How to find Topological Charge in the Quark-Gluon Plasma

Or from



То

Topological charge flucutations, D. Leinweber



Tracks in TPC of STAR

And back!

Kharzeev, McLerran & HJW, Nucl. Phys. A803, 227 (2008) HJW, J.Phys.G35, 104012 (2008) Fukushima, Kharzeev & HJW, Phys.Rev.D78, 074033 (2008) Kharzeev & HJW, arXiv:0907.5007



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Three (out of many) Intriguing Properties of QCD

Topological Charge

Axial anomaly

P- and CP-odd effects

Can we see their effects in heavy ion collisions?

The vacuum of the gluon sector of QCD has non trivial structure



 $v = \text{complicated formula}(A_i) = 0, \pm 1, \pm 2...$

Winding number = Topological invariant.

Smooth deformations cannot change winding number.

Need to go out of pure gauge=energy to change winding number.



mathworld.org

Transitions between vacua: $Q = \frac{g^{2}}{22\pi^{2}} \int d^{4}x F^{a}_{\mu\nu} F^{\tilde{\mu}\nu}_{a}$ topological charge <u>Sphaleron</u> Q=-1energy $v = N_{CS} = -3$ $\frac{1}{\text{Instanton}} \stackrel{0}{Q} = 1$ 3 -2 2 Instantons: Configuration with finite action. Tunneling through barrier Suppression of rate at large temperature 't Hooft ('76), Pisarski and Yaffe ('80) $\Gamma = \lim_{V, t \to \infty} \frac{1}{Vt} \langle Q^2 \rangle \sim \exp(-\frac{8\pi}{\sigma^2})$ $\Gamma(T=0) \approx (180 \,\mathrm{MeV})^4$ Euclidean topological susceptibility

<u>Sphaleron:</u> Configuration with finite energy. Real time. Go over barrier. Only possible at finite temperature, <u>rate not suppressed!</u> Manton ('83), Manton and Klinkhamer ('84), McLerran, Mottola and Shaposhnikov ('88)

 $\Gamma \sim 385 \,\alpha_S^5 \,T^4$

Bödeker, Moore and Rummukainen ('00), several transitions per fm⁻³ per fm/c

Also AdS/CFT sphaleron Son & Starinets ('02)

Minkowski topological susceptibility

Also topological charge from Glasma

HIC = Colliding sheets of color

Taking into account collision geometry + out of thermal equilibrium effects.



Krasnitz, Kharzeev, Venugopalan ('02) Lappi, McLerran ('06)

Longitudinal Chromo-Electric and Chromo-Magnetic fields just after collision

QCD: Gluon fields can have topological charge $Q = \frac{g^2}{32\pi^2} \int d^4 x F^a_{\mu\nu} \tilde{F^a_a}$

Belavin, Polyakov, Schwartz and Tyupkin ('75)



Non-perturbative physics

D. Leinweber, Topological charge flucutations

Average over time and space vanishes

 $\langle Q \rangle = 0$

But fluctuations not

$$\langle Q^2 \rangle \neq 0$$

How does topological charge deal with quarks?

The Axial Anomaly (= quantum mech. sym. breaking)











Steinberger ('49)

Schwinger ('51)

Bell, Jackiw ('69)

Adler ('69)

$$j_5^{\mu}(x) = \langle \bar{\psi}(x) \gamma^{\mu} \gamma^5 \psi(x) \rangle$$

Axial current in the chiral limit is not conserved

QED

$$\partial_{\mu} j_5^{\mu} = -\frac{e^2}{8\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Pion decay to two photons $\pi^0 \rightarrow \gamma \gamma$

QCD $\partial_{\mu} j_5^{\mu} = -\frac{g^2}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a$

