

# From Nuclear Forces To Nuclei

EMMI Physics Day



TECHNISCHE  
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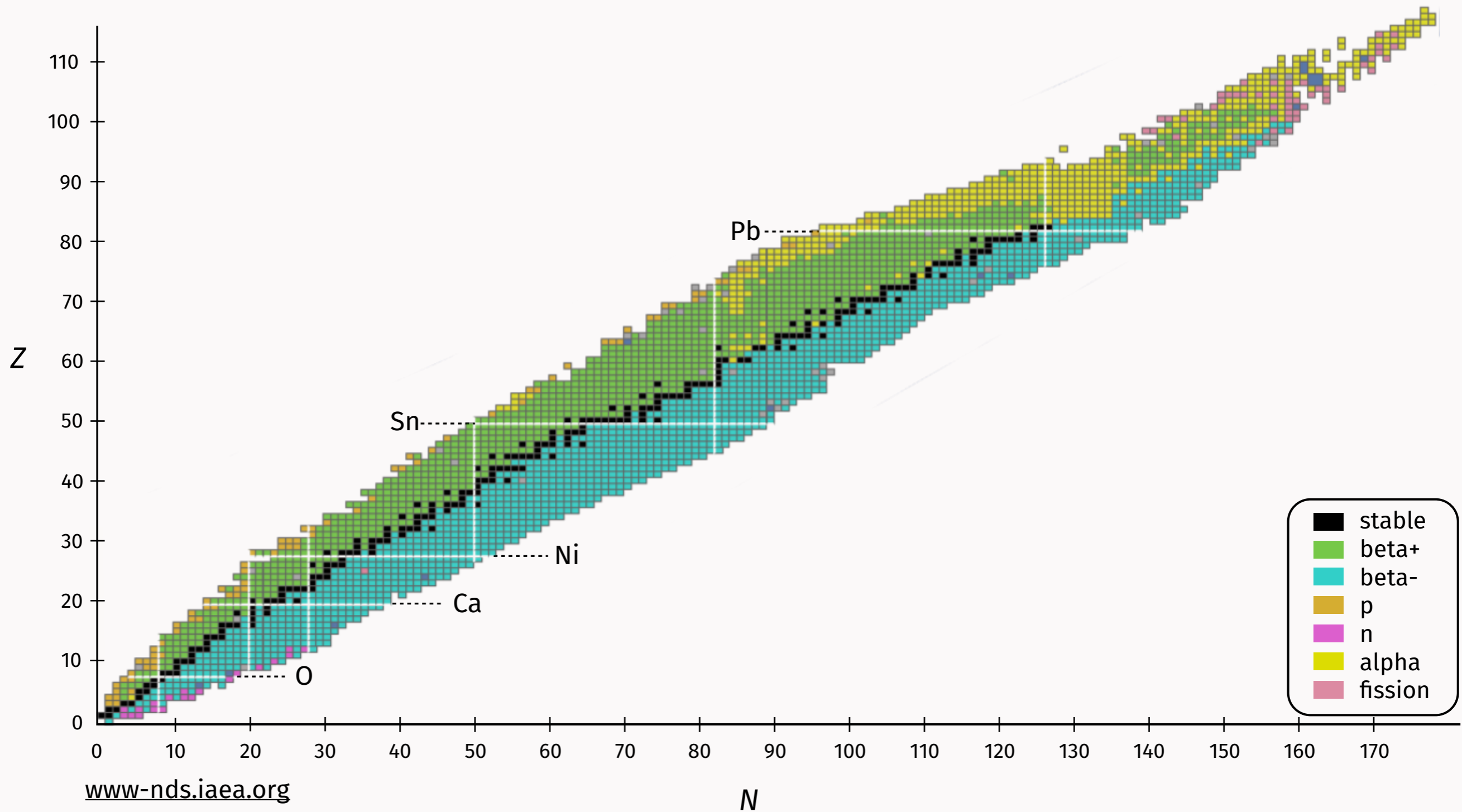
Joel E. Lynn

November 28, 2017

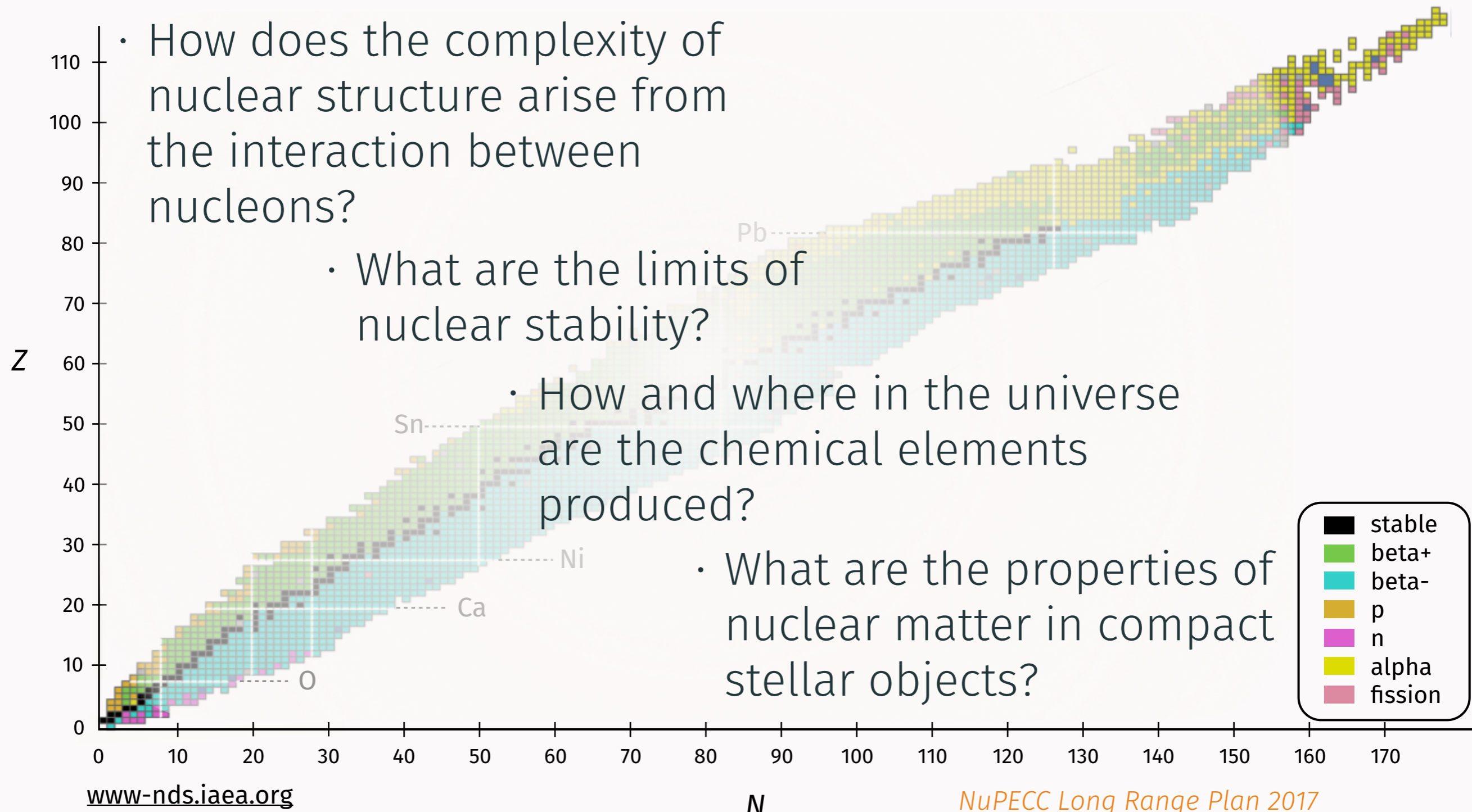
# Motivation

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# The Nuclear Landscape



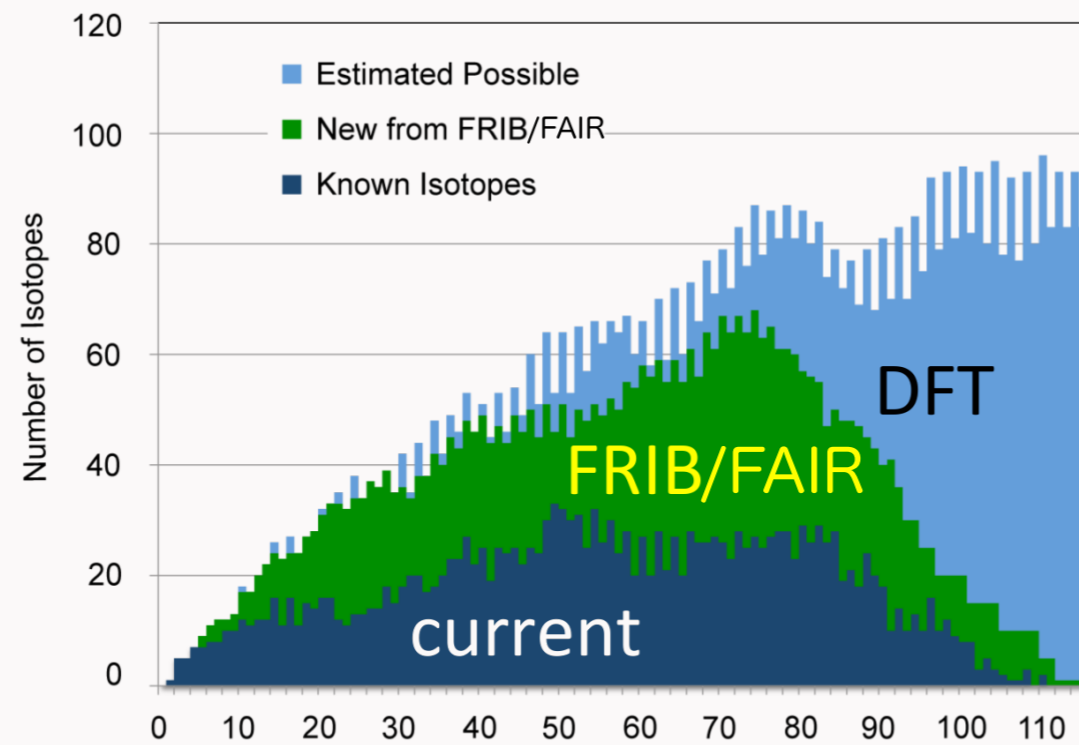
# The Nuclear Landscape



# Extending The Nuclear Landscape

Cutting-edge experimental results

Rare-isotope facilities



adapted from A. B. Balentekin et al.,  
Mod. Phys. Lett. A **29**, 1430010 (2014)

Neutron-star mergers

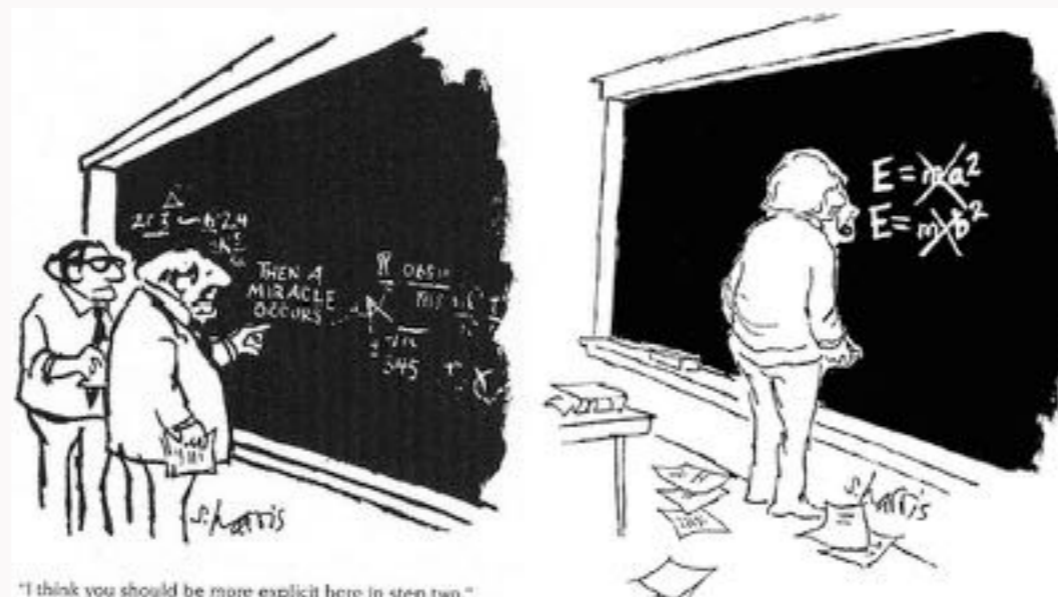


adapted from M. McLaughlin, APS  
Physics Viewpoint, October 16, (2017)

# What Can Theory Offer?

Nuclear theory has experienced a renaissance in the past few decades thanks (in part) to two developments.

1. Advances in *ab initio* many-body methods.
2. Chiral effective field theory (EFT) for nuclear interactions.



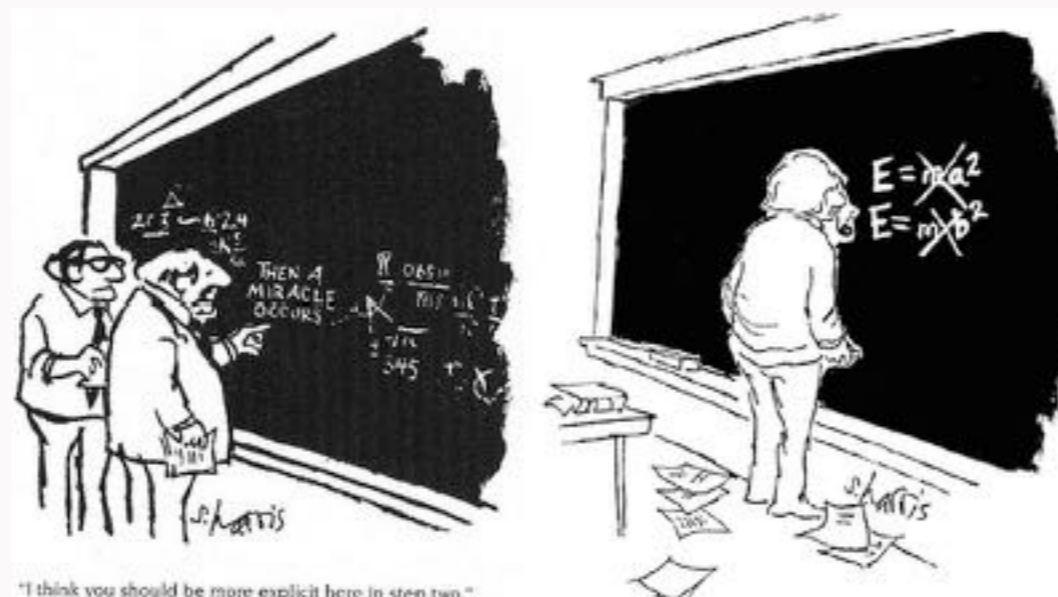
"I think you should be more explicit here in step two."

# What Can Theory Offer?

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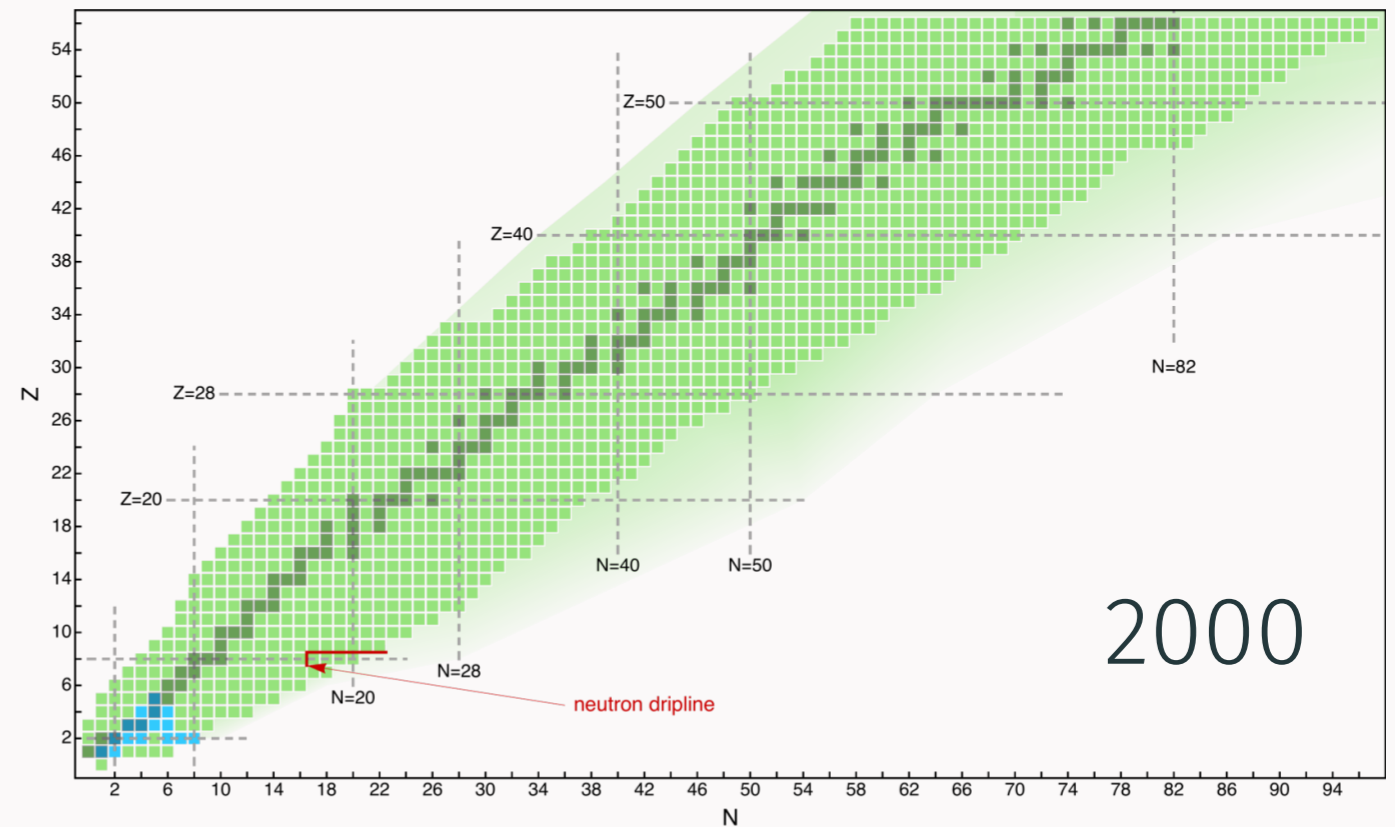
1. Advances in *ab initio* many-body methods.
2. Chiral effective field interactions.

work with protons + neutrons  
&  
controlled approximations

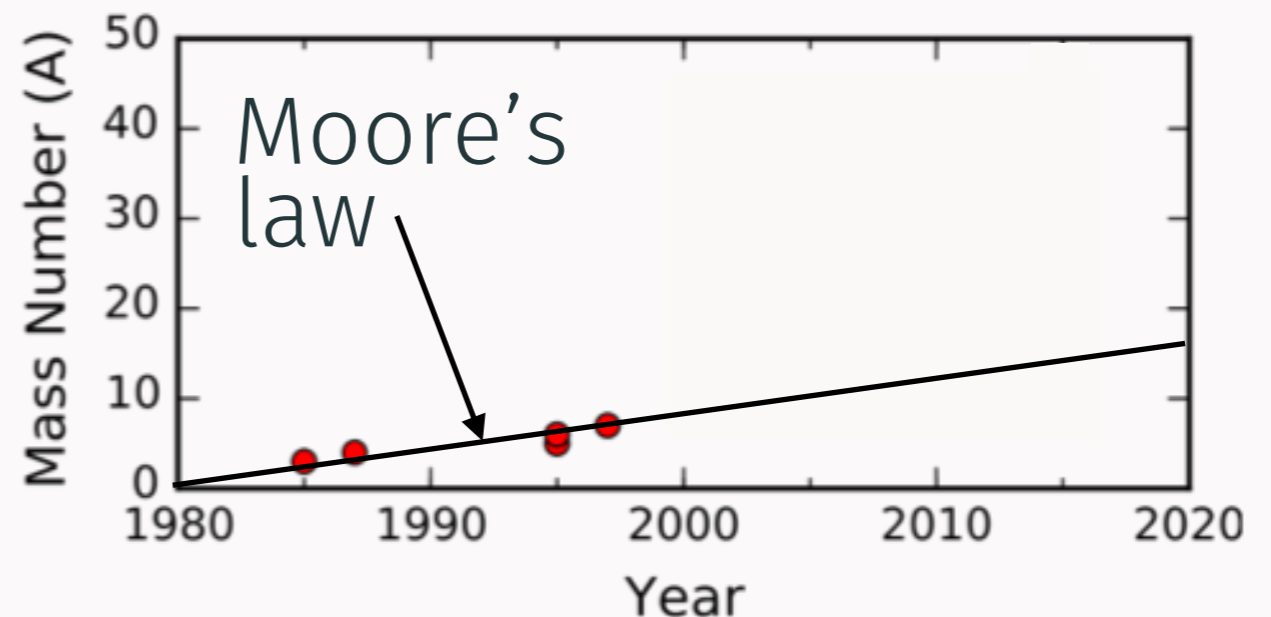


# Reach Of *Ab Initio* Methods

- 1980s & 1990s:  
Exact methods (exponential scaling) e.g. *Green's Function Monte Carlo Method (GFMC)*, *No-Core Shell Model*. Limited by Moore's law -  $A < 10, 12$
- 2000s and beyond:  
New methods (polynomial scaling) e.g. *Coupled cluster*, *auxiliary-field diffusion Monte Carlo (AFDMC)*. Closed-shell nuclei around up to  $A = 40$ .



from H. Hergert et al., Phys. Rep. **621**, 165 (2016)

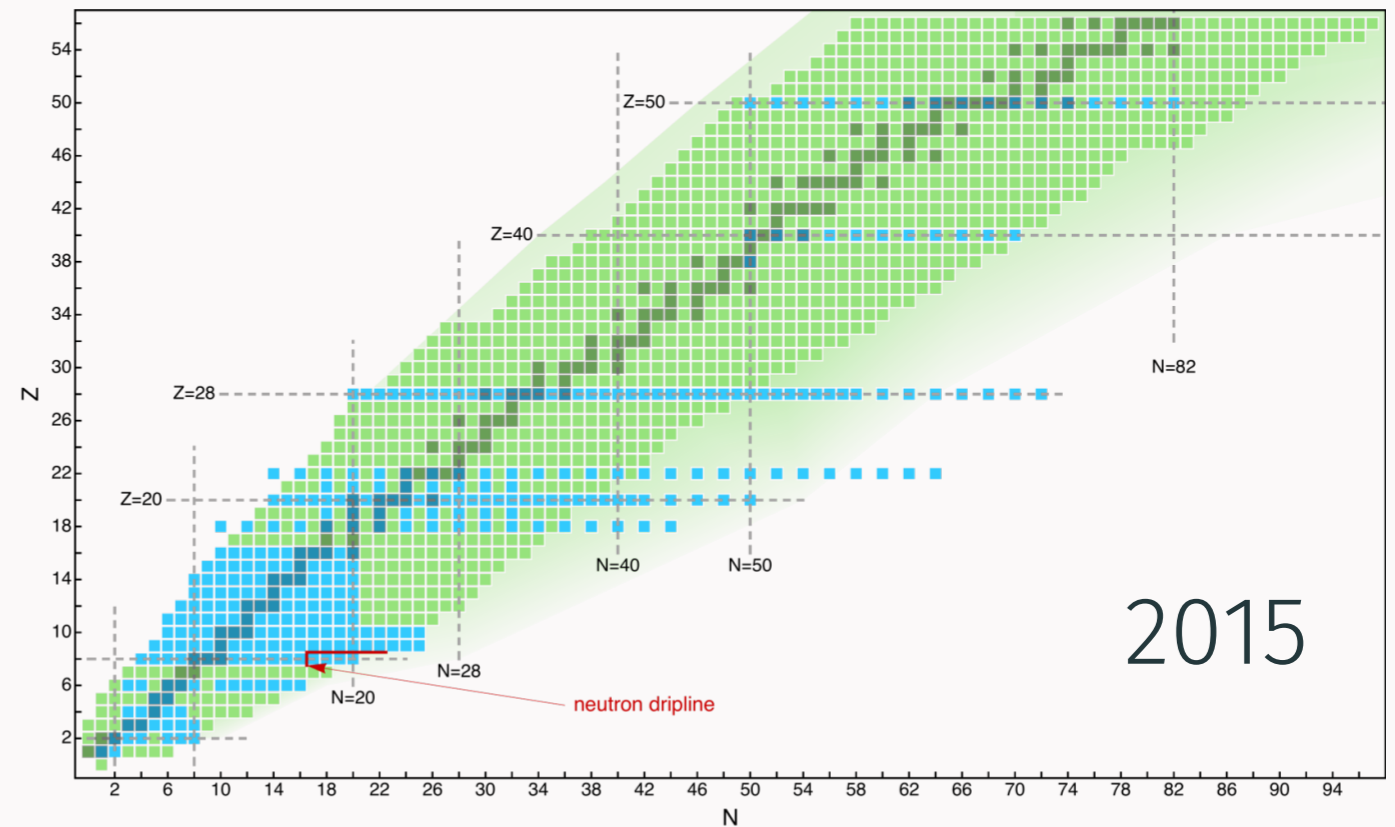


adapted from from G. Hagen et al., Nat. Phys. **12**, 186 (2016)

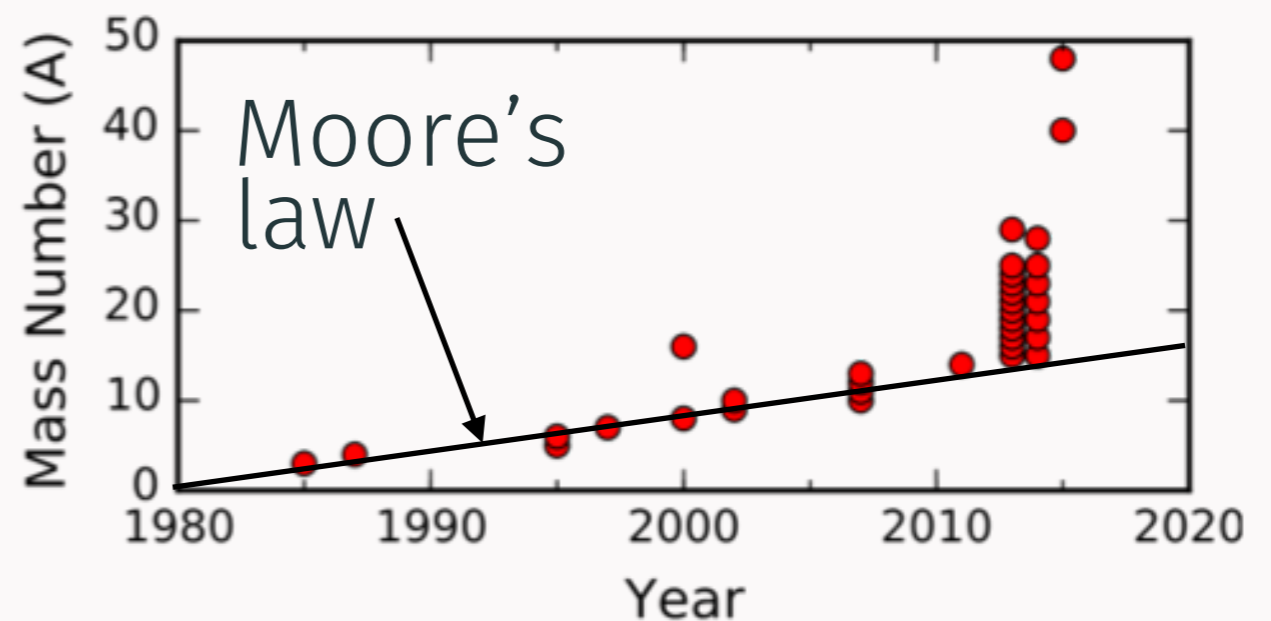


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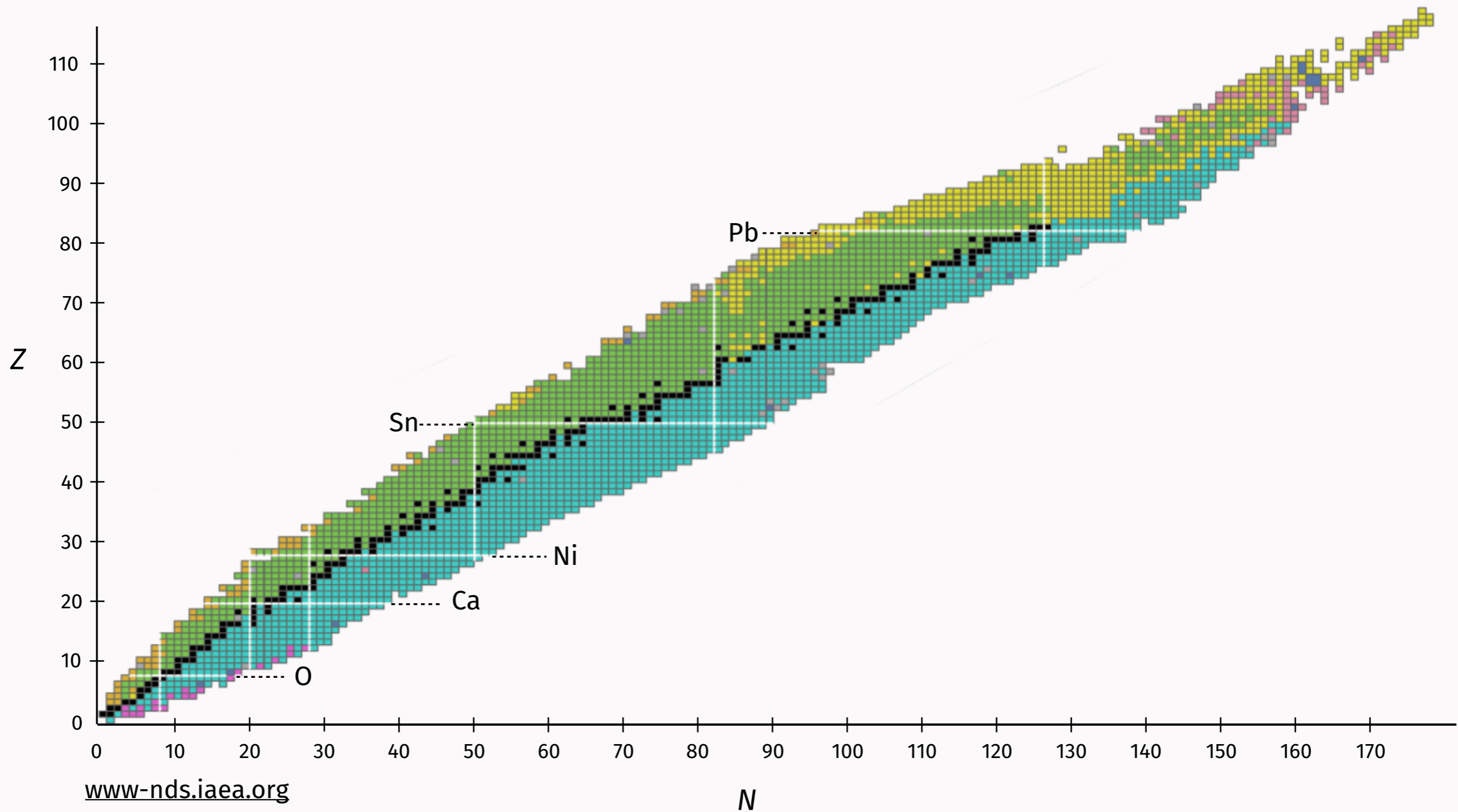


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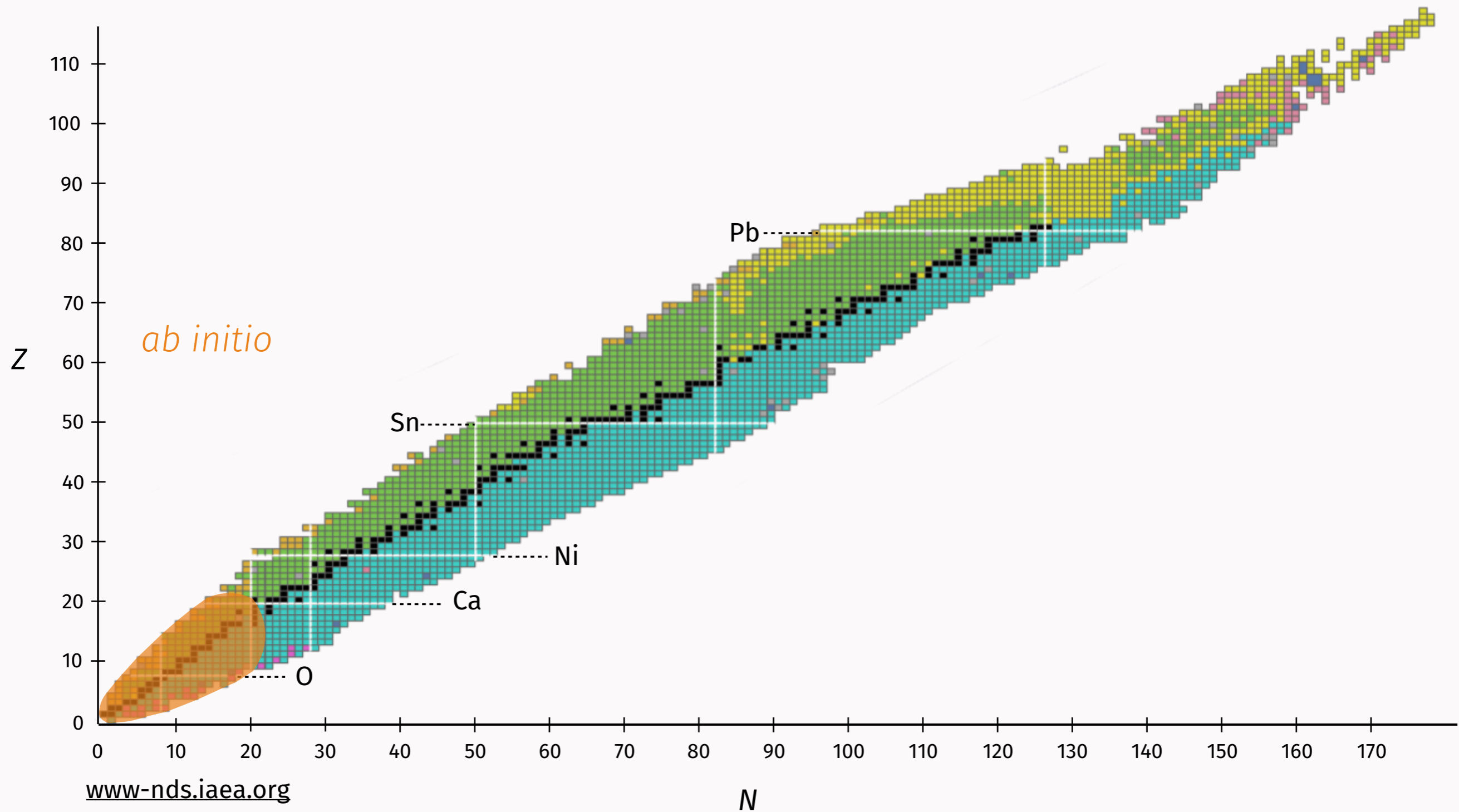


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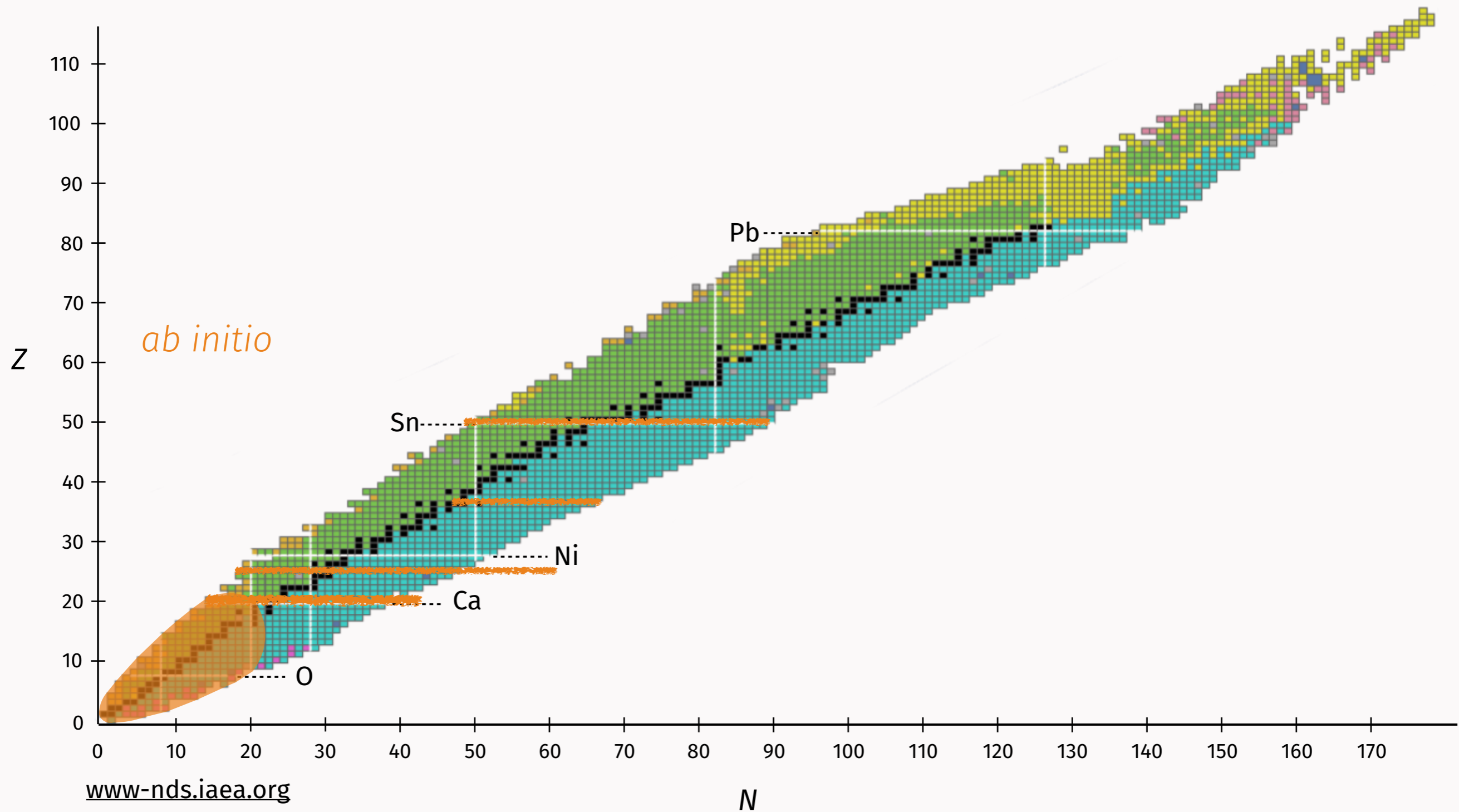
# The Nuclear Landscape



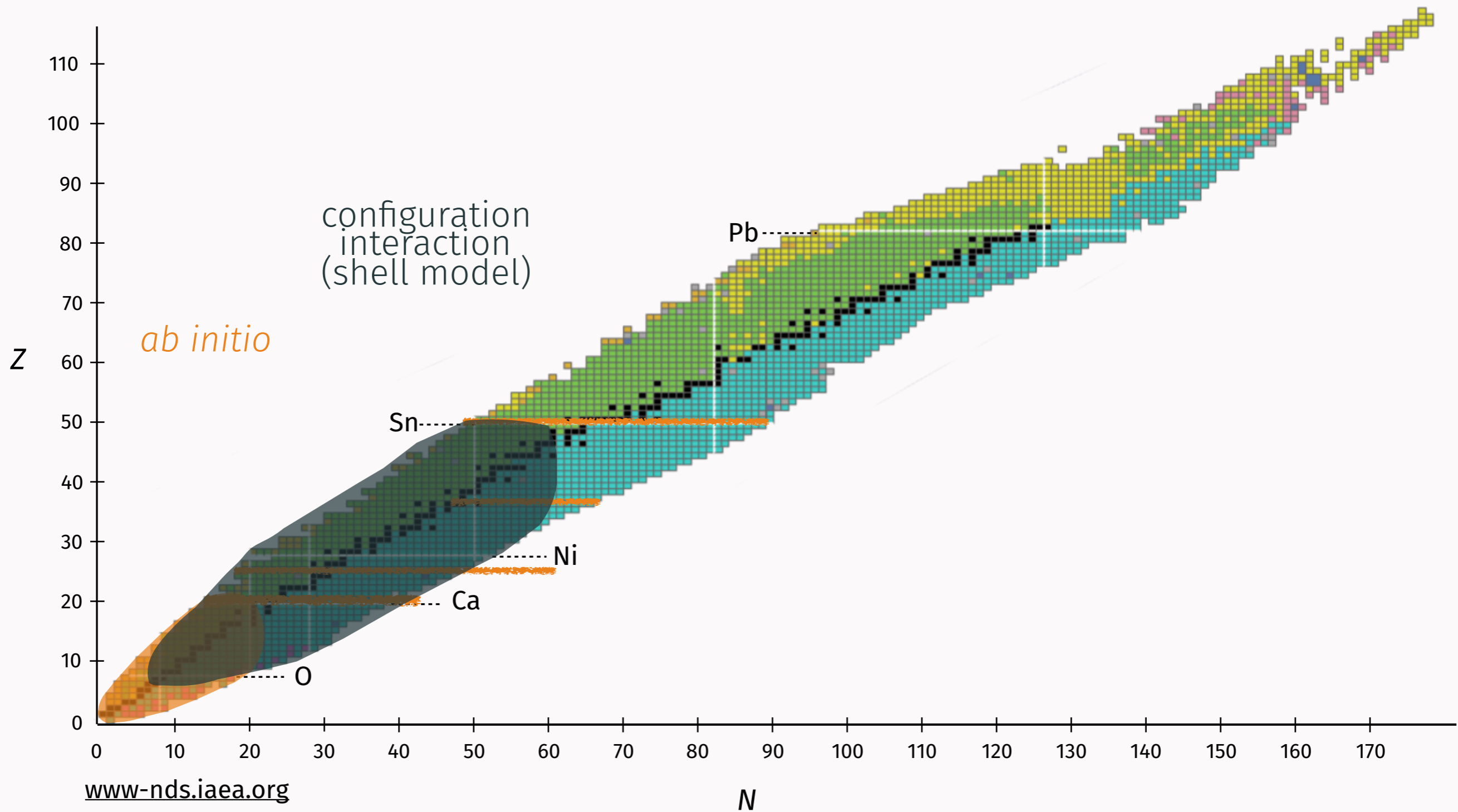
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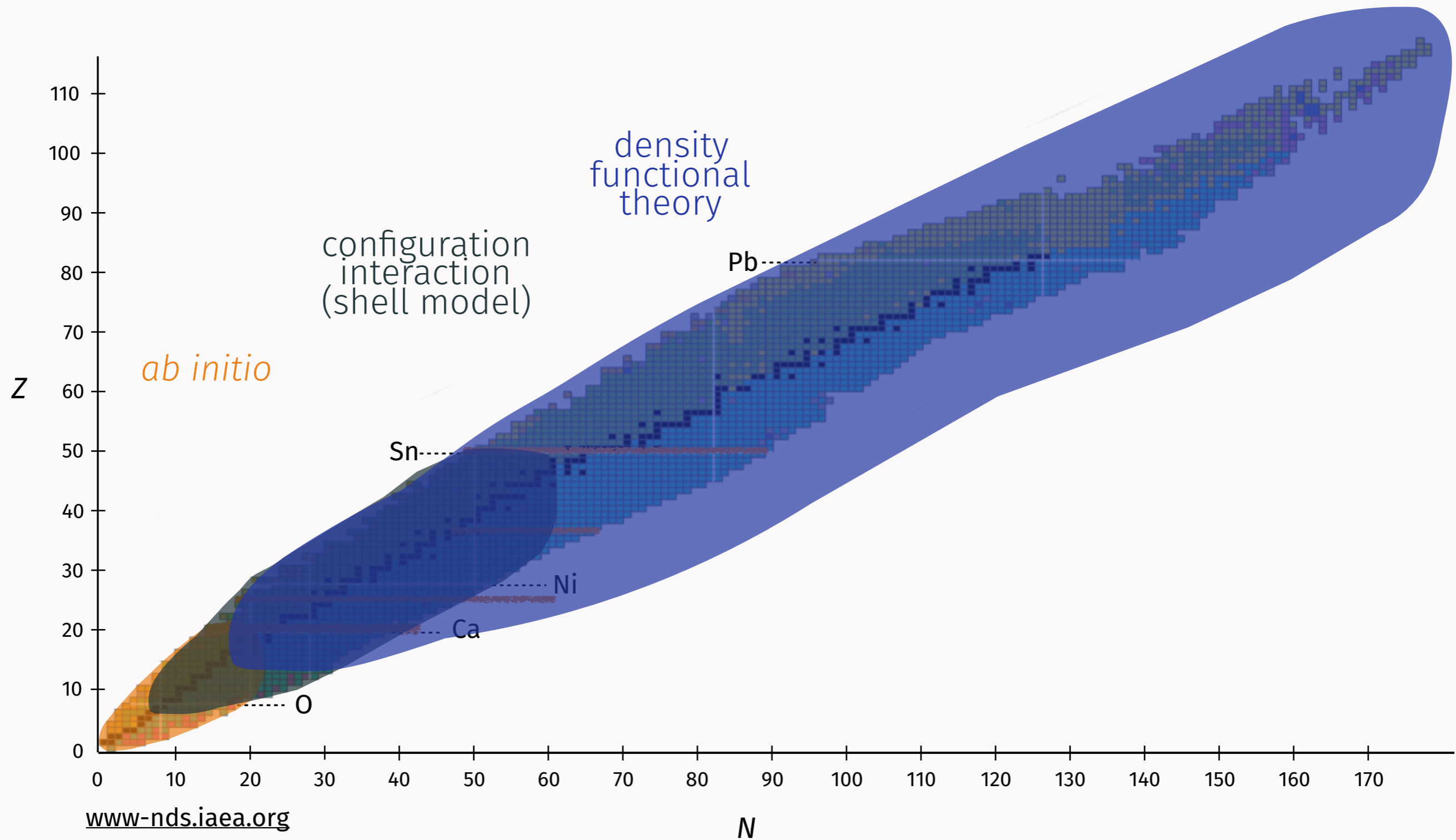
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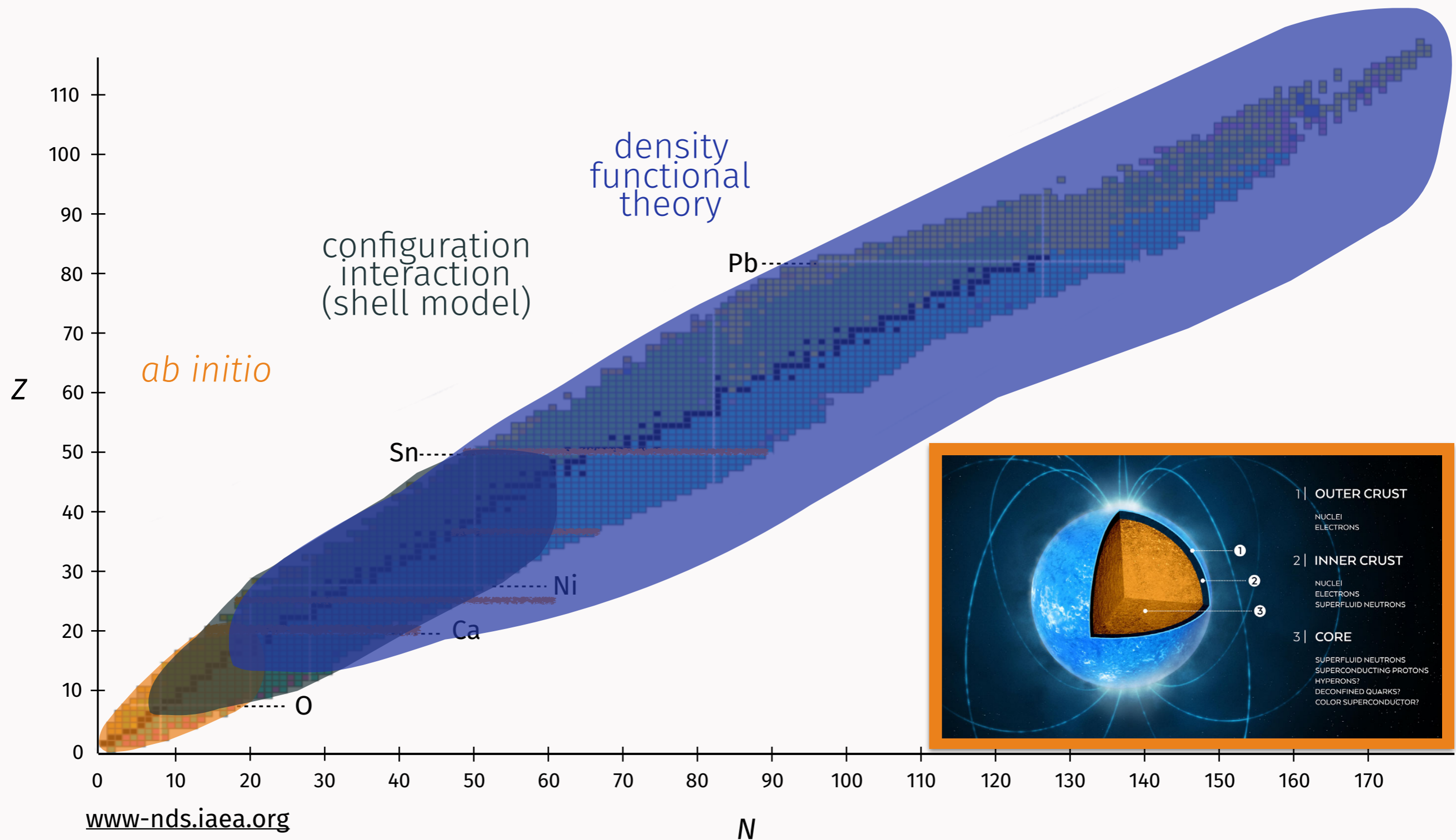
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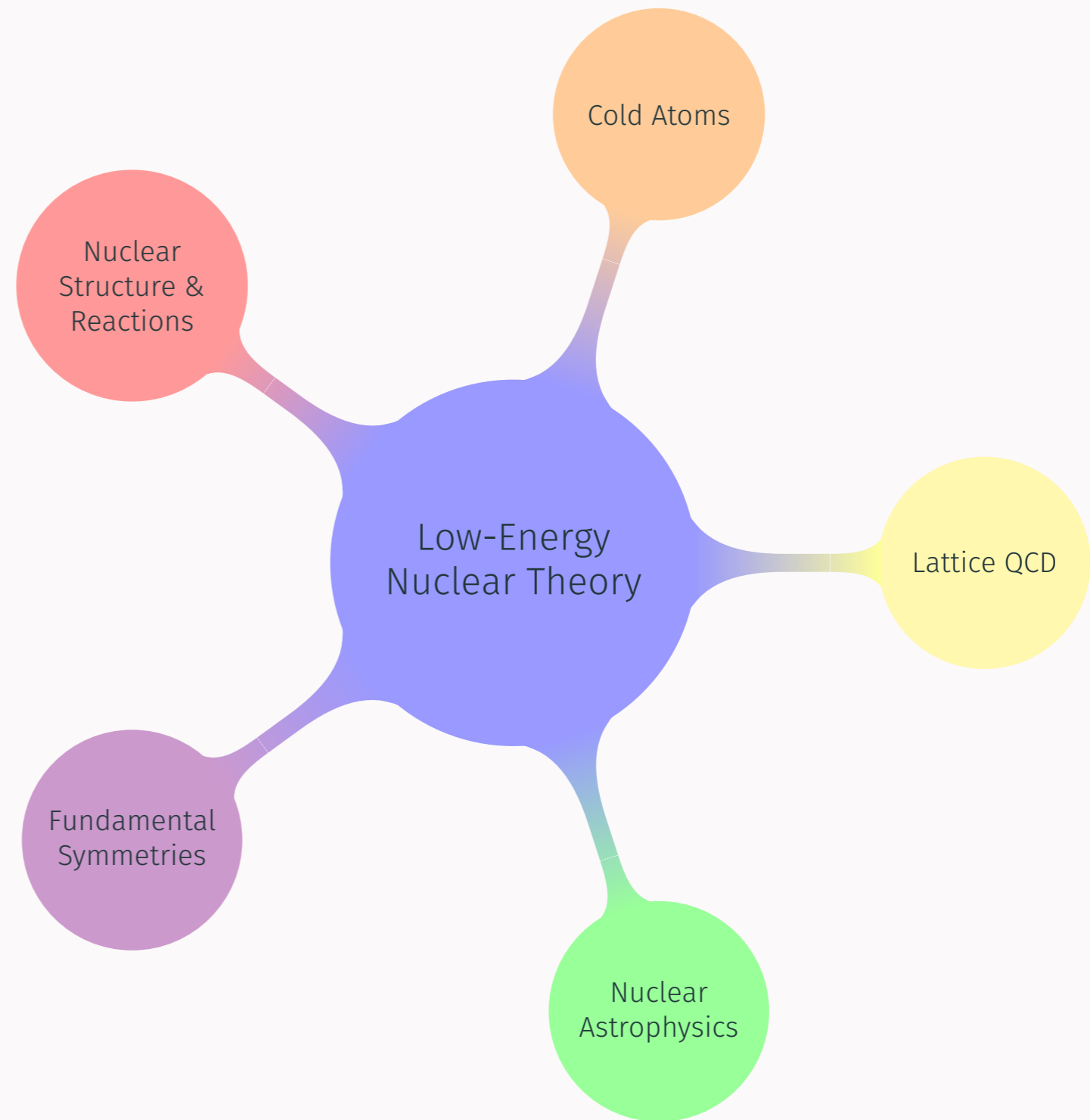


# The Nuclear Landscape



# Motivation

Low-energy nuclear theory sits in a privileged position, connecting many research areas.

