

Superfluid Helium-3: Universal Concepts for Condensed Matter and the Big Bang

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GSI Kolloquium, Darmstadt; May 9, 2017

Periodic table of the elements

Noble gas																				
1	H	2																		
3	Li	Be																		
11	Na	Mg	3																	
19	K	Ca	Sc	4	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr		
37	Rb	Sr	Y	22	Ti	23	24	25	26	27	28	29	31	32	33	34	35	36		
55	Cs	Ba	La	40	Zr	41	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
87	Fr	Ra	Ac	58-71	72	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
				90-103	104	Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn	Nh	Fl	Mc	Lv	Ts	Og
Lanthanoide																				
Actinoide																				
Ce Pr Nd Pm Sm Eu Gd Tb Dy Ho Er Tm Yb Lu																				
Th Pa U Np Pu Am Cm Bk Cf Es Fm Md No Lr																				
 natural, stable  experimentally produced  discovered at GSI  verified at GSI																				

Helium: after hydrogen the most abundant element in the universe

Helium

Two stable Helium isotopes: ^4He , ^3He

^4He : air, oil wells, ...

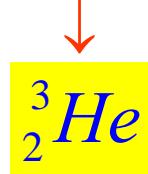
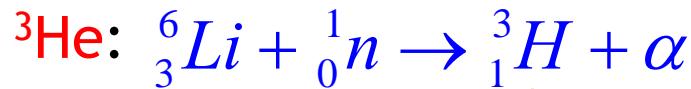
Janssen/Lockyer/Secci (1868)



$$\frac{{}^4\text{He}}{\text{air}} \approx 5 \times 10^{-6}$$

$$\left| \frac{{}^3\text{He}}{{}^4\text{He}} \right|_{\text{air}} \approx 1 \times 10^{-6}$$

Ramsay (1895)
Cleveit (UO_2)



Research on macroscopic samples
of ${}^3\text{He}$ only since 1947

${}^4\text{He}$: Coolant, Welding, Balloons

- ${}^3\text{He}$:
- Contrast agent in medicine
 - Neutron detectors
 - ${}^3\text{He}$ - ${}^4\text{He}$ dilution refrigerators (quantum computers!)

Helium

Atoms: spherical, hard core diameter $\sim 2.5 \text{ \AA}$

Interaction:

- hard sphere **repulsion**
- van der Waals dipole/multipole **attraction**

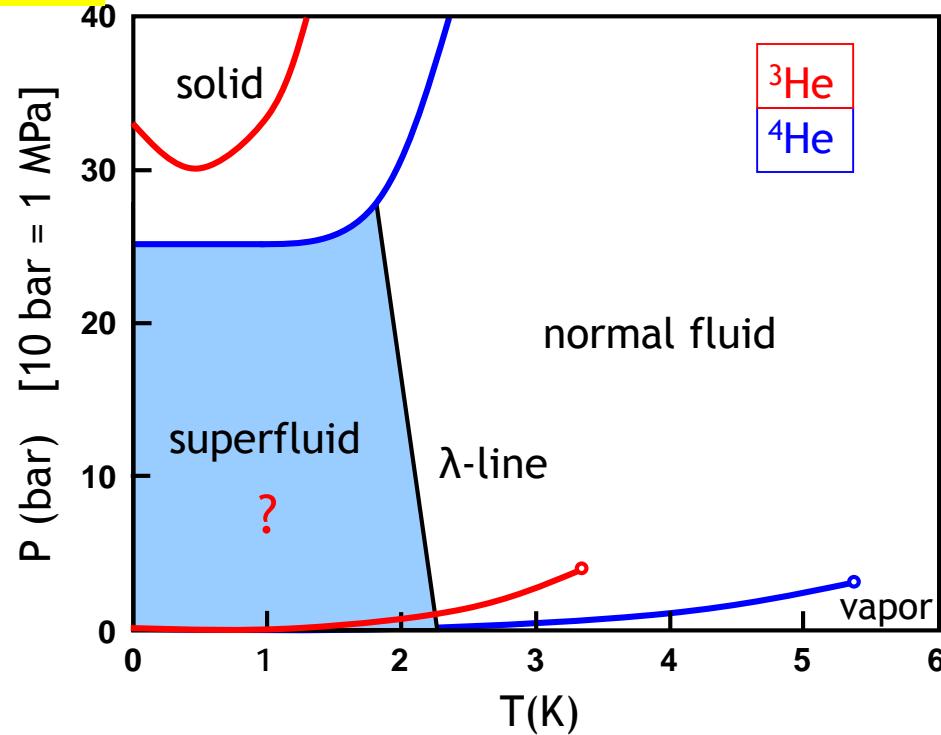
Boiling point: 4.2 K, ${}^4\text{He}$ Kamerlingh Onnes (1908)

3.2 K, ${}^3\text{He}$ Sydoriak *et al.* (1949)

Dense, simple liquids

$\left\{ \begin{array}{l} \text{isotropic} \\ \text{short-range interactions} \\ \text{extremely pure} \end{array} \right.$

Helium



- Atoms:
- spherical shape → weak attraction
 - light mass → strong zero-point motion

$T \rightarrow 0, P \lesssim 3$ MPa: Helium remains liquid

$$\lambda \propto \frac{\hbar}{\sqrt{k_B T}} \xrightarrow{T \rightarrow 0} \text{quantum phenomena on a macroscopic scale}$$

Helium

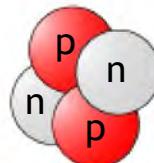
^4He

^3He

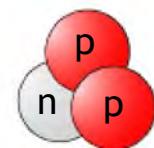
Electron shell:

$2 e^-$, $S = 0$

Nucleus:



$$S = 0$$



$$S = \frac{1}{2}\hbar$$

Atom(!) is a



Phase transition

$$T_\lambda = 2.2 \text{ K}$$

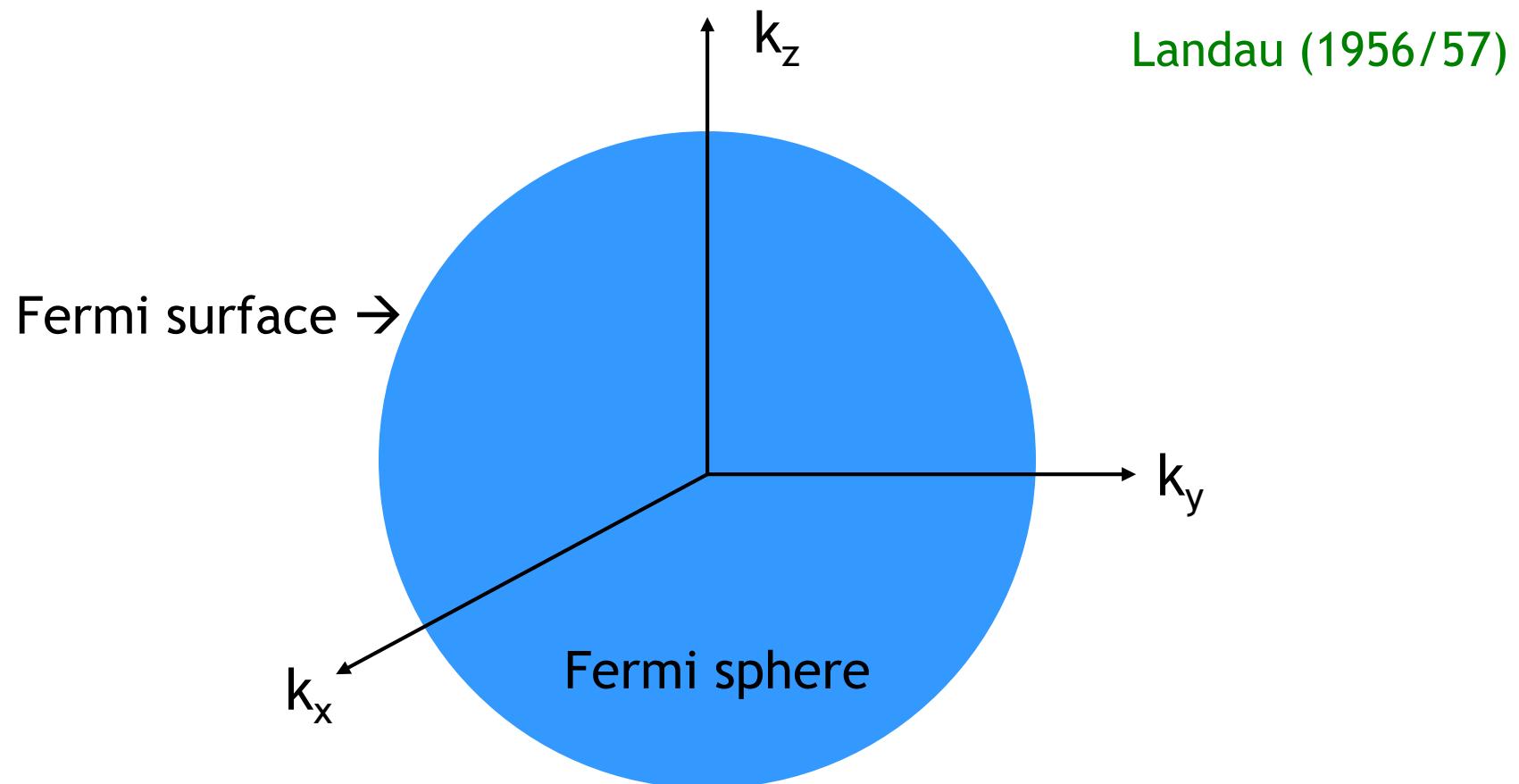
Bose-Einstein condensation → superfluid with frictionless flow

