BCS-BEC Crossover and Thermodynamics of Asymmetric Nuclear Matter: A Mean Field Analysis

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Based on: S.J. Mao, X.G. Huang and P.F. Zhuang, Phys.Rev.C 79,034304(2009) [arXiv:0812.0674]

Outline

- Introduction and Motivation
- BCS-BEC Crossover at T=0
- Thermodynamics at high density and T>0
- Summary and outlook

Introduction and Motivations

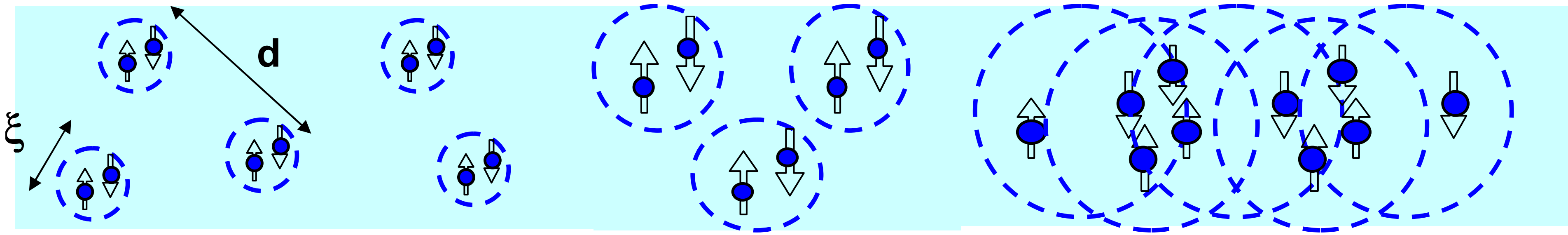
BCS : weak interacting, symmetric pairing

Beyond BCS

Lessons from cold atomic gas 1: BCS-BEC Crossover

Leggett 1980, Nozieres & Schmitt-Rink 1985, … …

BEC	Crossover	BCS
Interaction: Strong	Continuous change, no phase transion	Interaction: Weak
In real space: Bound pair (boson)		In real space: No bound pair
In momentum space: No Fermi Surface		In momentum space: Fermi Surface



Pair size / average distance

$\xi/d<<1$ | $\xi/d \sim 1$ | $\xi/d>1$

Pairing gap / Fermi energy

$\Delta/e_F>1$ | $\Delta/e_F \sim 1$ | $\Delta/e_F<<1$

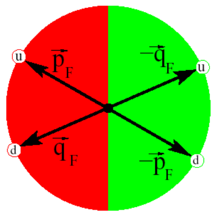
Cf. Matsuo

Introduction and Motivations

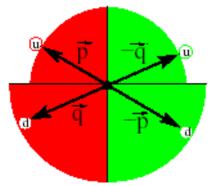
Lessons from cold atomic gas 2: Asymmetric superfluid

Sarma 1963, Fulde & Ferrel 1964, Larkin & Ovchinnikov 1964, Liu & Wilczek 2003, Muther & Sedrakian 2002, Bedaque, Caldas & Rupak 2003, … …

Symmetric BCS pairing, say, n_a=n_b:



Asymmetric pairing, say, n_a>n_b:



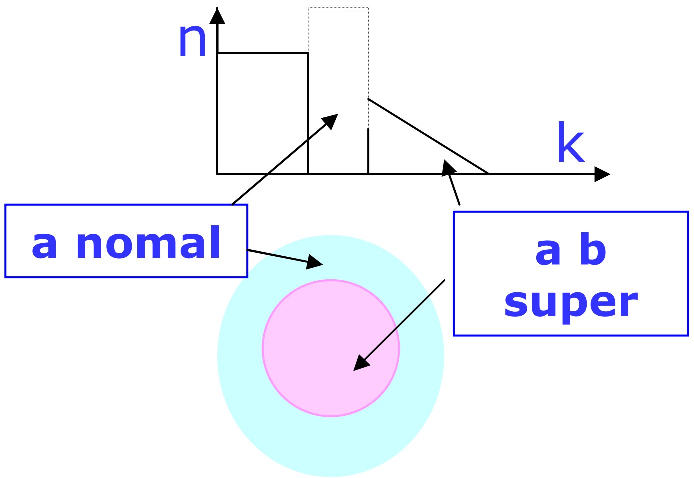
Basic question: what is the ground state of a asymmetric fermion system?

