Spectral Signature of the FFLO order parameter in one-dimensional optical lattices

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Haus Bergkranz, Riezlern, Austria

Outline

- Imbalanced Fermionic system: theory and experiments
- Mean-field approximation for 1D systems: lattice Bogoliubov-deGennes (BdG) formalism
- Radio-Frequency (RF) spectroscopy of a 1D imbalanced gas.

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Fermi systems with imbalanced spin population

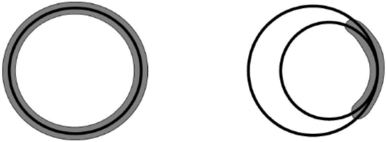
Breakdown of superconductivity in a magnetic field:

$\mu_B H \approx \Delta$ *Pauli limit*

Clogstone PRL, **9**, 266 (1962)

non-BCS pairing

- FFLO (Fulde-Ferrel-Larkin-Ovchinnikov) state:
  non-uniform order parameter



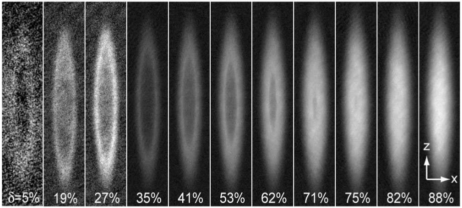
$k_{F\uparrow} = k_{F\downarrow}, \quad Q = 0$ $\qquad$ $k_{F\uparrow} \neq k_{F\downarrow}, \quad Q = k_{F\uparrow} - k_{F\downarrow}$

$\Delta(\mathbf{r}) = \Delta_0 e^{i\,2\mathbf{q}.\mathbf{r}}$ (single mode)

- FFLO observation in superconductors is still under debate:
  - H.A. Radovan *et.al*, Nature **425**, 51 (2003).
  - A. Bianchi *et.al*, PRL **91**, 187004 (2003).
  - K. Kakuyanagi *et.al*, PRL **44**, 047602 (2005).

Insight from quantum gases

$$\delta = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$



M.W. Zwierlein *et.al*, Science **311**, 492 (2006)
M.W. Zwierlein *et.al*, Nature **442**, 54 (2006)
Y. Shin *et.al*, PRL **97**,030401 (2006).

Population imbalance $\Longleftrightarrow$ magnetic field in a superconductor.

How to detect FFLO in Fermi gases?

Till now, there is no clear manifestation of FFLO pairing mechanism in Fermi gases.

Proposal: using noise correlation measurement

Topic Review: Kun Yang, cond-mat/0603190

M. Rizzi *et.al*, Phys. Rev. B **77**, 245105 (2008)

Recent work: J.M. Edge and N.R. Cooper, arXiv:0906.1801
(collective modes)

Our proposal: using RF-spectroscopy to detect FFLO phase

Hubbard description for a trapped 1D Fermi gas

$$\mathcal{H} = -t \sum_{i,\sigma} \left(\hat{c}^{\dagger}_{i,\sigma}\, \hat{c}_{i+1,\sigma} + h.c \right) - U \sum_{i} \hat{n}_{i\uparrow}\, \hat{n}_{i\downarrow} + \sum_{i,\sigma} \left(V_i^{ext} - \mu_{\sigma} \right) \hat{c}^{\dagger}_{i,\sigma}\, \hat{c}_{i,\sigma}$$

$$\hat{n}_{i\sigma} = \hat{c}^{\dagger}_{i,\sigma}\, \hat{c}_{i,\sigma}$$

$$V_i^{ext} = V_0 \left(i - \frac{L}{2} \right)^2$$ external trapping potential

Validity of mean-field approx. in 1D

In 1D long-range order is absent (**Luttinegr liquid paradigm**) so the mean-field approx is not valid as 3D. Specially the value of order parameter deviates from the exact solution.