$$T_c = ???$$

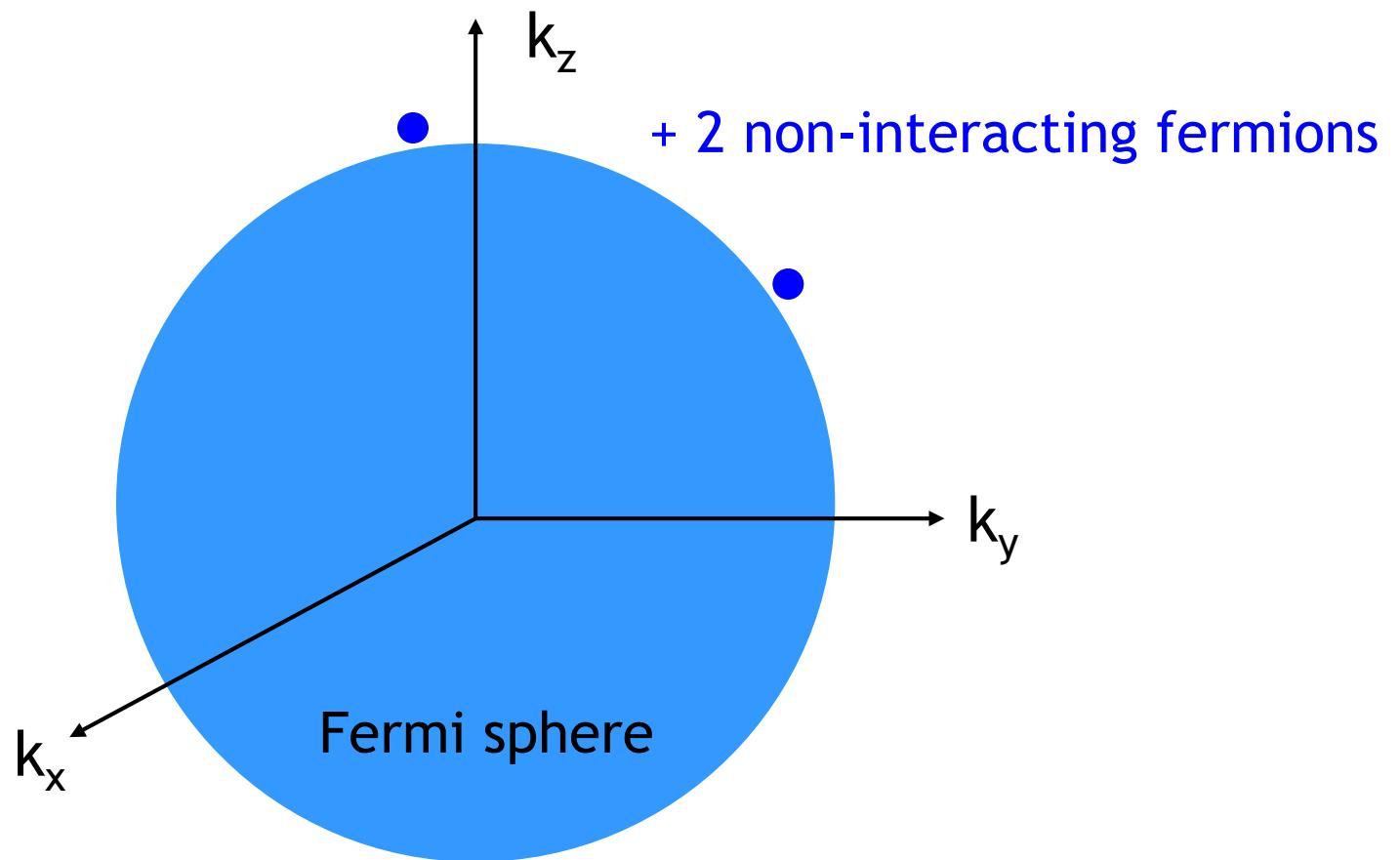
Fermion →

Quantum liquids

Interacting Fermions (Fermi liquid): Ground state

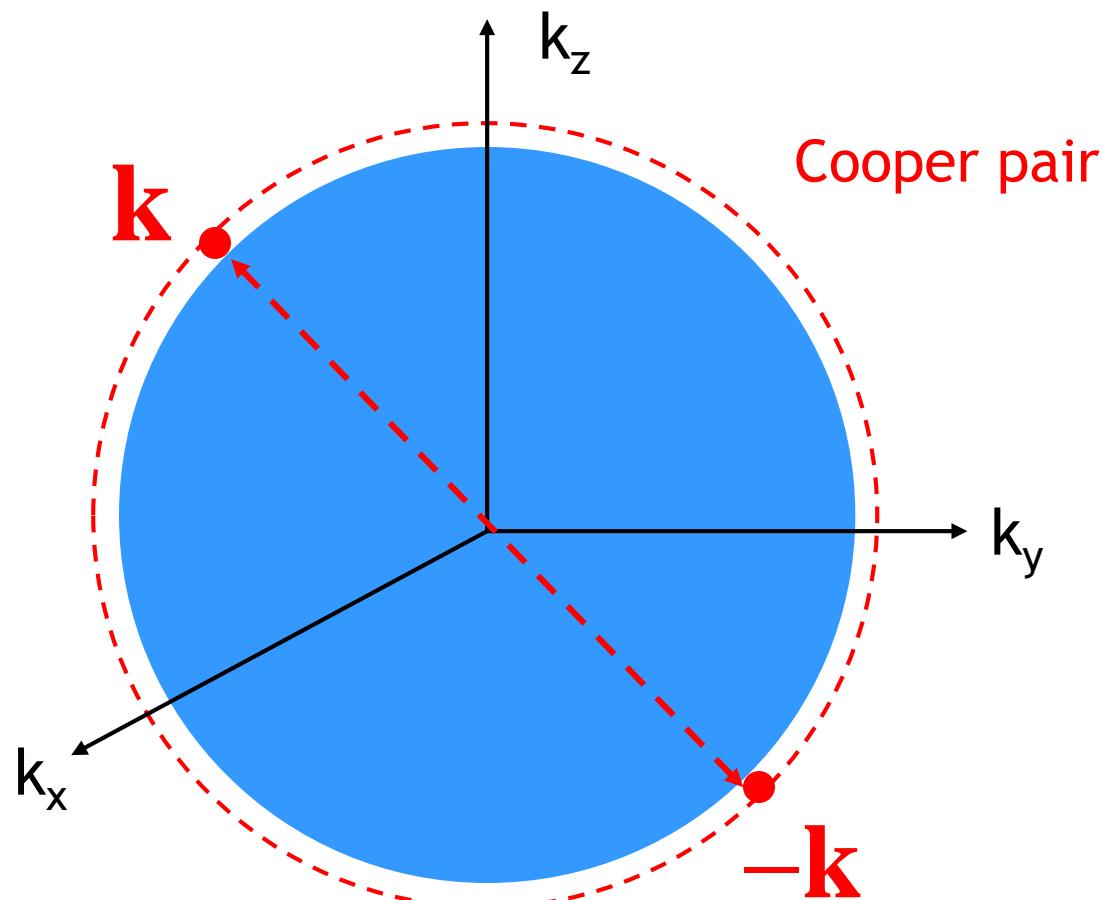


Instability of Fermi liquid



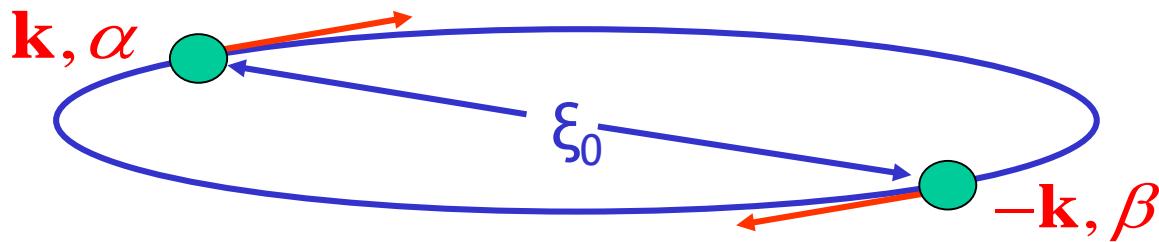
Arbitrarily weak attraction \Rightarrow instability

Cooper (1956)



Universal fermionic property

Arbitrarily weak attraction \Rightarrow Cooper pair $(\mathbf{k}, \alpha; -\mathbf{k}, \beta)$



$$\Psi_{L=0,2,4,\dots} = \psi(\mathbf{r}) |\uparrow\downarrow - \downarrow\uparrow\rangle$$

Antisymmetry

$S=0$ (singlet)

$$\begin{aligned} \Psi_{L=1,3,5,\dots} = & \psi_+(\mathbf{r}) |\uparrow\uparrow\rangle \\ & + \psi_0(\mathbf{r}) |\uparrow\downarrow + \downarrow\uparrow\rangle \\ & + \psi_-(\mathbf{r}) |\downarrow\downarrow\rangle \end{aligned}$$

$S=1$ (triplet)

$L = 0$ (“s-wave”): isotropic pair wave function

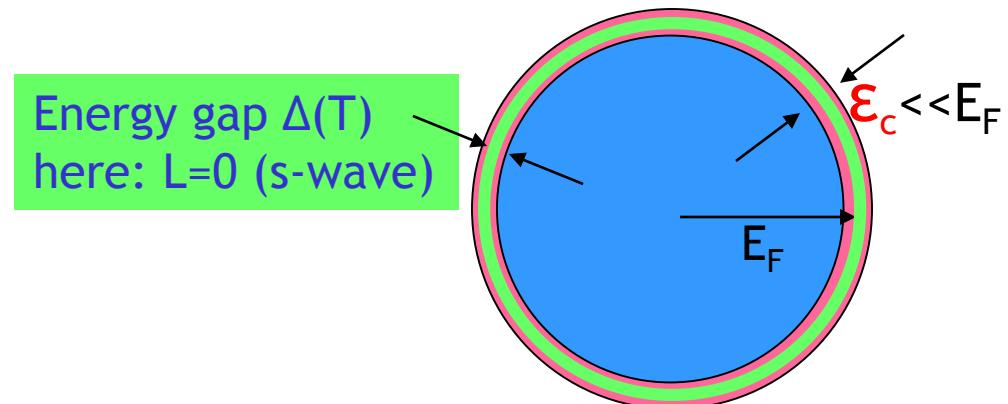
$L > 0$ (“p,d,f,... -wave”): anisotropic pair wave function

${}^3\text{He}$: Strongly repulsive interaction $\rightarrow L > 0$ expected

BCS theory

Bardeen, Cooper, Schrieffer (1957)

Generalization to macroscopically many Cooper pairs



→ Pair condensate
with transition temperature

$$T_c = 1.13 \epsilon_c \exp(-1/N(0)|V_L|)$$

in weak coupling theory

ϵ_c, V_L : Magnitude ? Origin ? → T_c ?

