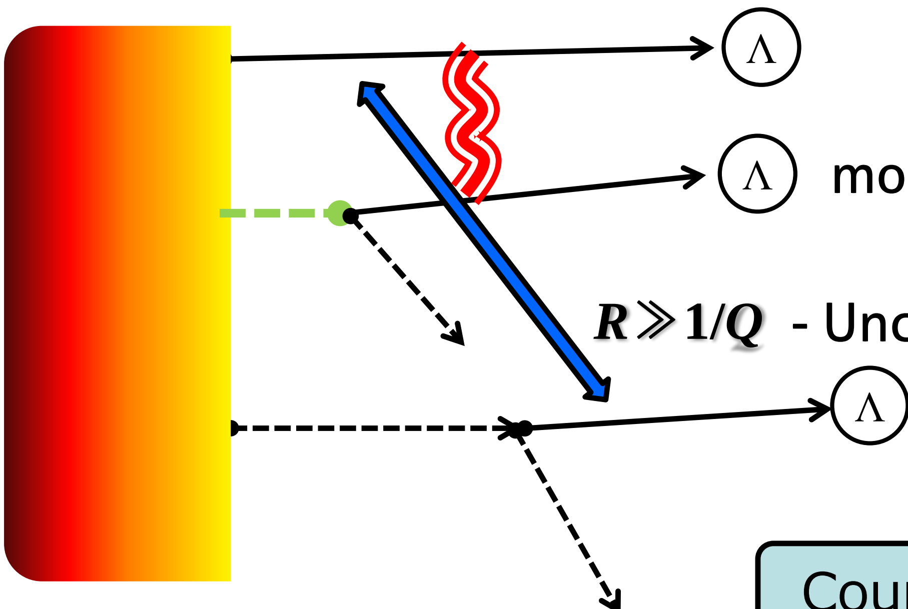
Σ^0/Λ =0.278 (p+Be data), Ξ/Λ = 0.15 (RHIC)→ $\Lambda^{dir}/\Lambda^{tot}$ = 0.67

$$C(Q) \rightarrow 1 + \left(\frac{\Lambda^{\rm dir}}{\Lambda^{\rm tot}}\right)^2 (C(Q)-1)$$

⇧

$(0.67)^2$

(0.52 if including Ξ)



modification of R

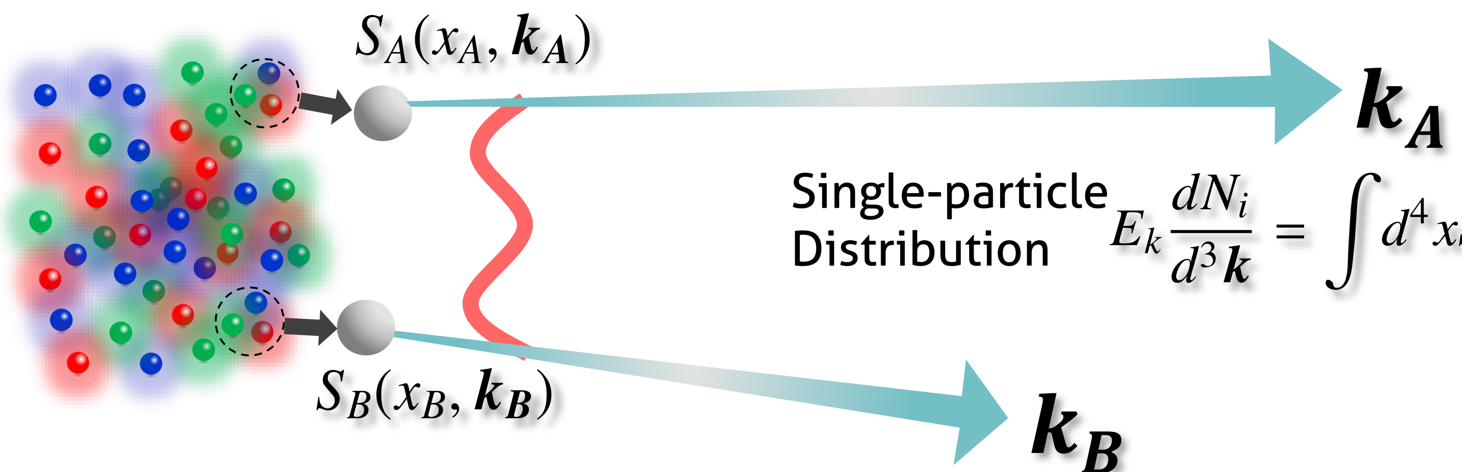
$R \gg 1/Q$ - Uncorrelated

cannot be resolved

$$C_{AB}(Q) = \frac{N_{\rm pair}^{\rm correlated}(Q)}{N_{\rm pair}^{\rm uncorrelated}(Q)}$$

