Non Abelian Gauge Fields and Cold Atoms: Schrödinger, Dirac and beyond

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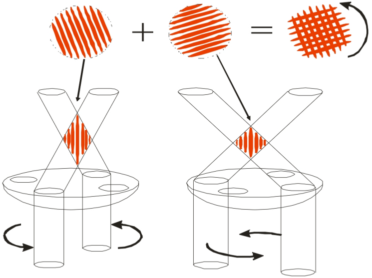
outline

1. Effective Fields
2. Howto: Optically Induced non-Abelian Fields
3. Constant non-Abelian SU(2) Field
   - Several Effects on Cold Atoms
   - Interactions ?
4. ...QCD and QGP
5. State of Art: Experiments

Effective Fields in Cold Atoms

example

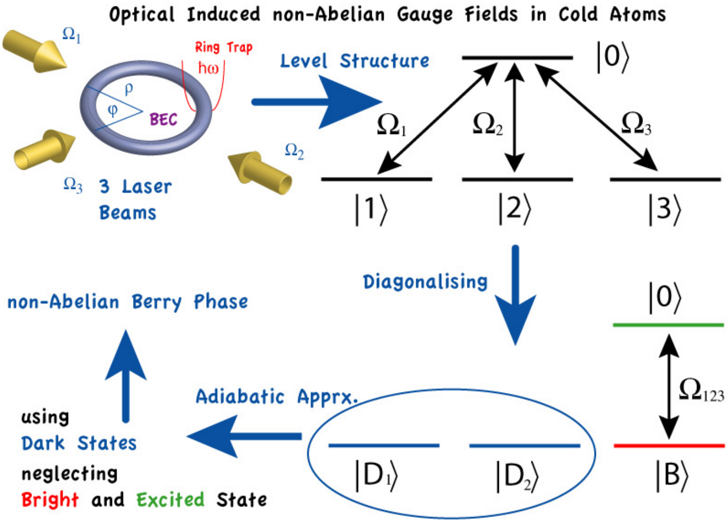
$$H_{rot} = H_{lab} - \Omega \cdot L$$
$$= \frac{(p - A)^2}{2m} + \frac{1}{2}(\omega^2 - \Omega^2)r^2$$



several schemes

- rotating frame (pic. from Ch. Foot, Oxford)
- tunneling in optical lattice (Lewenstein group)
- optically induced (Berry phase)

Tripod System: 2 Dark States

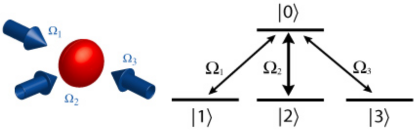


$$|\Phi\rangle = \sum_{i=1,2} \psi_i(\mathbf{r})|D_i(\mathbf{r})\rangle$$

Tripod System

three laser beams

$$\Omega_1 = \Omega \sin\theta \, e^{-i\kappa x}$$
$$\Omega_2 = \Omega \sin\theta \, e^{i\kappa x}$$
$$\Omega_3 = \Omega \, 2\cos\theta \, e^{-i\kappa y}$$
$$\Omega = \sqrt{|\Omega_1|^2 + |\Omega_2|^2 + |\Omega_3|^2}$$



- Ω_i: transition rates in int. pict. (and rotating wave appr.)
- due to spacial extension: $\Omega(r)$

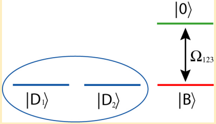
Quantum Optics

two dark states, one bright state, one excited state

$$|D_1\rangle \sim \mathrm{e}^{-\mathrm{i}\kappa y}\left(\mathrm{e}^{\mathrm{i}\kappa x}|1\rangle - \mathrm{e}^{-\mathrm{i}\kappa x}|2\rangle\right)$$