Thanksgiving 1971: Transition in ${}^3\text{He}$ at $T_c = 0.0026 \text{ K}$

Osheroff, Richardson, Lee (1972)
Osheroff, Gully, Richardson, Lee (1972)

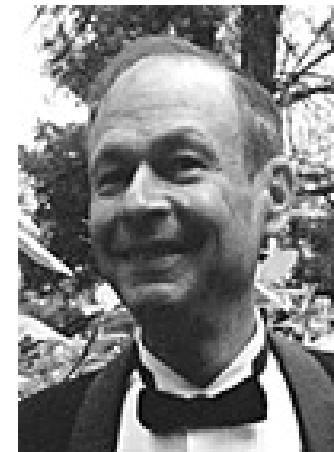
The Nobel Prize in Physics 1996
"for their discovery of superfluidity in helium-3"



David M. Lee
Cornell (USA)



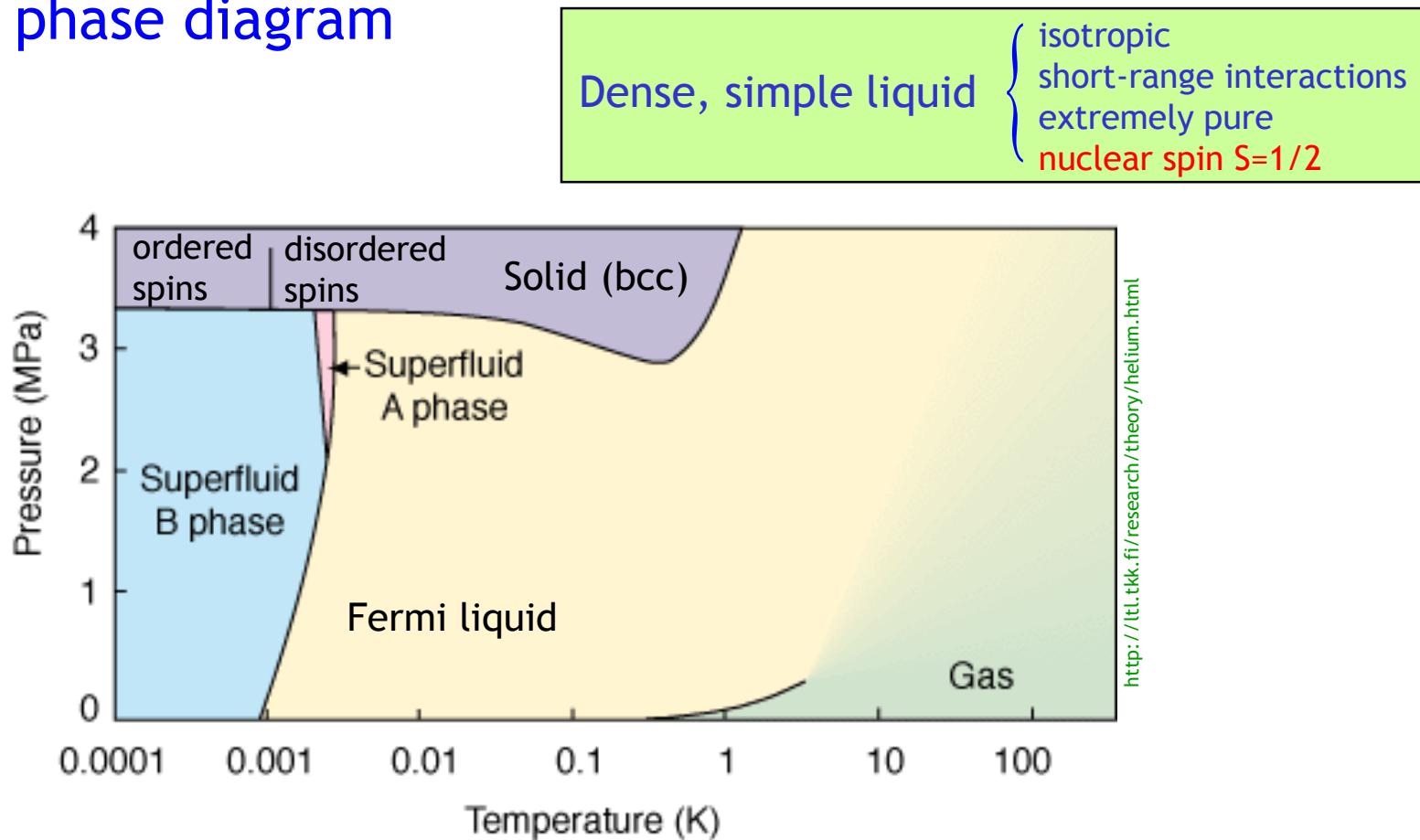
Douglas D. Osheroff
Stanford (USA)



Robert C. Richardson
Cornell (USA)

Phase diagram of Helium-3

P-T phase diagram

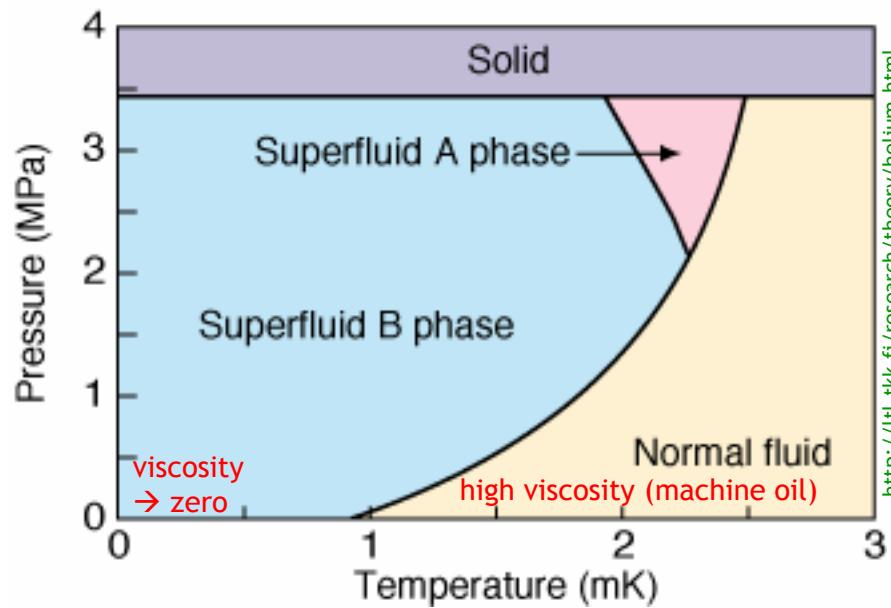


Phase diagram of Helium-3

P-T phase diagram

Dense, simple liquid

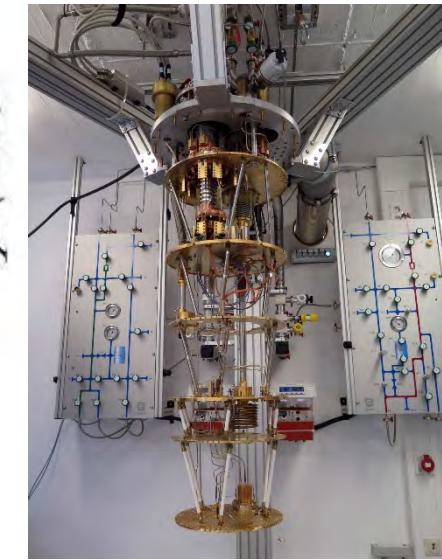
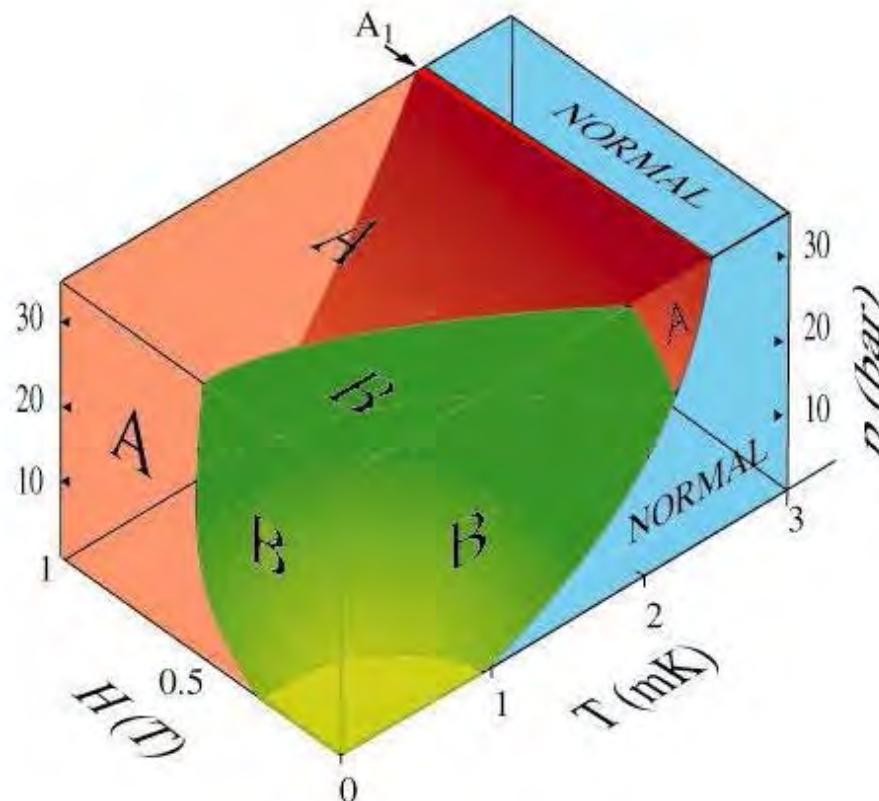
{ isotropic
short-range interactions
extremely pure
nuclear spin $S=1/2$



Phase diagram of Helium-3

P-T-H phase diagram

<http://ltl.tkk.fi/images/archive/ab.jpg>



Millikelvin Cryostat WMI Garching

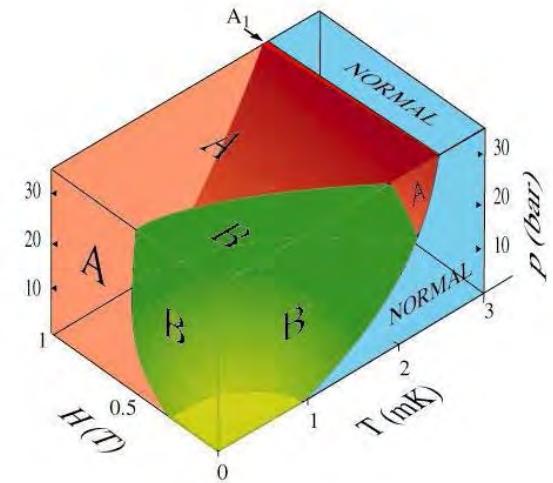
“Very (ultra) low temperatures”: $T \ll T_{\text{boiling}} \sim 3 \text{ K}$
and $\ll T_{\text{backgr. rad.}} \sim 3 \text{ K}$

Superfluid phases of ^3He

Experiment: Osheroff, Richardson, Lee, Wheatley, ...

Theory: Leggett, Wölfle, Mermin, ...

