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Color superconductivity in dense quark matter

For a review, see

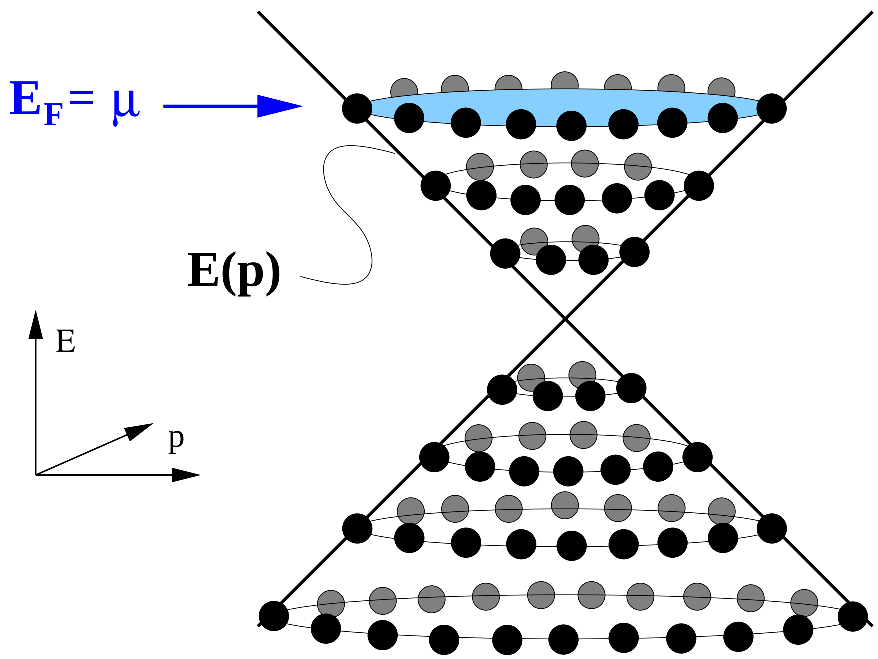
M. Alford, K. Rajagopal, T. Schäfer, A. Schmitt, Rev. Mod. Phys. 80, 1455 (2008)

- **Basics of (color) superconductivity**
- **Color-flavor locking (CFL) – highest densities**
- **Stressed pairing – below CFL densities**
- **Transport properties – quark matter in compact stars**

- **Part I**

- **Basics of (color) superconductivity**
- Color-flavor locking (CFL) – highest densities
- Stressed pairing – below CFL densities
- Transport properties – quark matter in compact stars

• Basics of (color) superconductivity (page 1/5)



free energy $\Omega = E - \mu N$

- no interactions: add fermion at $E = \mu$ without cost
- attractive interaction: add pair with gain
- pairs condense $\rightarrow$ superconductivity

This Bardeen-Cooper-Schrieffer (BCS) argument holds for electrons in metal, ^{3}He atoms, . . .,
and quarks in quark matter

- **Basics of (color) superconductivity (page 2/5)**
  Electromagnetic vs. color superconductor

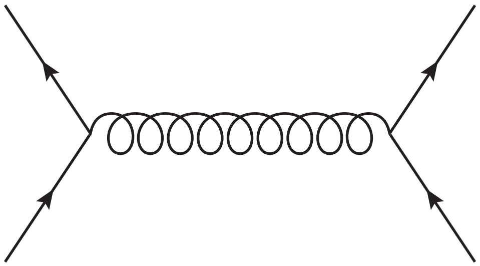
	Where?	**What?**	**Attractive force**	**Cooper pairs**	**Meissner effect**
"usual" superconductor	metals, alloys	ion lattice & electrons	phonons	electrons	photon
color superconductor	**neutron stars**	**quarks & gluons**	**gluons**	**quarks**	**gluons (and photon)**$^{(*)}$

$^{(*)}$ Most color superconductors are not electromagnetic superconductors ("rotated electromagnetism")

Exception: Spin-1 color superconductors
A. Schmitt, Q. Wang, D. H. Rischke, PRL 91, 242301 (2003)

• Basics of (color) superconductivity (page 3/5): attractive quark-quark interaction

- one-gluon exchange



attractive in antisymmetric antitriplet channel $[\bar{\mathbf{3}}]^a_c$

$$SU(3)_c : \qquad [\mathbf{3}]_c \otimes [\mathbf{3}]_c = [\bar{\mathbf{3}}]^a_c \oplus [\mathbf{6}]^s_c$$

- flavor space

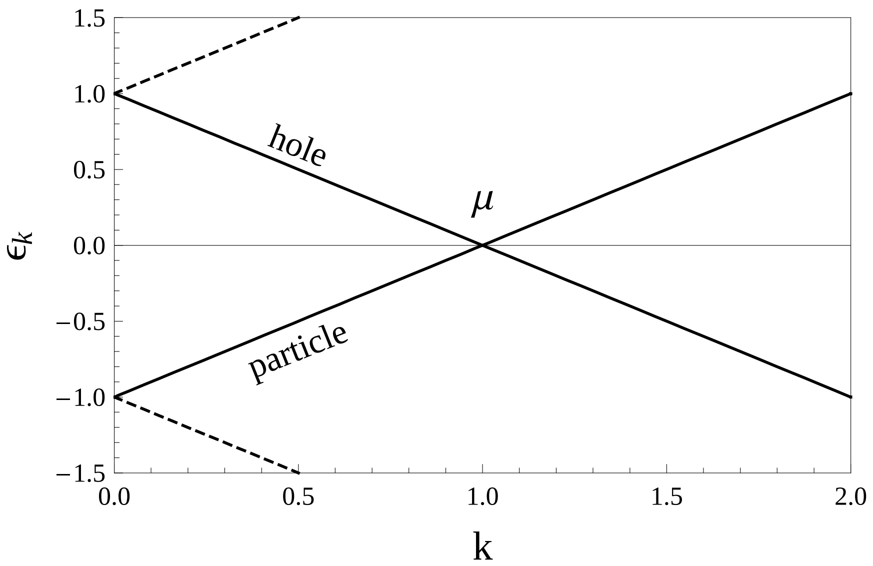
$$SU(3)_f : \qquad [\mathbf{3}]_f \otimes [\mathbf{3}]_f = [\bar{\mathbf{3}}]^a_f \oplus [\mathbf{6}]^s_f$$

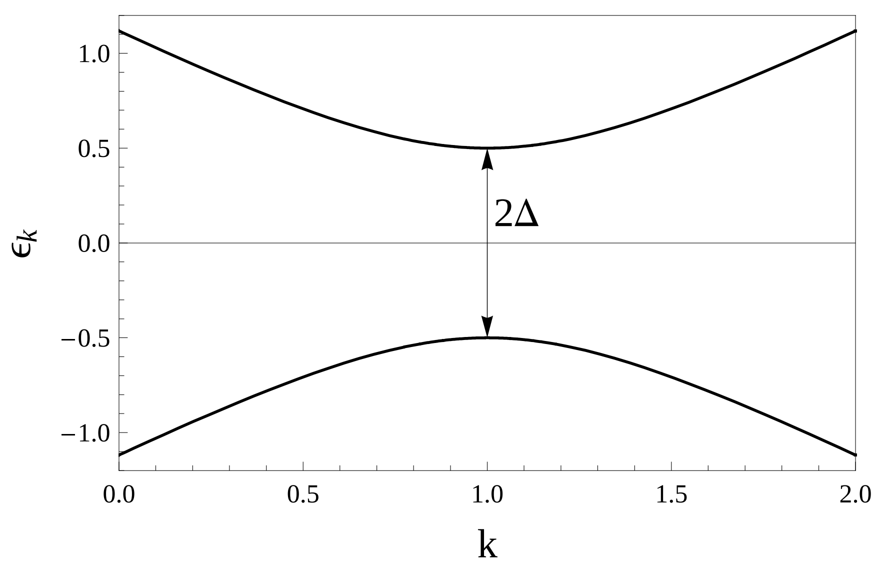
- order parameter (for spin-0 pairing):

$$\langle \psi_i^\alpha C\gamma_5 \psi_j^\beta \rangle \; \propto \; \epsilon^{\alpha\beta A}\, \epsilon_{ijB}\, \phi^A_B \; \in \; [\bar{\mathbf{3}}]^a_c \otimes [\bar{\mathbf{3}}]^a_f$$

• Basics of (color) superconductivity (page 4/5): energy gap

Fermion dispersions



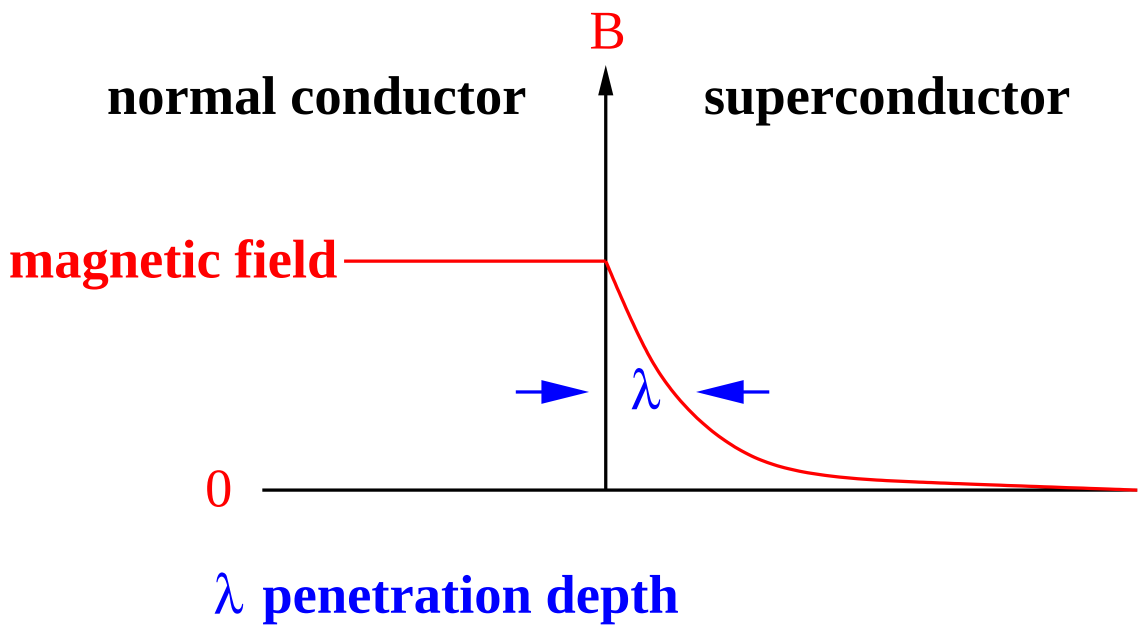


$$\epsilon_k = \pm(k - \mu) \qquad\qquad \epsilon_k = \pm\sqrt{(k-\mu)^2 + \Delta^2}$$

$\longrightarrow$ suppression of specific heat, viscosity, neutrino emissivity, etc.