F. Marsiglio, PRB **55**, 575 (1997)

Mean-field Hamiltonian for 1D interacting trapped Fermi gas

$$
\begin{aligned}
\mathcal{H}_{mf} &= -t\sum_{i,\sigma}\left(\hat{c}^{\dagger}_{i,\sigma}\,\hat{c}_{i+1,\sigma} + h.c\right) + \sum_{i}\left(\Delta_i \hat{c}^{\dagger}_{i,\uparrow}\,\hat{c}^{\dagger}_{i,\downarrow} + h.c\right) \\
&+ \sum_{i,\sigma}\left\{\left(V_i^{ext} - \mu_\sigma\right) - \underbrace{U\bar{n}_{i,\bar{\sigma}}}_{\text{Hartree term: neglect}}\right\}\hat{c}^{\dagger}_{i,\sigma}\,\hat{c}_{i,\sigma}
\end{aligned}
$$

$$
\Delta_i \equiv -U < \hat{c}_{i,\downarrow}\,\hat{c}_{i,\uparrow} > \qquad \text{local pairing gap}
$$

discrete Bogoliubov-deGennes (BdG) formalism

- Bogoliubov transformation

$$\hat{c}_{i,\sigma} = \sum_{\alpha} \left(u_{\alpha i \sigma} \hat{\gamma}_{\alpha\sigma} - \sigma v^{\star}_{\alpha i \sigma} \hat{\gamma}^{\dagger}_{\alpha\bar{\sigma}} \right)$$

- diagonalized H

$$\mathcal{H}_{mf} = E_0 + \sum_{\alpha\sigma} E_{\alpha\sigma}\, \hat{\gamma}^{\dagger}_{\alpha\sigma}\, \hat{\gamma}_{\alpha\sigma}$$

- BdG equations

$$\sum_{j=1}^{L} \begin{pmatrix} \mathcal{H}^{\sigma}_{ij} & \Delta_{ij} \\ \Delta_{ij} & -\mathcal{H}^{\bar{\sigma}}_{ij} \end{pmatrix} \begin{pmatrix} u_{\alpha j \sigma} \\ v_{\alpha j \bar{\sigma}} \end{pmatrix} = E_{\alpha\sigma} \begin{pmatrix} u_{\alpha i \sigma} \\ v_{\alpha i \bar{\sigma}} \end{pmatrix}$$

$$\begin{aligned} \mathcal{H}^{\sigma}_{ij} &= -t\, \delta_{i,j\pm 1} + \left(V^{ext}_i - \mu_\sigma \right) \delta_{ij} \\ \Delta_{ij} &= \Delta_i\, \delta_{ij} \end{aligned}$$

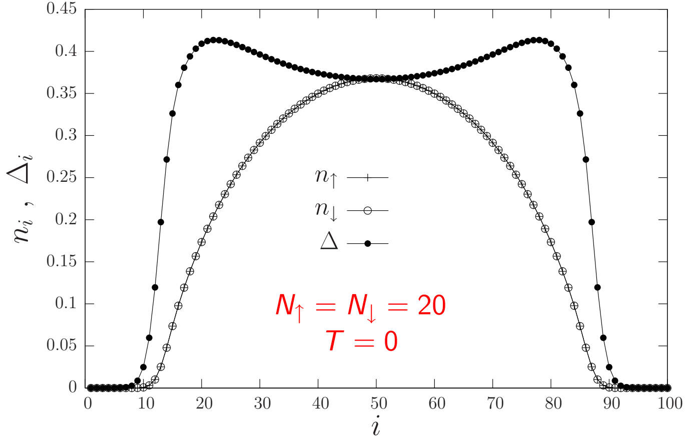
Self-consistent eqns. for spin-resolved density and gap

$$n_i^\sigma = \sum_{\alpha=1}^{L} \left[|u_{\alpha i\sigma}|^2 \, n_{\mathrm{F}}(E_{\alpha\,\sigma}) + |v_{\alpha i\sigma}|^2 \, n_{\mathrm{F}}(-E_{\alpha\,\bar{\sigma}}) \right]$$

$$\Delta_i = -U \sum_{\alpha=1}^{L} \left[u_{\alpha i\uparrow} \, v_{\alpha i\downarrow} n_{\mathrm{F}}(E_{\alpha\,\uparrow}) - u_{\alpha i\downarrow} \, v_{\alpha i\uparrow} n_{\mathrm{F}}(-E_{\alpha\,\downarrow}) \right]$$