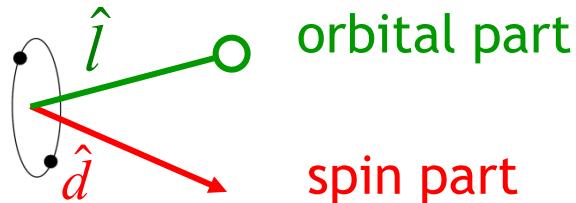
→ $L=1$, $S=1$ (“p-wave, spin-triplet”) in all 3 phases



Attraction due to spin fluctuations

Anderson, Brinkman (1973)

→ anisotropy directions
in every ^3He Cooper pair

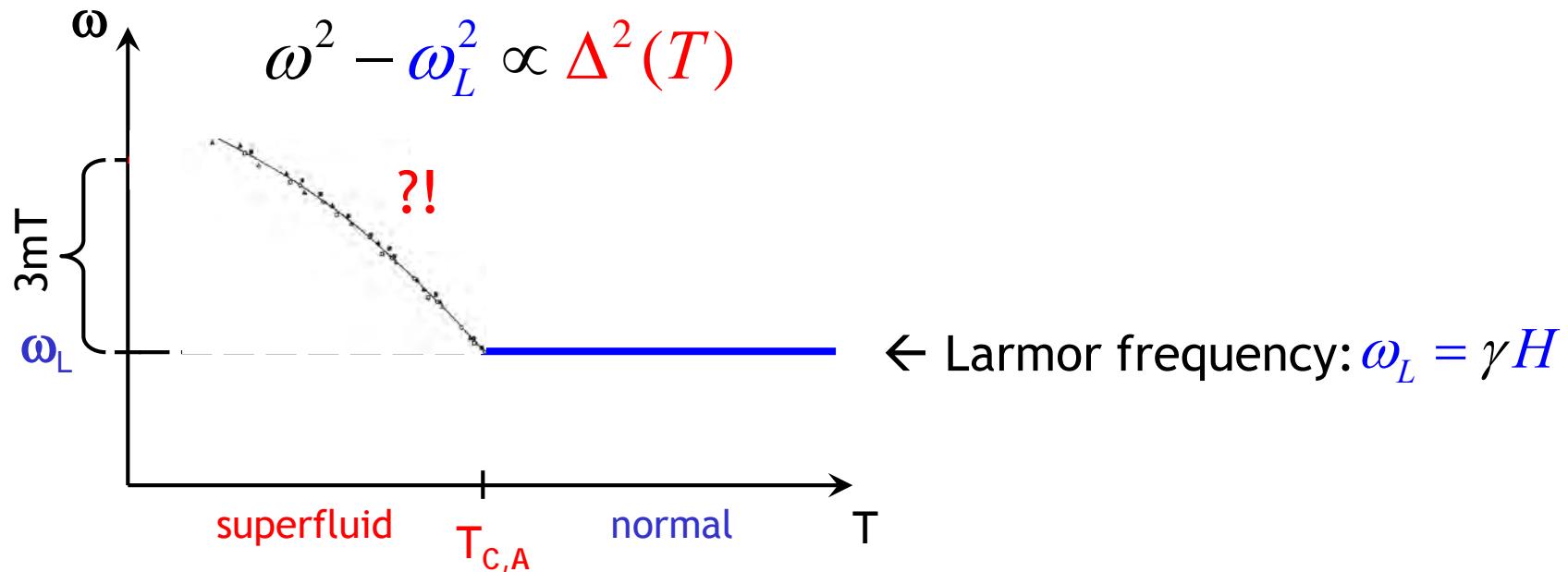


orbital part
spin part

... and a mystery!

NMR experiment on nuclear spins $I=\frac{1}{2}\hbar$

Osheroff *et al.* (1972)



Shift of ω_L \longleftrightarrow spin-nonconserving interactions

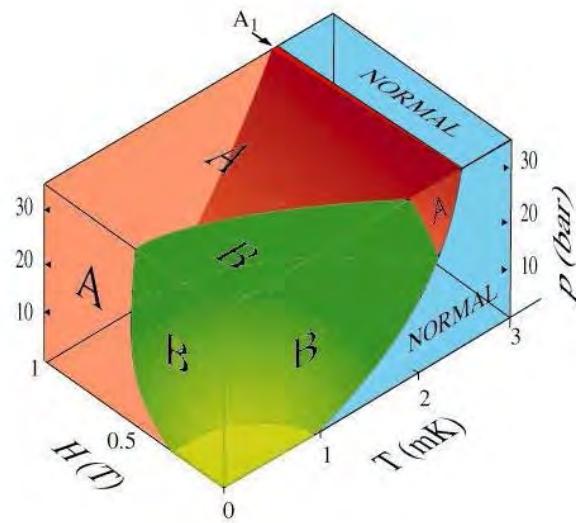


\rightarrow nuclear dipole interaction $g_D \sim 10^{-7} K \ll T_c$

Origin of frequency shift ?!

Leggett (1973)

The superfluid phases of ${}^3\text{He}$

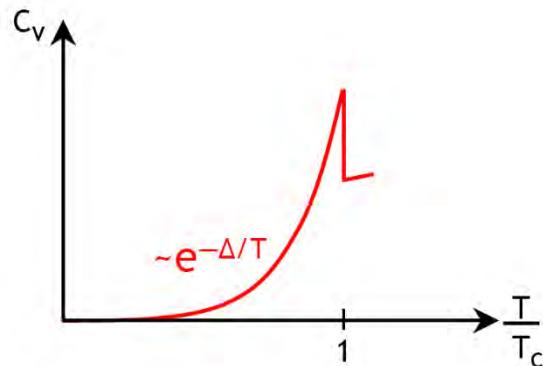
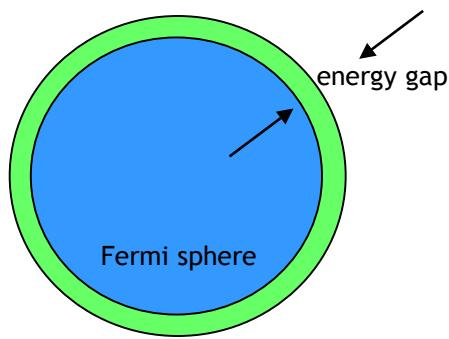
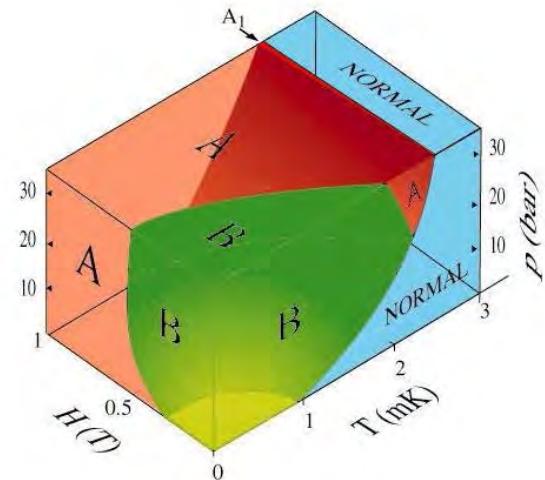


B-phase

All spin states $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow + \downarrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$ occur equally

$$\Delta(\mathbf{k}) = \Delta_0$$

Balian, Werthamer (1963)
Vdovin (1963)



“(pseudo-) isotropic state“ \leftrightarrow s-wave superconductor

Weak-coupling theory: stable for all $T < T_c$

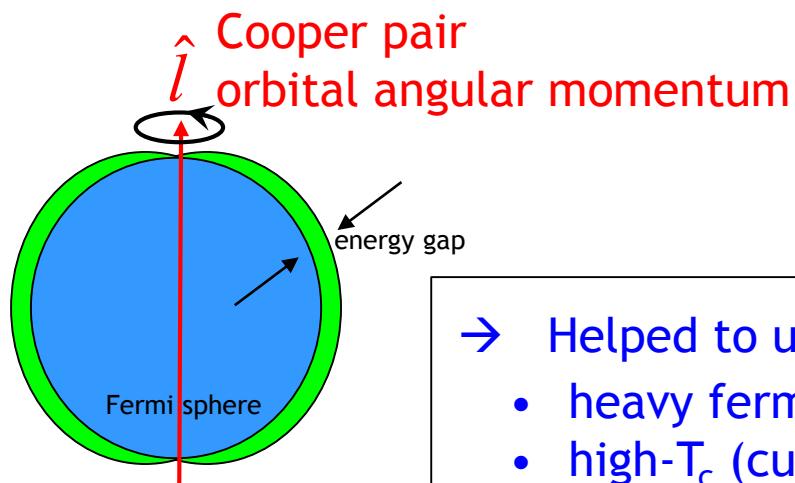
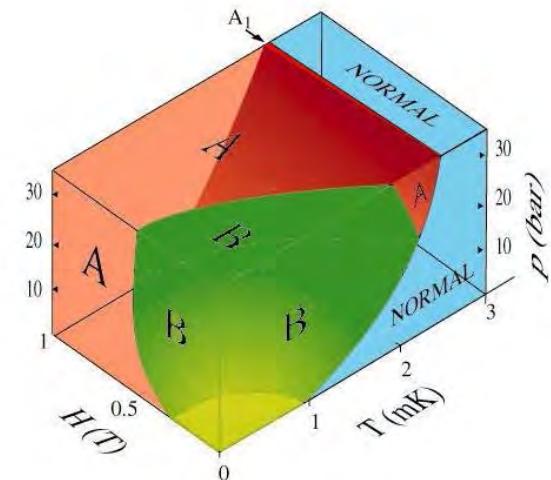
A-phase

Spin states $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$ occur equally

→ strong gap anisotropy

$$\Delta(\hat{k}) = \Delta_0 \sin(\hat{k}, \hat{l})$$

Anderson, Morel (1961)



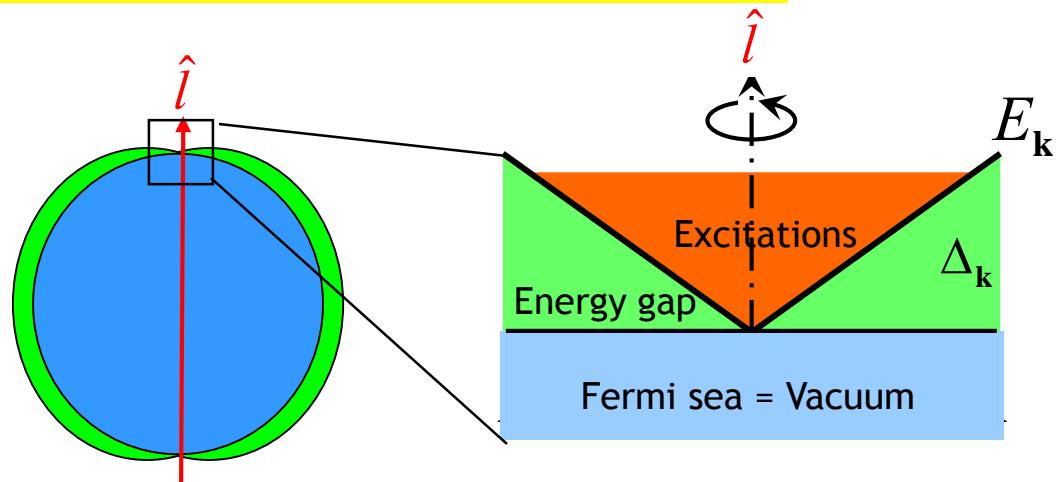
Energy gap with point nodes
“axial (chiral) state”

- Helped to understand unconventional pairing in
- heavy fermion superconductors (CeCu_2Si_2 , UPt_3 , ...)
 - high- T_c (cuprate) superconductors

Strong-coupling effect

$^3\text{He}-\text{A}$: Spectrum near nodes

Volovik (1987)



$$E_{\mathbf{k}}^2 = v_F^2 (k - k_F)^2 + \Delta_0^2 \sin^2(\hat{\mathbf{k}}, \hat{\mathbf{l}}) = g^{ij} p_i p_j \quad \text{Lorentz invariance}$$

$$e = \begin{cases} +1 & \hat{\mathbf{k}} \parallel +\hat{\mathbf{l}} \\ -1 & \hat{\mathbf{k}} \parallel -\hat{\mathbf{l}} \end{cases} \quad \begin{array}{l} \text{chirality "up"} \\ \text{chirality "down"} \end{array}$$