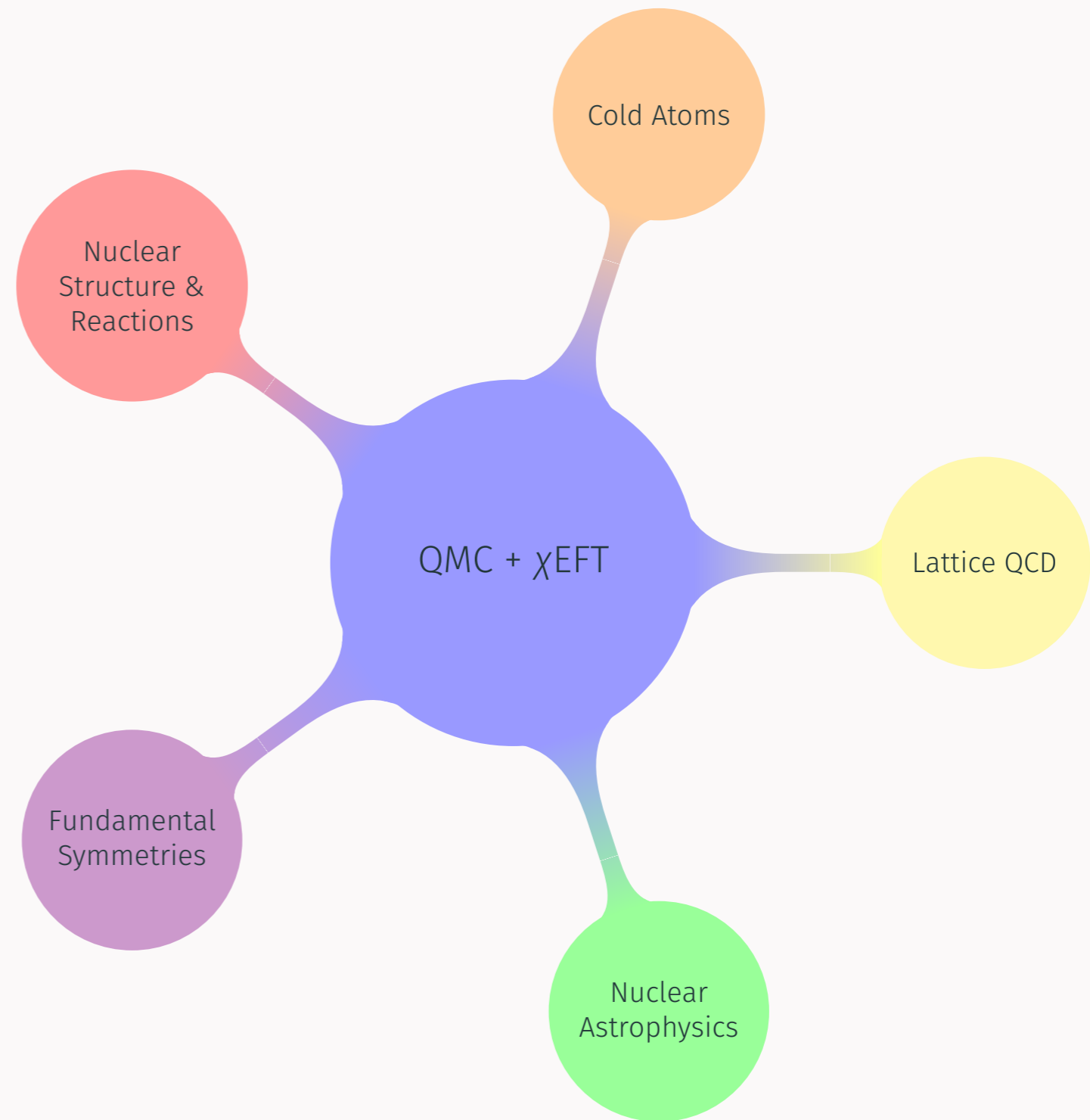




# Motivation

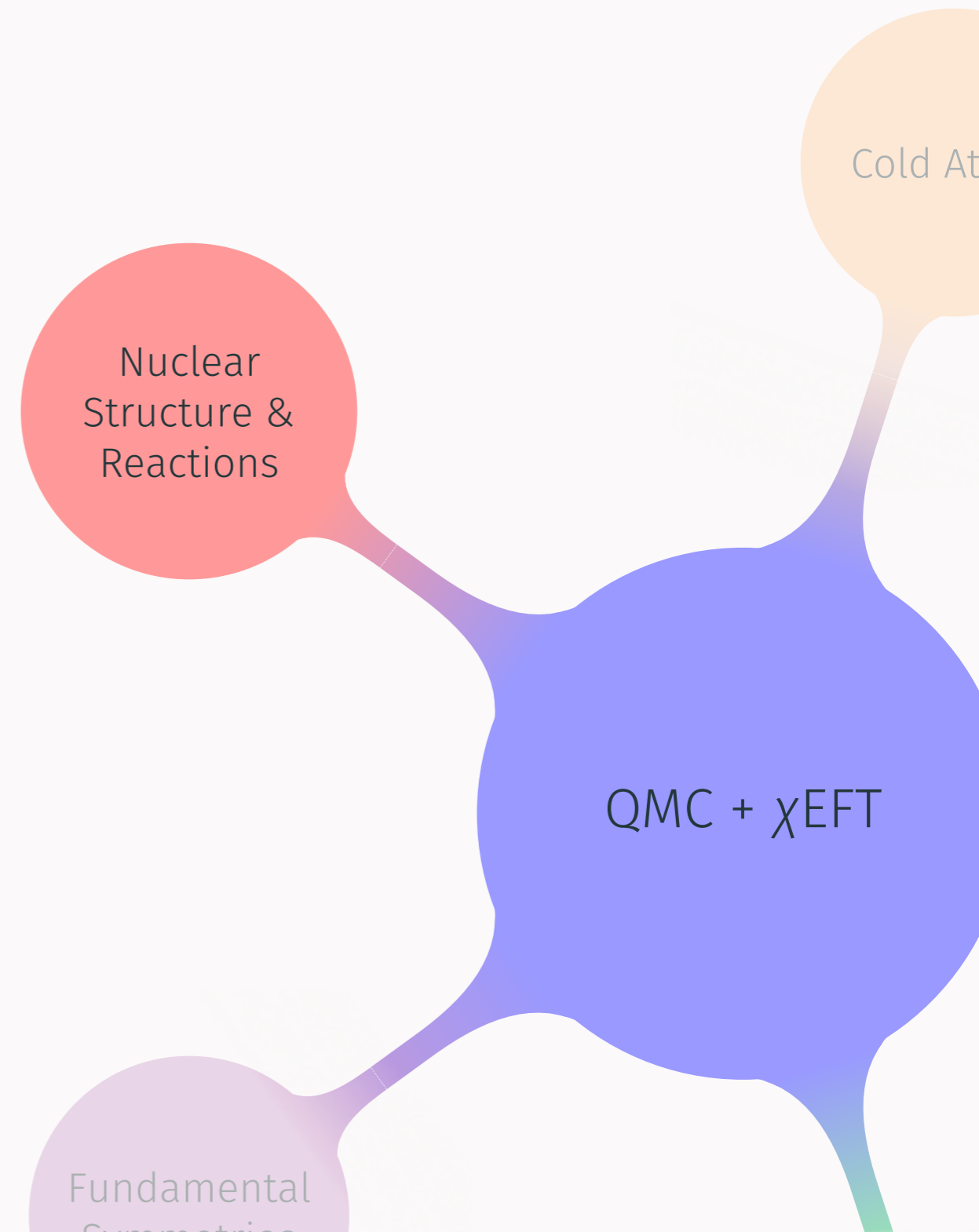
Low-energy nuclear theory sits in a privileged position, connecting many research areas.

Quantum Monte Carlo (QMC) methods with chiral effective field theory ( $\chi$ EFT) interactions is a compelling piece of the puzzle!



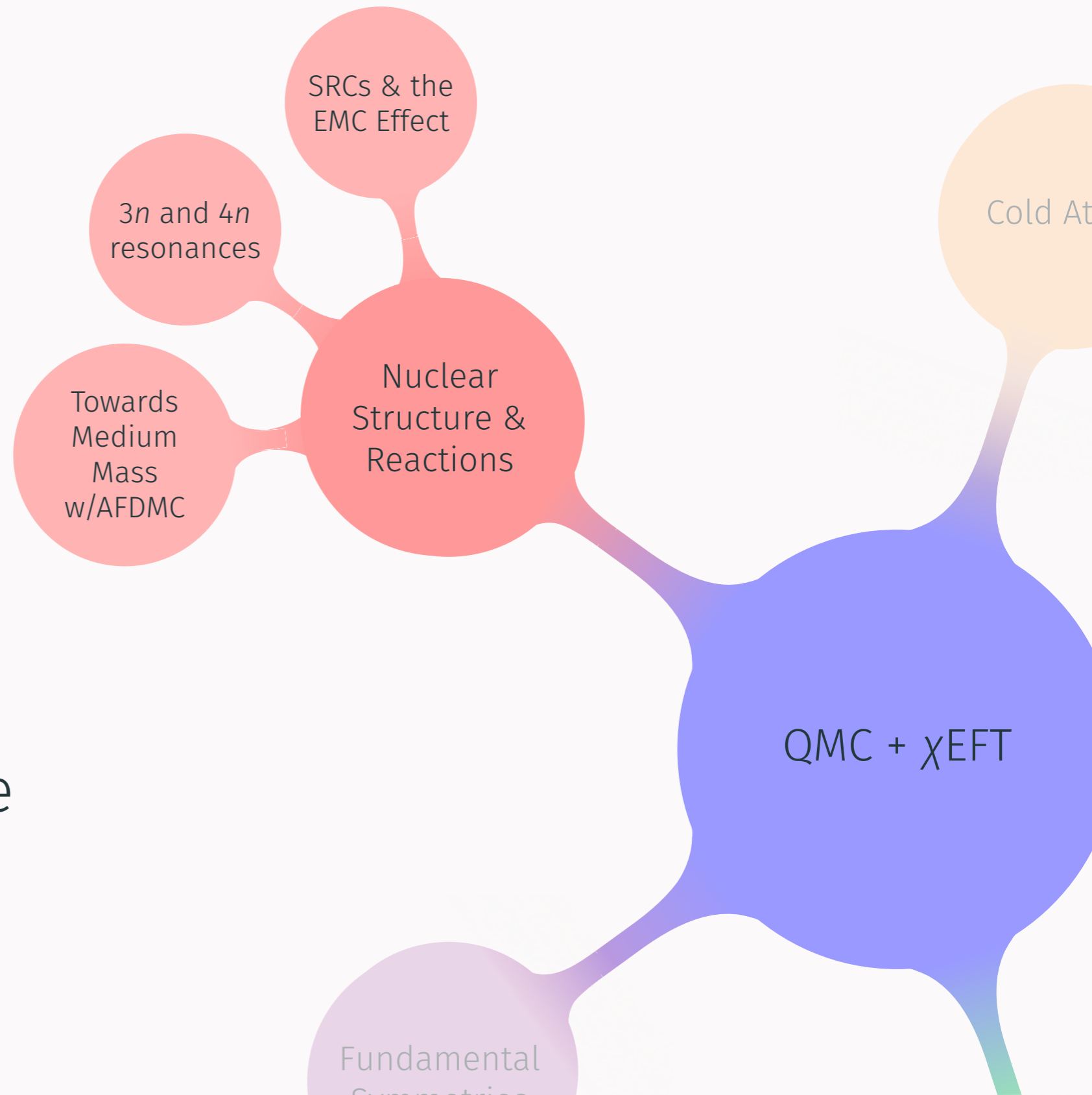
# Outline

- Quantum Monte Carlo Methods
- Chiral EFT



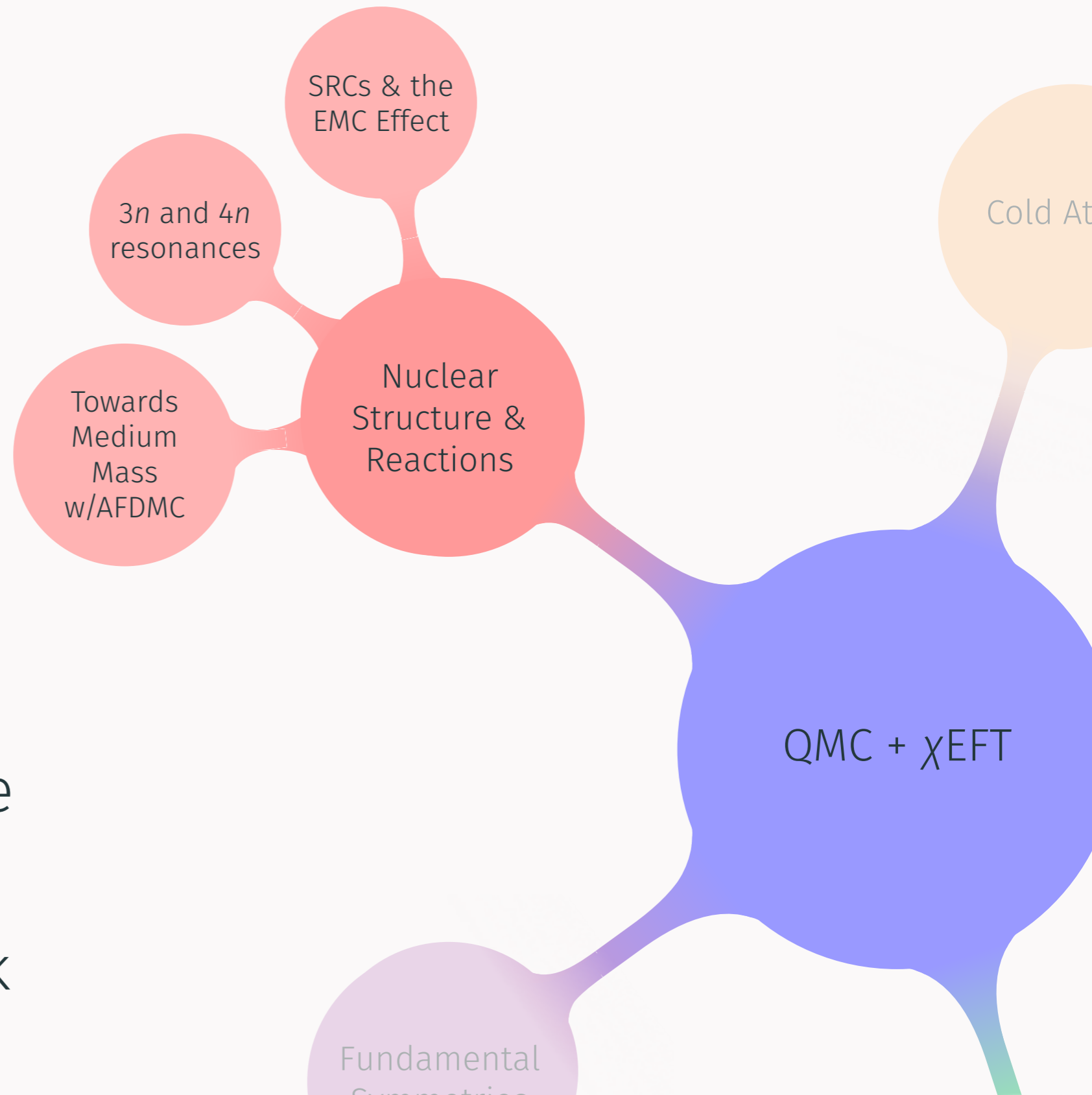
# Outline

- Quantum Monte Carlo Methods
- Chiral EFT
- Towards medium-mass nuclei with the AFDMC method
- $3n$  &  $4n$  resonances
- Short-range correlations and the EMC effect



# Outline


- Quantum Monte Carlo Methods
- Chiral EFT
- Towards medium-mass nuclei with the AFDMC method
- $3n$  &  $4n$  resonances
- Short-range correlations and the EMC effect
- Summary & Outlook



# Quantum Monte Carlo (QMC) Methods

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# QMC Methods - Variational Monte Carlo (VMC) Method

1. Start with a trial wave function  $\Psi_T$  and generate a random position:  $\mathbf{R} = \mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A$ .
2. Metropolis algorithm: Generate new positions  $\mathbf{R}'$  based on the probability  $P = \frac{|\Psi_T(\mathbf{R}')|^2}{|\Psi_T(\mathbf{R})|^2}$ .  $\rightarrow$  
3. Invoke the variational principle:  $E_T = \frac{\langle \Psi_T | H | \Psi_T \rangle}{\langle \Psi_T | \Psi_T \rangle} > E_0$ .

# QMC Methods - Diffusion Monte Carlo Method

- The wave function is imperfect:  $|\Psi_T\rangle = \sum_{i=0}^{\infty} \alpha_i |\Psi_i\rangle$  .
- Propagate in imaginary time to project out the ground state  $|\Psi_0\rangle$  .

$$\begin{aligned} |\Psi(\tau)\rangle &= e^{-(H-E_T)\tau} |\Psi_T\rangle \\ &= e^{-(E_0-E_T)\tau} \left[ \alpha_0 |\Psi_0\rangle + \sum_{i \neq 0} \alpha_i e^{-(E_i-E_0)\tau} |\Psi_i\rangle \right]. \end{aligned}$$

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$$|\Psi(\tau)\rangle \xrightarrow{\tau \rightarrow \infty} |\Psi_0\rangle.$$



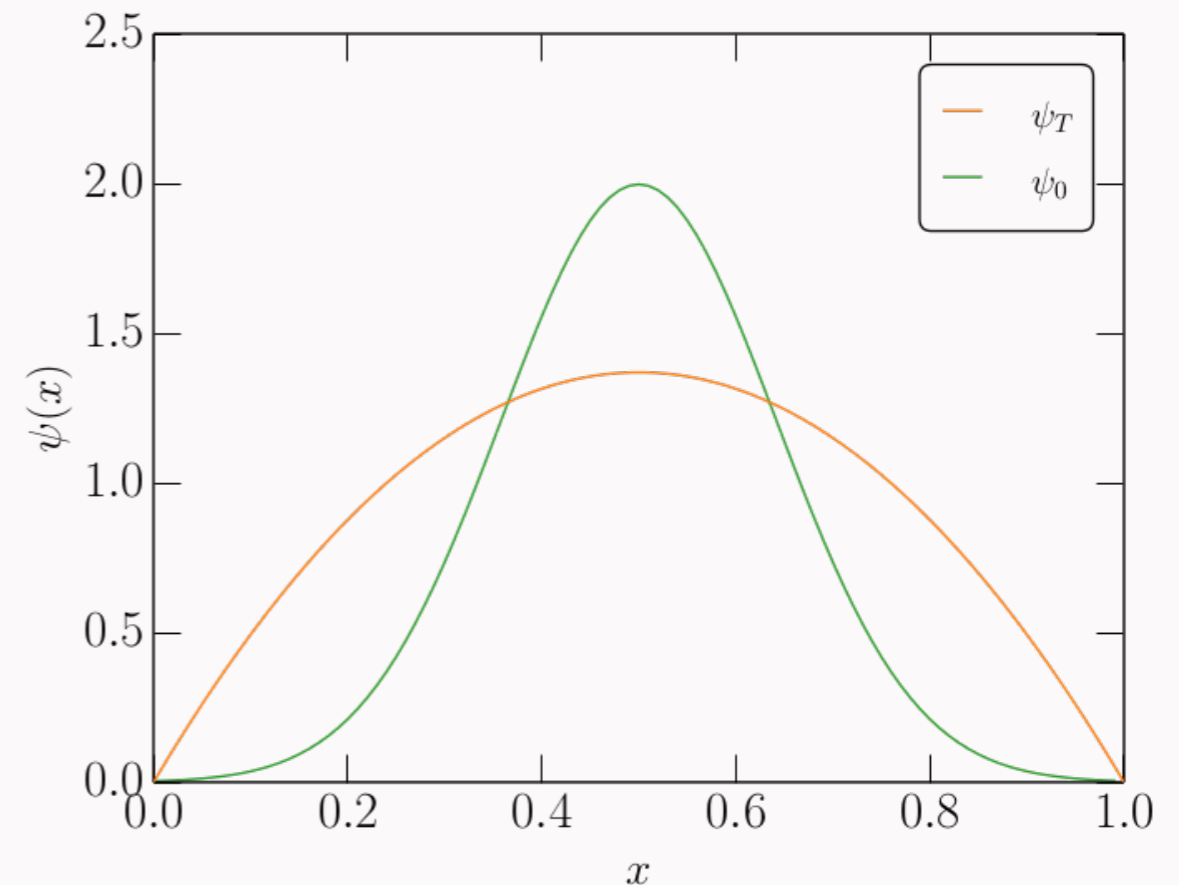
# QMC Methods - An Example

$$H = \frac{p^2}{2} + \frac{1}{2}\omega^2 x^2$$

$$\psi_0(x) = \left(\frac{\omega}{\pi}\right)^{1/4} e^{-\omega x^2/2}$$

Trial wave function; e.g.

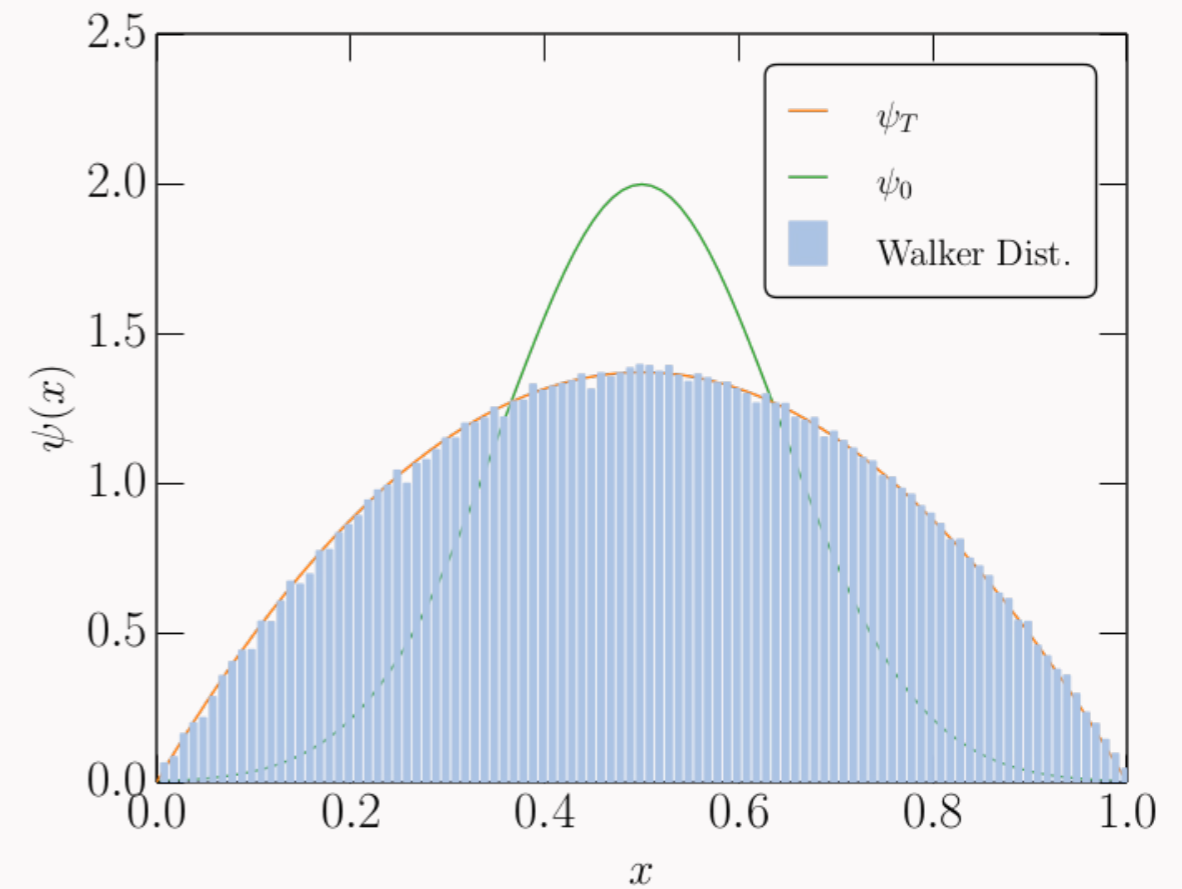
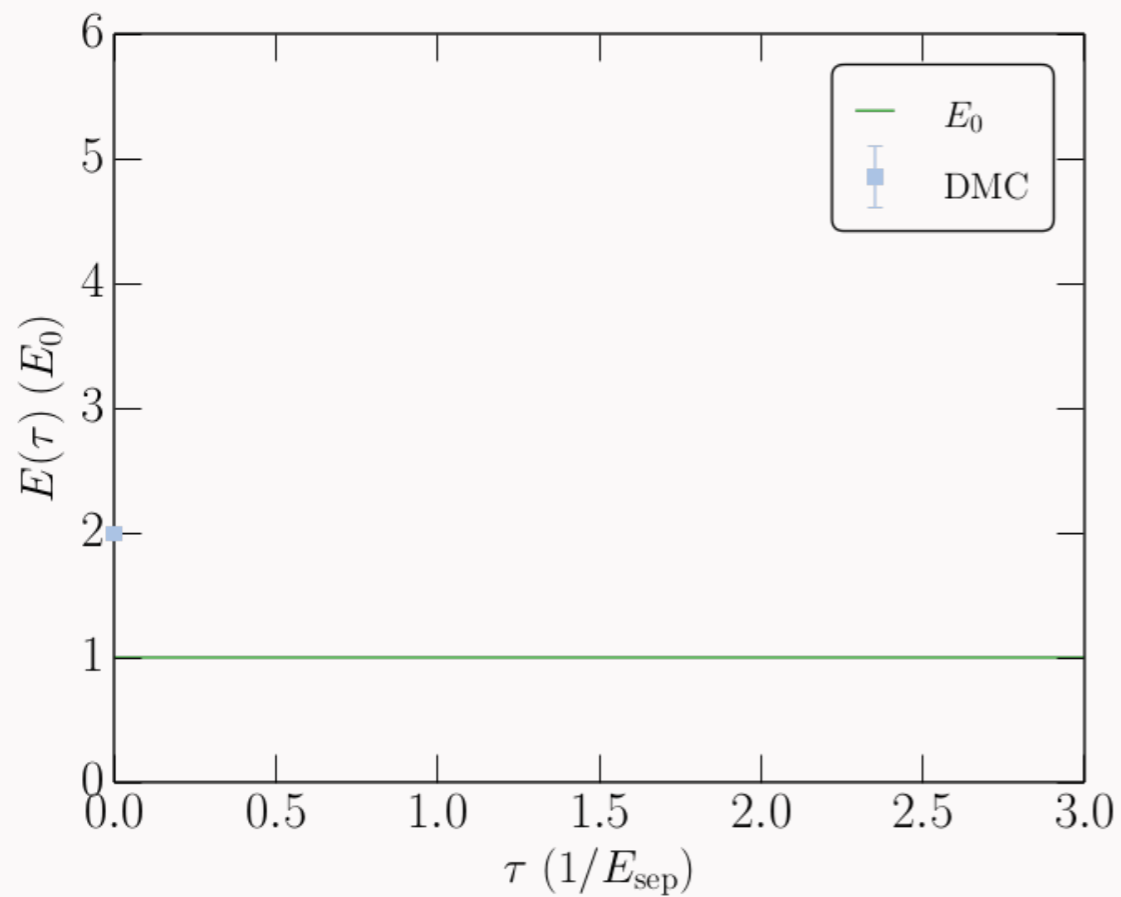
$$\Psi_T(x) = \sqrt{30}x(1-x).$$



# QMC Methods - An Example

Imaginary-time evolution:

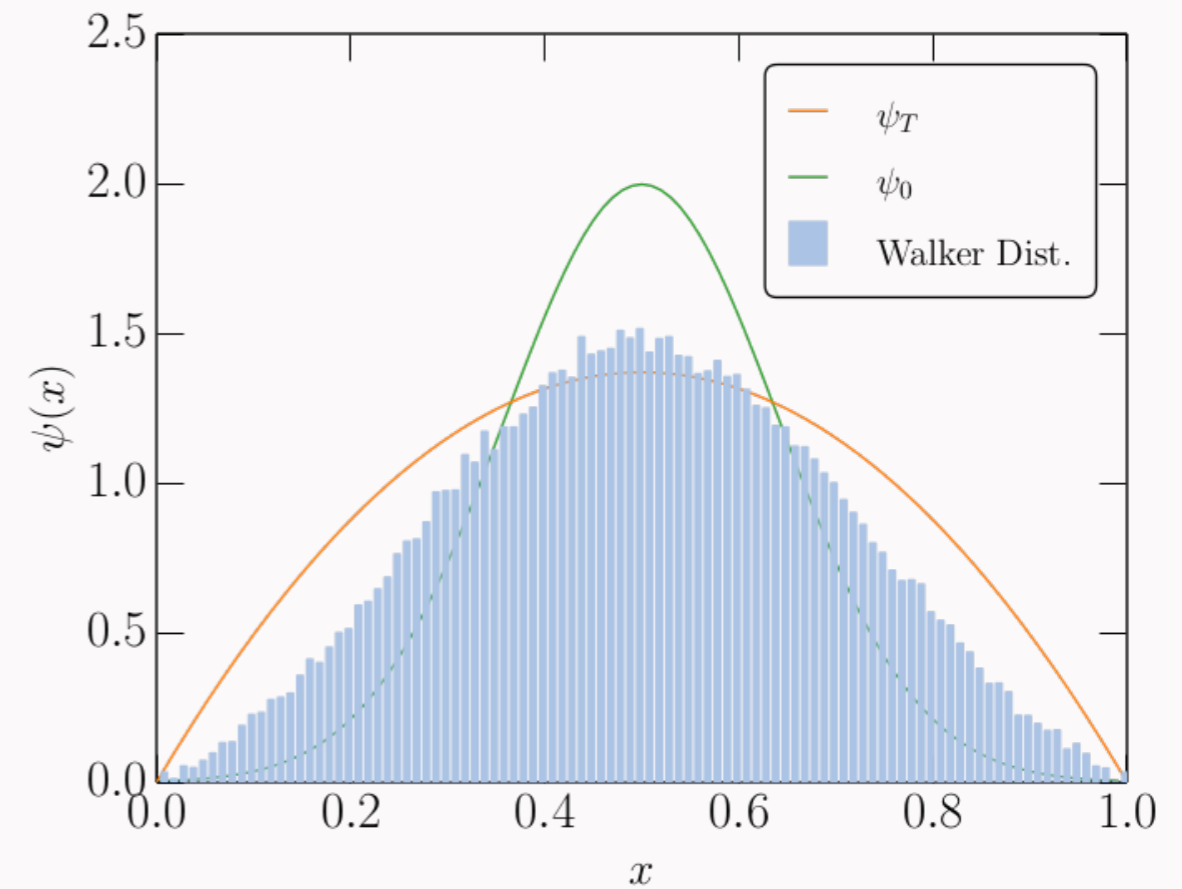
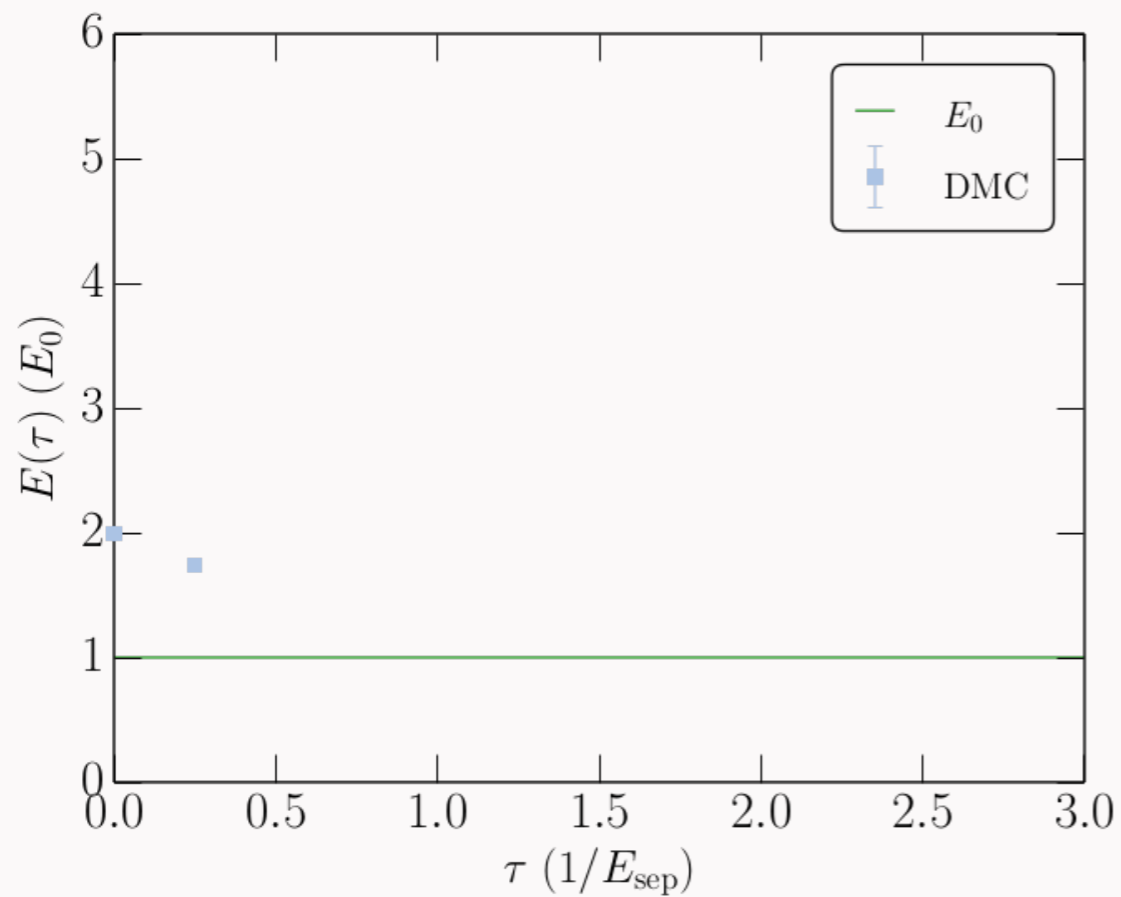
$$\tau = 0.00$$



# QMC Methods - An Example

Imaginary-time evolution:

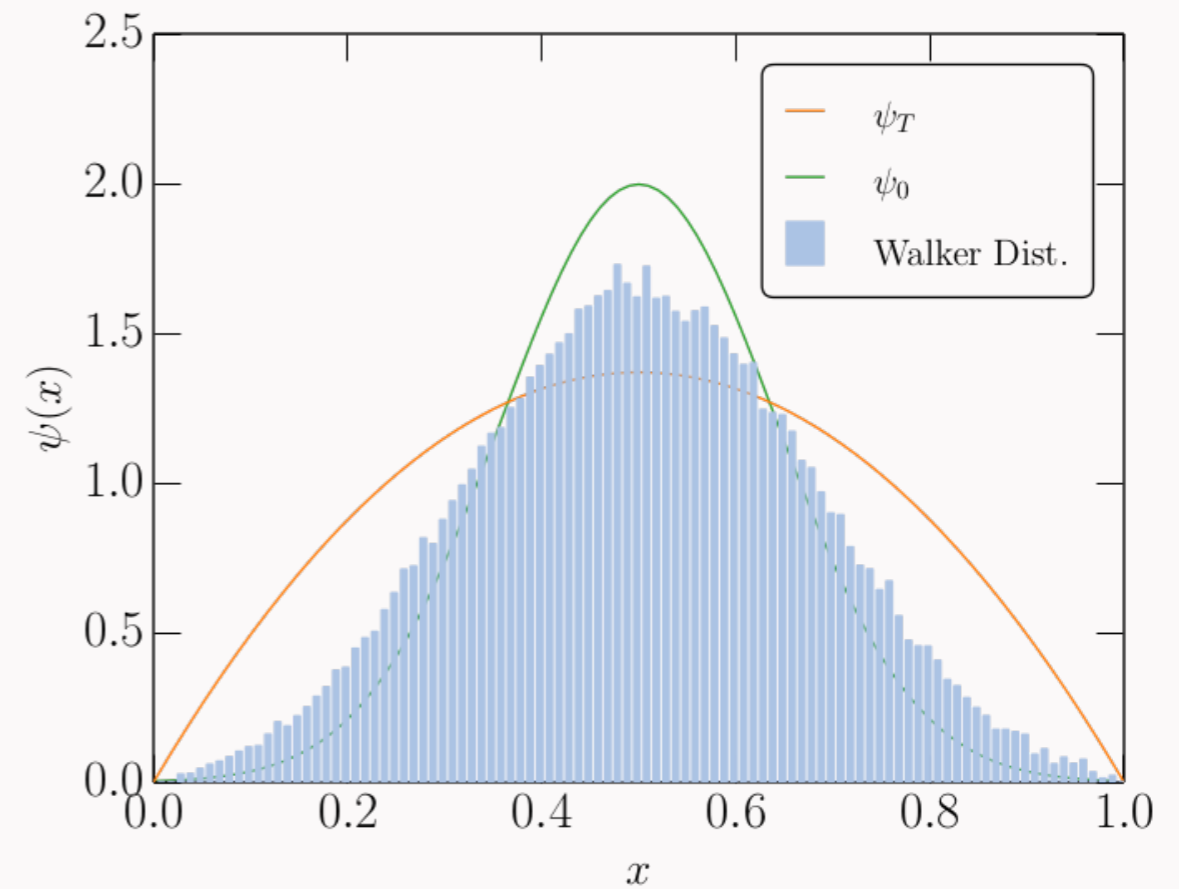
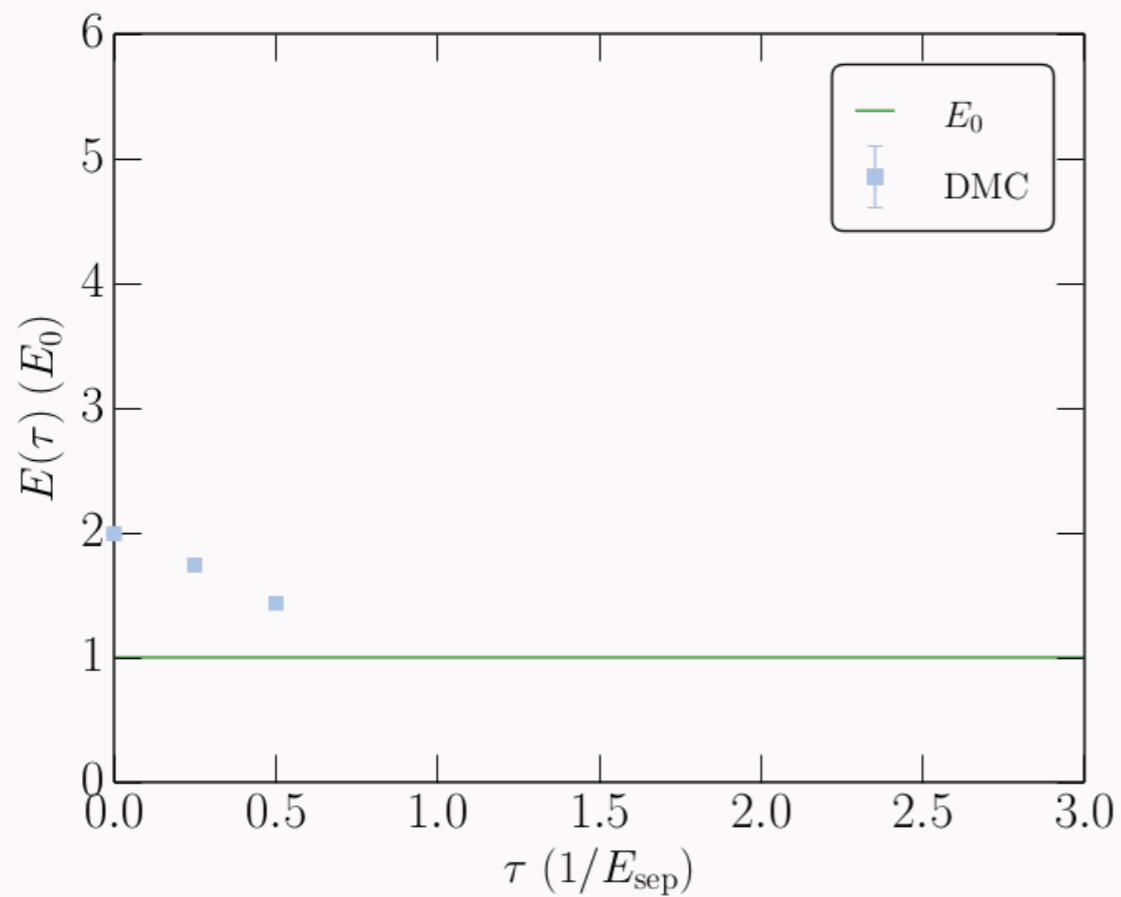
$$\tau = 0.25$$



# QMC Methods - An Example

Imaginary-time evolution:

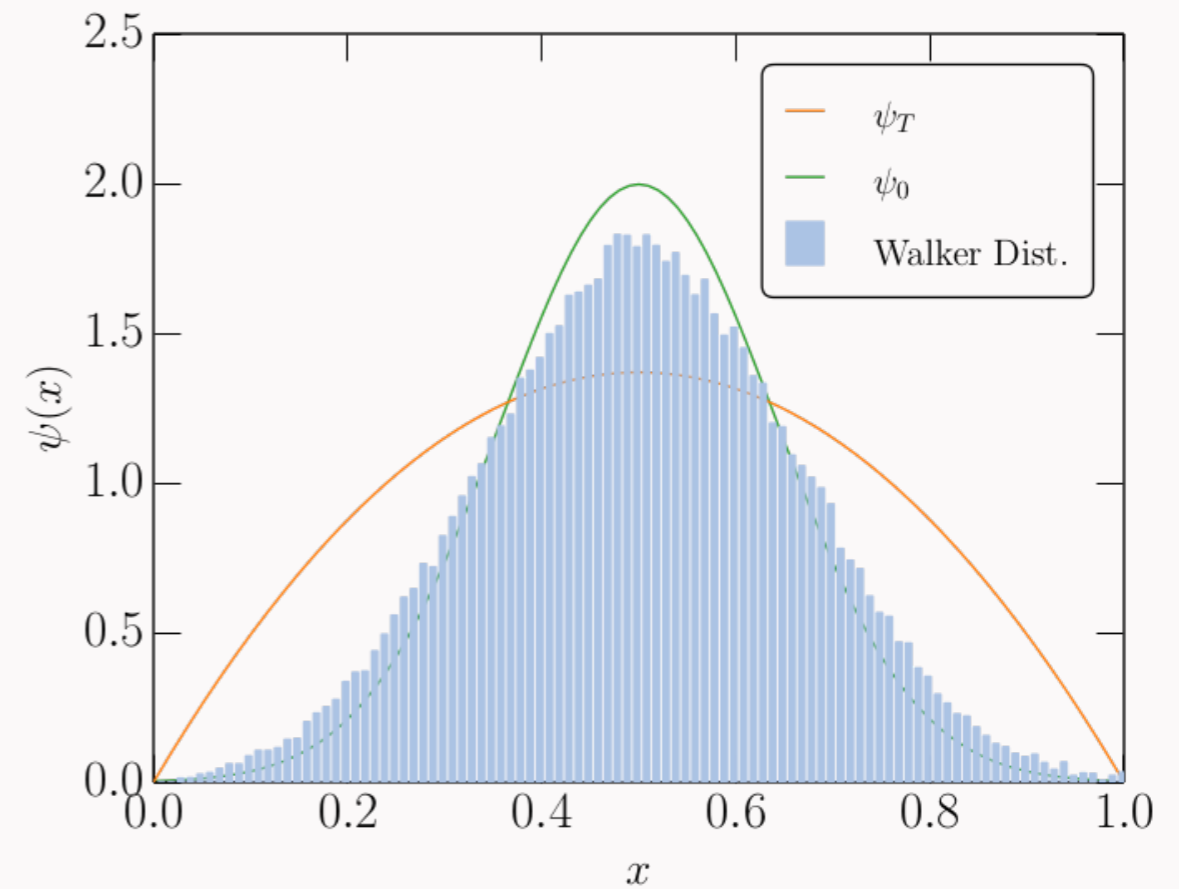
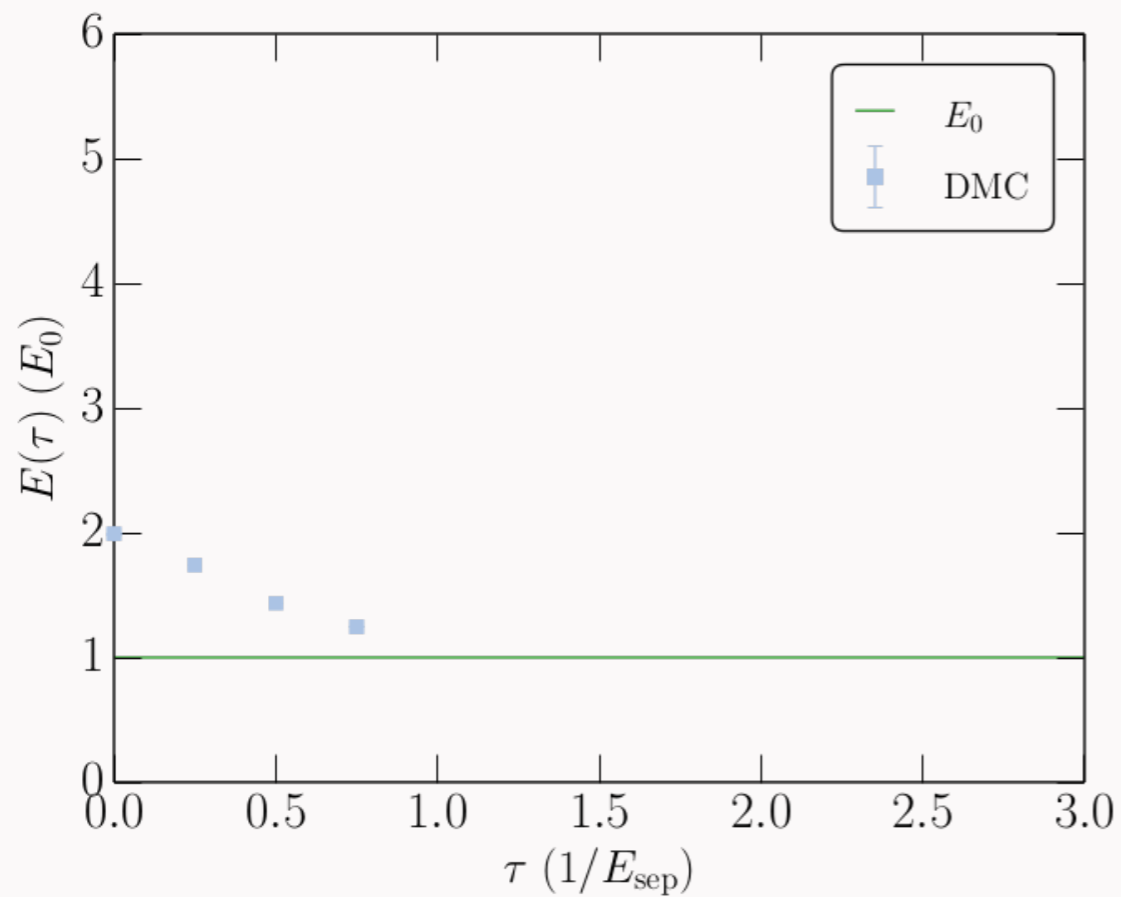
$$\tau = 0.50$$



# QMC Methods - An Example

Imaginary-time evolution:

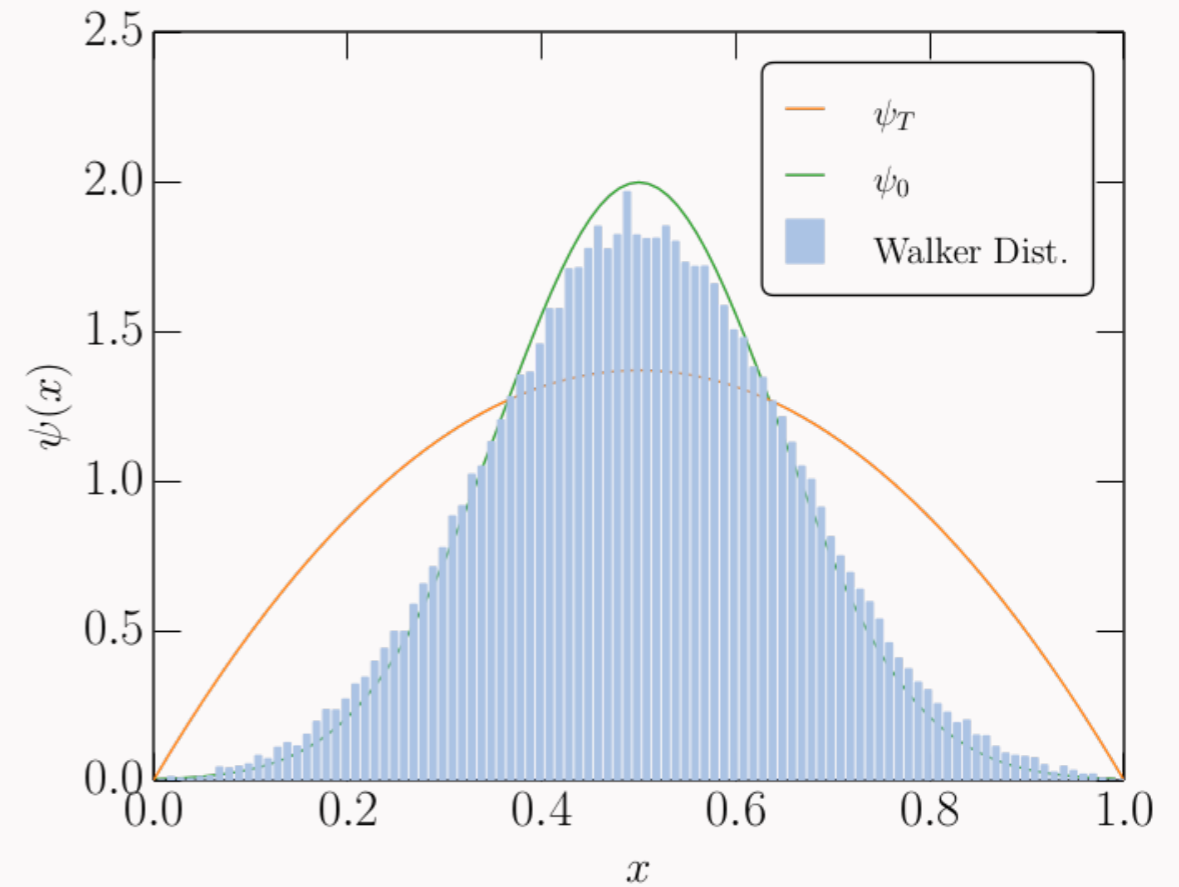
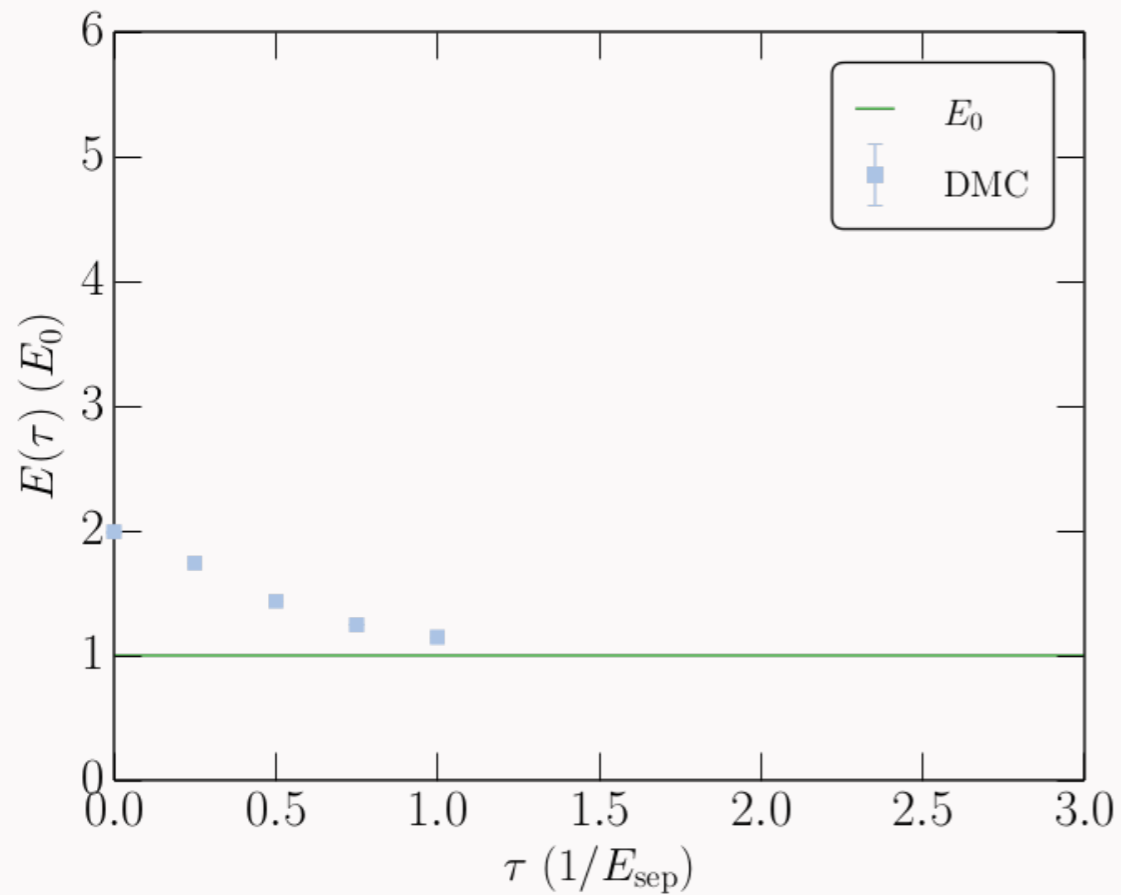
$$\tau = 0.75$$



# QMC Methods - An Example

Imaginary-time evolution:

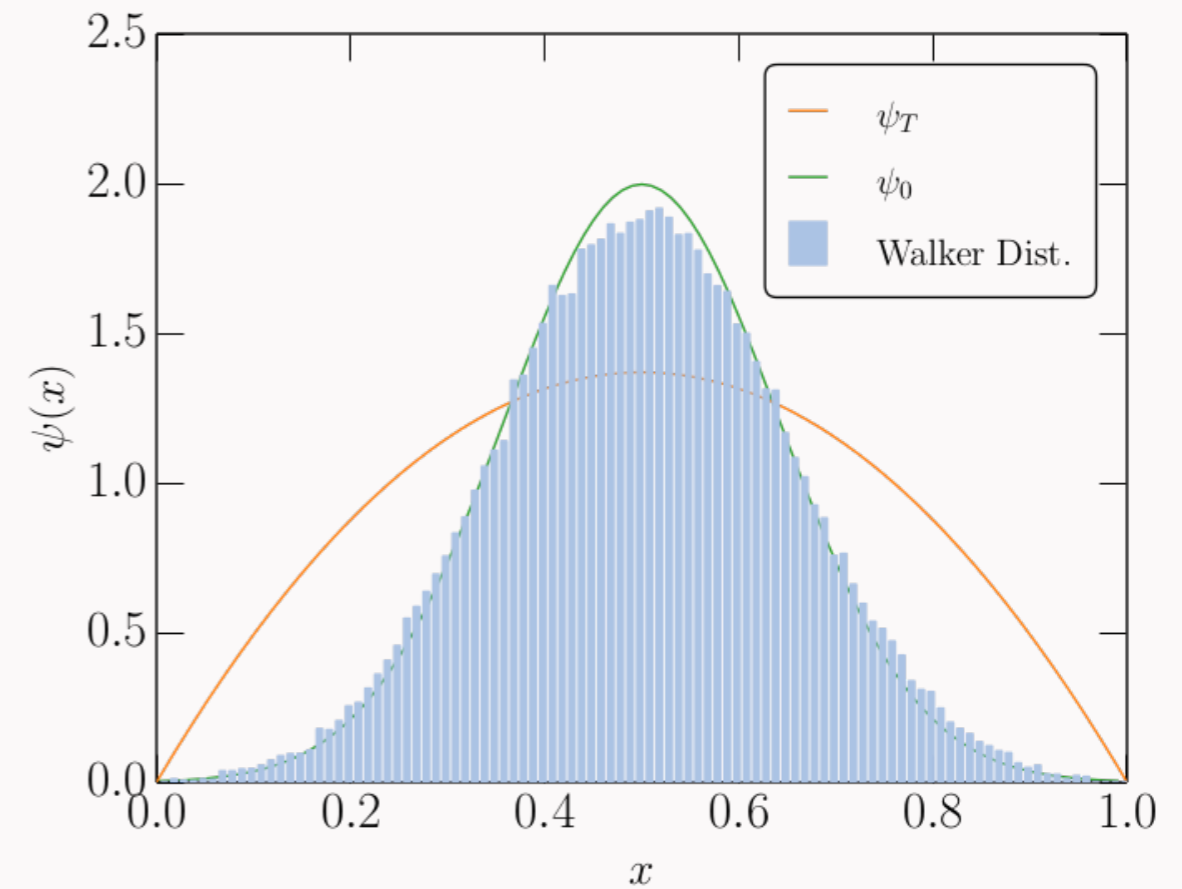
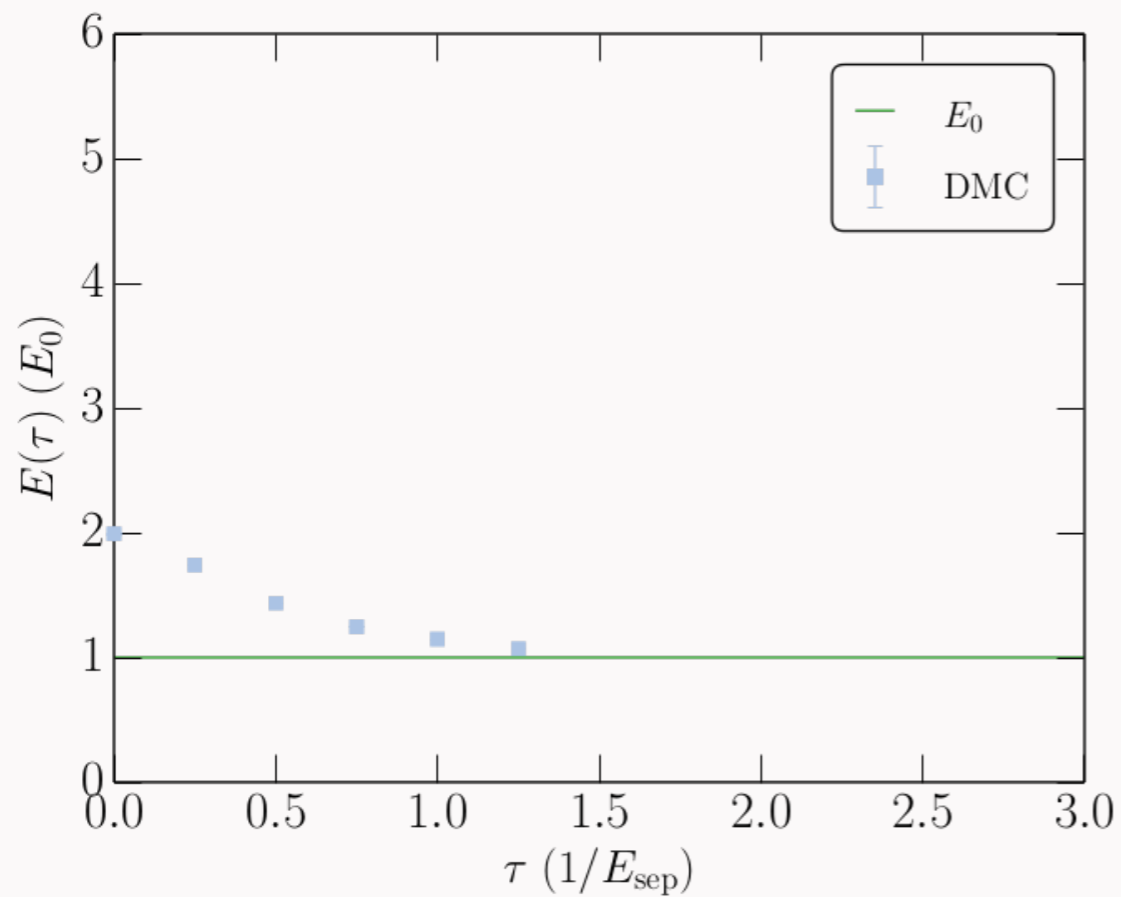
$$\tau = 1.00$$



