Hit position error estimation for the CBM Silicon Tracking System

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Introduction: Silicon Tracking System

Main STS task is to reconstruct tracks with:

- high momentum resolution ($\Delta p/p \approx 1.5\,\%$ for $p > 1\,\mathrm{GeV}$);
- high track reconstruction efficiency ($> 96\,\%$ for $p > 1\,\mathrm{GeV}$).

This leads to the requirements:

- **high spatial resolution** $\Rightarrow$ high granularity;
- low material budget.

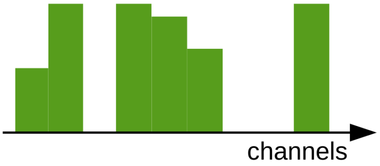
Design decision: 8 stations, double-sided Si sensors in 1 T magnetic field, r/o electronics outside of the acceptance connected to the sensors with thin microcables.

Spatial resolution are limited with:

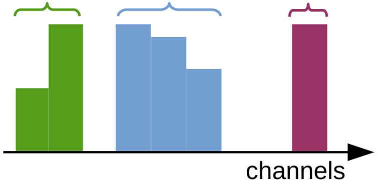
- multiple scattering;
- **intrinsic detector resolution**.

Cluster finding

Fired channels:



Clusters:



- Neighbouring digis (which presumably originate from the same incident particle) makes a cluster;
- Estimate cluster centre using measured charges q_i.

Cluster position finding algorithm

Centre-Of-Gravity algorithm (COG):

$$x_{\rm rec} = \frac{\Sigma x_i q_i}{\Sigma q_i}$$

x_i – the coordinate of ith strip,
q_i – its charge,
$i = 1..n$ – the strip index in the n-strip cluster.

COG is biased: $\langle x_{\rm true} - x_{\rm rec}\rangle \equiv \langle \Delta x\rangle \neq 0$ for $n \geq 2$ at fixed q_2/q_1.

An **unbiased algorithm**:
2-strip clusters:

$$x_{\rm rec} = 0.5\,(x_1 + x_2) + \frac{p}{3}\frac{q_2 - q_1}{\max(q_1, q_2)}, \quad p \text{ – strip pitch;}$$

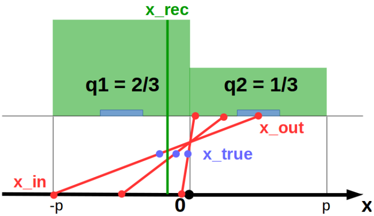
n-strip clusters (Analog head-tail algorithm[1]):

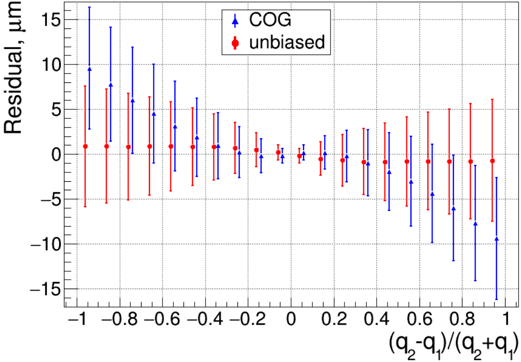
$$x_{\rm rec} = 0.5\,(x_1 + x_n) + \frac{p}{2}\frac{\min(q_n, q) - \min(q_1, q)}{q}, \quad q = \frac{1}{n-2}\sum_{i=2}^{n-1} q_i.$$

[1] R. Turchetta, "Spatial resolution of silicon microstrip detectors", 1993

COG vs Unbiased cluster position finding algorithm.