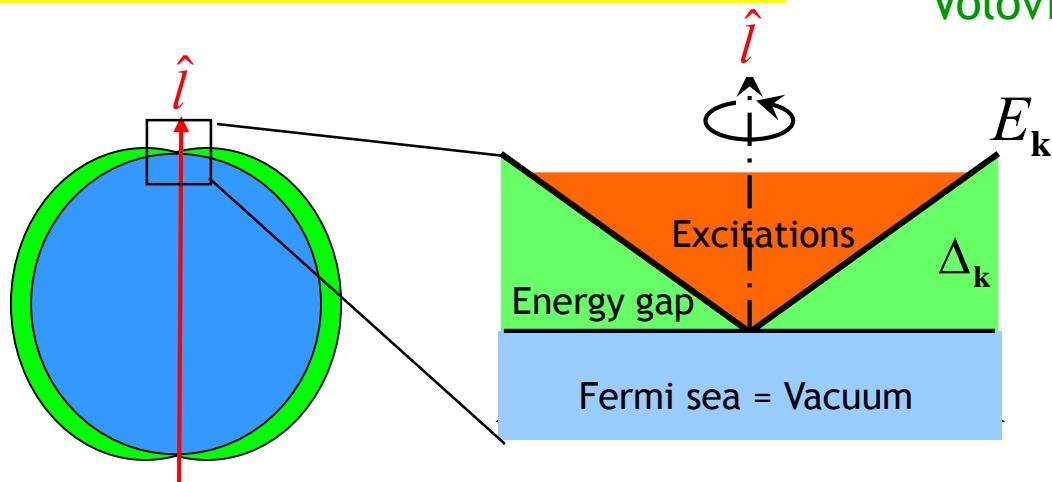
$$g^{ij} = v_F^2 l_i l_j + \left(\frac{\Delta}{k_F} \right)^2 (\delta_{ij} - l_i l_j)$$

$$\mathbf{A} = k_F \hat{\mathbf{l}}$$

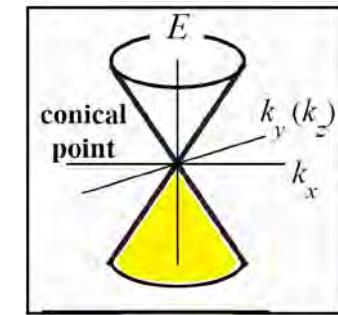
$$\mathbf{p} = \mathbf{k} - e\mathbf{A}$$

$^3\text{He}-\text{A}$: Spectrum near nodes

The Universe in a Helium Droplet,
Volovik (2003)



\Leftrightarrow



Fermi point: spectral flow
of fermionic charge

$$E_{\mathbf{k}}^2 = v_F^2 (k - k_F)^2 + \Delta_0^2 \sin^2(\hat{\mathbf{k}}, \hat{\mathbf{l}}) = g^{ij} p_i p_j$$

Lorentz invariance

$$e = \begin{cases} +1 & \hat{\mathbf{k}} \parallel +\hat{\mathbf{l}} \quad \text{chirality "up"} \\ -1 & \hat{\mathbf{k}} \parallel -\hat{\mathbf{l}} \quad \text{chirality "down"} \end{cases}$$

$$g^{ij} = v_F^2 l_i l_j + \left(\frac{\Delta}{k_F} \right)^2 (\delta_{ij} - l_i l_j)$$

\Leftrightarrow Massless, chiral leptons, e.g., neutrino $E(\mathbf{p}) = cp$

→ Chiral (Adler) anomaly measured

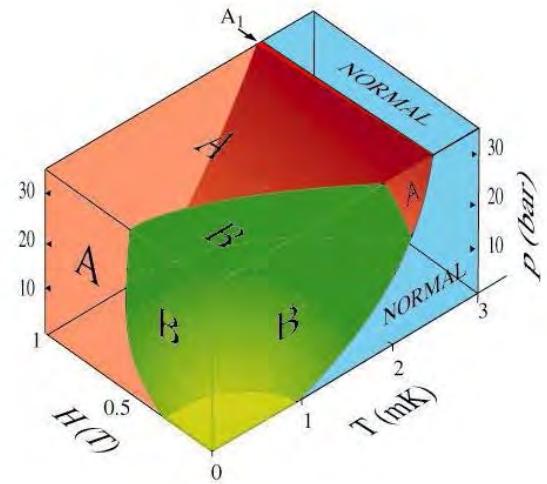
Bevan *et al.* (1997)

A₁-phase

In finite magnetic field

Only spin state $|\uparrow\uparrow\rangle$

Long-range ordered magnetic liquid



Cooper pairing of Fermions vs. Bose-Einstein condensation

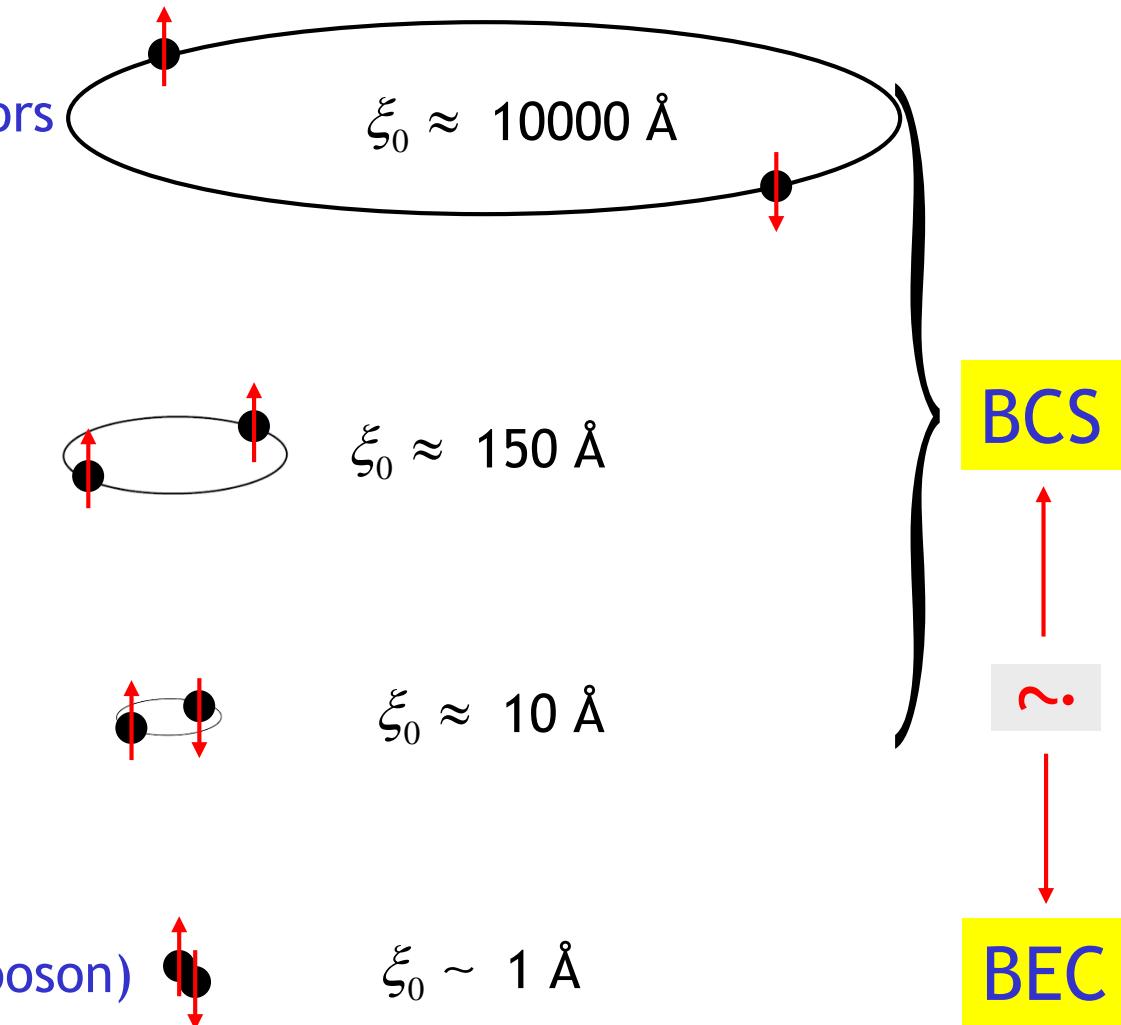
Cooper pair: “Quasi-boson”

Conventional superconductors

Superfluid ^3He

High- T_c superconductors

Superfluid ^4He :
Tightly packed fermions (boson)



New insights from BEC of cold atoms

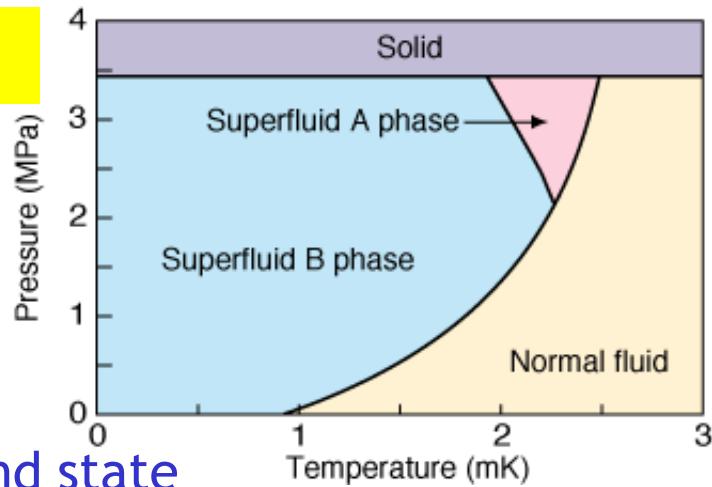
Leggett (1980)

Broken Symmetries & Long-Range Order



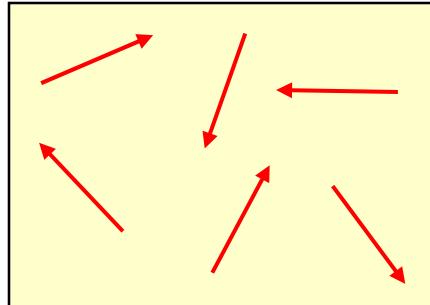
Broken Symmetries & Long-Range Order

Normal ${}^3\text{He}$ \leftrightarrow ${}^3\text{He-A}$, ${}^3\text{He-B}$:
 2^{nd} order phase transition