Critical temperature (in BCS theory): $T_c \simeq 0.57\Delta$

- **Basics of (color) superconductivity (page 5/5): Meissner effect**

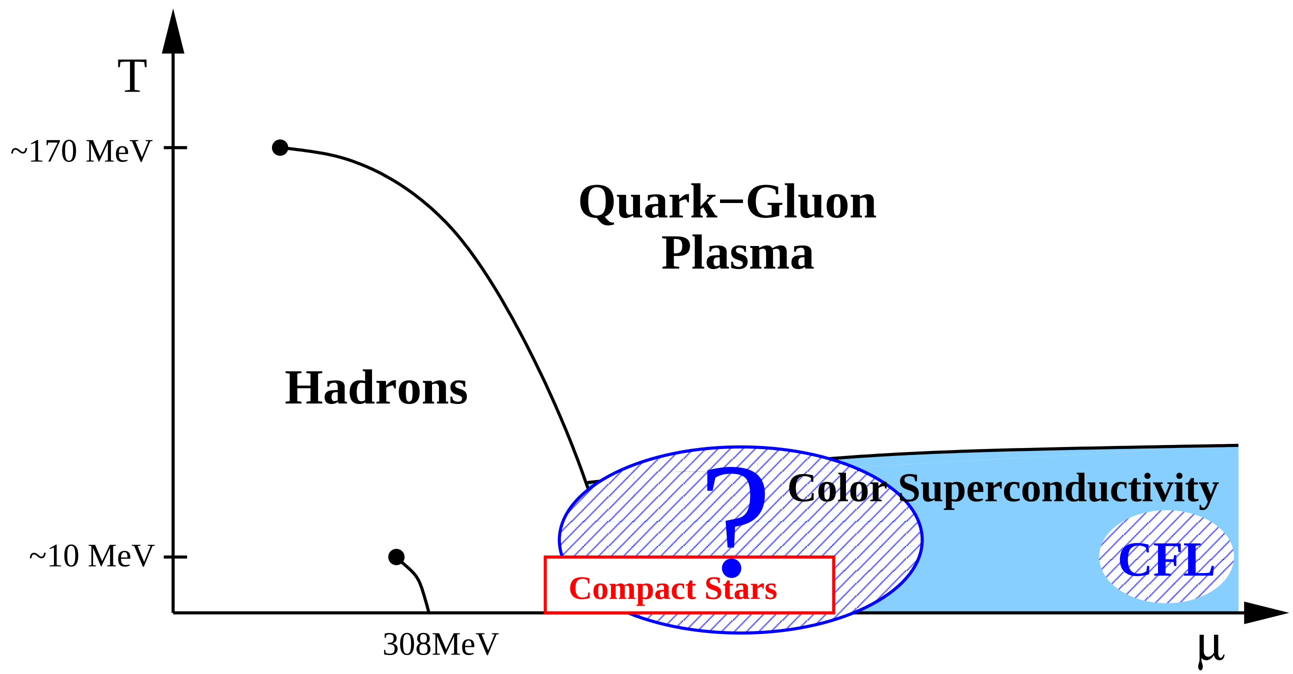


- spontaneous symmetry breaking: $U(1)_{\mathrm{em}} \to \mathbb{Z}_2$
- Anderson-Higgs mechanism: photon Meissner mass m_M

$$m_M = \frac{1}{\lambda}\,, \qquad B \propto e^{-m_M r}$$

- gluon Meissner masses in color superconductors

• QCD phase diagram (page 1/3): Known and unknown territories



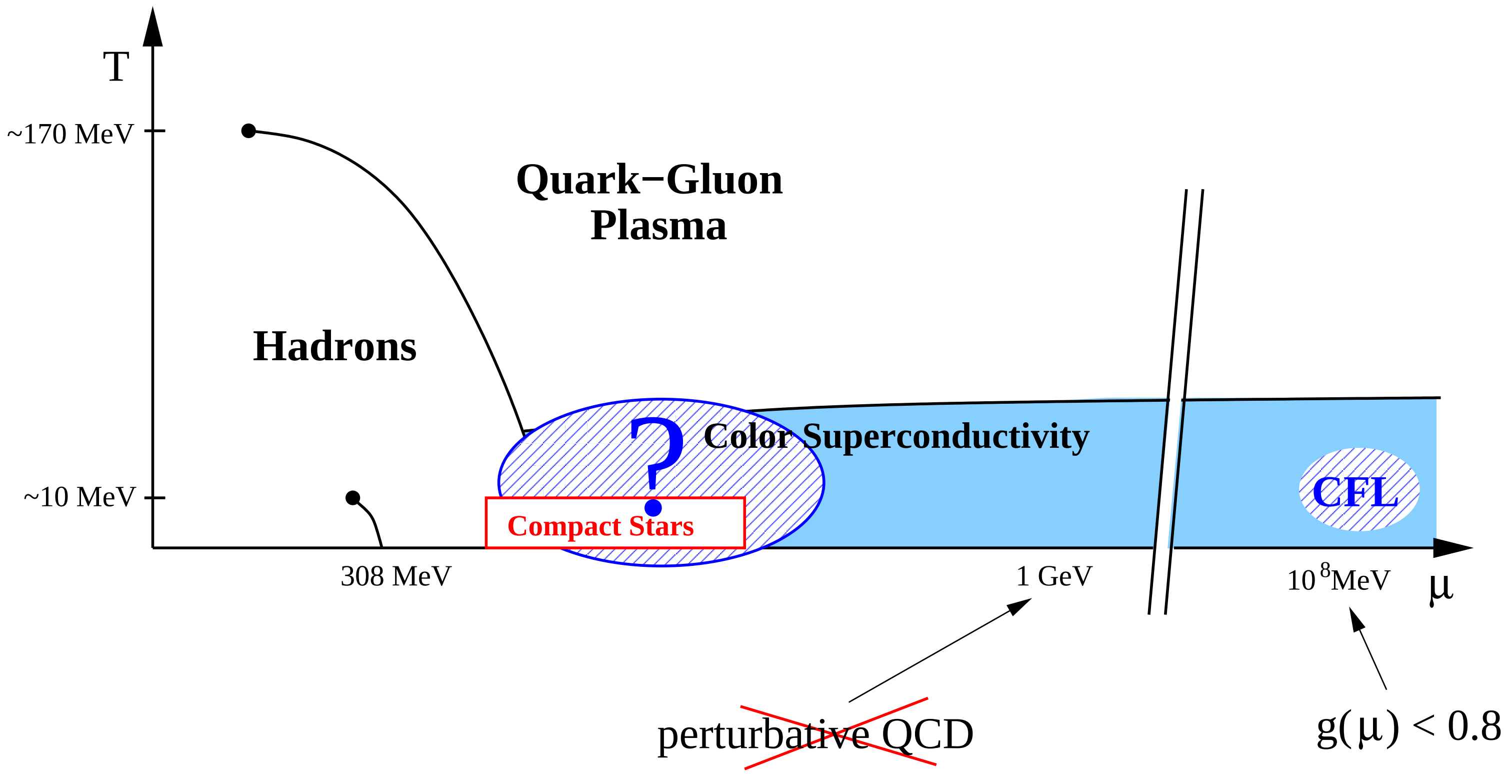
High densities:

- rigorous theoretical control
- no nonperturbative gaps in our understanding

Moderate densities:

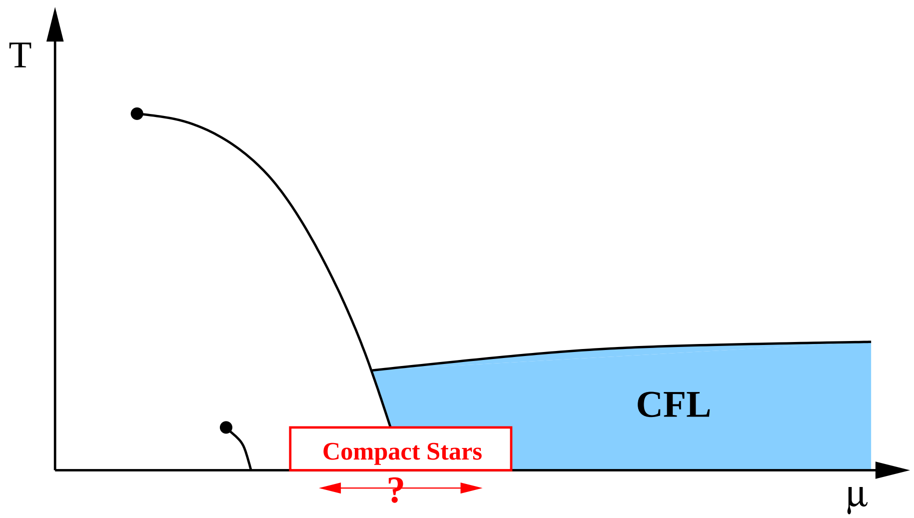
- perturbative QCD not valid
- strange mass & neutrality: stress on Cooper pairing

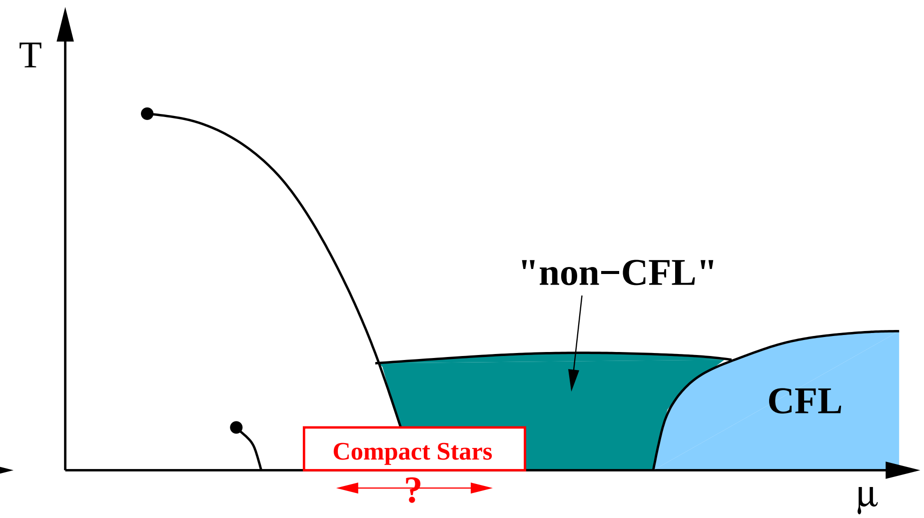
- **QCD phase diagram (page 2/3): Validity of perturbative QCD**



- here: always $N_f = 3$ (ignore heavy quarks)
- $N_f > 3$: T. Schäfer, NPB 575, 269 (2000)

- **QCD phase diagram (page 3/3): 2 Possible scenarios**





CFL superseded by nuclear matter:

- effective theory for CFL in strongly-coupled regime
- CFL matter in the core of compact stars?

CFL superseded by “non-CFL” matter:

- complicated phase structure?
- rely on Nambu-Jona-Lasinio-type models

Question:

What is the ground state of deconfined quark matter at moderate densities (in the interior of compact stars)?

