Recent thermal model developments
connection of (anti-)nuclei to critical observables

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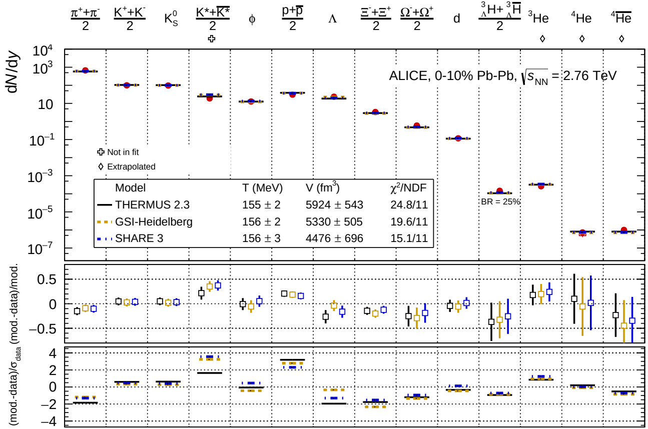
- Thermal fits, excluded volume effects, and role of light nuclei
- Recent HRG model developments based on lattice QCD

EMMI Workshop on anti-matter, hyper-matter and exotica production at LHC
Turin, Italy

November 6, 2017

Thermal model and light nuclei

- Thermal model: Particles, including light nuclei, stem from thermally equilibrated source. Described by HRG model



ALICE Collaboration, 1710.07531

- Another approach: coalescence model (not covered in this talk) See tomorrow's talks by C.-M. Ko, U. Heinz, A. Botvina

Excluded-volume effects

Excluded volume model: a schematic way to include repulsive interactions between hadrons in HRG [D. Rischke et al., Z. Phys. C (1991); G. Yen et al., PRC (1997)]

$$n_i^{ev} = \frac{n_i^{id}}{1 + \sum_j v_j n_j^{id} e^{-v_j P/T}} \, e^{-v_i P/T}$$

Each hadron assigned an “eigenvolume” $v_i = \frac{16\pi}{3} r_i$, where r_i is radius parameter[1]

Most common (and simplest) case: identical v_i for all hadrons, e.g. $r_i = r = 0.3$ fm [A. Andronic et al., 1201.0693]

In this case EV effects cancel out in hadron yield ratios and thermal fits are unaffected

EV interactions also often used for light nuclei in nuclear matter, as a mechanism for cluster dissolution at high densities
Lattimer, Swesty, Nucl. Phys. A (1991); Shen et al., nucl-th/9806095; S. Typel, EPJ (2016)

[1] r_i value should not be identified a hard-core radius. This cannot be done due to quantum mechanical effects.

Excluded-volume effects

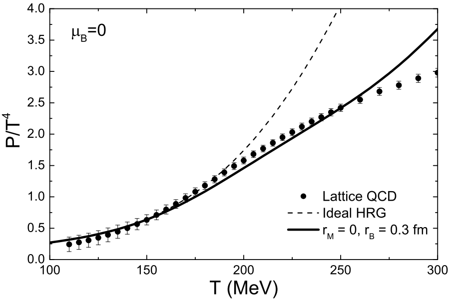
There is no reason for all radii parameters to be constant

Scattering phase shifts suggest significant repulsive interactions between baryons

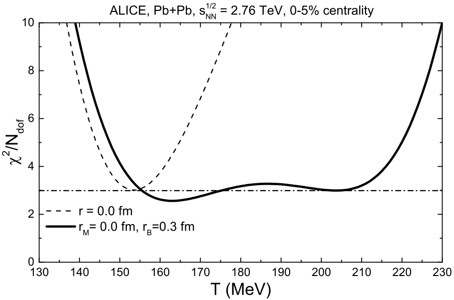
They do *not* yield as much evidence for meson-meson or meson-baryon repulsion [Prakash, Venugopalan, Nucl. Phys. A (1992); P.M. Lo et al., 1703.00306]

An example: $r_M = 0$ fm, $r_B = 0.3$ fm[1]

Pressure p/T^4



ALICE data fit (just hadrons)



As soon interactions are switched on thermal fits may look quite different[2]

More dramatic effects with some other parametrizations, e.g. bag model

[1] This parametrization first studied in Andronic et al., 1201.0693

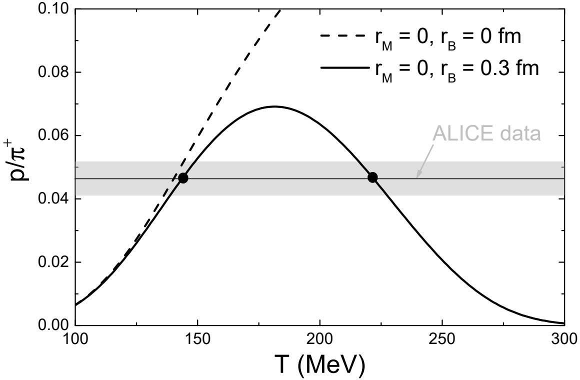
[2] V.V., H. Stoecker, 1512.08046

Origin of the two minima

Where does the 2nd minimum come from?