$T < T_c$: higher order, lower symmetry of ground state

I. Ferromagnet



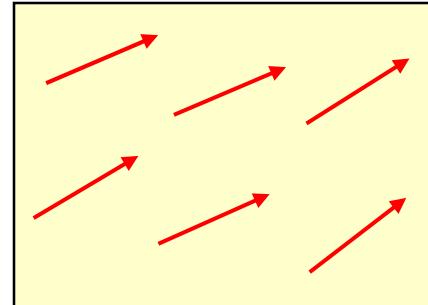
$$T > T_c$$

Average magnetization:

$$\langle \mathbf{M} \rangle = 0$$

Symmetry group:

$$\text{SO}(3)$$



$$T < T_c$$

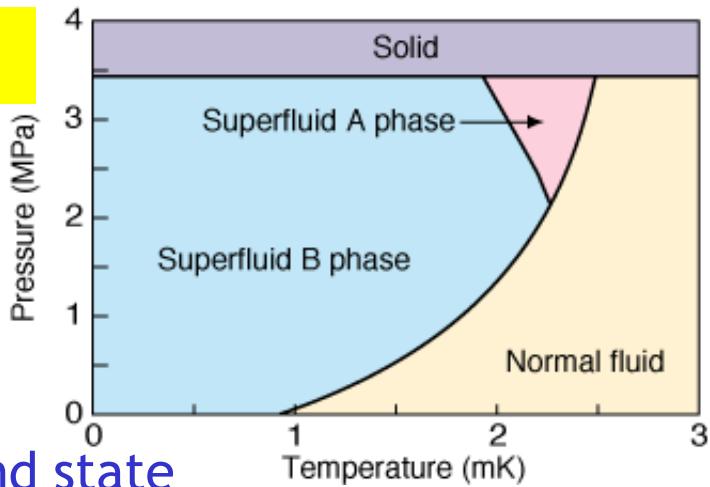
$\langle \mathbf{M} \rangle \neq 0$ Order parameter

$$\text{U}(1) \subset \text{SO}(3)$$

$T < T_c$: $\text{SO}(3)$ rotation symmetry in spin space spontaneously broken

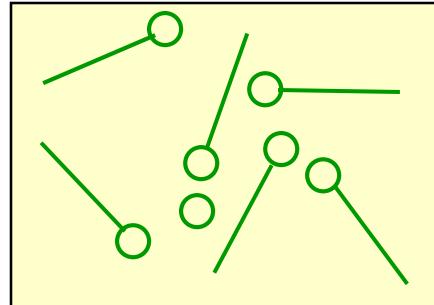
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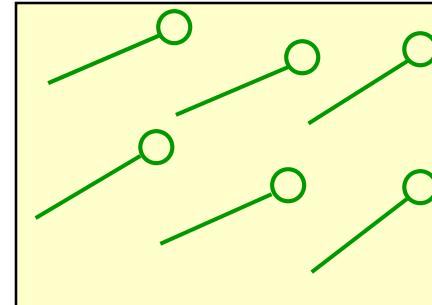


$T < T_c$: higher order, lower symmetry of ground state

II. Liquid crystal



$$T > T_c$$



$$T < T_c$$

Symmetry group:

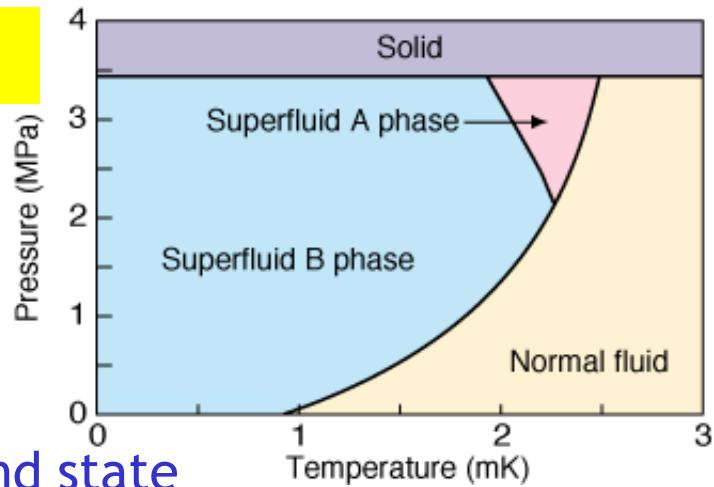
$$\text{SO}(3)$$

$$\text{U}(1) \subset \text{SO}(3)$$

$T < T_c$: $\text{SO}(3)$ rotation symmetry in real space spontaneously broken

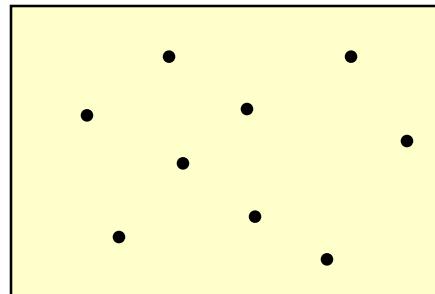
Broken Symmetries & Long-Range Order

Normal ${}^3\text{He} \leftrightarrow {}^3\text{He-A}, {}^3\text{He-B}$:
2nd order phase transition



$T < T_c$: higher order, lower symmetry of ground state

III. Conventional superconductor



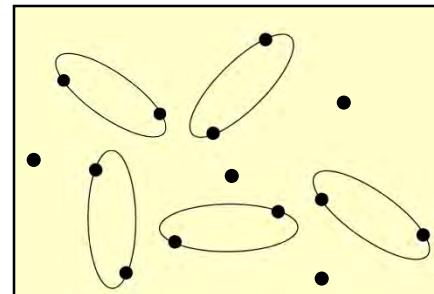
$$T > T_c$$

Pair amplitude $\langle c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger \rangle = 0$

Gauge transf. $c_{\mathbf{k}\sigma}^\dagger \rightarrow c_{\mathbf{k}\sigma}^\dagger e^{i\varphi}$: gauge invariant

Symmetry group

$\text{U}(1)$



$$T < T_c$$

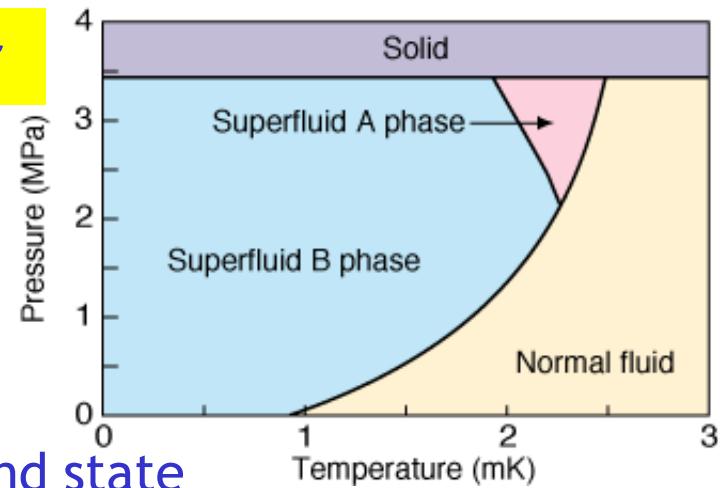
$\Delta e^{i\phi}$ complex order parameter

not gauge invariant

—

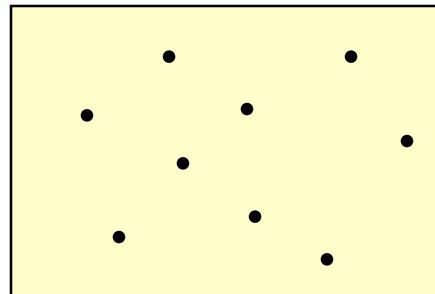
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Normal $^3\text{He} \leftrightarrow ^3\text{He-A}, ^3\text{He-B}$:
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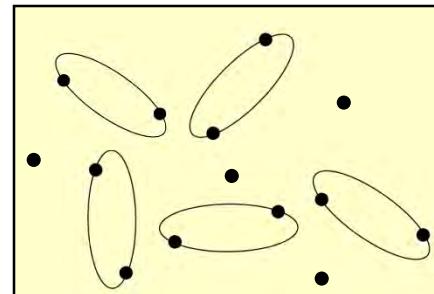


$T < T_c$: higher order, lower symmetry of ground state

III. Conventional superconductor



$T > T_c$



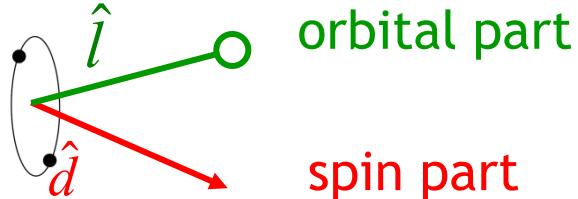
$T < T_c$

$T < T_c$: U(1) “gauge symmetry” spontaneously broken

Broken symmetries in superfluid ^3He

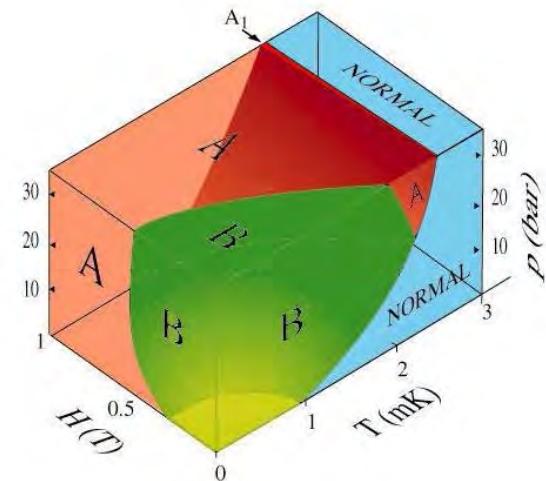
$L=1, S=1$ in all 3 phases

Cooper pair:



orbital part

spin part



Quantum coherence in $\left\{ \begin{array}{l} \text{phase (complex order parameter)} \\ \text{anisotropy direction in real space} \\ \text{anisotropy direction in spin space} \end{array} \right.$

Superfluid,
liquid crystal
magnet

Characterized by $2 \times (2L + 1) \times (2S + 1) = 18$ real numbers

3x3 order parameter matrix $A_{ij\mu}$

$\text{SO}(3)_S \times \text{SO}(3)_L \times \text{U}(1)_\varphi$ symmetry spontaneously broken Leggett (1975)

$\simeq \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_Y$ for electroweak interactions Pati, Salam (1974)

Broken symmetries in superfluid ^3He

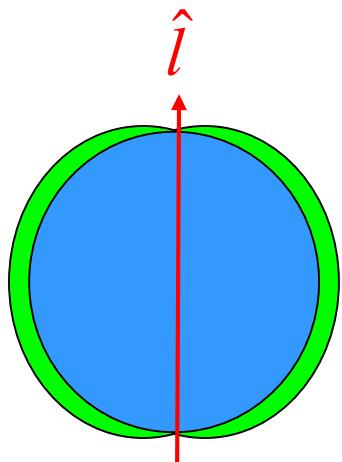
Mineev (1980)
Bruder, Vollhardt (1986)

$^3\text{He-A}$

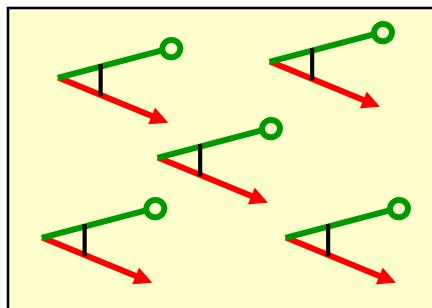
$\text{SO}(3)_S \times \text{SO}(3)_L \times \text{U}(1)_\varphi$ symmetry broken

$$\downarrow \quad \quad \quad \downarrow \quad \quad \quad \downarrow$$
$$\text{U}(1)_{S_z} \times \text{U}(1)_{L_z - \varphi}$$