Note: these relations are exact in chiral limit

Axial U(1) symmetry breaking

 $m_\eta' \gg m_\pi$, m_K

Topological charge induces chirality

<u>Chirality:</u> difference between number of quarks + antiquarks with right- and left-handed helicity

$$N_{5} = \# \left(\mathbf{q}_{\mathsf{R}} + \# \left(\mathbf{\bar{q}}_{\mathsf{R}} - \# \left(\mathbf{q}_{\mathsf{L}} - \# \left(\mathbf{\bar{q}}_{\mathsf{L}} - \# \left(\mathbf{\bar{q}}_{\mathsf{L}$$

$$\Delta N_{5} \equiv [N_{R} - N_{L}]_{t=\infty} - [N_{R} - N_{L}]_{t=-\infty} = -2Q$$

Axial Anomaly: <u>Topological charge induces chirality</u>

Total number right- and left-handed fermions can differ in each event globally.

$$N_R \neq N_L$$

Imbalance

Event-by-event parity- (P) and charge-parity (CP) violation.

Chirality and Helicity

<u>Right-handed chirality:</u> $\psi_R = \frac{1}{2}(1 + \gamma_5)\psi$

Left-handed chirality: $\psi_L = \frac{1}{2}(1 - \gamma_5)\psi$

<u>Left-handed helicity:</u> $\frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|} = -1$

 $\frac{\vec{p}}{\vec{b}} = -1$ Right-handed helicity: $\frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|} = 1$

In the chiral limit

Particle (P) with Left/Right-handed chirality has Left/Right-handed helicity Antiparticle (AP) with Left/Right-handed chirality has Right/Left-handed helicity

$$N_{5} = \int \mathrm{d}^{3} x \langle \bar{\psi} \gamma^{0} \gamma^{5} \psi \rangle = \int \mathrm{d}^{3} x [\langle \psi_{R}^{+} \psi_{R} \rangle - \langle \psi_{L}^{+} \psi_{L} \rangle]$$

 $N_5 = N_R - N_L =$ $\frac{\# (P - AP) \text{ with } RH \text{ chirality } - \# (P - AP) \text{ with } LH \text{ chirality}}{\# (P + AP) \text{ with } RH \text{ helicity } - \# (P + AP) \text{ with } LH \text{ helicity}}$

$$[N_{R} - N_{L}]_{t=\infty} - [N_{R} - N_{L}]_{t=-\infty} = -2N_{f}Q$$

How to find topological in Heavy Ion Collisions

Or from



 $\langle Q^2 \rangle \neq 0$





And back!

What does a magnetic field do to quarks

A magnetic field will align the spins, depending on their electric charge

No Magnetic Field: No polarization

Magnetic field: Polarization

The momenta of the quarks align along the magnetic field A quark with right-handed helicity will have momentum opposite to a left-handed one In this way the magnetic field can <u>distinguish</u> between <u>right</u> and <u>left</u>





R

What does a magnetic field do with chirality (generated by topological charge)

A magnetic field will align the <u>spins</u>, depending on their electric charge

No Magnetic Field: No polarization

Magnetic field: Polarization B



Positively charged particles move parallel the magnetic field

Negatively charged particles move to antiparallel to magnetic field

An electromagnetic current is created along the magnetic field

The Chiral Magnetic Effect

- 1. Topological charge induces Chirality
- 2. In presence of <u>Magnetic field</u> this induces an Electromagnetic Current along Magnetic Field.
- 3. In finite volume this causes <u>separation of positive from negative charge</u>



Valid for full polarization, what about smaller fields?



If a system has Chirality, Fermi-surfaces Right- and Left-handed fermions differ, imbalance.

