Efimov Physics with Fermions

H.-W. Hammer

Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics

Universität Bonn

bctp Bethe Center for Theoretical Physics

Bundesministerium für Bildung und Forschung

Deutsche Forschungsgemeinschaft DFG

Collaborators: E. Braaten, D. Canham, D. Kang, L. Platter, ...

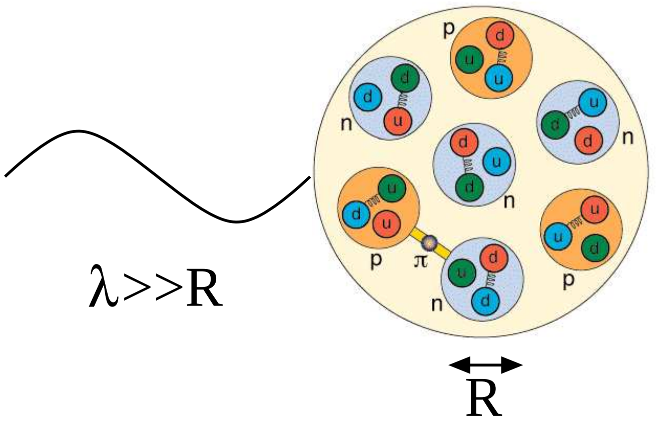
Agenda

- Introduction
- Resonant Interactions and Efimov Physics
- Effective Field Theory for Large Scattering Length
- Applications
  - Cold atoms
  - Infrared limit cycle in QCD?
  - Halo nuclei
- Summary and Outlook

Review article: Braaten, HWH, Phys. Rep. **428** (2006) 259

Effective Theory

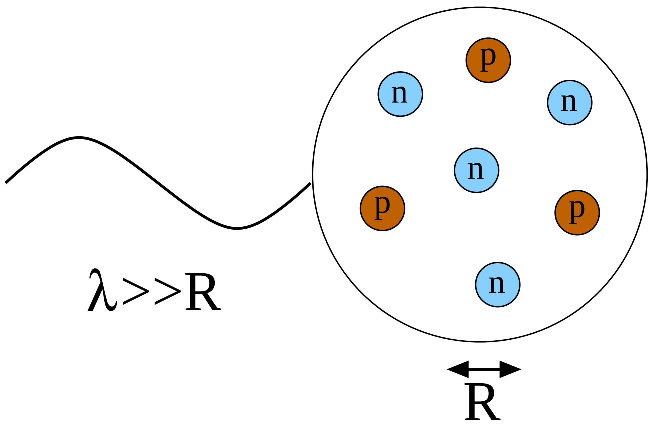
- Separation of scales:
  $1/k = \lambda \gg R$
- Limited resolution at low energy:
  $\longrightarrow$ expand in powers of kR



Effective Theory

universitätbonn

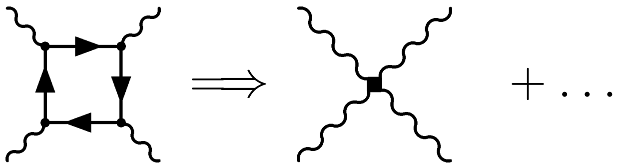
- Separation of scales:
  $1/k = \lambda \gg R$
- Limited resolution at low energy:
  $\longrightarrow$ expand in powers of kR



- Short-distance physics not resolved
  $\longrightarrow$ capture in low-energy constants using renormalization
  $\longrightarrow$ include long-range physics explicitly
- Systematic, model independent $\rightarrow$ error estimates
- Classic example: light-light-scattering (Euler, Heisenberg, 1936)

  Simpler theory for $\omega \ll m_e$:

  $\mathcal{L}_{QED}[\psi, \bar{\psi}, A_\mu] \rightarrow \mathcal{L}_{eff}[A_\mu]$



Exploiting Limited Resolution

- Painting at the limit of resolution of the human eye

G. Seurat, A Sunday on La Grande Jatte

Large Scattering Length

universitätbonn

- Resonant interaction $\Longrightarrow$ large scattering length a
- Natural expansion parameter: $\ell/|a|$, $k\ell$,... $(\ell \sim r_e, l_{vdW}, ...)$

Universal properties: $B_2 = \dfrac{\hbar^2}{ma^2} + \ldots \quad (a < 0)$

- Atomic physics:
  - ^{4}He: $a \approx 104$ Å $\gg r_e \approx 7$ Å $\sim l_{vdW} \longrightarrow B_d \approx 100$ neV
  - Feshbach resonances $\Longrightarrow$ variable scattering length
- Nuclear physics: S-wave NN-scattering, halo nuclei,...
  - $^1S_0$, $^3S_1$: $|a| \gg r_e \sim 1/m_\pi \longrightarrow B_d \approx 2.2$ MeV
  - ^{6}He: $2n$ separation energy $\approx$ 973 keV
- Particle physics: $X(3872)$ as a $D^0 D^{0*}$ molecule? $(J^{PC} = 1^{++})$

$$m_X - (m_{D^0} + m_{D^{0*}}) = (-0.3 \pm 0.4) \text{ MeV}$$

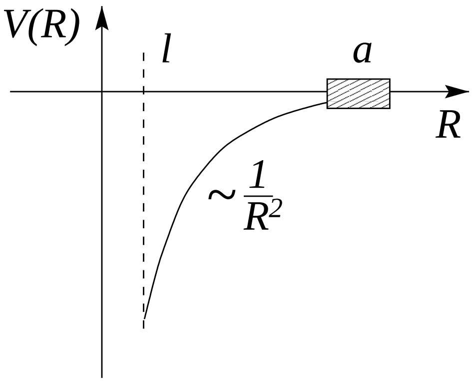
Efimov Physics

universität bonn

(V. Efimov, Phys. Lett. **33B** (1970) 563)

- Three-body system with large scattering length a
- Hyperspherical coordinates: $R^2 = (r_{12}^2 + r_{13}^2 + r_{23}^2)/3$
- Schrödinger equation simplifies for $|a| \gg R \gg l$:

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial R^2} + \frac{s_0^2 + 1/4}{R^2}\right] f(R) = \underbrace{-\frac{\hbar^2\kappa^2}{m}}_{E} f(R)$$



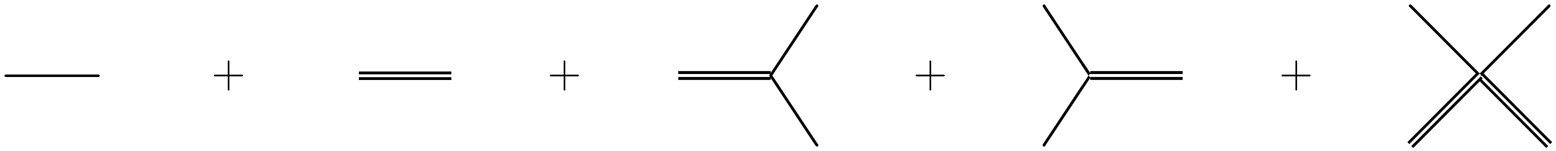
- Singular Potential: renormalization required
- Boundary condition at small R: breaks scale invariance