# QMC Methods - An Example

Imaginary-time evolution:

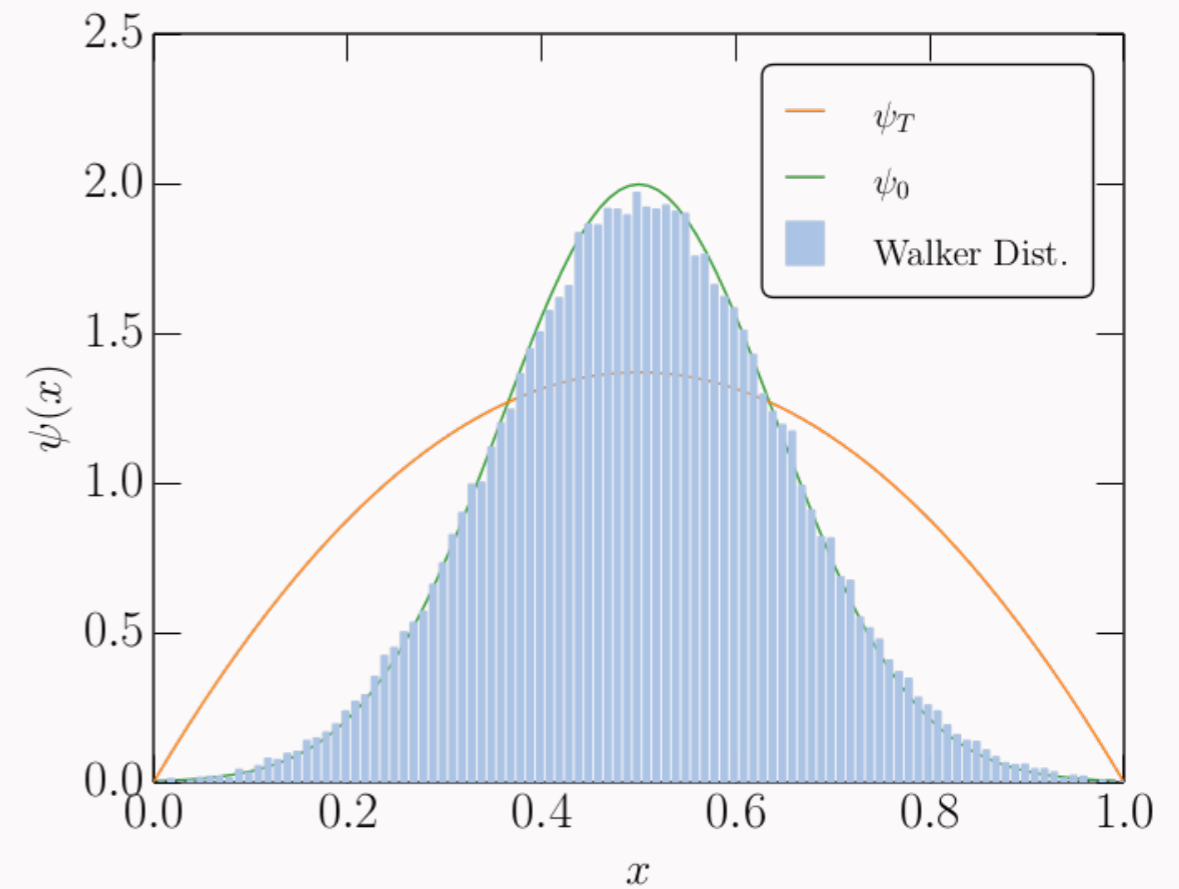
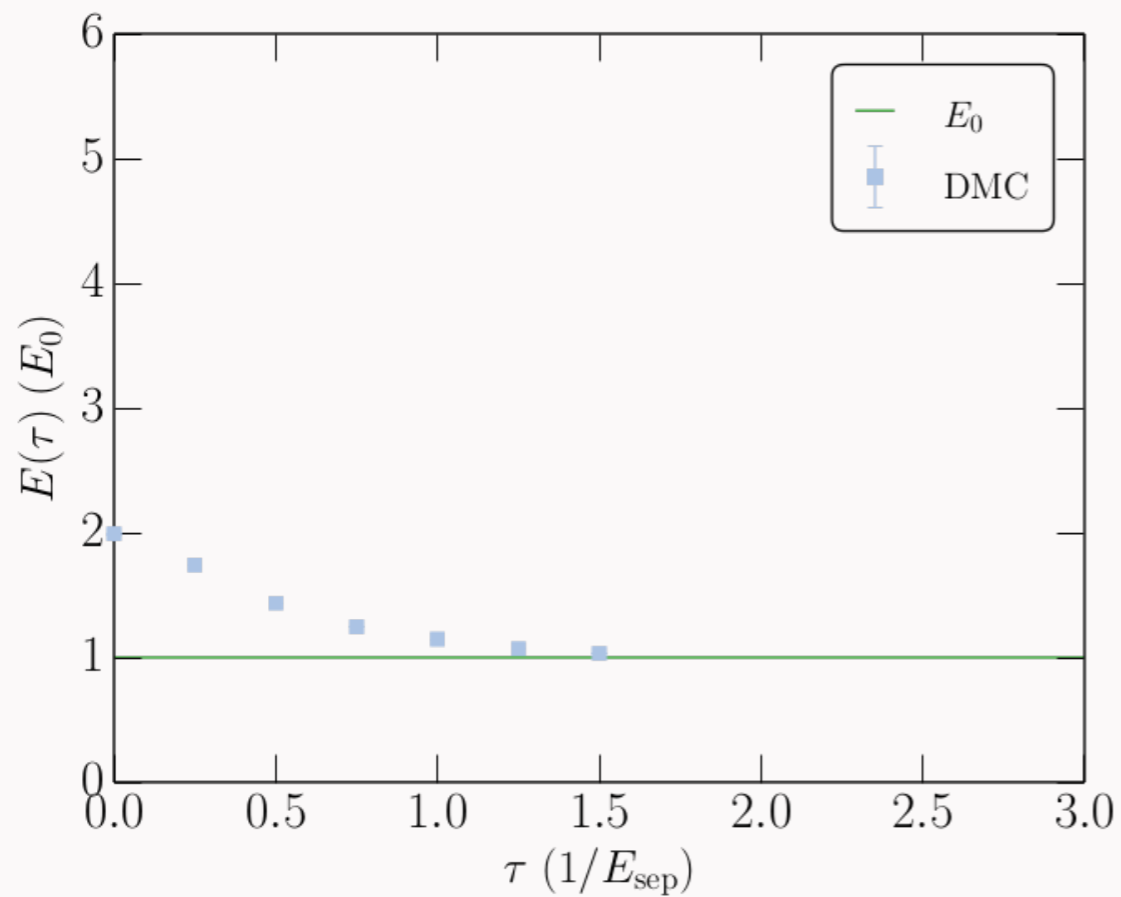
$$\tau = 1.25$$



# QMC Methods - An Example

Imaginary-time evolution:

$$\tau = 1.50$$

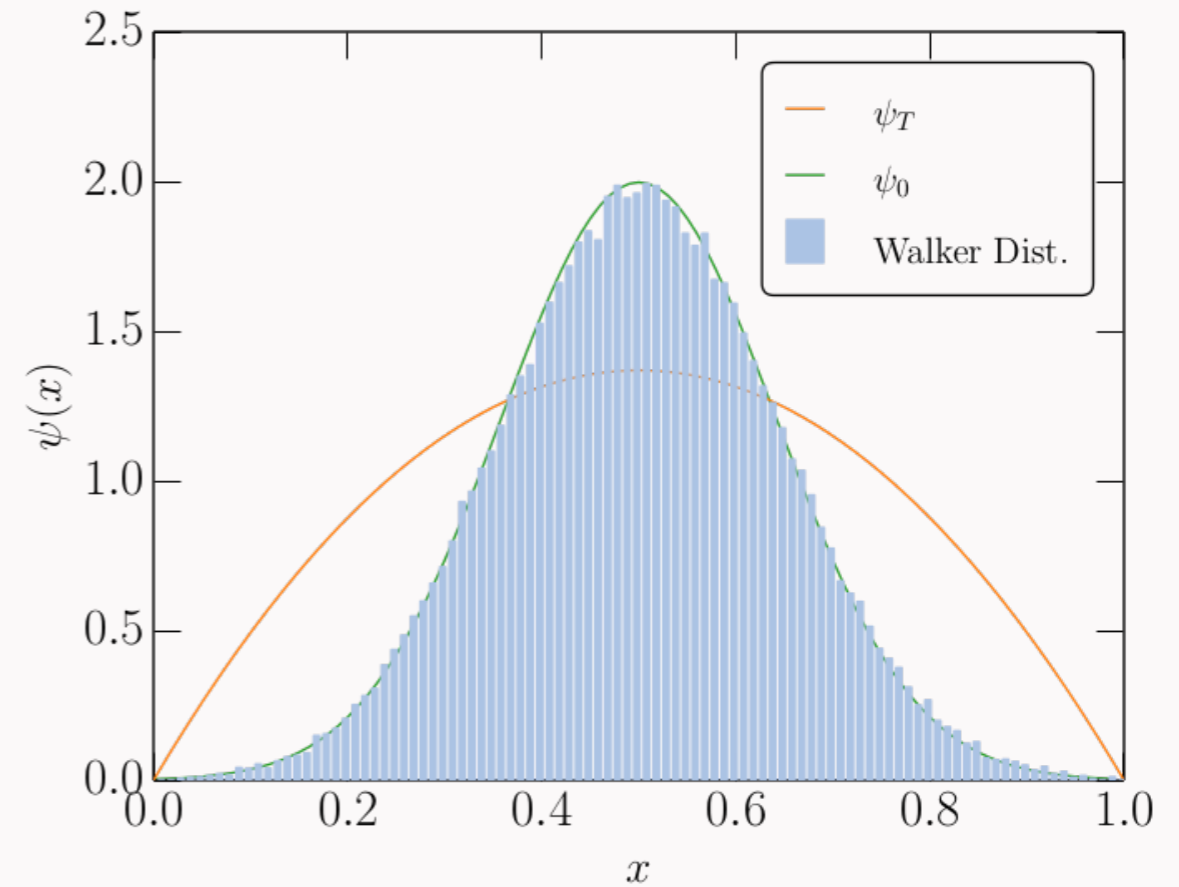
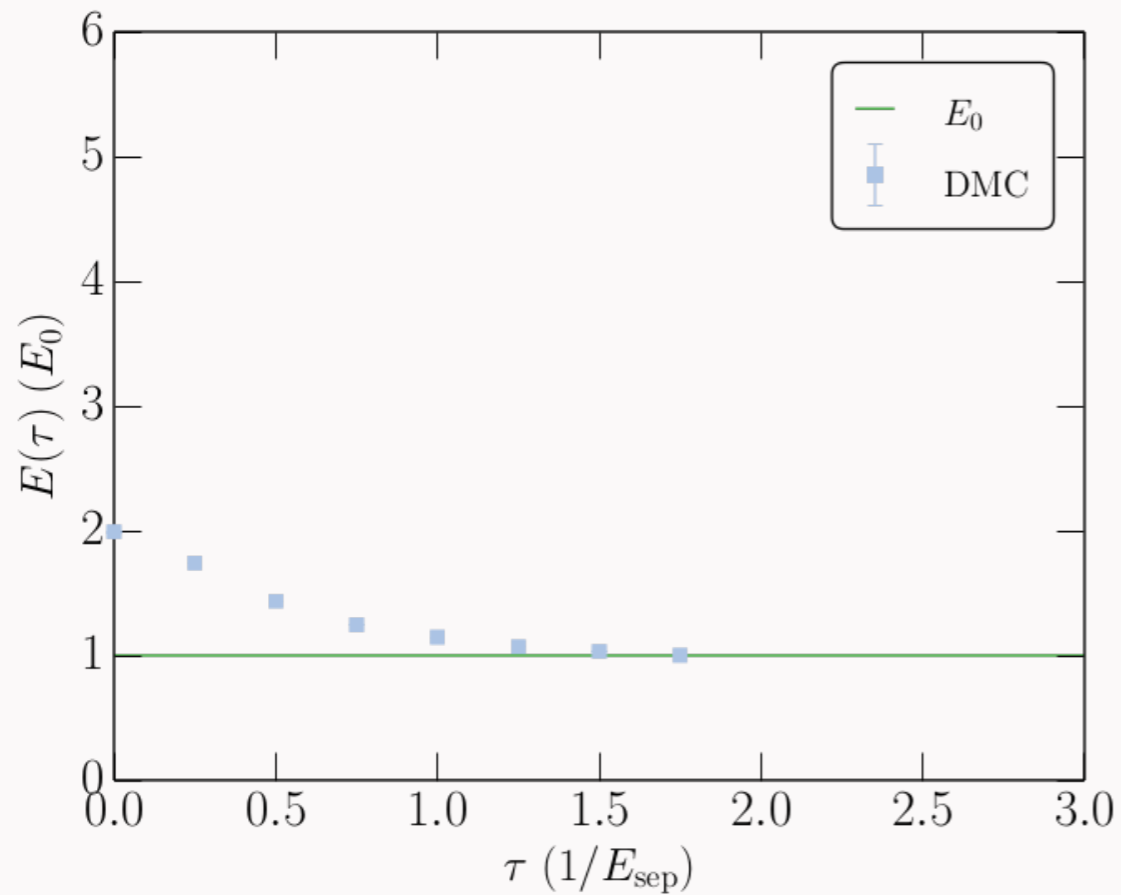




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Imaginary-time evolution:

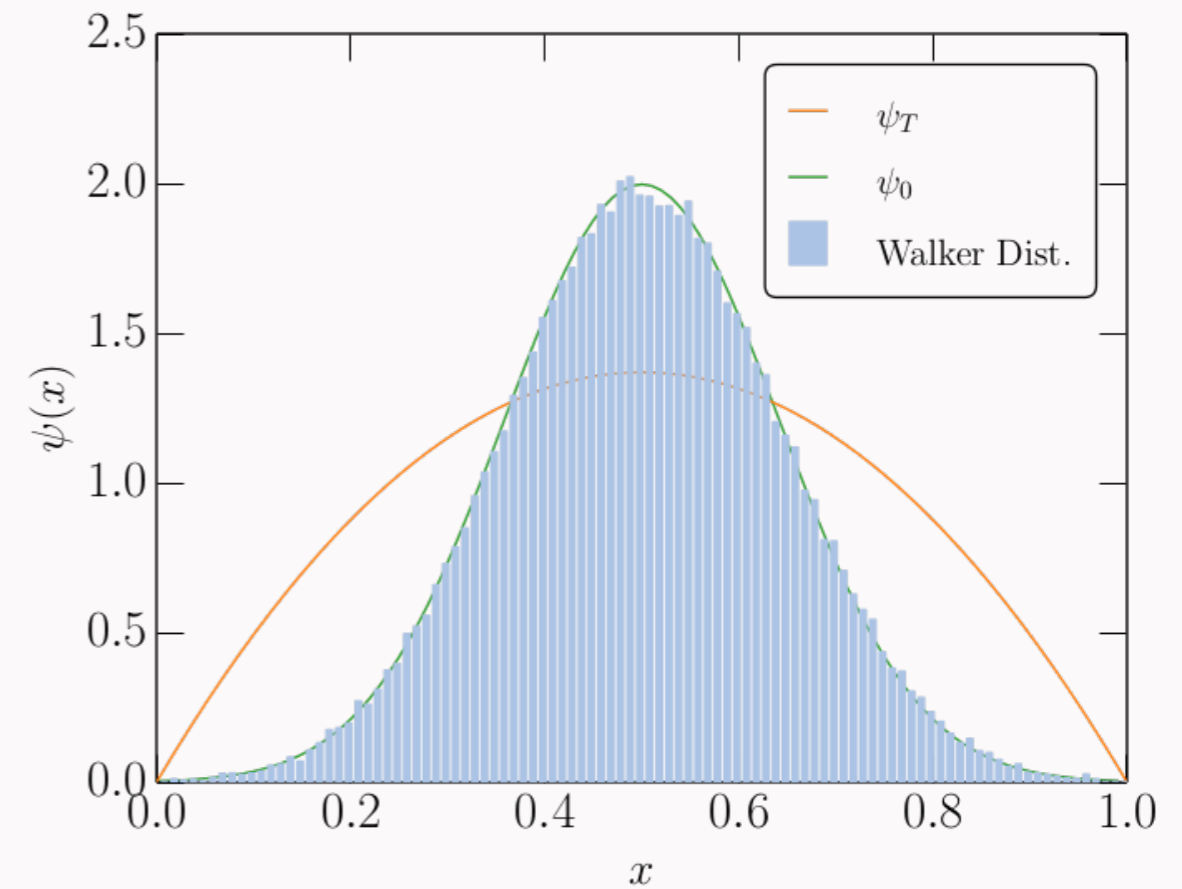
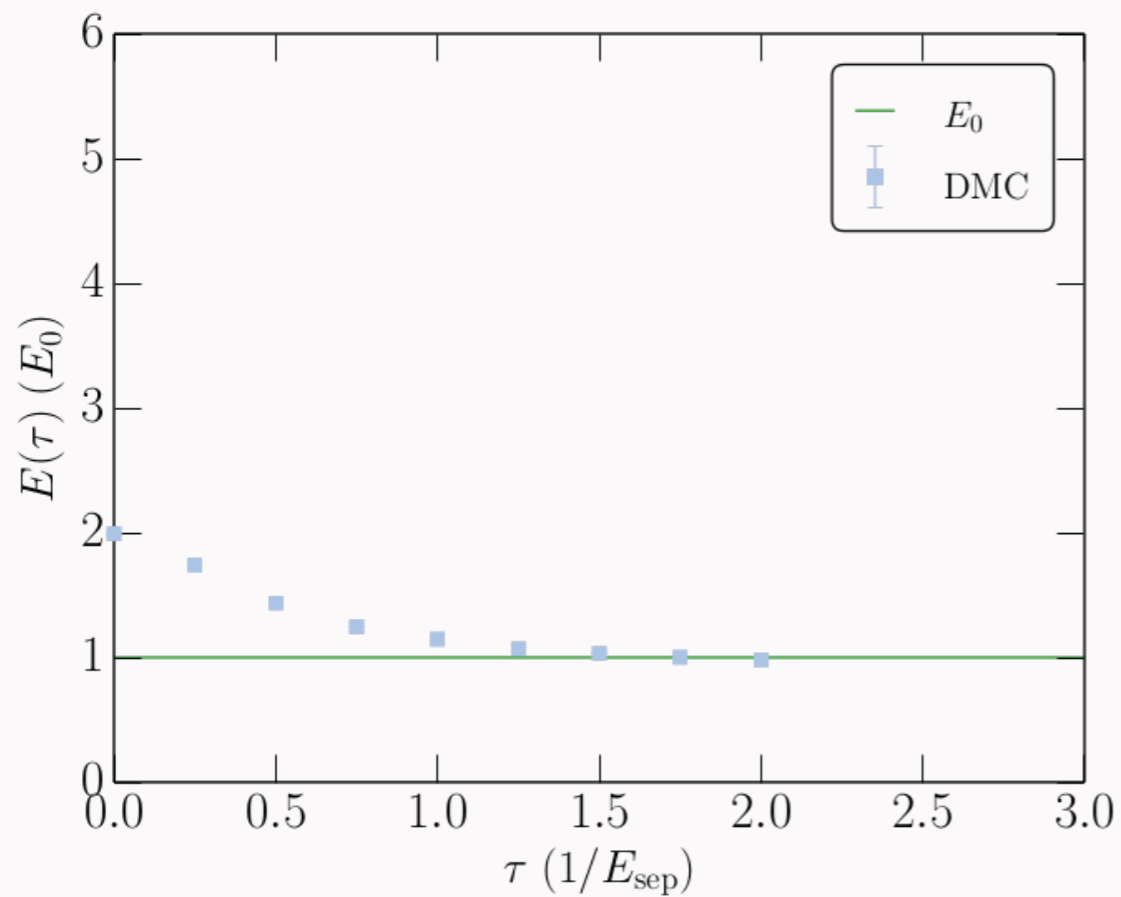
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Imaginary-time evolution:

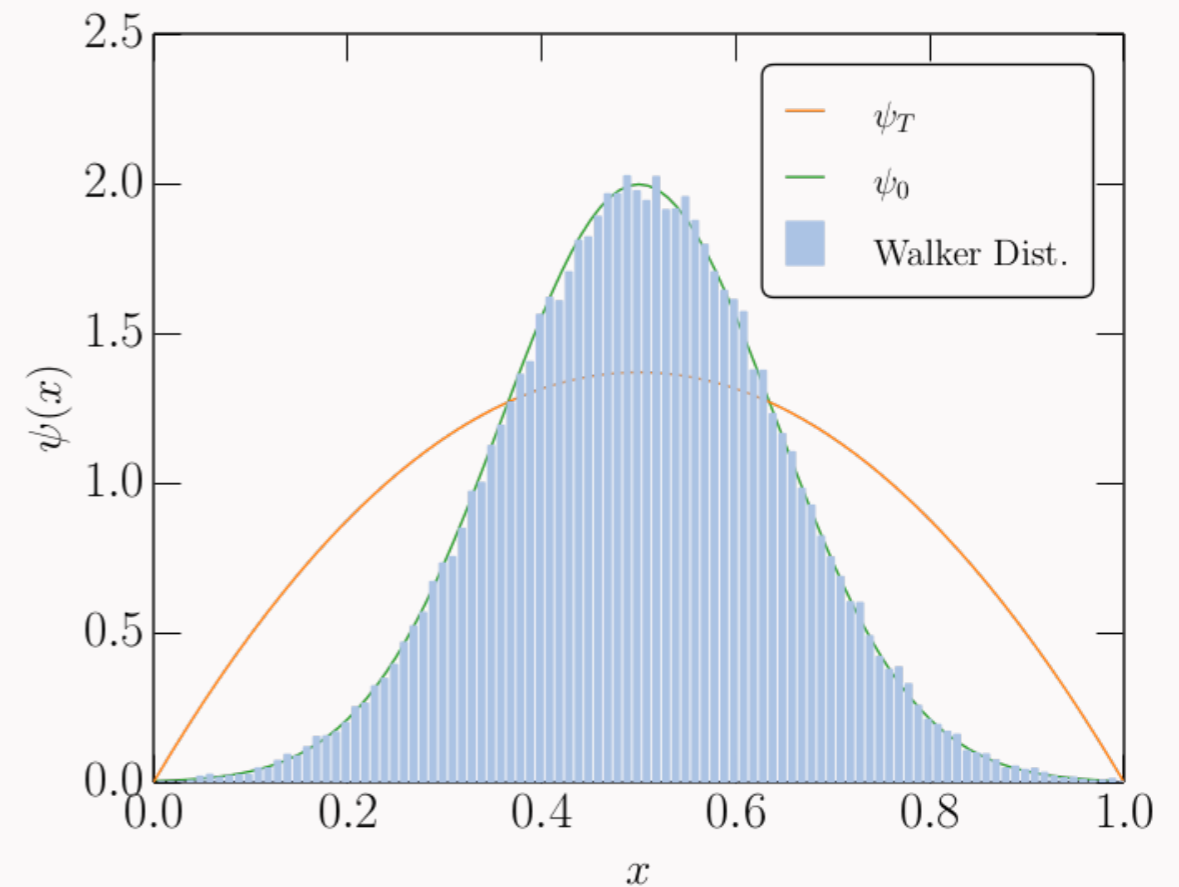
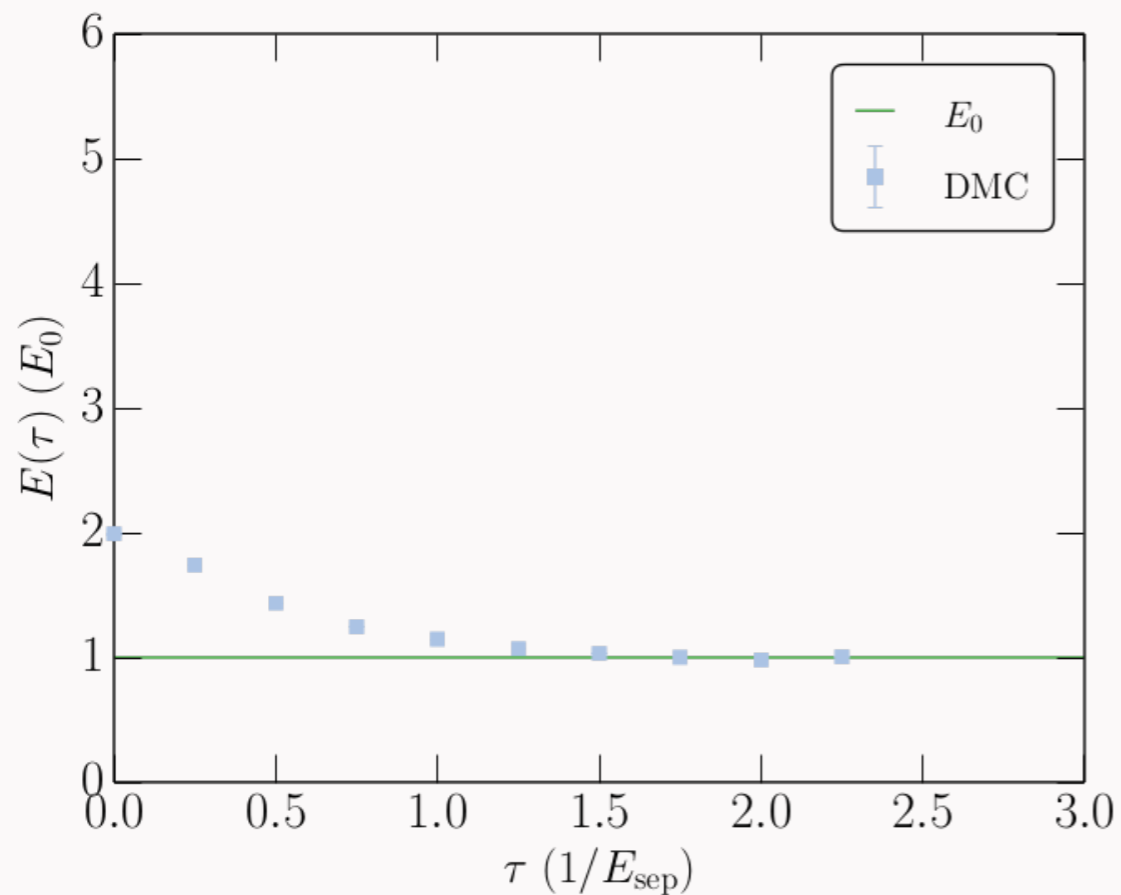
$$\tau = 2.00$$



# QMC Methods - An Example

Imaginary-time evolution:

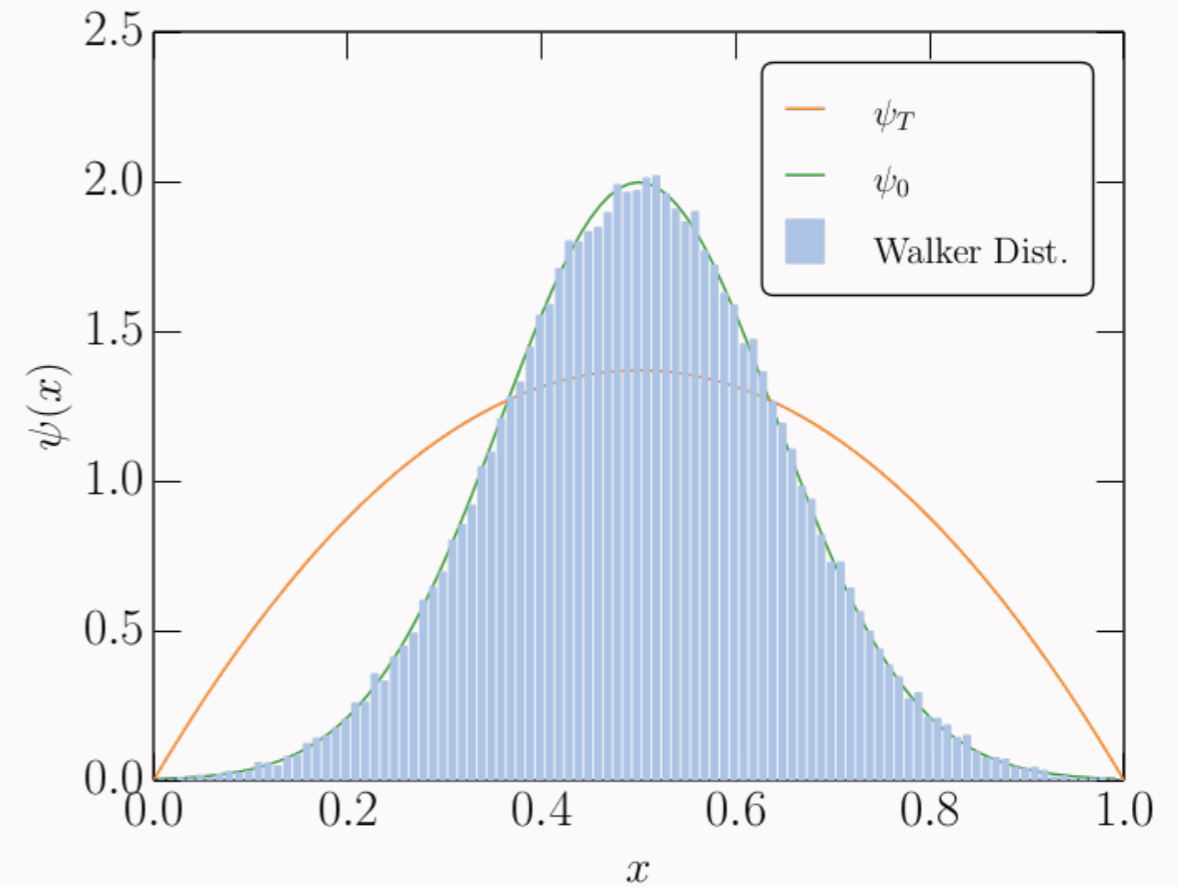
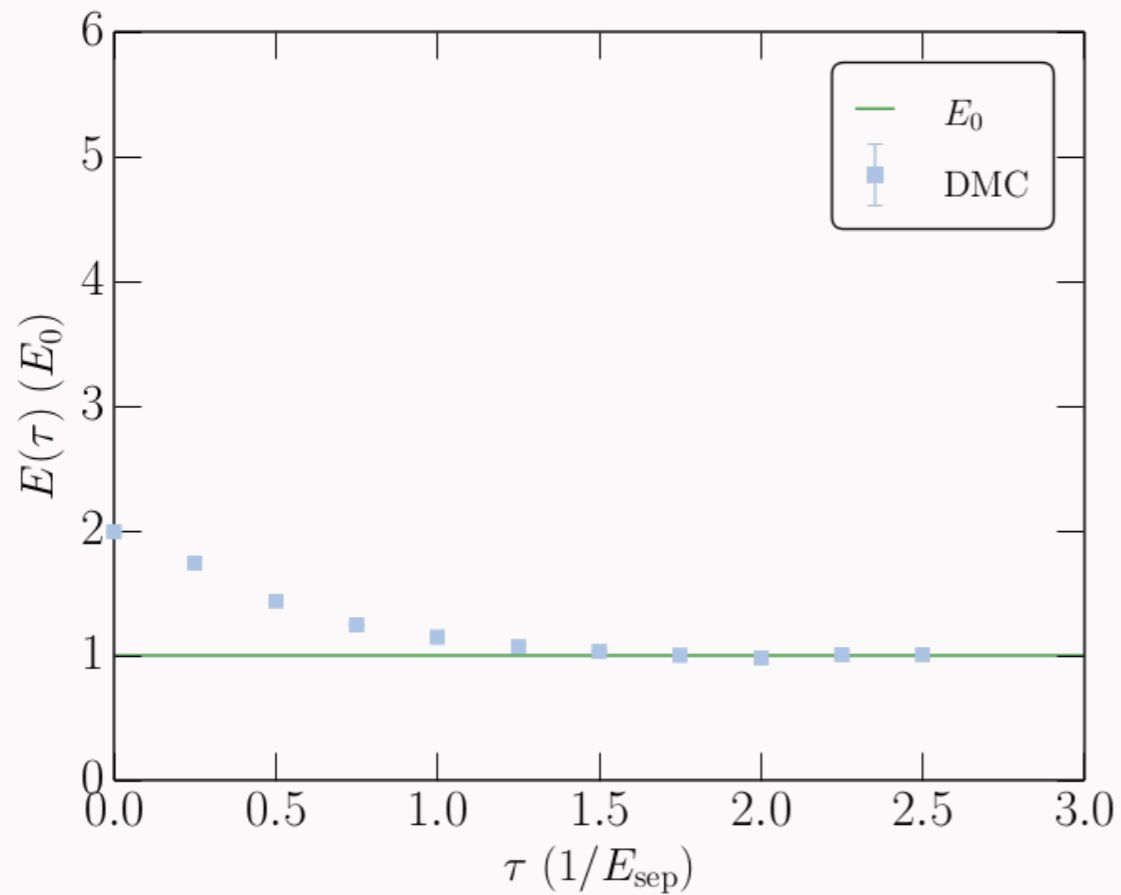
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# QMC Methods - An Example

Imaginary-time evolution:

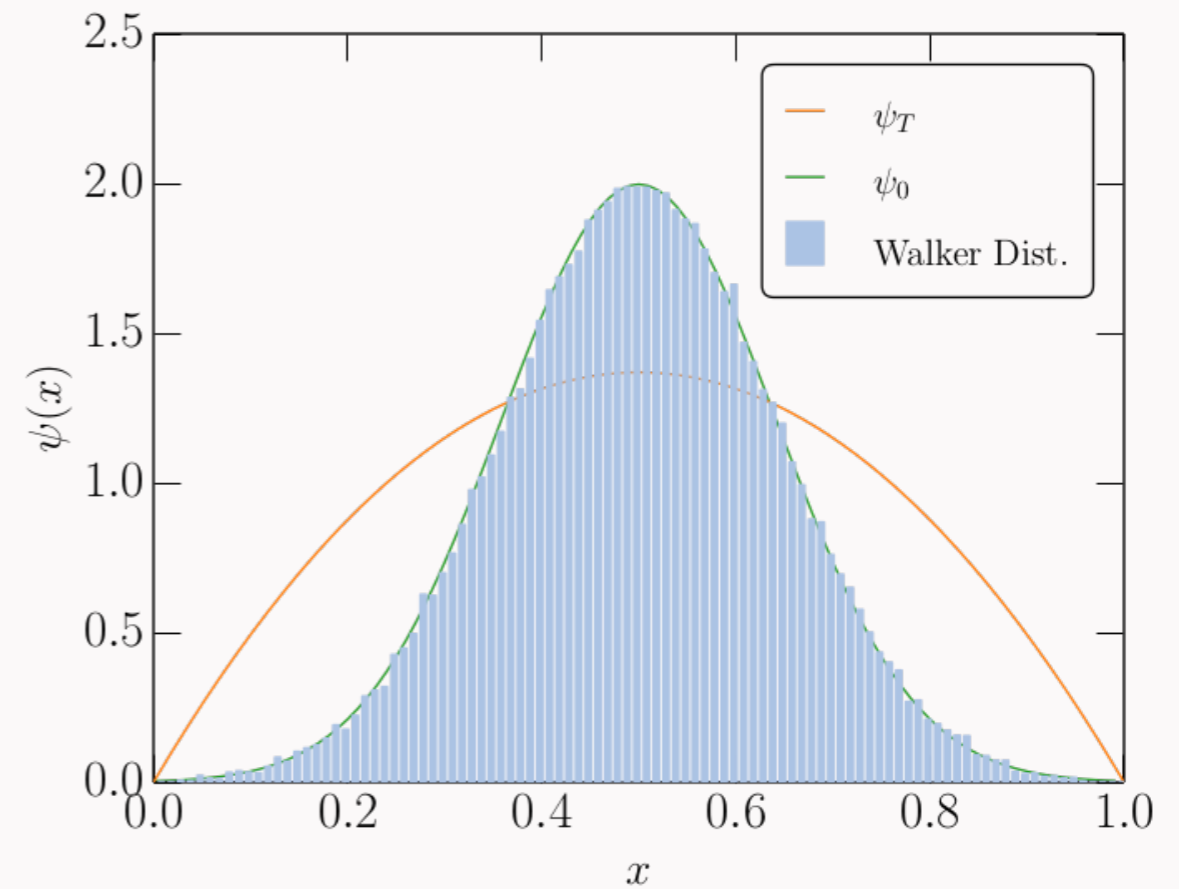
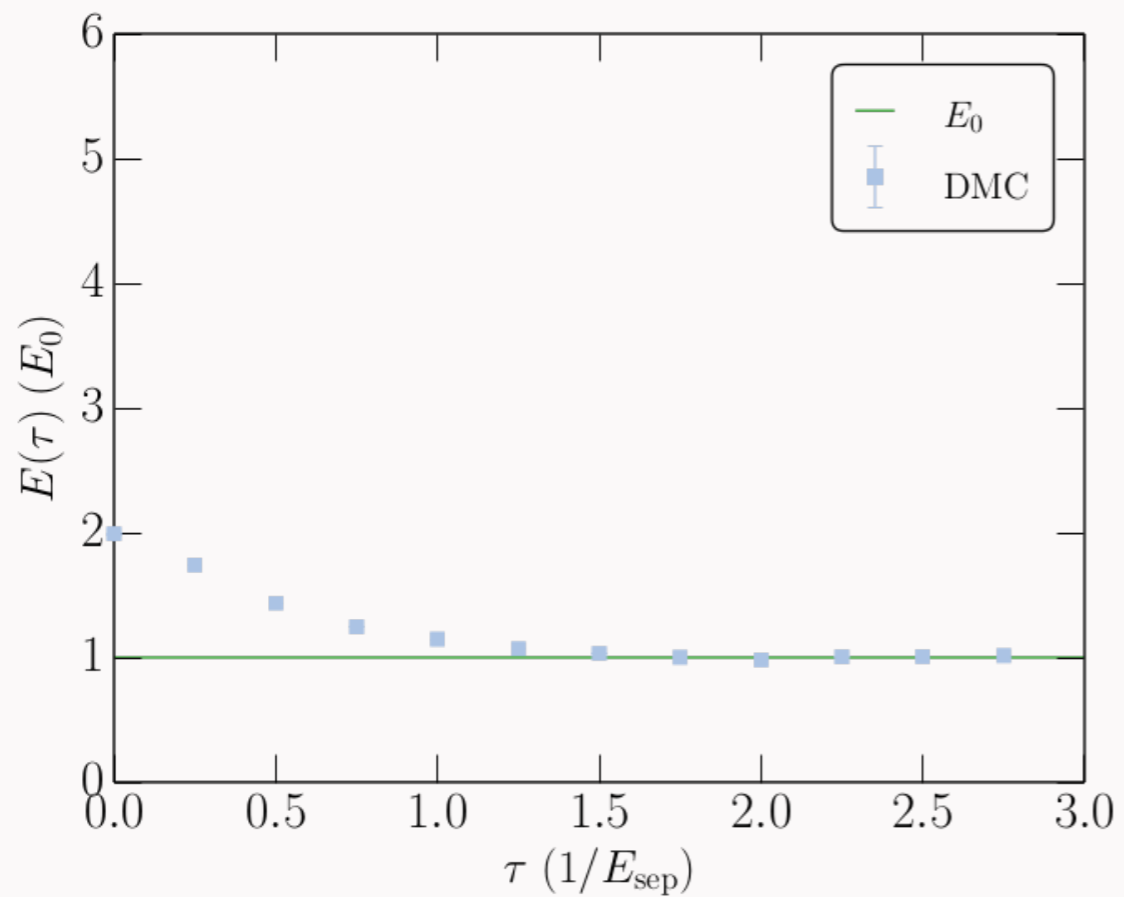
$$\tau = 2.50$$



# QMC Methods - An Example

Imaginary-time evolution:

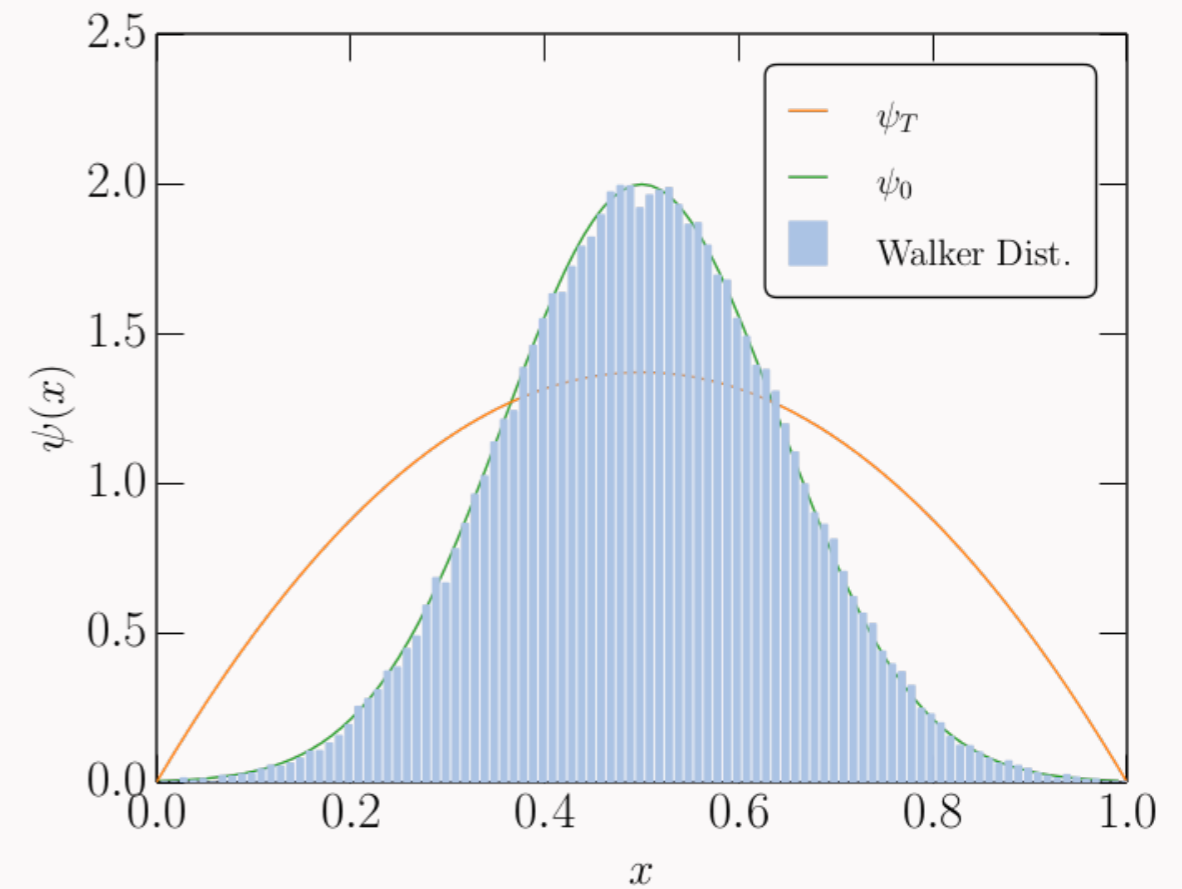
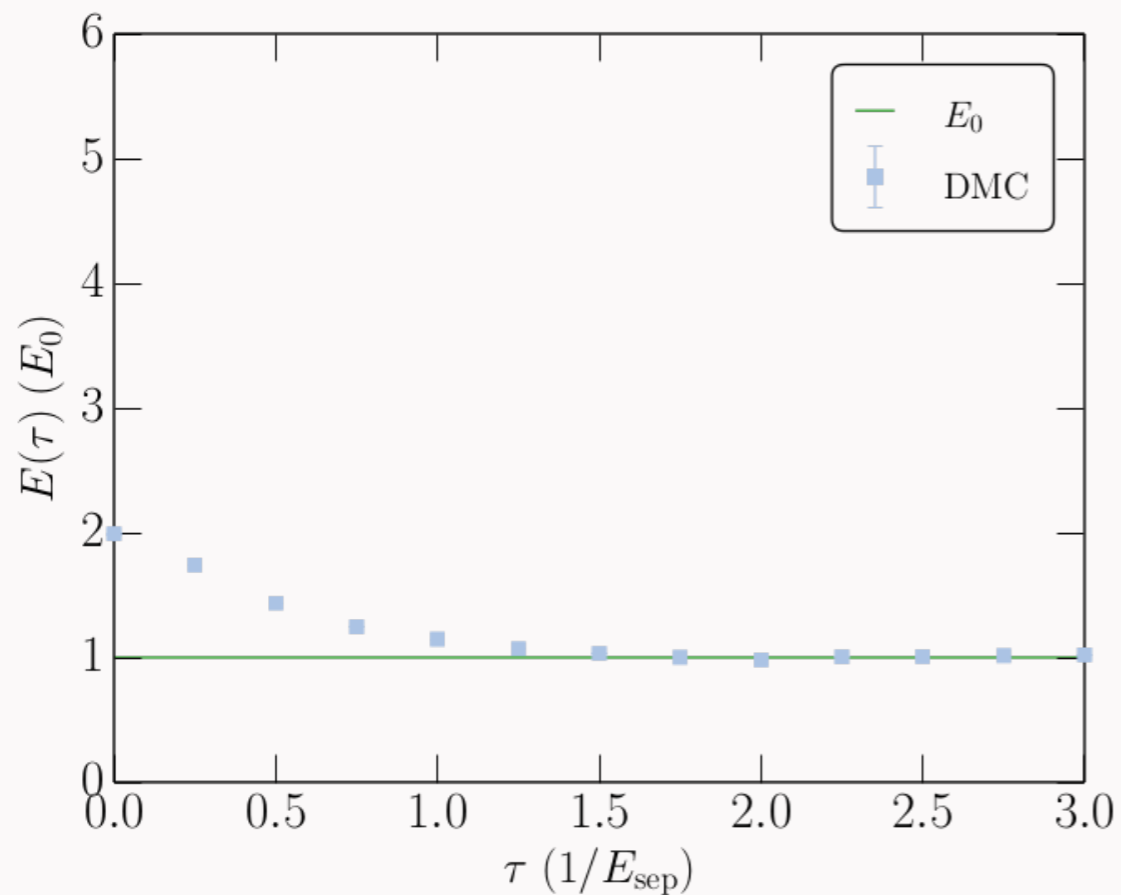
$$\tau = 2.75$$



# QMC Methods - An Example

Imaginary-time evolution:

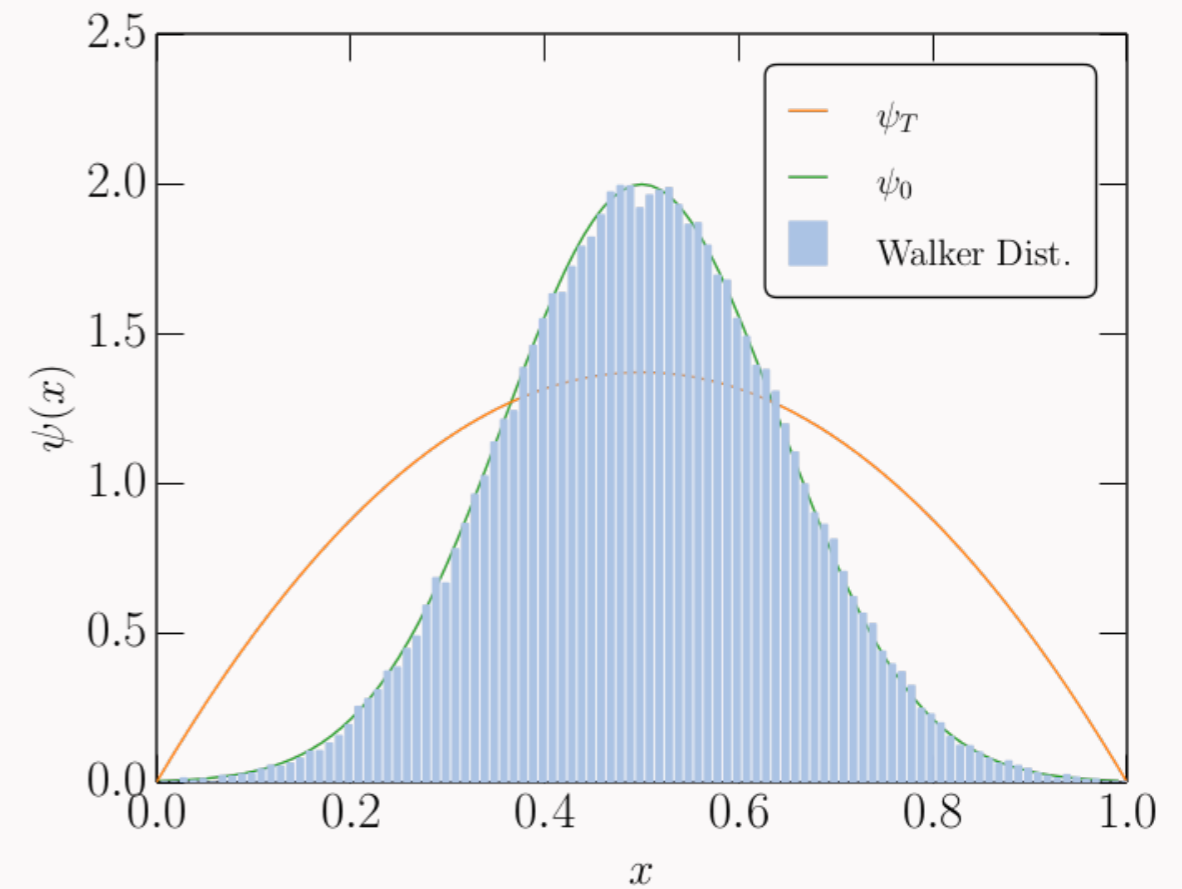
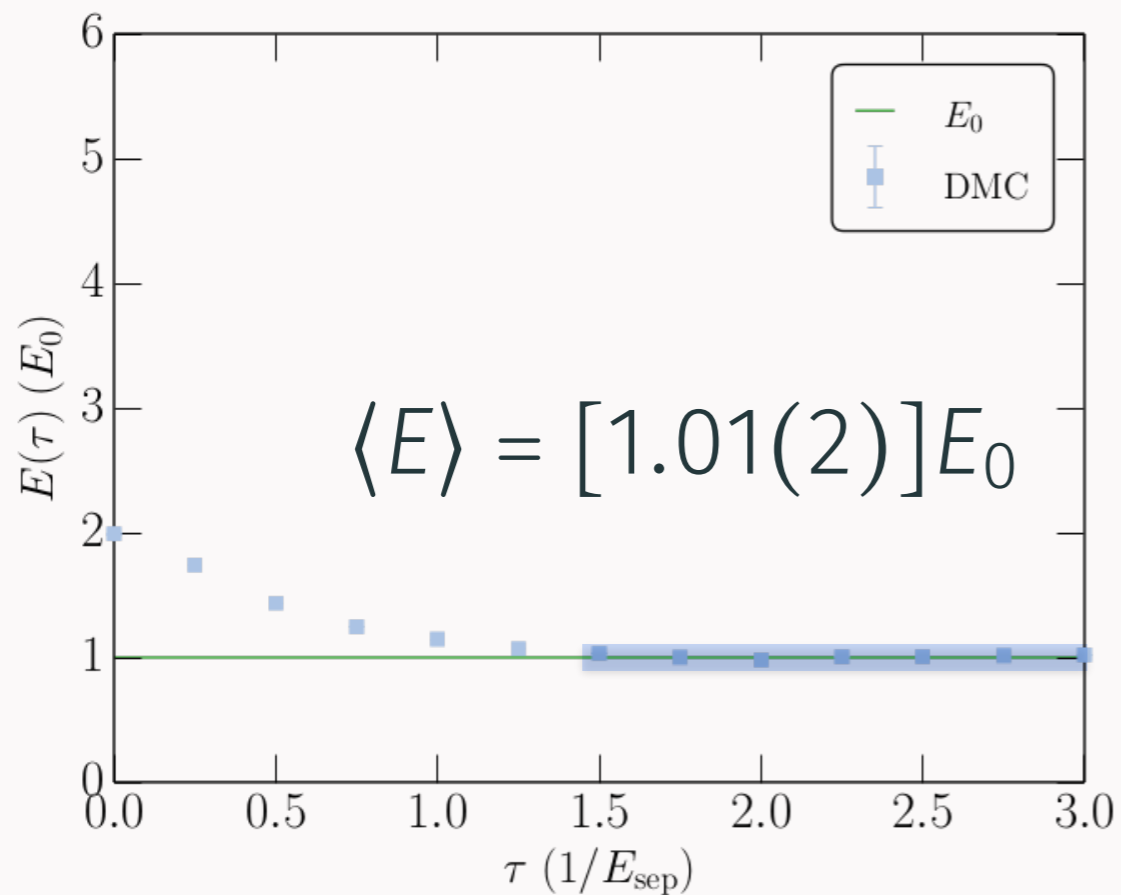
$$\tau = 3.00$$



# QMC Methods - An Example

Imaginary-time evolution:

$$\tau = 3.00$$



# QMC Methods - Compare/Contrast GFMC & AFDMC

Green's function Monte Carlo (GFMC)

Auxiliary-field diffusion Monte Carlo (AFDMC)

- $|\Psi_T\rangle \sim 3A$  coordinates &  $2^A \binom{A}{Z}$  complex amplitudes:  
*Exponential scaling.*

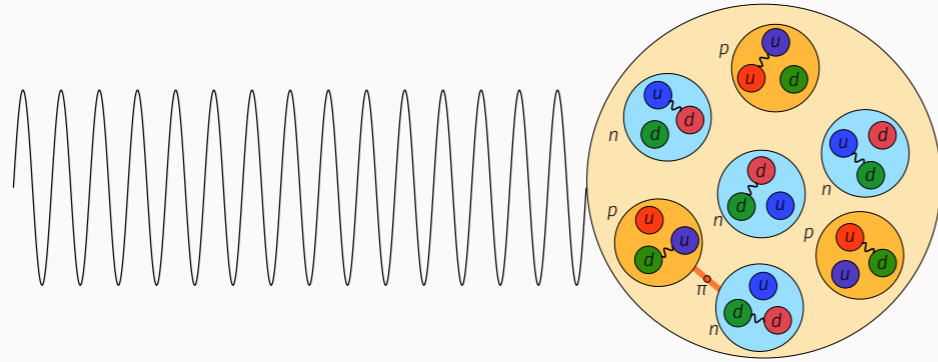
- $|\Psi_T\rangle \sim 3A$  coordinates &  $4A$  complex amplitudes ( $|n\uparrow\rangle, |n\downarrow\rangle, |p\uparrow\rangle, |p\downarrow\rangle$ ):  
*Polynomial scaling.*



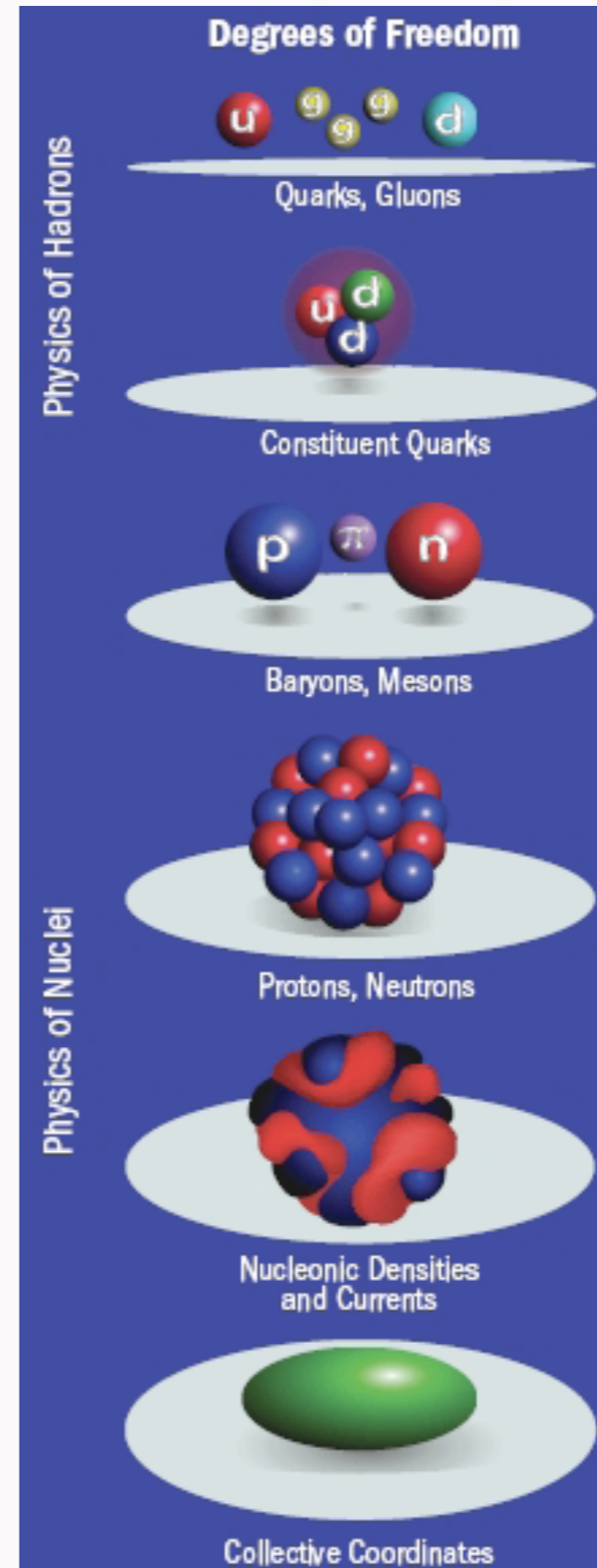
# Chiral Effective Field Theory (EFT)

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# Chiral EFT



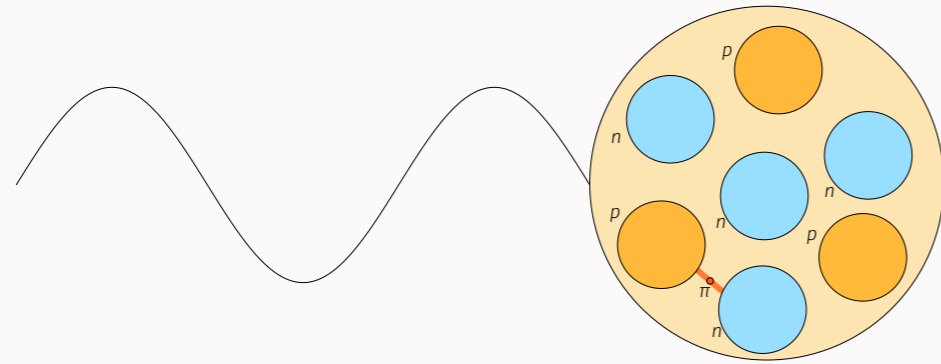
- If probed at high energies, substructure is resolved.
- At low energies, details are not resolved.
- Can replace fine structure by something simpler (think of multipole expansion): low-energy observables unchanged.