1. **Theoretical approach: start from CFL and ask "what is next phase down in density?" (if not hadronic matter)**
2. **Phenomenological approach: "guess" possible phase, compute its properties and compare with astrophysical observations**
3. **Tabletop approach: learn from parallels to cold fermionic atoms in optical trap**

- **Part II**

  - Basics of (color) superconductivity
  - **Color-flavor locking (CFL) – highest densities**
  - Stressed pairing – below CFL densities
  - Transport properties – quark matter in compact stars

- **On safe grounds: Asymptotically large density**

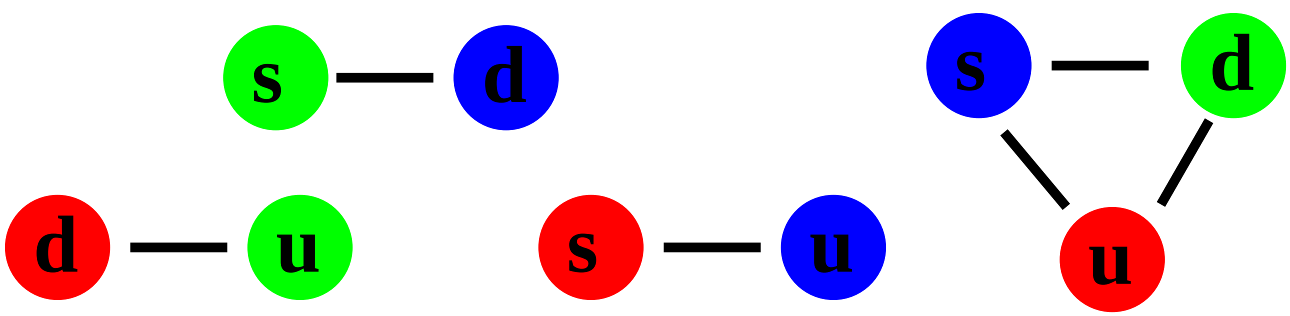
$$0 \simeq m_s \simeq m_u \simeq m_d \ll \mu \qquad \text{all quark masses negligible}$$

"color-flavor locked phase (CFL)"

M. Alford, K. Rajagopal, F. Wilczek, NPB 537, 443 (1999)

$$\phi^A_B = \delta^A_B \quad \Rightarrow \quad \langle \psi^\alpha_i C \gamma_5 \psi^\beta_j \rangle \; \propto \; \epsilon^{\alpha\beta A} \, \epsilon_{ijA}$$

$$\Rightarrow \quad SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \rightarrow \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}} \times \mathbb{Z}_2$$



• Properties of CFL (page 1/3)

(1) chiral symmetry breaking

- usual chiral symmetry breaking: LR pairing $\langle \bar{\psi}_R \psi_L \rangle$
- here: LL, RR pairing $\langle \psi_R \psi_R \rangle$, however

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \rightarrow \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}}$$

- chiral symmetry broken through "locking" to color
- octet of pseudo-Goldstone modes K^0, $K^{\pm}$, π^0, ...
- quark-hadron continuity? T. Schäfer, F. Wilczek, PRL 82, 3956 (1999)

• Properties of CFL (page 2/3)

(2) superfluidity

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}}$$

- *exactly massless* Goldstone mode ϕ
- vortices in rotating CFL M. M. Forbes, A. R. Zhitnitsky, PRD 65, 085009 (2002)

(3) rotated electromagnetism

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}}$$

- Cooper pairs neutral under $\tilde{Q} = Q + \frac{2}{\sqrt{3}}T_8$
- photon-gluon mixing with (small) mixing angle

$$\cos^2\theta = \frac{3g^2}{3g^2 + 4e^2} \simeq 1$$

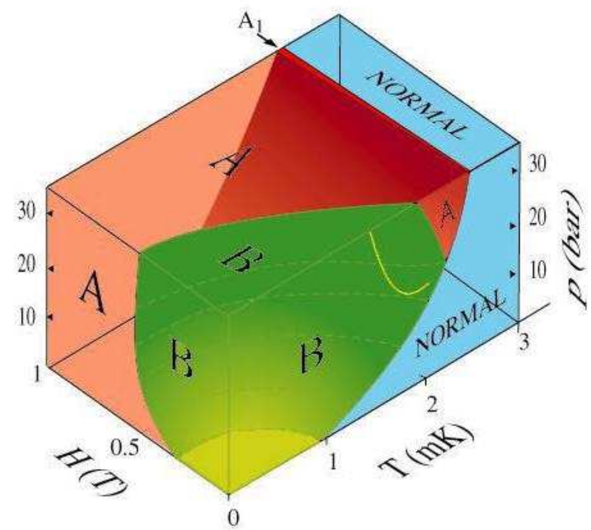
- **Why CFL is favored (page 1/2)**
  - general order parameter

$$\langle \psi_i^\alpha C \gamma_5 \psi_j^\beta \rangle \;\propto\; \epsilon^{\alpha\beta A}\, \epsilon_{ijB}\, \phi_B^A$$

  - complex 3×3 matrix $\phi_B^A \rightarrow$ *huge configuration space?!*
    - cf. **superfluid ^{3}He**

$$SO(3)_L \times SU(2)_S \times U(1)$$

    - 3×3 order parameter in angular momentum L, spin S
    - **A phase**: $\phi_B^A = \delta^{A3}(\delta_{B1} + i\delta_{B2})$
    - **B phase**: $\phi_B^A = \delta_B^A$

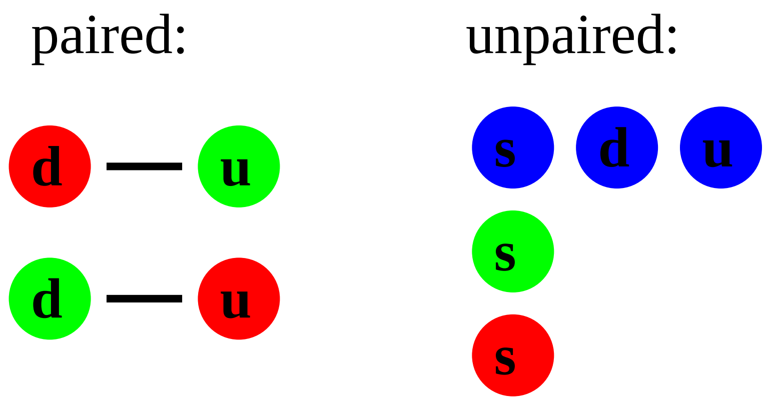


- **Why CFL is favored (page 2/2)**

  - *simple at high densities!* $\longrightarrow$ full $SU(3)_c \times SU(3)_f$ symmetry
  - can restrict to diagonal ϕ^A_B

$$\forall \phi \;\; \exists U \in SU(3)_c \, , \; V \in SU(3)_f : \quad U^T \phi V \text{ diagonal}$$

  - then $\phi^A_B = \delta^A_B$ is only phase where **all quarks pair**
  - for instance **2SC phase** $\phi^A_B = \delta^{A3}\delta_{B3}$

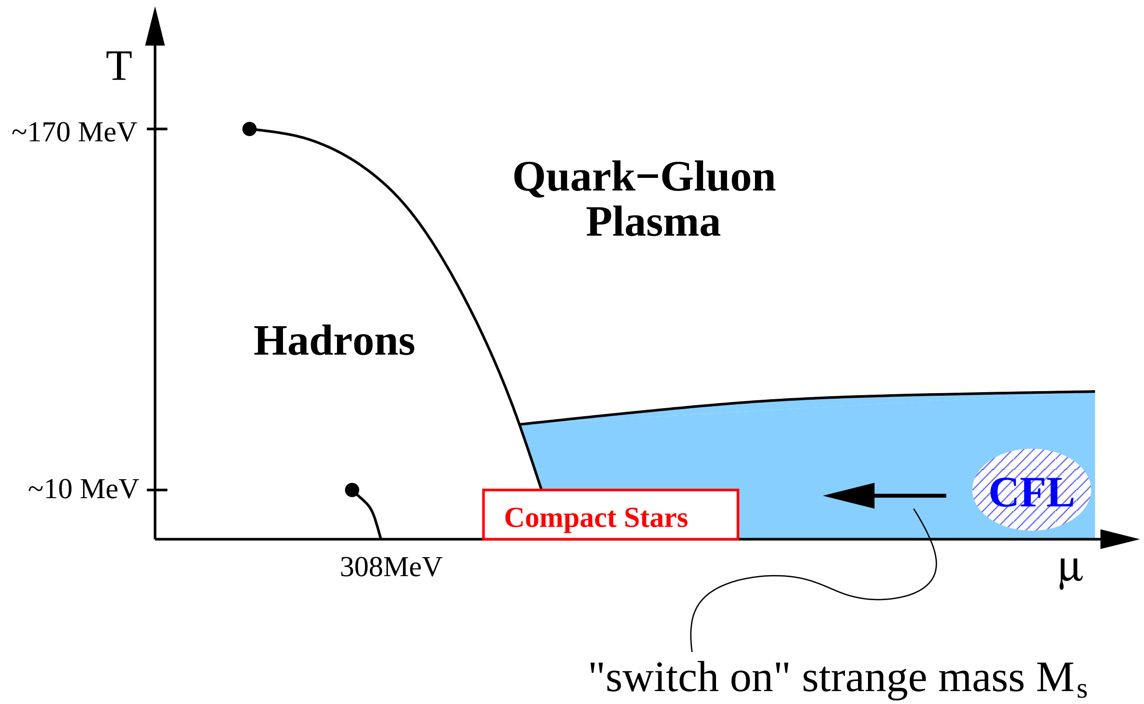


- **Part III**

- Basics of (color) superconductivity
- Color-flavor locking (CFL) – highest densities
- **Stressed pairing – below CFL densities**
- Transport properties – quark matter in compact stars

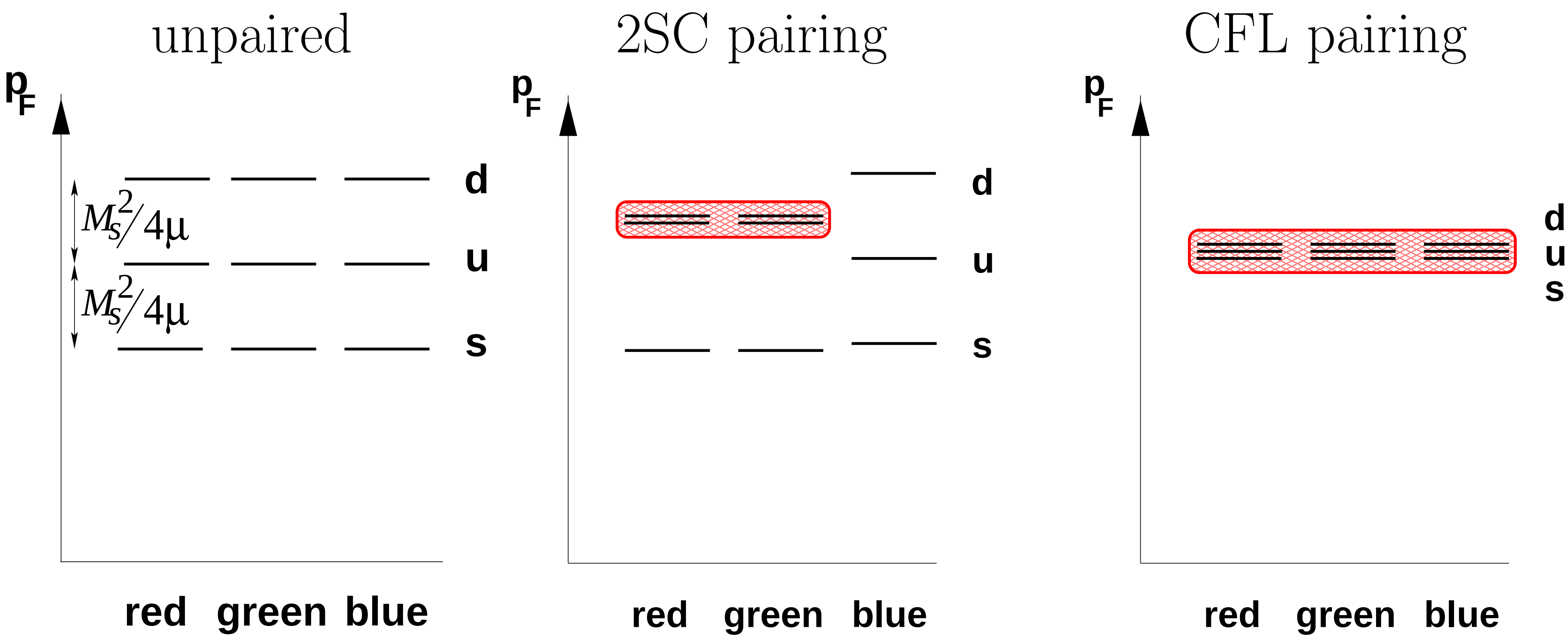
- **Going down in density: Large, but not asymptotically large, densities**