This can be described by a chiral chemical potential μ_5 Study equilibrium response to Magnetic Field

Computing the induced current (1)

Introduce Chiral Chemical Potential μ_5 to obtain nonzero Chirality Study equilibrium response to Magnetic Field $\vec{B} = B(x, y)\hat{z}$

$$J^{\mu} = e \int d^{3}x \langle \bar{\psi} \gamma^{\mu} \psi \rangle \qquad j^{3} = e \langle \bar{\psi} \gamma^{3} \psi \rangle = e \langle \phi_{R}^{+} \sigma^{3} \phi_{R} \rangle - e \langle \phi_{L}^{+} \sigma_{3} \phi_{L} \rangle$$

Current is number density of right-handed particles in Lowest Landau level minus density of left-handed particles in lowest Landau Level

density of states ~ floor
$$\left[\frac{e\Phi}{2\pi}\right] \approx \frac{eBL^2}{2\pi}$$
 $\Phi = \int dx dy B(x, y)$

$$J_3 = \frac{e^2 B L^3}{2 \pi^2} \mu_5$$
 Independent of mass, temperature
and baryon chemical potential

4 alternative ways to compute current: Chern-Simons term Thermodynamic potential, Energy Conservation, Linear respons (time dep mag field)

Kharzeev, Fukushima & HJW ('08) See also: Metlitsky and Zhitnitsky ('05)

Computing the induced current (2)

Introduce Chiral Chemical Potential μ_5 to obtain nonzero Chirality Study equilibrium response to Magnetic Field $J^{\mu} = \int d^3x \langle \bar{\psi} \chi^{\mu} \psi \rangle$

- 1. Consider Parallel Electric and Magnetic Fields
- 2. Chirality is generated by the EM anomaly with rate
- 3. Moving particles from one to other Fermi Surface μ_2
- 4. Energy has to be delivered by current, energy conservation gives

$$\frac{d(N_R - N_L)}{dt} = \frac{e^2}{2\pi^2} \int d^3 x \vec{E} \cdot \vec{B}$$

$$\int \frac{d(N_R - N_L)}{dt} = \mu_5 \frac{e^2}{2\pi^2} \int d^3 x \vec{E} \cdot \vec{B}$$

$$\int d^3 x \vec{j} \cdot \vec{E} = \mu_5 \frac{e^2}{2\pi^2} \int d^3 x \vec{E} \cdot \vec{B}$$
Nielsen and Ninomiya ('83)

5. Take limit Electric field -> 0

$$J_3 = \frac{e B L^3}{2 \pi^2} \mu_5$$

The Chiral Magnetic Effect: QCD anomaly provides chirality EM anomaly provides current

Kharzeev, Fukushima & HJW ('08)

See paper this and 4 other methods to arrive at this result

Result for the induced current

$$J_3 = \frac{e^2 B L^3}{2 \pi^2} \mu_5$$



Kharzeev, Fukushima & HJW ('08)

Current as a function of Chirality $J = \frac{e^2 B L^3}{2 \pi^2} \mu_5$ Express μ_5 in terms of N_5 (neglecting gluonic correction

Express μ_5 in terms of N_5 (neglecting gluonic corrections)



High temperature and small magnetic field approx. (dashed line) valid for QCD

$$\mu_5 = \frac{3n_5}{T^2 + \mu^2 / \pi^2}$$

$$N_5 = -2N_f Q$$

$$V = -\frac{3e^2}{\pi^2} \frac{Q}{T^2 + \mu^2 / \pi^2} B \sum_{f} q_{f}^2$$

Kharzeev, Fukushima & HJW ('08)

See also lattice QCD work by P.V. Buividovich et al. (arXiv:0907.0494)

How to find topological charge in the quark-gluon plasma



And back!

The Chiral Magnetic Effect in Heavy Ion Collisions

Event by event P- and CP-violation



More positively charged quarks implies more positively charged hadrons

Kharzeev ('06), Kharzeev & Zhitnitsky ('07), Kharzeev, McLerran & HJW ('07)

Magnetic Field in Heavy Ion Collisions



Magnetic field falls off rapidly: early time dynamics

Current in time-dep. magnetic field

Chiral Magnetic Conductivity σ for nonzero frequencies. $j = \sigma B$

$$\sigma(\omega) = \lim_{p^i \to 0} \frac{1}{2i p^i} \epsilon^{ijk} \tilde{\Pi}_{R}^{jk}(\omega, p)$$



Anti-symmetric part of off-diagonal photon polarization tensor (nonzero in presence of μ_5)

$$\sigma(\omega=0) \equiv \sigma_0 = \frac{e^2}{2\pi^2}\mu_5$$

Constant chirality in time-dep mag. field

1 loop result, valid at high T.