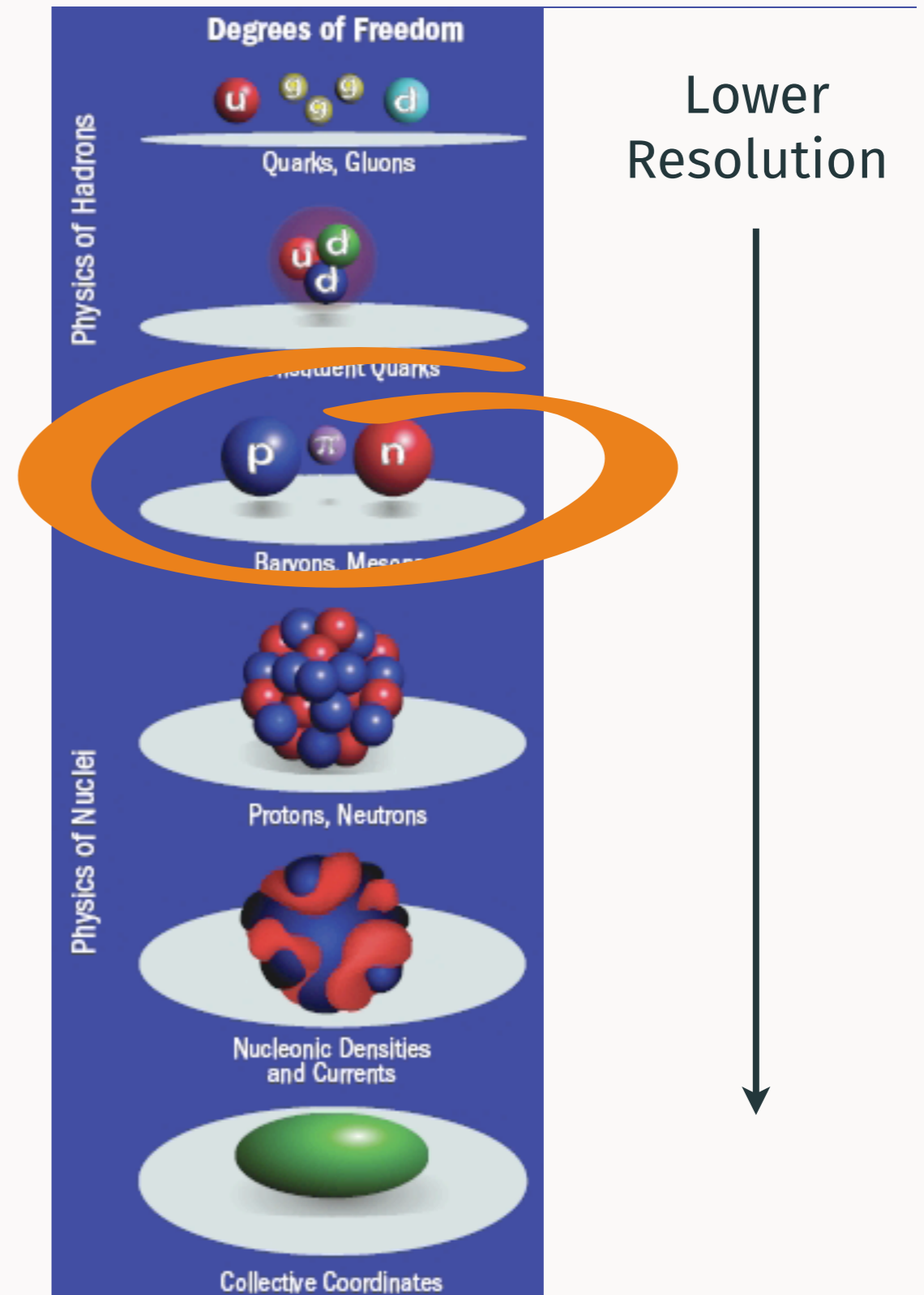
Lower  
Resolution



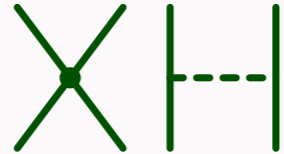
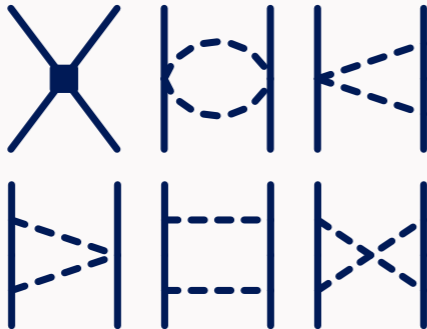
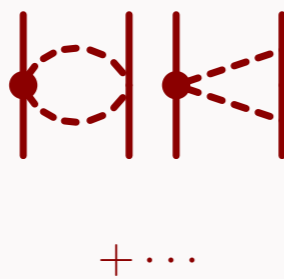
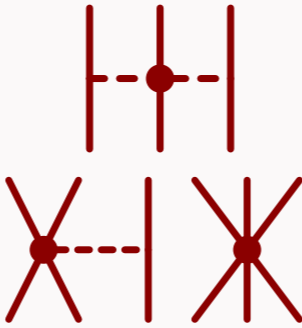

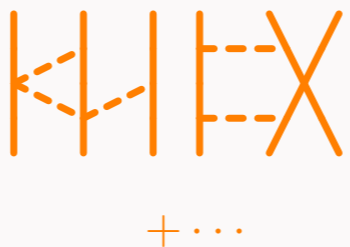
# Chiral EFT



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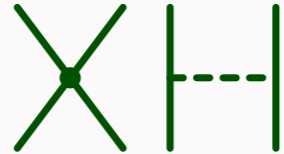
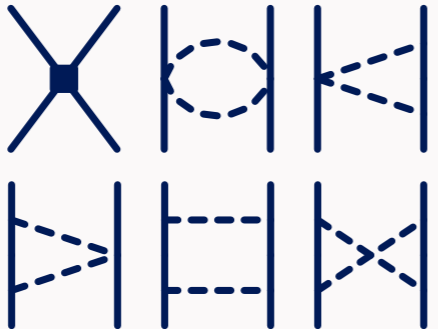


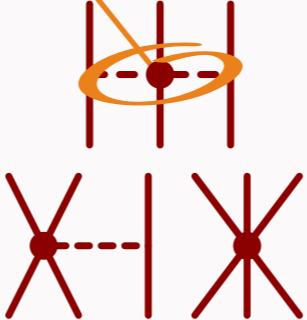

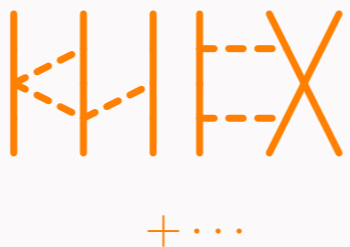


# Chiral EFT

	$NN$	$NNN$
LO $\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^0$		-
NLO $\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^2$		-
N <sup>2</sup> LO $\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^3$		
N <sup>3</sup> LO $\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^4$		

- Chiral EFT: Expand in powers of  $Q/\Lambda_b$ .  
 $Q \sim m_\pi \sim 100 \text{ MeV}$   
 $\Lambda_b \sim 500 \text{ MeV}$
- Long-range physics:  $\pi$  exchanges.
- Short-range physics: Contacts  $\times$  LECs.
- Many-body forces & currents enter systematically.

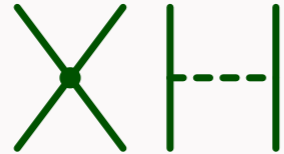
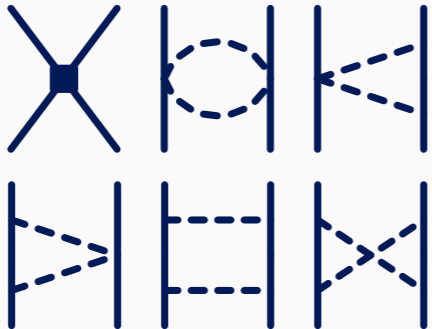

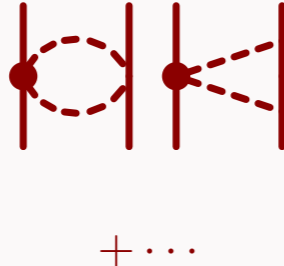
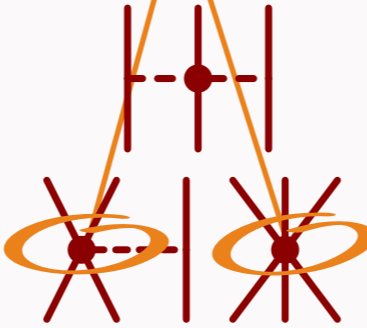

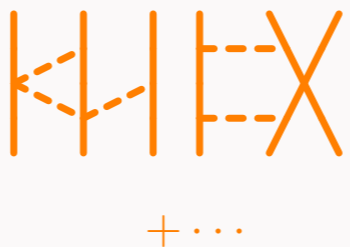
# Chiral EFT

		NN	NNN
LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^0$		-
NLO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^2$		
N <sup>2</sup> LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^3$		
N <sup>3</sup> LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^4$		

Same LECs in NN & 3N sectors!

- Chiral EFT: Expand in powers of  $Q/\Lambda_b$ .  
 $Q \sim m_\pi \sim 100 \text{ MeV}$   
 $\Lambda_b \sim 500 \text{ MeV}$
- Long-range physics:  $\pi$  exchanges.
- Short-range physics: Contacts  $\times$  LECs.
- Many-body forces & currents enter systematically.

# Chiral EFT

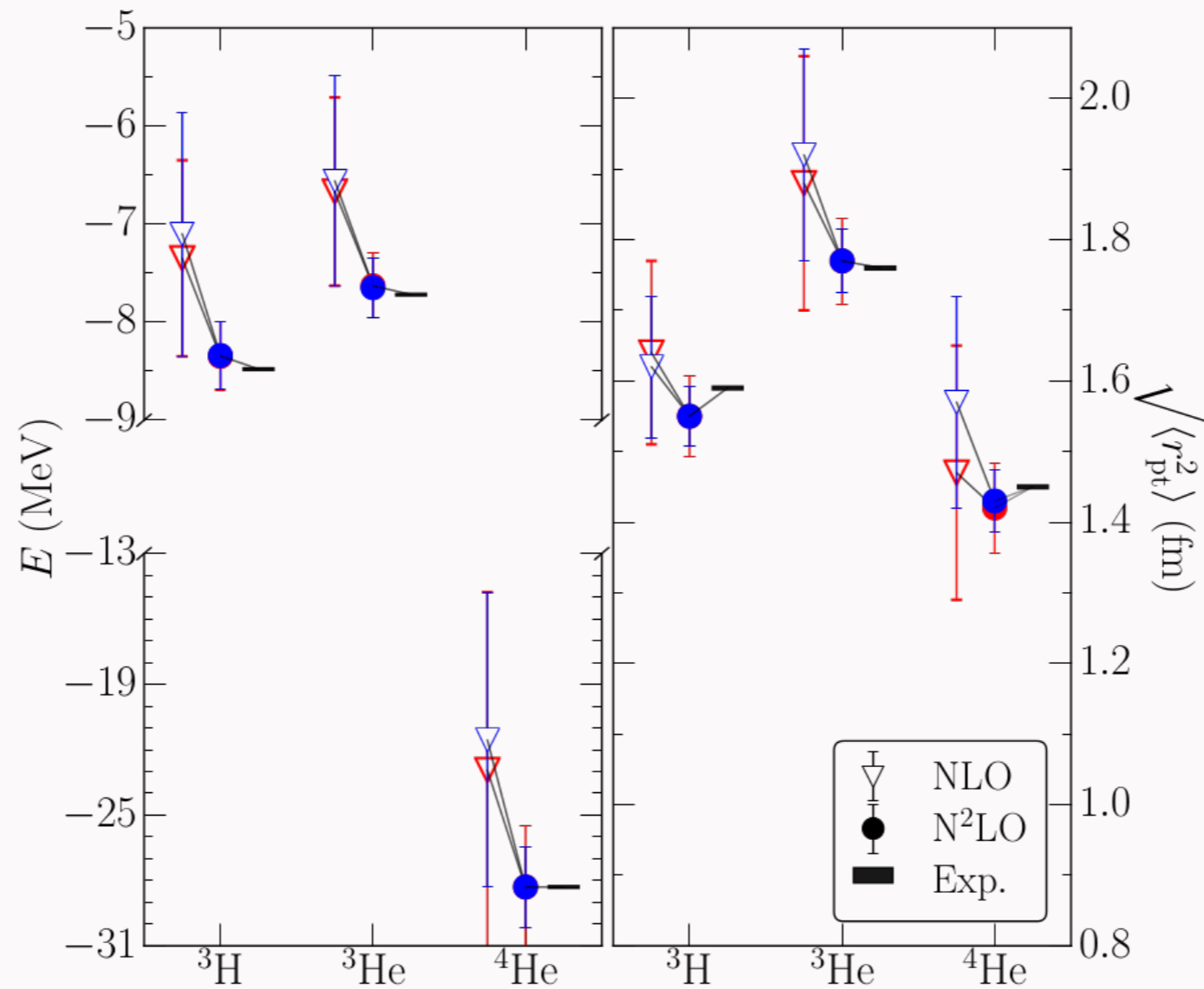
		NN	NNN
LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^0$		-
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N <sup>2</sup> LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^3$		
N <sup>3</sup> LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^4$		

Fit to few-body data.

- Chiral EFT: Expand in powers of  $Q/\Lambda_b$ .  
 $Q \sim m_\pi \sim 100 \text{ MeV}$   
 $\Lambda_b \sim 500 \text{ MeV}$
- Long-range physics:  $\pi$  exchanges.
- Short-range physics: Contacts  $\times$  LECs.
- Many-body forces & currents enter systematically.

# First Results

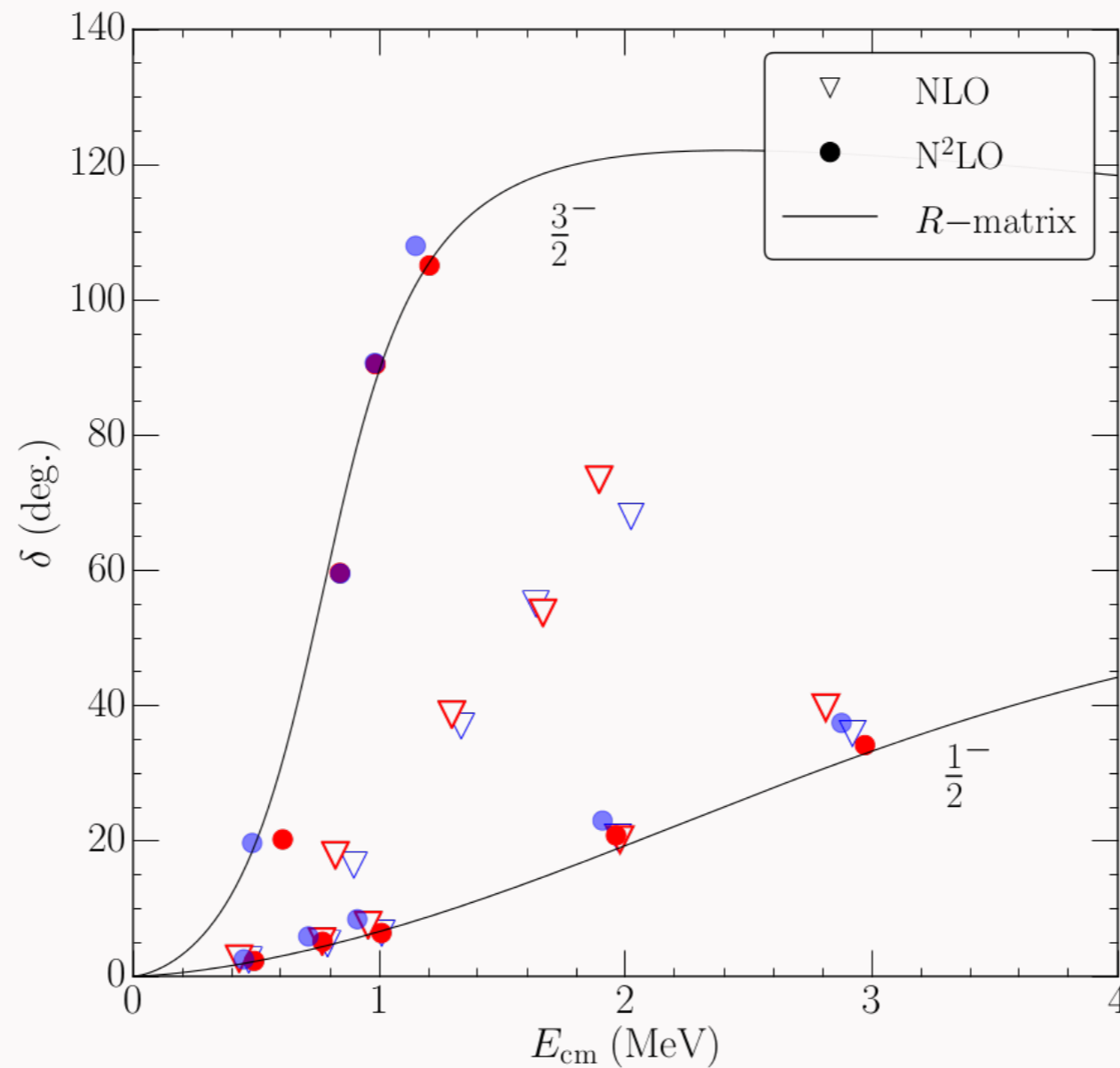
One consistent approach:  
Light Nuclei



JEL et al, PRL 116, 062501 (2016)

# First Results

One consistent approach:  
 $n$ - $\alpha$  Elastic Scattering

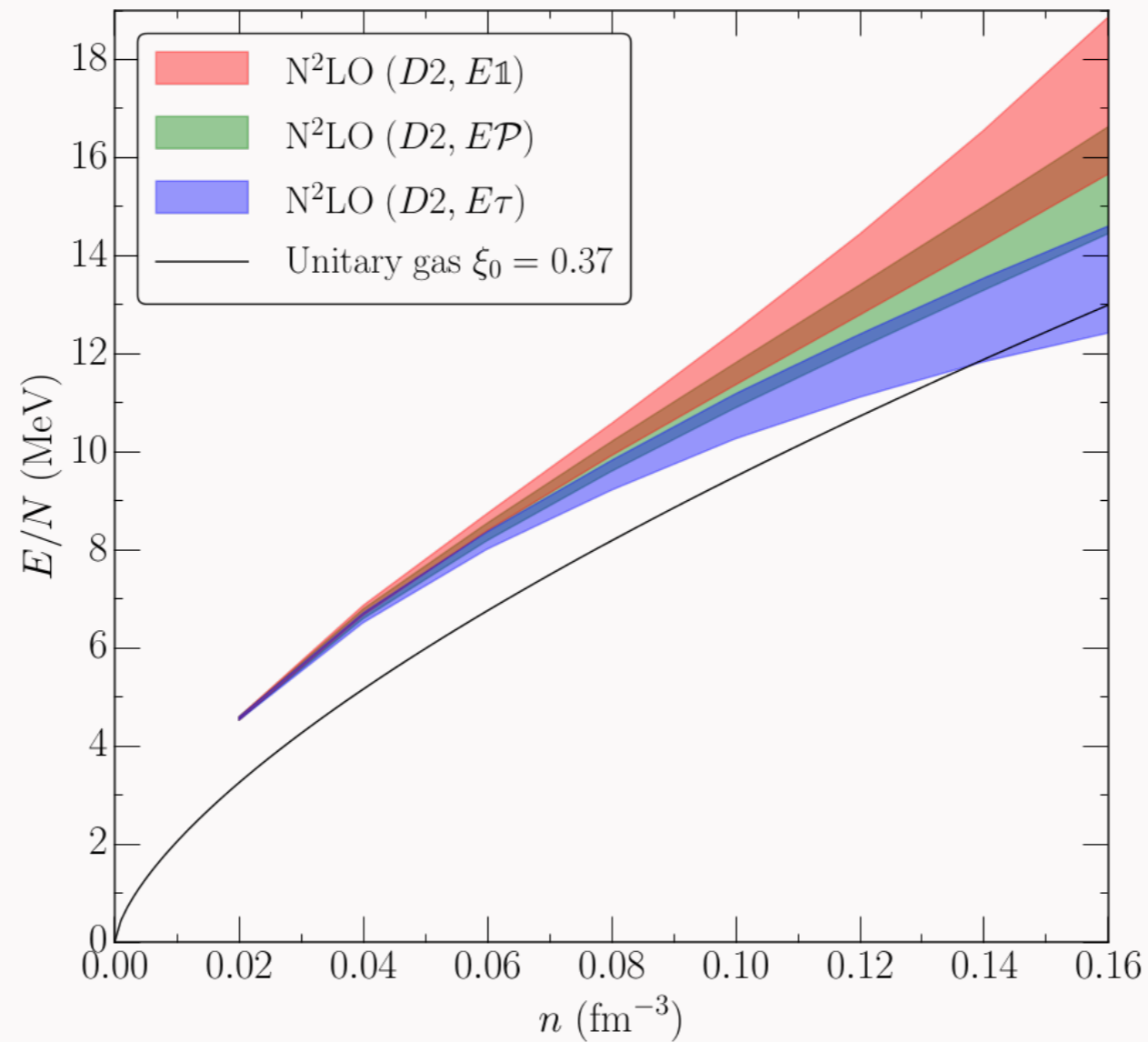


JEL et al, PRL 116, 062501 (2016)



# First Results

One consistent approach:  
Neutron Matter



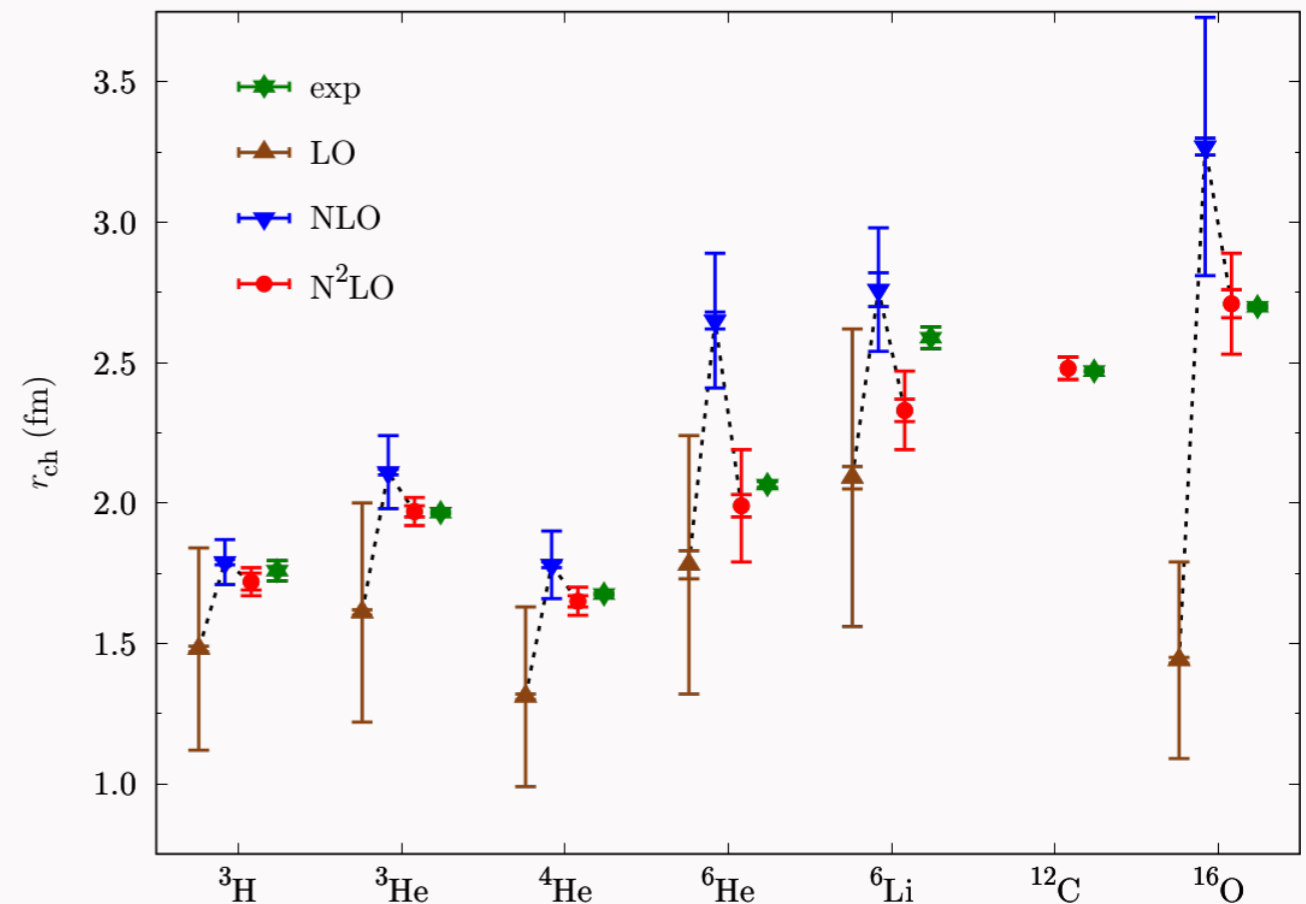
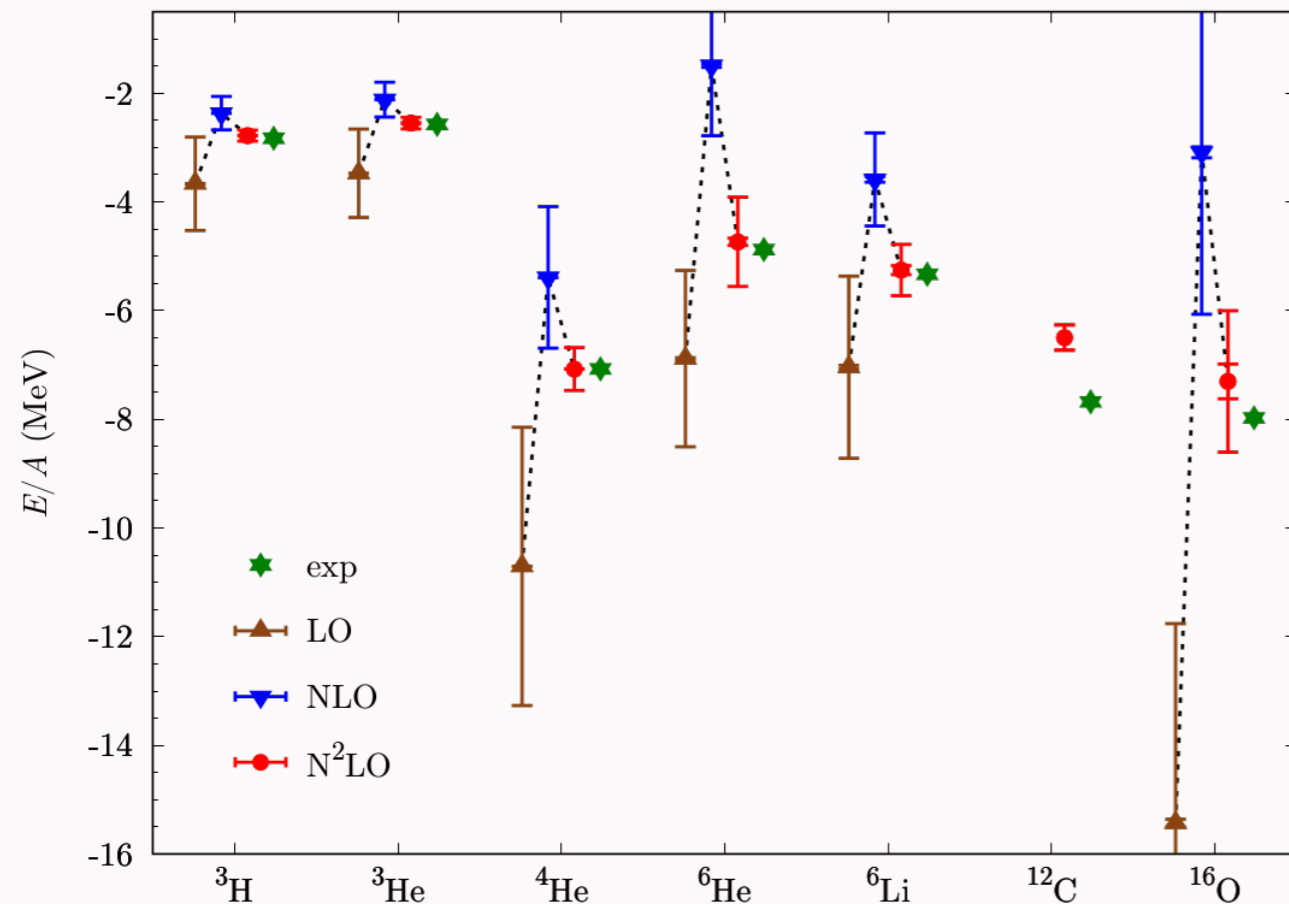
JEL et al, PRL 116, 062501 (2016)

# Recent Advances In AFDMC Calculations

---

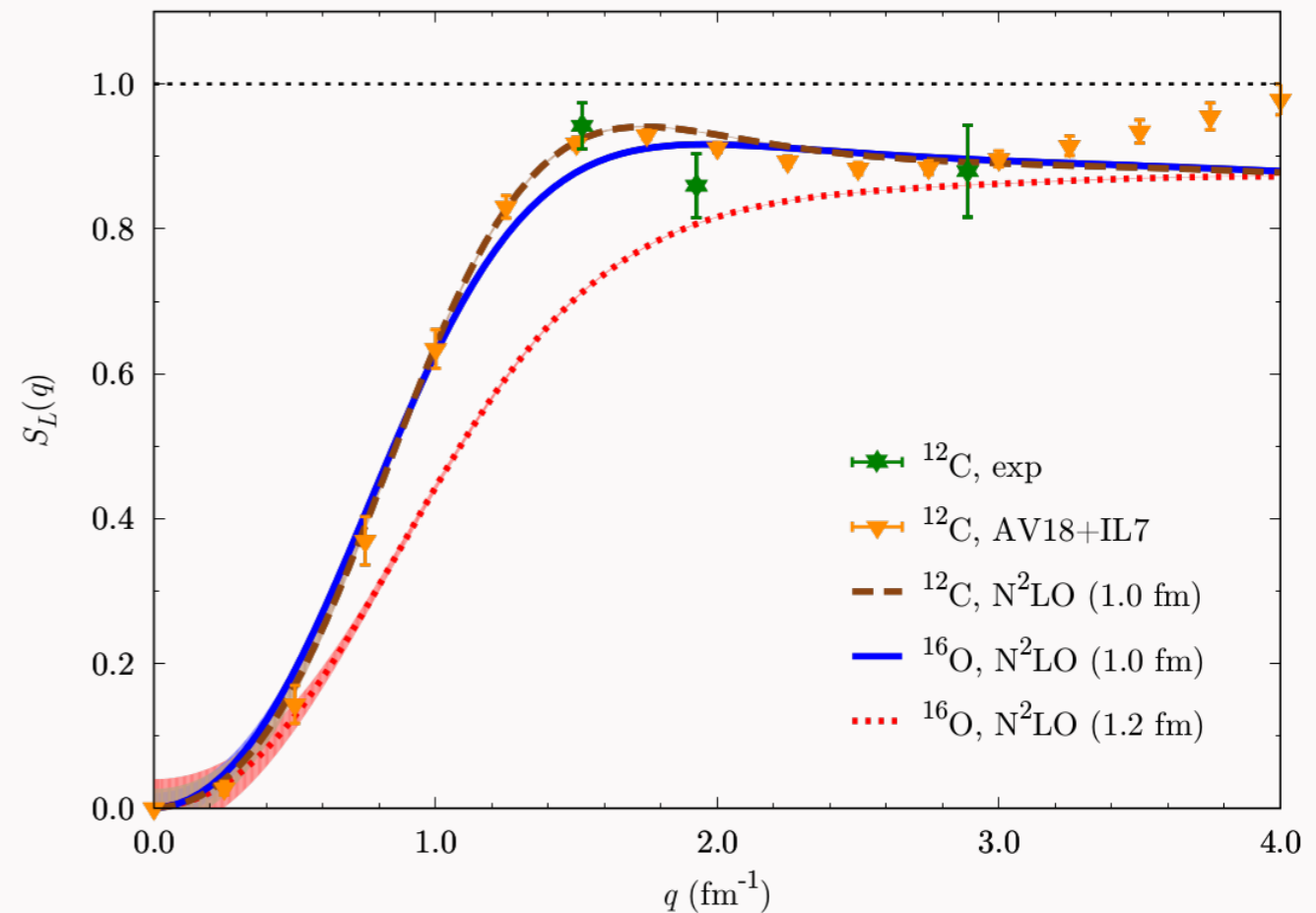
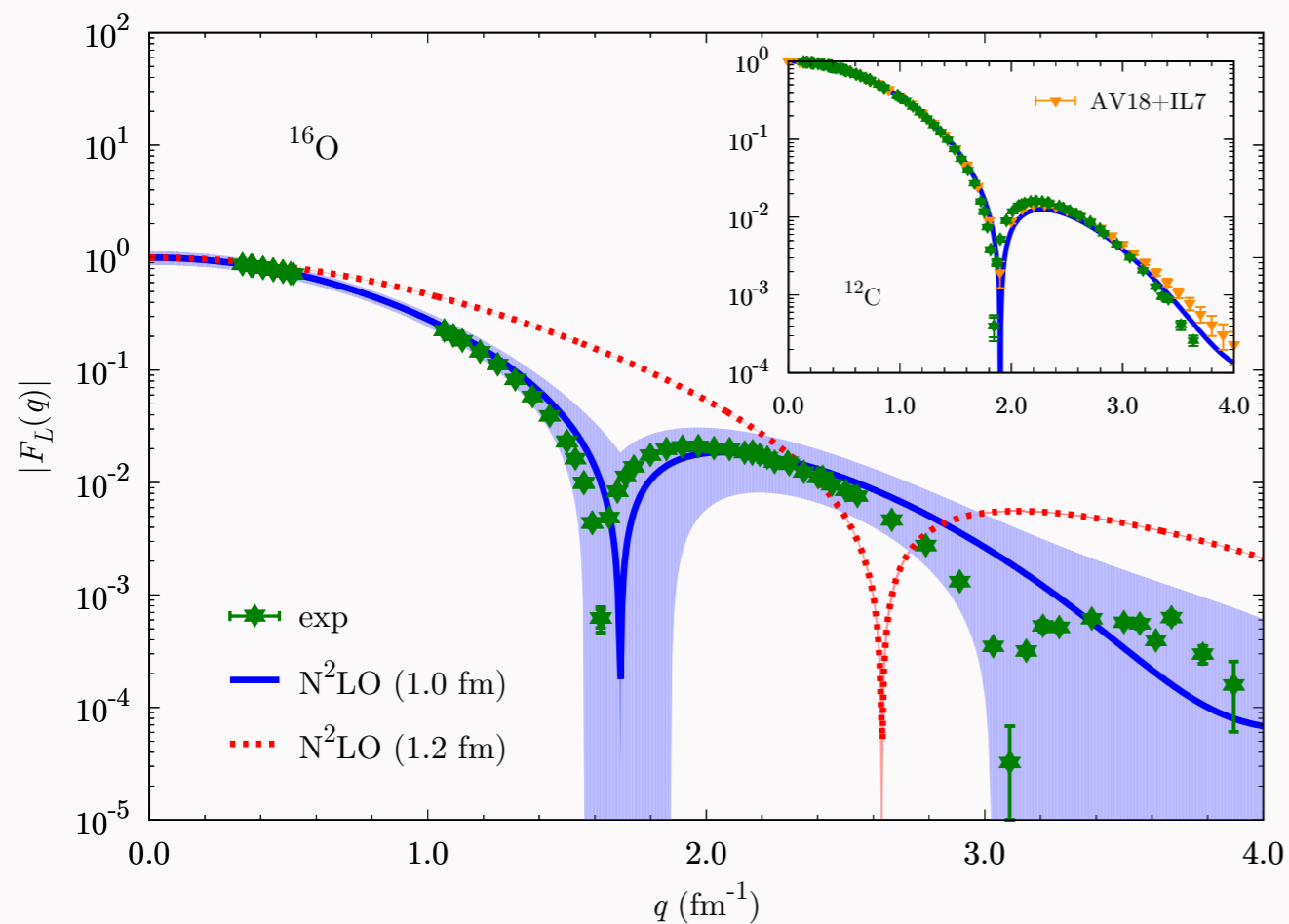
# AFDMC Results

Energies and charge radii of selected nuclei up to  $^{16}\text{O}$  well reproduced.



# AFDMC Results

Charge form factors and Coulomb sum rules also well reproduced.



# $3n, 4n$ Resonances

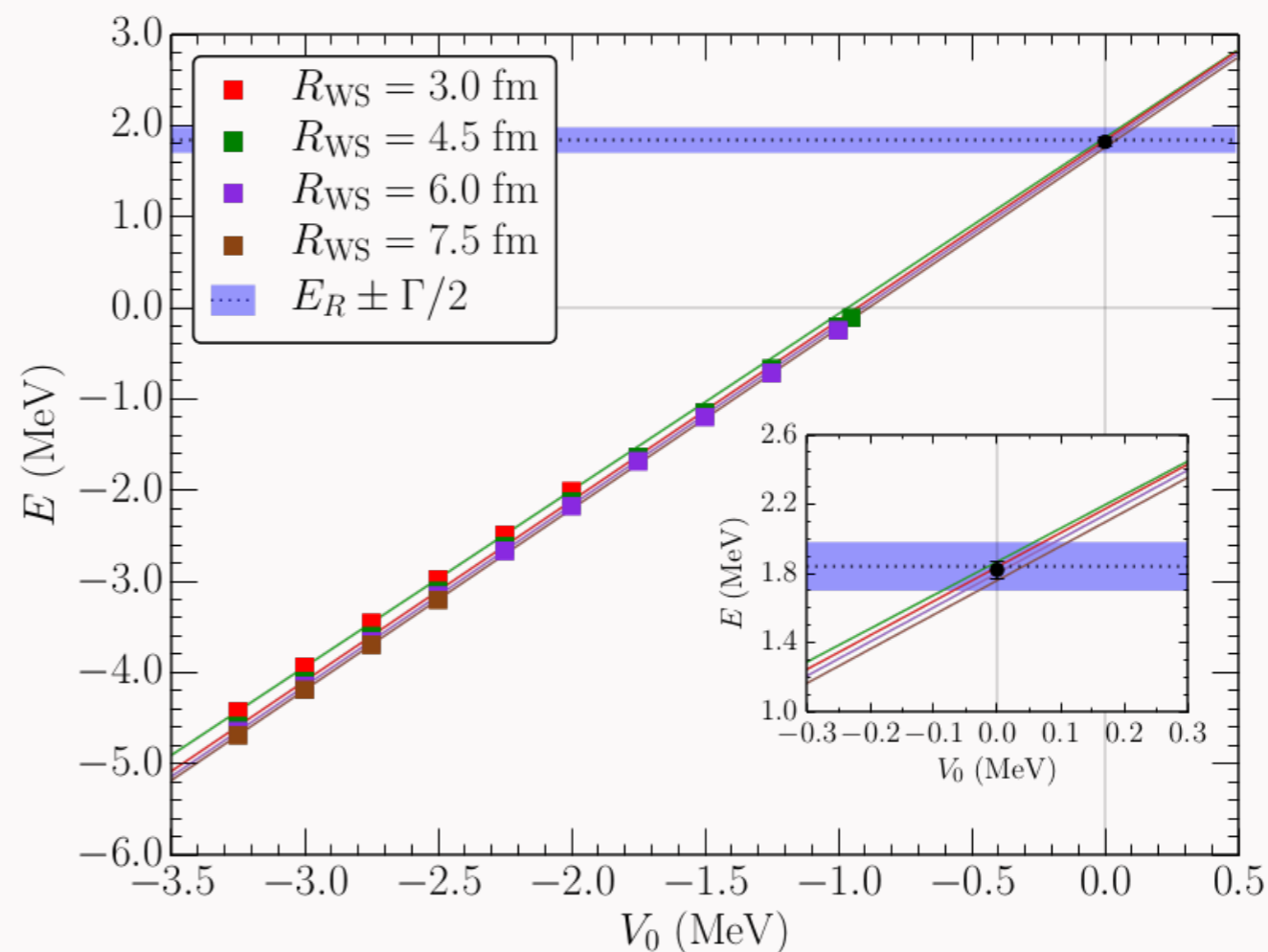
---

# A Two-Body Test

A simple  $S$ -wave potential + Woods-Saxon:

$$V(r) = V_1 e^{-\left(\frac{r}{R_1}\right)^2} + V_2 e^{-\left(\frac{r-r_2}{R_2}\right)^2}$$

$$V_{\text{WS}}(r) = V_0 / [1 + e^{(r-R_{\text{WS}})/a}]$$



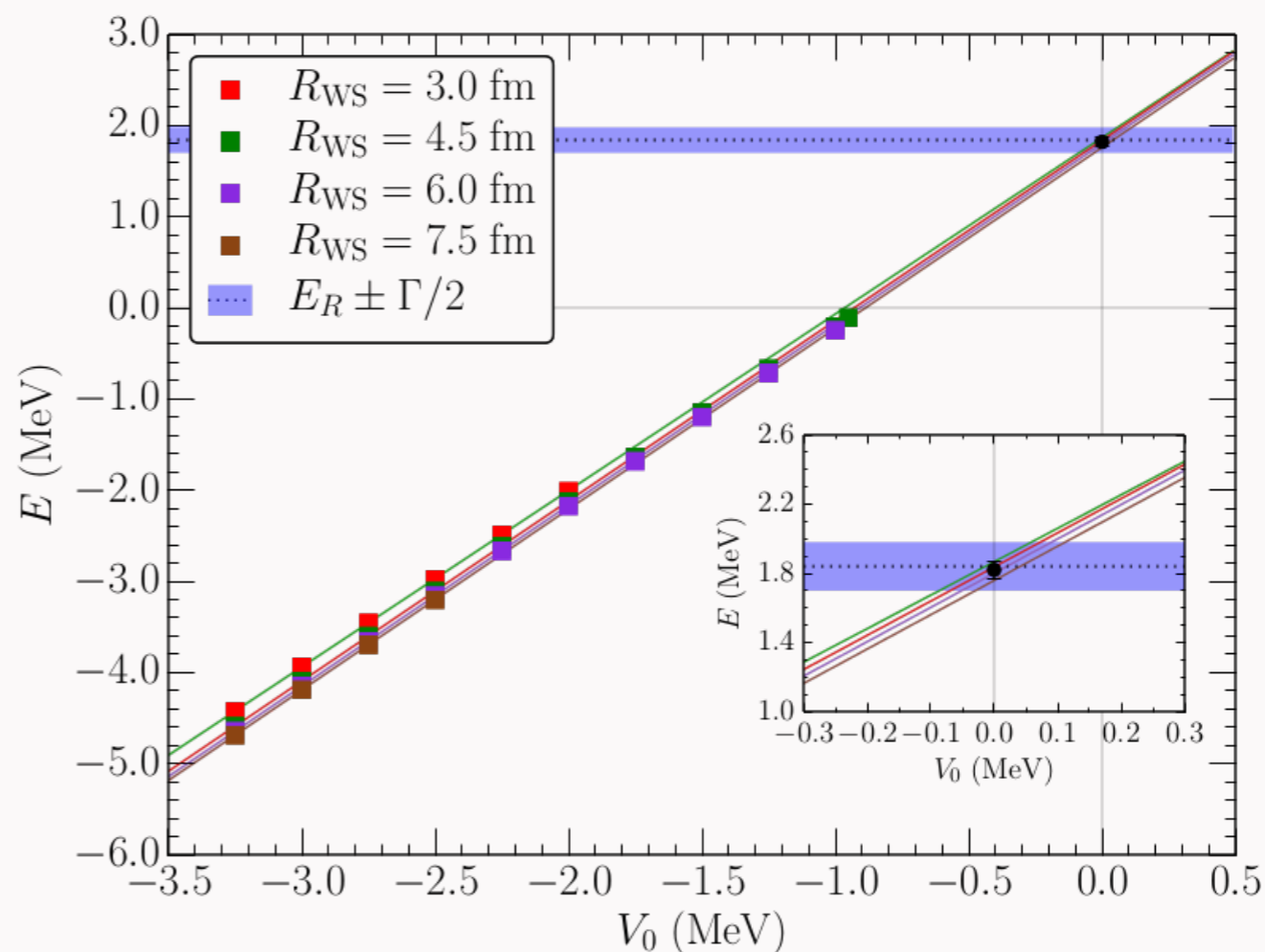
# A Two-Body Test

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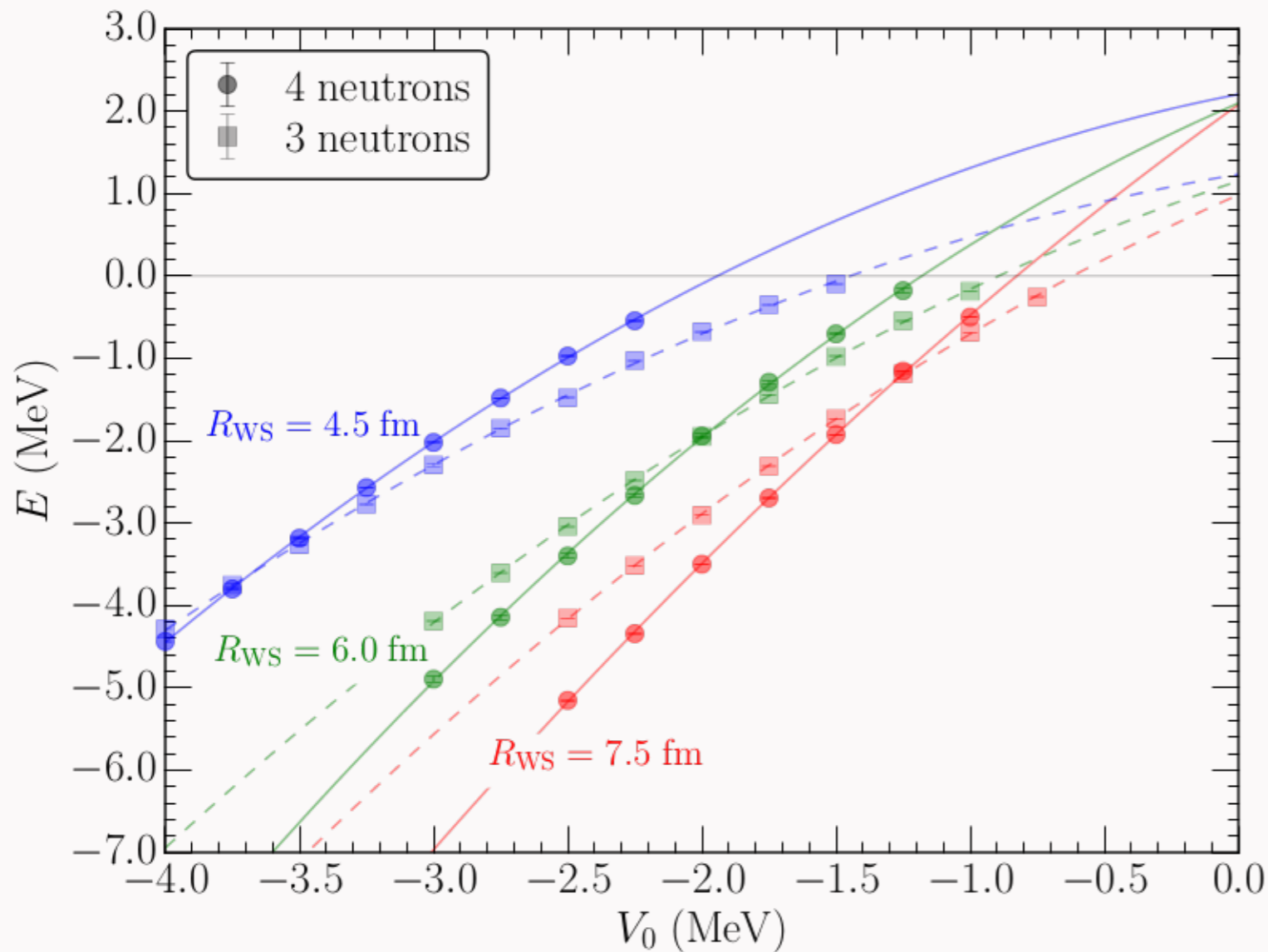
- Different Woods-Saxon radii:  
Independence of trap geometry.
- Extrapolations give 1.83(5) MeV. (Compare to 1.84 MeV).



# Neutrons In A Trap

Now confine 3 & 4 neutrons in the external potential.

$$H = - \sum_i \frac{\hbar^2}{2m} \nabla_i^2 + \sum_i V_{\text{WS}}(r_i) + \sum_{i < j} V_{ij} + \sum_{i < j < k} V_{ijk},$$

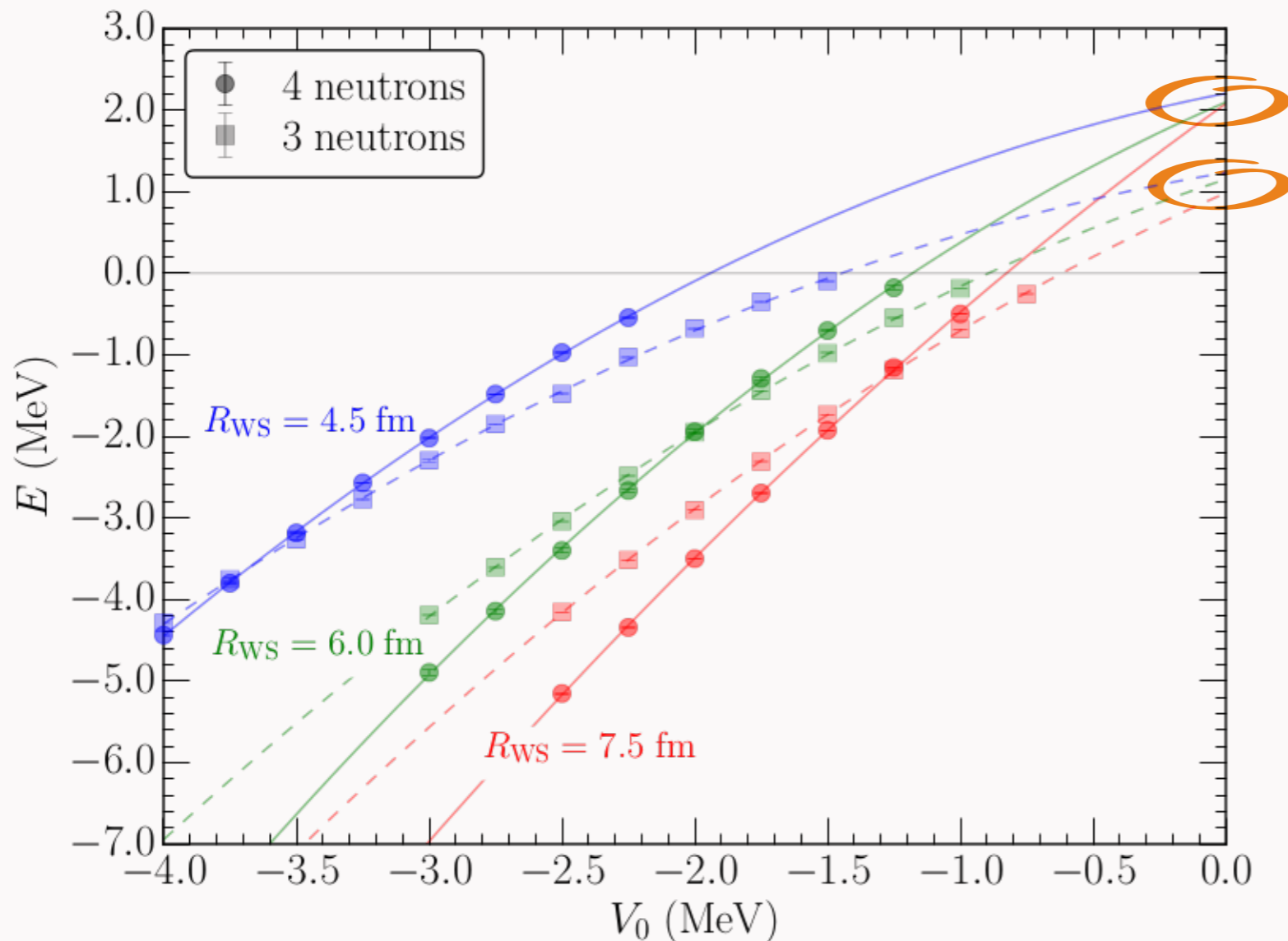




# Neutrons In A Trap

Now confine 3 & 4 neutrons in the external potential.