- strange mass $M_s \simeq 120\,\mathrm{MeV}$ no longer $\ll \mu \simeq 400\,\mathrm{MeV}$



- at given chemical potentials μ, μ_e: $M_s \neq 0$ reduces p_F^s
- electric neutrality: increase in p_F^d to compensate
- **Fermi momenta "try" to split apart**

- p_F**'s splitting apart**

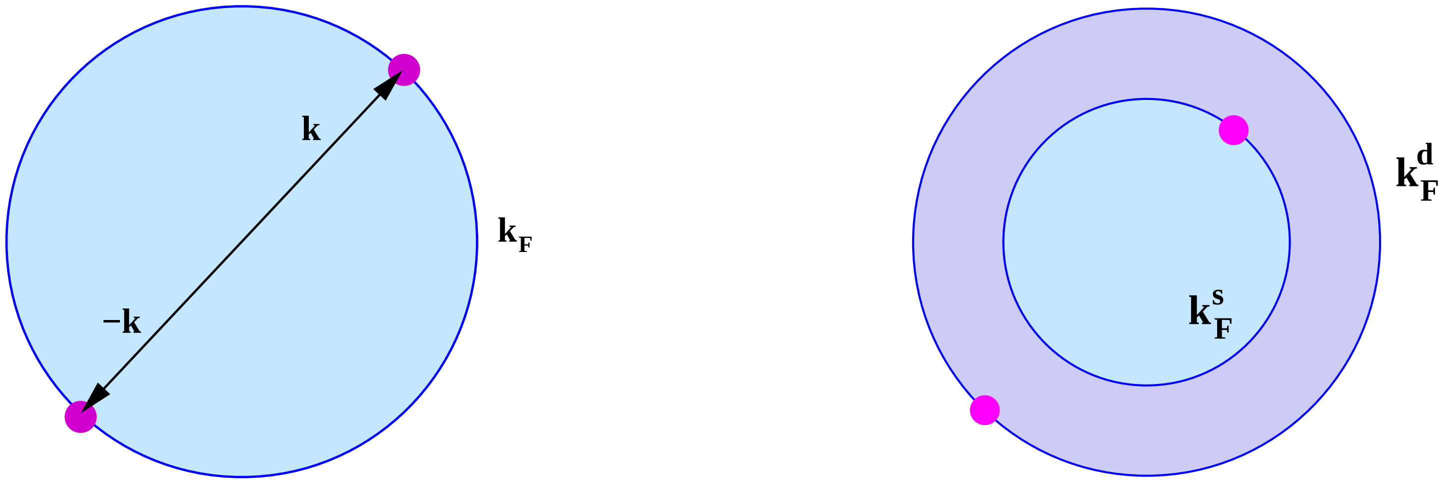


- any pairing pattern most "comfortable" with M_s and neutrality?
- stressed pairing is **unavoidable**!

K. Rajagopal, A. Schmitt, PRD 73, 045003 (2006)

• Stressed Cooper pairing: a general phenomenon (page 1/2)

BCS-pairing: split Fermi momenta:



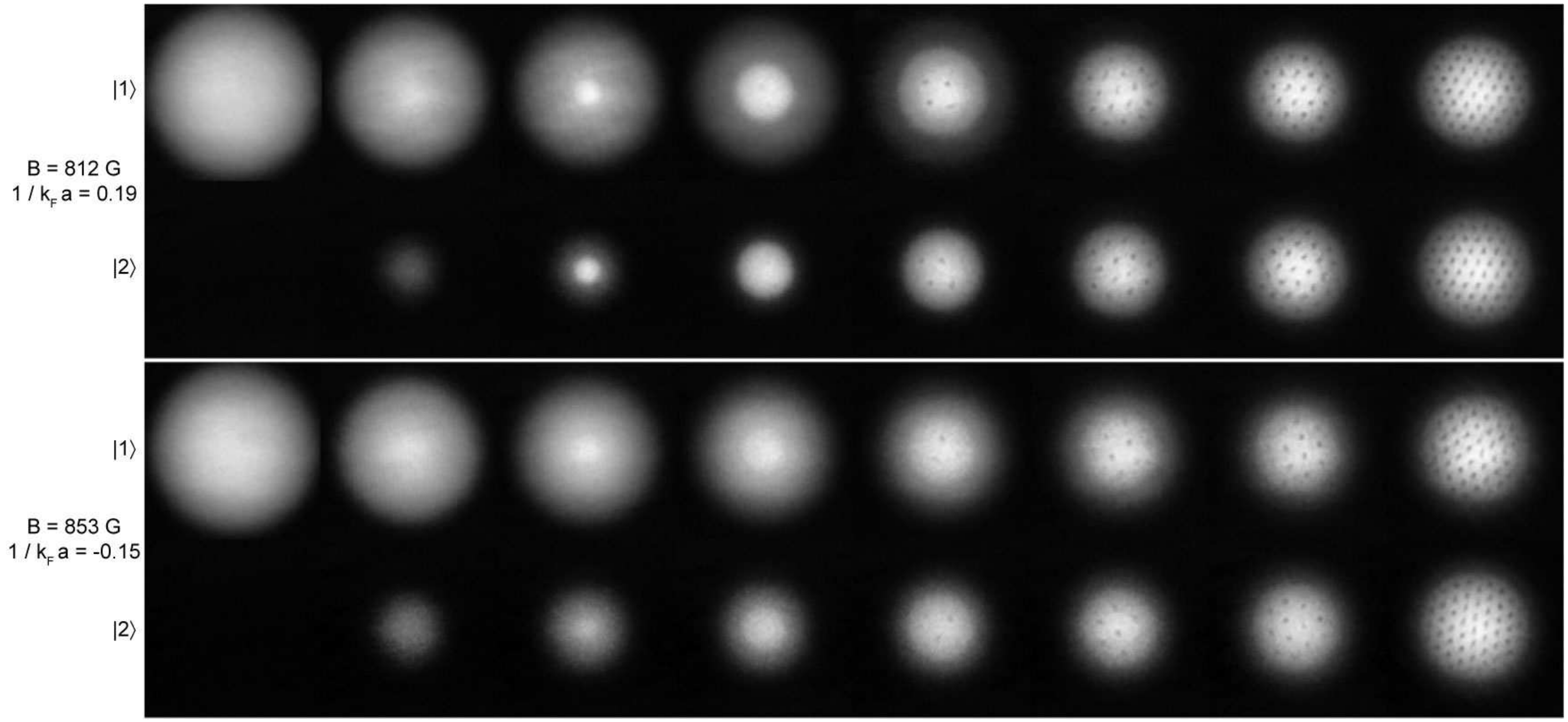
- for instance: two species (spin states) of cold fermionic atoms
  - Experiments:

    Y. Chin, M.W. Zwierlein, C.H. Schunck, A. Schirotzek, W. Ketterle, PRL 97, 030401 (2006)

    G.B. Partridge, W. Li, R.I. Kamar, Y. Liao, R.G. Hulet, Science 311, 503 (2006)
  - Theory (review):

    D.E. Sheehy, L. Radzihovsky, Ann. Phys. 322, 1790 (2007)

- **Stressed Cooper pairing: a general phenomenon (page 2/2)**

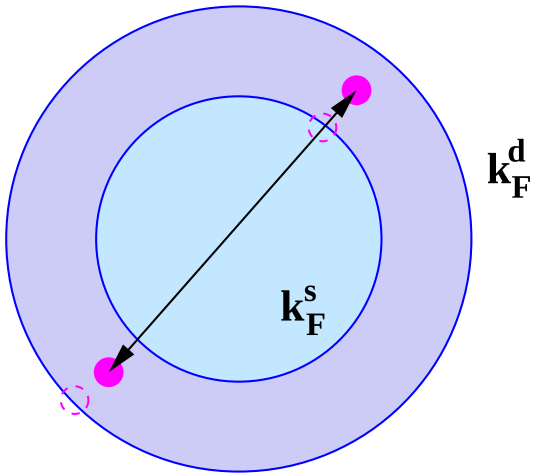


M.W. Zwierlein, A. Schirotzek, C.H. Schunck, W. Ketterle, Science 311, 492 (2006)

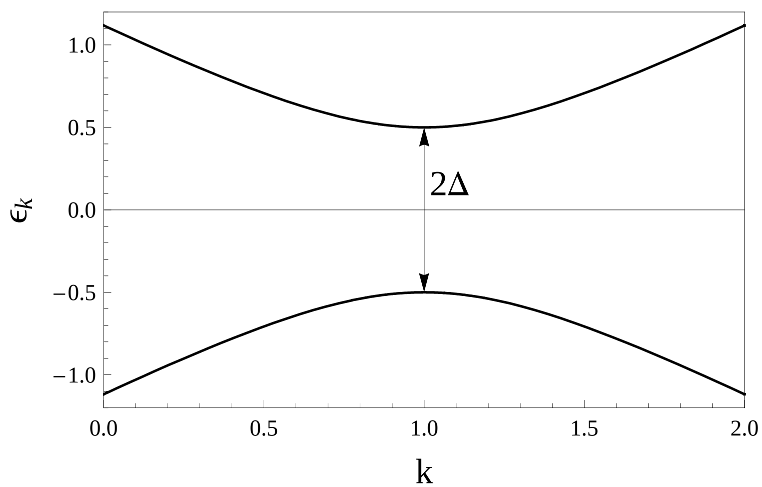
- **phase separation** of superfluid and normal components
- phase separation unlikely in quark matter (local color neutrality!)