  $\Longrightarrow$ dependence of observables on 3-body parameter (and a)
- EFT formulation: boundary condition $\Rightarrow$ 3-body interaction

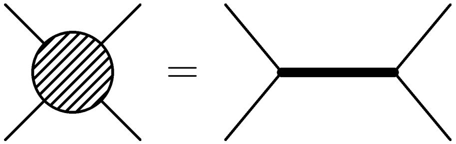
Two-Body System in EFT

- Effective Lagrangian

$$\mathcal{L}_d = \psi^\dagger \left(i\partial_t + \frac{\vec{\nabla}^2}{2m} \right) \psi + \frac{g_2}{4} d^\dagger d - \frac{g_2}{4} (d^\dagger \psi^2 + (\psi^\dagger)^2 d) - \frac{g_3}{36} d^\dagger d \psi^\dagger \psi + ..$$



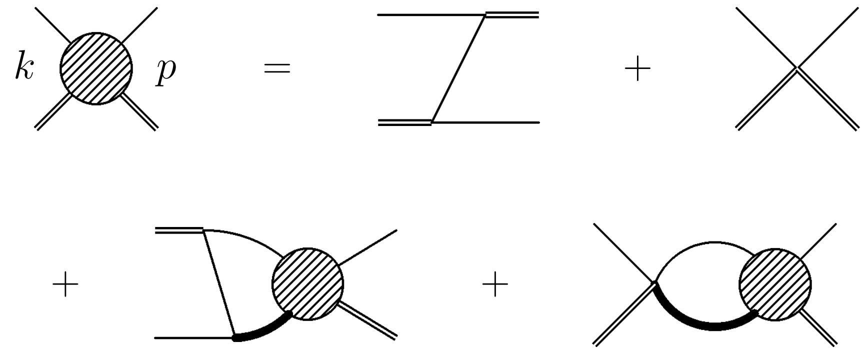
- Interacting dimeron propagator $\longrightarrow$ sum bubbles

- Two-body amplitude $\mathcal{T}_2(k,k)$:  $\propto \frac{1}{1/a - ik} + \ldots$

- Matching: $g_2 \longleftarrow a, B_d$
- RG fixed points of g_2: $a = 0$ and $a = \infty$
- Higher order corrections: perturbation theory

Three-Body System in EFT

- Three-body equation :

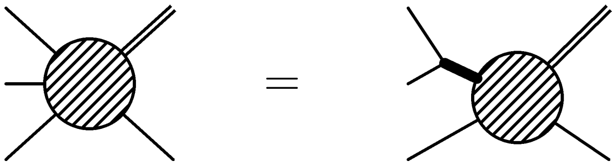


$$\mathcal{T}_3(k,p) = M(k,p) + \frac{4}{\pi}\int_0^{\Lambda} dq\, q^2\, M(q,p) D_d(q)\, \mathcal{T}_3(k,q)$$

with $M(k,p) = \underbrace{F(k,p)}_{\text{1-atom exchange}} \quad \underbrace{-\frac{g_3}{9g_2^2}}_{H(\Lambda)/\Lambda^2}$

($g_3 = 0$, $\Lambda \to \infty \longrightarrow$ Skorniakov, Ter-Martirosian '57)

- Three-body recombination:



Renormalization

universitätbonn

- Observables must be independent of regulator/cutoff Λ

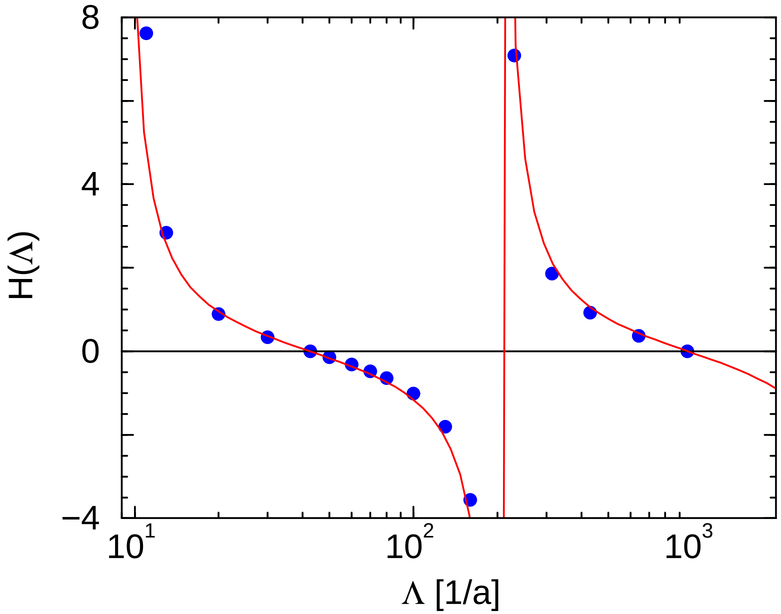
$\Rightarrow$ Running coupling $H(\Lambda)$

- $H(\Lambda)$ periodic: limit cycle

$$\Lambda \to \Lambda\, e^{n\pi/s_0} \approx \Lambda(22.7)^n$$

(cf. Wilson, 1971)

- Full scale invariance broken to discrete subgroup



$$H(\Lambda) = \frac{\cos(s_0 \ln(\Lambda/\Lambda_*) + \arctan(s_0))}{\cos(s_0 \ln(\Lambda/\Lambda_*) - \arctan(s_0))}\,, \qquad s_0 \approx 1.00624$$

- Limit cycle $\Longleftrightarrow$ Discrete scale invariance
- Matching: $\Lambda_* \longleftarrow B_t,\, K_3, \ldots \longrightarrow \kappa_*, a_*, a'_*$

Discrete Scale Invariance

universitätbonn

- Similarity to Matrjoschka doll

  $\longrightarrow$ "Russian Doll Renormalization"

- Other examples
  - $1/r^2$ potential in QM
  - Field theory models

    (Wilson, Glazek, LeClair et al.,...)
  - Turbulence, earthquakes, financial markets,...