Consider p/π ratio in the EV model

$$\frac{n_p^{ev}}{n_\pi^{ev}} = \frac{n_p^{id}}{n_\pi^{id}} \, e^{(v_\pi - v_p)P/T}$$



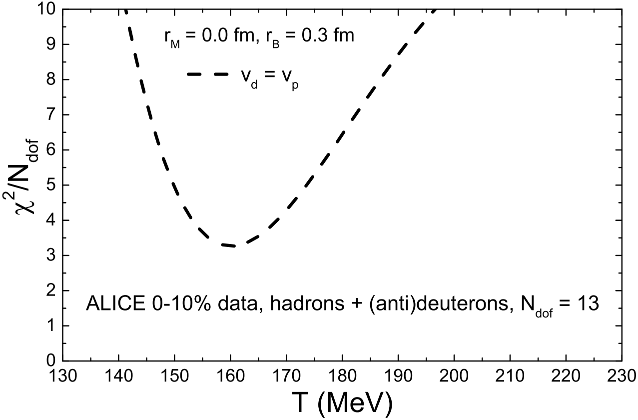
Non-monotonic behavior when $v_\pi < v_p$ which yields two solutions

L.M. Satarov et al., 1610.08753

Adding the light nuclei

Let us now add deuteron into the fit

First assume for simplicity $v_d = v_p$, i.e. $r_p = r_d = 0.3$ fm

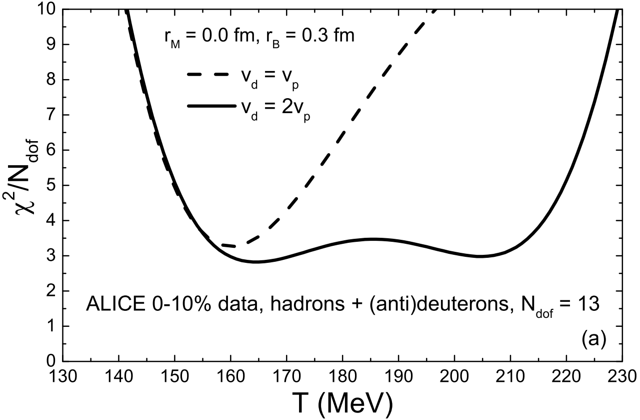


Thermal fits are stabilized?!

V.V., H. Stoecker, 1610.02346

Adding the light nuclei

A more realistic scenario: $v_d = 2\,v_p$



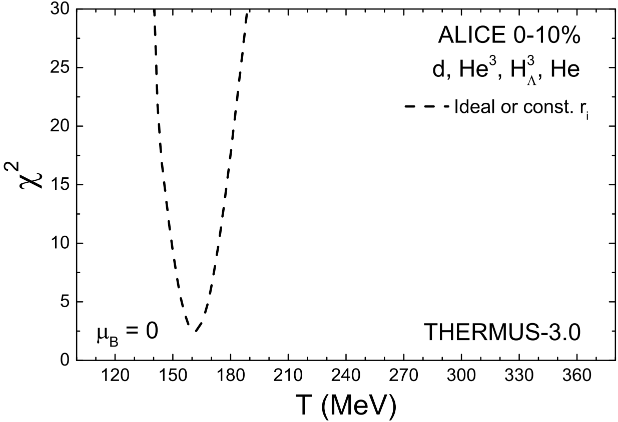
The 2nd minimum strikes again

V.V., H. Stoecker, 1610.02346

Fitting light nuclei only

One could forget about the hadrons and fit just the light nuclei

Advantage: No dependence on high-mass resonance spectrum and feeddown[1]



Ideal HRG (or $v_i = \text{const.}$): $T_f = 160 \pm 5$ MeV

[1] A. Andronic et al., 1710.09425

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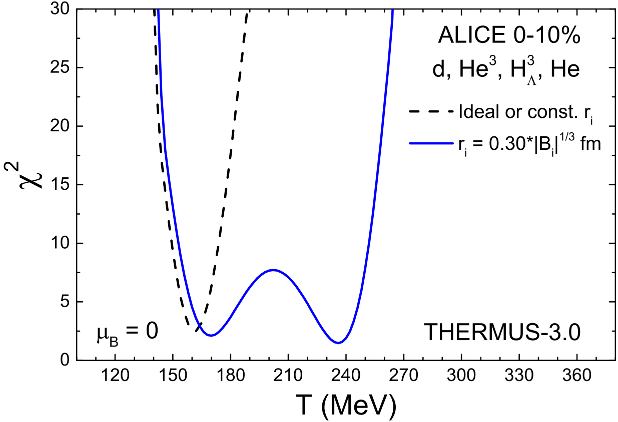