M. Alford, C. Kouvaris, K. Rajagopal, PRL 92, 222001 (2004)

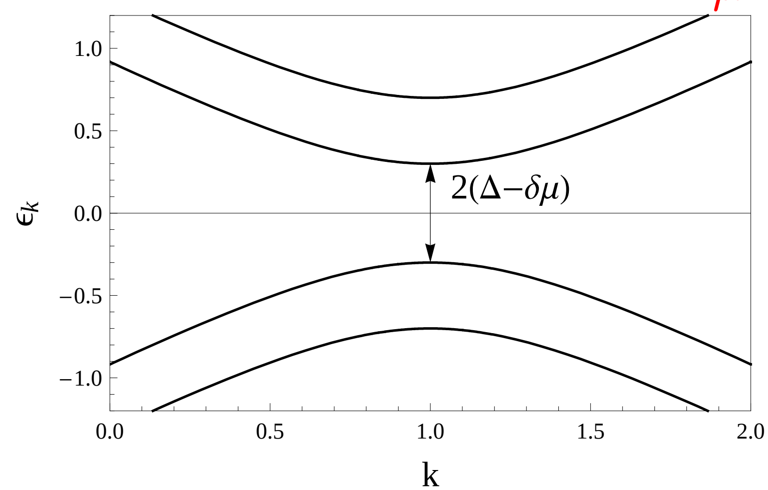
• CFL pairing with small stress



- create common Fermi surface:
  cost in free energy
  $\sim \delta p_F^2\,\mu^2 \sim M_s^4$
- form pairs:
  gain in free energy $\sim \Delta^2\mu^2$
- CFL survives for $\Delta \gtrsim \dfrac{M_s^2}{\mu}$



$$\epsilon_k = \pm\sqrt{(k-\mu)^2+\Delta^2}$$

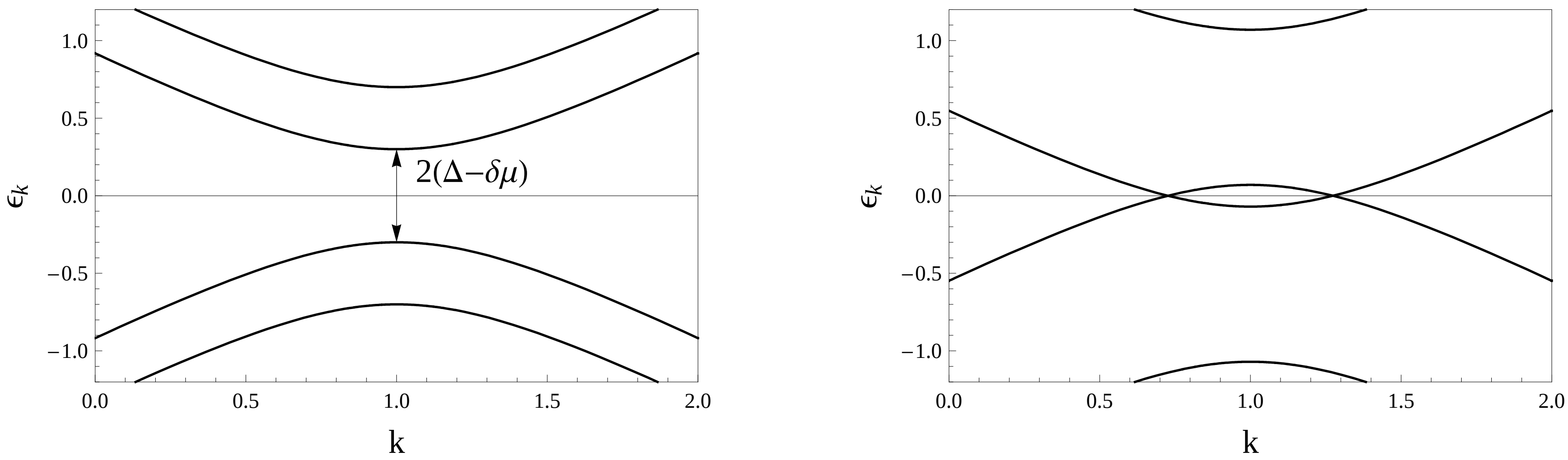


$$\epsilon_k = \pm\left(\sqrt{(k-\bar{\mu})^2+\Delta^2}\pm\delta\mu\right)$$

• Large stress: gapless CFL?

$\longrightarrow$ "gapless CFL" for $\delta\mu > \Delta$

M. Alford, C. Kouvaris, K. Rajagopal, PRL 92, 222001 (2004)



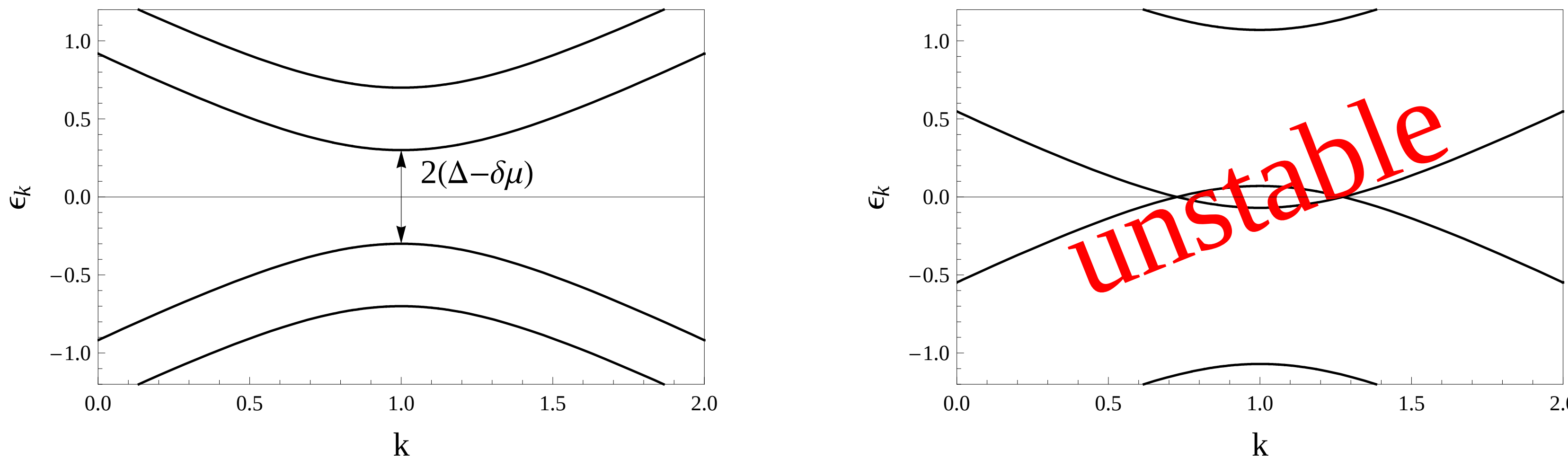
$\longrightarrow$ or: "breached" pairing

E. Gubankova, W. V. Liu and F. Wilczek, PRL 91, 032001 (2003)

- **Large stress: gapless CFL? No!**

$\longrightarrow$ "gapless CFL" for $\delta\mu > \Delta$

M. Alford, C. Kouvaris, K. Rajagopal, PRL 92, 222001 (2004)



$\longrightarrow$ or: "breached" pairing

E. Gubankova, W. V. Liu and F. Wilczek, PRL 91, 032001 (2003)

$\longrightarrow$ **chromomagnetic instability** (at $T = 0$)

M. Huang, I. A. Shovkovy, PRD 70, 051501 (2004)

R. Casalbuoni, R. Gatto, M. Mannarelli, G. Nardulli, M. Ruggieri, PLB 605, 362 (2005)

$\longrightarrow$ ground state must be different $\longrightarrow$ **currents**

• Less (and less symmetric) pairing (page 1/4)

Kaon condensation "CFL-K^0"

P. F. Bedaque and T. Schäfer, NPA 697, 802 (2002)

- chiral field

$$\Sigma = \phi_L^\dagger \phi_R$$

- pure CFL: $\Sigma = \mathbf{1}$
- kaon condensation $\Rightarrow \Sigma = e^{i\varphi T_6}$ (relative L/R rotation)

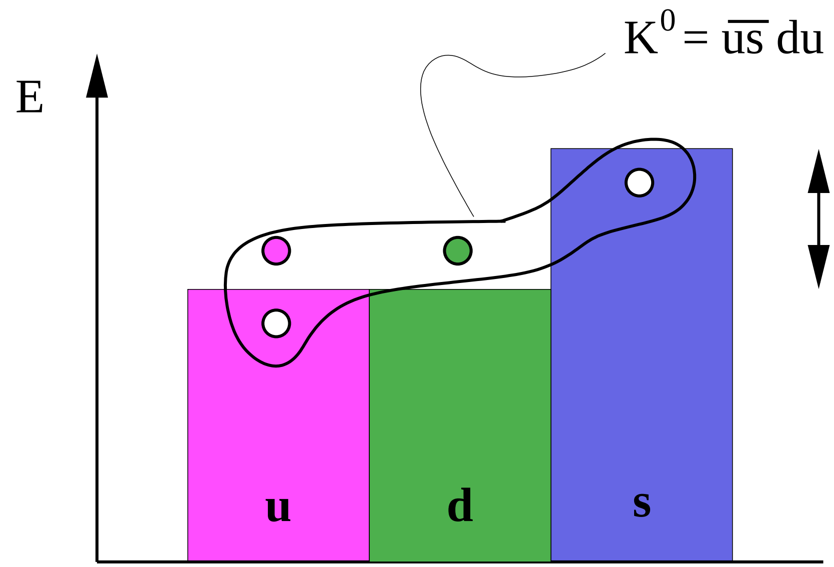
- in other words:

  create kaon with mass

  $m^2_{K^0} = a\, m_d(m_s + m_u) \ll \Delta^2$

  from $0 \rightarrow \bar{s} + u + \bar{u} + d$

  $(a \sim \Delta^2/\mu^2)$



- **Less (and less symmetric) pairing (page 2/4)**

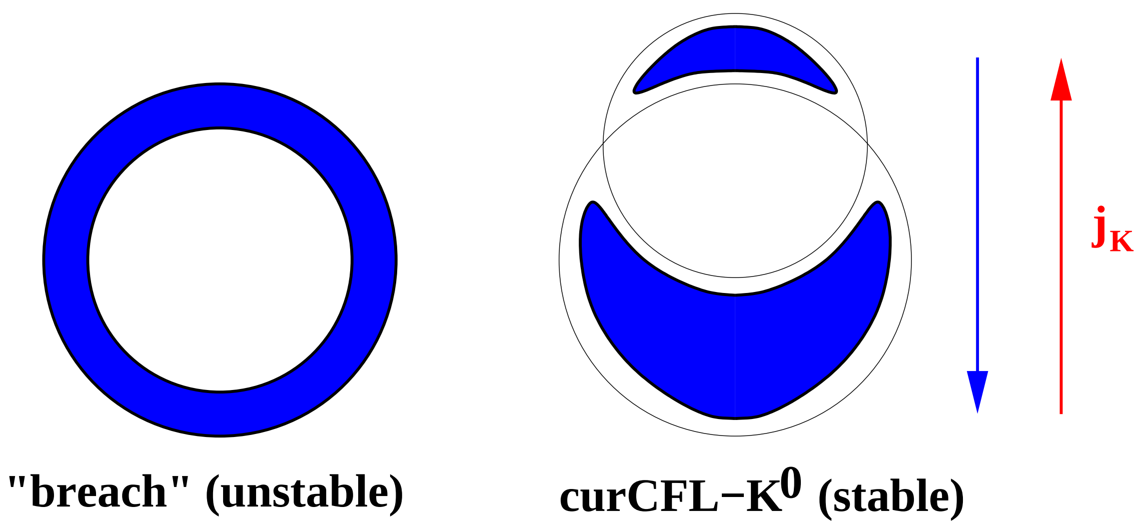
(Super)currents in CFL: "curCFL-K^0"

T. Schäfer, PRL 96, 012305 (2006); A. Kryjevski, PRD 77, 014018 (2008)

A. Schmitt, NPA 820, 49C (2009)

$$\phi_L(\mathbf{x}) = \Delta\, e^{i\mathbf{j}_K\cdot\mathbf{x}\,T_8}\; e^{i(\varphi/2)T_6}$$
$$\phi_R(\mathbf{x}) = \Delta\, e^{i\mathbf{j}_K\cdot\mathbf{x}\,T_8}\; e^{-i(\varphi/2)T_6}$$

- "anisotropic breach"
- stable and unstable Fermi surface topologies:



• Less (and less symmetric) pairing (page 3/4)

More currents in CFL: crystalline structures (LOFF)

M. Alford, J. Bowers, K. Rajagopal, PRD 63, 074016 (2001)

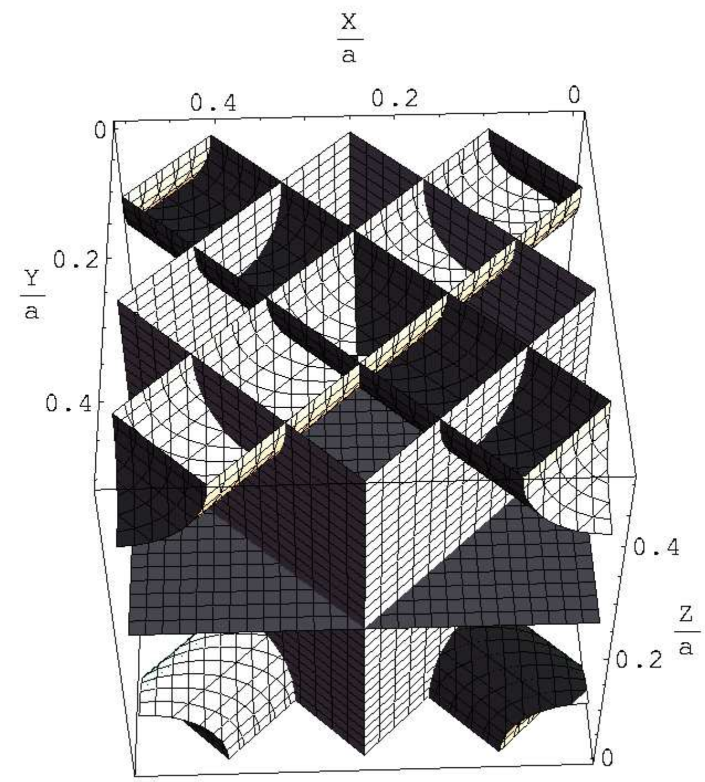
M. Mannarelli, K. Rajagopal and R. Sharma, PRD 73, 114012 (2006)

$$\langle ud\rangle \sim \Delta_3 \sum_a \exp(2i\mathbf{q}_3^a \cdot \mathbf{x})$$

$$\langle us\rangle \sim \Delta_2 \sum_a \exp(2i\mathbf{q}_2^a \cdot \mathbf{x})$$

$$\langle ds\rangle \simeq 0$$

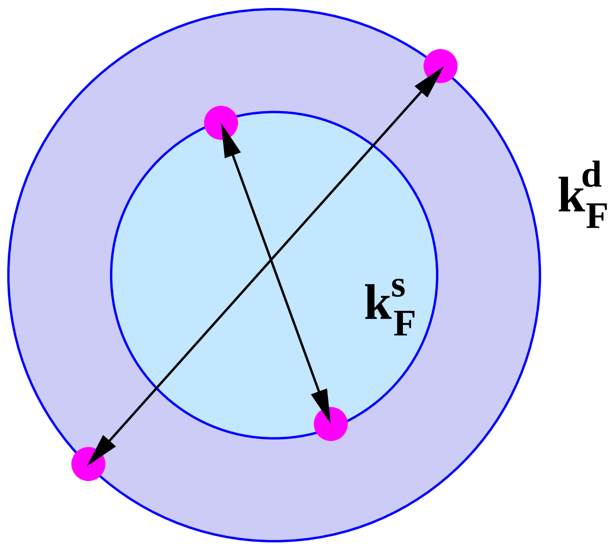
- here: "CubeX"
- $\{\mathbf{q}_3\}$, $\{\mathbf{q}_2\}$ each contain 4 vectors, together pointing to the corners of a cube



$\Delta_3(\mathbf{x}), \Delta_2(\mathbf{x})$

- **Less (and less symmetric) pairing (page 4/4)**

Last resort: single flavor pairing



- need $J = 1$ Cooper pairs

$$\phi \in [\bar{\mathbf{3}}]_c^a \otimes [\mathbf{3}]_J^s$$

- different possible phases:

  Color-spin locking (CSL),

  A-phase, polar phase ...

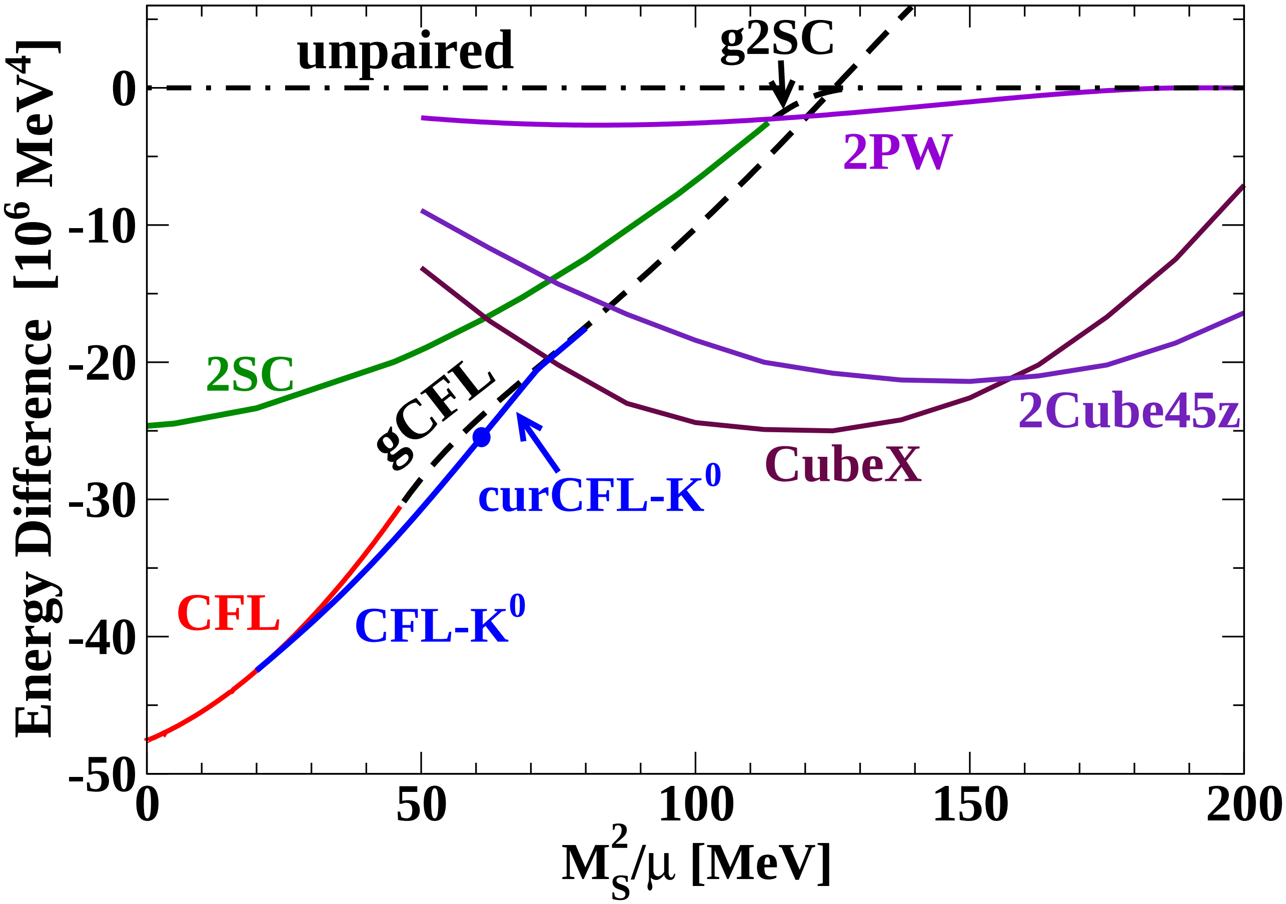
  T. Schäfer, PRD 62, 094007 (2000)

  A. Schmitt, PRD 71, 054016 (2005)

- preferred phase at high densities: CSL
- gap much smaller than in spin-0 phases:

$$\Delta_{J=1} \lesssim 10^{-2} \Delta_{J=0}$$

- **Stressed pairing: free energy comparison**



here: $\Delta_{\mathrm{CFL}} = 25$ MeV

(pert. QCD: $\Delta_{\mathrm{CFL}} \simeq 20$ MeV, NJL: $\Delta_{\mathrm{CFL}} \simeq (20-100)$ MeV).

- **Part VI**

- Basics of (color) superconductivity
- Color-flavor locking (CFL) – highest densities
- Stressed pairing – below CFL densities
- **Transport properties – quark matter in compact stars**

• Color superconductivity in compact stars (I)

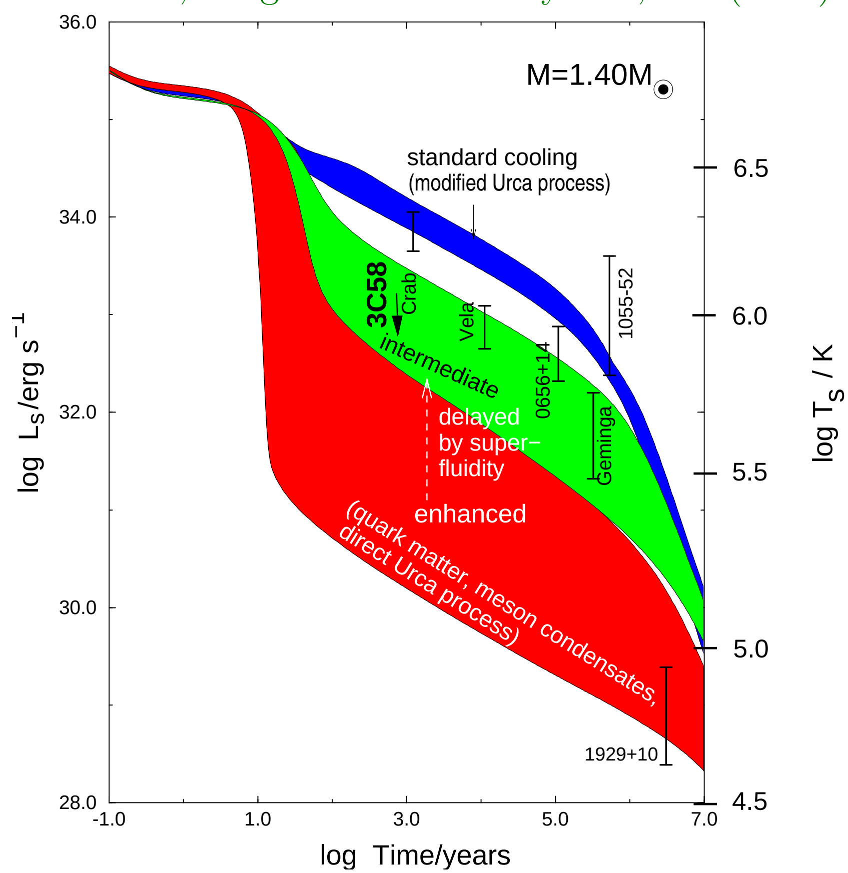
Cooling of the star

- **neutrino emissivity**

  $T \sim 10\,\mathrm{MeV} \to 0.1\,\mathrm{keV}$

  $(t \sim 0 \to 10^6\,\mathrm{yr})$

F. Weber, Prog. Part. Nucl. Phys. 54, 193 (2005)



unpaired quark matter Iwamoto, PRL 44, 1637 (1980) **CFL** Jaikumar, Prakash, Schäfer, PRD 66, 063003 (2002) **2SC** Jaikumar, Roberts, Sedrakian, PRC 73, 042801 (2006) **spin-1** Schmitt, Shovkovy, Wang, PRD 73, 034012 (2006) **LOFF** Anglani, Nardulli, Ruggieri, Mannarelli, PRD 74, 074005 (2006)