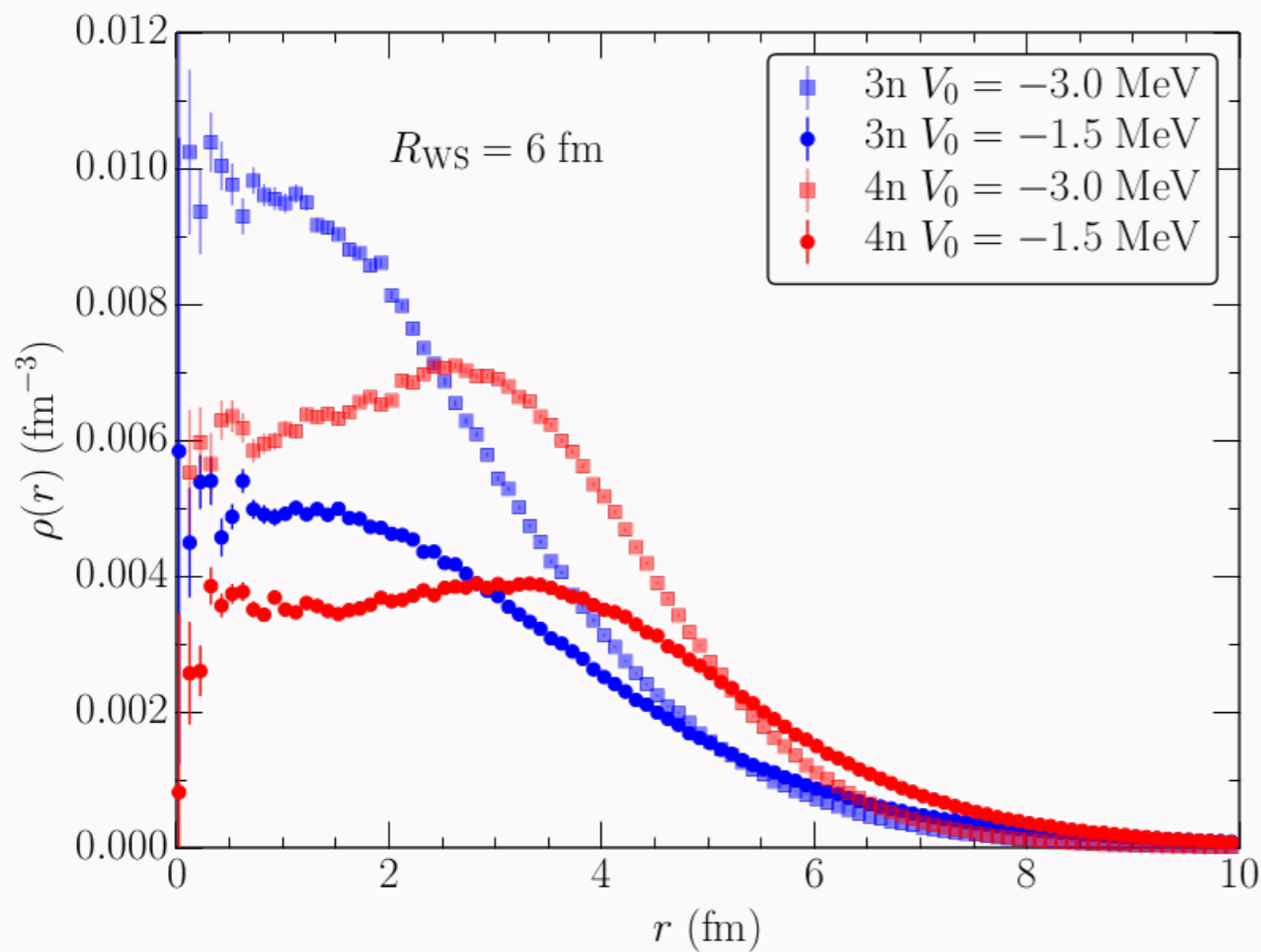
$$H = - \sum_i \frac{\hbar^2}{2m} \nabla_i^2 + \sum_i V_{\text{WS}}(r_i) + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk},$$



- $E_{4n} = 2.1(2)$  MeV,  
 $E_{3n} = 1.1(2)$  MeV.
- ${}^3n$  resonance lower than  ${}^4n$  resonance!

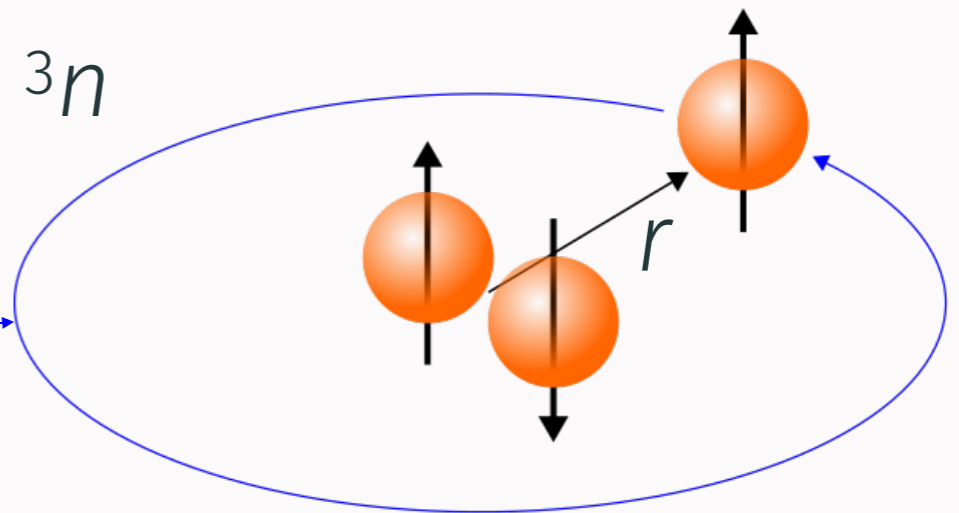
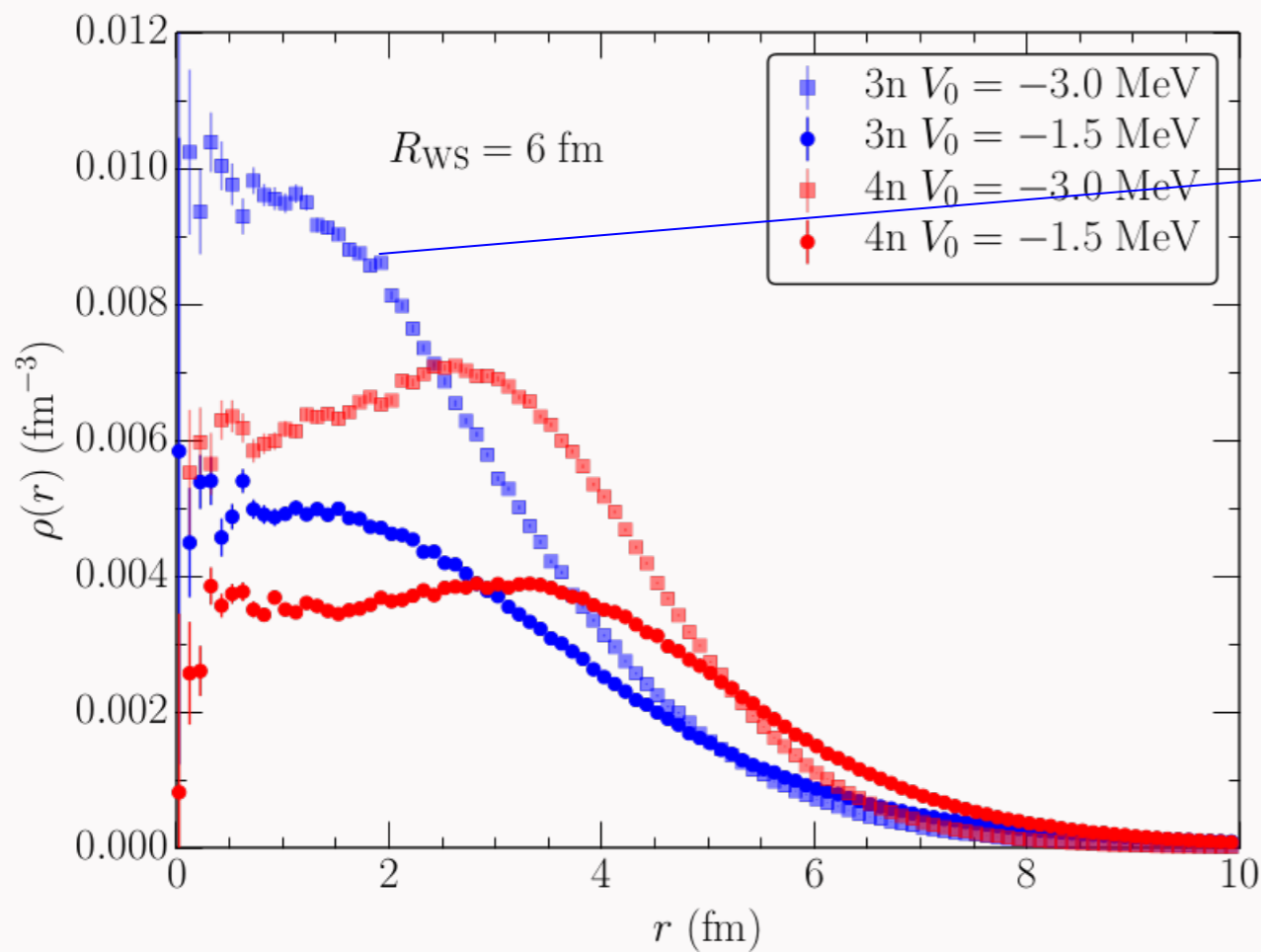
# One-Body Densities

- The  $^3n$  and  $^4n$  systems are very dilute.
- $^3n$  and  $^4n$  systems show different short-distance structure.



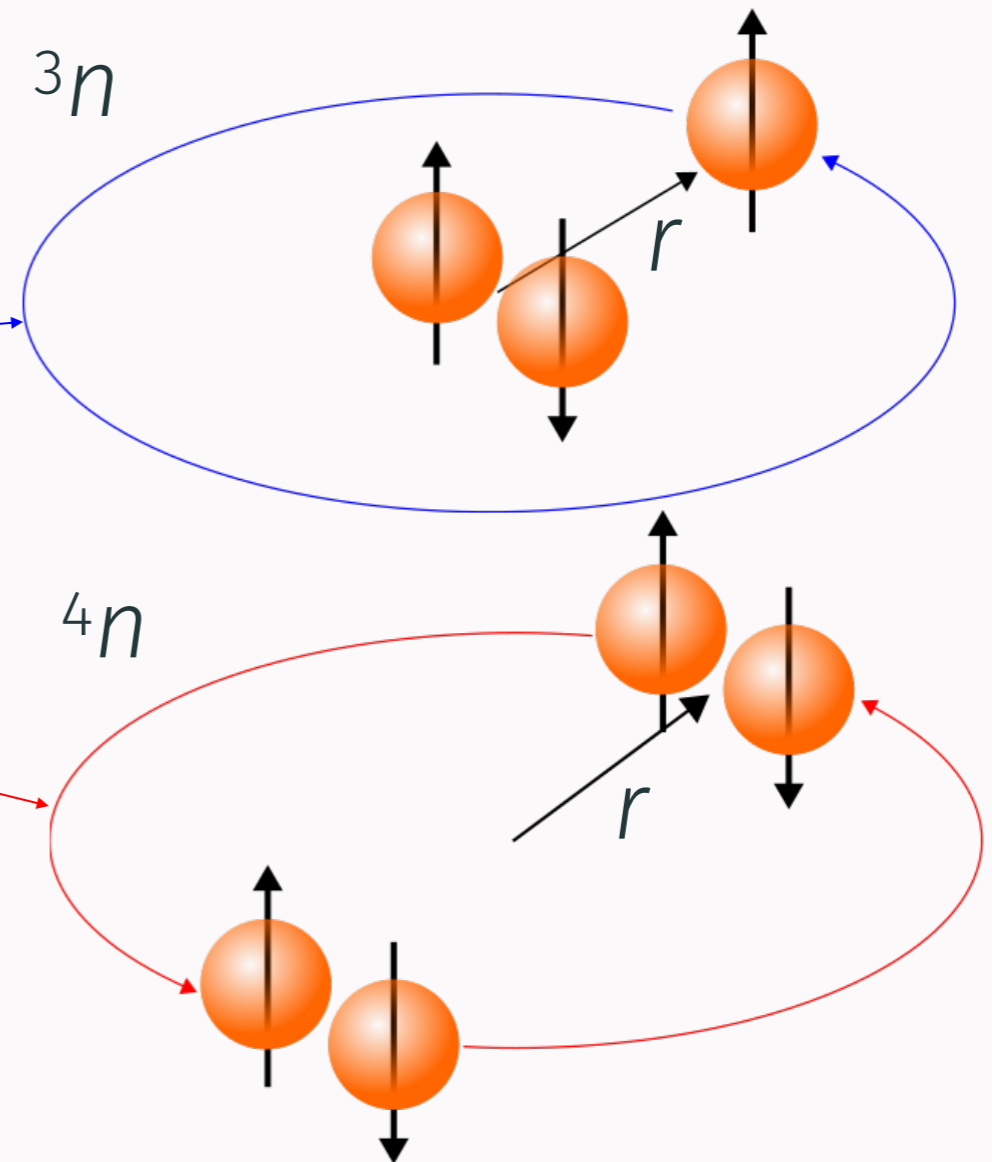
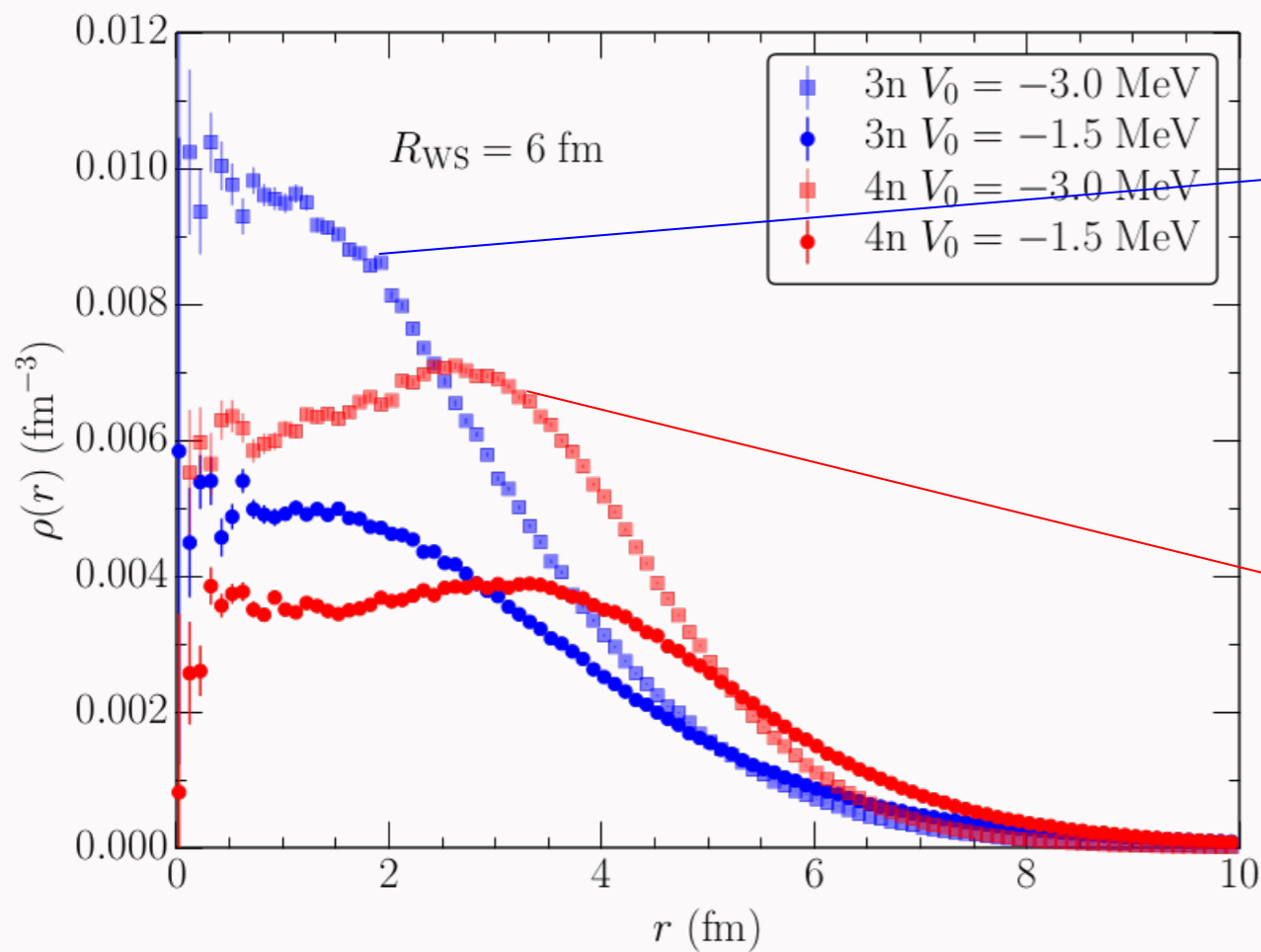
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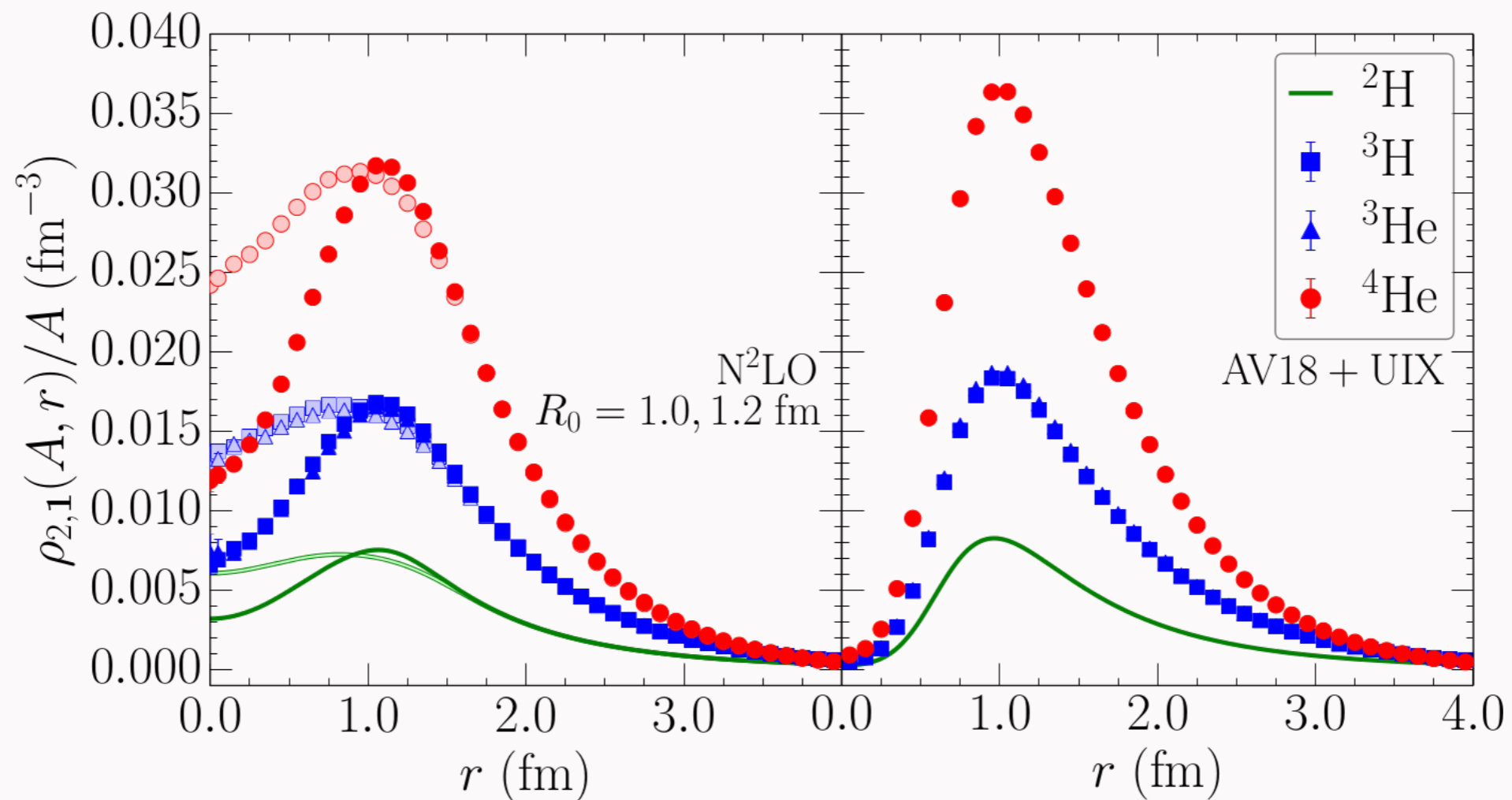
# Short-Range Correlations (SRCs) & EMC Effect

---

# Two-Body Densities

$$\rho_{2,1}(A, r) \equiv \frac{1}{4\pi r^2} \langle \Psi_0 | \sum_{i < j} \delta(r - r_{ij}) | \Psi_0 \rangle$$

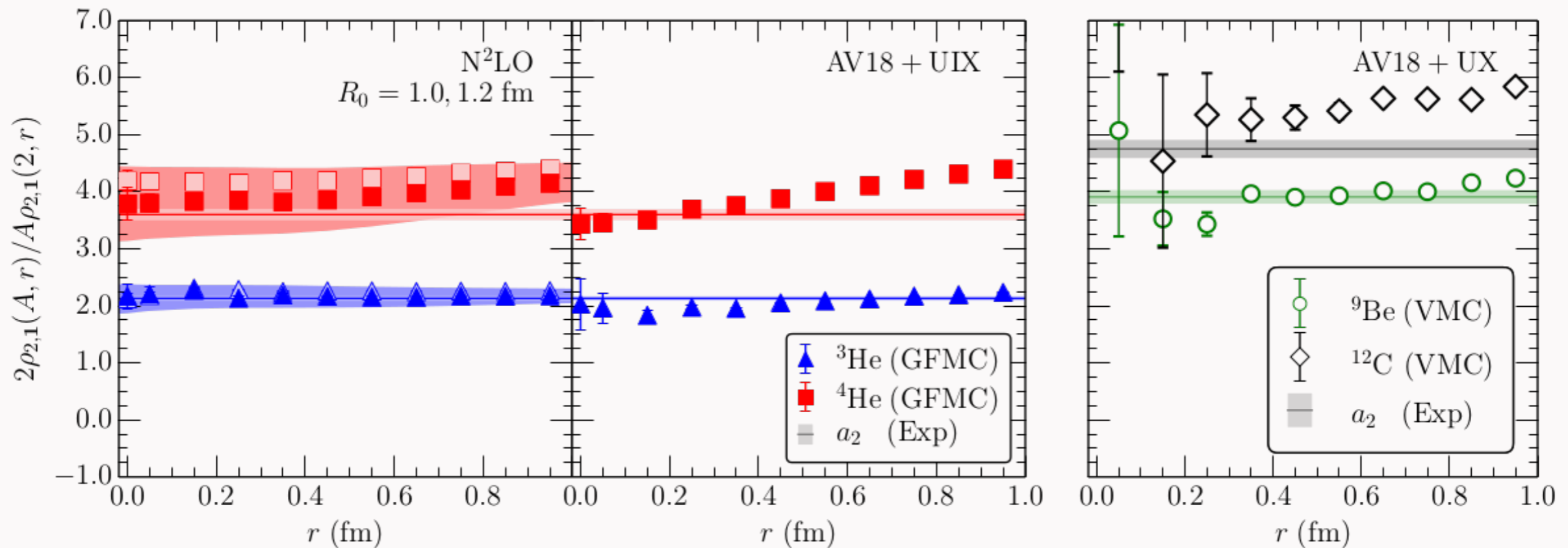
Scale and scheme dependent



# SRC Correlation Factors

$$a_2 \equiv \lim_{r \rightarrow 0} \frac{2\rho_{2,1}(A, r)}{A\rho_{2,1}(2, r)}$$

Scale and scheme *independent!*

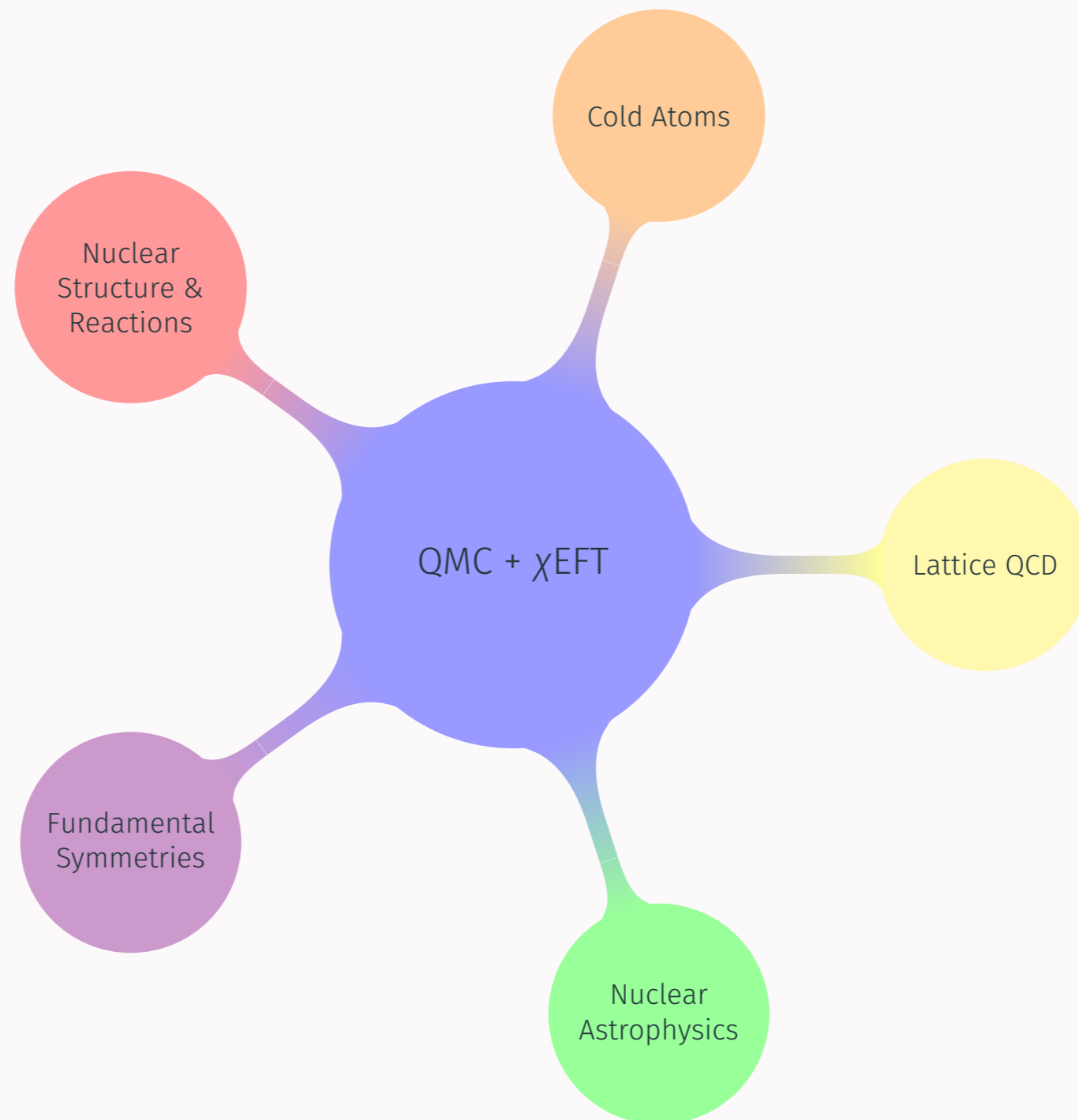


# Summary

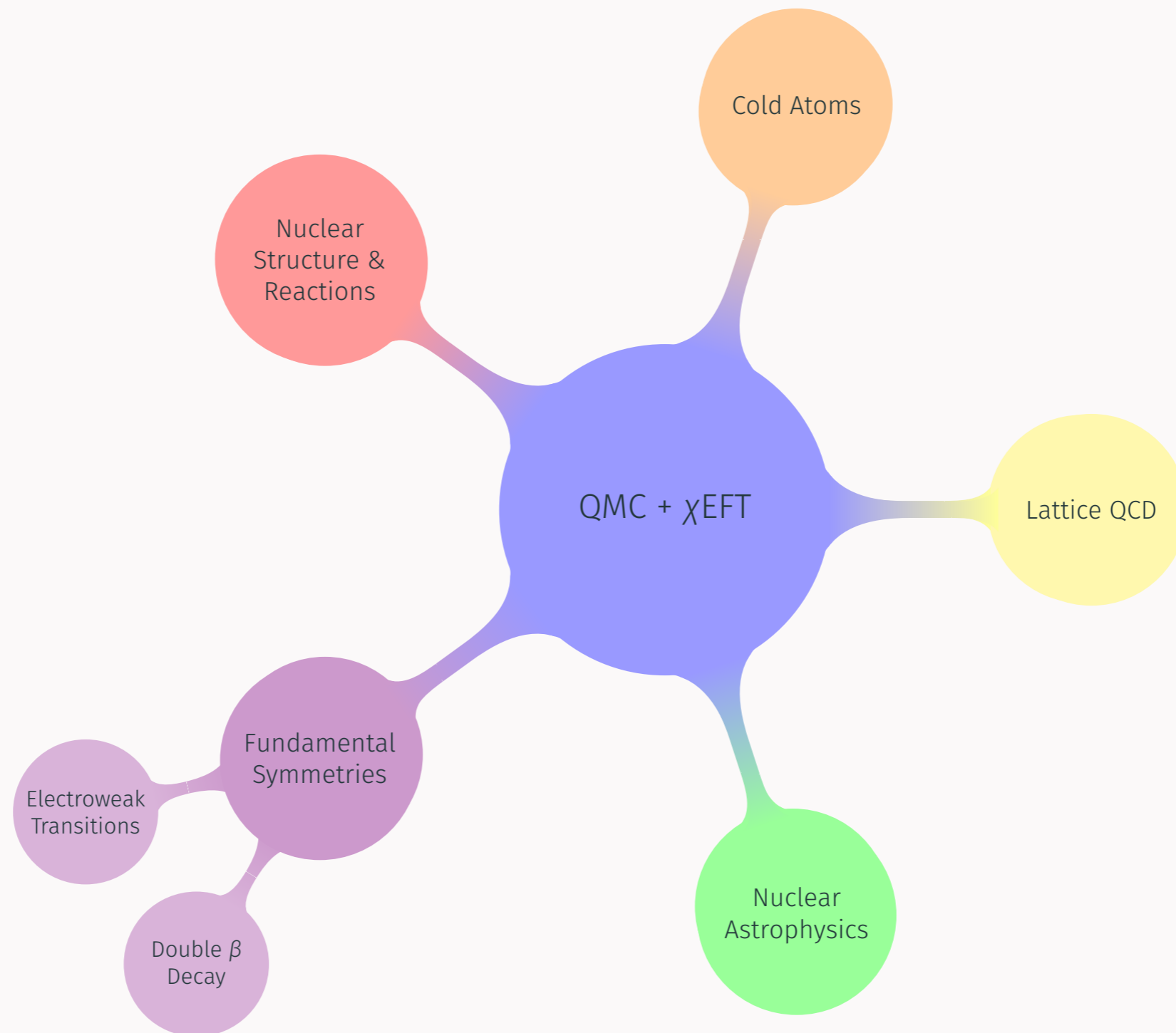
- An exciting time in nuclear physics thanks to new experiments, many-body methods, and chiral EFT.
- QMC methods with chiral EFT interactions: A powerful set of tools to advance nuclear physics.
- AFDMC is beginning to reach towards *ab initio* medium-mass nuclei.
- A  $3n$  resonance might be lower in energy than a  $4n$  resonance and might be observable as well.
- We can make scheme- and scale-independent predictions for SRC scaling factors.



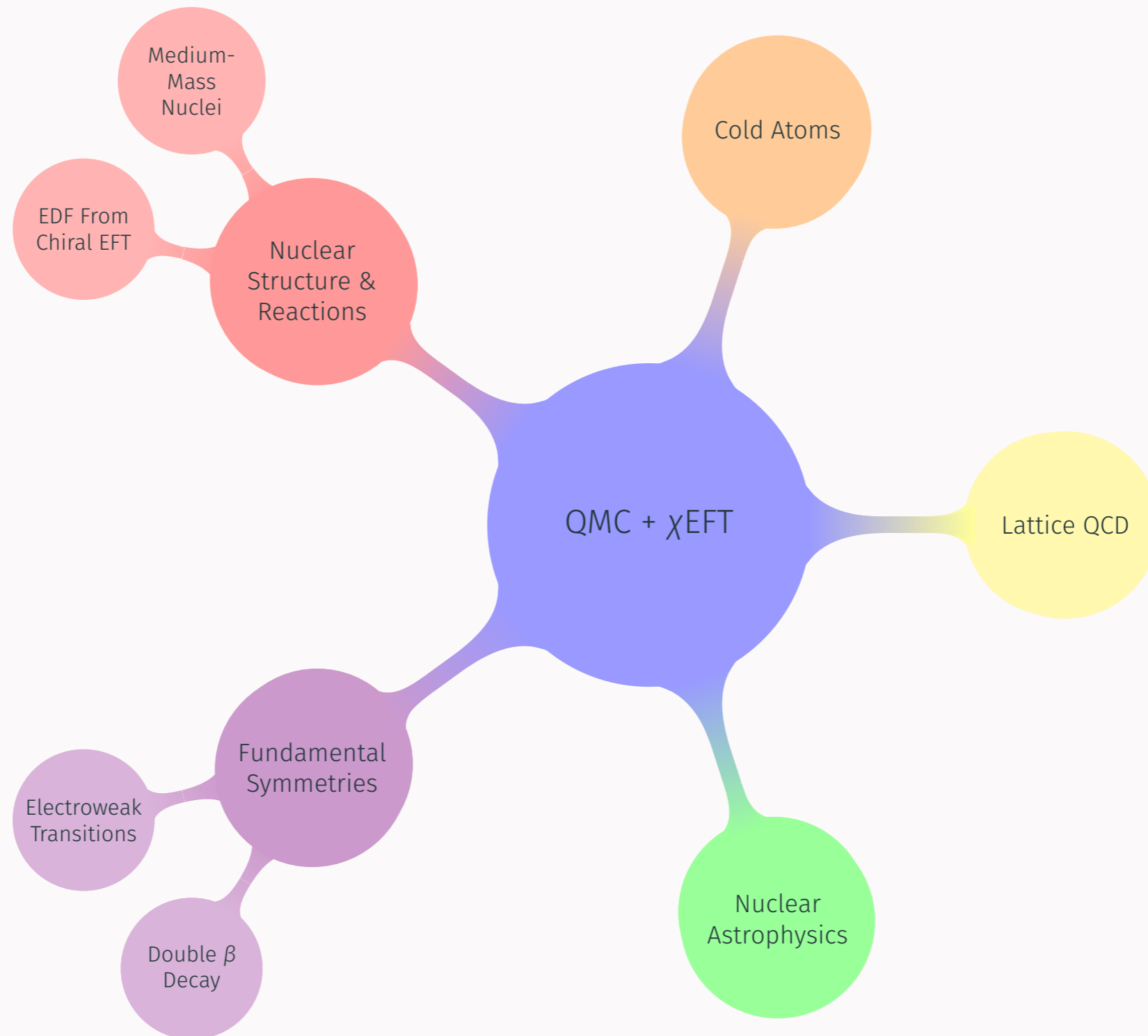
# Outlook



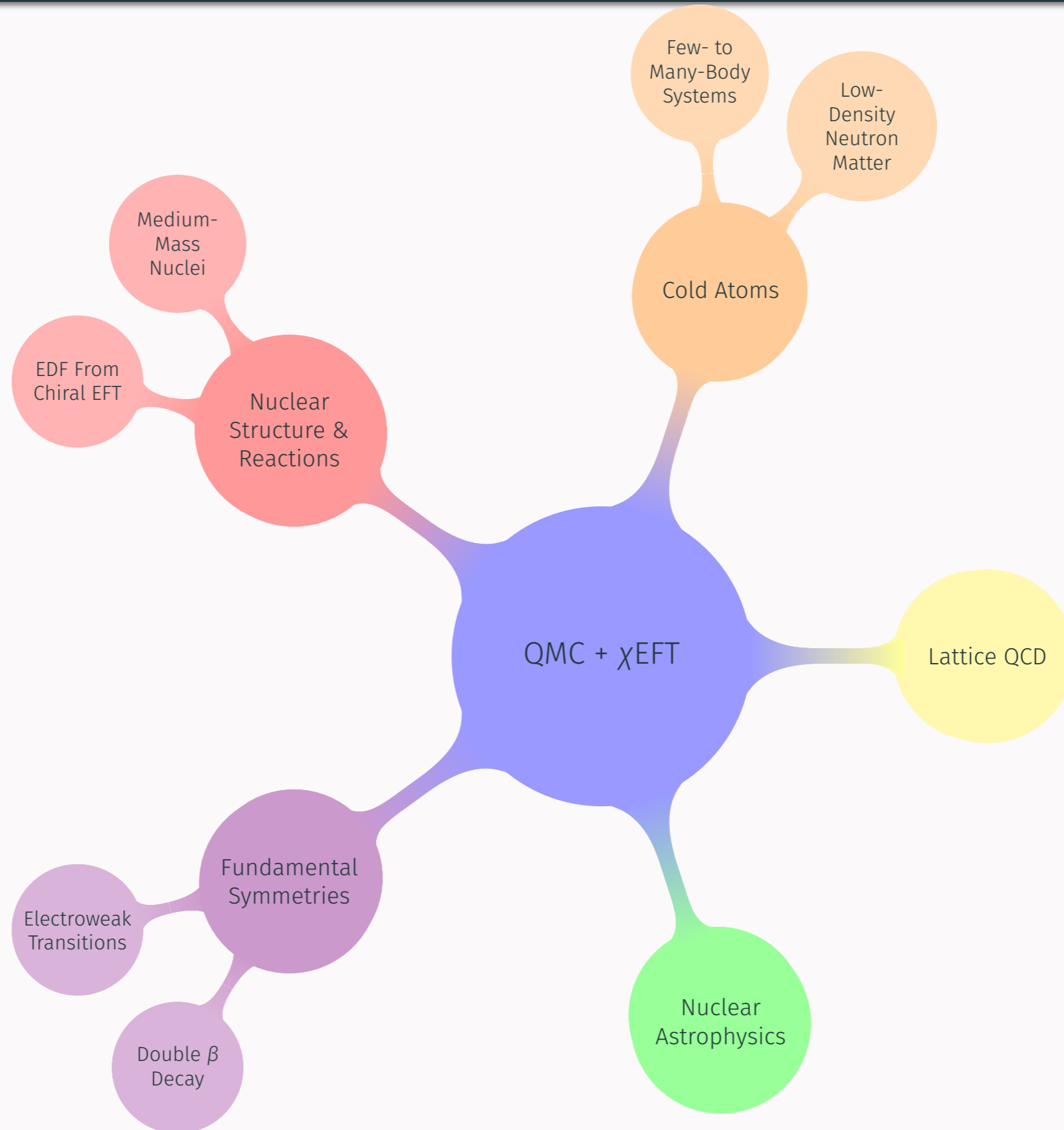
# Outlook



# Outlook



# Outlook



# Acknowledgments

## Collaborators

- P. Klos  
H.-W. Hammer  
A. Schwenk



- J. Carlson,  
S. Gandolfi,  
D. Lonardoni



- I. Tews



- J.-W. Chen



- A. Gezerlis



- K. E. Schmidt



- W. Detmold



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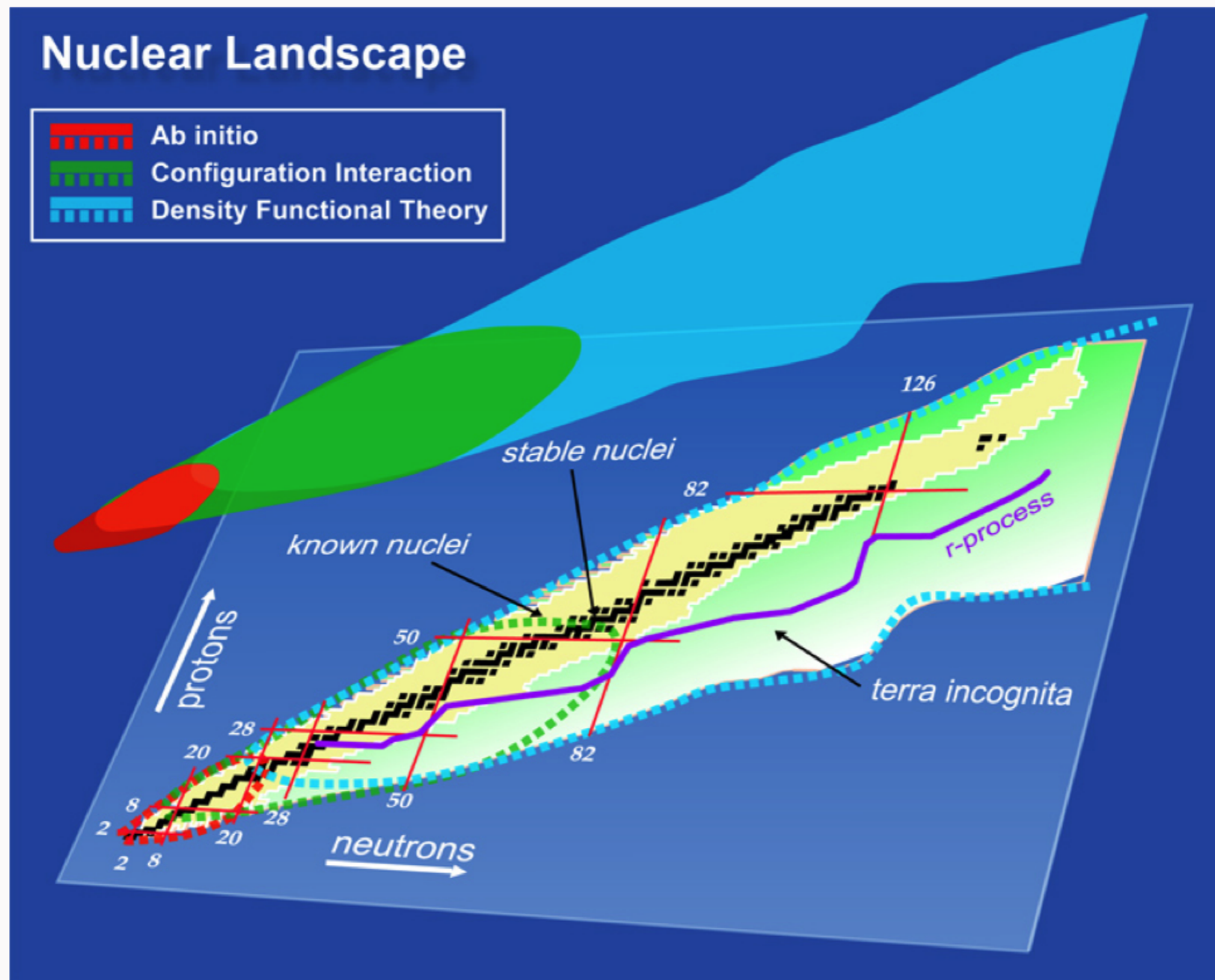


- W. Detmold

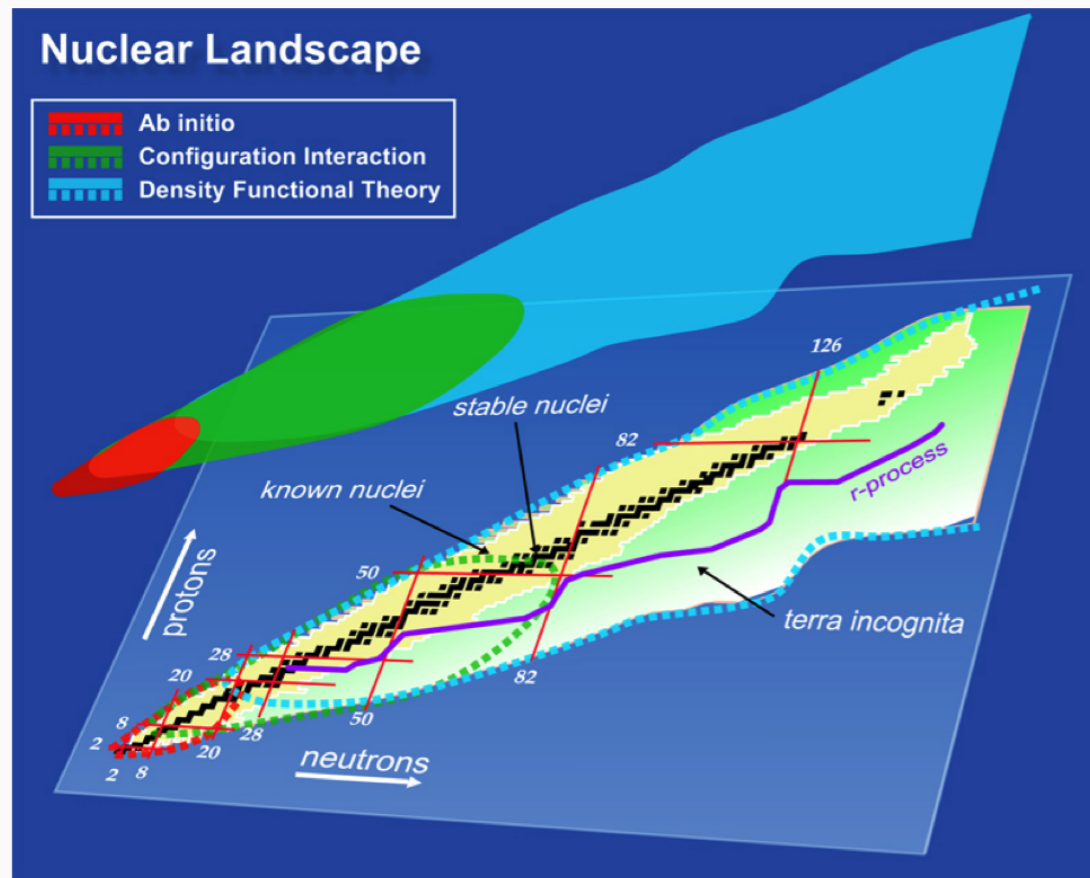


**Thank you for your attention!**

# Reach Of *Ab Initio* Methods

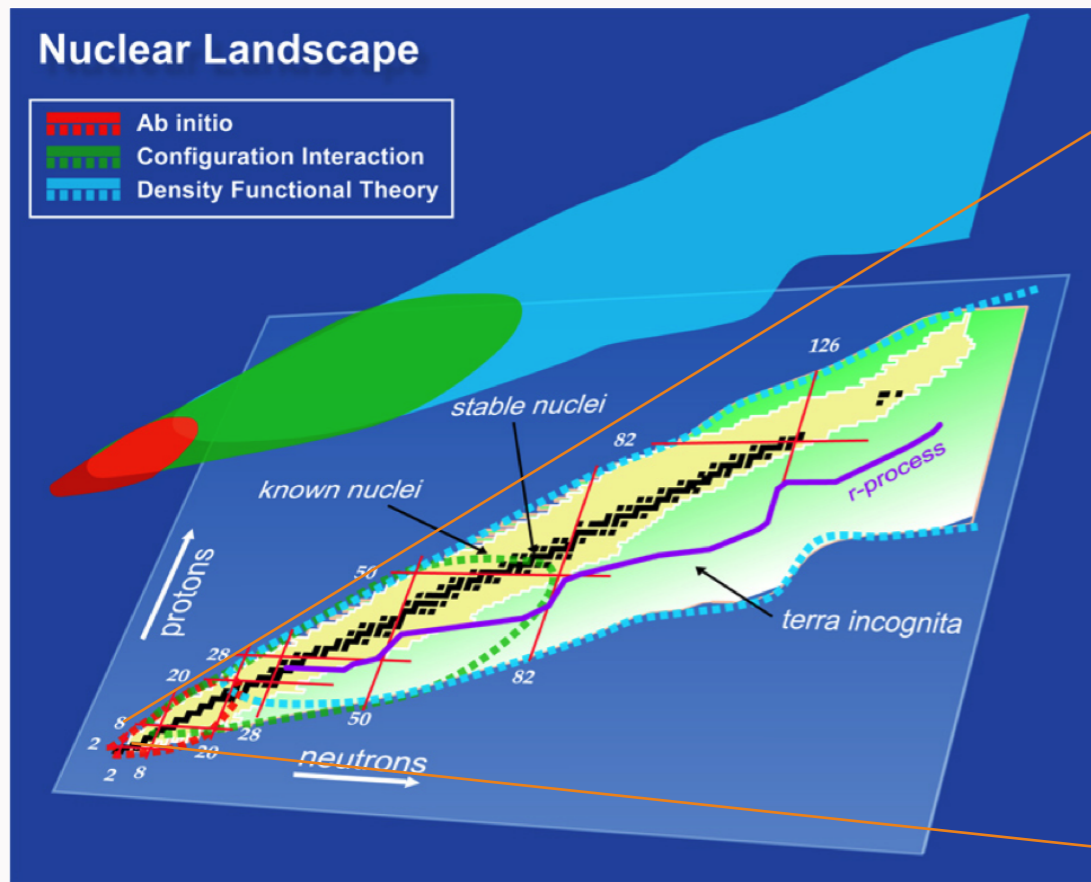


# Reach Of *Ab Initio* Methods

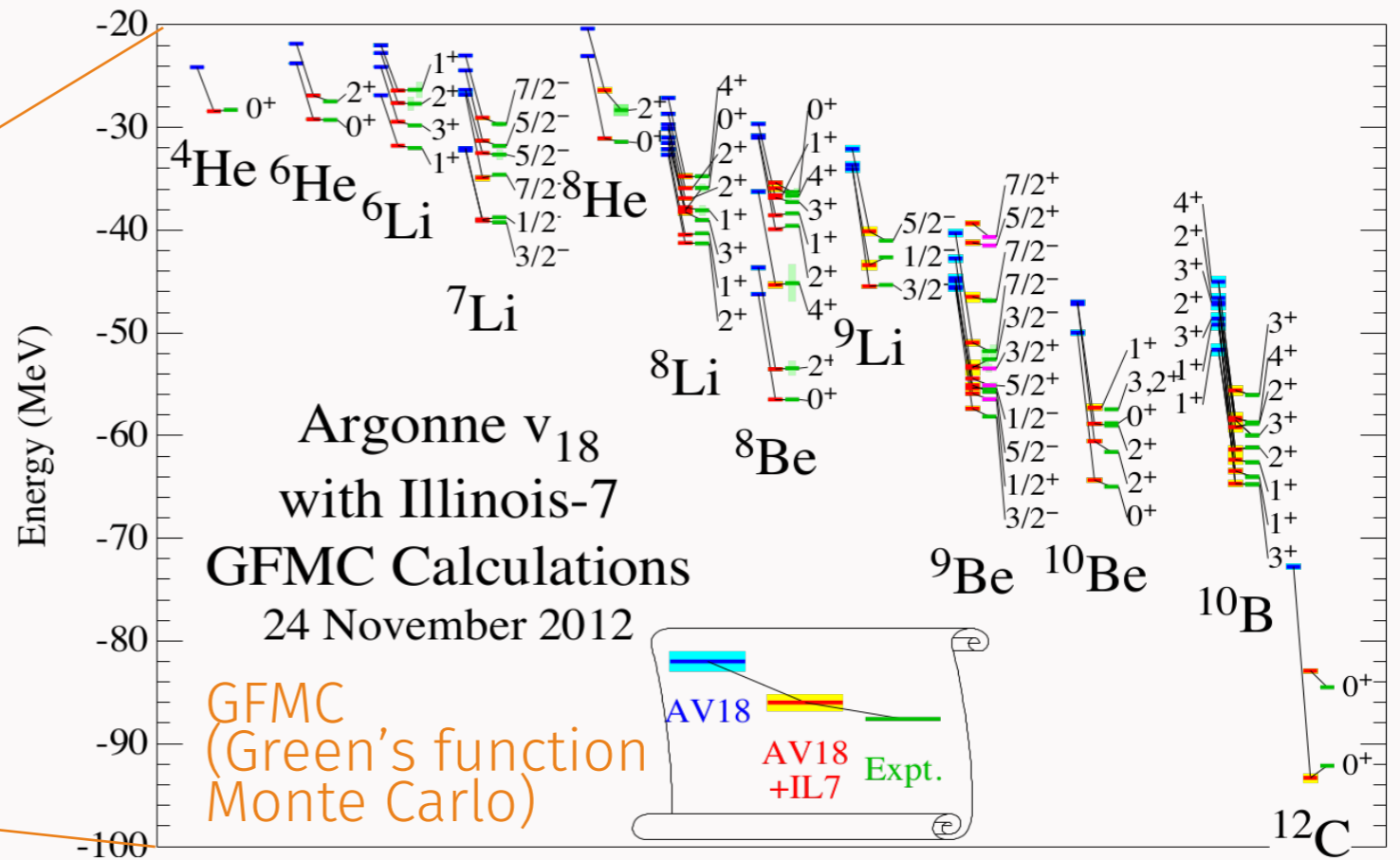




# Reach Of *Ab Initio* Methods



from H. Nam et al., J. Phys. Conf. Ser. 402, 012033 (2012)