Ideal HRG (or v_i = const.): $T_f = 160 \pm 5$ MeV

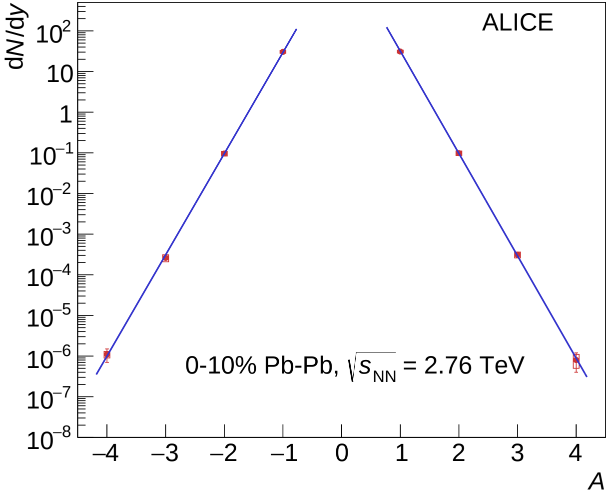
EV-HRG with $v_i = v|A_i|$: $T_f = 160 - 250$ MeV

Disadvantage: Fits are even more sensitive to EV corrections

[1] A. Andronic et al., 1710.09425

Penalty factor

Alternatively, one could consider penalty factor p_f

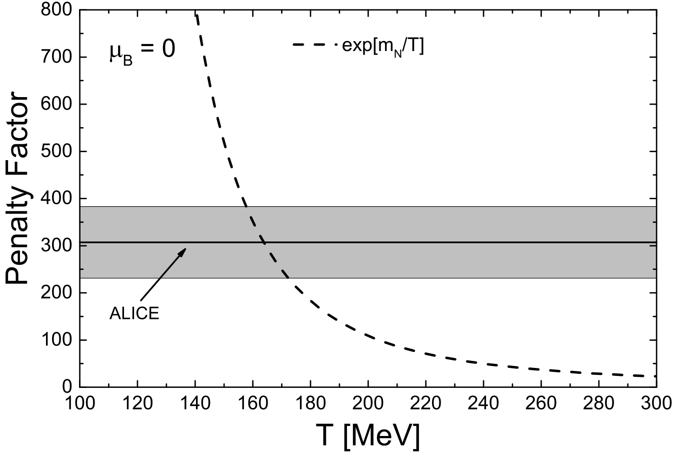


For 0-10% Pb+Pb collisions penalty factor is around 300

ALICE Collaboration, 1506.08951, 1710.07531

Penalty factor in thermal model

In Ideal HRG $p_f \simeq \exp\left(\frac{m_N - \mu_B}{T}\right)$

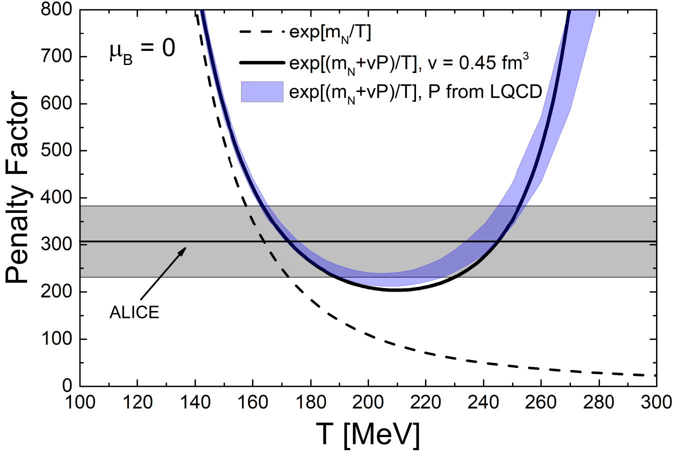


$\mu_B = 0$ and $p_f \sim 300 \quad \Rightarrow \quad T_f \sim 160$ MeV

Penalty factor in thermal model: EV effects

In EV HRG $n_i^{ev} \propto \exp\left(-v_i \frac{p}{T}\right)$

Assuming $v_i = v\,|A_i|$ one obtains $p_f \simeq \exp\left(\frac{m_N - \mu_B + vP}{T}\right)$



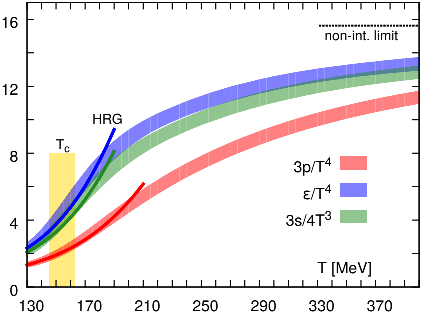
Calculation done for $r_M = 0$, $r_p = 0.3$ fm ($v \simeq 0.45$ fm^3)

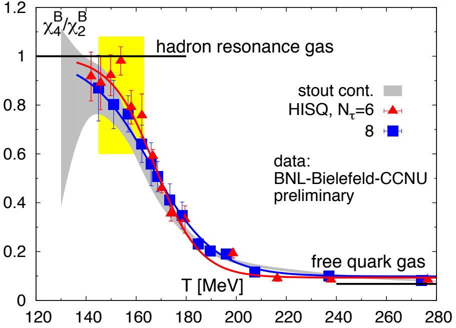
Data no longer point to a unique temperature value

Recent HRG model developments

QCD equation of state at $\mu = 0$

In the last few years, rich amount of lattice data





- Rapid breakdown of ideal HRG model in crossover region for description of susceptibilities[1]
- Often interpreted as clear signal of deconfinement...
- But what is the role of hadronic interactions beyond those in ideal HRG?