- **Neutrino emissivity**

CFL

- (pseudo-)**Goldstone processes**

$$\pi^{\pm}, K^{\pm} \rightarrow e^{\pm} + \bar{\nu}_e$$
$$\pi^0 \rightarrow \nu_e + \bar{\nu}_e$$
$$\varphi + \varphi \rightarrow \varphi + \nu_e + \bar{\nu}_e$$

$\Rightarrow$ very small emissivity

$$\epsilon_\nu \sim \frac{G_F^2 T^{15}}{f^2 \mu^4}$$

Non-CFL

- **direct Urca process**

$$u + e \rightarrow d + \nu_e$$
$$d \rightarrow u + e + \bar{\nu}_e$$

$\Rightarrow$

$$\epsilon_\nu \simeq \frac{457}{630} \alpha_s G_F^2 T^6 \mu_e \mu_u \mu_d$$

(unpaired quark matter)

- non-CFL (2SC, LOFF, spin-1, ...): $\rightarrow$ **ungapped quarks**
- possible exception (?): CSL

• Color superconductivity in compact stars (II)

magnetic fields

- spin-0 **no Meissner effect**

  M.G. Alford, J. Berges, K. Rajagopal,

  NPB 571, 269 (2000)

- spin-1 **Meissner effect**

  A. Schmitt, Q. Wang, D.H. Rischke,
  PRL 91, 242301 (2003)

- "magnetic" CFL

  E.J. Ferrer, V. de la Incera, C. Manuel,

  PRL 95, 152002 (2005)

- de Haas-van Alphen oscillations

  K. Fukushima and H. J. Warringa,
  PRL 100, 032007 (2008)
  J. L. Noronha and I. A. Shovkovy,
  PRD 76, 105030 (2007)

- ferromagnetism in the **curCFL-**K^0 phase

  D. T. Son and M. A. Stephanov, PRD 77, 014021 (2008)

• Color superconductivity in compact stars (III)

glitches

- **need vortex pinning at some crystal**
- maybe: neutron superfluidity + ion lattice in crust
- however: inconsistent with precession of star on $\sim$ yr time scale

  B. Link, arXiv:astro-ph/0608319

- better (?): crystalline color superconductivity
- **large shear modulus of CubeX** (and 2Cube45z)

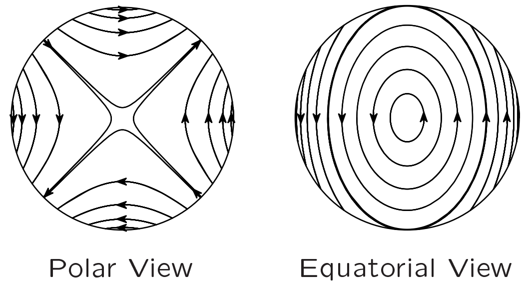
  M. Mannarelli, K. Rajagopal, R. Sharma, PRD 76, 074026 (2007)

$$\nu = 3.96 \times 10^{33} \text{erg/cm}^3 \left(\frac{\Delta}{10 \text{ MeV}}\right)^2 \left(\frac{\mu}{400 \text{ MeV}}\right)^2$$

- 20 – 100 times more rigid than neutron star crust

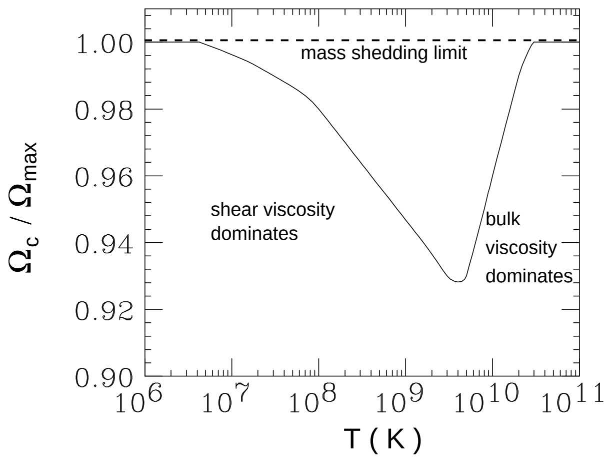
• Color superconductivity in compact stars (IV)

r-mode instability



L. Lindblom, arXiv:astro-ph/0101136

- **r-modes:**
  non-radial pulsation modes
- **grow unstable**
  in a perfect-fluid rotating star
  $\longrightarrow$ emission of gravitational waves
- **spin down**
  the star drastically and quickly (within days)
- fast rotating stars are observed!
  $\omega \simeq 1\text{ms}^{-1}$
- must be some damping mechanism $\longrightarrow$ bulk/shear viscosity



• What is bulk viscosity?

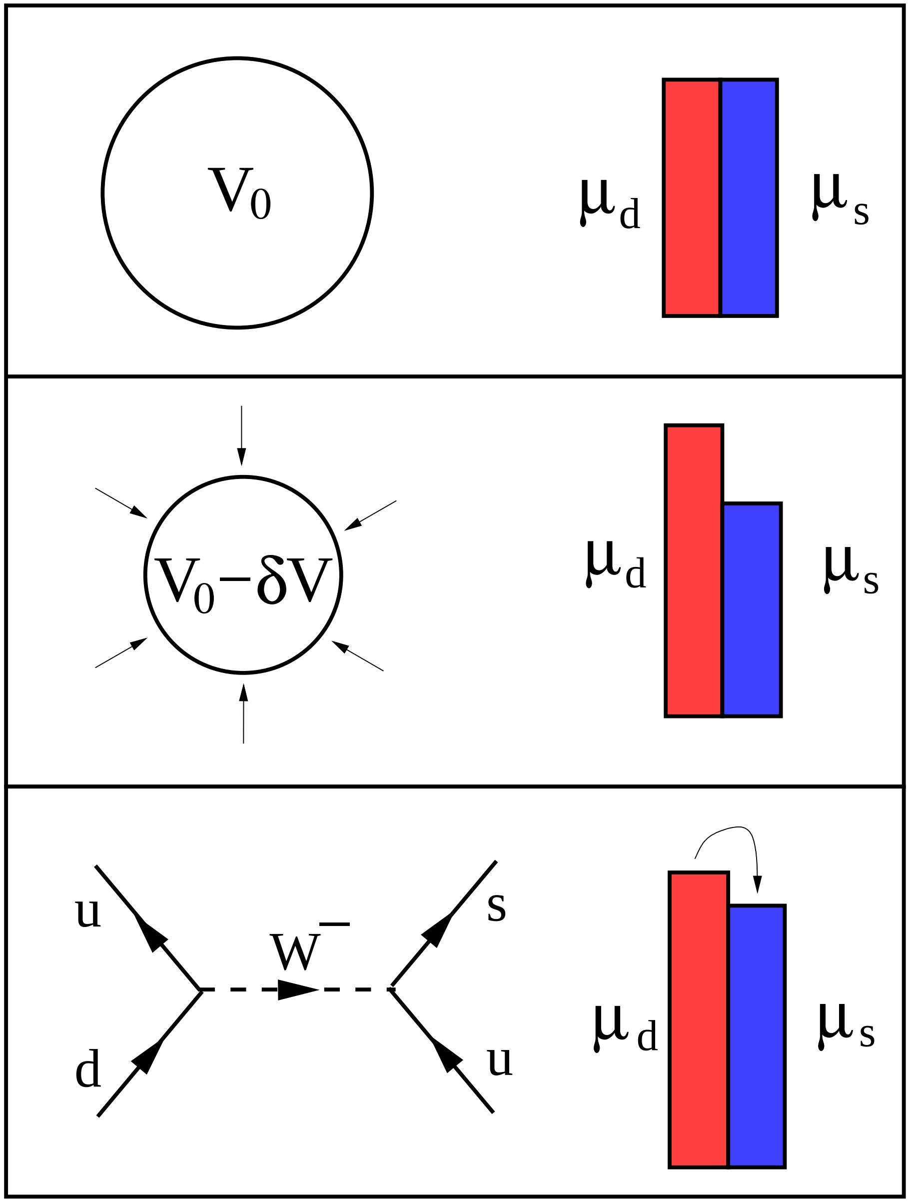
- volume oscillation
  $\longrightarrow$ chemical
  non-equilibrium