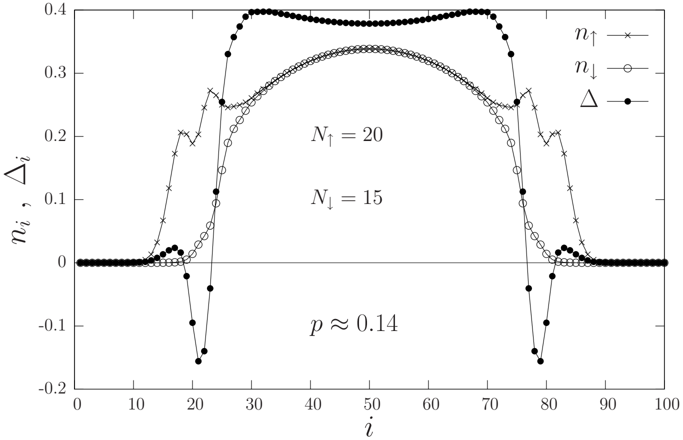
$$\sum_i n_i^\sigma = N_\sigma$$

Unpolarized gas



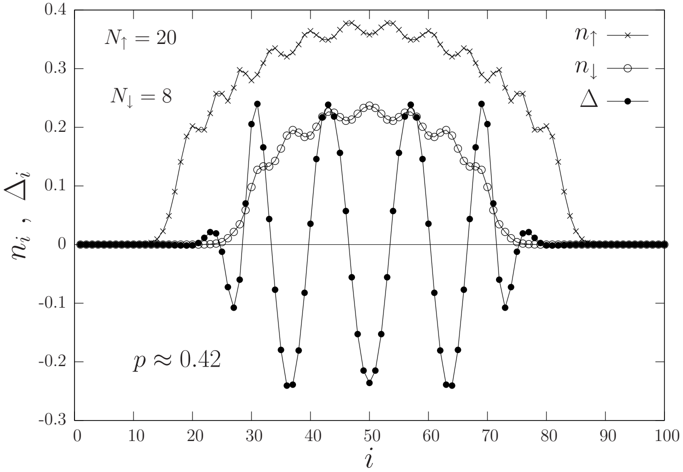
BCS-type of pairing: almost constant order-parameter.

Imbalanced gas



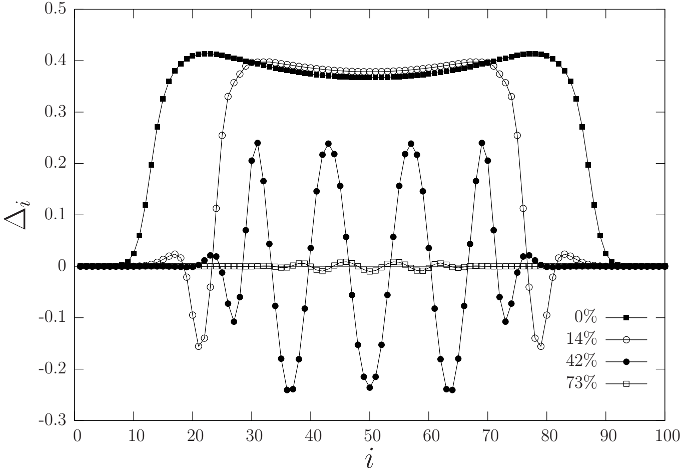
BCS pairing at the center and FFLO-type at the edges.

Imbalanced gas



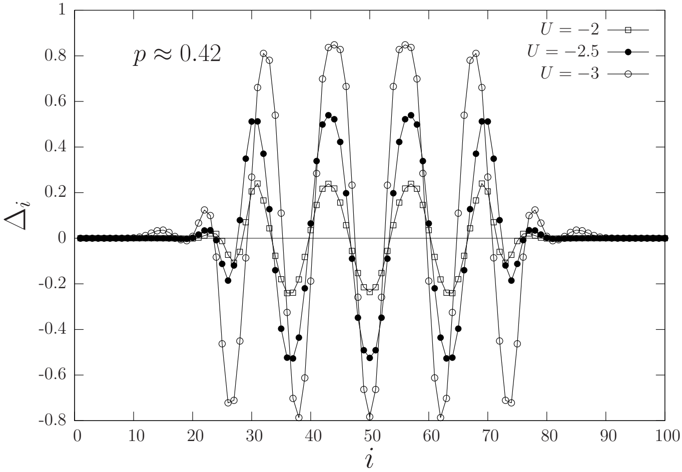
Minority component determines the extrema for order-parameter.