Unconventional pairing

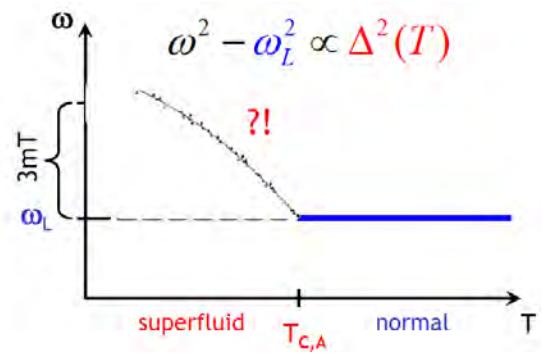


Cooper pairs



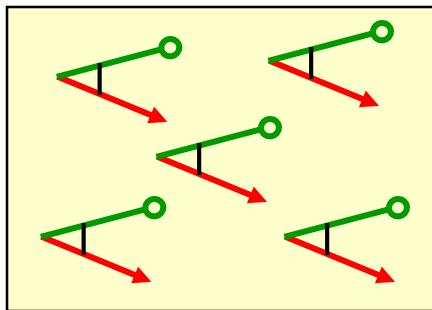
Fixed absolute orientation

... solution of the NMR mystery



Superfluid ^3He - a quantum amplifier

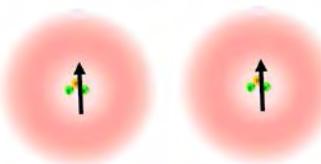
Cooper pairs in $^3\text{He}-\text{A}$



Fixed absolute orientation

What fixes the relative orientation of \hat{d}, \hat{l} ?

→ Interaction of nuclear dipoles ("spin-orbit coupling") :

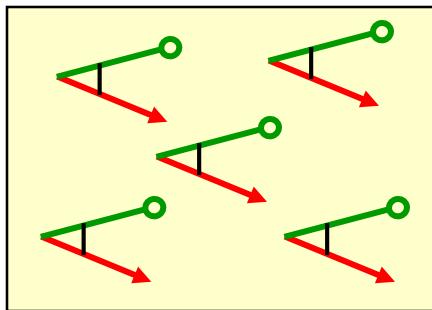


Dipole-dipole coupling of ^3He nuclei: $g_D \sim 10^{-7} K \ll T_C$

Unimportant ?!

Superfluid ^3He - a quantum amplifier

Cooper pairs in $^3\text{He-A}$

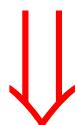


Fixed absolute orientation

- Long-range order in \hat{d}, \hat{l}
- $g_D \sim 10^{-7} K$: tiny, but lifts degeneracy of relative orientation

Quantum \downarrow coherence

\hat{d}, \hat{l} locked in all Cooper pairs at a fixed angle

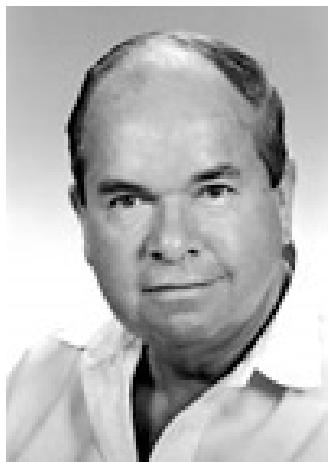


NMR frequency increases: $\omega^2 = (\gamma H)^2 + g_D \Delta^2(T)$ Leggett (1973)

→ Nuclear dipole interaction is macroscopically measurable

The Nobel Prize in Physics 2003

"for pioneering contributions to the theory of superconductors
and superfluids"



Alexei A. Abrikosov
USA and Russia

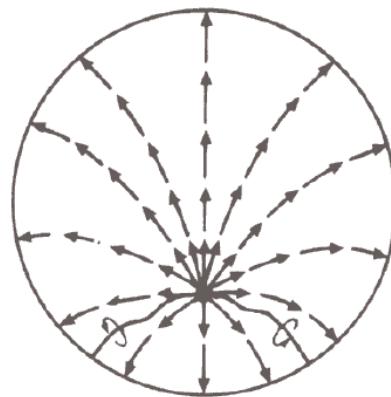


Vitaly L. Ginzburg
Russia



Anthony J. Leggett
UK and USA

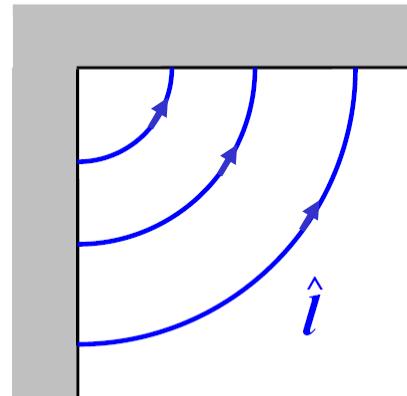
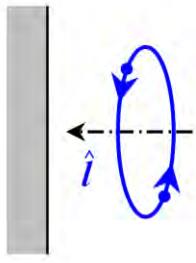
Order-parameter textures and topological defects



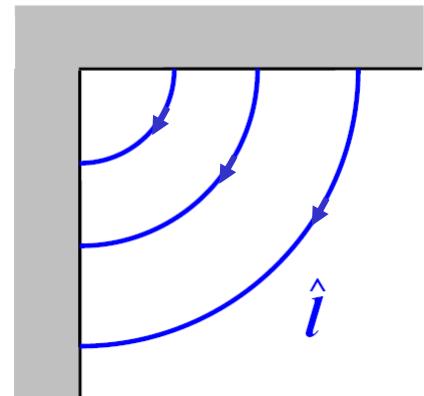
Order-parameter textures in $^3\text{He-A}$

Orientation of the anisotropy directions \hat{d}, \hat{l} :

1) Walls $\rightarrow \hat{l}$



or



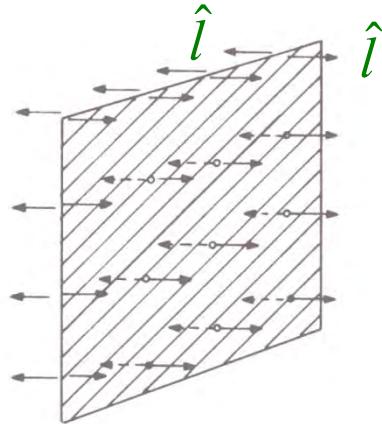
Chirality exp. Confirmed:
Walmsley, Golov (2012)

2) Magnetic field $\rightarrow \hat{d}$

\rightarrow *Textures in \hat{d}, \hat{l} \leftrightarrow liquid crystals*

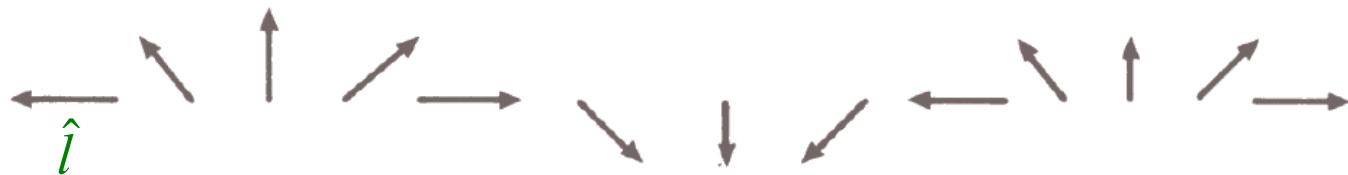
Order-parameter textures and topological defects in ${}^3\text{He}-\text{A}$

D=2: domain walls in \hat{d} or \hat{l}



Cannot be removed
by local surgery
→ topological defect

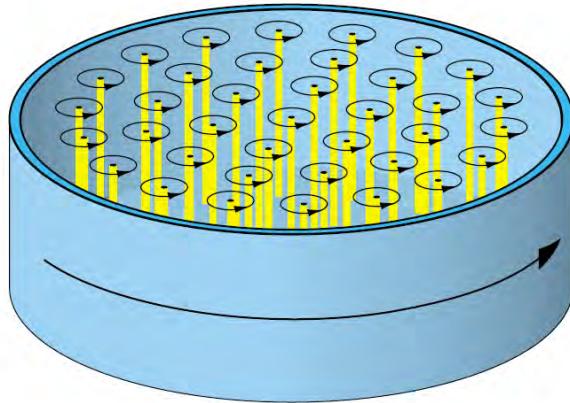
Single domain wall



Domain wall lattice

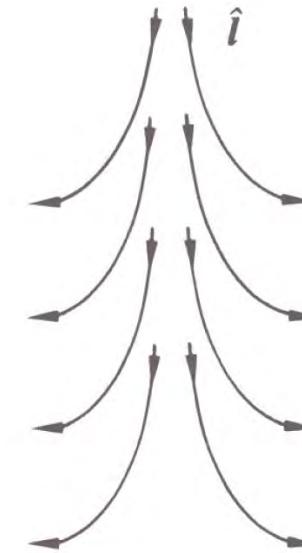
Order-parameter textures and topological defects in ${}^3\text{He-A}$

D=1: Vortices



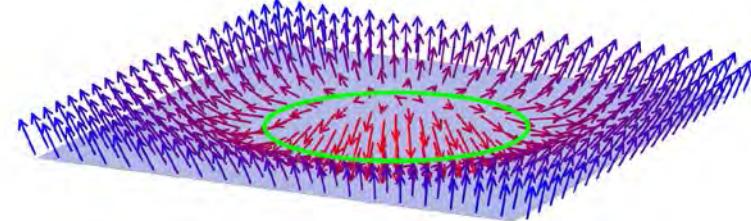
Vortex formation by rotation

<http://ltl.tkk.fi/research/theory/vortex.html>



e.g., Mermin-Ho vortex
(non-singular)

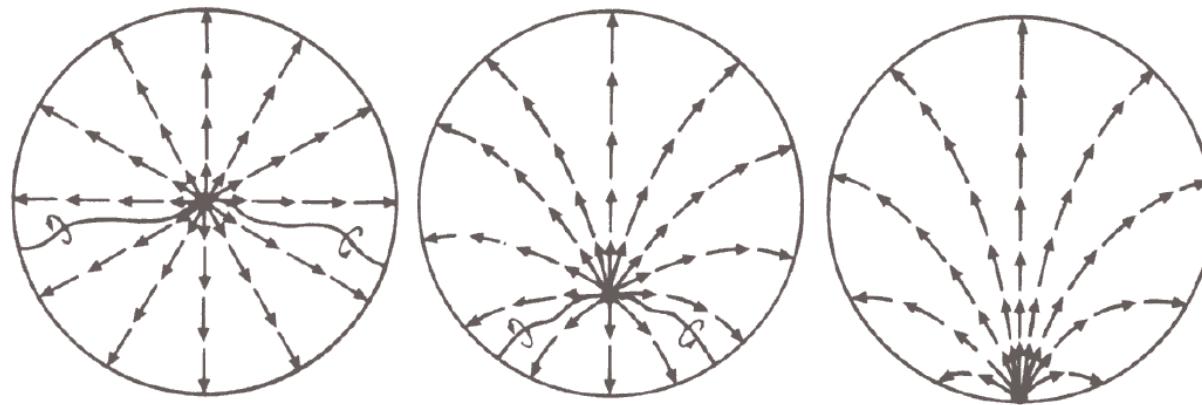
Thin film of ${}^3\text{He-A}$ (chiral)