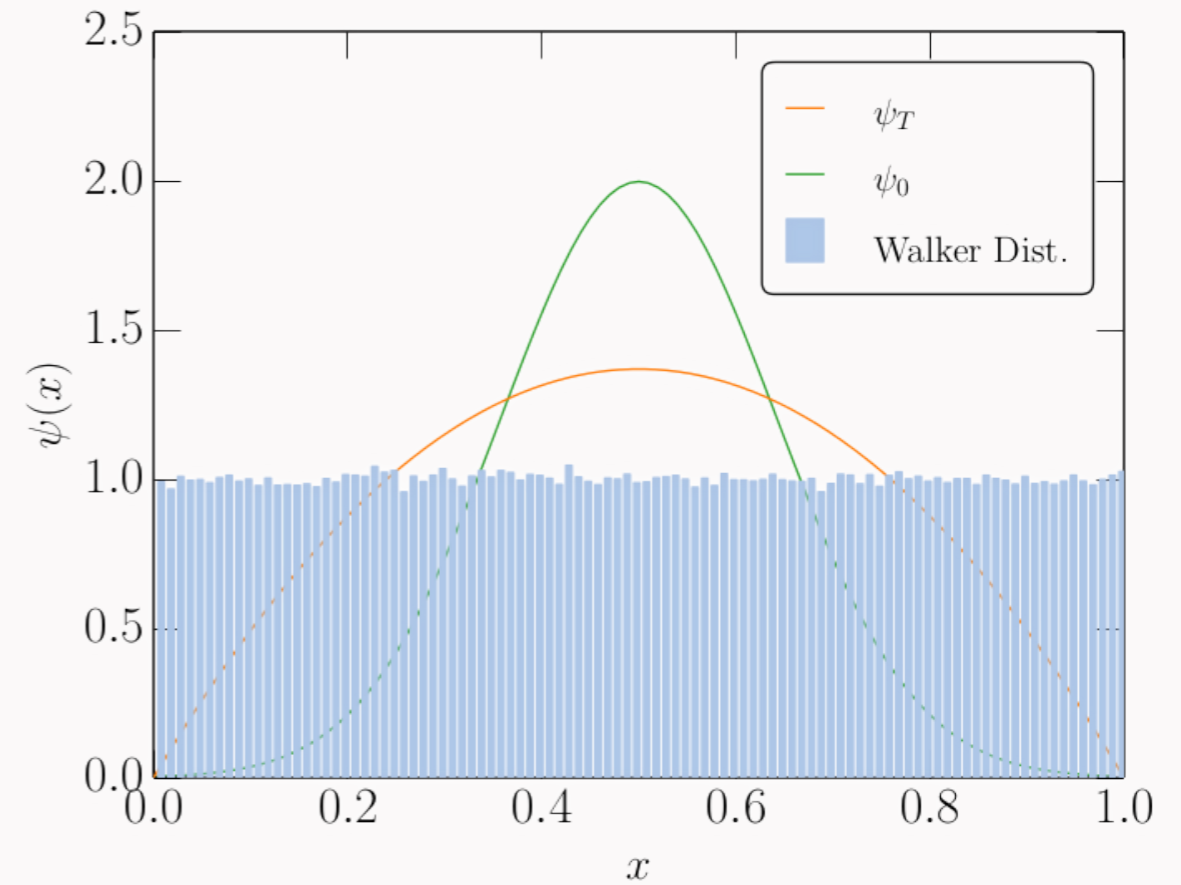
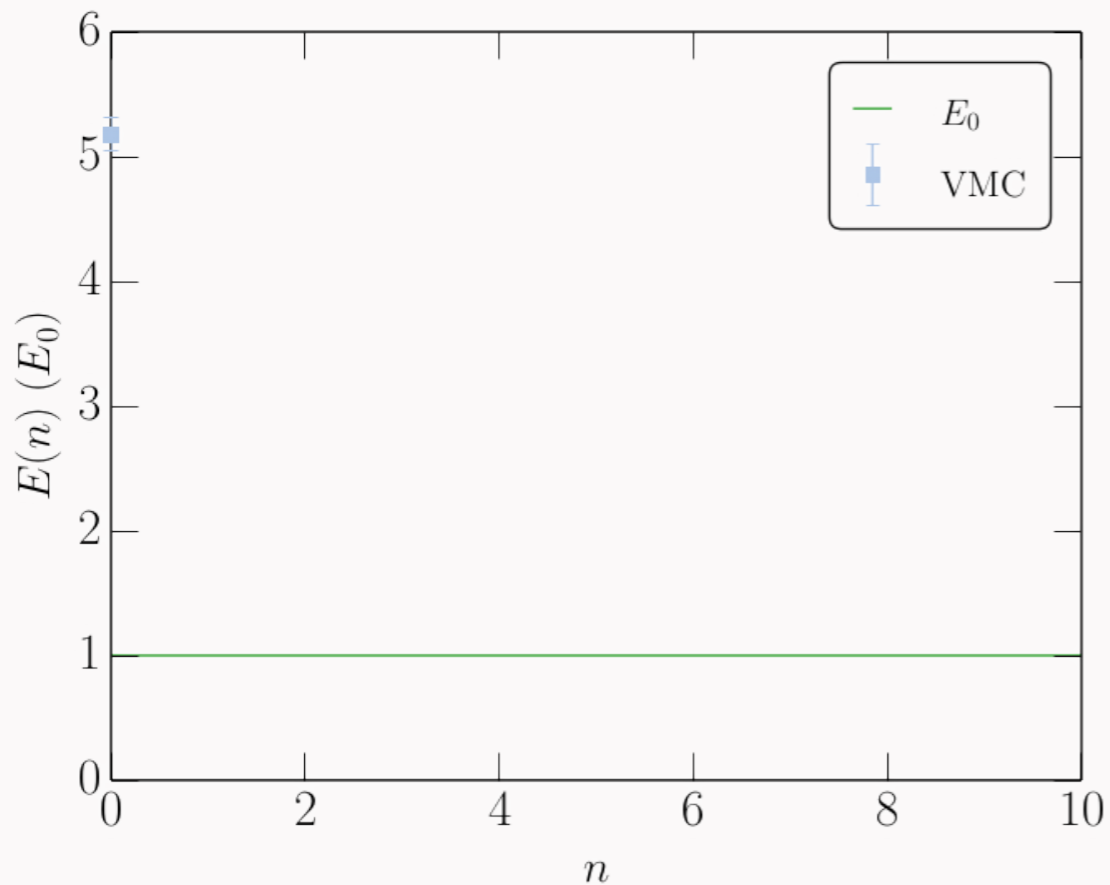
from the Argonne National Lab Group

# QMC Methods - An Example

First, VMC equilibration:

$$\frac{\langle \psi_T | H | \psi_T \rangle}{\langle \psi_T | \psi_T \rangle}$$

Metropolis step  $n = 0$

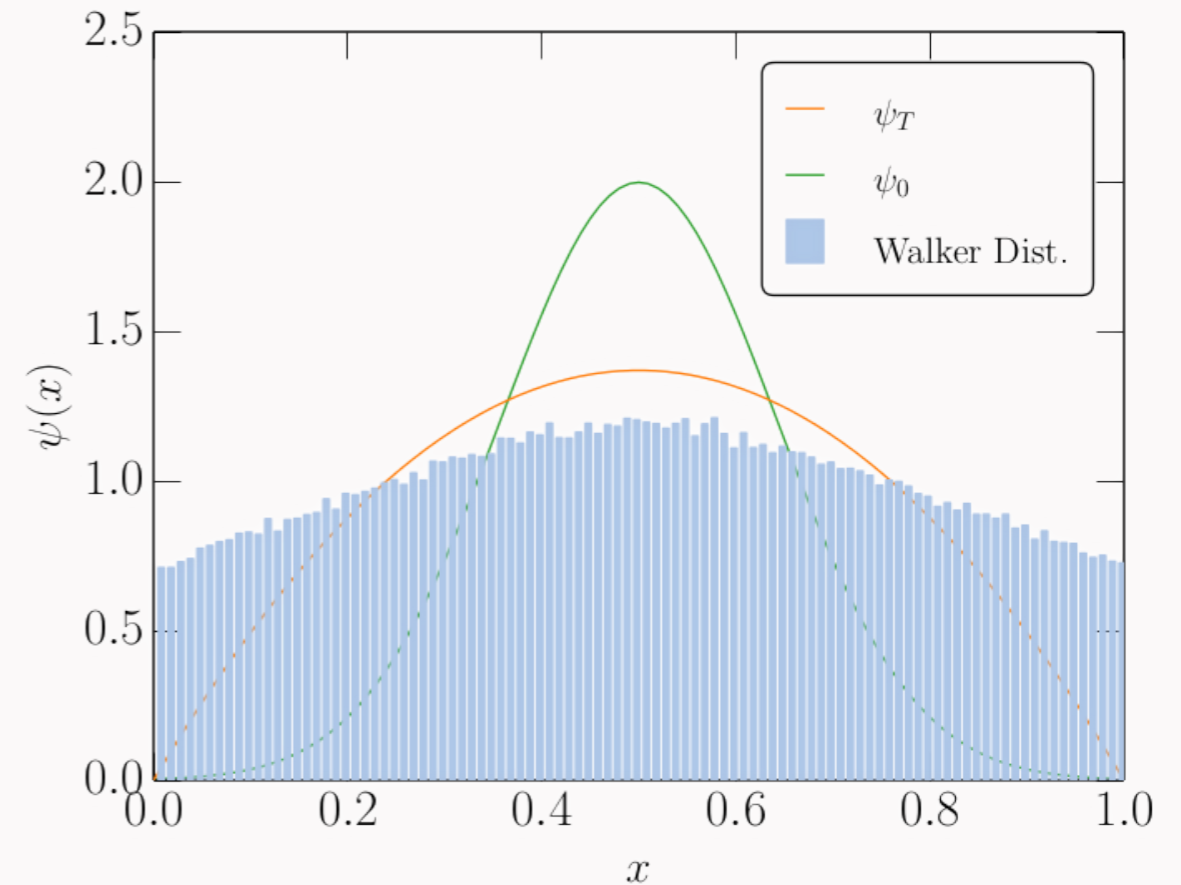
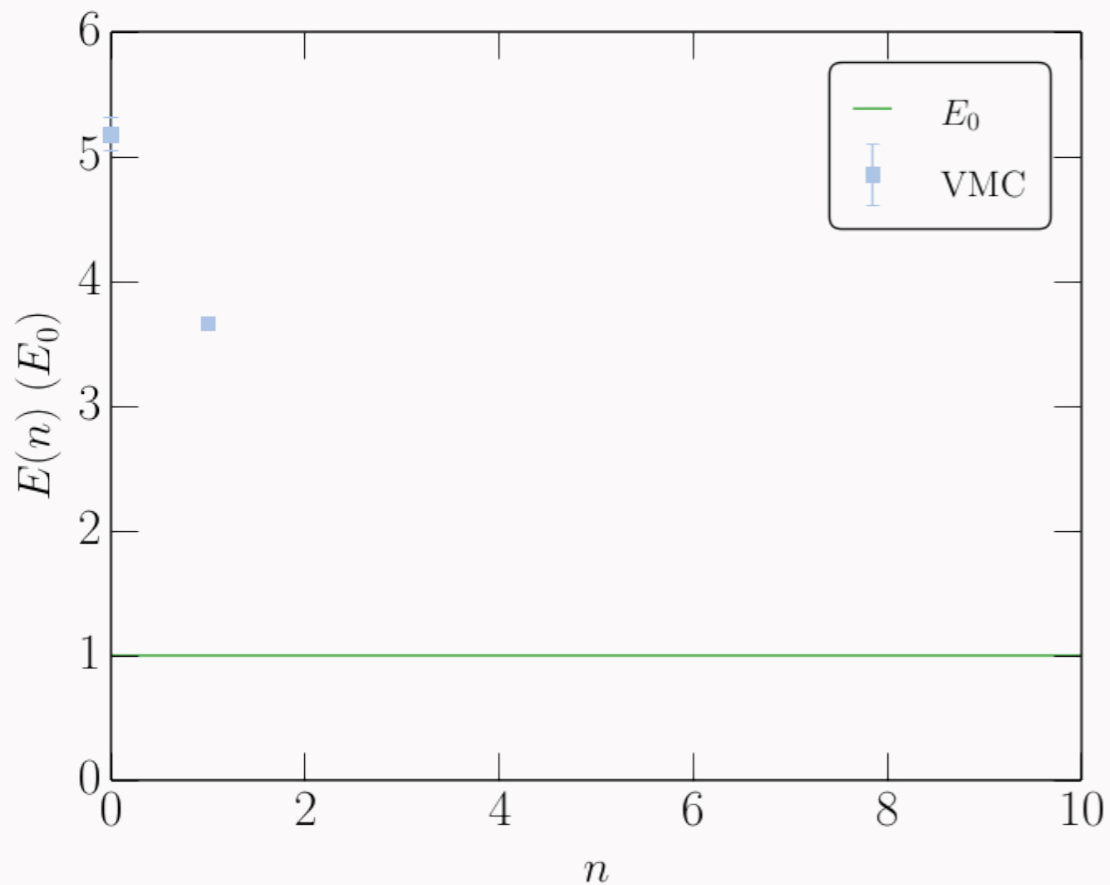


# QMC Methods - An Example

First, VMC equilibration:

$$\frac{\langle \psi_T | H | \psi_T \rangle}{\langle \psi_T | \psi_T \rangle}$$

Metropolis step  $n = 1$

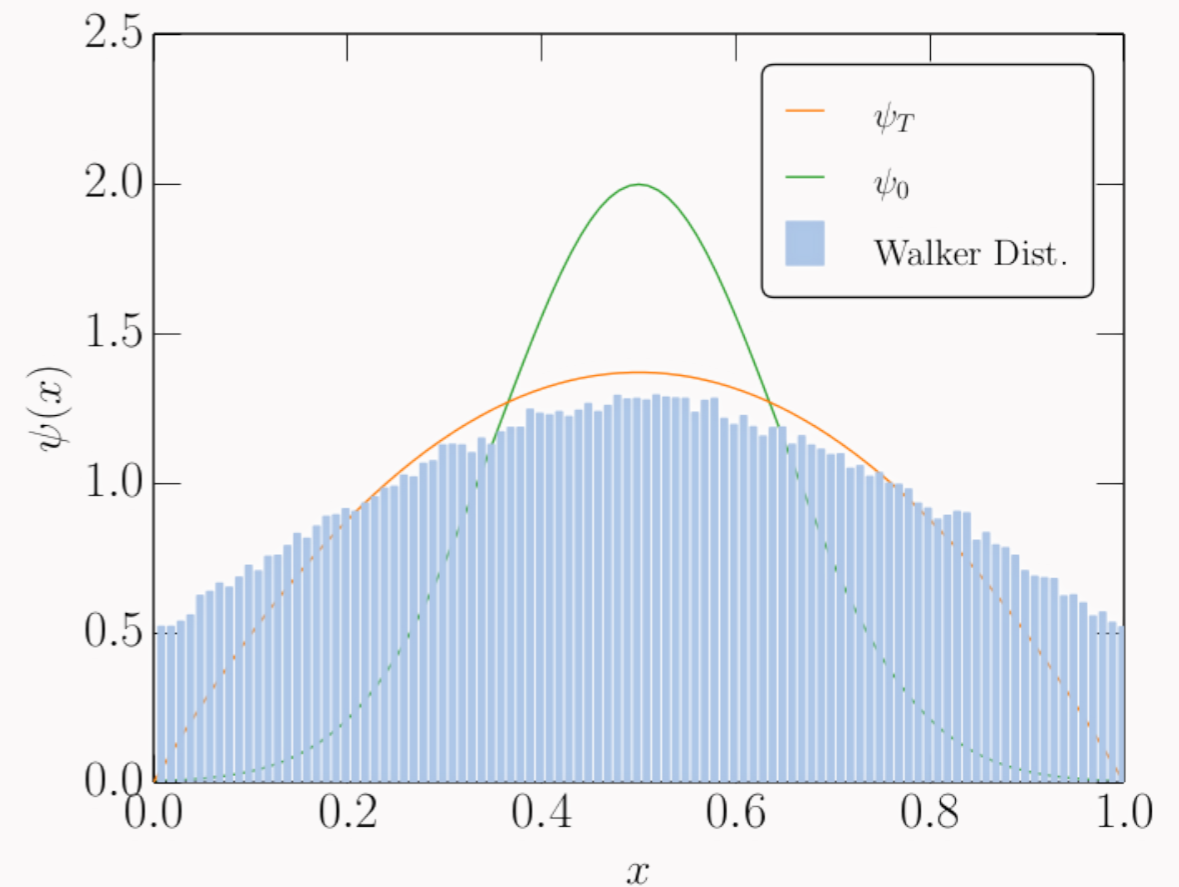
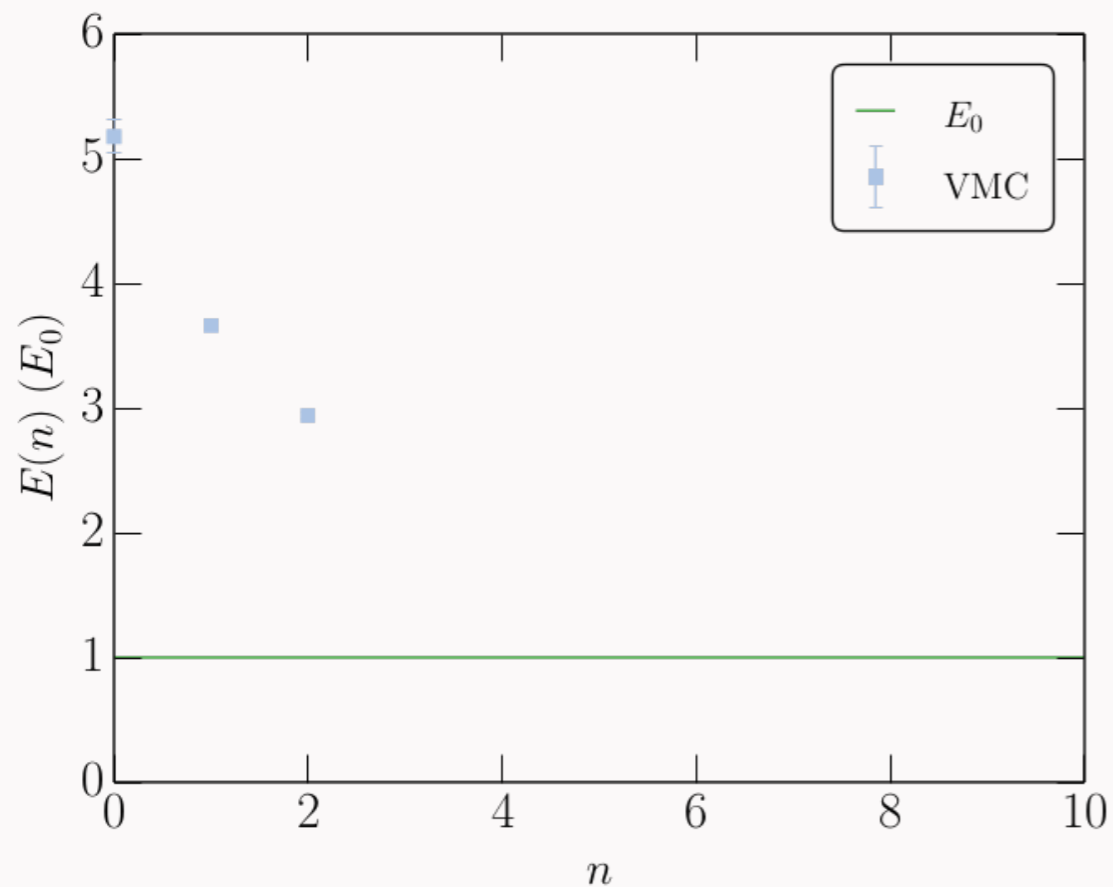


# QMC Methods - An Example

First, VMC equilibration:

$$\frac{\langle \psi_T | H | \psi_T \rangle}{\langle \psi_T | \psi_T \rangle}$$

Metropolis step  $n = 2$

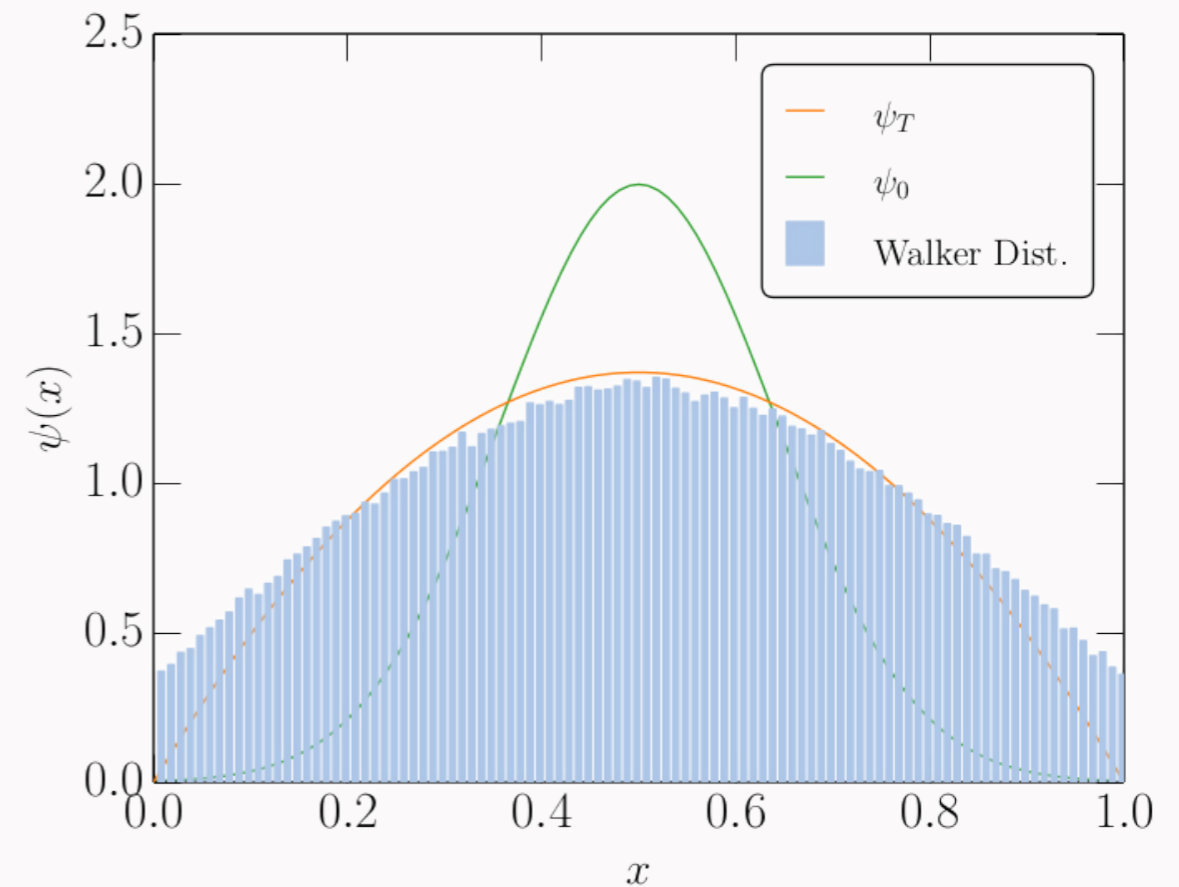
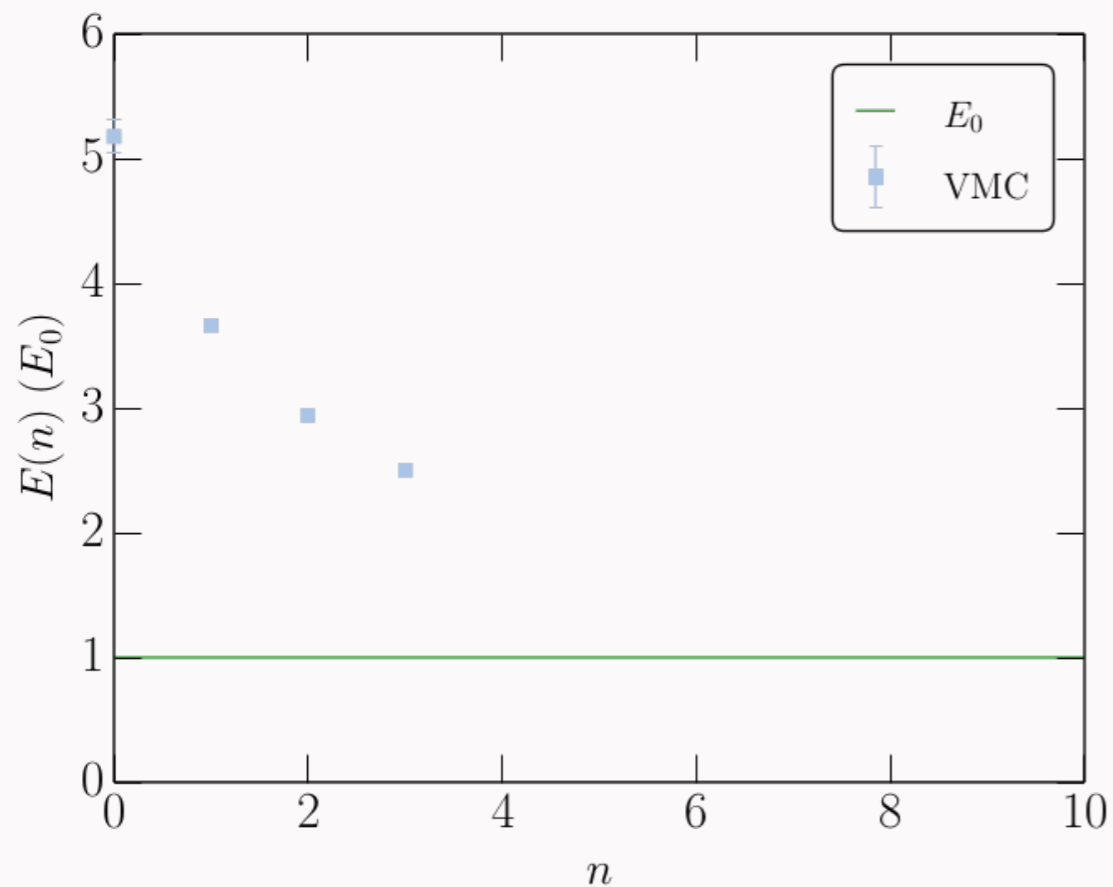


# QMC Methods - An Example

First, VMC equilibration:

$$\frac{\langle \psi_T | H | \psi_T \rangle}{\langle \psi_T | \psi_T \rangle}$$

Metropolis step  $n = 3$

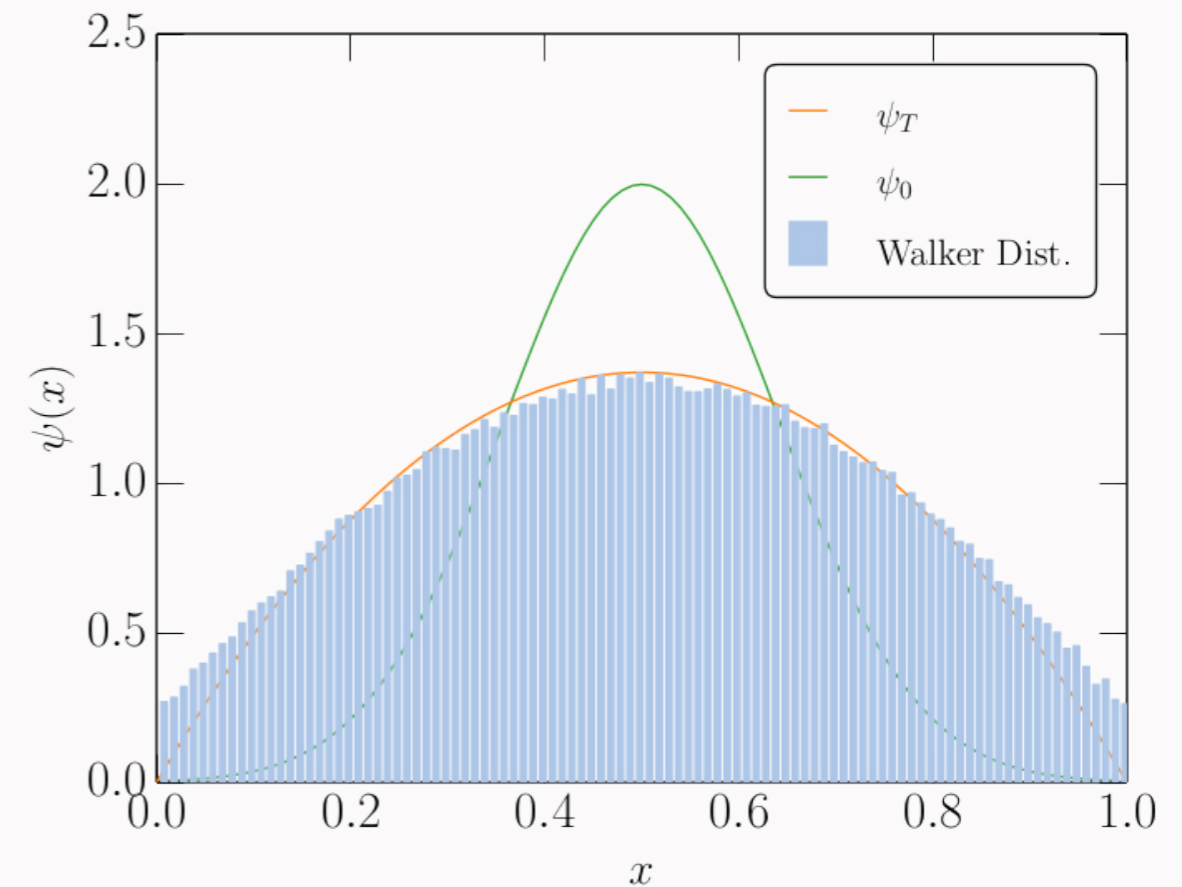
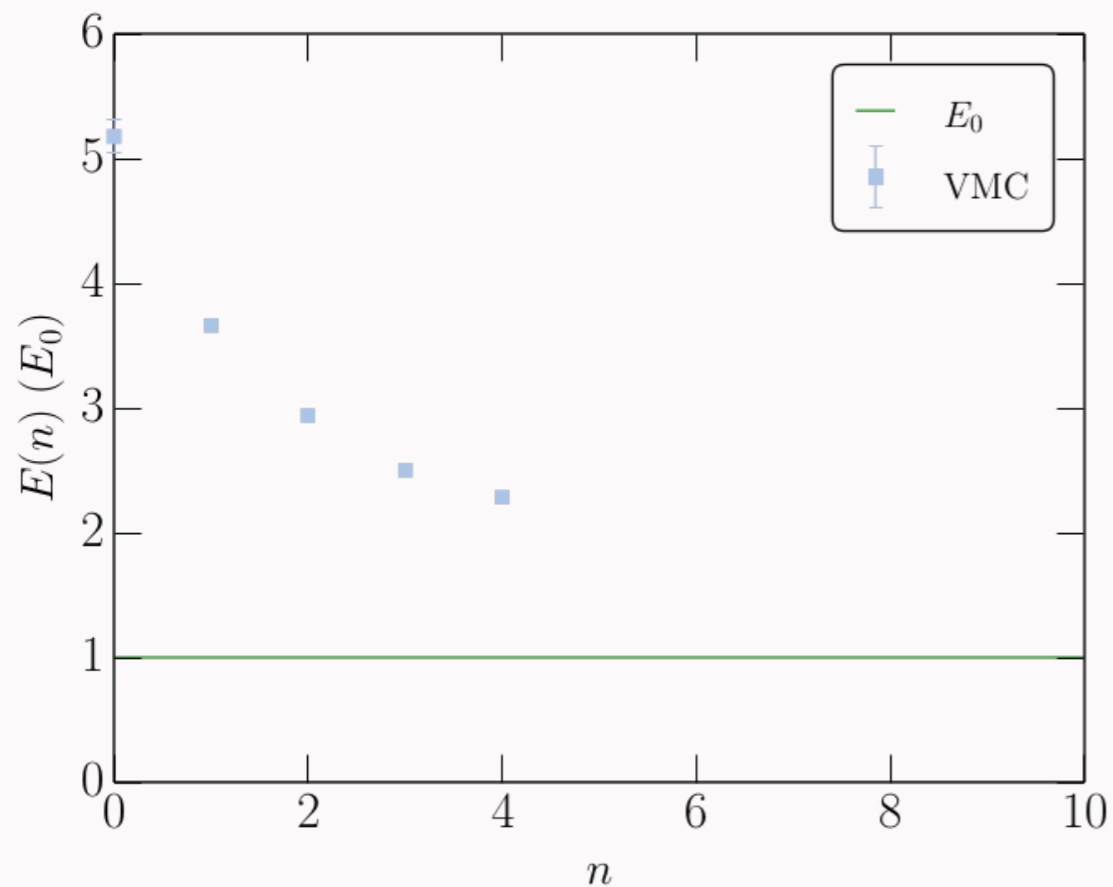


# QMC Methods - An Example

First, VMC equilibration:

$$\frac{\langle \psi_T | H | \psi_T \rangle}{\langle \psi_T | \psi_T \rangle}$$

Metropolis step  $n = 4$

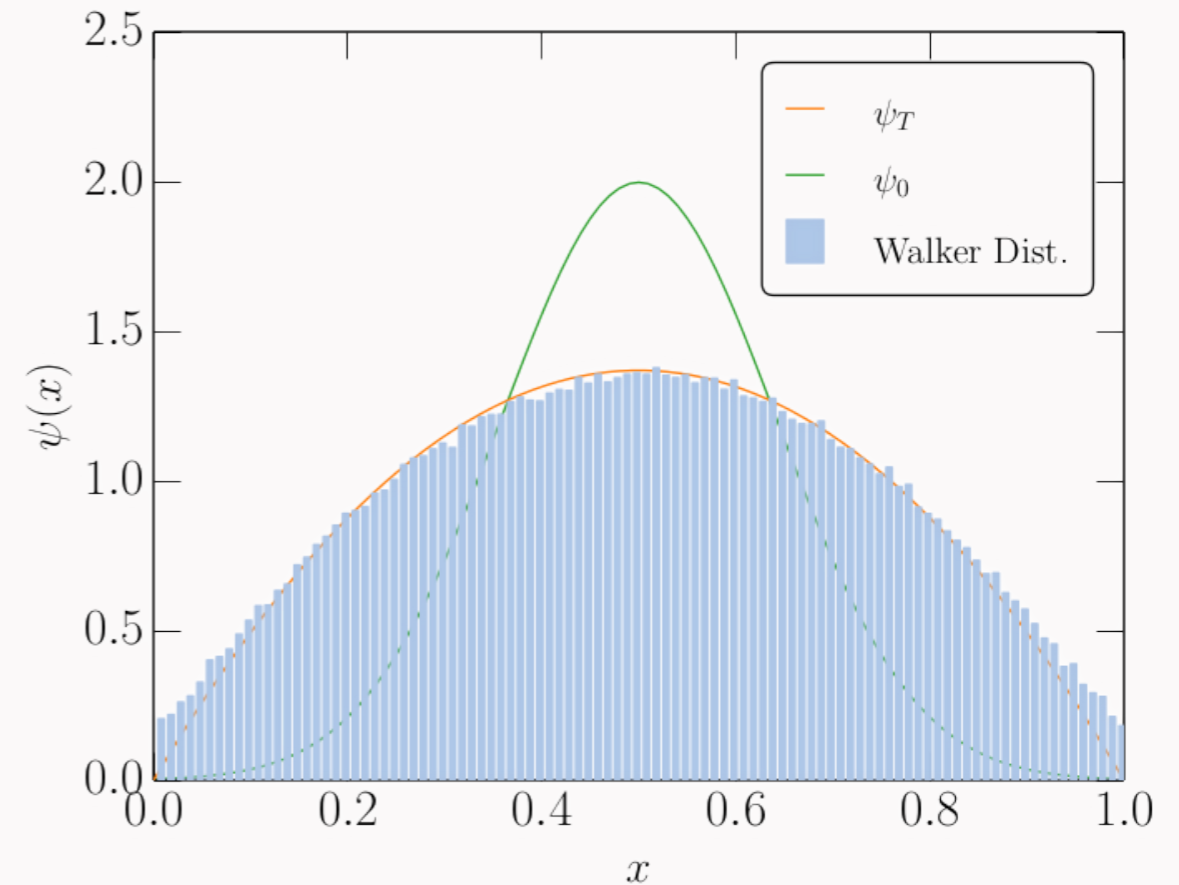
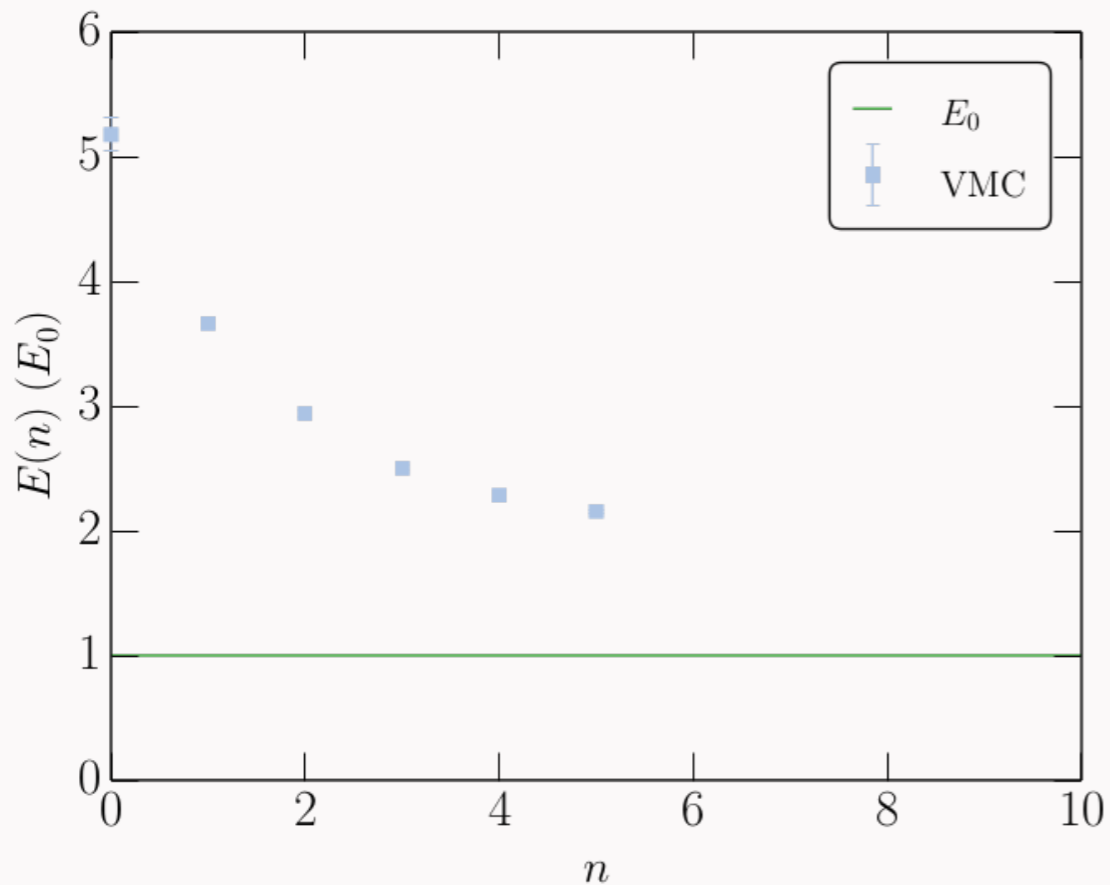


# QMC Methods - An Example

First, VMC equilibration:

$$\frac{\langle \psi_T | H | \psi_T \rangle}{\langle \psi_T | \psi_T \rangle}$$

Metropolis step  $n = 5$

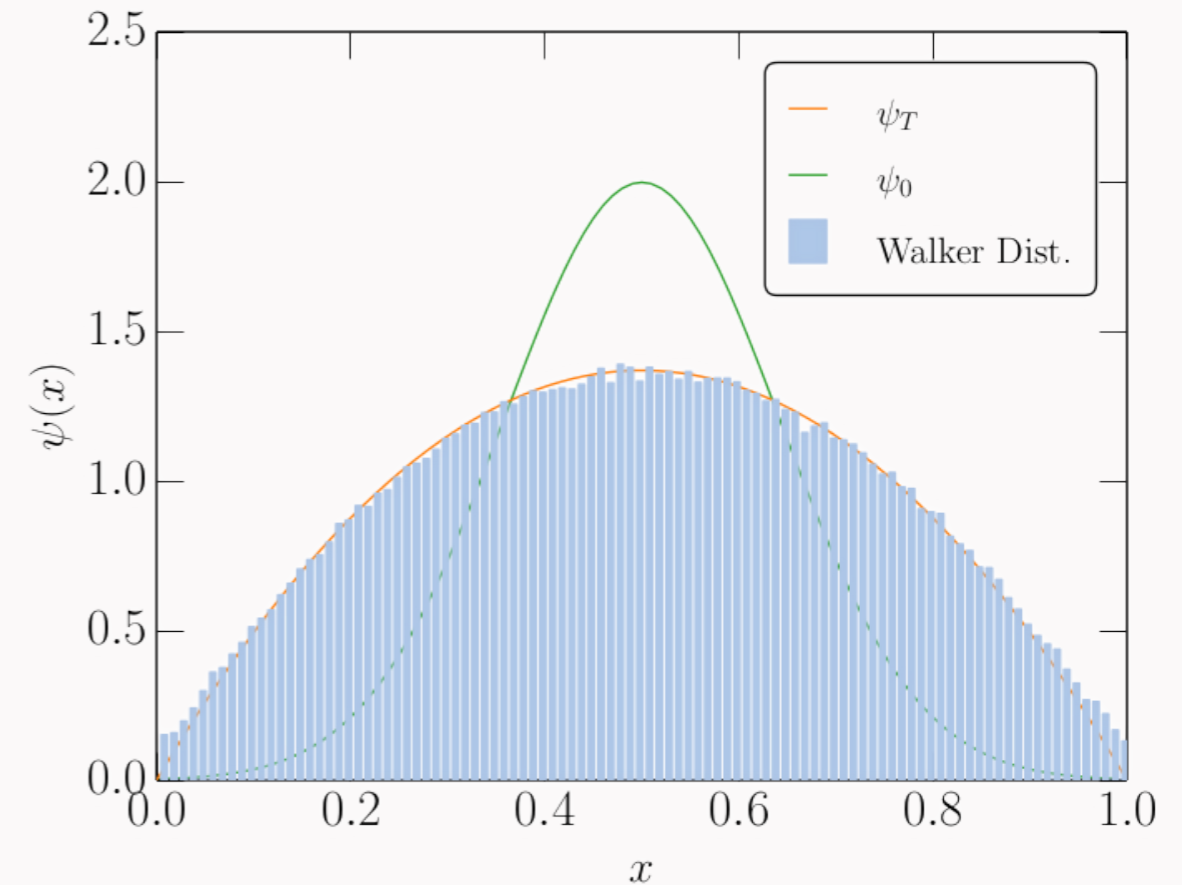
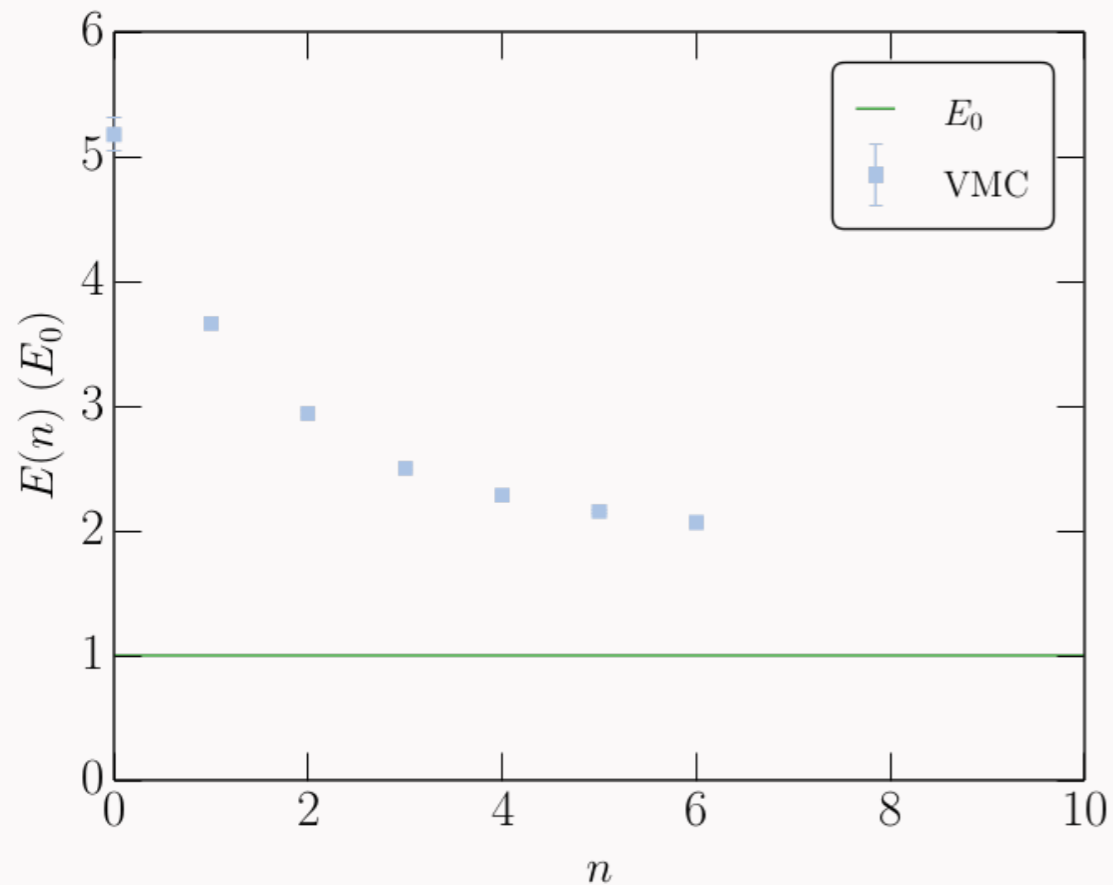


# QMC Methods - An Example

First, VMC equilibration:

$$\frac{\langle \psi_T | H | \psi_T \rangle}{\langle \psi_T | \psi_T \rangle}$$

Metropolis step  $n = 6$



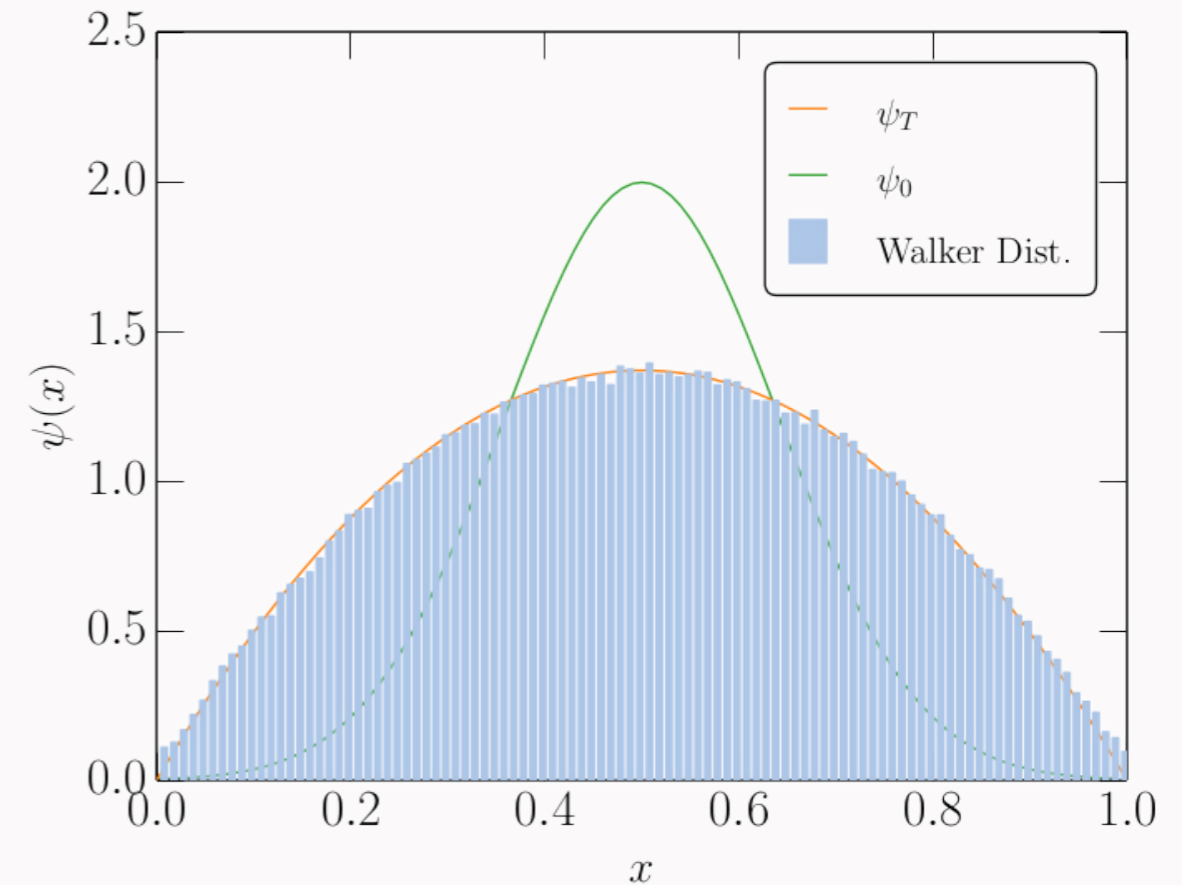
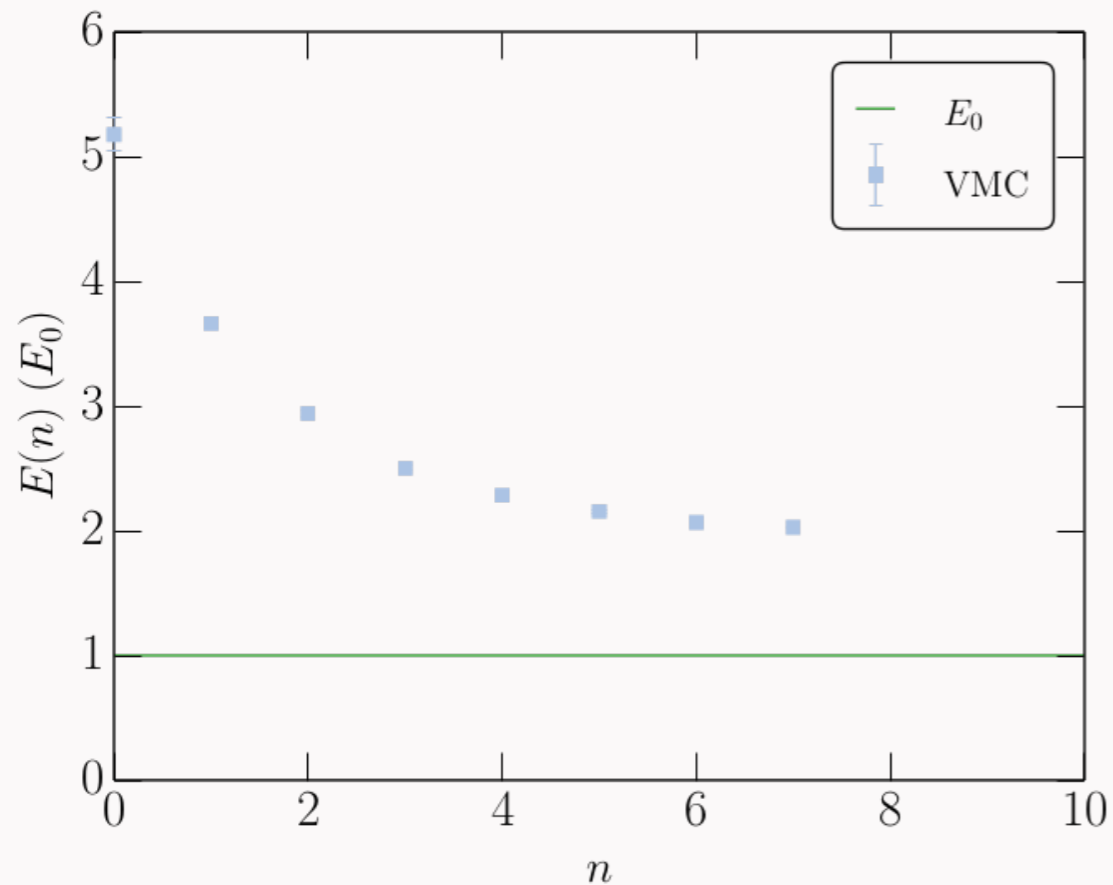


# QMC Methods - An Example

First, VMC equilibration:

$$\frac{\langle \psi_T | H | \psi_T \rangle}{\langle \psi_T | \psi_T \rangle}$$

Metropolis step  $n = 7$

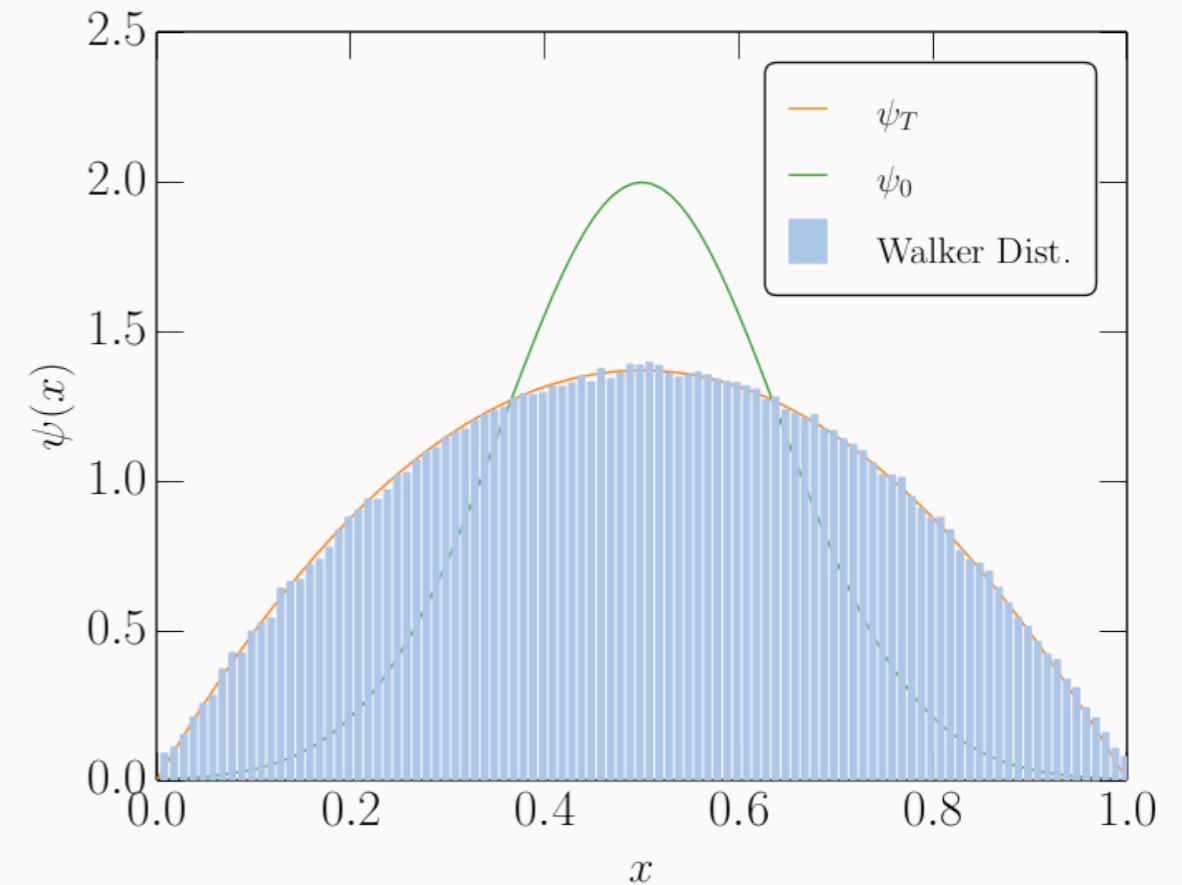
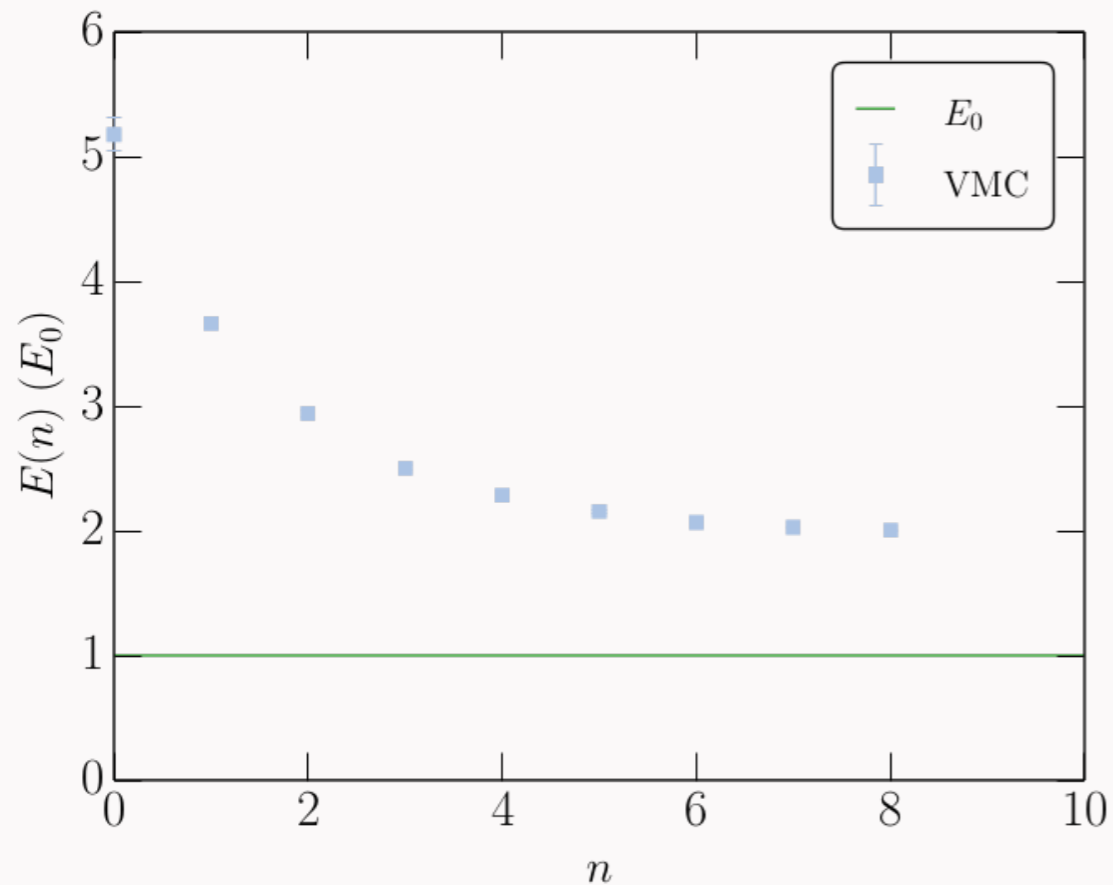