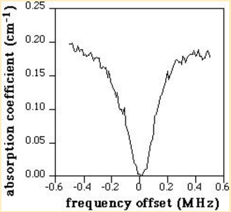
$$|D_2\rangle \sim \mathrm{e}^{-\mathrm{i}\kappa y}\cos\theta\left(\mathrm{e}^{\mathrm{i}kx}|1\rangle - \mathrm{e}^{-\mathrm{i}kx}|2\rangle\right) - \sin\theta|3\rangle$$

$$|B\rangle \sim \left[\frac{1}{\Omega}(\Omega_1^*|1\rangle + \Omega_2^*|2\rangle + \Omega_3^*|3\rangle)\right]$$

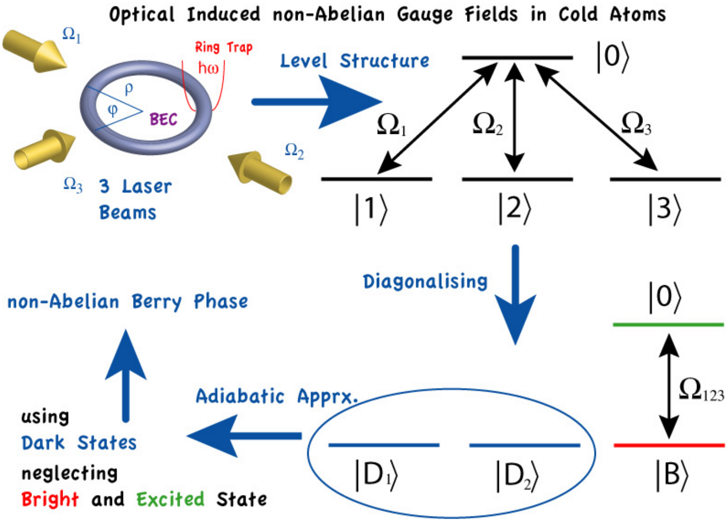


Dark State?

- eigenstate with eigenvalue 0
- no absorption
- spacial dependence



Tripod System: 2 Dark States



$$|\Phi\rangle = \sum_{i=1,2} \psi_i(\mathbf{r})|D_i(\mathbf{r})\rangle$$

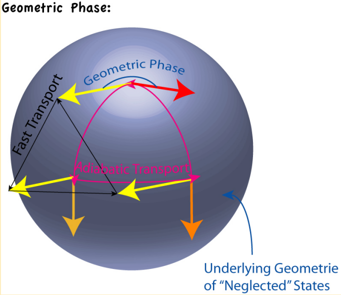
Berry phase

Coupling between internal and external states

- adiabatic approximation $|\Phi\rangle = \sum_{\mathrm{i}=1,2} \psi_{\mathrm{i}}(\mathbf{r}) \otimes |\mathrm{D}_{\mathrm{i}}(\mathbf{r})\rangle$
- but $\hat{\mathbf{P}}_{|\Phi\rangle} \neq \hat{\mathbf{p}}_{\Psi} \otimes \hat{\mathbf{p}}_{|D\rangle}$

  $\langle \mathrm{D}|\hat{\mathbf{P}}_{\Phi}|\Phi\rangle = \langle \mathrm{D}| - \mathrm{i}\hbar\nabla \left[\psi_{\mathrm{i}}(\mathbf{r}) \otimes |\mathrm{D}_{\mathrm{i}}(\mathbf{r})\rangle\right]$

  $= -\mathrm{i}\hbar\nabla\Psi(\mathbf{r}) + \mathsf{A}(\mathbf{r})\Psi(\mathbf{r})$



- Gauge Potential $\mathbf{A}_{ij} = -\mathrm{i}\hbar\langle \mathrm{D}_i(\mathbf{r})|\nabla \mathrm{D}_j(\mathbf{r})\rangle$

Gauge Potential

Center of Mass Wave Function Ψ

$$i\hbar\frac{\partial}{\partial t}\Psi = \left[\frac{1}{2m}(\mathbf{p} - \mathsf{A})^2 + \Phi + V\right]\Psi, \qquad \Psi = (\Psi_1, \Psi_2)^{\mathrm{T}}$$