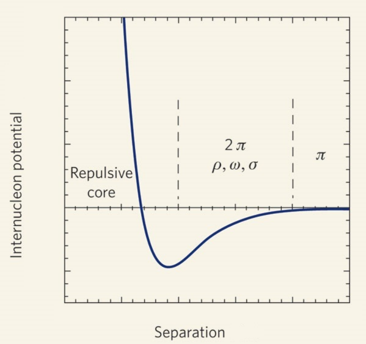
[1] Ding, Karsch, Mukherjee, IJMPE 24, 1530007 (2015)
HotQCD Collaboration: 1407.6387; 1701.04325; 1708.04897
Wuppertal-Budapest Collaboration: 1112.4416, 1309.5258, 1507.04627

Nucleon-nucleon interaction

Many hadronic interactions described by resonance formation... however

Nucleon-nucleon potential:

- Repulsive core at small distances
- Attraction at intermediate distances
- No resonance structure
- Suggestive similarity to vdW interactions
- Could nuclear matter be described by vdW equation?



Nuclear matter with quantum van der Waals (QvdW) equation

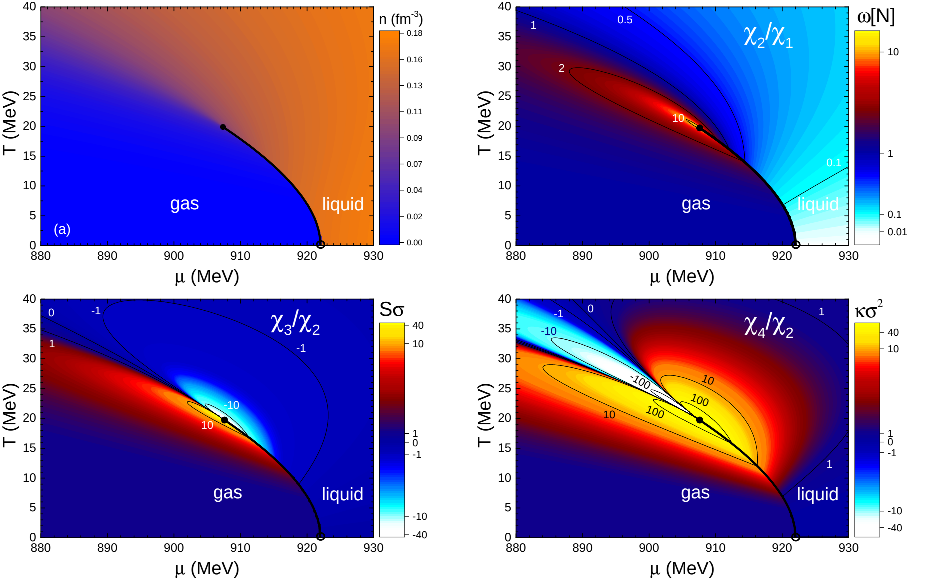
$$p(T,n) \; = \; p_q^{\rm id}\left(T, \frac{n}{1-bn}\right) - a\,n^2$$

$$E/A = -16 \text{ MeV}, \;\; n_0 = 0.16 \text{ fm}^{-3} \quad \Rightarrow \quad a_{NN} = 329 \text{ MeV fm}^3, \;\; b_{NN} = 3.42 \text{ fm}^3$$

V.V., Anchishkin, Gorenstein, PRC '15; Redlich, Zalewski, APPB '16.

QvdW gas of nucleons: (T, μ) plane

(T, μ) plane: structure of critical fluctuations $\chi_i = \partial^i (p/T^4)/\partial(\mu/T)^i$



V.V., D. Anchishkin, M. Gorenstein, R. Poberezhnyuk, PRC 91, 064314 (2015)

van der Waals interactions in hadron resonance gas

Let us now include nuclear matter physics into HRG...

(Q)vdW-HRG model

- Identical vdW interactions between all baryons
- Baryon-antibaryon, meson-meson, meson-baryon vdW terms neglected
- Baryon vdW parameters extracted from ground state of nuclear matter ($a = 329$ MeV fm^3, $b = 3.42$ fm^3)

Three independent subsystems: mesons + baryons + antibaryons

$$p(T, \boldsymbol{\mu}) = P_M(T, \boldsymbol{\mu}) + P_B(T, \boldsymbol{\mu}) + P_{\bar{B}}(T, \boldsymbol{\mu}),$$

$$P_M(T, \boldsymbol{\mu}) = \sum_{j \in M} p_j^{\rm id}(T, \mu_j) \quad \text{and} \quad P_B(T, \boldsymbol{\mu}) = \sum_{j \in B} p_j^{\rm id}(T, \mu_j^{B*}) - a\, n_B^2$$

In this simplest setup model is essentially "parameter-free"

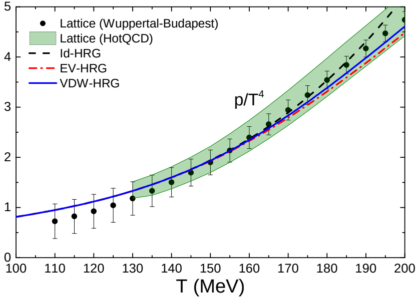
Crucial point: EV/vdW terms for baryon-baryon pairs only