Result can be systematically improved by taking into 2 loop corrections.

Strong interactions: fast response.

Even very quickly changing magnetic field gives rise to sizable current.

The Chiral Magnetic Effect in Heavy Ion Collisions



Topological charge **Q** fluctuates anywhere in the QGP

Measure: variances -> nonzero Event-by-Event P- & CP-violation

The Chiral Magnetic Effect is a near the surface effect

Medium causes screening

Variance of charge difference between upper and lower side reaction plane:

$$\langle \Delta_{\pm}^2 \rangle = 2 \int_{t_i}^{t_f} \mathrm{d} t \int_V \mathrm{d}^3 x \ \Gamma \ [\xi_{\pm}^2(x_{\perp}) + \xi_{\pm}^2(x_{\perp})] \ (\sum_f q_f^2 e B \rho)^2$$

Time & Volume integralRate of creationScreeningSquare of ChangeOverlap regionTopological chargeFunctionsCharge difference

Estimate magnitude relative asymmetry for large impact parameter 10⁻⁴ with 1-2 orders of magnitude uncertainty.



A possible result of the Chiral Magnetic Effect in Gold-Gold collisions at 130 GeV per nucleon



 $\frac{1}{N_{\pm}} \frac{d N_{\pm}}{d \phi} = \frac{1}{2\pi} + a_{\pm} \sin(\phi - \Psi_{RP}) + v_2 \cos[2(\phi - \Psi_{RP})] + \dots$



Features of the Chiral Magnetic Effect

200

100

Quarks and Gluons

conductor?

Net Baryon Density

Critical point

Hadrons

Nuclei

Order parameter for Confinement / Deconfinement Confined quarks cannot be separated.

Order parameter for Chiral Symmetry Breaking / Restoration Chirality will be removed by the chiral condensate (L-R pairing). Mass term in anomaly.

Hence no QGP implies: no Chiral Magnetic Effect [emperature T [MeV]

Test: Energy scan

<u>The correlators are proportional to Z²</u> Chiral magnetic effect: QCD + EM

Test: use nuclei with same A and different Z, isobars.¹ Need to be stable + almost 100% natural abundance. Calcium-40 (Z=20) compared to Argon-40 (Z=18). Expected: 24.5% increase







Backup slides

Computing the induced current

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Nielsen and Ninomiya ('83)

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Topological Susceptibility in Euclidean space-time

$$\boldsymbol{\Gamma} = \lim_{V,t\to\infty} \frac{1}{Vt} \langle [N_{\rm CS}(t) - N_{\rm CS}(t=0)]^2 \rangle = \frac{1}{V} \langle Q^2 \rangle$$

$$Q = \frac{g^2}{32\pi^2} \int d^4 x F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a$$

$$\mathbf{X} = \frac{1}{V_4} \langle Q^2 \rangle = \int \mathrm{d}^4 x \langle q(x) q(0) \rangle$$

$$\Gamma \sim \exp[-S] \sim \exp[-\frac{8\pi^2}{g^2}|Q|]$$

Large suppression at large temperatures..



Del Debbio, Panagopolous & Vicari ('02)

$$\frac{2N_f}{f_\pi^2} \chi = m_{\eta'}^2 + m_{\eta}^2 - 2m_K^2 \qquad \chi(T=0) \approx (180 \,\mathrm{MeV})^4$$



Sphaleron: Configuration with finite energy. Go over barrier. Only possible at finite temperature, <u>rate not suppressed</u>, <u>look for it in QGP!</u> Manton ('83), Manton and Klinkhamer ('84), McLerran, Mottola and Shaposhnikov ('88)

 $\Gamma \sim 385 \alpha_S^5 T^4$

Bödeker, Moore and Rummukainen ('00), several transitions per fm⁻³ per fm/c

Winding in real-time is very different from winding in Euclidean space-time. See Arnold and McLerran ('88). "The sphaleron strikes back" for a nice discussion.