    (cf. Sornette, Phys. Rep. **297** (1998) 239)

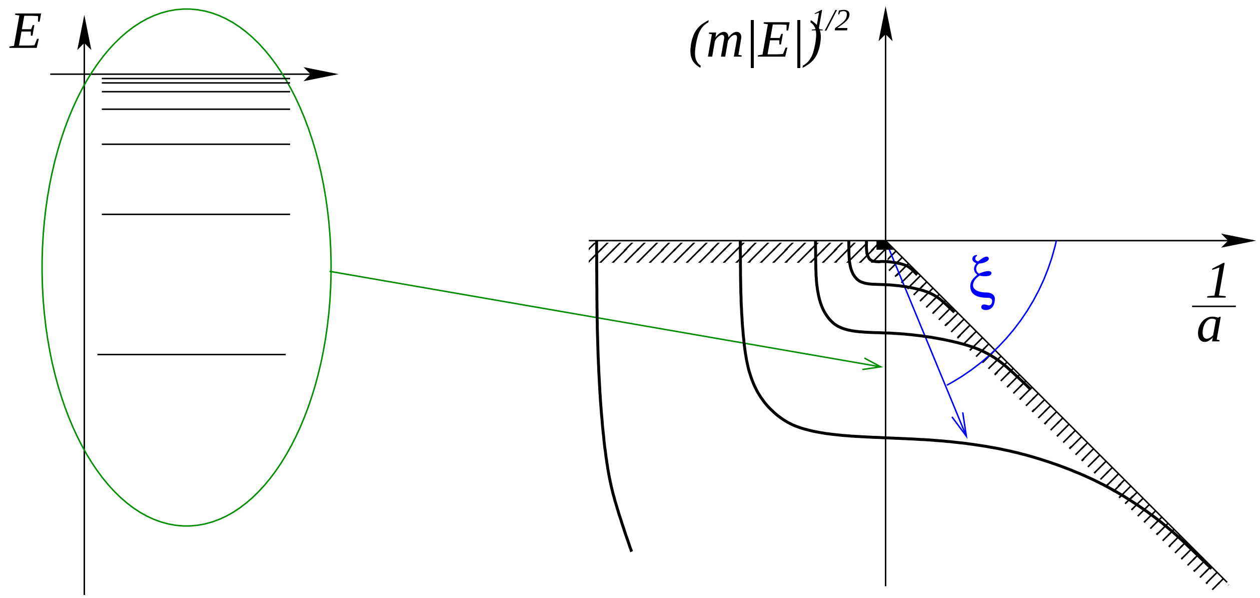
- Observable Consequences?

  $\Longrightarrow$ universal correlations, Efimov effect, log-periodic dependence on the scattering length,...

Efimov Effect

- Universal spectrum of three-body states

  (V. Efimov, Phys. Lett. **33B** (1970) 563)

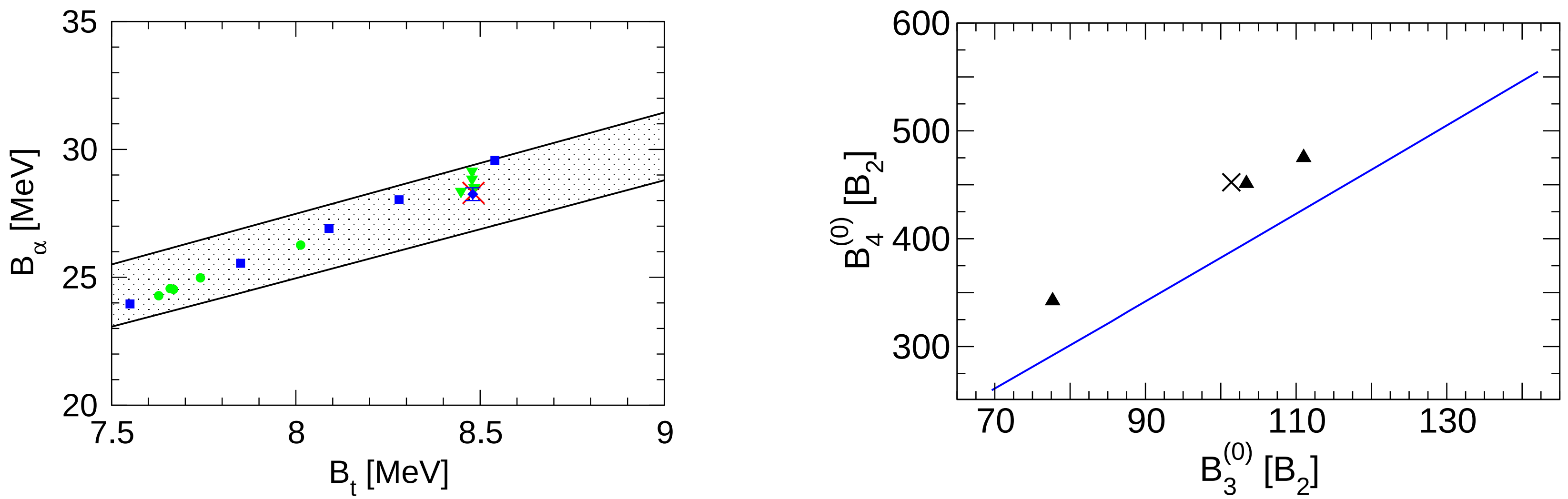


- Discrete scale invariance for fixed angle ξ
- Geometrical spectrum für $1/a \to 0$ (⇒ “Efimov effect”)

$$B_3^{(n)}/B_3^{(n+1)} \xrightarrow{1/a\to 0} 515.035\ldots$$

Universal Correlations

- 2 Parameters at LO $\Rightarrow$ 3-body observables are correlated
  $\Longrightarrow$ Phillips line (Phillips, 1968)
- No four-body parameter at LO (Platter, HWH, Meißner, 2004)
  $\Rightarrow$ 4-body observables are correlated $\Longrightarrow$ Tjon line



- Variation of 3-body parameter generates correlations
- Nuclear physics: Λ dependence of V_{low-k} (Bogner et al., 2004)
- Tjon line also at NLO (Kirscher et al., 2009)

More on the 4-Body System

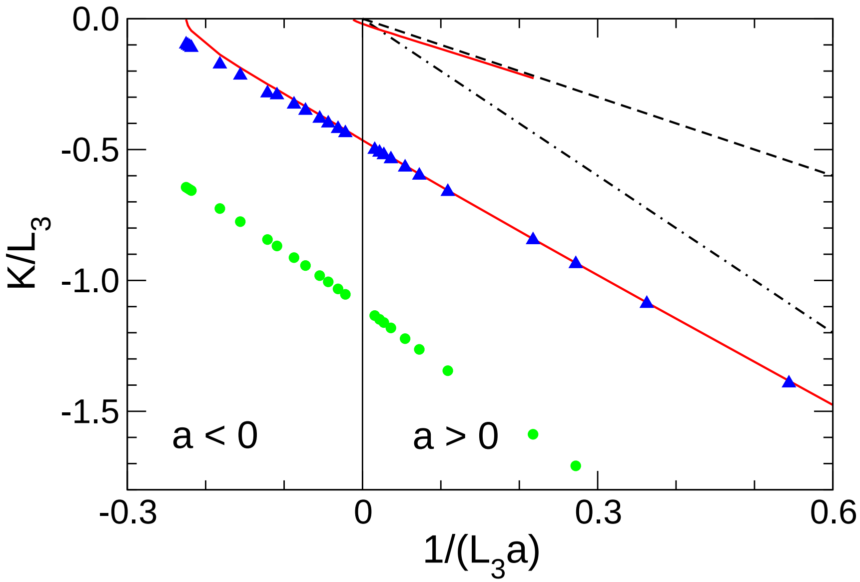
- Universal properties of 4-body system with large a
  - Bound state spectrum
  - Scattering Observables
- "Efimov-plot": 4-body bound state spectrum as function of $1/a$

$$K = \mathrm{sign}(E)\sqrt{m|E|}$$

$$B_4^{(0)} = 5B_3^{(0)} \qquad (1/a \equiv 0)$$

$$B_4^{(1)} = 1.01B_3^{(0)}$$

(Platter, HWH, EPJA **32** (2007) 113)