Additional Potentials

$$\mathsf{A}_{n,m} = i\hbar\langle D_n(\mathbf{r})|\nabla D_m(\mathbf{r})\rangle$$

$$\Phi_{n,m} = \frac{1}{2m}\sum_l \mathsf{A}_{n,l}\mathsf{A}_{l,m}$$

$$V_{n,m} = \langle D_n(r)|\hat{V}_{\text{ext.}}|D_m(r)\rangle$$

Gauge Transformation

$$\Psi \rightarrow U(\mathbf{r})\Psi$$

$$A \rightarrow U(\mathbf{r})AU^\dagger(\mathbf{r}) - i\hbar\left(\nabla U(\mathbf{r})\right)U^\dagger(\mathbf{r})$$

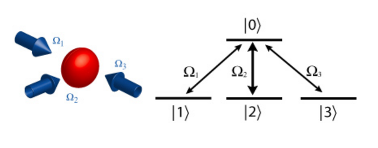
$$\Phi \rightarrow U(\mathbf{r})\Phi U^\dagger(\mathbf{r})$$

Gauge Potential

Center of Mass Wave Function Ψ

$$i\hbar\frac{\partial}{\partial t}\Psi = \left[\frac{1}{2m}(\mathbf{p}-\mathrm{A})^2 + \Phi + V\right]\Psi, \qquad \Psi = (\Psi_1, \Psi_2)^{\mathrm{T}}$$

Additional Potentials



constant non-Abelian

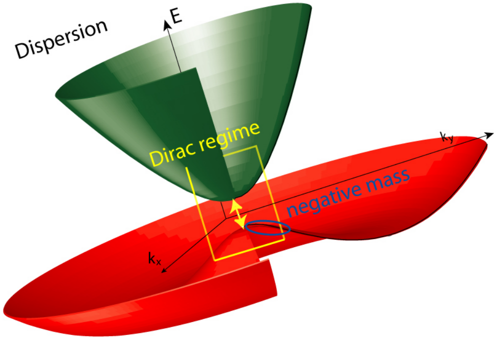
- Using Pauli Matrices

$$\mathrm{A} = \hbar\kappa(\mathbf{e}_x\sigma_x + \mathbf{e}_y\sigma_y)$$

- $\Phi_1 \neq \Phi_2$
- $B = \hbar^2\kappa\sigma_z\mathbf{e}_z$

Dispersion Relation

- two eigenstates $\Psi_{\pm}$
- non-trivial dispersion relation
- Dirac regime, negative mass regime
- negative reflection



Non-Abelian SU(2) Effects in Cold Atoms

2D: modified Schrödinger

- non-Abelian Landau Levels
- Veselago lenses
- non specular reflexion
- *Bogoliubov excitation spectrum* [g]

Spin Orbit Coupling

- Spintronics

1D: Dirac

- Dirac regime
- Zitterbewegung
- *Solitons* [g]
- *Kleintunneling*

Ring Geometry [g]

- Internal Josephson effect
- Spacial oscillations
- persistent currents

together with
Gediminas Juzeliūnas, Frank Zimmer, Luis Santos, Andreas Jakob

Interactions

only s-wave scattering

$$U_{interaction}(r - r') = g\,\delta(r - r')$$

Gross-Pitaevskii eq.

$$H = \left[\frac{1}{2m}(\mathbf{p} - \mathrm{A})^2 + \Phi + V\right] + g(|\Psi_1|^2 + |\Psi_2|^2)$$

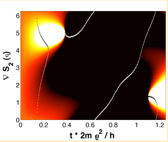
phenomenological

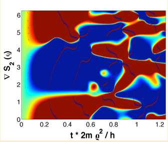
$$g = \frac{4\pi\hbar^2}{m}a$$

a: Scattering length

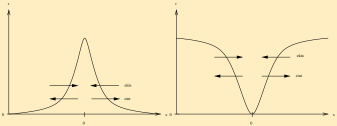
Interaction influence

disturbing: currents

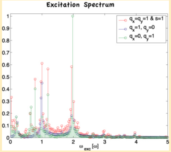


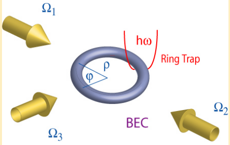


useful: soliton



neutral: eigenstates, excitations...