Order-parameter evolution



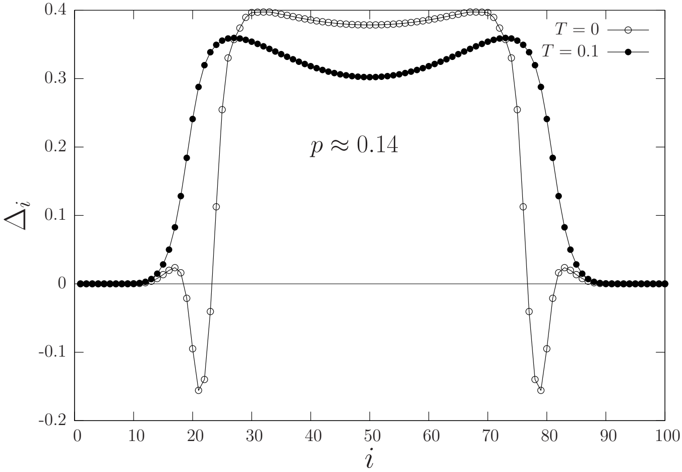
The oscillations become weaker and more confined upon increasing p.

Effect of interaction



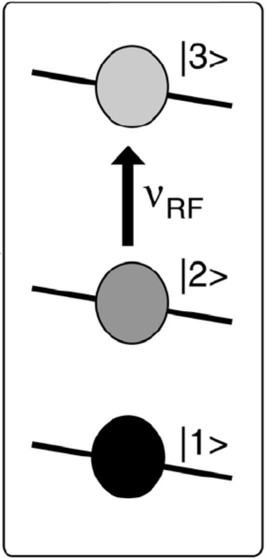
Stronger interaction enhance the oscillations.

Finite temperature effect



The oscillation disappear for high-T.

RF spectroscopy, tool for probing superfluid excitation gap



Original idea: using laser light to make a transition from an initially interactive state to a third empty state.

Goal: to detect BCS ground state and measure superconducting (-fluidity) order parameter.

P. Törma and P. Zoller, PRL **85**, 487 (2000)

RF spectroscopy of Fermi gases

Observation of pairing gap:

Experiment: Rudi Grimm group, Science **305**, 1128 (2004)

Theory: Päivi Törmä group, Science **305**, 1131 (2004)

Latest RF experiments

C.H. Schunck *et.al*, Science **316**, 867 (2007)

C.H. Schunck *et.al*, Nature **454**, 739 (2008)

J.T. Stewart *et.al*, Nature **454**, 744 (2008)

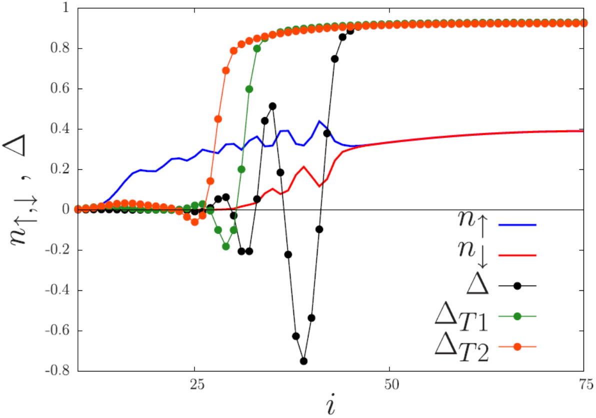
A. Schirotzek *et.al*, Phys.Rev.Lett **102**, 230402 (2009)