# QMC Methods - An Example

First, VMC equilibration:

$$\frac{\langle \psi_T | H | \psi_T \rangle}{\langle \psi_T | \psi_T \rangle}$$

Metropolis step  $n = 8$

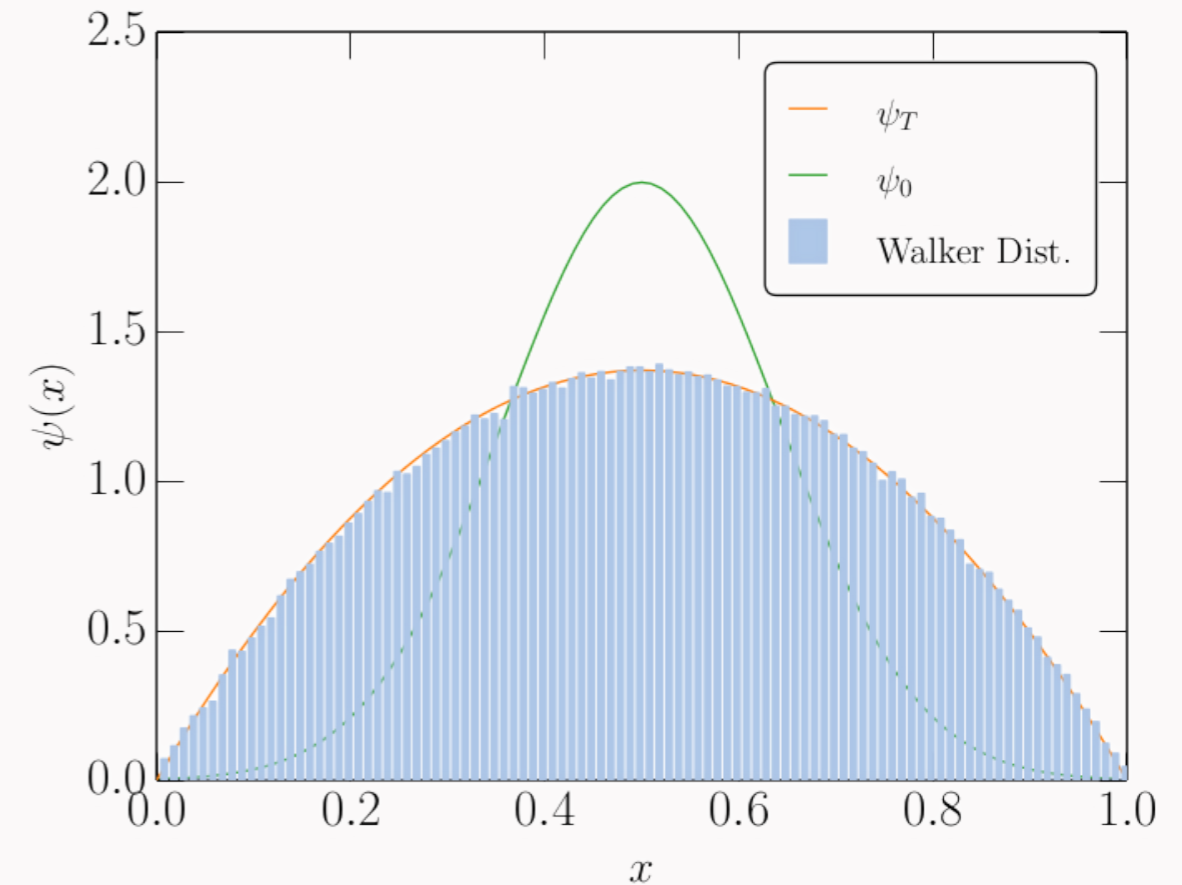
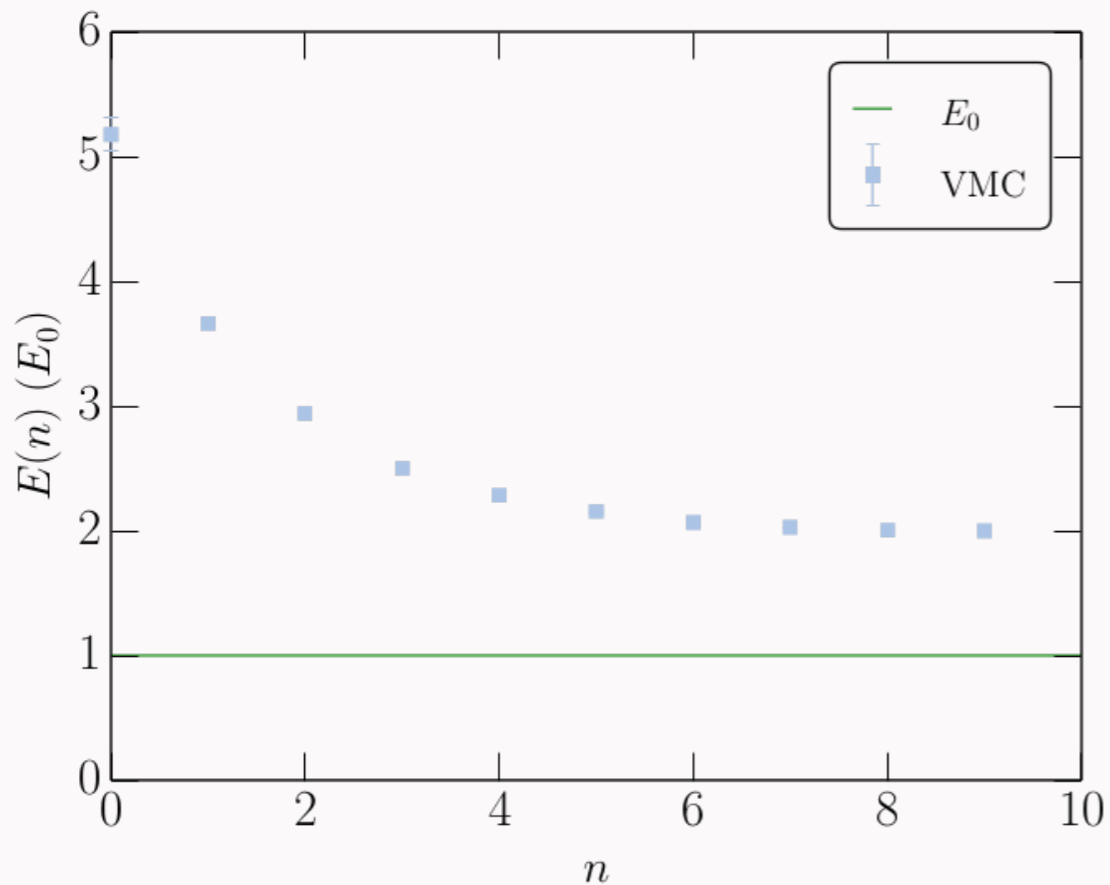


# QMC Methods - An Example

First, VMC equilibration:

$$\frac{\langle \psi_T | H | \psi_T \rangle}{\langle \psi_T | \psi_T \rangle}$$

Metropolis step  $n = 9$

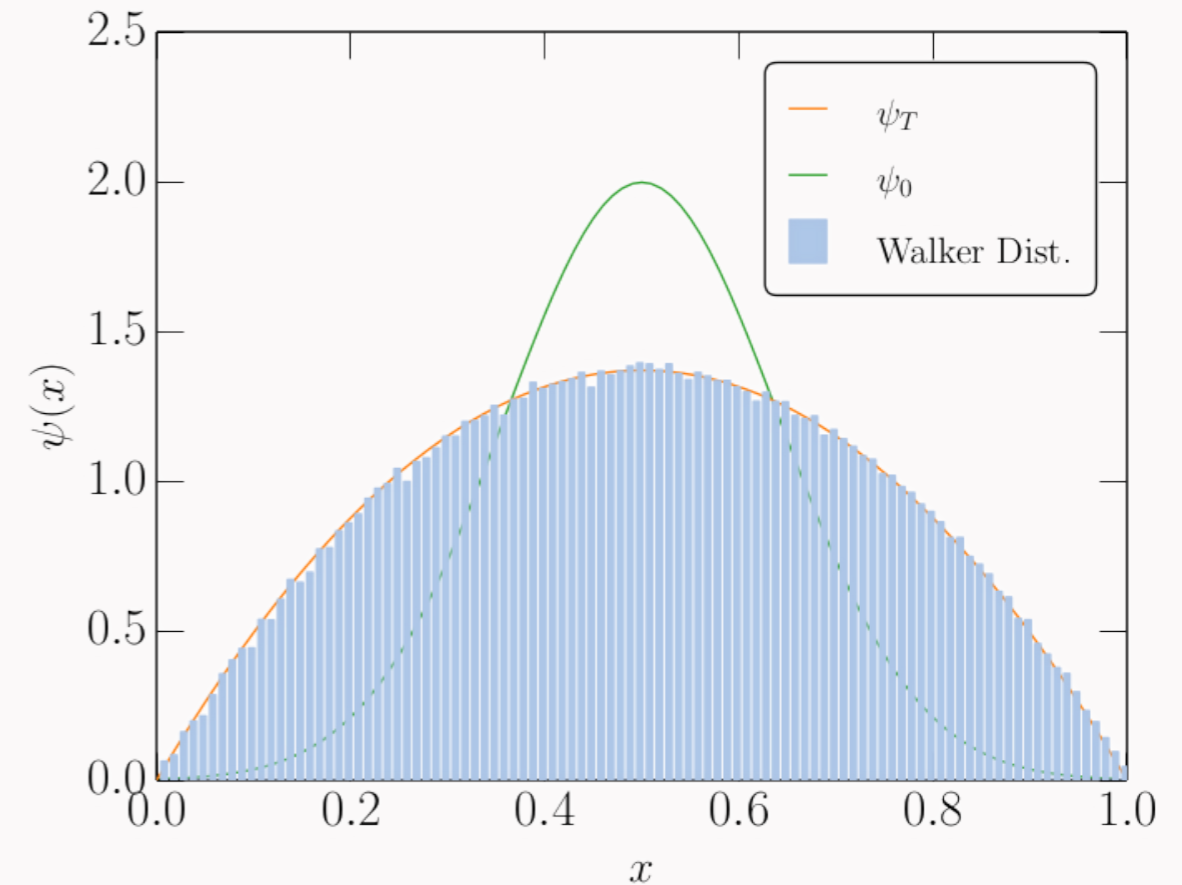
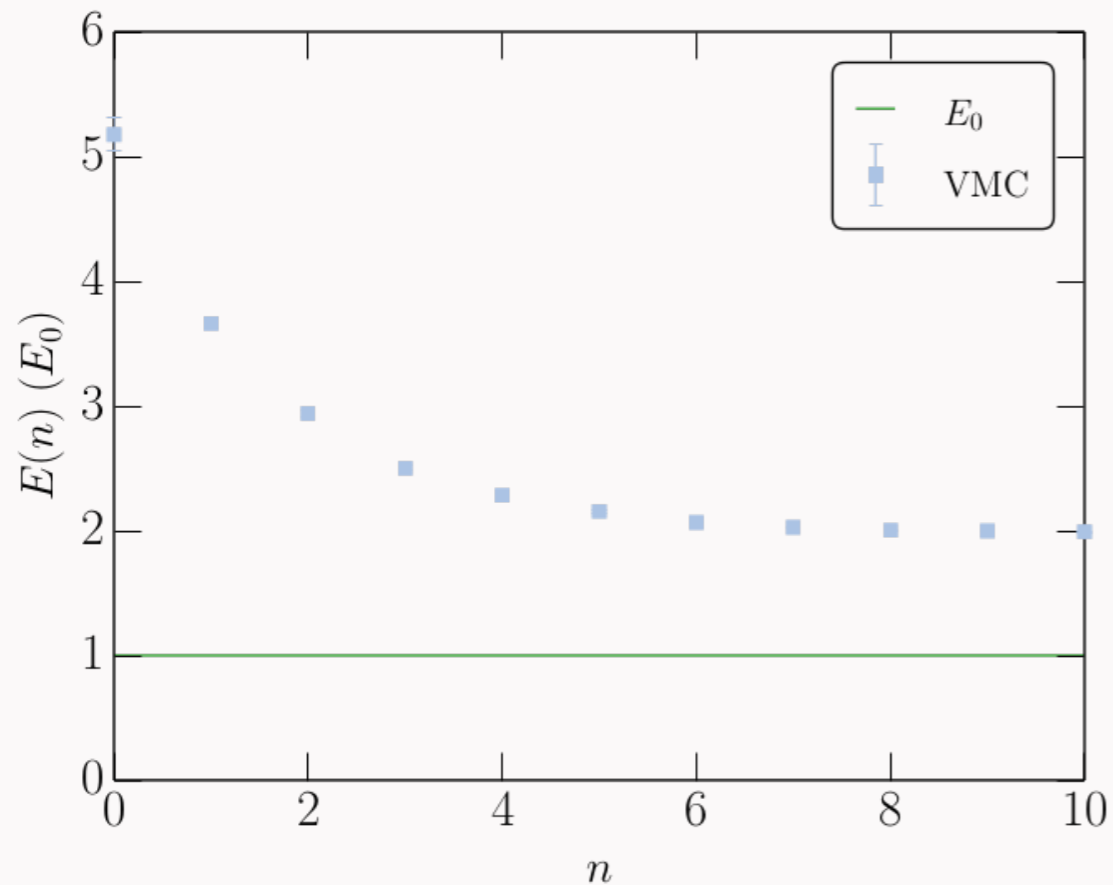


# QMC Methods - An Example

First, VMC equilibration:

$$\frac{\langle \psi_T | H | \psi_T \rangle}{\langle \psi_T | \psi_T \rangle}$$

Metropolis step  $n = 10$



Local construction possible<sup>1</sup> up to N<sup>2</sup>LO.

Definitions.

$$\mathbf{q} = \mathbf{p} - \mathbf{p}', \quad \mathbf{k} = \mathbf{p} + \mathbf{p}'$$

Regulator:

$$f(p, p') = e^{-(p/\Lambda)^n} e^{-(p'/\Lambda)^n}$$

Contacts:

$$\propto \mathbf{q} \text{ and } \mathbf{k}$$

<sup>1</sup>A. Gezerlis et al, PRL **111** 032501 (2013); JEL et al, PRL **113** 192501 (2014); A. Gezerlis et al, PRC **90** 054323 (2014)

# Chiral EFT

Local construction possible<sup>1</sup> up to N<sup>2</sup>LO.

Definitions.

$$\mathbf{q} = \mathbf{p} - \mathbf{p}', \mathbf{k} = \mathbf{p} + \mathbf{p}'$$

Regulator:

~~$$f(\mathbf{p}, \mathbf{p}') = e^{-(\mathbf{p}/\Lambda)^n} e^{-(\mathbf{p}'/\Lambda)^n}$$~~

$$\rightarrow f_{\text{long}}(r) = 1 - e^{-(r/R_0)^4} : R_0 = 1.0, 1.1, 1.2 \text{ fm.}$$


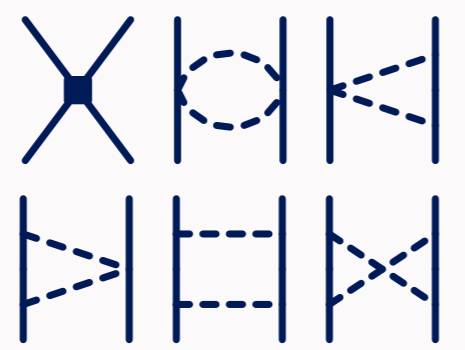
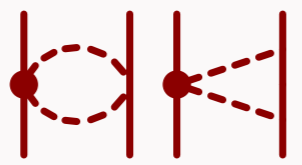
Contacts:

~~$$\propto \mathbf{q} \text{ and } \mathbf{k}$$~~


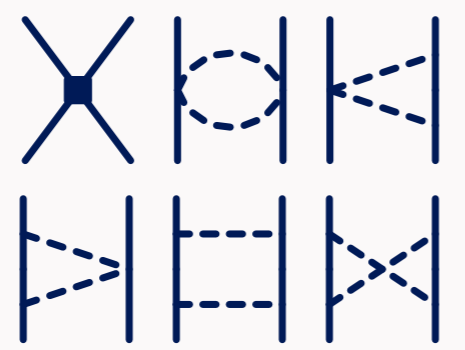
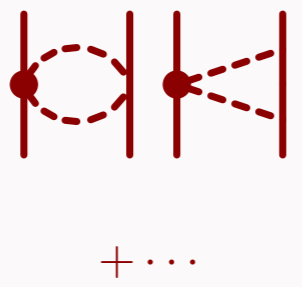
→ Choose contacts  $\propto \mathbf{q}$  (As much as possible!)

<sup>1</sup>A. Gezerlis et al, PRL **111** 032501 (2013); JEL et al, PRL **113** 192501 (2014); A. Gezerlis et al, PRC **90** 054323 (2014)

# Chiral EFT

		$NN$
LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^0$	
NLO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^2$	
N <sup>2</sup> LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^3$	 + ...

# Chiral EFT

		NN
LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^0$	
NLO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^2$	
N <sup>2</sup> LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^3$	

$$V_{\text{cont}}^{(0)} = \alpha_1 + \alpha_2(\sigma_1 \cdot \sigma_2) + \alpha_3(\tau_1 \cdot \tau_2) + \alpha_4(\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2)$$


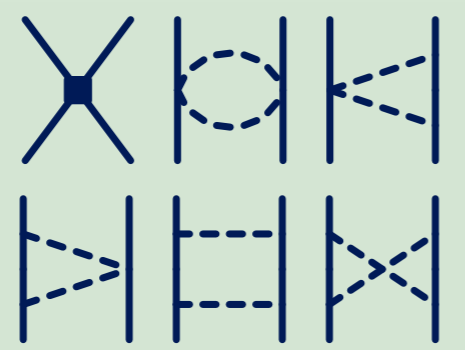
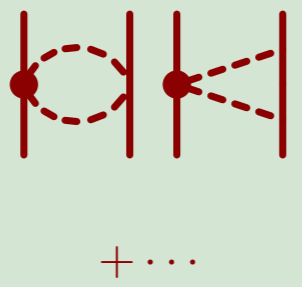
Pauli Exclusion Principle

→ Only two independent contacts!

$$V_{\text{cont}}^{(0)} = C_S + C_T(\sigma_1 \cdot \sigma_2)$$


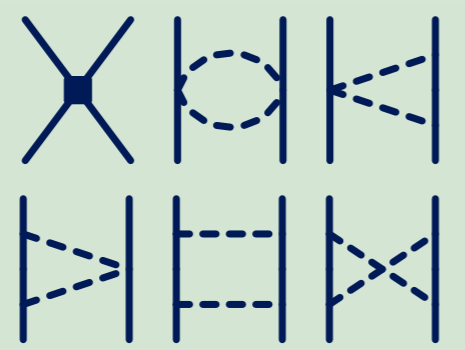
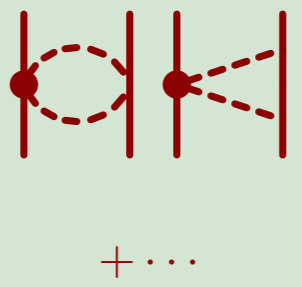


# Chiral EFT

		NN
LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^0$	
NLO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^2$	
N <sup>2</sup> LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^3$	

$$\begin{aligned}
 V_{\text{cont}}^{(2)} = & \gamma_1 q^2 + \gamma_2 q^2 (\sigma_1 \cdot \sigma_2) \\
 & + \gamma_3 q^2 (\tau_1 \cdot \tau_2) + \gamma_4 q^2 (\sigma_1 \cdot \sigma_2) (\tau_1 \cdot \tau_2) \\
 & + \gamma_5 k^2 + \gamma_6 k^2 (\sigma_1 \cdot \sigma_2) + \gamma_7 k^2 (\tau_1 \cdot \tau_2) \\
 & + \gamma_8 k^2 (\sigma_1 \cdot \sigma_2) (\tau_1 \cdot \tau_2) \\
 & + (\sigma_1 + \sigma_2) (\mathbf{q} \times \mathbf{k}) (\gamma_9 + \gamma_{10} (\tau_1 \cdot \tau_2)) \\
 & + (\sigma_1 \cdot \mathbf{q}) (\sigma_2 \cdot \mathbf{q}) (\gamma_{11} + \gamma_{12} (\tau_1 \cdot \tau_2)) \\
 & + (\sigma_1 \cdot \mathbf{k}) (\sigma_2 \cdot \mathbf{k}) (\gamma_{13} + \gamma_{14} (\tau_1 \cdot \tau_2))
 \end{aligned}$$

# Chiral EFT

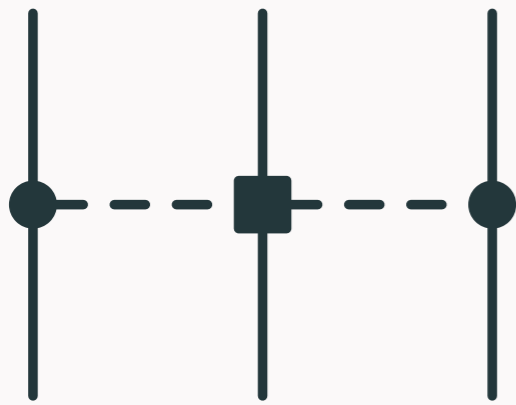
		NN
LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^0$	
NLO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^2$	
N <sup>2</sup> LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^3$	

$$\begin{aligned}
 V_{\text{cont}}^{(2)} = & \gamma_1 q^2 + \gamma_2 q^2 (\sigma_1 \cdot \sigma_2) \\
 & + \gamma_3 q^2 (\tau_1 \cdot \tau_2) + \gamma_4 q^2 (\sigma_1 \cdot \sigma_2) (\tau_1 \cdot \tau_2) \\
 & + \gamma_5 k^2 + \gamma_6 k^2 (\sigma_1 \cdot \sigma_2) + \gamma_7 k^2 (\tau_1 \cdot \tau_2) \\
 & + \gamma_8 k^2 (\sigma_1 \cdot \sigma_2) (\tau_1 \cdot \tau_2) \\
 & + (\sigma_1 + \sigma_2) (\mathbf{q} \times \mathbf{k}) (\gamma_9 + \gamma_{10} (\tau_1 \cdot \tau_2)) \\
 & + (\sigma_1 \cdot \mathbf{q}) (\sigma_2 \cdot \mathbf{q}) (\gamma_{11} + \gamma_{12} (\tau_1 \cdot \tau_2)) \\
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 \end{aligned}$$

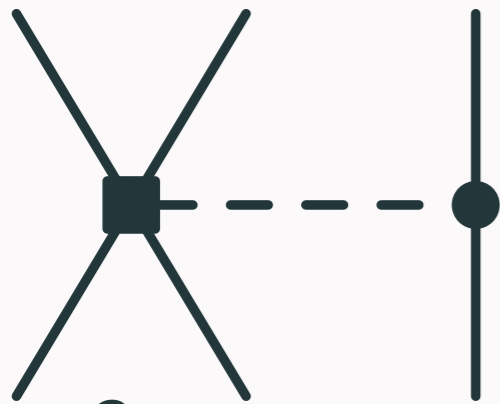
# Three-Nucleon Interactions

---

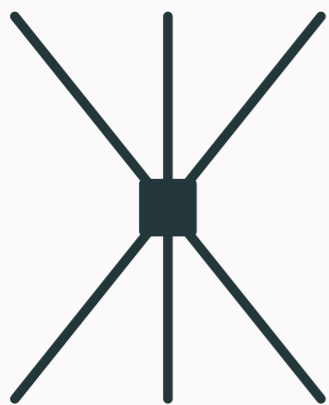
# Three-Nucleon Interaction



$C_1, C_3, C_4$

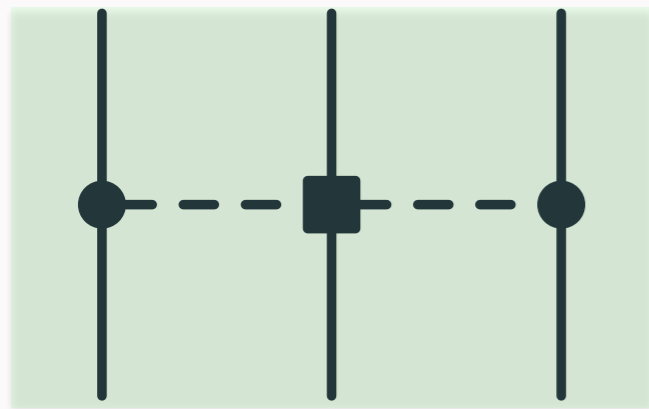


$C_D$



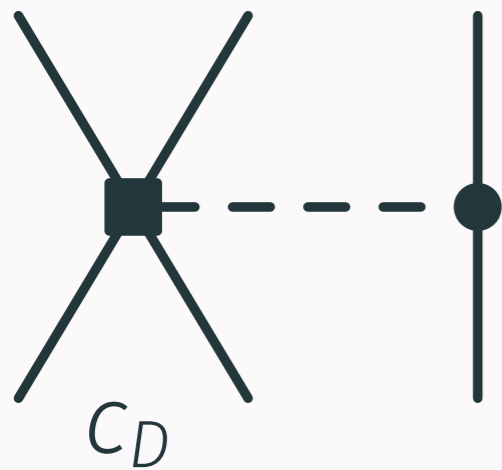
$C_E$

# Three-Nucleon Interaction



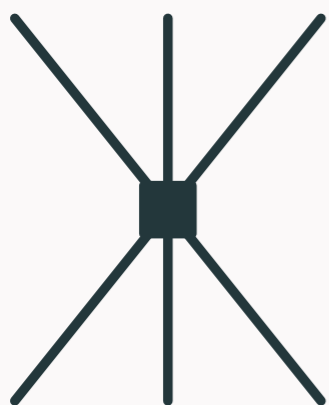
$C_1, C_3, C_4$

$$\mathcal{F} \left\{ \begin{array}{c} \bullet \text{---} \blacksquare \text{---} \bullet \\ | \quad | \quad | \\ C_1 \end{array} \right\} \rightarrow \sim \text{Tucson-Melbourne } a' \text{ Term}$$



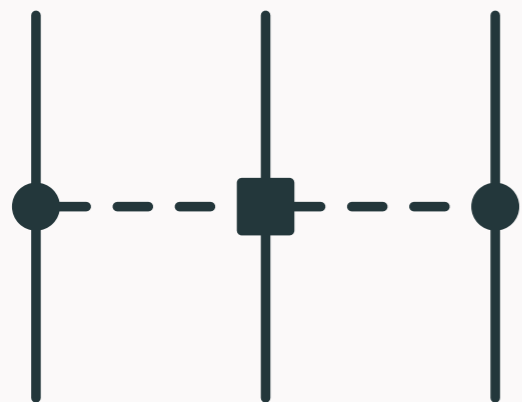
$C_D$

$$\mathcal{F} \left\{ \begin{array}{c} \bullet \text{---} \blacksquare \text{---} \bullet \\ | \quad | \quad | \\ C_3, C_4 \end{array} \right\} \rightarrow \sim \text{Fujita-Miyazawa}$$



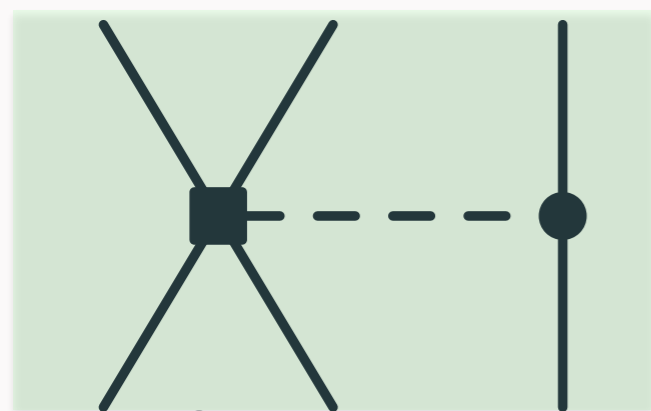
$C_E$

# Three-Nucleon Interaction



$C_1, C_3, C_4$

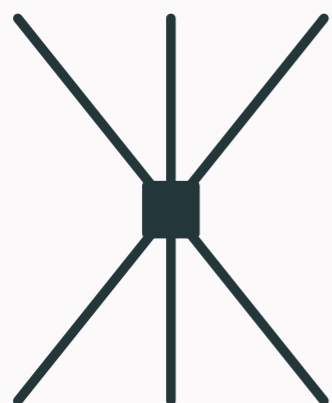
$$\mathcal{F} \left\{ \begin{array}{c} \text{---} \bullet \text{---} \blacksquare \text{---} \bullet \text{---} \\ | \quad | \quad | \\ \text{---} \end{array} \right\} \rightarrow \sim \text{Tucson-Melbourne } a' \text{ Term}$$



$C_D$

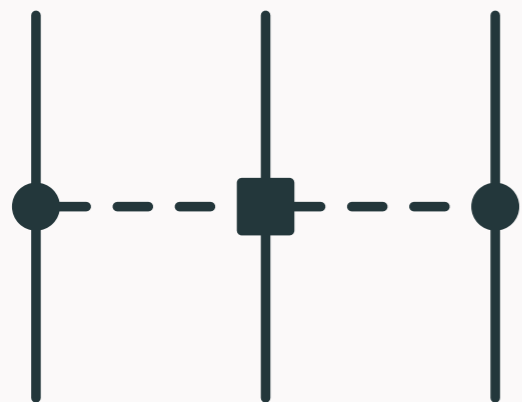
$$\mathcal{F} \left\{ \begin{array}{c} \text{---} \bullet \text{---} \blacksquare \text{---} \bullet \text{---} \\ | \quad | \quad | \\ \text{---} \end{array} \right\} \rightarrow \sim \text{Fujita-Miyazawa}$$

$$\mathcal{F} \left\{ \begin{array}{c} \text{---} \bullet \text{---} \blacksquare \text{---} \bullet \text{---} \\ | \quad | \quad | \\ \text{---} \end{array} \right\} \rightarrow 1\pi\text{-Exchange + Contact}$$



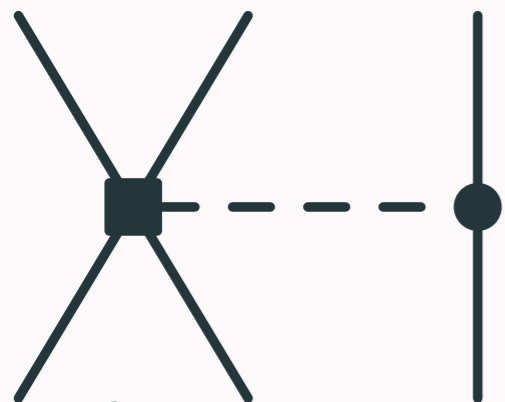
$C_E$

# Three-Nucleon Interaction



$C_1, C_3, C_4$

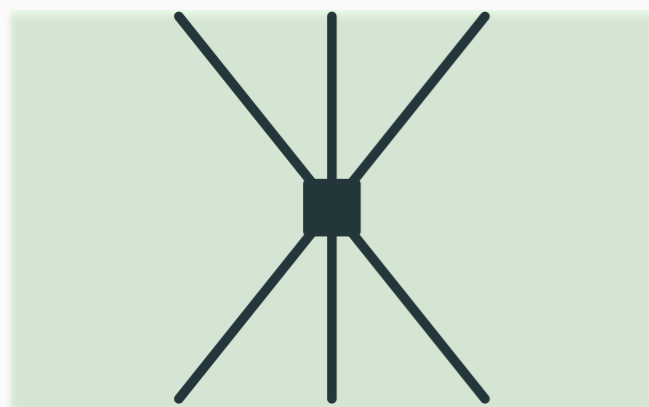
$$\mathcal{F} \left\{ \begin{array}{c} \text{Diagram} \\ C_1 \end{array} \right\} \rightarrow \sim \text{Tucson-Melbourne } a' \text{ Term}$$



$C_D$

$$\mathcal{F} \left\{ \begin{array}{c} \text{Diagram} \\ C_3, C_4 \end{array} \right\} \rightarrow \sim \text{Fujita-Miyazawa}$$

$$\mathcal{F} \left\{ \begin{array}{c} \text{Diagram} \\ C_D \end{array} \right\} \rightarrow 1\pi\text{-Exchange} + \text{Contact}$$



$C_E$

$$\mathcal{F} \left\{ \begin{array}{c} \text{Diagram} \\ C_E \end{array} \right\} \rightarrow \text{Contact}$$

# Fitting $c_D$ And $c_E$

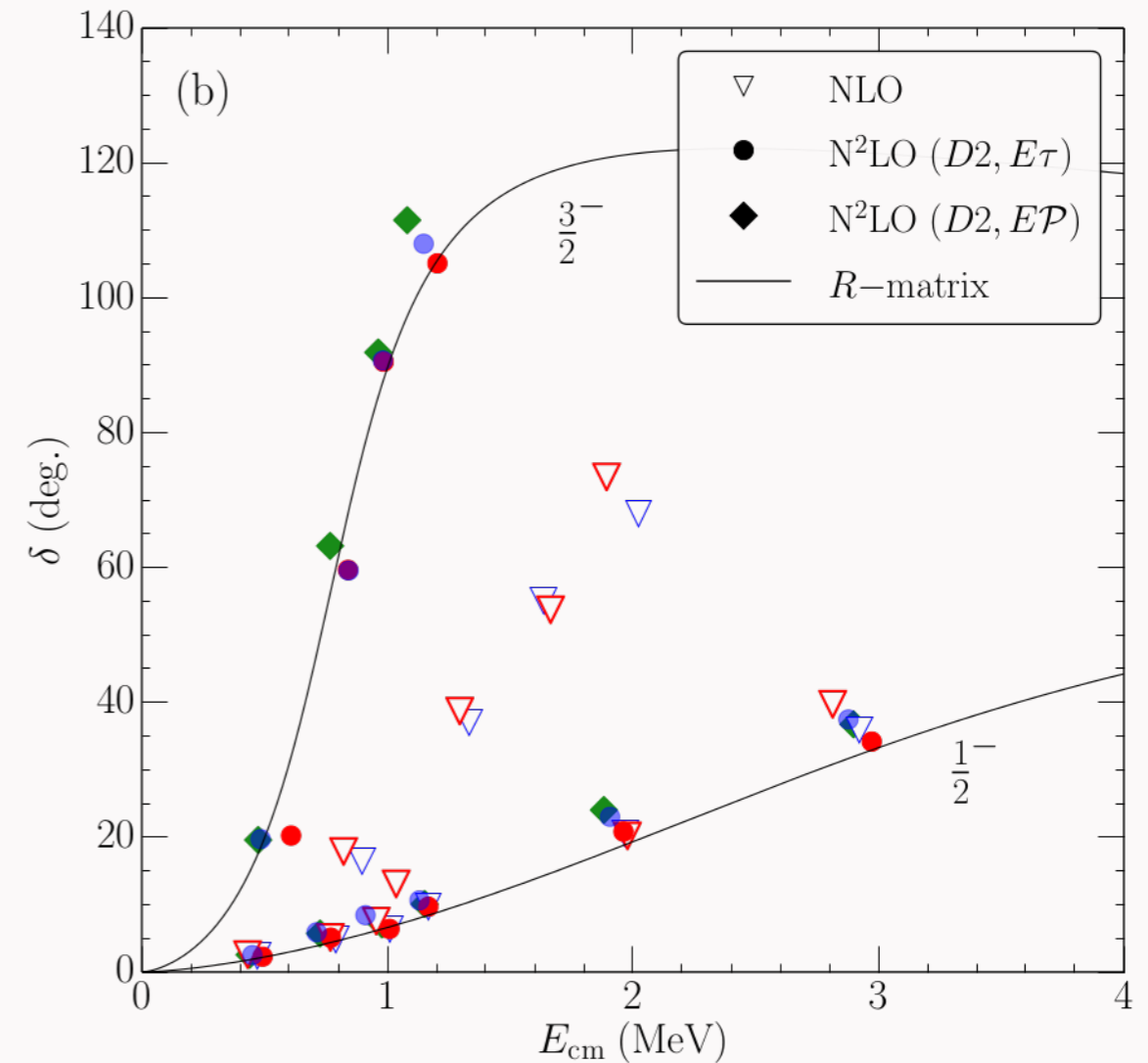
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# Choosing Observables

What to fit  $c_D$  and  $c_E$  to?

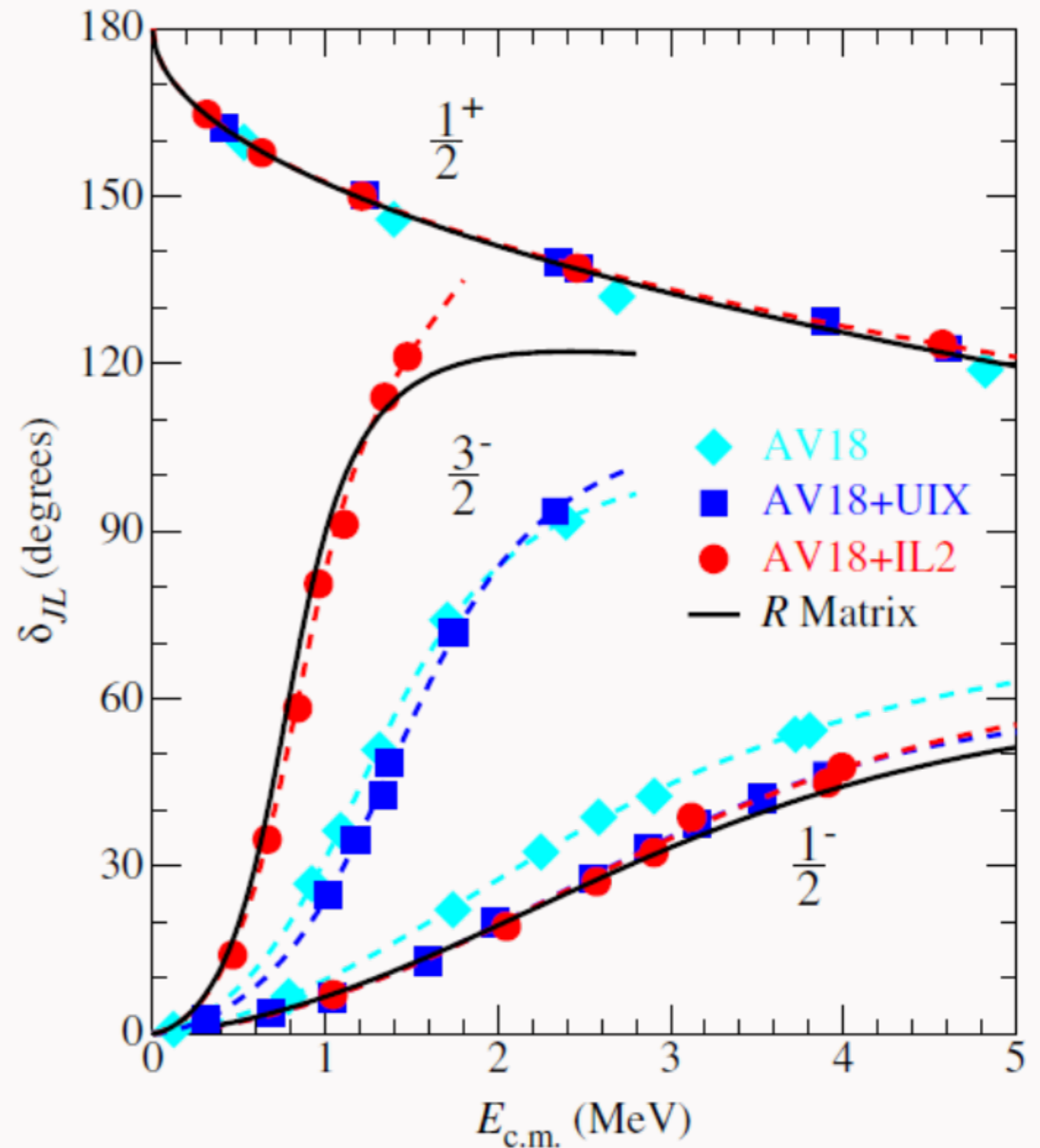
- Uncorrelated observables.
- Probe properties of light nuclei:  ${}^4\text{He } E_B$ .
- Probe  $T = 3/2$  physics:  $n$ - $\alpha$  scattering phase shifts.



JEL et al, PRL 116, 062501 (2016)

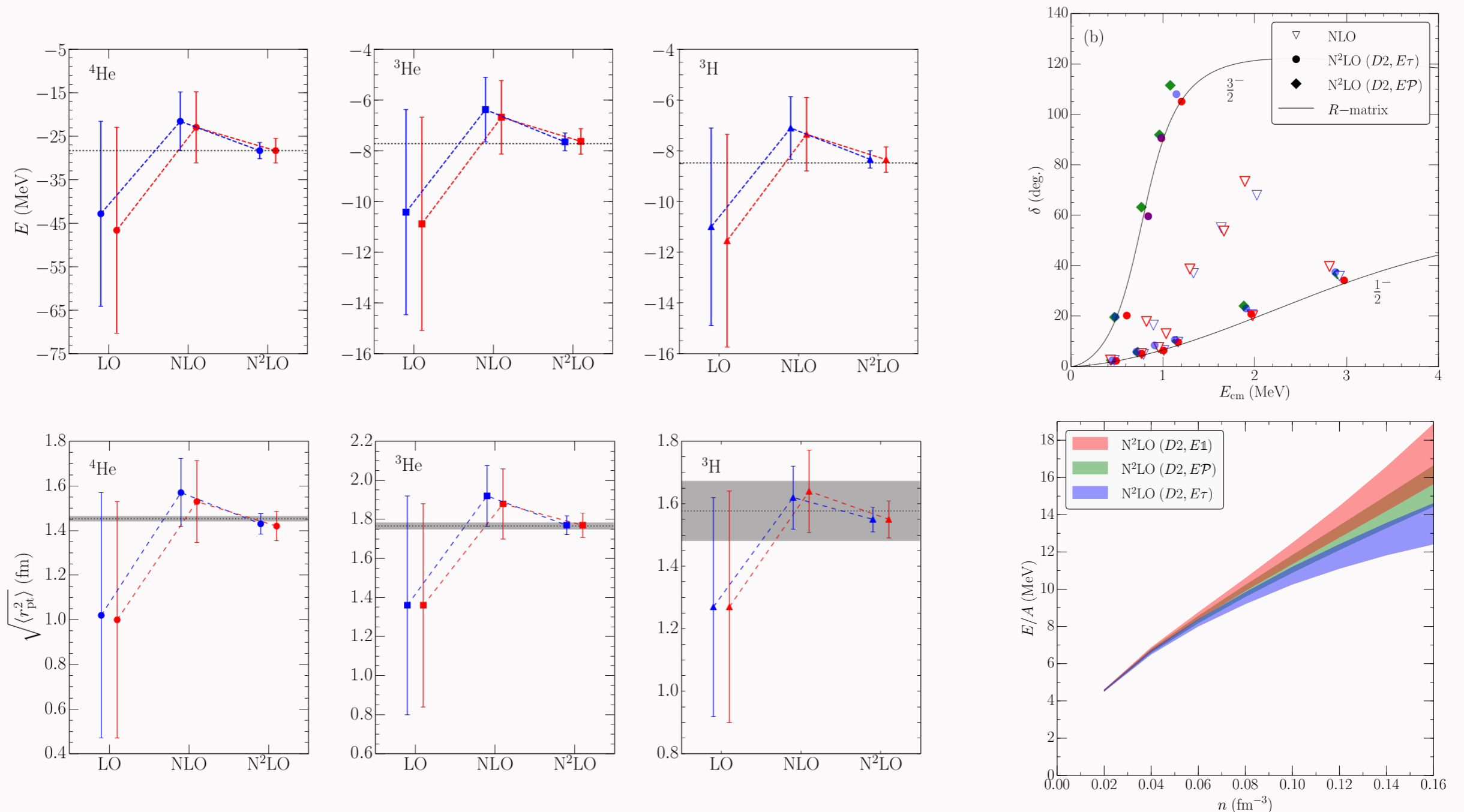
# $n$ - $\alpha$ Scattering - Details

- Results<sup>2</sup> showed need for greater spin-orbit splitting than was provided for by the Urbana IX (UIX) 3N interaction.
- Interpretation was  $T=3/2$  component in Illinois 3N interaction was necessary. (?)



# Results

A simultaneous description of properties of light nuclei,  $n$ - $\alpha$  scattering and neutron matter is possible.



# Uncertainty Analysis

Sources of uncertainty include

- Systematic uncertainty due to truncation of the chiral expansion
- Uncertainty in knowledge of  $\pi N$  LECs (long range)
- Uncertainty in knowledge of LECs for contacts (short range)
- Uncertainties in experimental data or partial-wave analysis.

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- Uncertainties in experimental data or partial-wave analysis.

# Uncertainty Analysis

Define

$$X^{(i)} = X^{(0)} + \Delta X^{(2)} + \dots + \Delta X^{(i)},$$
$$\Delta X^{(2)} = X^{(2)} - X^{(0)}, \quad \Delta X^{(i)} = X^{(i)} - X^{(i-1)}, \quad i > 2.$$

Expected size

$$\Delta X^{(i)} = \mathcal{O}(Q^i X^{(0)}).$$

Then,

$$\delta X^{(0)} = Q^2 |X^{(0)}|, \quad \delta X^{(i)} = \max(Q^{i+1} |X^{(0)}|, Q^{i+1-j} |\Delta X^{(j)}|), \quad 2 \leq j \leq i.$$
$$Q = \max(p/\Lambda_b, m_\pi/\Lambda_b).$$

# Four Neutrons: A Recent History

---

2002

2003

2005

Experiment  
Theory



F. M. Marqués et al. Phys. Rev. C **65**, 052501.

Experimental claim of a bound tetraneutron from detection of neutron clusters from  $^{14}\text{Be}$  fragmentation.

~6 events!

Experiment  
Theory

2002

2003

2005

# Experiment Theory

2002      2003      2005

1) C. A. Bertulani and V. Zelevinsky, J. Phys. G **29**, 2431 (2003),  
2) N. K. Timofeyuk J. Phys. G **29**, L9 (2003).  
No bound tetraneutron using 1) a dineutron-dineutron molecule  
model and 2) a toy *NN* potential.

# Experiment Theory

2002      2003      2005

- 1) C. A. Bertulani and V. Zelevinsky, J. Phys. G **29**, 2431 (2003),
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No bound tetraneutron using 1) a dineutron-dineutron molecule model and 2) a toy  $NN$  potential.

S. C. Pieper Phys. Rev. Lett. **90**, 252501.

Modern nuclear Hamiltonians cannot tolerate a bound tetraneutron.

But...

*"This suggests that there might be a  ${}^4n$  resonance near 2 MeV"*

# Experiment Theory

2002      2003      2005

R. Lazauskas and J. Carbonell, *Phys. Rev. C* **72**, 034003.  
Complex scaling w/ Reid 93 potential (*NN* only!)  
Low-lying  $^4n$  resonance not seen.

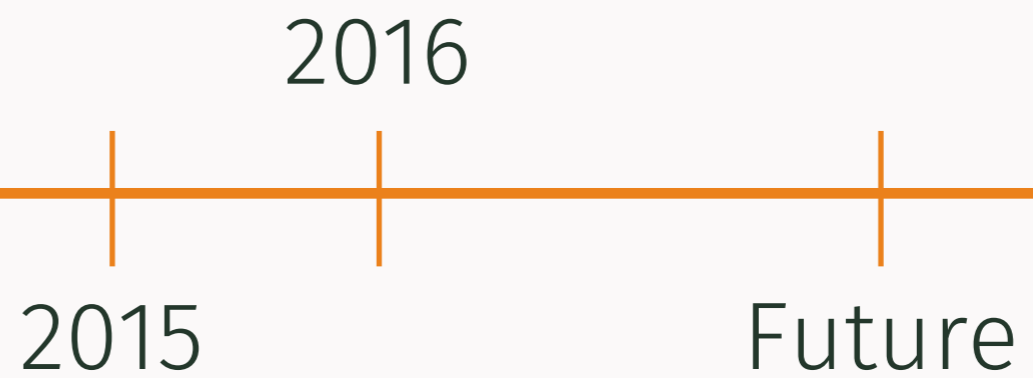
2002

2003

2005

Experiment  
Theory

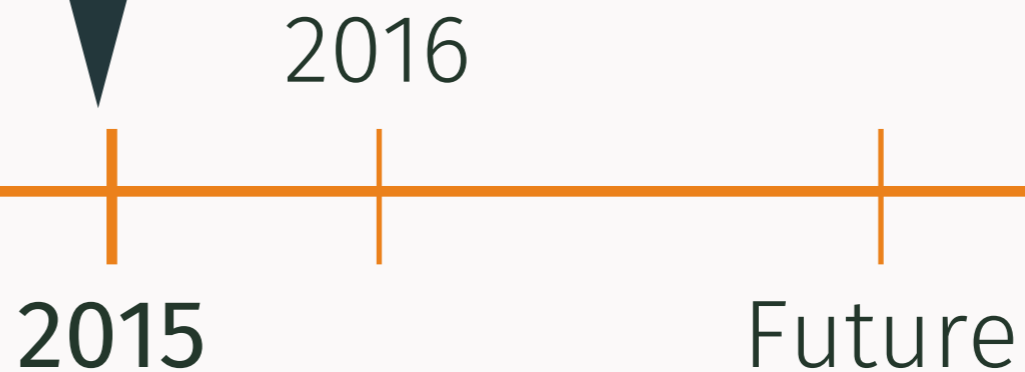
Experiment  
Theory



K. Kisamori et al., Phys. Rev. Lett. **116**, 044006.

A recent double-charge-exchange reaction  ${}^8_2\text{He} + {}^4_2\text{He} \rightarrow {}^8_4\text{Be} + 4n$  measurement at the RIKEN radioactive ion beam factory (RIBF) suggests a tetraneutron resonance at  $0.83 \pm 0.65(\text{stat}) \pm 1.25(\text{syst})$  MeV.

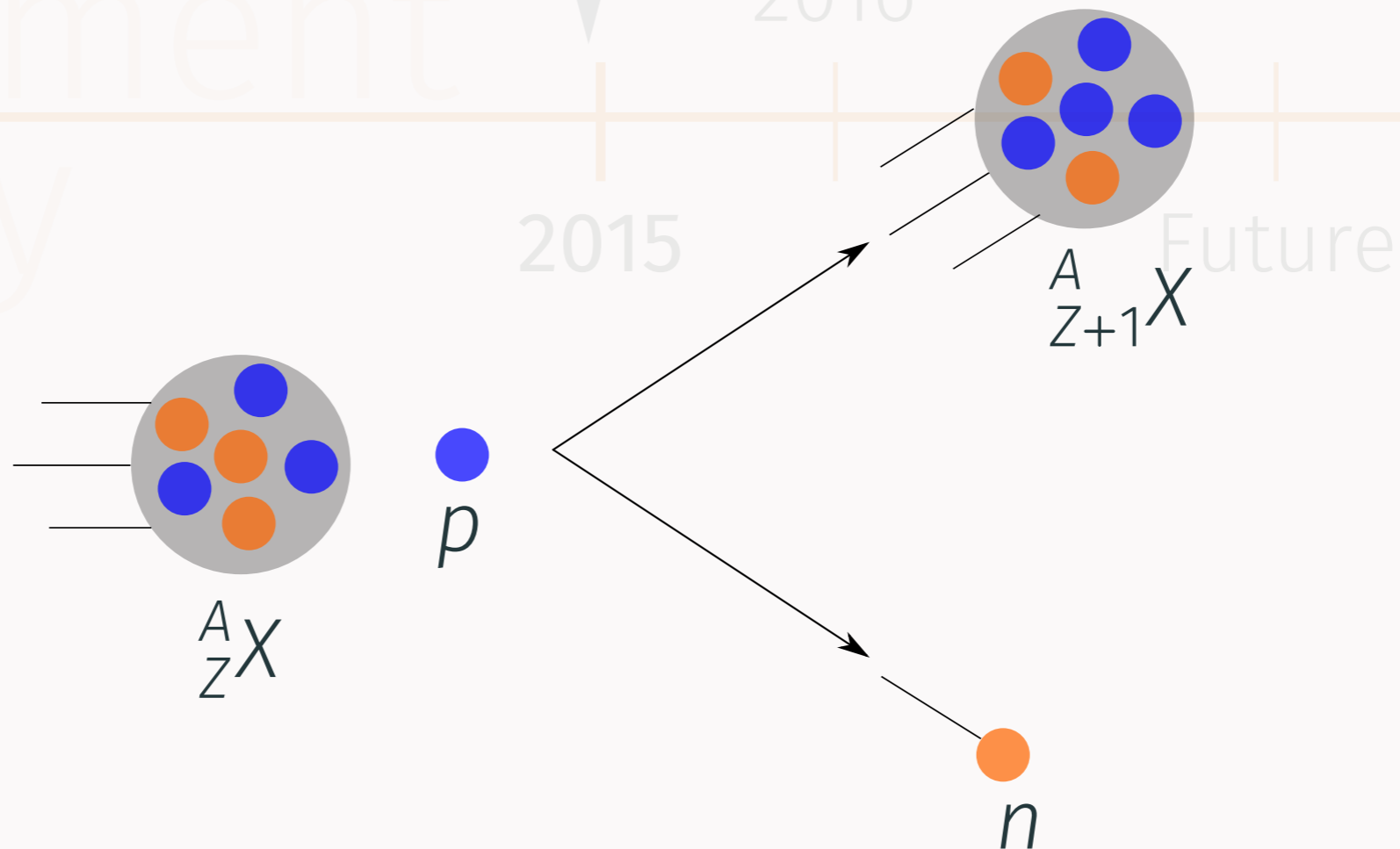
Experiment  
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Single-charge-exchange reaction

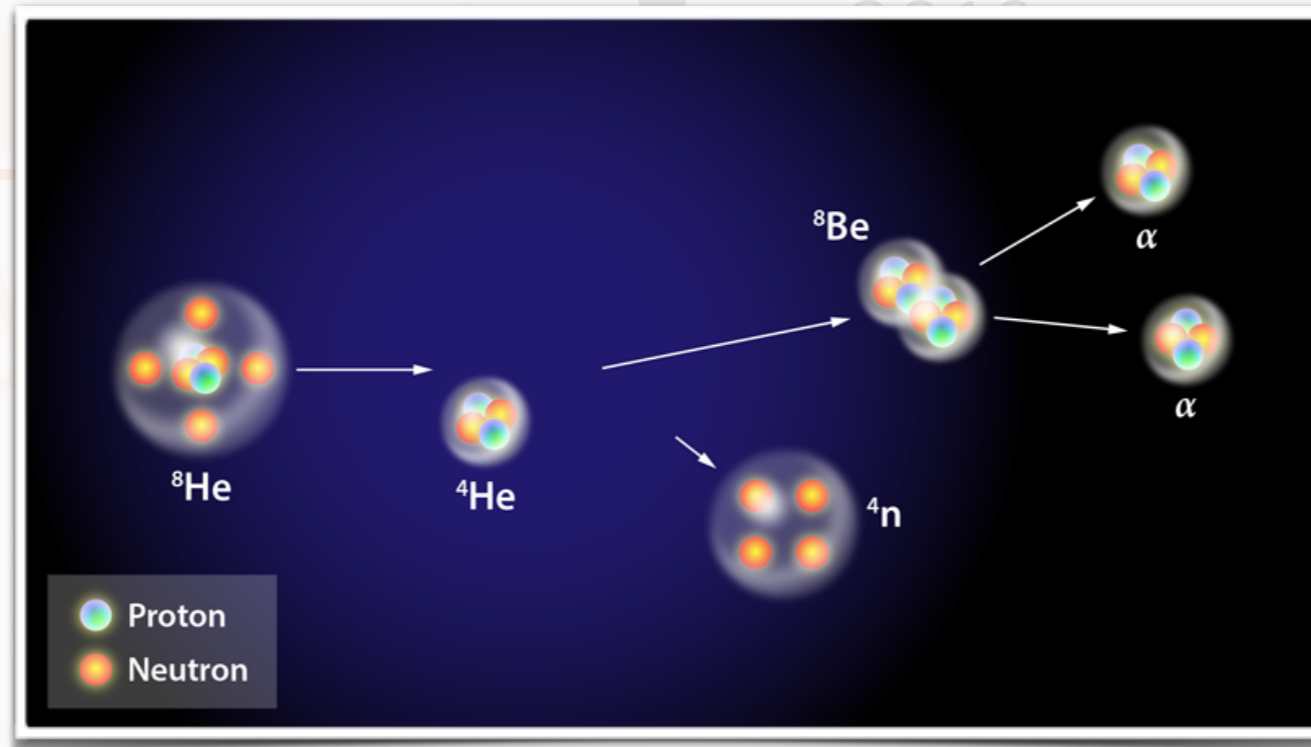




K. Kisamori et al., Phys. Rev. Lett. **116**, 044006.

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Double-charge-exchange reaction

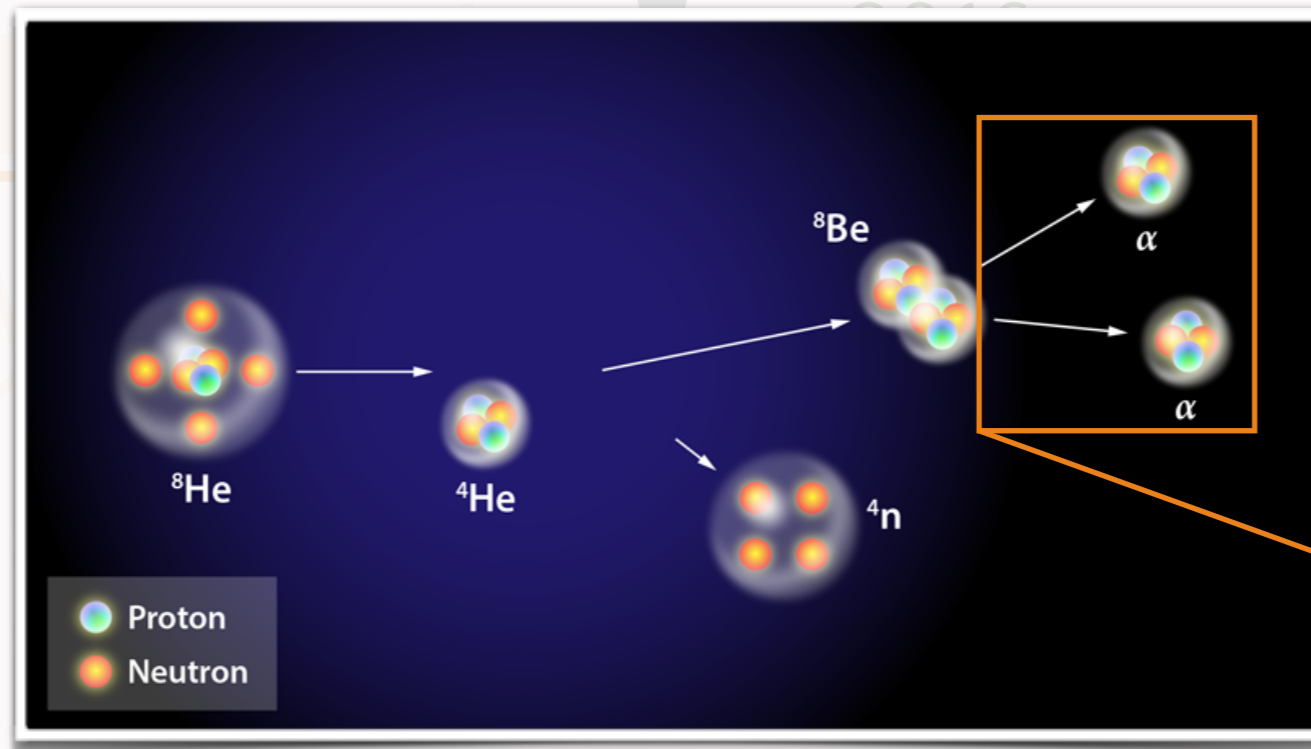


APS/Alan Stonebraker

K. Kisamori et al., Phys. Rev. Lett. **116**, 044006.

A recent double-charge-exchange reaction  ${}^8_2\text{He} + {}^4_2\text{He} \rightarrow {}^8_4\text{Be} + 4n$  measurement at the RIKEN radioactive ion beam factory (RIBF) suggests a tetraneutron resonance at  $0.83 \pm 0.65(\text{stat}) \pm 1.25(\text{syst})$  MeV.