Counted

Final State Interaction



Single-particle Distribution $E_k \frac{dN_i}{d^3\boldsymbol{k}} = \int d^4x S_i(x, \boldsymbol{k})$

For simplicity, spherical and static thermal source is considered.

$$E_k \frac{dN_i}{d^3\boldsymbol{k}} = * \int d^4x E_k e^{-E_k/T} \exp\left(-\frac{x^2+y^2+z^2}{2R^2}\right)\delta(t-t_0)$$

Small Q – Low energy scattering → S-wave dominant

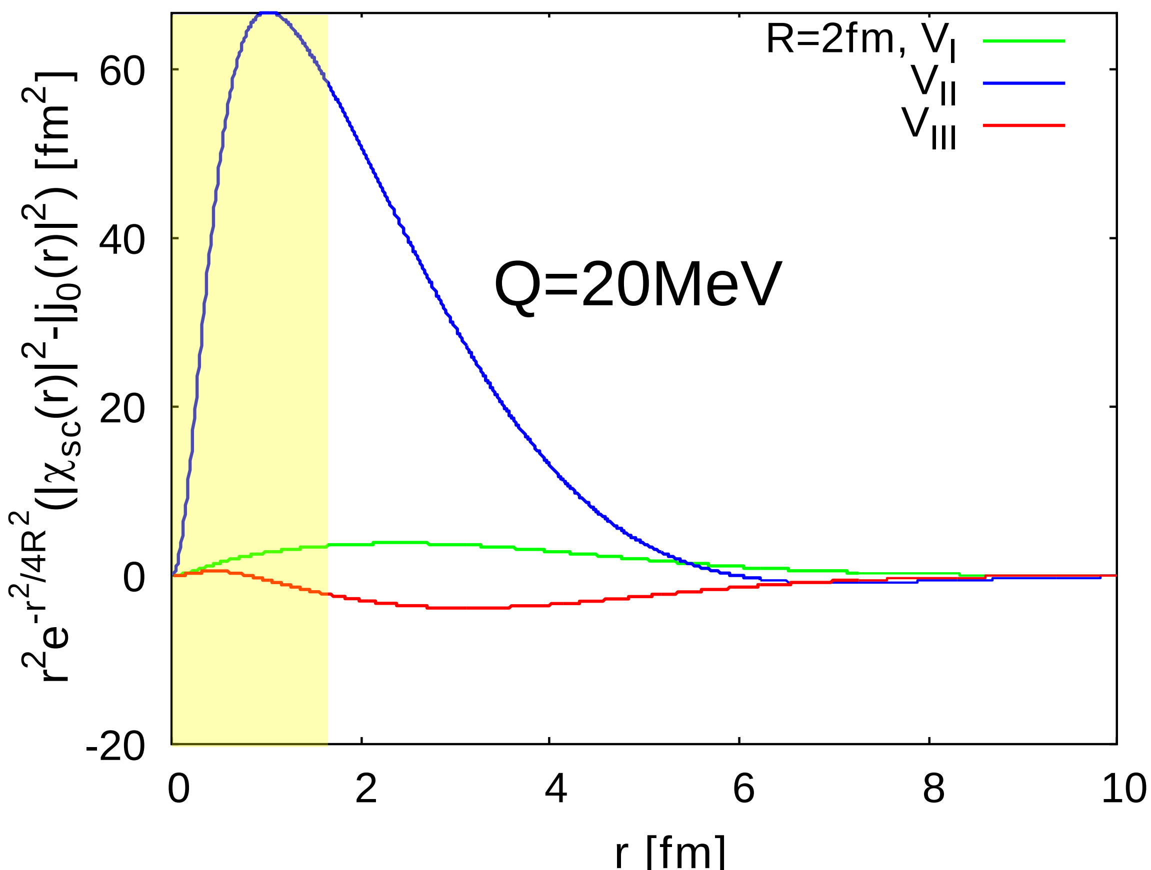
$$\psi_Q^{\mathrm{rel}}(r) = e^{i\boldsymbol{Q}\cdot\boldsymbol{r}} - j_0(Qr) + \chi_Q(r)$$ Replacing S-wave component of the wave func.