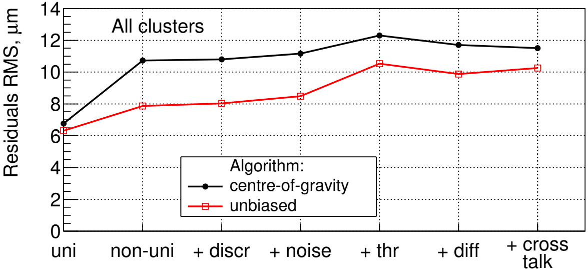
2-strip clusters example





Ideal detector model & uniform energy loss.
Error bars: RMS of the residual distribution.
$q_{1,2}$ – measured charges on the strips.

Comparison of residuals: COG vs Unbiased algorithms



1000 minimum bias Au+Au events at 10 AGeV are simulated with the realistic STS geometry.

- The unbiased algorithm is faster and simplifies the hit position error estimation.

Hit position error: introduction and motivation

Importance: A reliable estimate of the hit position error $\Rightarrow$
get proper track χ^2 $\Rightarrow$
discard ghost track candidates $\Rightarrow$
improve the signal/background and keep the efficiency high.

Method: Calculations from basic principles and independent of:
simulated residuals;
measured spatial resolution.

Verification: hit pull distribution: shape and width;
track χ^2 distribution: mean value.

Hit position error: basic ideas

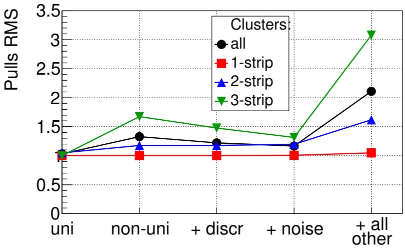
$$\sigma^2 = \sigma^2_{\mathrm{alg}} + \sum_i \left(\frac{\partial x_{\mathrm{rec}}}{\partial q_i}\right)^2 \sum_{\mathrm{sources}} \sigma^2_j,$$

σ_{alg} – an error of the cluster position finding algorithm;
σ_j – errors of the charge registration at one strip, among them already included:

- $\sigma_{\mathrm{noise}} = \mathrm{Equivalent\ Noise\ Charge};$
- $\sigma_{\mathrm{discr}} = \frac{\mathrm{dynamic\ range}}{\sqrt{12}\ \mathrm{number\ of\ ADC}};$
- $\sigma_{\mathrm{non-uni}}$ is estimated assuming:
  - registered charge corresponds to the most probable value of the energy loss;
  - incident particle is ultrarelativistic ($\beta\gamma \gtrsim 100$).
- σ_{diff} is negligible in comparison with other effects.

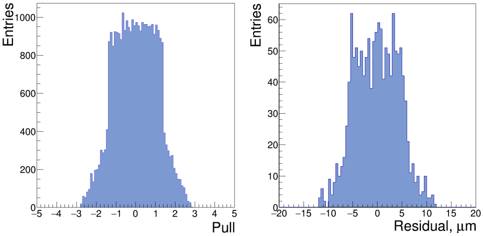
Verification: hit pull distribution

Width



1000 mbias events Au+Au @ 10 AGeV

Shape

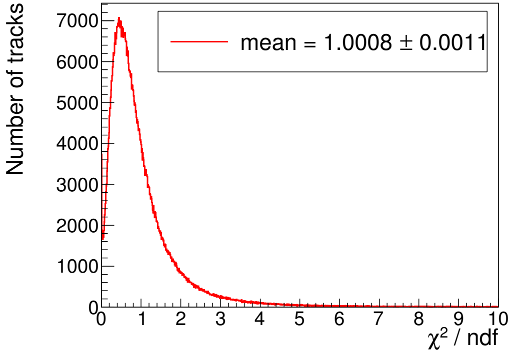


Ideal detector, 2-strip clusters,

residuals at fixed: $\dfrac{|q_2 - q_1|}{\max(q_1, q_2)}$.

- $\text{pull} = \dfrac{\text{residual}}{\text{error}}$;
- pull distribution width must be ≈ 1;
- pull distribution shape must reproduce residual shape.

Verification: track χ^2 distribution



10 000 minimum bias events Au+Au @ 10 AGeV

- χ^2/ndf distribution for tracks: mean value must be ≈ 1.