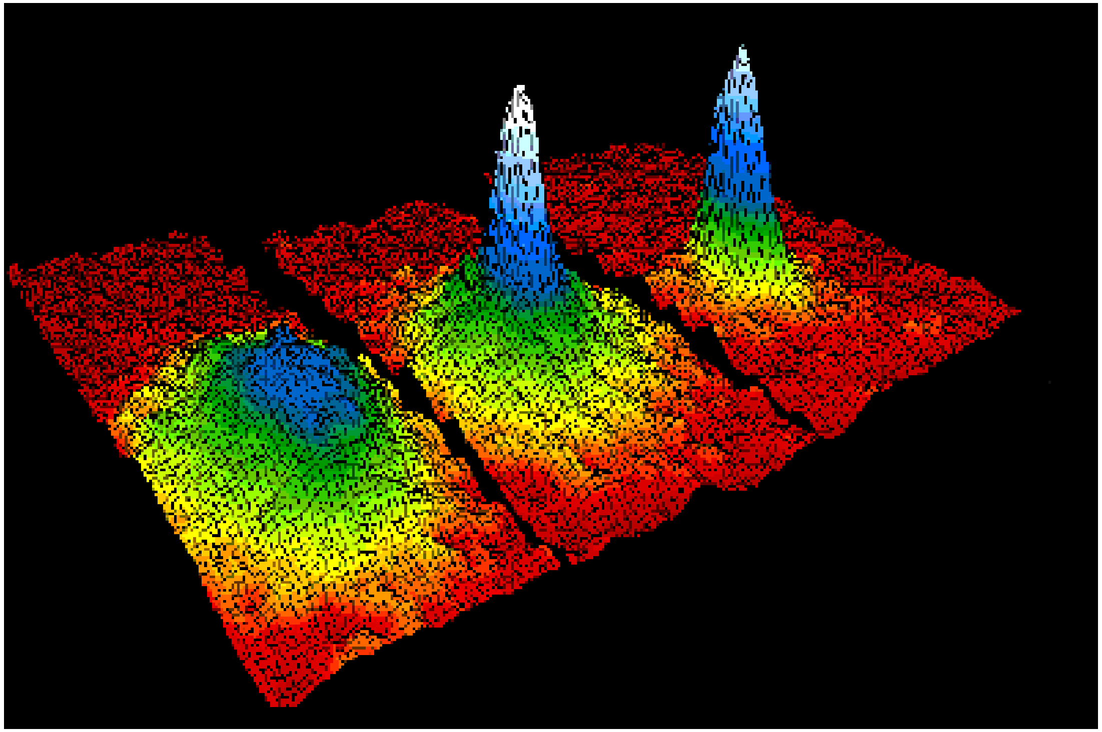
- Signature in Cs loss data

von Stecher, D'Incao, Greene, Nature Physics **5** (2009) 417

Ferlaino, Knoop, Berninger, Harm, D'Incao, Nägerl, Grimm, PRL **102** (2009) 140401

Efimov Physics in Cold Atoms

- Velocity distribution ($T =$ 400 nK, 200 nK, 50 nK)

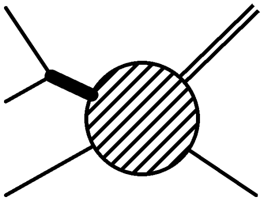


(Source: http://jilawww.colorado.edu/bec/)

- Few-body loss rates provide window on Efimov physics
- Variable scattering length via Feshbach resonances

Three-Body Recombination

universität bonn



- Recombination into weakly-bound dimer:

  3 atoms $\rightarrow$ dimer + atom $\Rightarrow$ loss of atoms

- Recombination constant: $\dot{n}_A = -3\,\alpha\, n_A^3$ (gas)

- Scattering length dependence for $a > 0$:

  (Nielsen, Macek, 1999; Esry, Greene, Burke, 1999; Bedaque, Braaten, HWH, 2000)

$$\alpha \approx 67.1 \sin^2\left[s_0 \ln(a\kappa_*) + 1.16\right] \frac{\hbar a^4}{m}\,, \qquad s_0 \approx 1.00624..$$

- Alkali atoms form deeply-bound dimers

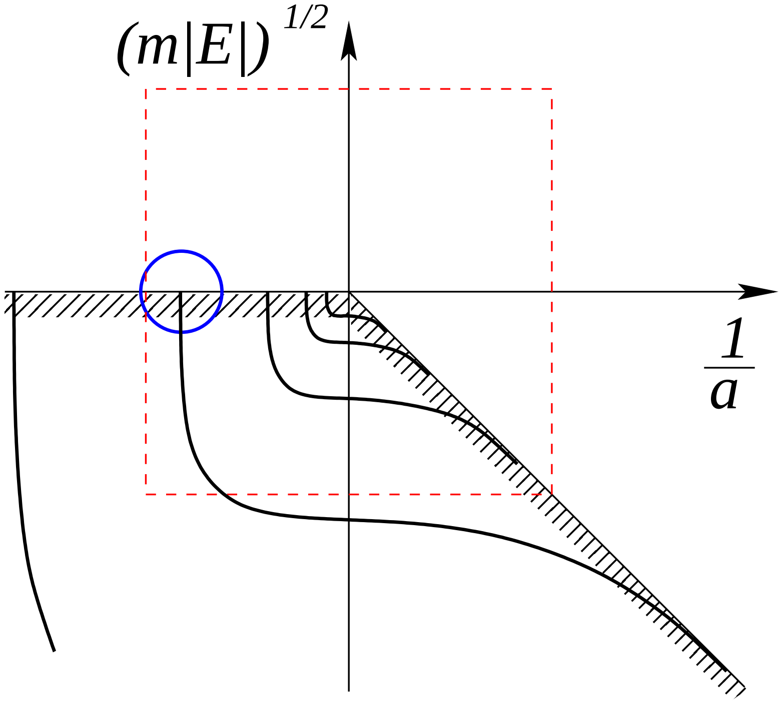
- Modification from deep dimers: Efimov states aquire width