$$\to e^{i\boldsymbol{Q}\cdot\boldsymbol{r}} + \frac{f^*_{\ell=0}(Q)}{r}e^{-iQr} \quad (r \to \infty)$$ Asymptotic form

Correlation vs Wave function: Interaction

$$C_{AB}(Q) - 1 = \frac{4\pi}{(2\pi R^2)^3}\int dr r^2 S^{\rm rel}(r)[|\chi_Q(r)|^2 - |j_0(Qr)|^2]$$

$$S^{\rm rel}(r) \to (\pi R^2)^{3/2}\exp\left(-\frac{r^2}{4R^2}\right)$$ Static/Spherical Source



V_I : Weak attraction

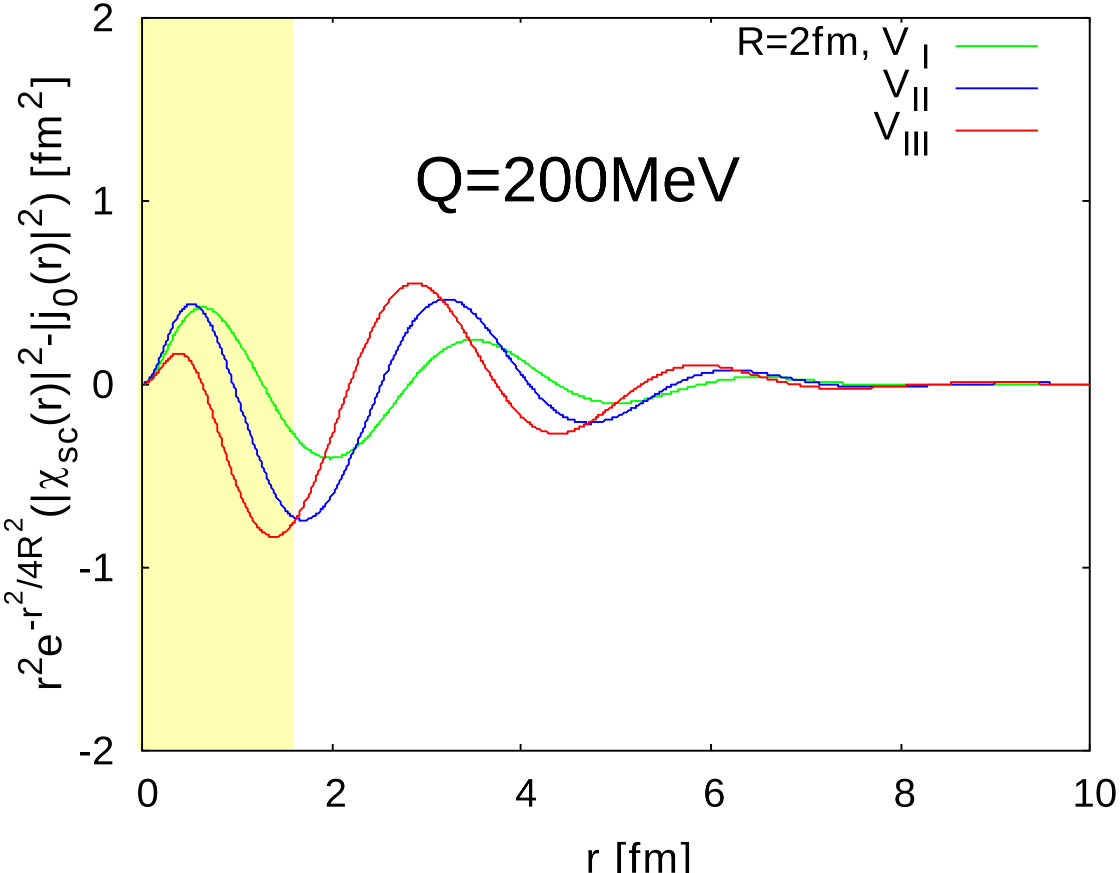
V_{II}: Stronger attraction (close to unitary limit)