V.V., M. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

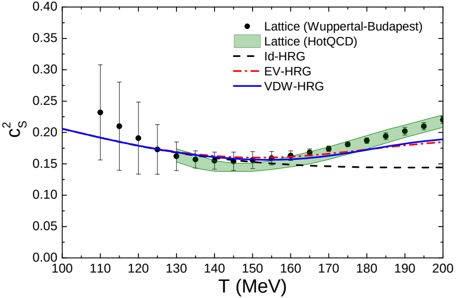
vdW-HRG at $\mu_B = 0$: thermodynamics

Comparison of vdW-HRG with lattice QCD at $\mu_B = 0$

Pressure p/T^4



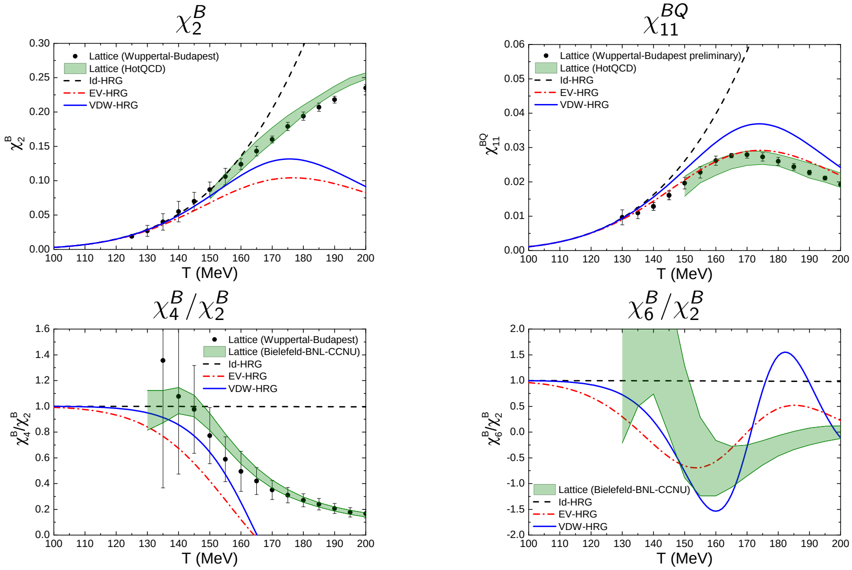
Speed of sound $c_s^2 = \frac{dp}{d\varepsilon}$



- vdW-HRG does not spoil existing agreement of Id-HRG with LQCD despite significant excluded-volume interactions between baryons
- Not surprising: matter meson-dominated at $\mu_B = 0$
- No acausal behavior

V.V., M. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

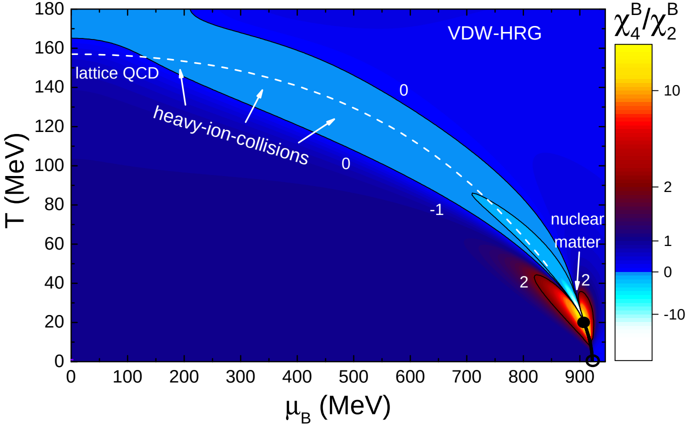
vdW-HRG at $\mu_B = 0$: susceptibilities



- Quantitative features of QCD captured by vdW-HRG
- Extrapolation from cold NM to high T overestimates interaction effects

vdW-HRG at finite μ_B

Net-baryon fluctuations in T-μ_B plane: χ_4^B/χ_2^B



- Almost no effect in Id-HRG, only Fermi statistics...
- Rather rich structure for vdW-HRG, huge effect of vdW interactions!
- Fluctuations seen at RHIC are remnants of nuclear liquid-gas PT?

V.V., M. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)
see also A. Mukherjee et al., 1611.10144

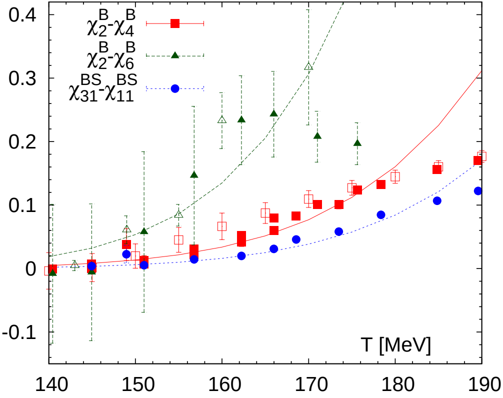
Alternative: repulsive mean field

HRG with repulsive mean-field interactions between baryons

Huovinen, Petreczky, 1708.00879

$$p_B + \bar{p}_B = T\,(n_B + \bar{n}_B) + \frac{K}{2}(n_B^2 + \bar{n}_B^2),$$

where $K = 450$ MeV fm^3 based on empirical NN phase shifts.