$$\mu_d - \mu_s \neq 0$$

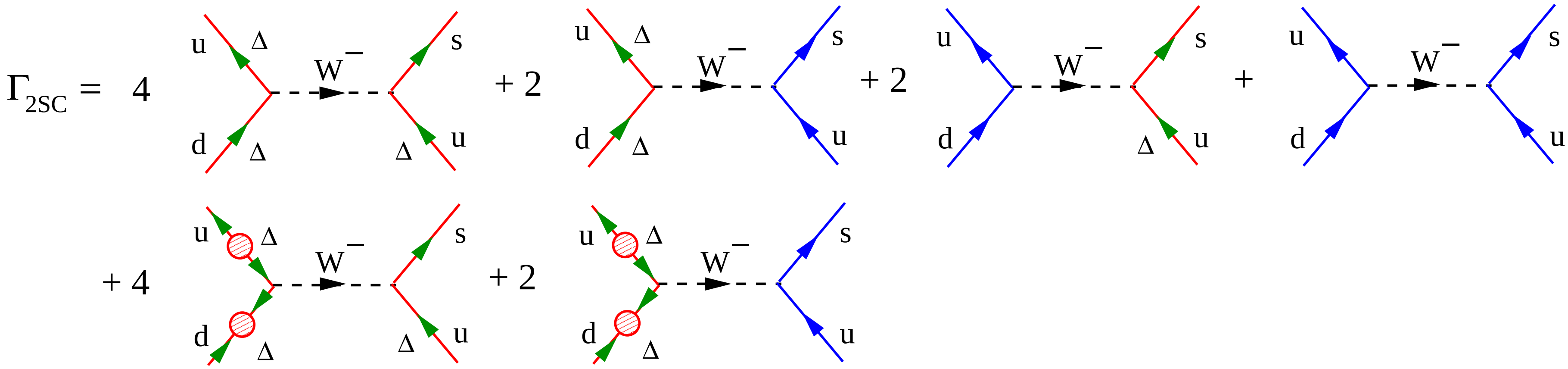
- re-equilibration via

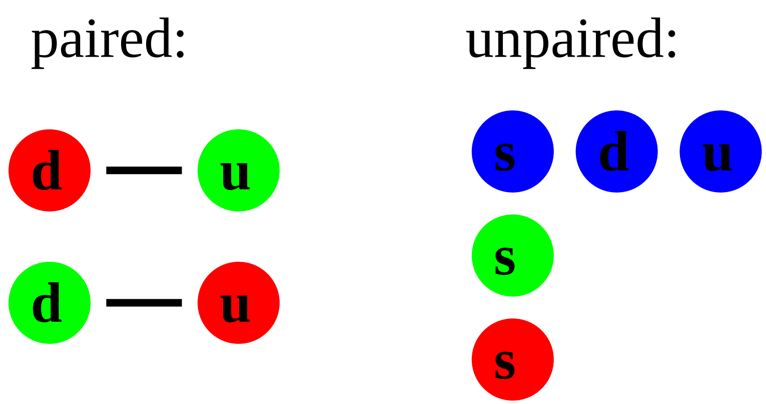
$$\boxed{u + d \leftrightarrow u + s}$$

- **resonance phenomenon**:
  external oscillation
  vs. microscopic rate



- **Compute rate for $u + d \leftrightarrow u + s$ in 2SC**





small temperatures,

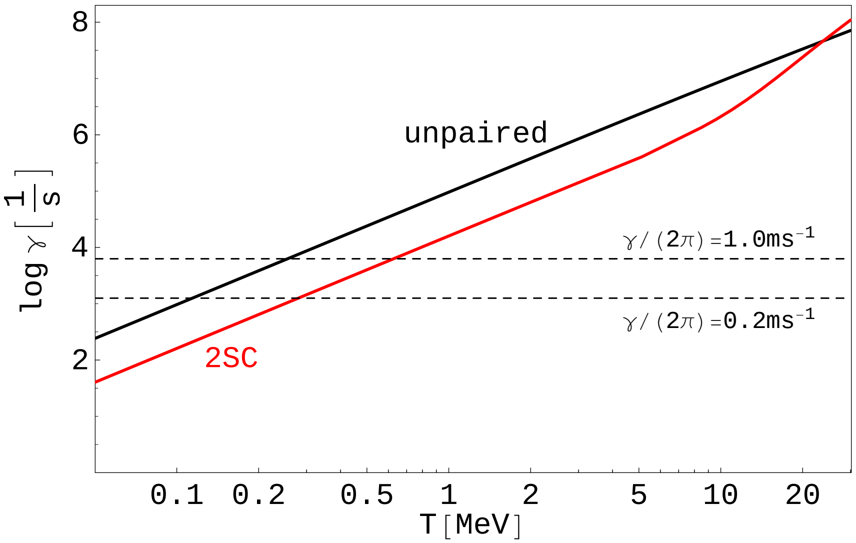
$$T \ll T_c \simeq 30\text{MeV}$$

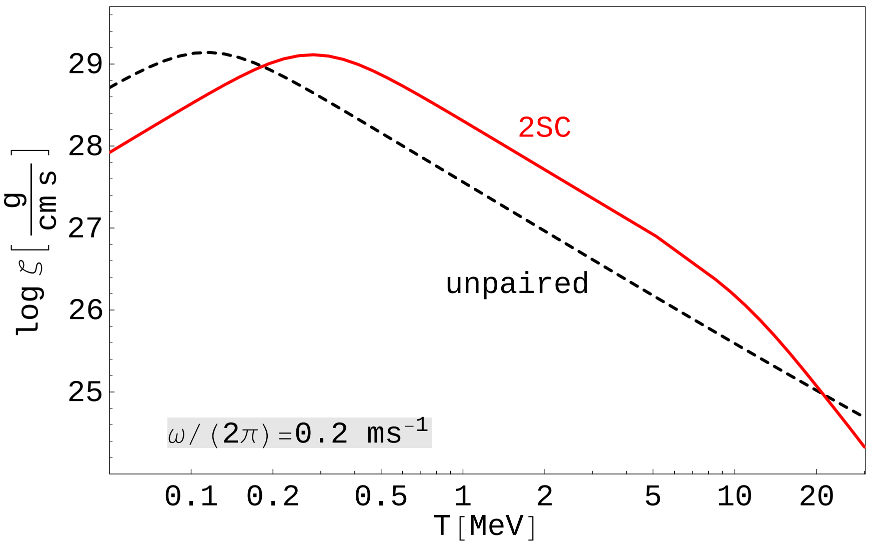
$$\Gamma_{2SC} = \frac{1}{9}\Gamma_{\text{unpaired}}$$

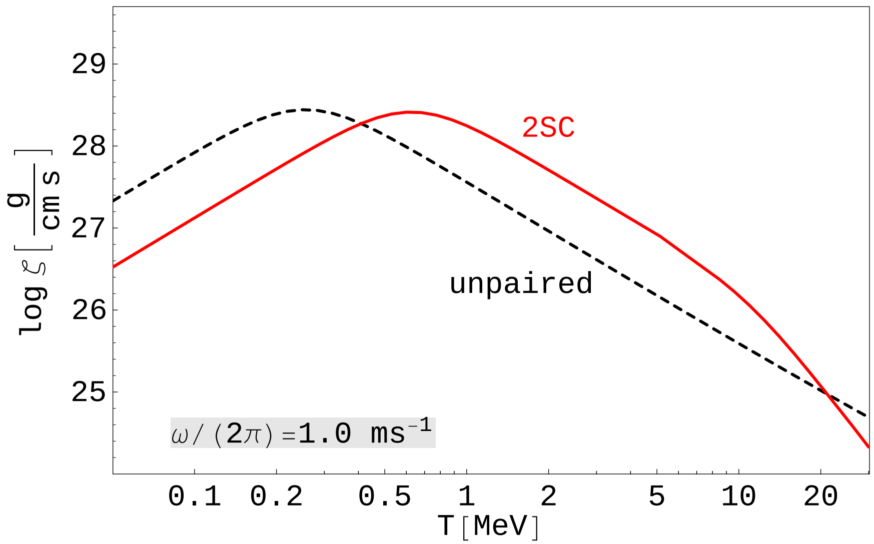
due to **exponential suppression**
$\exp(-\Delta/T)$ of gapped modes

• Results for bulk viscosity

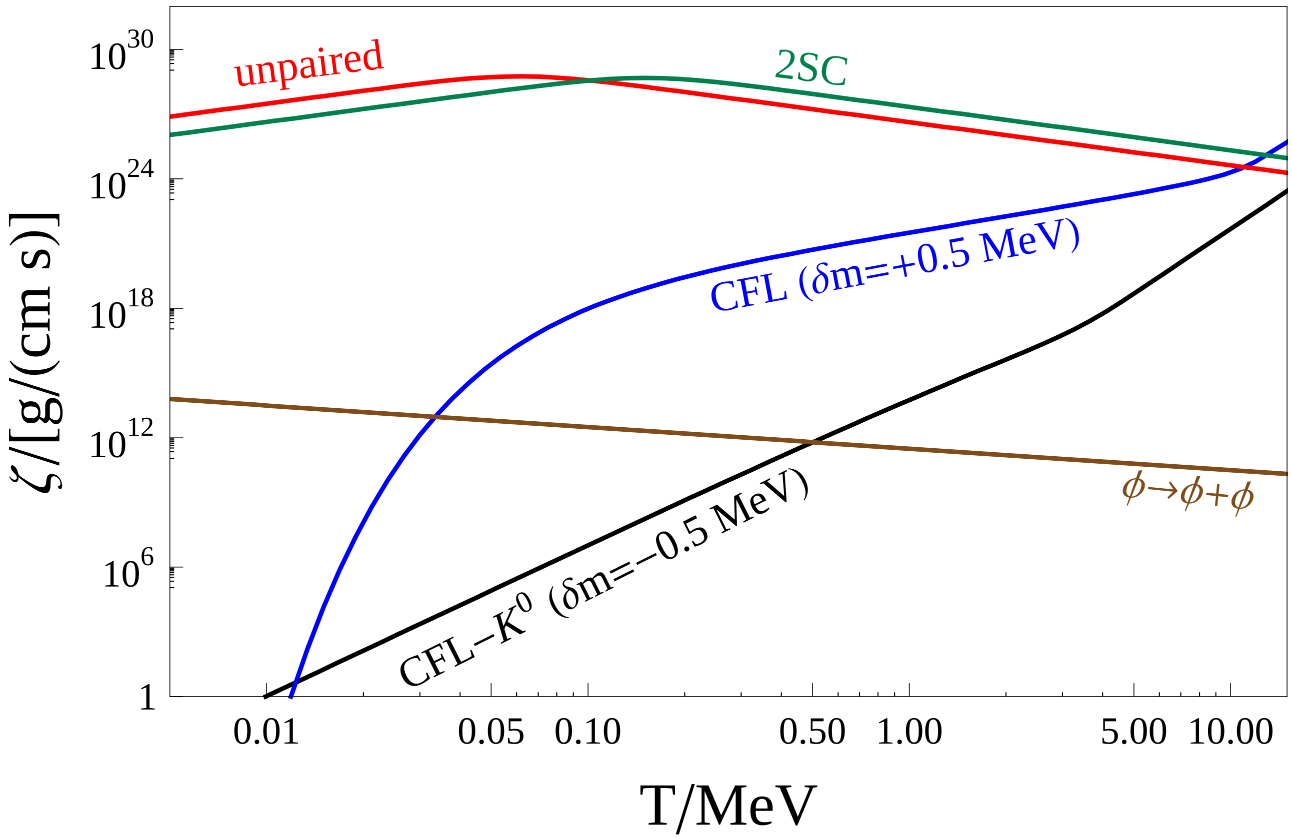
$$\zeta = \alpha \frac{\gamma}{\gamma^2 + \omega^2}$$







• Quark matter bulk viscosity: different phases



$\omega/(2\pi) = 1\,\mathrm{ms}^{-1}$

$\mu = 400\,\mathrm{MeV}$

$\delta m \equiv m_{K^0} - \mu_{K^0}$

unpaired J. Madsen, PRD 46, 3290 (1992)

2SC M.G. Alford, A. Schmitt, JPG 34, 67-101 (2007)

CFL from $K^0 \leftrightarrow \phi + \phi$ M.G. Alford, M. Braby, S. Reddy, T. Schafer, PRC 75, 055209 (2007)

CFL-K^0 from $K^0 \leftrightarrow \phi + \phi$ M.G. Alford, M. Braby, A. Schmitt, JPG 35, 115007 (2008)

CFL from $\phi \leftrightarrow \phi + \phi$ C. Manuel, F. Llanes-Estrada, JCAP 0708, 001 (2007)

See also: Spin-1 B.A. Sa'd, I.A. Shovkovy, D.H. Rischke, PRD 75, 065016 (2007)

Vortices in CFL M. Mannarelli, C. Manuel, B. A. Sa'd, PRL 101, 241101 (2008)

- **Summary**

  - **3-flavor quark matter** at asymptotically high densities is in the **Color-Flavor locked (CFL)** state
  - **CFL** quark matter
    - breaks chiral symmetry
    - is a superfluid
    - is a transparent insulator
  - phase(s) between CFL and hadronic matter (if there is/are any) is/are unknown
    - **large coupling**
    - **stressed pairing** renders ground state complicated
  - **transport properties** dominated by **Goldstone modes** in CFL ($\rightarrow$ small ν-emissivity, specific heat, bulk viscosity ...) and **ungapped quarks** in non-CFL ($\rightarrow$ large emissivity ...)

• Outlook/open questions

- QCD at high densities: **BEC – BCS crossover**?
  (however: quark bound states contain 3 quarks, not 2)
- insight from **large-N_c QCD**:
  competition/coexistence with **quarkyonic phase**?

  L. McLerran, R. D. Pisarski, NPA 796, 83 (2007)
- **second critical endpoint** at low T in QCD phase diagram?

  T. Hatsuda, M. Tachibana, N. Yamamoto, G. Baym, PRL 97, 122001 (2006)
- superfluid properties of CFL-K^0, curCFL-K^0;
  multicomponent superfluid?

  M.G. Alford, A. Schmitt, in preparation