RF spectra

$$
\begin{aligned}
J_{\uparrow/\downarrow}(\delta, K) = & - & 2\pi \sum_{\alpha=1}^{L} \Bigg[\Bigg| \sum_{i=1}^{L} v_{\alpha i \downarrow/\uparrow} \, v^{\mathrm{non}}_{K i \uparrow\downarrow} \Bigg|^2 n_{\mathrm{F}}(-E_{\alpha\downarrow/\uparrow}) \\
& \times & \delta(E_{\alpha\downarrow/\uparrow} + \epsilon_K - \delta - \mu_{\uparrow/\downarrow}) \\
& + & \Bigg| \sum_{i=1}^{L} u_{\alpha i \uparrow/\downarrow} \, v^{\mathrm{non}}_{K i \uparrow\downarrow} \Bigg|^2 n_{\mathrm{F}}(E_{\alpha\uparrow/\downarrow}) \\
& \times & \delta(E_{\alpha\uparrow/\downarrow} - \epsilon_K + \delta + \mu_{\uparrow/\downarrow}) \Bigg]
\end{aligned}
$$

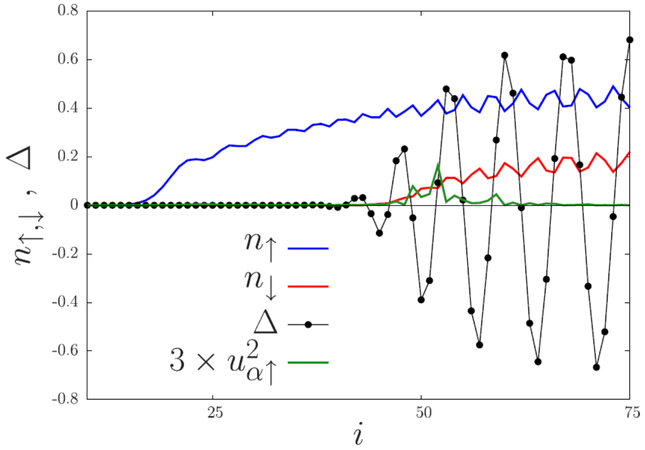
$$\delta = \omega_{\mathrm{rf}} - (\omega_{\mathrm{f}} - \omega_{\uparrow/\downarrow}) \qquad \epsilon_K = \mu_{\uparrow\downarrow} - E_{K\downarrow\uparrow} \qquad J_{\uparrow/\downarrow}(\delta) = \sum_{K=1}^{L} J_{\uparrow/\downarrow}(\delta, K)$$

density and gap profiles for more realistic system size

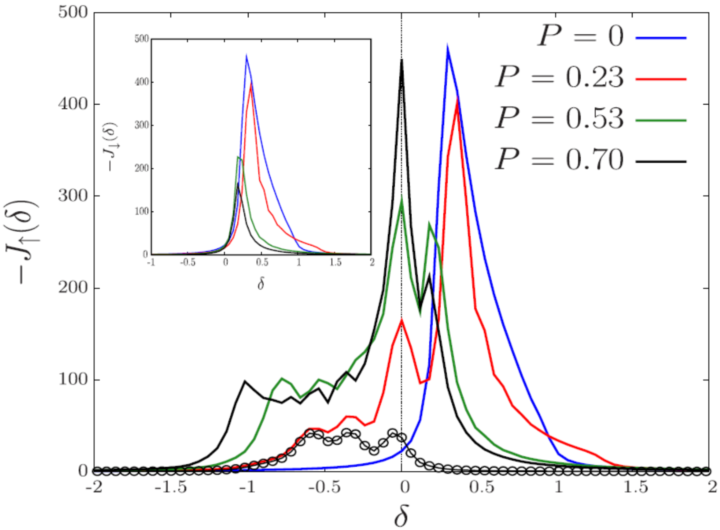


$$U = -3$$
$$V_0 = 5 \times 10^{-4}$$
$$T1 = 0.075$$
$$T2 = 0.1$$
$$L = 150$$
$$N_\uparrow = 40$$
$$N_\downarrow = 25$$