Summary

For the Silicon Tracking System of the CBM experiment:

- Two **cluster position finding algorithms** were implemented: Centre-Of-Gravity and the unbiased. The last:
  - gives similar residuals as the Centre-Of-Gravity algorithm;
  - simplifies position error estimation.
- Developed method of the **hit position error estimation** yields correct errors, that was verified with:
  - hit pulls distribution (width and shape);
  - track χ^2/ndf distribution.

Summary

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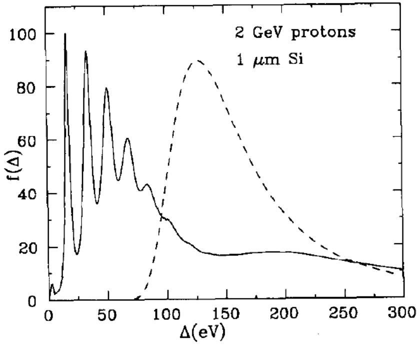
  - hit pulls distribution (width and shape);
  - track χ^2/ndf distribution.

Thank you for your attention!

Detector response model:

- non-uniform energy loss in sensor: *divide a track into small steps and simulate energy losses in each of them using Urban model*[1] *;*
- drift of created charge carriers in planar electric field
- movement of e-h pairs in magnetic field (Lorentz shift)
- diffusion
- cross-talk due to interstrip capacitance
- modeling of the read-out chip

[1] K. Lassila-Perini and L. Urbán (1995)

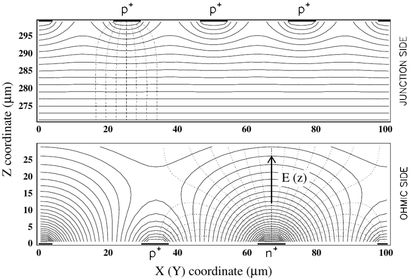


Energy losses of $2\,\mathrm{GeV}$ *protons in* $1\,\mu\mathrm{m}$ *of Si (solid line)*[2]*.*

[2] *H. Bichsel (1990)*

Detector response model:

- non-uniform energy loss in sensor
- drift of created charge carriers in planar electric field:
  non-uniformity of the electric field is negligible in 90% of the volume;
- movement of e-h pairs in magnetic field (Lorentz shift)
- diffusion
- cross-talk due to interstrip capacitance
- modeling of the read-out chip

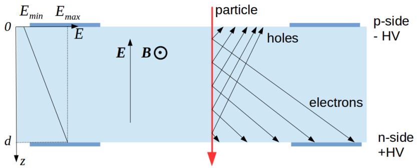


Calculated electric field for sensors with strip pitch $25.5\,\mu\mathrm{m}$ *on the p-side and* $66.5\,\mu\mathrm{m}$ *on the n-side*[1].

[1] *S. Straulino et al. (2006)*

Detector response model:

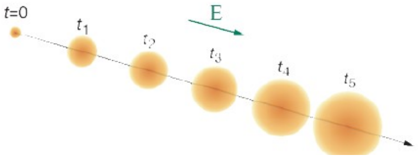
- non-uniform energy loss in sensor
- drift of created charge carriers in planar electric field
- movement of e-h pairs in magnetic field (Lorentz shift):
  taking into account the fact that Lorentz shift depends on the mobility, which depends on the electric field, which depends on the z-coordinate of charge carrier;
- diffusion
- cross-talk due to interstrip capacitance
- modeling of the read-out chip



Lorentz shift for electrons and holes in Si sensor.