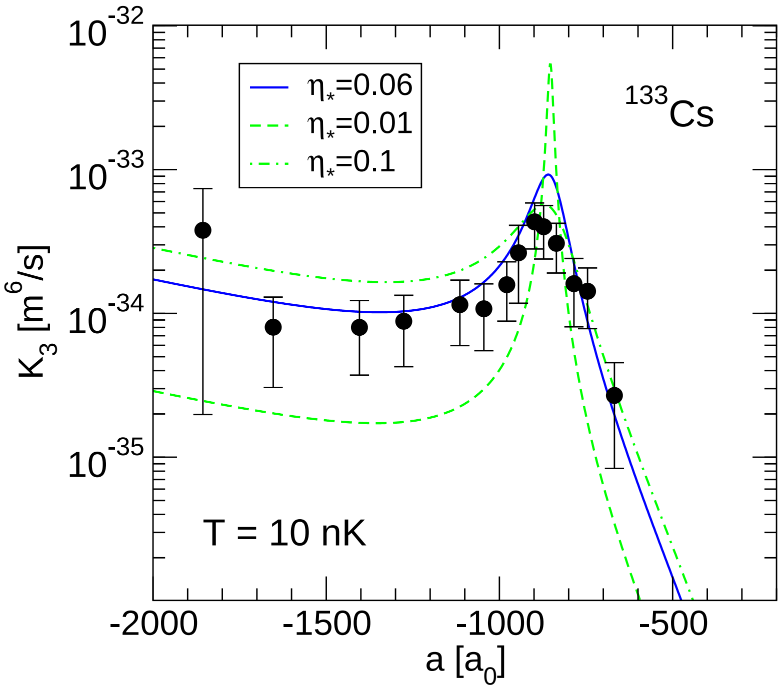
  $\Longrightarrow \qquad \kappa_* \rightarrow \kappa_* \exp(i\eta_*/s_0)$ (Braaten, HWH, 2004)

- Recombination into deep dimers

Efimov States in ^{133}Cs

- Experimental evidence for Efimov states in ^{133}Cs
  (Kraemer et al. (Innsbruck), Nature **440** (2006) 315)
  - Identification via 3-body recombination rate



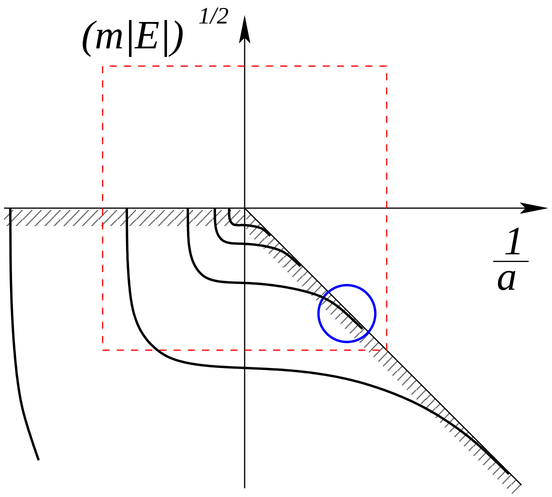


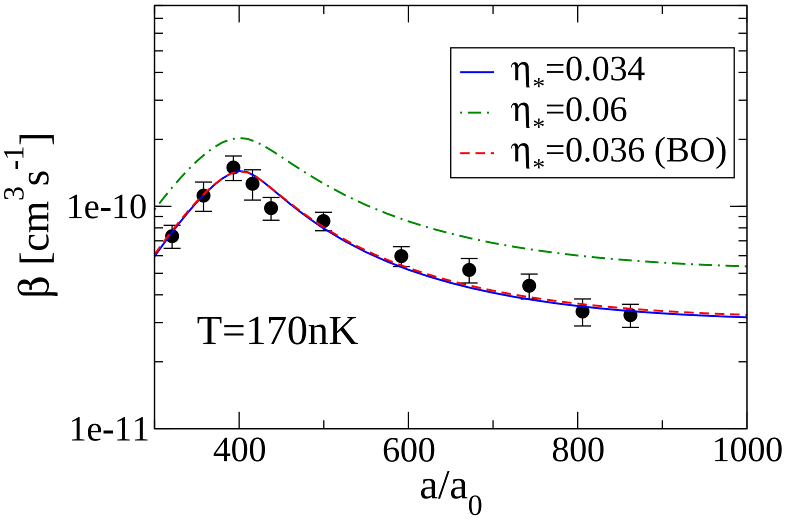
$a < 0$: $\quad a'_* = -875 a_0$

- Finite temperature effects small for $T \lesssim 200$ nK
  (Braaten, HWH, Kang, Platter, 2008)

Dimer Relaxation in ^{133}Cs

- Dimer Relaxation: atom + dimer $\rightarrow$ atom + deep dimer (energetic)
- Relaxation constant: $$\frac{dn_A}{dt} = \frac{dn_D}{dt} = -\beta\, n_A\, n_D$$
- Recent experiment: Knoop et al. (Innsbruck), Nature Physics **5** (2009) 227
- Finite temperature $T \sim T_c$: Bose-Einstein average
- Include trap geometry: $\mu_i \longrightarrow \mu_i - m_i \bar{\omega}^2 r^2/2, \quad i = a, d$

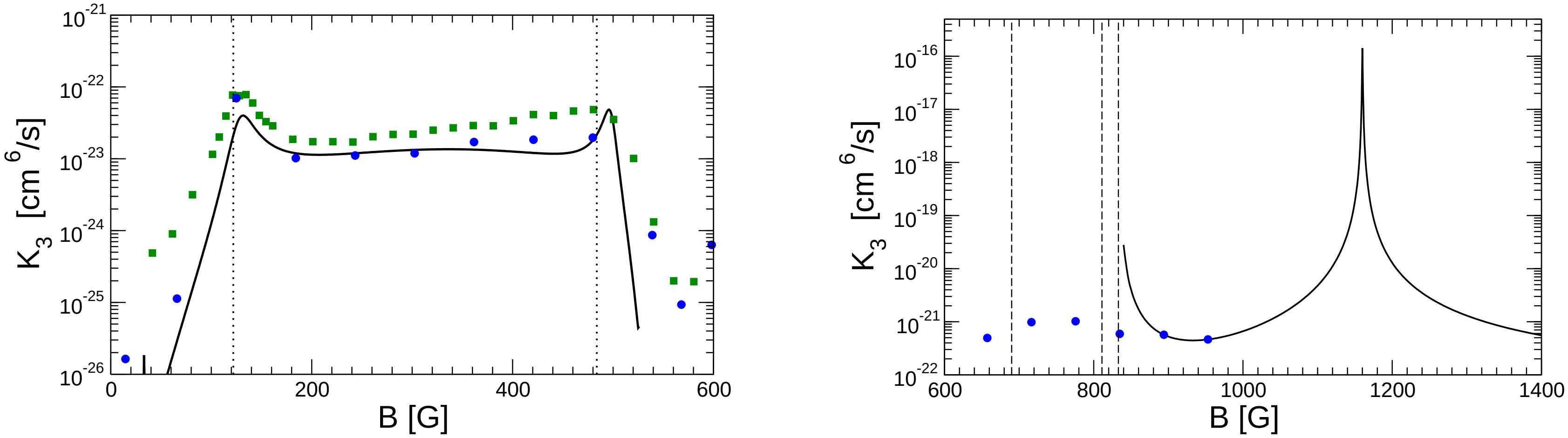




Helfrich, HWH, EPL **86** (2009) 53003

Efimov States in ^{6}Li

- Efimov effect for fermions $\Rightarrow \geq 3$ spin states
- Experimetal evidence for Efimov states in ^{6}Li
  - ■ Ottenstein et al. (Heidelberg), Phys. Rev. Lett. **101** (2008) 203202
  - ● Huckans et al. (Penn State), Phys. Rev. Lett. **102** (2009) 165302