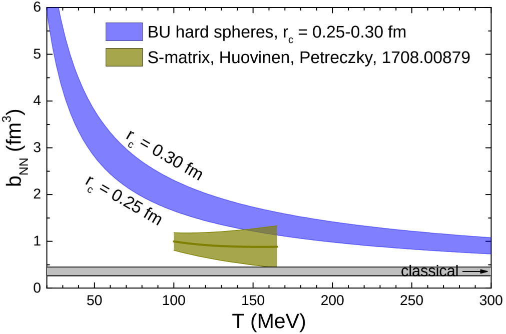


"Signals of deconfinement" interpreted in terms of repulsive baryonic interactions

Hard-core repulsion: classical vs Beth-Uhlenbeck

Nucleon $\lambda_{dB} = \sqrt{2\pi/(mT)} \simeq 1.3$ fm at $T = 150$ MeV, comparable to r_c

QM approach to NN hard-core repulsion: Beth-Uhlenbeck (BU) formula



$$\delta_{J,T}(\varepsilon) = \arctan\left\{\frac{j_L[2r_c q(\varepsilon)]}{y_L[2r_c q(\varepsilon)]}\right\}$$

To be compared with

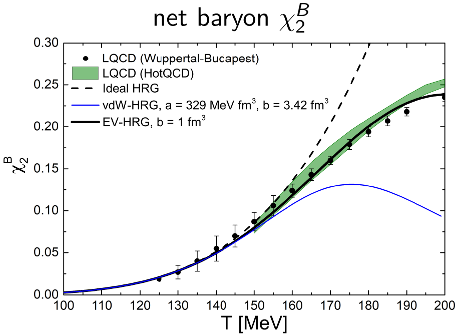
Classical EV model

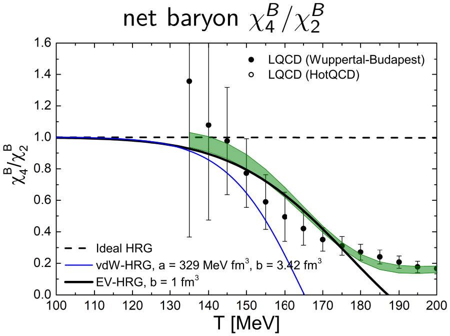
$$p = p^{\mathrm{id}}(T, \mu - bp), \qquad b = \frac{16\pi r_c^3}{3}$$

- Classical approach with $r_c \simeq 0.25$-0.3 fm[1] underestimates EV by factor 3-4
- Radius parameter in EV model is very different from actual hard-core radius!
- Correcting for residual attraction, one arrives at $b \simeq 1$ fm^3 at $T \sim 150$ MeV

[1] NN-scattering data analysis: R. B. Wiringa et al., Phys. Rev. C **51**, 38 (1995)
V.V, A. Motornenko, M. Gorenstein, H. Stoecker, 1710.00693

Revised LQCD comparison





Conclusion:

Empirical and LQCD evidence for net repulsive EV-type baryonic interactions with

$$b \simeq 1 \text{ fm}^3 \text{ in the crossover region}$$

Not much evidence for significant repulsion between other hadron pairs

Imaginary μ_B

QCD observables at imaginary μ_B

QCD thermodynamics with relativistic cluster/virial expansion:

Pressure: $$\frac{p(T,\mu_B)}{T^4} = \sum_{k=0}^{\infty} p_k(T) \cosh\left(\frac{k\,\mu_B}{T}\right),$$

Net baryon density: $$\frac{\rho_B(T,\mu_B)}{T^3} = \sum_{k=1}^{\infty} b_k(T) \sinh\left(\frac{k\,\mu_B}{T}\right), \quad b_k(T) \equiv k\, p_k(T)$$

Lattice QCD is problematic at real μ but tractable at imaginary μ

$\mu_B \to \mathrm{i}\tilde{\mu}_B \quad \Rightarrow \quad$ QCD observables obtain trigonometric Fourier series form

Pressure: $$\frac{p(T,\mathrm{i}\tilde{\mu}_B)}{T^4} = \sum_{k=0}^{\infty} p_k(T) \cos\left(\frac{k\,\tilde{\mu}_B}{T}\right),$$

Net baryon density: $$\frac{\rho_B(T,\mathrm{i}\tilde{\mu}_B)}{T^3} = \mathrm{i}\sum_{k=1}^{\infty} b_k(T) \sin\left(\frac{k\,\tilde{\mu}_B}{T}\right), \quad b_k(T) \equiv k\, p_k(T)$$