Detector response model:

- non-uniform energy loss in sensor
- drift of created charge carriers in planar electric field
- movement of e-h pairs in magnetic field (Lorentz shift)
- diffusion:
  integration time is bigger than the drift time: estimate the increase of the charge carrier cloud during the whole drift time using Gaussian low;
- cross-talk due to interstrip capacitance
- modeling of the read-out chip



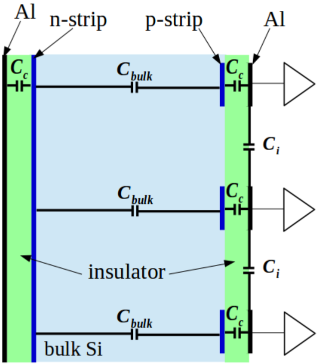
Increasing of charge cloud in time.

Detector response model:

- non-uniform energy loss in sensor
- drift of created charge carriers in planar electric field
- movement of e-h pairs in magnetic field (Lorentz shift)
- diffusion
- cross-talk due to interstrip capacitance:

$$Q_{\text{neib strip}} = \frac{Q_{\text{strip}} C_{\text{i}}}{C_{\text{c}} + C_{\text{i}}};$$

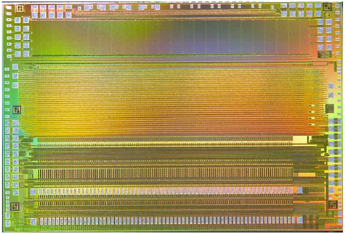
- modeling of the read-out chip



Simplified double-sided silicon microstrip detector layout.

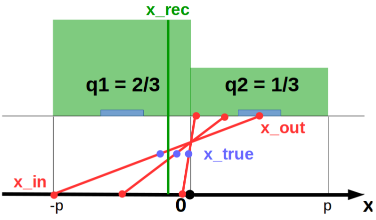
Detector response model:

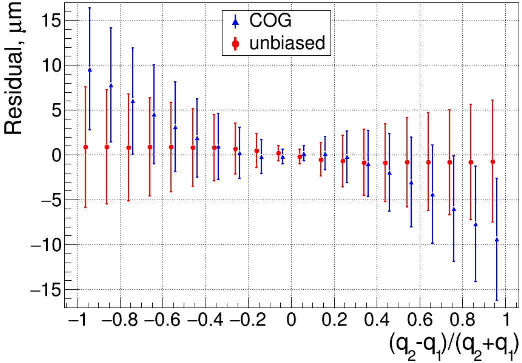
- non-uniform energy loss in sensor
- drift of created charge carriers in planar electric field
- movement of e-h pairs in magnetic field (Lorentz shift)
- diffusion
- cross-talk due to interstrip capacitance
- modeling of the read-out chip:
    - *noise: + Gaussian noise to the signal in fired strip;*
    - *threshold;*
    - *digitization of analog signal;*
    - *time resolution;*
    - *dead time.*



STS-XYTER read-out chip for the CBM Silicon Tracking System.

Residuals comparison for 2 CPFAs: 2-strip clusters





Ideal detector model & uniform energy loss.
Error bars: RMS of the residual distribution.
$q_{1,2}$ – measured charges on the strips.

Unbiased cluster position finding algorithm (CPFA), n-strip clusters

formula for **unifrom** energy loss:

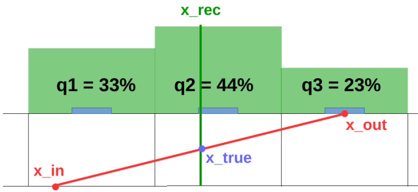
$$x_{\text{rec}} = 0.5\,(x_1 + x_n) + \frac{p}{2}\frac{q_n - q_1}{q},$$

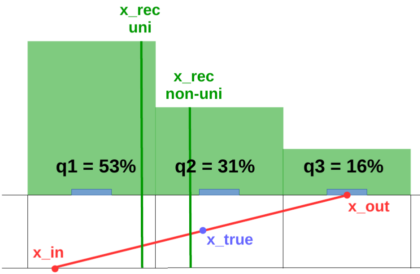
$$q = \frac{1}{n-2}\sum_{i=2}^{n-1} q_i;$$

formula for **non-uniform** energy loss (head-tail algorithm[1]):

$$x_{\text{rec}} = 0.5\,(x_1 + x_n) + \frac{p}{2}\frac{\min(q_n, q) - \min(q_1, q)}{q},$$