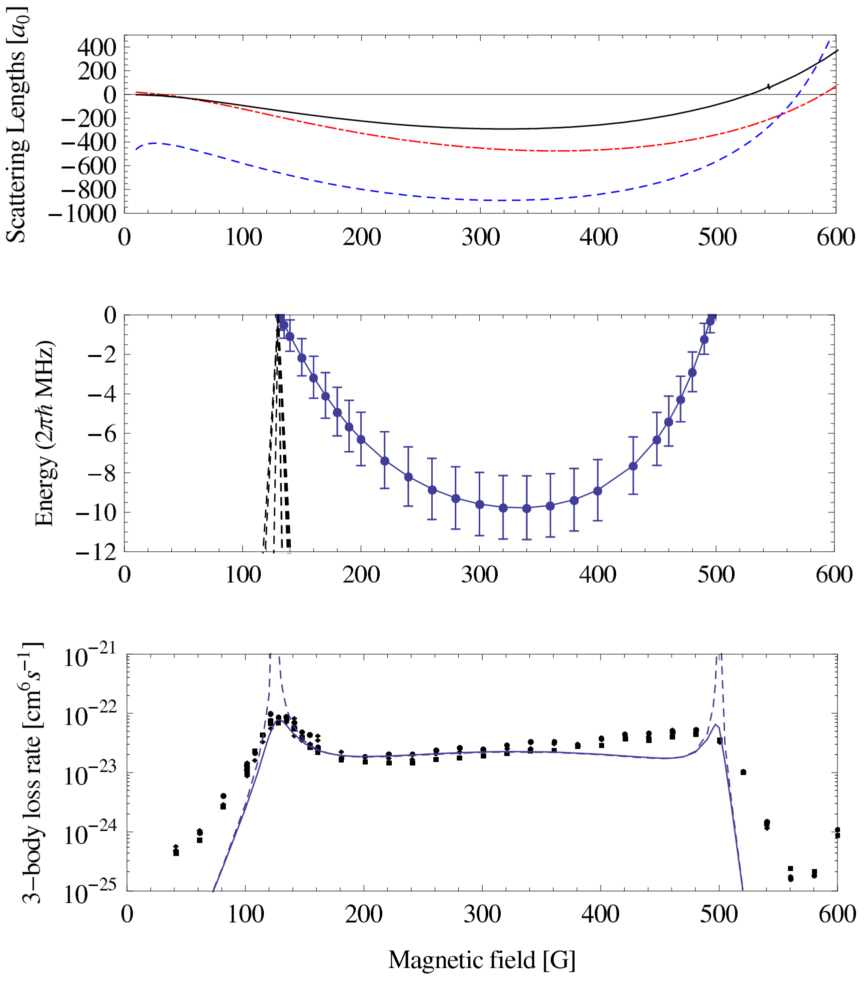


(Braaten, HWH, Kang, Platter, arXiv:0811.3578)

- Systematic normalization error: 70-90%
- Possible resonance around $B \approx 1160$ G
- Other approaches: Schmidt et al., arXiv:0812.1191; Naidon et al., arXiv:0811:4086

Alternative Approaches

- Solution of Faddeev equations in hyperspherical formalism
  (Naidon, Ueda, arXiv:0811:4086)
- Functional renormalization group with explicit trion field
  (Schmidt, Floerchinger, Wetterich, arXiv:0812.1191)
- Good agreement among different approaches
  (figure taken from Naidon, Ueda)



Recent Experimental Progress

- Efimov resonances in a mixture of ^{41}K and ^{87}Rb atoms
  (Barontini et al. (Florence), arXiv:0901:4584)

$\Rightarrow$ Connected K-Rb-Rb resonances for $a > 0$ and $a < 0$

- Efimov spectrum in ultracold potassium atoms near a Feshbach resonance (Zaccanti et al. (Florence), arXiv:0904.4453)

$\Rightarrow$ Observation of first two states of an Efimov spectrum

- Observation of universality in ultracold ^{7}Li three-body recombination (Gross et al. (Ramat-Gan), arXiv:0906.4731v1)

Infrared Limit Lycle in QCD?

- Large scattering lengths in NN interaction imply:

  QCD close to critical trajectory for infrared limit cycle

- Conjecture:

  > QCD can be tuned to critical trajectory by small changes in the up- and down-quark masses

  (Braaten, HWH, 2003)

- Quark mass dependence of low-energy constants

  $\longrightarrow$ Chiral EFT with explicit pions

  $\longrightarrow$ Extrapolation in quark masses possible

  Beane, Bedaque, Savage, van Kolck, 2002

  Beane, Savage, 2003

  Epelbaum, Meißner, Glöckle, 2003

Extrapolation in Quark Masses

universitätbonn

- Gell-Mann-Oakes-Renner relation: $m_\pi^2 \propto (m_u + m_d)$
- Chiral NN potential: $V = V_{OPE} + V_{TPE} + V_{contact}$

$$V_{OPE} = \left(-\frac{g_A^2}{4F_\pi^2} + \ldots \right) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q})}{\mathbf{q}^2 + m_\pi^2}$$

$$V_{contact} = \overline{C}_S + m_\pi^2 \overline{D}_S + (\overline{C}_T + m_\pi^2 \overline{D}_T) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \ldots$$

- $\overline{D}_{S,T}$ not known $\longrightarrow$ dimensional analysis estimate:
  (Epelbaum, Meißner, Glöckle, 2003)

$$-3.0 \; < \; F_\pi^2 \, \Lambda_\chi^2 \, \overline{D}_{S,T} \; < \; 3.0 \qquad \longrightarrow \text{error band in extrapolation}$$

- Extrapolation of a_t, a_s, B_d as function of $m_\pi^2 \propto (m_u + m_d)$

Extrapolation of Scattering Lengths

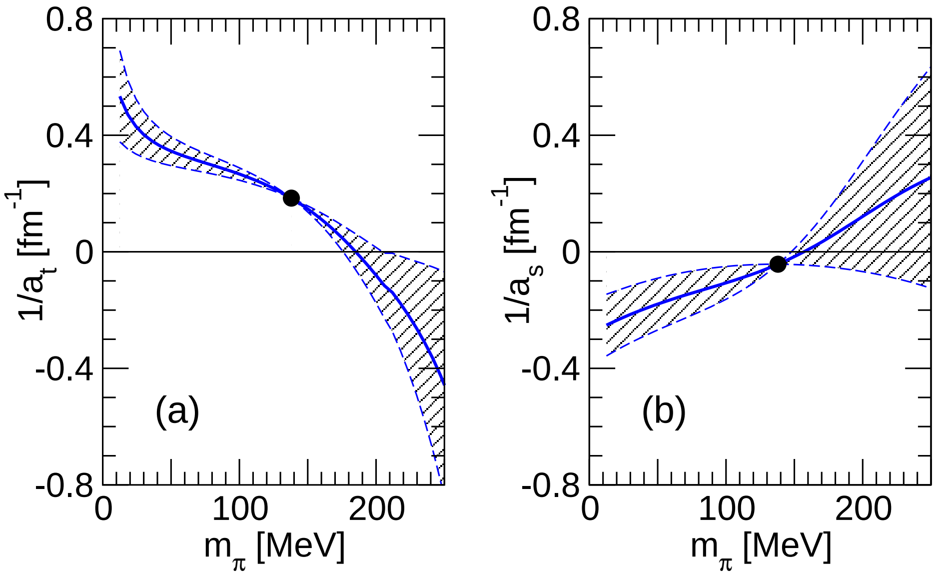
universitätbonn

- Remember: $m_\pi^2 \propto (m_u + m_d)$
- Limit cycle $\Rightarrow$ require

$$1/a_t = 1/a_s = 0$$

- Solutions exist for (NLO)