Skyrmion vortex Volovik (2003), Sauls (2013)

Order-parameter textures and topological defects in ${}^3\text{He}-\text{A}$

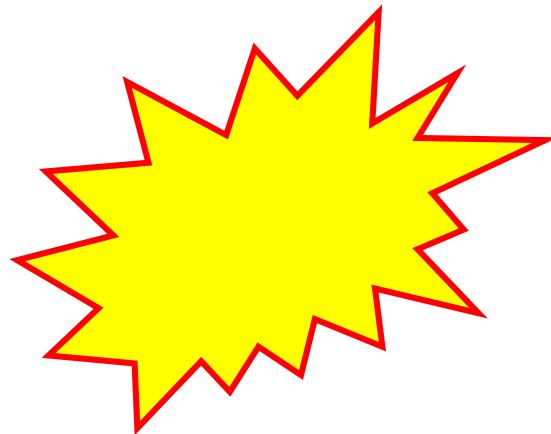
D=0: Monopoles



“Boojum” in \hat{l} -texture of ${}^3\text{He}-\text{A}$

- Defect formation by
- rotation
 - geometric constraints
 - rapid crossing through continuous phase transition

Big bang simulation in the low-temperature lab



Universality in continuous phase transitions



High symmetry,
short-range order

$T > T_c$



Spins:
para-
magnetic

Helium:
normal
liquid

Universe:
Unified forces
and fields

$T = T_c$

Phase transition

Broken symmetry,
long-range order

ferromagnetic superfluid

elementary
particles,
fundamental
interactions

Defects: domain
walls

vortices,
etc.

cosmic strings,
etc. Kibble (1976)

$T < T_c$

nucleation of galaxies?



Rapid thermal quench through 2nd order phase transition

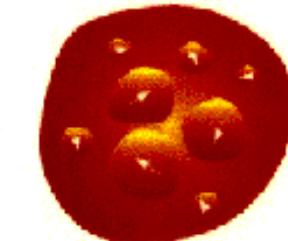
Bäuerle *et al.* (1996)

1. Local temperature $T \gg T_c$

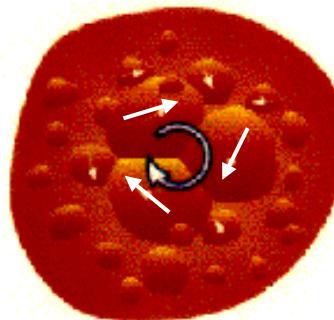


→ Expansion + rapid cooling

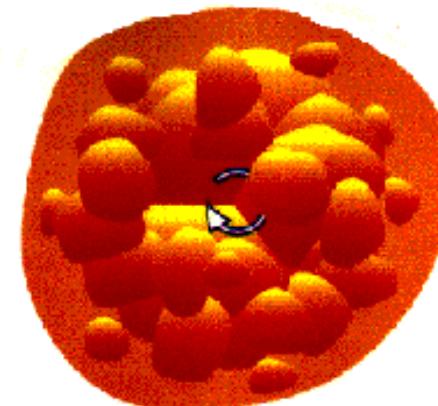
2. Nucleation of independently ordered regions



Clustering of ordered regions
→ Defects

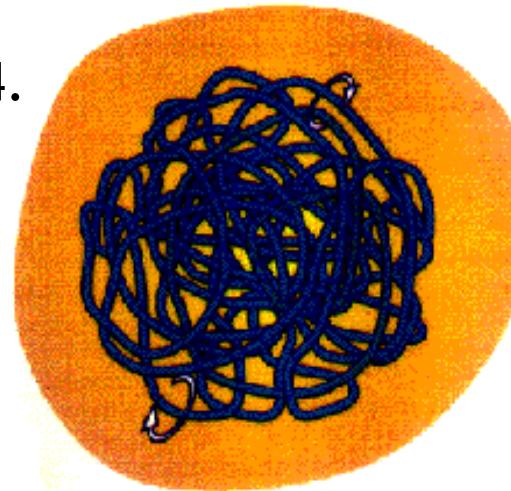


3.



Defects overlap

4.



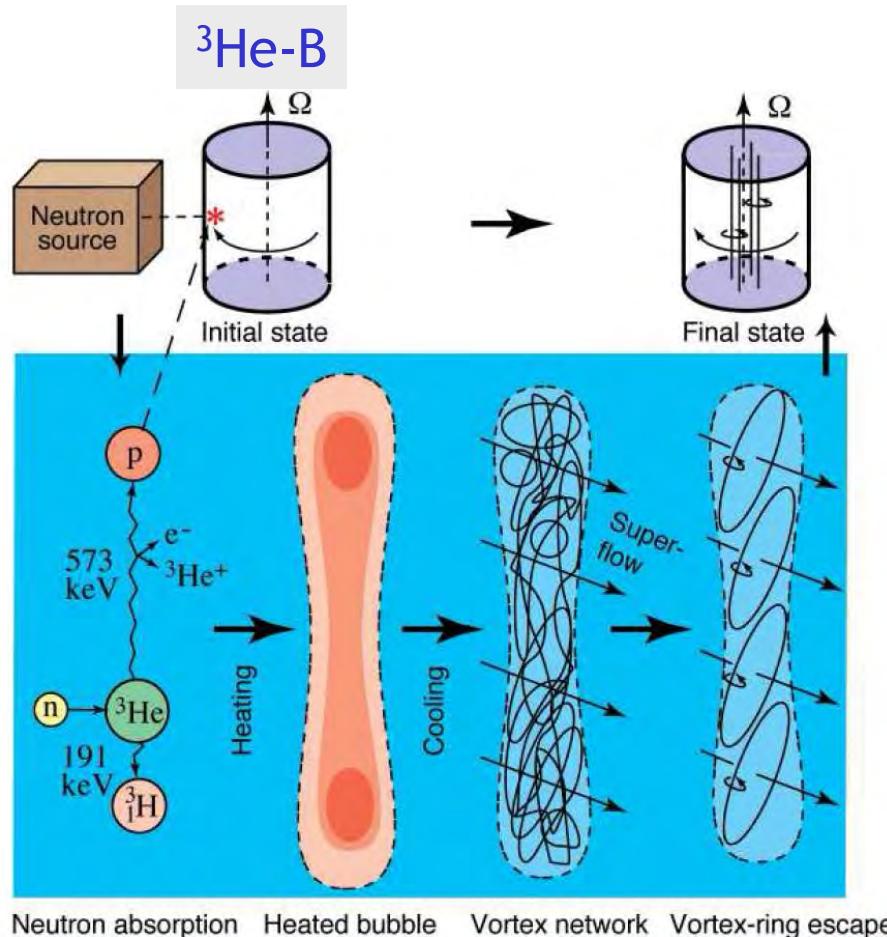
$T < T_c$:
Vortex tangle

Estimate of density of defects: Zurek (1985)

"Kibble-Zurek mechanism" of defect formation: How to test?

Big-bang simulation in the low-temperature laboratory

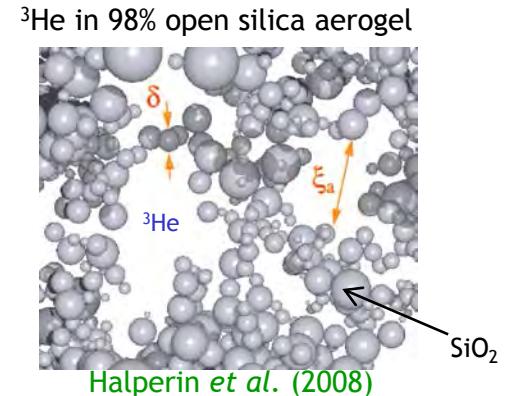
Grenoble: Bäuerle *et al.* (1996), Helsinki: Ruutu *et al.* (1996)



Measured vortex tangle density:
Quantitative support for Kibble-Zurek mechanism

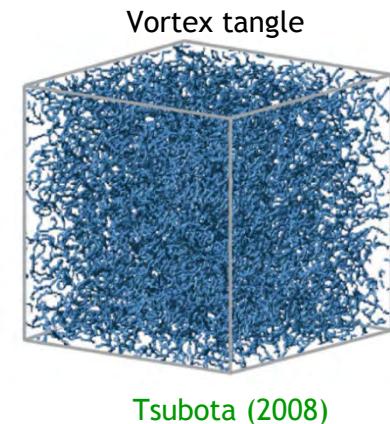
Current research on superfluid ^3He

1. Influence of disorder on superfluidity



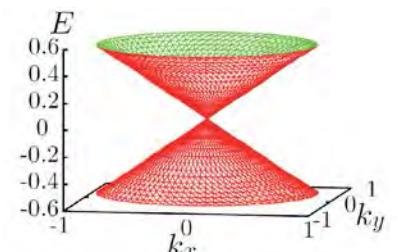
2. Quantum Turbulence (=Turbulence in the absence of viscous dissipation) Origin of dissipation in the absence of friction?

Test systems: $^4\text{He-II}$, $^3\text{He-B}$

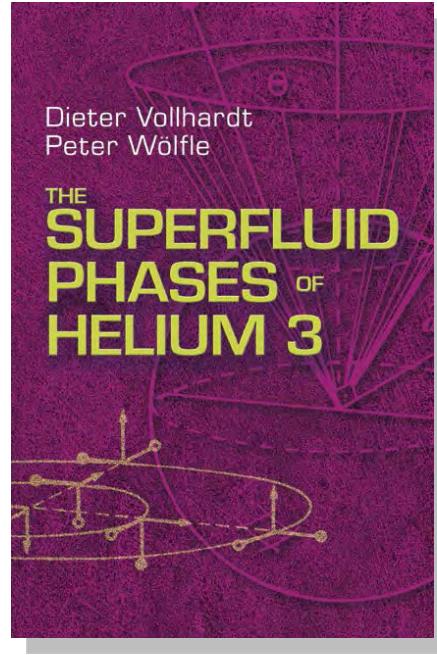


3. Majorana fermions (e.g., zero-energy Andreev bound states at surfaces in $^3\text{He-B}$)

Majorana cone for $^3\text{He-B}$ in a thick slab



Tsutsumi, Ichioka, Machida (2011)



The Superfluid Phases of Helium 3

D. Vollhardt and P. Wölfle

(Taylor & Francis, 1990), 656 pages

Reprinted by Dover Publications (2013)

Conclusion

Superfluid Helium-3:

- Anisotropic superfluid (p-wave, spin-triplet pairing)
 - Cooper pairs with internal structure
 - 3 different bulk phases with many novel properties
- Large symmetry group broken
 - Close connections with particle physics
 - Zoo of topological defects