Relation between current and topological charge

В

$$J_{3} = \frac{e B L^{2}}{2 \pi^{2}} \mu_{5} \qquad \mu_{5} = ? \qquad N_{5} = -2Q$$

$$n_{5} = \frac{\partial \Omega}{\partial \mu_{5}} \qquad \Omega = \frac{1}{-1} \log Z$$

At large temperatures and small magnetic fields,

 $\partial \mu_5$

We can take a free noninteracting gas of fermions. This can be improved.

$$\mu_5 = \frac{3n_5}{T^2 + \mu^2 / \pi^2} \qquad J_3 \approx \frac{3e^2}{\pi^2} \frac{Q}{T^2 + \mu^2 / \pi^2} B \sum_f q_f^2$$

Fluctuations in topological charge lead to fluctuations in the current

$$\langle J_3^2 \rangle \approx \left[\frac{3e^2}{\pi^2} \frac{1}{T^2 + \mu^2 / \pi^2} B \sum_f q_f^2\right]^2 \langle Q^2 \rangle$$

Kharzeev, Fukushima & HJW ('08)

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Kharzeev, Fukushima & HJW ('08)

See paper this and 4 other methods to arrive at this result

Suppression of +/- correlations



Preliminary data Au & Cu 62 GeV



Voloshin (STAR Collaboration) Quark Matter 2008

The Chiral Battery

EM Current created with constant magnetic field? -> No violation of energy conservation

Chirality means storage of energy

Imagine hypothetical material exactly described by massless Dirac equation.

This might be pure Science Fiction, but anyway.. at least useful analogy

We can store chirality=energy in this material by placing it in parallel electric and magnetic fields,

The Chiral Battery





Graphene comes close,

But is not the right material. Pseudospin instead of real spin Also need something 3-d.

The Chiral Battery

Charging: use axial anomaly, place battery in parallel electric and magnetic fields.

<u>Utilize:</u> apply (constant) magnetic field in the direction one wants the current to flow. The behavior of the current follows from our analysis.

Discharge: Voltage difference is created over the terminals, hence electric field, but now antiparallel to magnetic field.

The battery will also discharge by the axial anomaly.

<u>Capacity:</u> same order of magnitude as ordinary batteries.









Kharzeev, Fukushima & HJW ('08)

Measurements suggest

Preferential emission of charged particles in one direction perpendicular to reaction plane.

<u>Correlations between positively charged</u> <u>particles and negatively charged particles</u> on opposite sides.

Existence of screening effect.

About 1-3 % asymmetry



Sergei Voloshin (STAR Collaboration) Quark Matter 2008

Relative asymmetry increases for more peripheral collisions

Magnitude <u>asymmetry Cu-Cu and Au-Au very similar</u> both at 62 GeV and 200 GeV for all centralities.

Is it due to the Chiral Magnetic Effect or due to something else, and how to find out?



Features of the Chiral Magnetic Effect

- Magnitude of asymmetry estimate: gold-gold at 130 GeV at large impact parameter $a_{++} \sim 10^{-4}$ with large uncertainty

- Atomic Number (A) dependence is determined by initial time. A better computation (no pancake approximation) could give us this more accurately.

For now it seems that for intermediate energies we have $(Z/A)^2$ dependence, not completely certain: depends on dynamics

- The correlators are proportional to Z² Test: use nuclei with same A and different Z, isobars
- Particle species dependence
- Beam energy dependence is determined by initial time. A better computation (no pancake approximation) could give us this.

At LHC smaller asymmetries. Magnetic field decays faster.