[1] R. Turchetta, "Spatial resolution of silicon microstrip detectors", 1993





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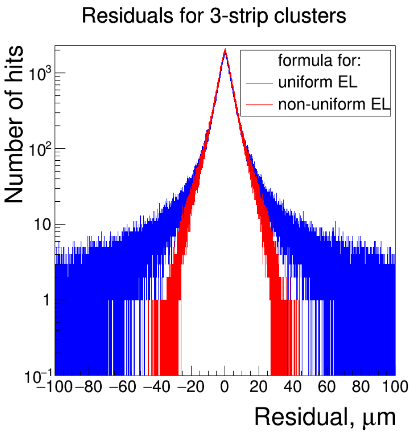

Estimation of hit position error

Hit position error: $\sigma^2 = \sigma^2_{\text{alg}} + \sum_i \left(\frac{\partial x_{\text{rec}}}{\partial q_i}\right)^2 \sum_{\text{sources}} \sigma_j^2,$

σ_{alg} – an error of the unbiased CPFA:

$$\sigma_1 = \frac{p}{\sqrt{24}}, \qquad \sigma_2 = \frac{p}{\sqrt{72}} \frac{|q_2 - q_1|}{\max(q_1, q_2)}, \qquad \sigma_{n>2} = 0.$$

σ_j – errors of the charge registration at one strip, among them already included:

- $\sigma_{\text{noise}} = \text{Equivalent Noise Charge};$
- $\sigma_{\text{discr}} = \frac{\text{dynamic range}}{\sqrt{12}\ \text{number of ADC}};$
- $\sigma_{\text{non-uni}}$ is estimated assuming:
  - registered charge corresponds to the most probable value of the energy loss;
  - incident particle is ultrarelativistic ($\beta\gamma \gtrsim 100$).
- σ_{diff} is negligible in comparison with other effects.

Error due to non-uniform energy loss

The contribution from the non-uniformity of energy loss is more difficult to take into account because the actual energy deposit along the track is not known. The following approximations allow a straightforward solution:

- the registered charge corresponds to the most probable value (MPV) of energy loss;
- the incident particle is ultrarelativistic ($\beta\gamma \gtrsim 100$).

The second assumption is very strong but it uniquely relates the MPV and the distribution width (Particle Data Group)

$$MPV = \xi[\mathrm{eV}] \times \left(\ln\left(1.057 \times 10^6 \xi[\mathrm{eV}]\right) + 0.2 \right).$$

Solving this with respect to ξ gives the estimate for the FWHM (S. Merolli, D. Passeri and L. Servoli, Journal of Instrumentation, Volume 6, 2011)

$$\sigma_{\mathrm{non}} = w/2 = 4.018\xi/2.$$

1-strip clusters: why not $\sigma_{method} = p/\sqrt{12}$?

In general, for **all** track inclinations:

- $N = \int\limits_{x_{in}} \int\limits_{x_{out}} P_1(x_{in}, x_{out}) dx_{in} dx_{out} = p^2$;
- $\sigma^2 = \frac{1}{N} \int\limits_{x_{in}} \int\limits_{x_{out}} P_1(x_{in}, x_{out}) dx_{in} dx_{out} \Delta x^2 = \frac{p^2}{24}$.

Particullary, for **perpendicular** tracks: $x_{in} = x_{out}$

- $N = \int\limits_{x_{in}} P_1(x_{in}, x_{out}) dx_{in} = p$;
- $\sigma^2 = \frac{1}{N} \int\limits_{x_{in}} P_1(x_{in}, x_{out}) dx_{in} \Delta x^2 = \frac{p^2}{12}$