V_{III}: Strong attraction (w/ bound state)

Correlation vs Wave function: Q

$$C_{AB}(Q) - 1 = \frac{4\pi}{(2\pi R^2)^3} \int dr r^2 S^{\rm rel}(r) [|\chi_Q(r)|^2 - |j_0(Qr)|^2]$$

$$\to (\pi R^2)^{3/2} \exp\left(-\frac{r^2}{4R^2}\right)$$ Static/Spherical Source



V_I : Weak attraction

V_{II}: Stronger attraction (close to unitary limit)

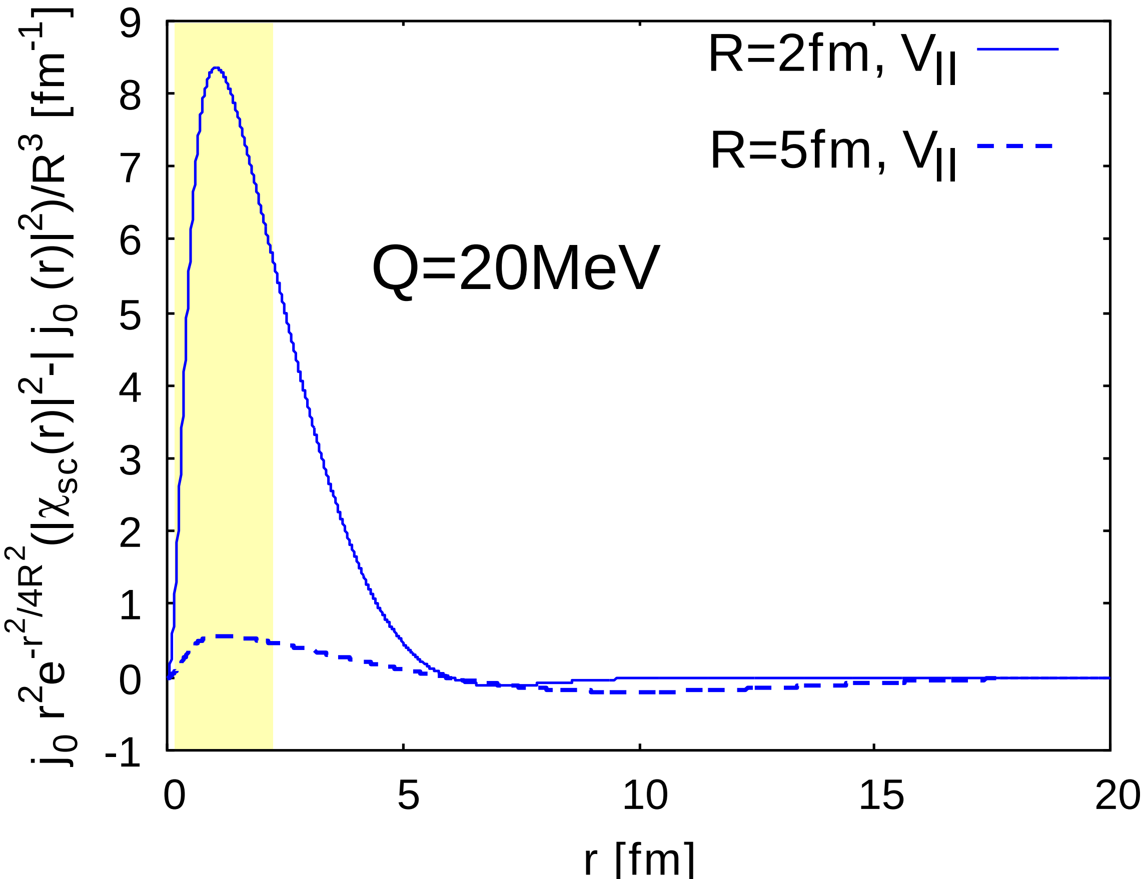
V_{III}: Strong attraction (w/ bound state)