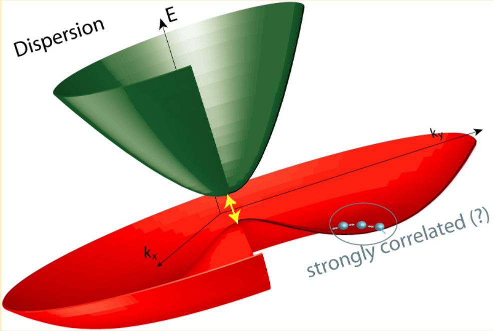




Beyond: QGP (?)

Strongly correlated regime

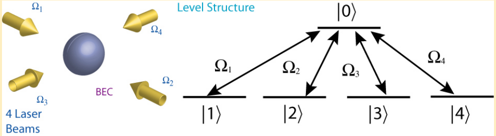
- $E = 0$
- momentum degeneration
- $|\hbar k| = \hbar k_0$
- $g = 0$: vortex GS
- g dominating ?



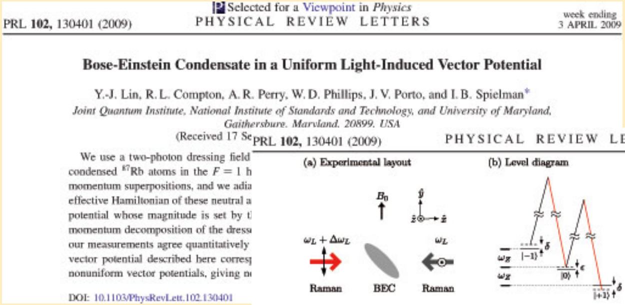
Beyond: QCD

QCD: SU(3) Gauge fields

- non relativistic?
- external A?
- confinement?
- asymptotic freedom?



Experimentally



Selected for a Viewpoint in *Physics*

PRL **102,** 130401 (2009) PHYSICAL REVIEW LETTERS week ending 3 APRIL 2009

Bose-Einstein Condensate in a Uniform Light-Induced Vector Potential

Y.-J. Lin, R. L. Compton, A. R. Perry, W. D. Phillips, J. V. Porto, and I. B. Spielman*

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(Received 17 Se

We use a two-photon dressing field
condensed [87]Rb atoms in the $F = 1$ h
momentum superpositions, and we adia
effective Hamiltonian of these neutral a
potential whose magnitude is set by t
momentum decomposition of the dress
our measurements agree quantitatively
vector potential described here corresp
nonuniform vector potentials, giving n

DOI: 10.1103/PhysRevLett.102.130401

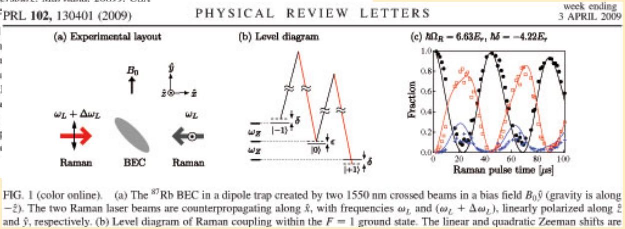

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FIG. 1 (color online). (a) The [87]Rb BEC in a dipole trap created by two 1550 nm crossed beams in a bias field $B_0\hat{y}$ (gravity is along $-\hat{z}$). The two Raman laser beams are counterpropagating along $\hat{x}$, with frequencies ω_L and $(\omega_L + \Delta\omega_L)$, linearly polarized along $\hat{z}$ and $\hat{y}$, respectively. (b) Level diagram of Raman coupling within the $F = 1$ ground state. The linear and quadratic Zeeman shifts are

Non Abelian Gauge Fields in Cold Atoms

Optically Induced SU(2)

- Berry phase with degenerated dark state sub manifold
- Bosons and Fermions
- Wide range of effects
- Non-trivial Dispersion
- Connection to QGP / QCD

References:

- M. Merkl et al. EPL 83 540002 (2008)
- A . Jacob et al. NJP 10 045022 (2008)
- J. Ruseckas et al. PRL 95, 010404 (2005)

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