Coefficients $b_k(T)$ can and are now being calculated in LQCD

Connection to light nuclei

At low T/densities QCD $\simeq$ ideal HRG with light nuclei

$$\frac{\rho_B^{\mathrm{hrg}}(T,\mu_B)}{T^3} = \underbrace{2\sum_{i\in B}\int dm\,\rho_i(m)\,\frac{d_i\,m^2}{2\pi^2 T^3}\,K_2\left(\frac{m}{T}\right)}_{b_1(T)}\sinh\left(\frac{\mu_B}{T}\right)$$

$$+\underbrace{4\sum_{i\in d,\{N\Lambda\},\dots}\frac{d_i\,m_i^2}{2\pi^2 T^3}\,K_2\left(\frac{m_i}{T}\right)}_{b_2(T)}\sinh\left(2\,\frac{\mu_B}{T}\right)$$

$$+\underbrace{6\sum_{i\in He^3,\dots}\frac{d_i\,m_i^2}{2\pi^2 T^3}\,K_2\left(\frac{m_i}{T}\right)}_{b_3(T)}\sinh\left(3\,\frac{\mu_B}{T}\right)$$

$$+\dots$$

Light nuclei induce positive $b_k(T)$ for $k \geq 2$,
which are otherwise zero in ideal Boltzmann HRG

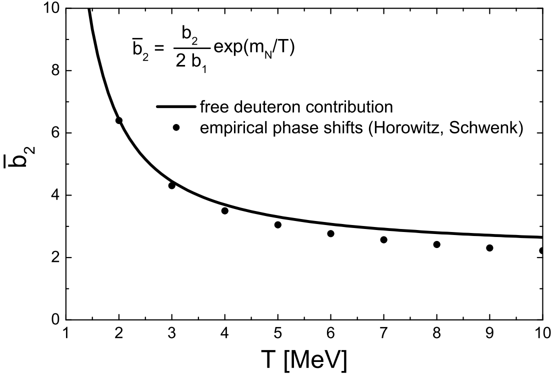
Connection to light nuclei: low temperatures

Even lower temperatures: interacting gas of nucleons

Free deuteron gas contribution to the reduced b_2 coefficient

$$\bar{b}_2(T) = \frac{b_2(T)}{b_1(T)} e^{m_N/T} = \frac{3}{2^{1/2}}\, e^{E_d/T}$$

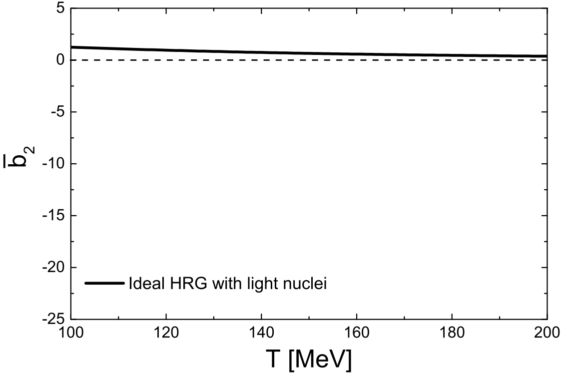
can be compared with the model-independent calculation for interacting nucleons employing empirical phase shifts [Horowitz, Schwenk, nucl-th/0507033]



$b_2(T)$ corresponds to free deuterons at $T \lesssim 10$ MeV

Connection to light nuclei: crossover region

Light nuclei in ideal HRG yield positive $\bar{b}_2(T)$ at crossover temperatures



[1] V.V., A. Pásztor, Z. Fodor, S.D. Katz, H. Stoecker, 1708.02852; S. Borsányi, QM2017

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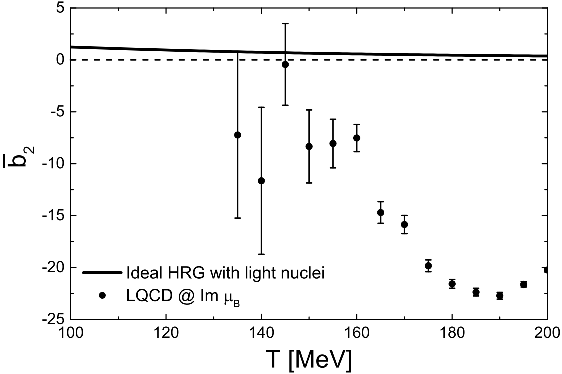


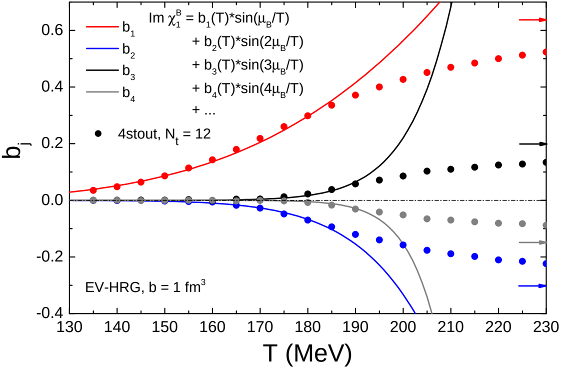
Contradicts imaginary μ_B lattice data[1]: $b_2(T) < 0$ for $T > 135$ MeV

Modification of ideal HRG model is required

[1] V.V., A. Pásztor, Z. Fodor, S.D. Katz, H. Stoecker, 1708.02852; S. Borsányi, QM2017