also the wave function for higher polarization

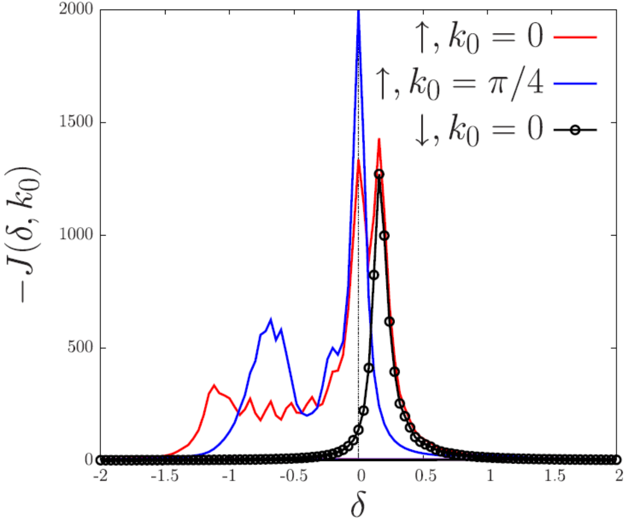


$$\alpha = 123$$
$$N_\uparrow = 40$$
$$N_\downarrow = 7$$



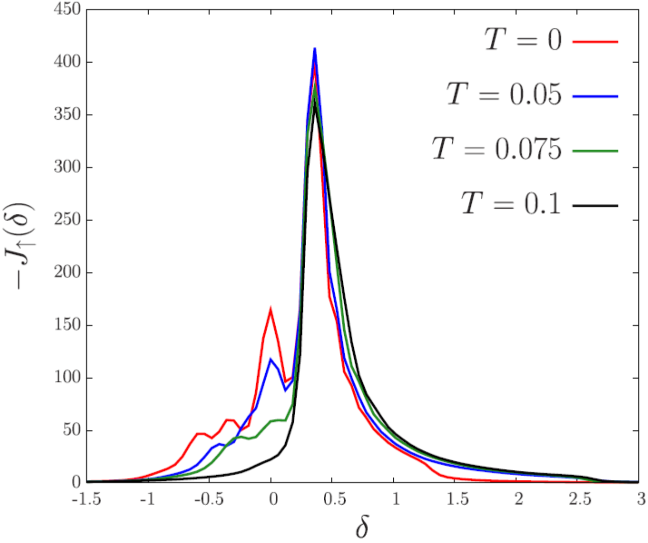
peaks at $\delta > 0$: paired atoms, peak at $\delta = 0$ unpaired majority.

Final state momentum-resolved spectra



$$J_{\uparrow/\downarrow}(\delta, k) = \sum_{K=1}^{L} \left| \sum_{i=1}^{L} e^{\imath(ki)} v^{\mathrm{non}}_{Ki\uparrow\downarrow} \right|^2 J_{\uparrow/\downarrow}(\delta, K)$$

T-dependence of majority component spectra

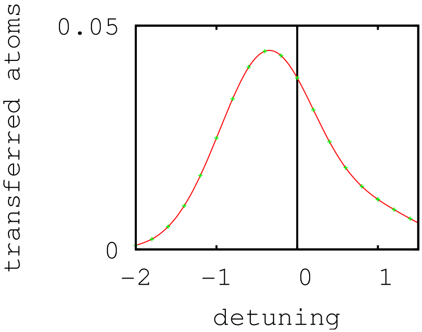


$T \gg \qquad \Longrightarrow$vanish order parameter$\Longrightarrow$disappear spectra at $\delta < 0$

validity of mean-field picture

comment about mean-field vs DMRG

R.A. Molina *et.al*, PRL, **102**, 168901 (2009)
M.R. Bakhtiari *et.al*, PRL, **102**, 168902 (2009)



calculated by Matrix Product States (MPS)

Summary and future plan

- FFLO signature of a 1D imbalanced gas at mean-field level
- finite temperature behavior of density and gap profile
- proposal on order parameter detection using RF spectra
- **Future plan:** FFLO with DMFT: higher D and arbitrary U

  initial work: M. Snoek *et.al*, NJP **10**, 093008 (2008)
- **Q: Does FFLO observation make the life easier?**