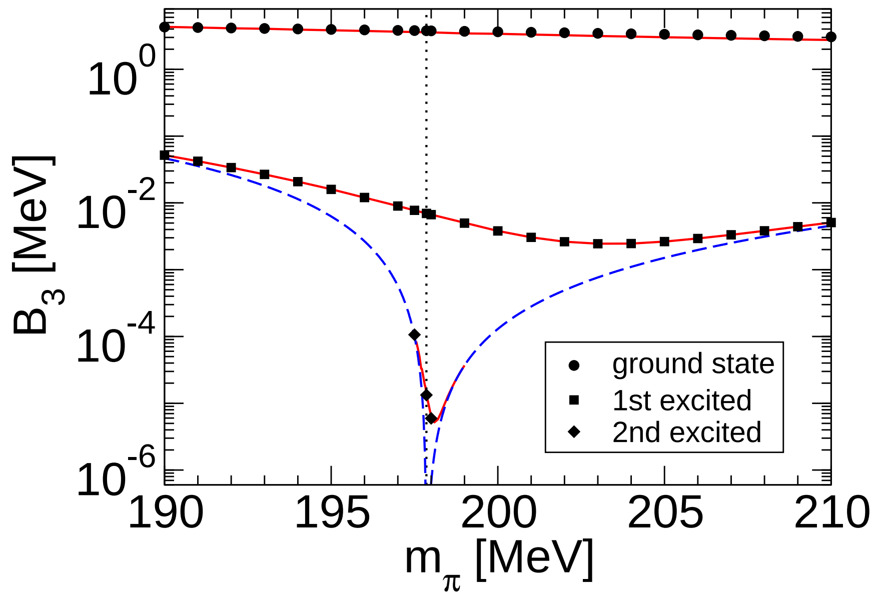
175 MeV $\lesssim m_\pi^{crit.} \lesssim$ 205 MeV



(Epelbaum, Meißner, Glöckle, NPA **714** ('03) 535)

- Vary m_u and m_d independently, but:
  only operators $\propto (m_u + m_d)$ at NLO
- Properties of limit cycle universal: study one of solutions in NLO ($m_\pi^{crit.} = 197.8577$ MeV) (Epelbaum, HWH, Meißner, Nogga, 2006)
- Beane & Savage: similar results for $m_\pi > (m_\pi)_{phys}$

Limit Cycle: Bound States



- Identification:

  $\longrightarrow$ Efimov effect for triton

  $\longrightarrow$ infinitely many excited states
- Calculate in chiral/universal EFT
- Properties:
  - $2N - N$ cluster structure close to threshold
  - $2N - 3N$ overlap vanishes at critical point
- Can use chiral EFT for extrapolations
- Effect observable in Lattice QCD calculations?

  (cf. Wilson, 2005)
- Variation of fundamental constants?

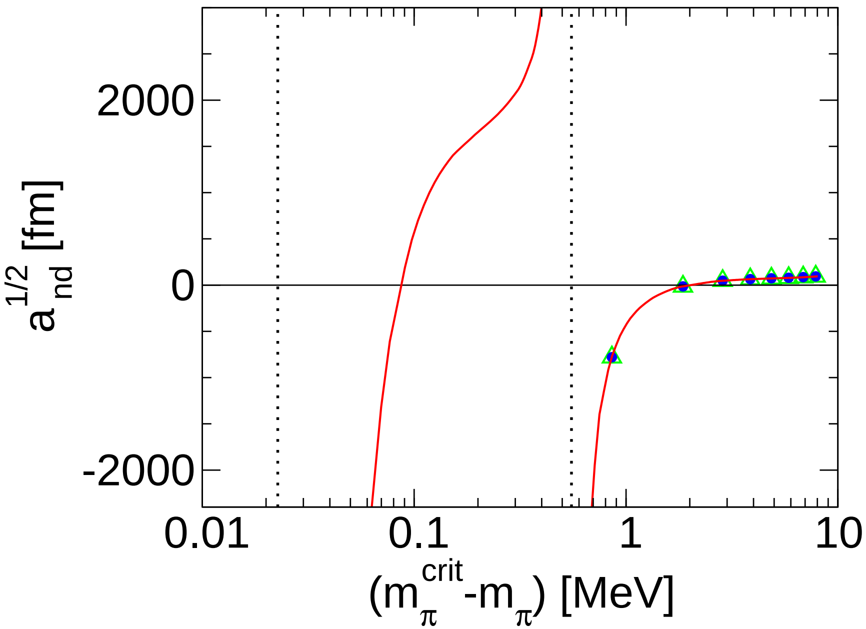
Limit Cycle: Scattering

- “Matching” close to critical point:

  chiral EFT $\longleftrightarrow$ universal EFT

- Approaches complementary: e.g. scattering observables in universal EFT

- Divergences:
  - at critical point
  - when new Efimov states appear



- N2LO in universal theory (including effective range corrections)

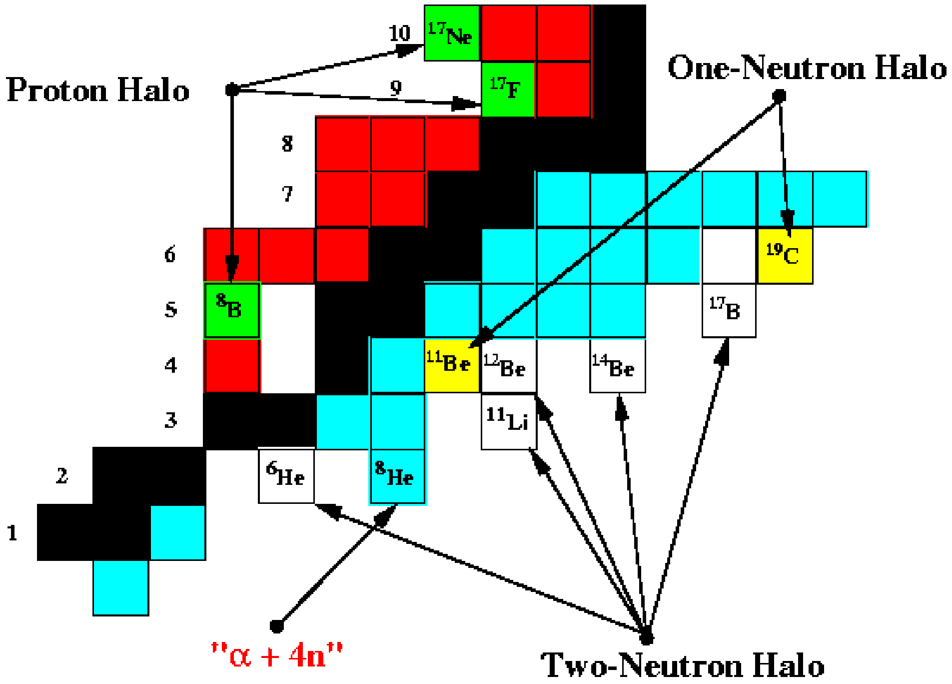
  (HWH, Phillips, Platter, 2007)