Imaginary μ_B and repulsive baryonic interactions

$$\frac{\rho_B(T, \mathrm{i}\tilde{\mu}_B)}{T^3} = \mathrm{i} \sum_{j=1}^{\infty} b_j(T) \sin\left(j\, \tilde{\mu}_B / T\right)$$



HRG with baryonic EV repulsion:

$$b_1^{\mathrm{ev}}(T) = 2\, \frac{\phi_B(T)}{T^3}$$

$$b_2^{\mathrm{ev}}(T) = -4\, [b\phi_B(T)]\, \frac{\phi_B(T)}{T^3}$$

$$b_3^{\mathrm{ev}}(T) = 9\, [b\, \phi_B(T)]^2\, \frac{\phi_B(T)}{T^3}$$

$$b_4^{\mathrm{ev}}(T) = -\frac{64}{3}\, [b\, \phi_B(T)]^3\, \frac{\phi_B(T)}{T^3}$$

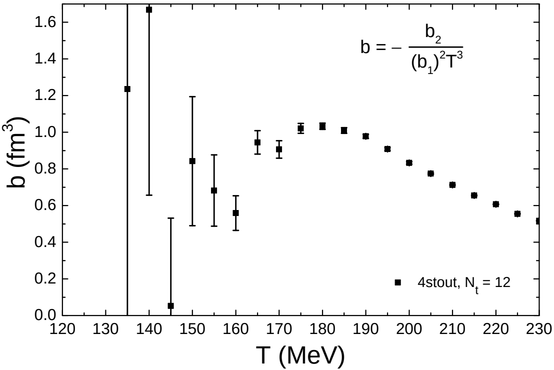
- Ideal HRG describes well $b_1(T)$ at small temperatures
- Non-zero $b_j(T)$ for $j \geq 2$ signal deviations from ideal HRG
- Addition of EV interactions between baryons reproduces lattice trend

V.V., A. Pásztor, Z. Fodor, S. Katz, H. Stoecker, 1708.02852; S. Borsányi, QM2017

“Excluded volume” parameter from imaginary μ_B data

“Excluded volume” parameter of BB interactions can be estimated from lattice

$$b(T) = -\frac{b_2(T)}{[b_1(T)]^2\, T^3}$$



$$b(T) \simeq 1 \text{ fm}^3 \text{ at } T < 190 \text{ MeV}$$

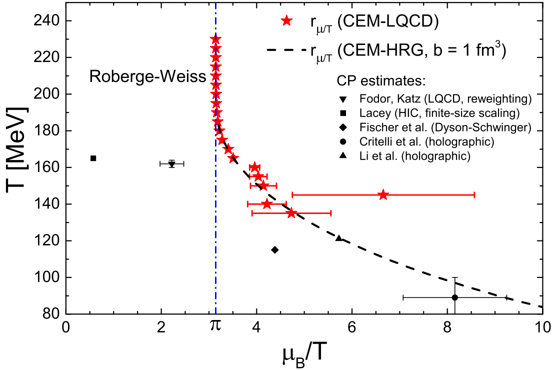
V.V., A. Pásztor, Z. Fodor, S.D. Katz, H. Stoecker, 1708.02852

Cluster expansion model for imaginary μ_B

EV-type expression, but matched to Stefan-Boltzmann limit

$$b_k(T) = \alpha_k^{SB} \frac{[b_2(T)]^{k-1}}{[b_1(T)]^{k-2}}, \qquad \alpha_k^{SB} = \frac{[b_1^{\mathrm{SB}}]^{k-2}}{[b_2^{\mathrm{SB}}]^{k-1}} \, b_k^{\mathrm{SB}}.$$

Predicts *all* $b_k(T)$, works very well for $b_3(T)$, $b_4(T)$, $\chi_k^B(T)$ from LQCD



Radius of convergence of Taylor series in μ_B/T sees Roberge-Weiss transition

V.V., J. Steinheimer, O. Philipsen, H. Stoecker, 1711.01261

Summary

- Proper modeling of hadronic interactions crucially important for thermal model applications
- Thermal model works very well for light nuclei yields. Only in ideal HRG, however, it does point to a unique freeze-out temperature.
- The van der Waals type interactions between baryons in HRG change qualitative behavior of fluctuations of conserved charges in the crossover region
- LQCD data at both, $\mu = 0$ and imaginary μ, points to overall repulsive baryonic interactions in the crossover region, with an average "eigenvolume" parameter $b \simeq 1$ fm^3
- Imaginary μ_B LQCD data show no evidence for existence of light nuclei at $T \sim 150$ MeV. Partial pressure in $|B| = 2$ sector is dominated by repulsive baryonic interactions.

Summary

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- The van der Waals type interactions between baryons in HRG change qualitative behavior of fluctuations of conserved charges in the crossover region
- LQCD data at both, $\mu = 0$ and imaginary μ, points to overall repulsive baryonic interactions in the crossover region, with an average "eigenvolume" parameter $b \simeq 1$ fm^3
- Imaginary μ_B LQCD data show no evidence for existence of light nuclei at $T \sim 150$ MeV. Partial pressure in $|B| = 2$ sector is dominated by repulsive baryonic interactions.

Thanks for your attention!

Backup slides

Cross-check of fits with other codes