Double-charge-exchange reaction

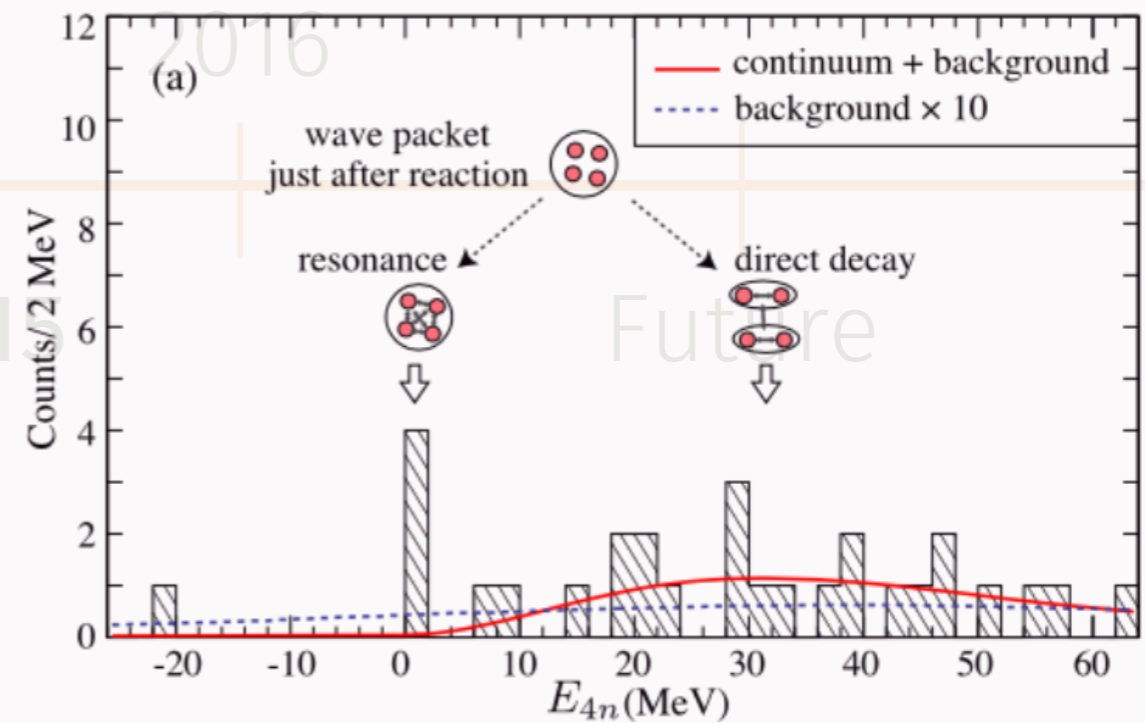
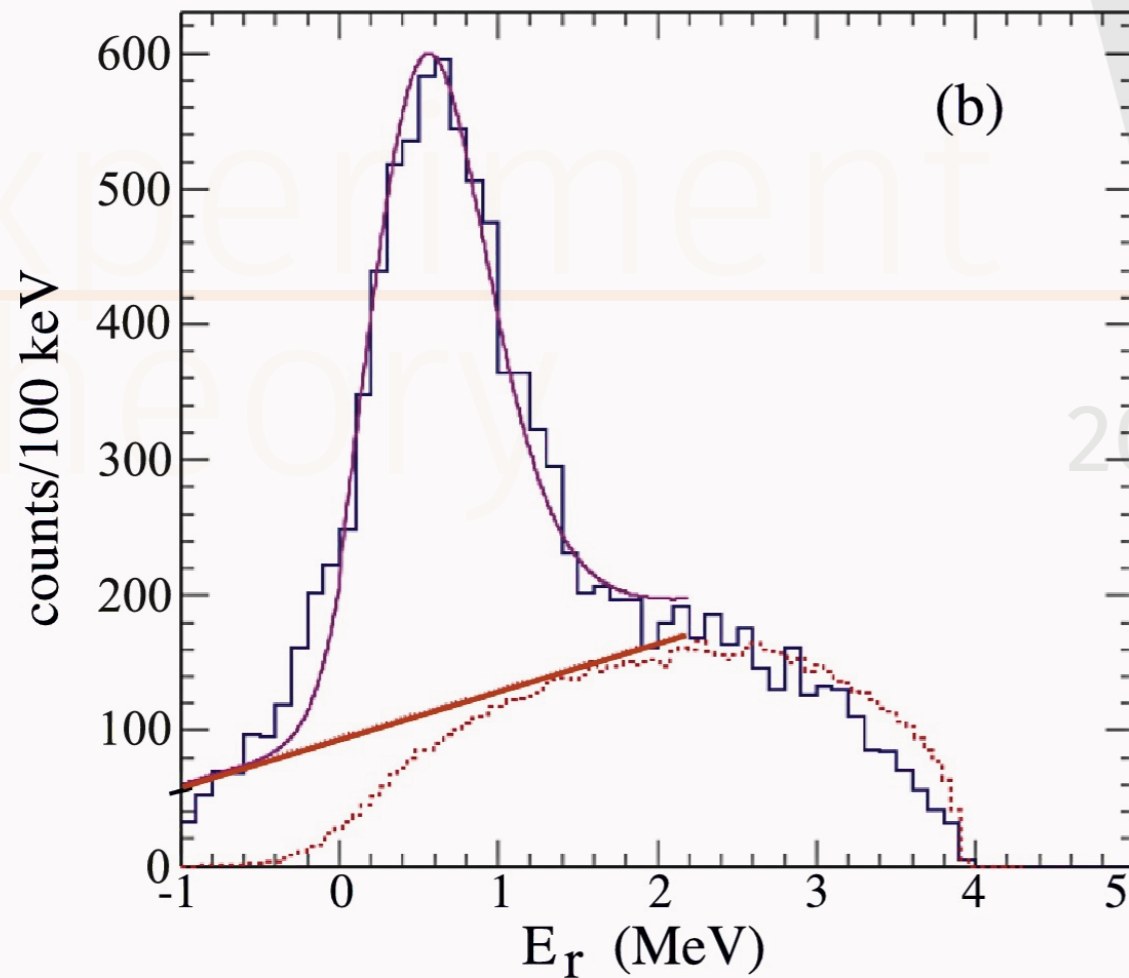


Measured

Know  $P_{8\text{He}}$ ,  $P_{\alpha}$ ,  $P_{\alpha'}$ , and  $P_{\alpha} \cdot P_{\alpha'}$ :  
Calculate “missing mass” spectrum of  $4n$ .

K. Kisamori et al., Phys. Rev. Lett. **116**, 044006.

A recent double-charge-exchange reaction  ${}^8_2\text{He} + {}^4_2\text{He} \rightarrow {}^8_4\text{Be} + 4n$  measurement at the RIKEN radioactive ion beam factory (RIBF) suggests a tetraneutron resonance at  $0.83 \pm 0.65(\text{stat}) \pm 1.25(\text{syst})$  MeV.

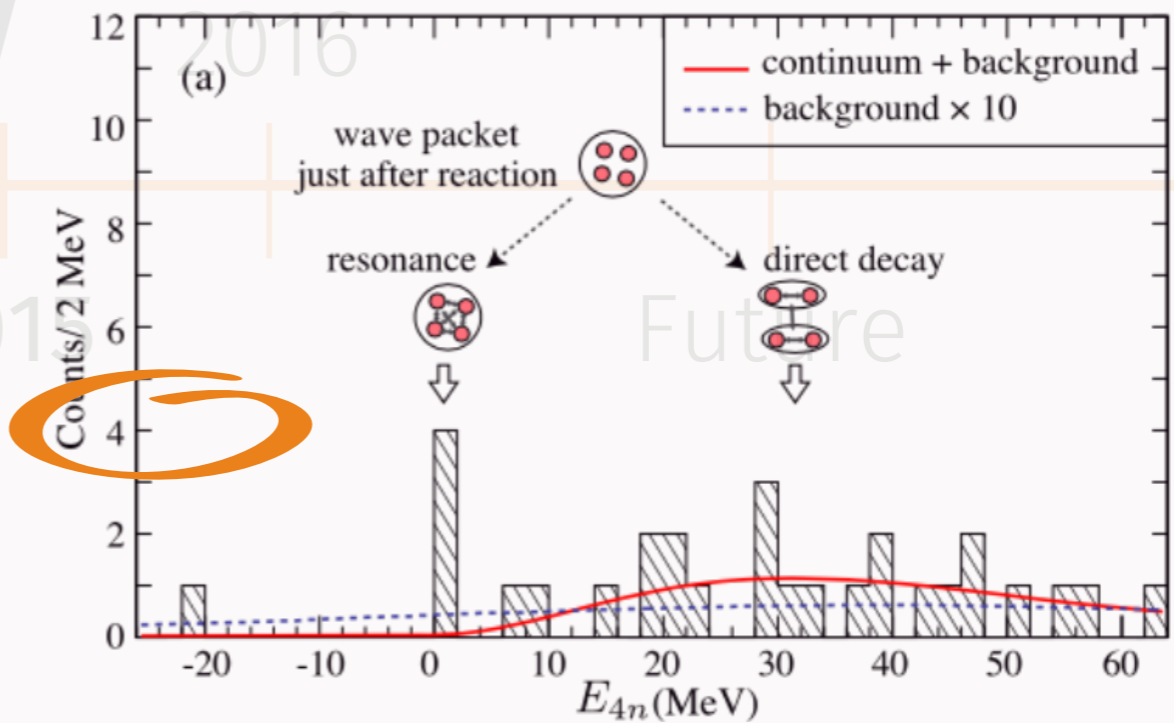
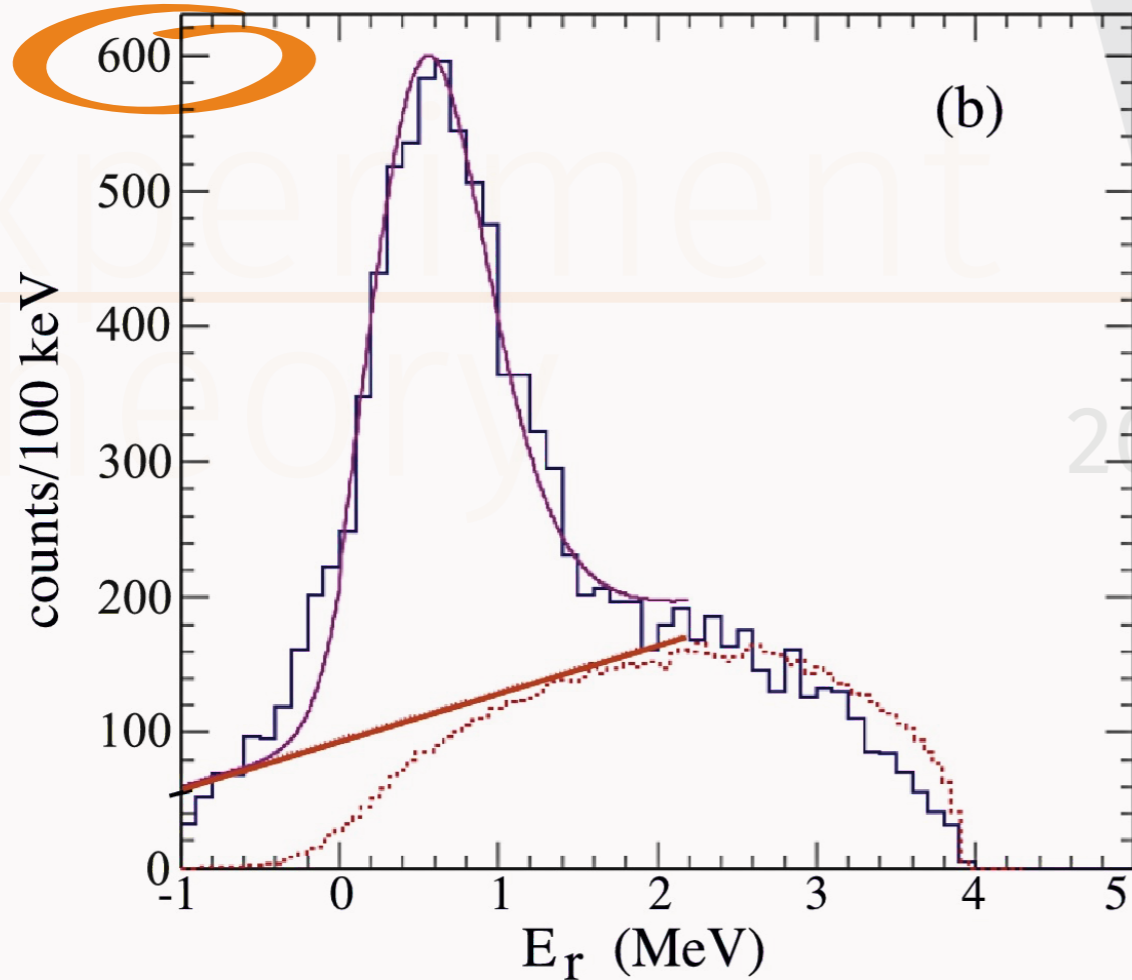


from A. Sanetullaev et al., Phys. Lett. B **755**, 481 (2016)

K. Kisamori et al., Phys. Rev. Lett. **116**, 044006.

A recent double-charge-exchange reaction  ${}^8_2\text{He} + {}^4_2\text{He} \rightarrow {}^8_4\text{Be} + 4n$  measurement at the RIKEN radioactive ion beam factory (RIBF) suggests a tetraneutron resonance at  $0.83 \pm 0.65(\text{stat}) \pm 1.25(\text{syst})$  MeV.

Relatively low statistics: More data needed!

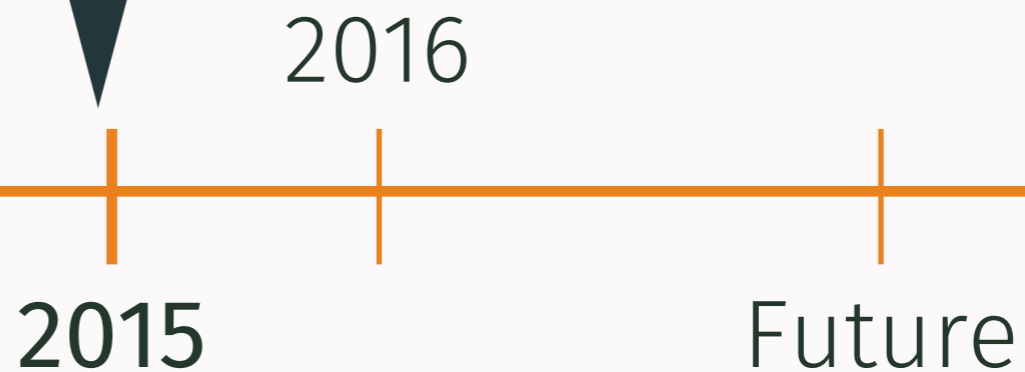


from A. Sanetullaev et al., Phys. Lett. B **755**, 481 (2016)

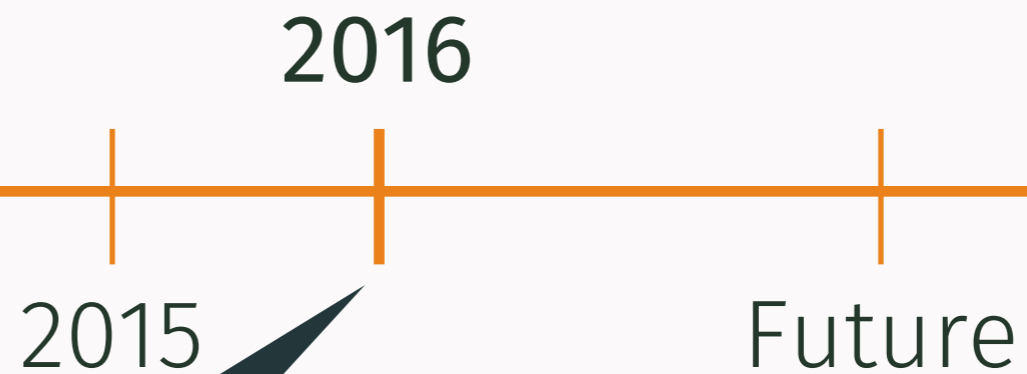
K. Kisamori et al., Phys. Rev. Lett. **116**, 044006.

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Experiment  
Theory



# Experiment Theory



E. Hiyama, R. Lazauskas, J. Carbonell, and M. Kamimura, *Phys. Rev. C* **93**, 044004.

Complex scaling w/ $AV8'$  potential + toy  $T = 3/2$   $3N$  interaction. Low-lying  $^4n$  resonance only possible if other well-known resonance structures in light nuclei are strongly perturbed.

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A. M. Shirokov, G. Papadimitriou, A. I. Mazur, R. Roth, J. P. Vary, *Phys. Rev. Lett.* **117**, 1825022.

No-Core Shell Model + Single-State Harmonic Oscillator Representation of Scattering equations. Compelling confirmation of a  ${}^4n$  resonance at 0.8 MeV with **JISP**  $NN$  interaction.

T. Aumann, D. Rossi, S.  
Shimoura, S. Paschalis  
et al., RIBF Experimental  
Proposal

NP1406-SAMURAI19.  
 ${}^8_2\text{He}(p, p\alpha){}^4n$

Experiment  
Theory

2015

2016

Future



T. Aumann, D. Rossi, S. Shimoura, S. Paschalis et al., RIBF Experimental Proposal

NP1406-SAMURAI19.  
 ${}^8_2\text{He}(p, p\alpha)^4n$

F. M. Marqués et al., RIKEN-RIBF proposal

“Many-neutron systems: search for superheavy  ${}^7\text{H}$  and its tetraneutron decay,” NP-1512-SAMURAI34.

Experiment  
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S. Shimoura et al., RIKEN-RIBF proposal “Tetraneutron resonance produced by exothermic double-charge exchange reaction,” NP1512-SHARAQ10.

Experiment  
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T. Aumann, D. Rossi, S. Shimoura, S. Paschalis et al., RIBF Experimental Proposal

NP1406-SAMURAI19.  
 ${}^8_2\text{He}(p, p\alpha)^4n$

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2015

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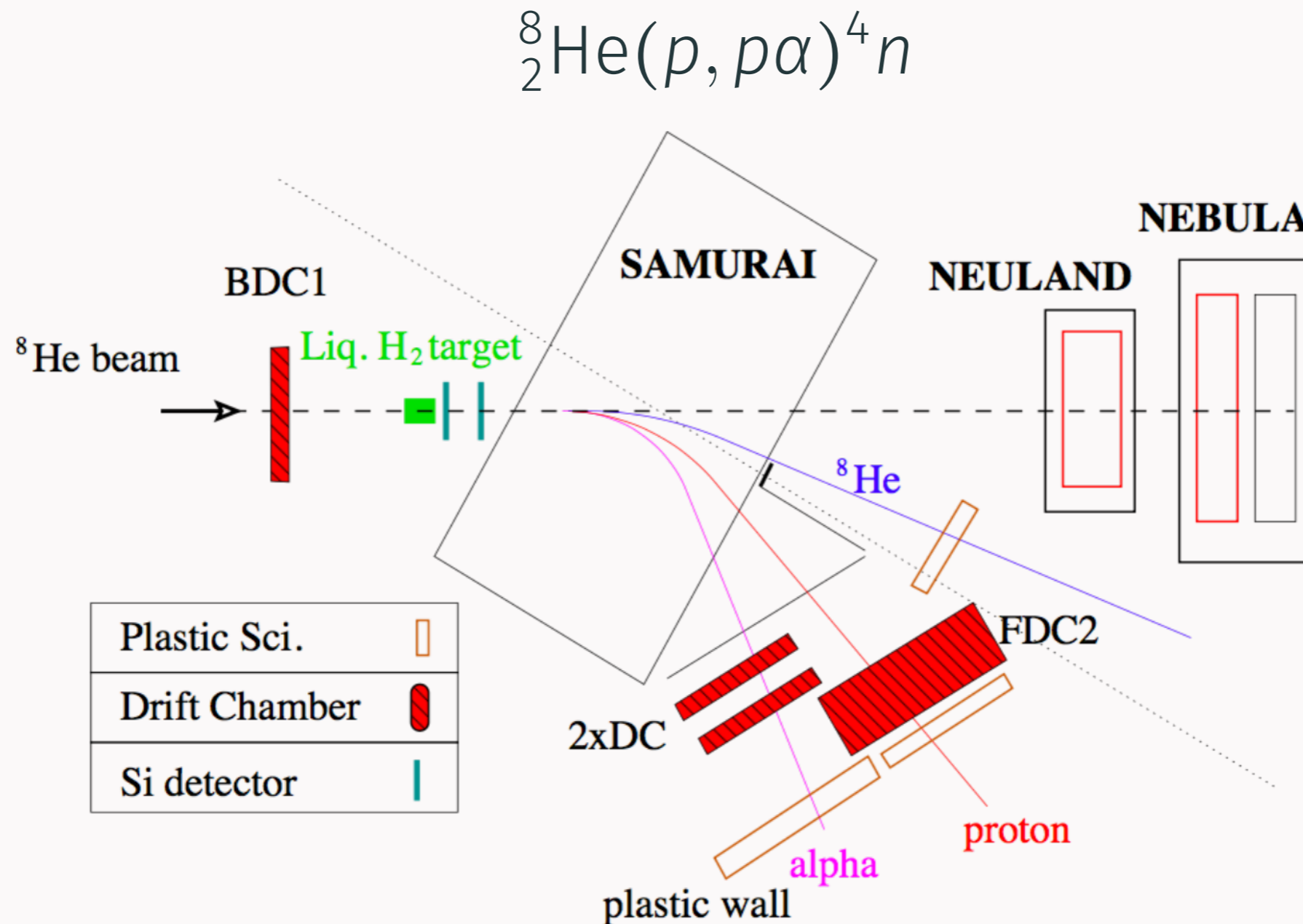
What's still missing?

An *ab initio* calculation with chiral  $NN$  and  $3N$  interactions.

Initial efforts using Quantum Monte Carlo calculations with chiral interactions.  
(This talk!)

# Motivation - Exp. Proposal Of SFB 1245 A06.4

Quasi-free alpha knockout reaction RIBF at RIKEN utilizing the so-called SAMURAI configuration

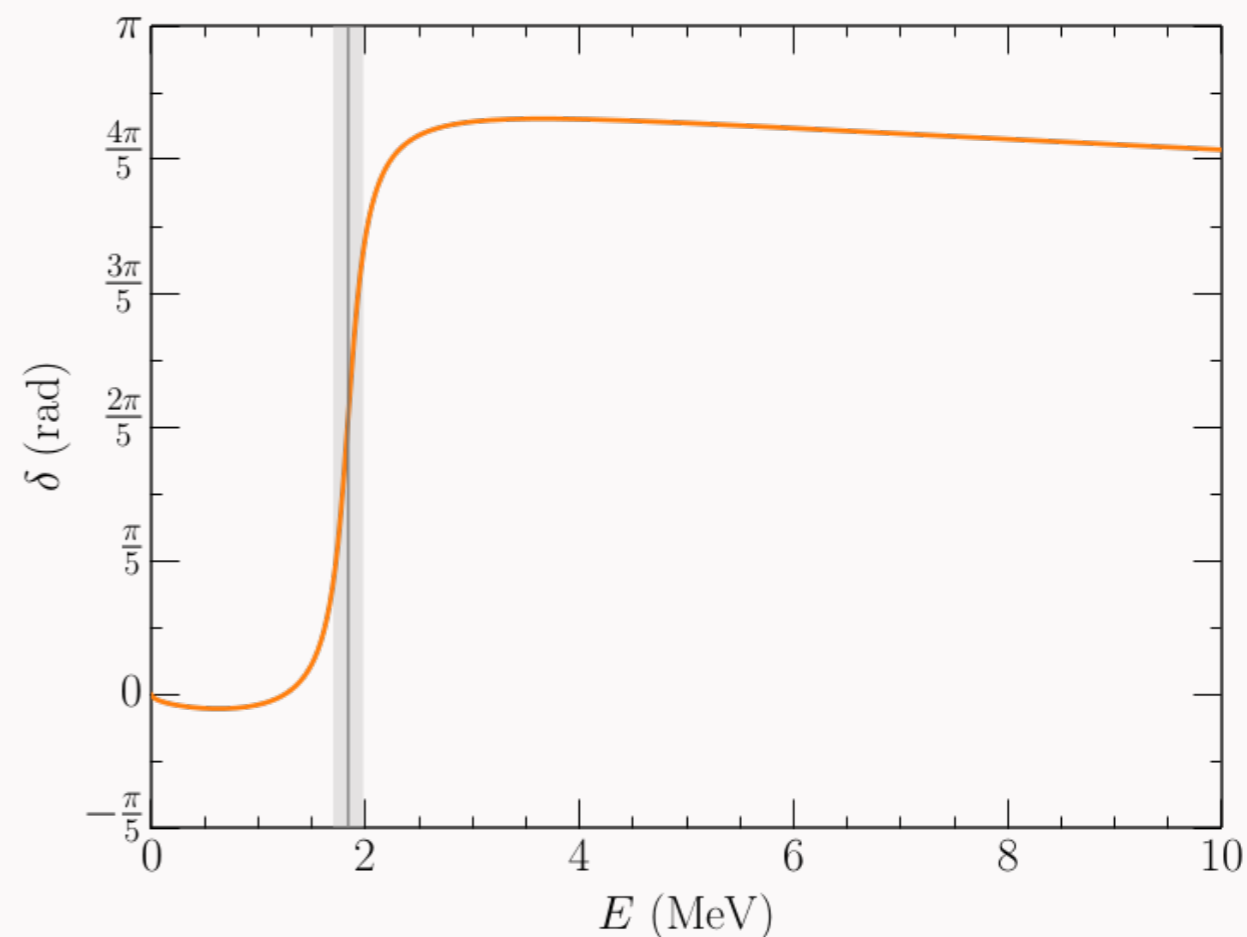
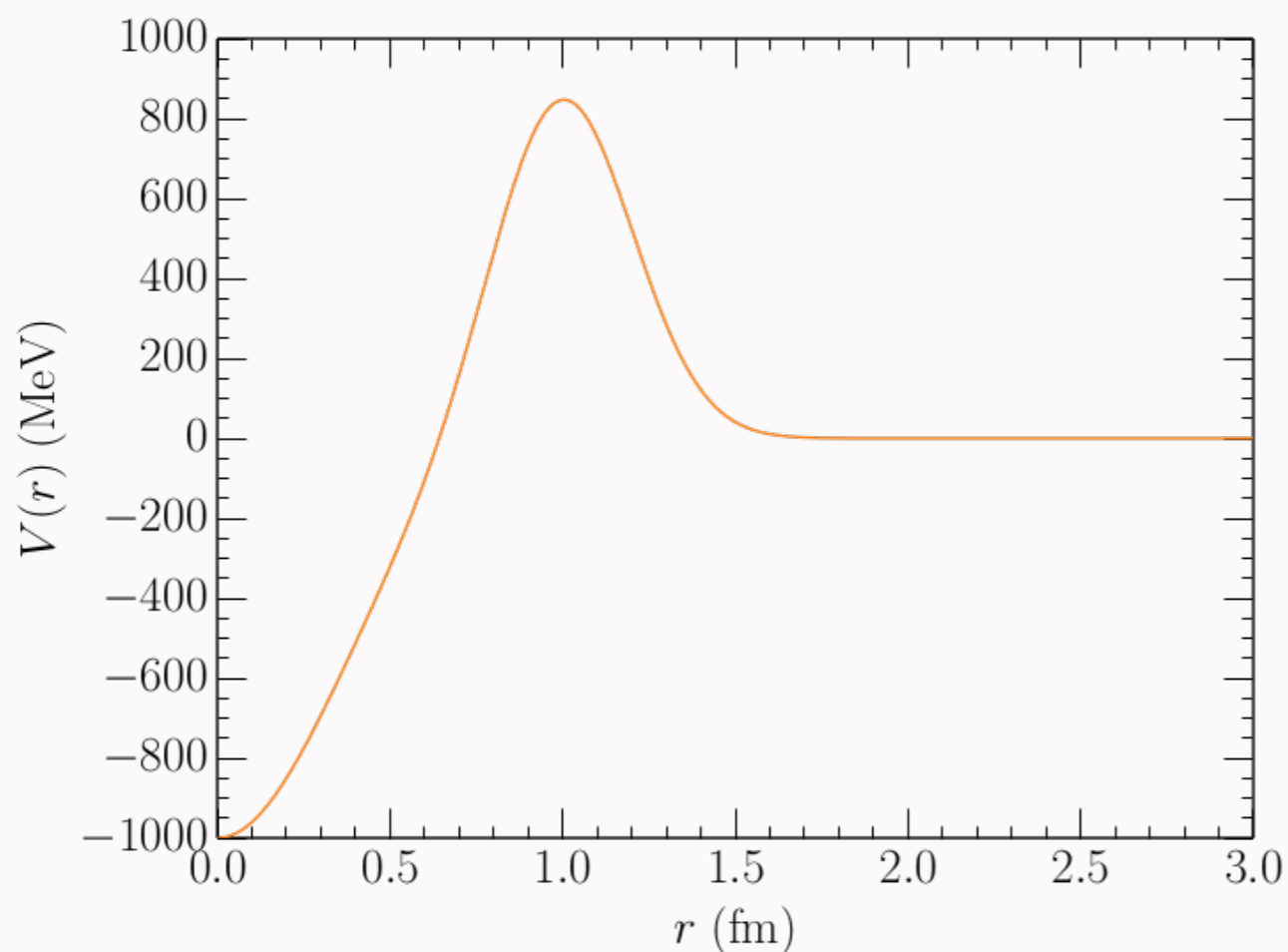


# A Two-Body Test

A simple  $S$ -wave potential:

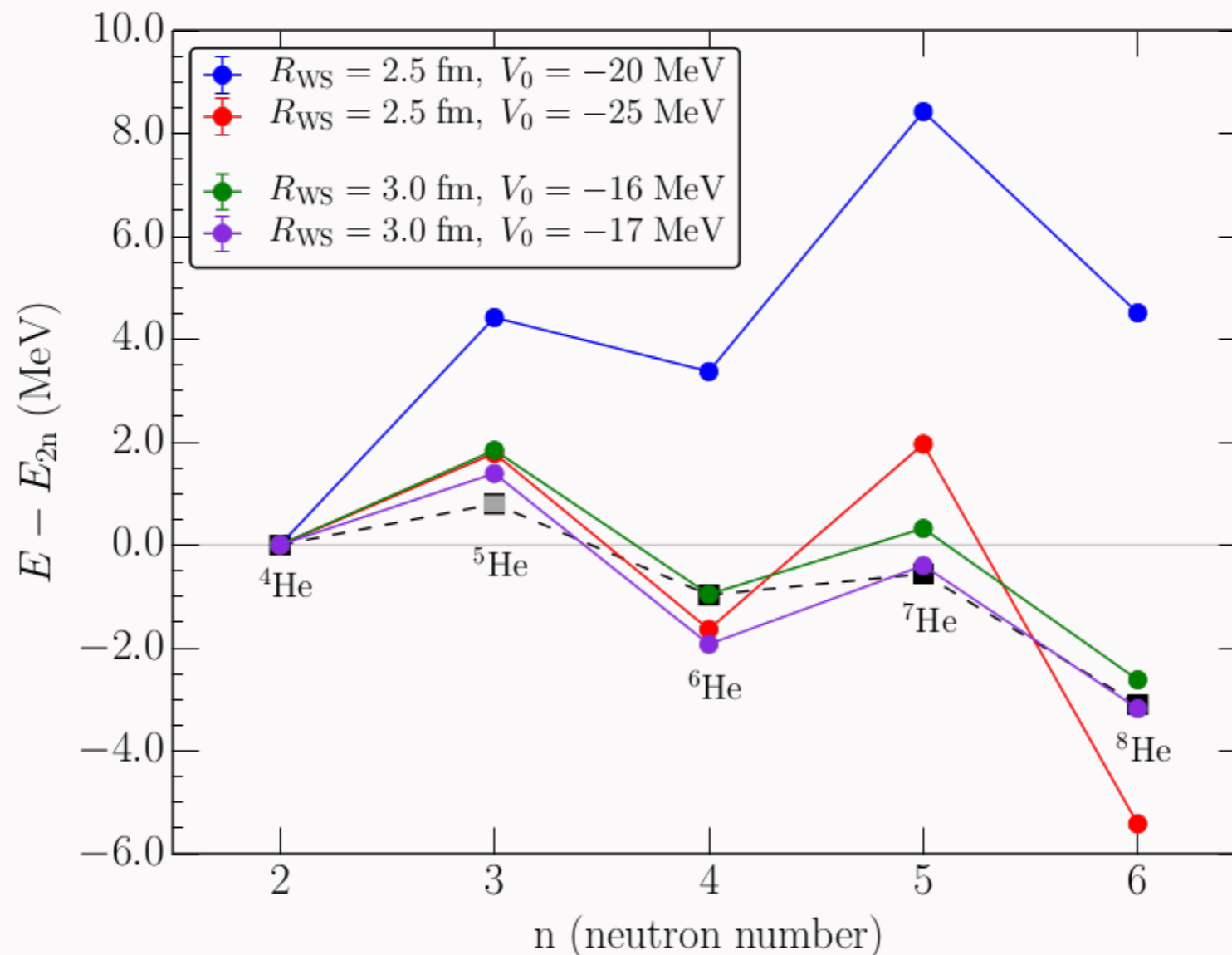
$$V(r) = V_1 e^{-\left(\frac{r}{R_1}\right)^2} + V_2 e^{-\left(\frac{r-r_2}{R_2}\right)^2}$$

$$E_R = 1.84 \text{ MeV}, \Gamma = 0.282 \text{ MeV}$$



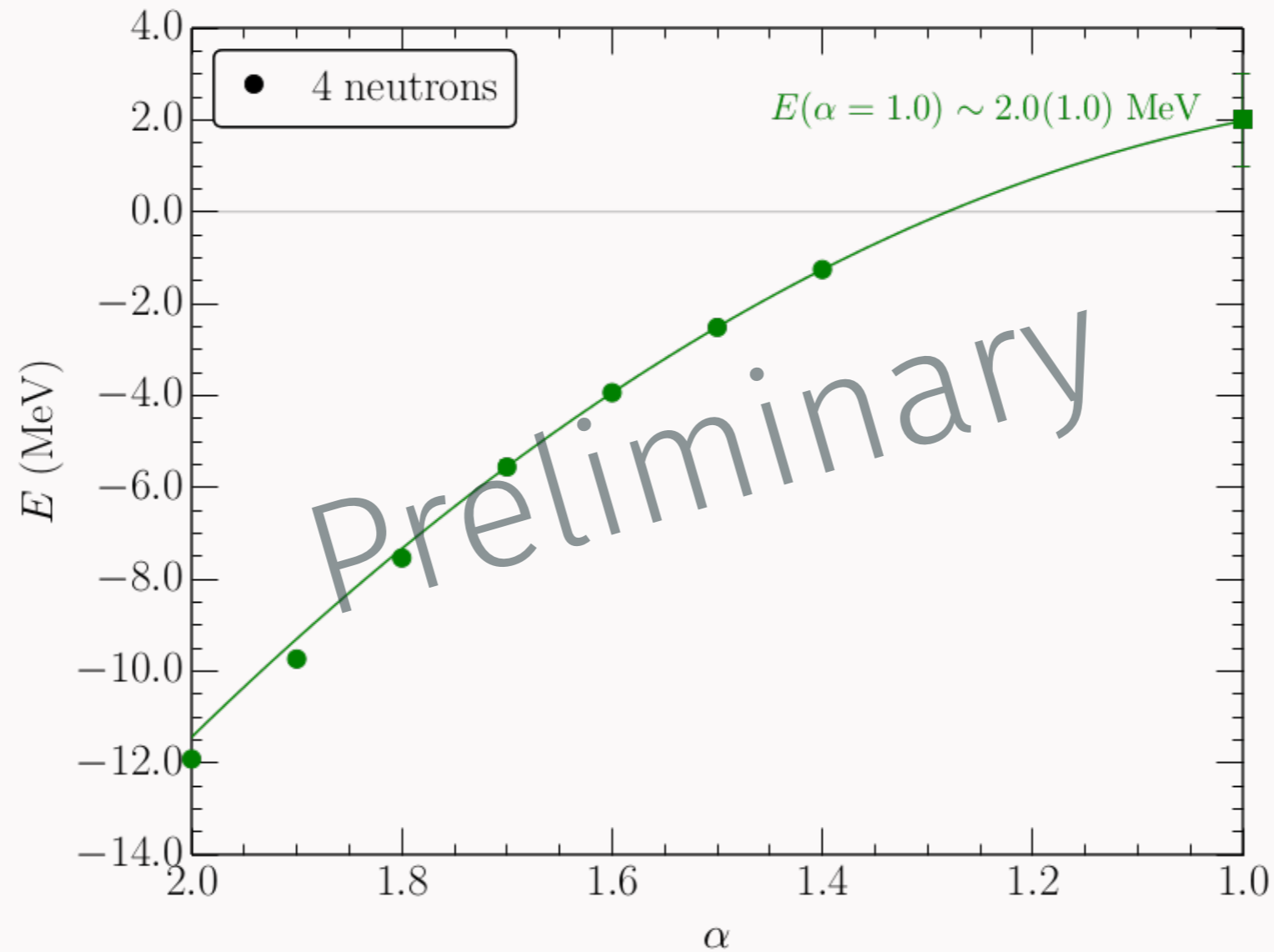
# Helium Chain

- That  ${}^3n$  is lower than  ${}^4n$  is not an artifact of the Woods-Saxon potential.
- In helium chain,  ${}^3n$  is always higher than  ${}^4n$ .



# Another Extrapolation\*

$$V \rightarrow \alpha V, \quad E_R = \lim_{\alpha \rightarrow 1} E(\alpha)$$



\* à la K. Fossez, J. Rotureau, N. Michel, and M. Płoszajczak arXiv:1612.01483

# Cold Atoms Connections

- Extrapolated energies for  ${}^3n$  and  ${}^4n$  are consistent with scaling like the number of pairs.  $E_{A_n} \sim \frac{A(A-1)}{2}$
- Mean-field interaction of dilute gas of spin-1/2 fermions:  $E_{MF}/A = \frac{k_F^2}{2m} \frac{2}{3\pi} (k_F a) \sim A \Rightarrow E_{MF} \sim A^2$
- Cold atomic gas experiments could determine if one-body density behavior is governed by large-scattering-length physics or details of nuclear interactions.



# Some History And Definitions

Deep inelastic scattering (DIS) cross section for EM interactions of charged leptons with nuclear targets:

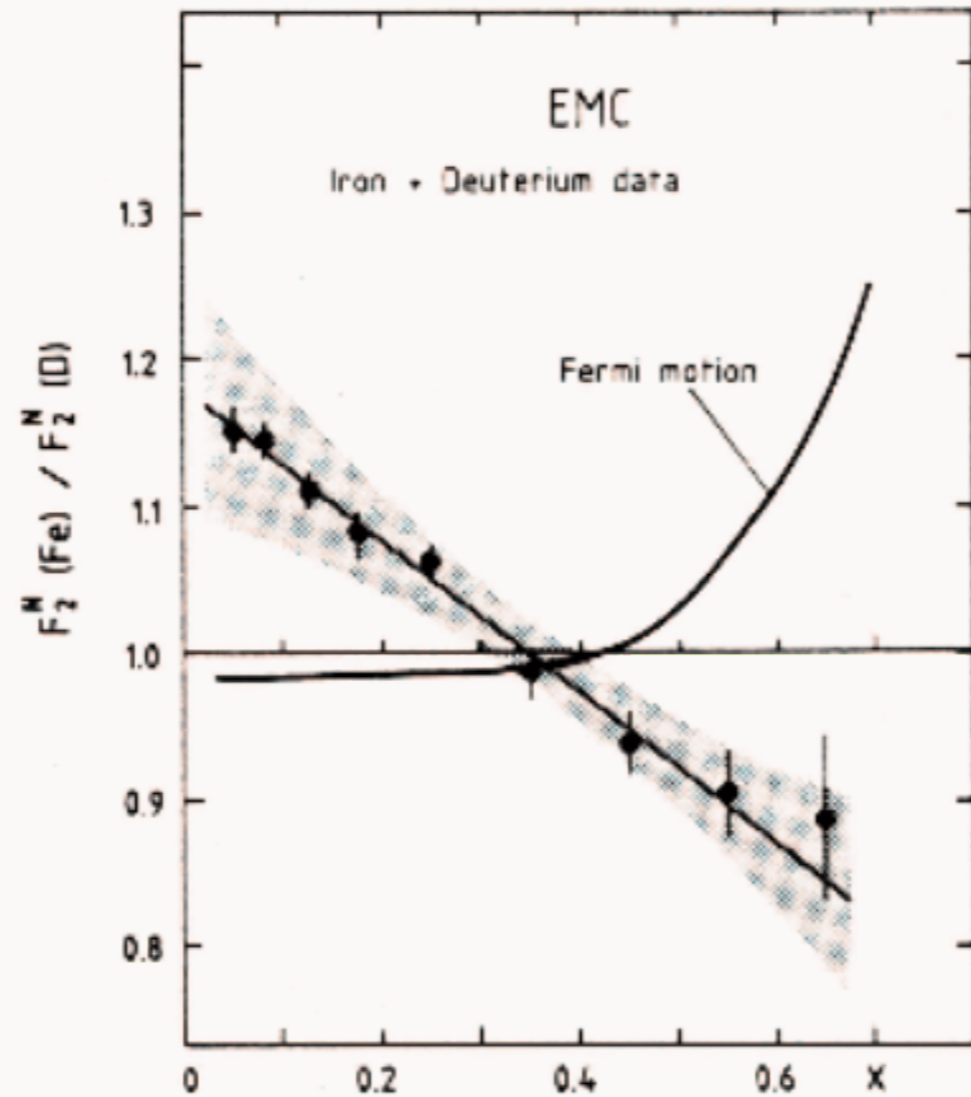
$$\frac{d^2\sigma}{dQ^2 dx} \propto \frac{4\pi\alpha^2}{Q^4} \frac{F_2^A(x, Q^2)}{x}$$

Bjorken  $x = Q^2/(2p \cdot q)$ , and  $Q^2 = -q^2$  are defined in terms of the target four-momentum  $p$  and the momentum transfer from the lepton to the target,  $q$ .

The ratio  $R_{\text{EMC}}(A, x) = \frac{2F_2^A(x, Q^2)}{AF_2^d(x, Q^2)} \sim \frac{2\sigma^A}{A\sigma^d}$  plays an important role.

# 1983 EMC Paper

One-picture/One-sentence summary



K. Rith, arXiv:1402.5000 [hep-ex] (2014)

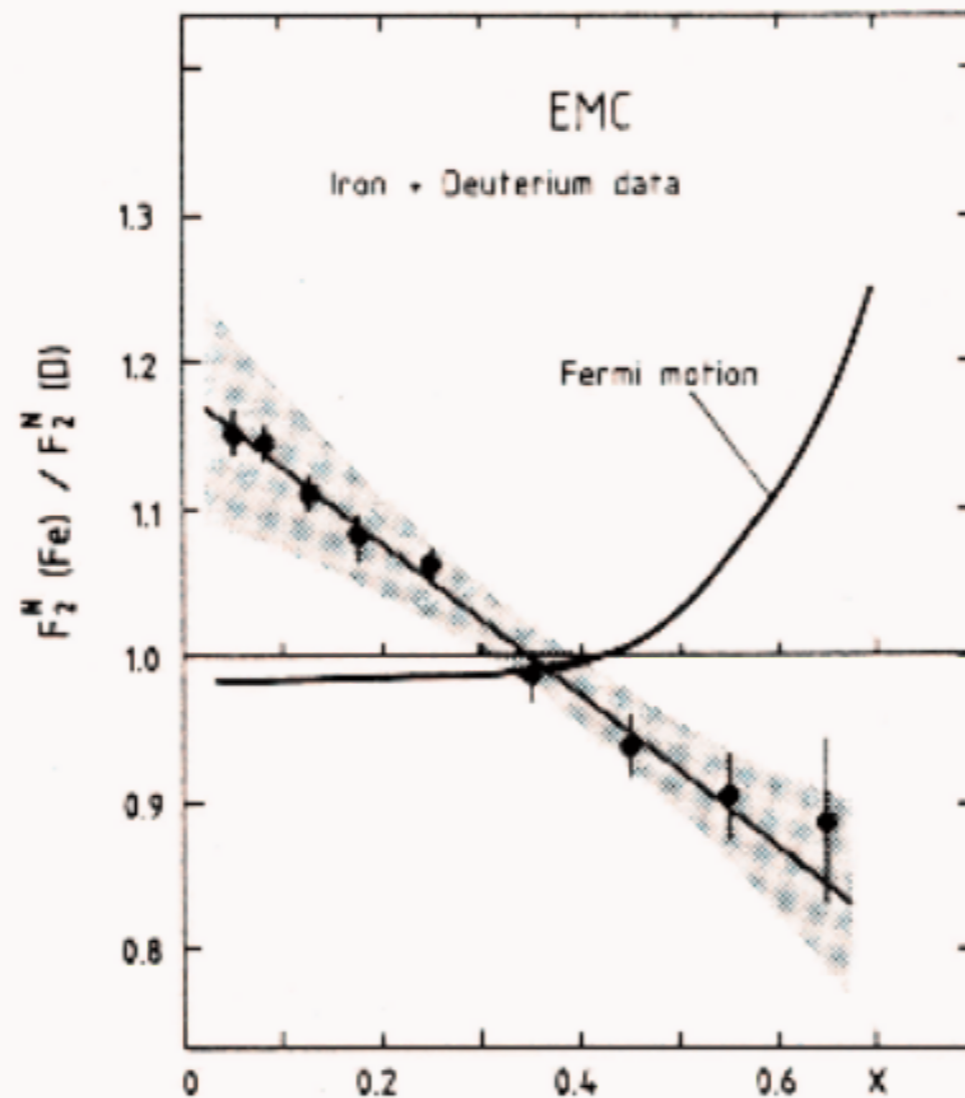
J. J. Aubert et al. (EMC), Phys. Lett. B. **123**, 275

“We are not aware of any published detailed prediction presently available which can explain the behaviour of these data.”

# 1983 EMC Paper

The strength of the EMC effect is given in terms of the slope:

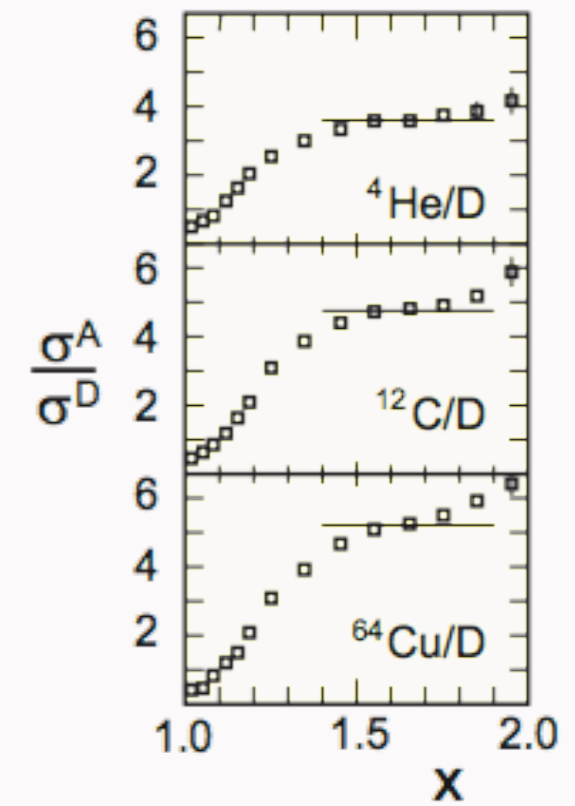
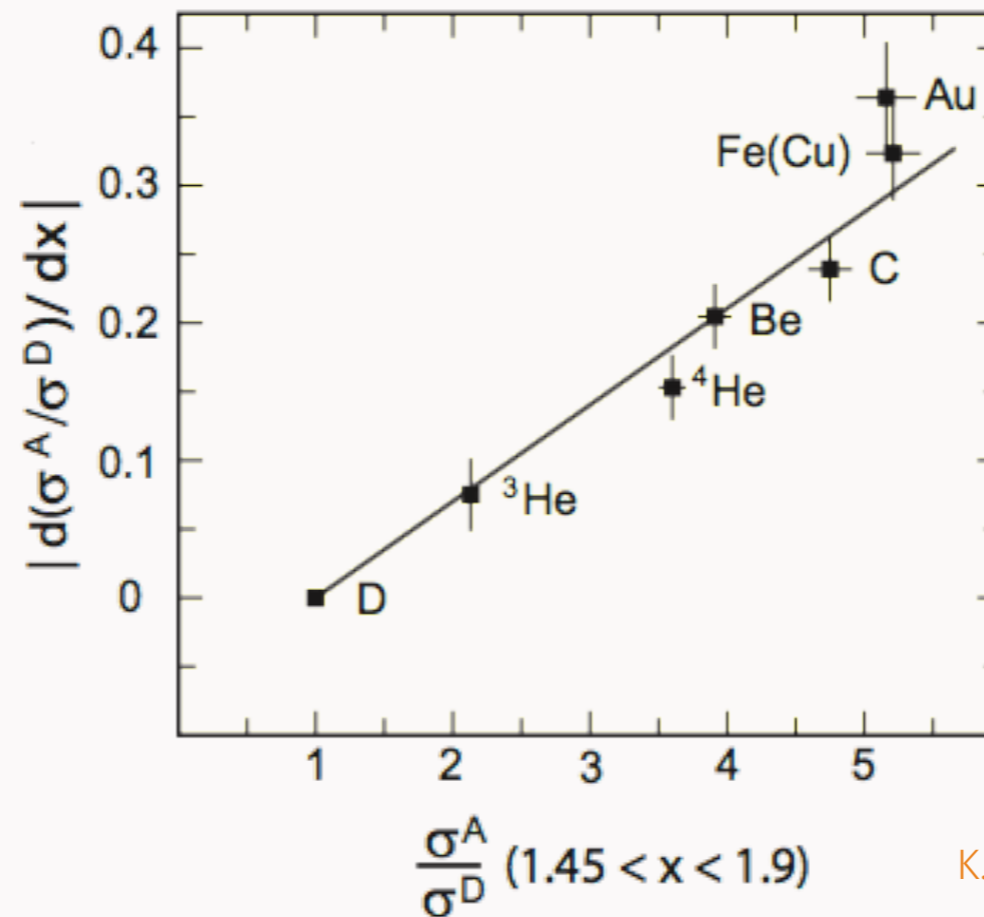
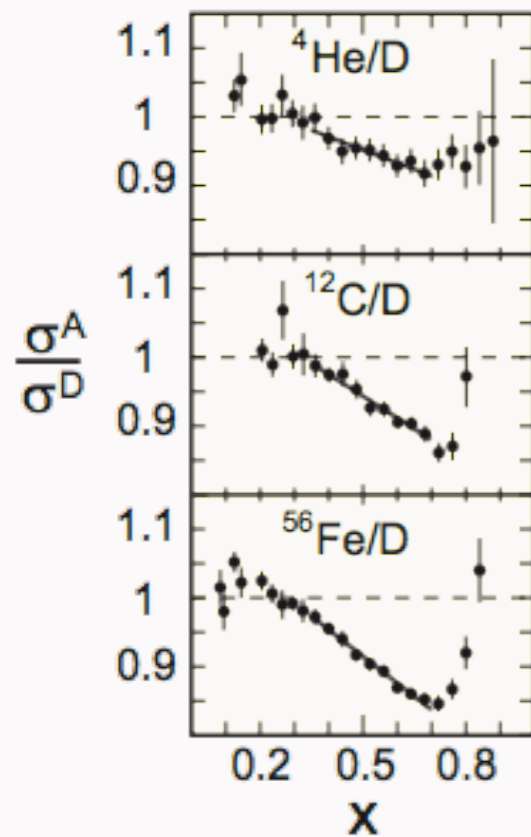
$$dR_{\text{EMC}}(A, x)/dx|_{0.35 < x < 0.7} \sim d(\sigma^A / \sigma^d) / dx|_{0.35 < x < 0.7}$$



# Short-Range Correlations And The EMC Effect

$$\text{SRC scaling factor } a_2(A, x) \equiv \frac{2\sigma^A}{A\sigma^d} \Big|_{1.5 < x < 2.0}$$

$$dR_{\text{EMC}}/dx \propto a_2$$



K. Rith, arXiv:1402.5000 [hep-ex] (2014)

# Implications Of EFT

J.-W. Chen & W. Detmold, Phys. Lett. B **625**, 165 (2005):

Structure functions factorize:  $F_2^A(x)/A = F_2^N(x) + g_2(A, \Lambda) f_2(x, \Lambda)$

$$g_2(A, \Lambda) = \frac{1}{A} \langle A | (N^\dagger N)^2 | A \rangle_\Lambda$$

J.-W. Chen, W. Detmold, JEL, A. Schwenk,  
arXiv:1607.03065 [hep-ph] (2016):

$$a_2(A, x > 1) = \frac{g_2(A, \Lambda)}{g_2(2, \Lambda)} \Rightarrow \frac{dR_{\text{EMC}}}{dx} \propto a_2.$$