Halo Nuclei

universitätbonn

- Low separation energy of valence nucleons: $B_{valence} \ll B_{core}, E_{ex}$

  $\longrightarrow$ close to "nucleon drip line" $\longrightarrow$ scale separation $\longrightarrow$ EFT

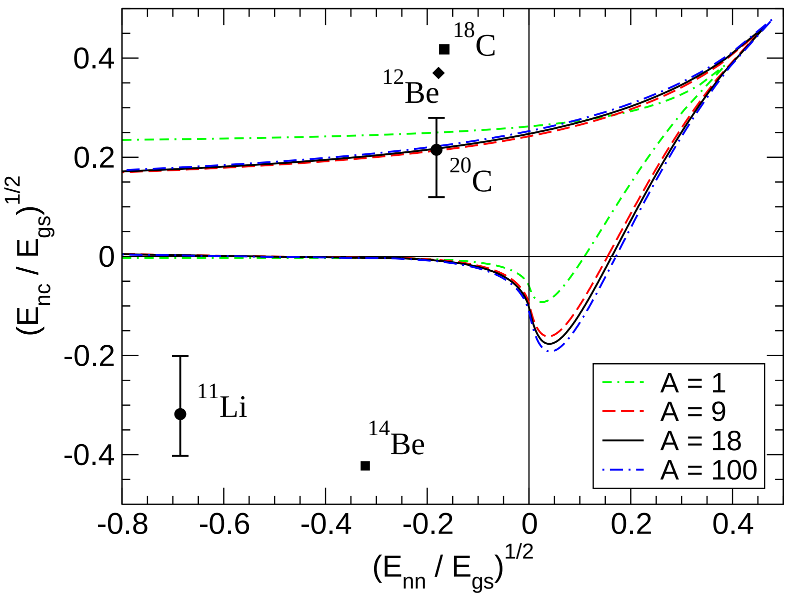


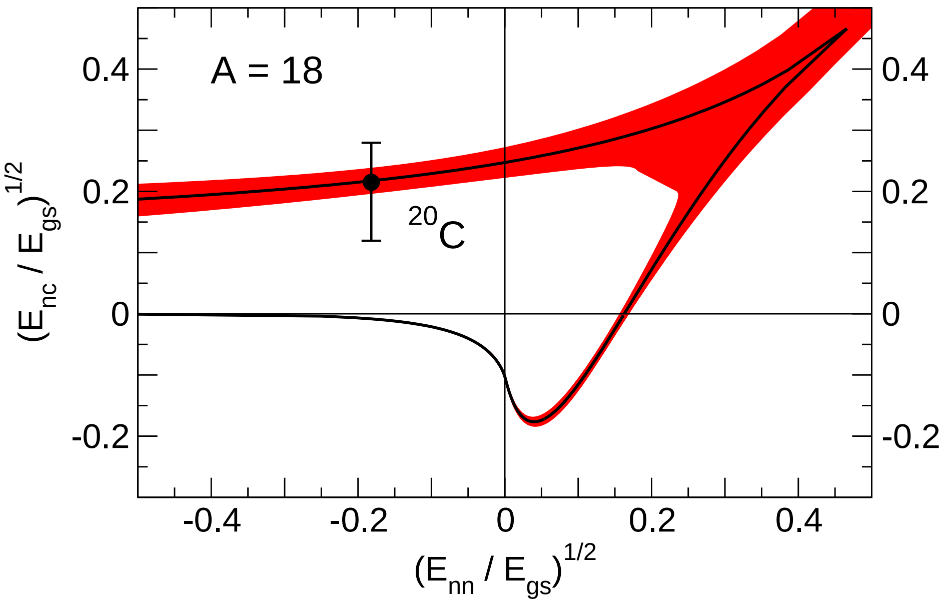
http://www.nupecc.org

- EFT for halo nuclei
  - $n\alpha$-System ("^{5}He") (Bedaque, Bertulani, HWH, van Kolck, 2002)
  - $\alpha\alpha$-System ("^{8}Be") (Higa, HWH, van Kolck, 2008)

3-Body Halos

- Examples: ^{14}Be $\longleftrightarrow$ ^{12}Be +n +n, ^{20}C $\longleftrightarrow$ ^{18}C +n +n
- “Effective" 3-body system: separation energy of valence nucleons small compared to binding energy of “core”
- Efimov effect in halo nuclei? $\Rightarrow$ excited states





Canham, HWH, Eur. Phys. J. A **37** (2008) 367

(cf. Amorim, Frederico, Tomio, 1997)

Form Factors and Radii

- Structure of halo nuclei $\rightarrow$ matter form factors, radii

nucleus	B_{nnc} [keV]	B_{nc} [keV]	$\sqrt{\langle r_{nn}^2 \rangle}$ [fm]	$\sqrt{\langle r_{nc}^2 \rangle}$ [fm]
^{14}Be	1120	-200.0	4.1 ± 0.5	3.5 ± 0.5
^{20}C	3506	161	2.8 ± 0.3	2.4 ± 0.3
	3506	530	3.0 ± 0.7	2.5 ± 0.6
	3506	60	2.8 ± 0.2	2.3 ± 0.2
^{20}C*	65 ± 6.8	60	42 ± 3	38 ± 3

Canham, HWH, Eur. Phys. J. A **37** (2008) 367

(cf. Yamashita, Tomio, Frederico, 2004)

- Input: TUNL Nuclear data evaluation project, ...
- Experiment: ^{14}Be $\rightarrow \sqrt{\langle r_{nn}^2 \rangle} = (5.4 \pm 1.0)$ fm

  (Marques et al., Phys. Rev. C **64** (2001) 061301)

Summary and Outlook

universitätbonn

- Universality at large scattering length
  - Limit cycle in three-body system $\Leftrightarrow$ Efimov physics
  - Universal correlations (Phillips, Tjon line,...)
- Applications in atomic, nuclear, and particle physics
  - Cold atoms close to Feshbach resonance
  - Halo nuclei
  - Scattering properties of the $X(3872)$
- Future directions:
  - Cold atoms: $N \geq 4$, 2d-systems, ...
  - Halo nuclei: reactions, external currents, ...
  - Hadronic molecules: universal properties, Efimov effect?
  - Three-nucleon system on the lattice: limit cycle in “deformed” QCD?