Difference in Small Q

Correlation vs Wave function: Size

$$C_{AB}(Q)-1=\frac{4\pi}{(2\pi R^2)^3}\int dr r^2 S^{\rm rel}(r)[|\chi_Q(r)|^2-|j_0(Qr)|^2]$$

$$(\pi R^2)^{3/2}\exp\left(-\frac{r^2}{4R^2}\right)$$ Static/Spherical Source



Large R: outside int. range

→determined by a_0, r_{eff}

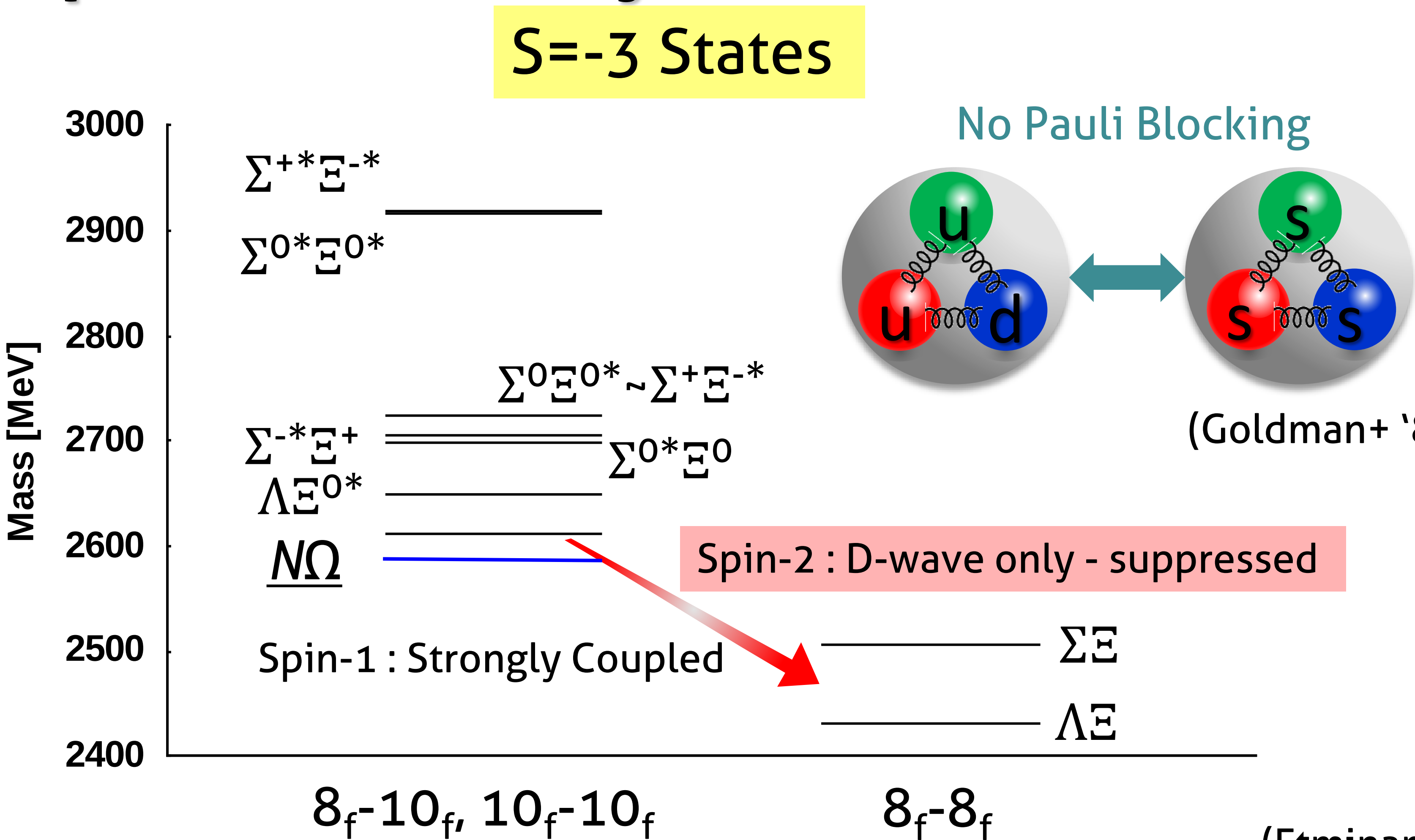
(Lednicky+'82)

C(Q) ~ $1/R^3$

Difference in Small Q

→ Enhanced in Small R

Spin-2 *NΩ* Dibaryon?



(Goldman+ '87)

(Etminan+ '14)

Lattice QCD : *NΩ* bound state at heavy π mass (HAL QCD)

Physical point also ! (preliminary, Iritani+'17))