No definite answer, depend on parameters, various candidates

Sarma or breached pairing phase: superfluid + normal homogeneously(unstalbe in weak coupling at low T)

Phase separation: superfluid+normal inhomogeneously

FFLO phase: Cooper pair has nonzero momentum(only stable in weak coupling)
Sheehy & Radzihovsky 2006



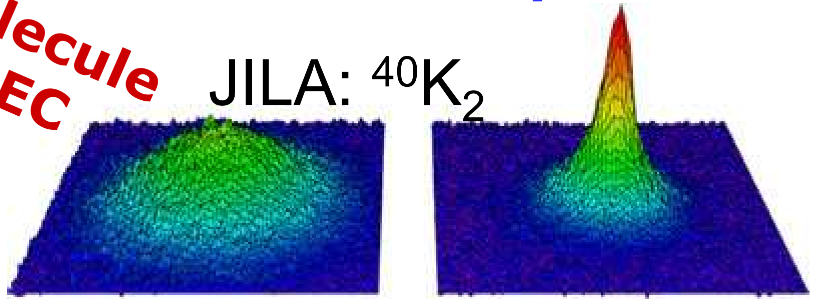
$\Delta \sim \sin kx$ or $\exp(ikx)$

Introduction and Motivations

Experiments of Cold Atoms

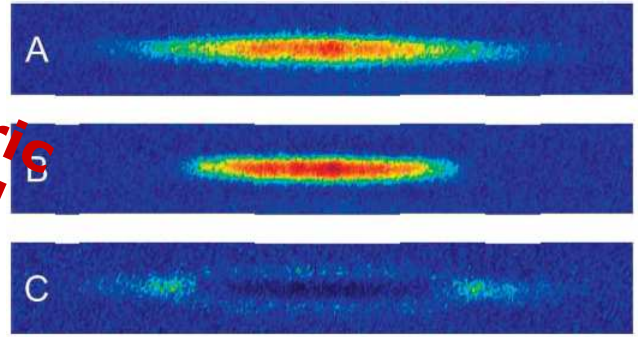
Molecule BEC

JILA: $^{40}K_2$

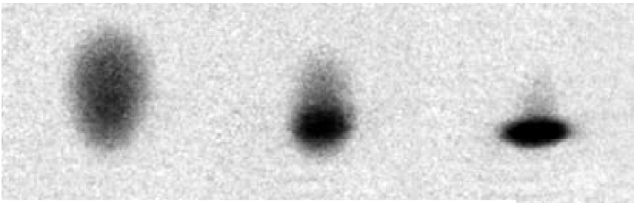


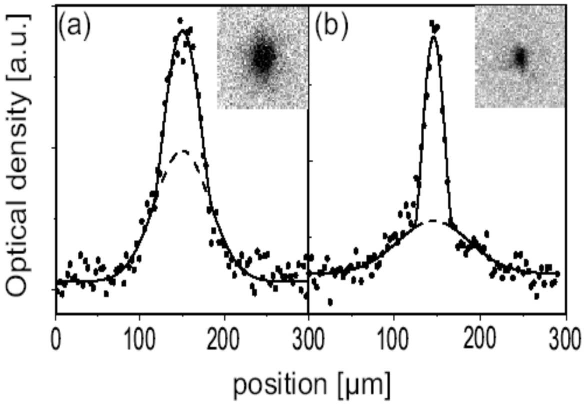
Asymmetric superfluid

Rice $^{6}Li_2$

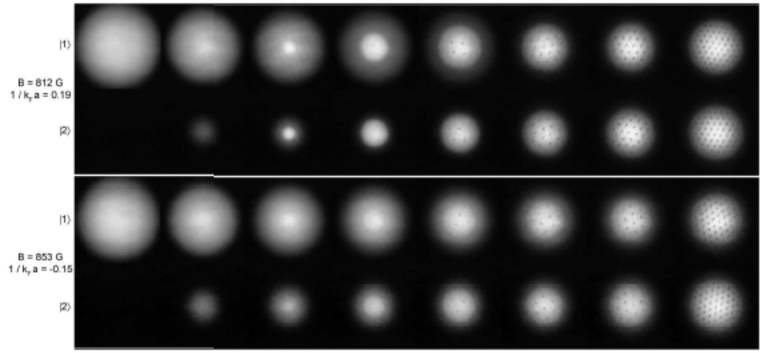


MIT $^{6}Li_2$





ENS $^{6}Li_2$



MIT $^{6}Li_2$

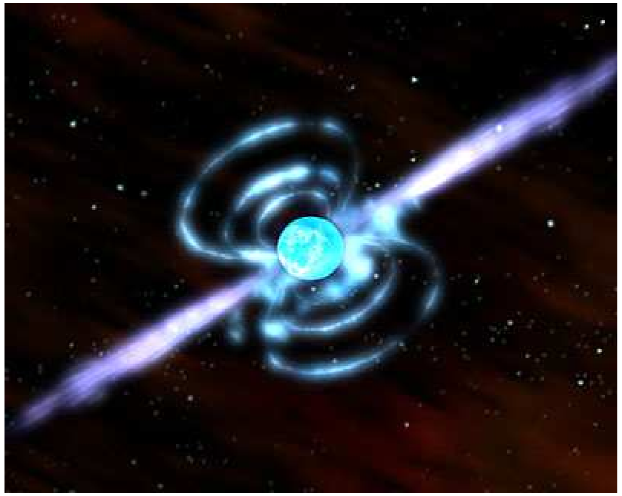
... ...

Introduction and Motivations

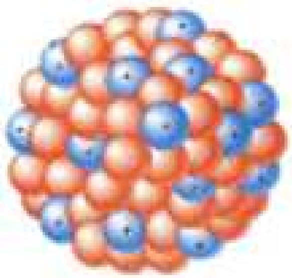
Nucleon-Nucleon (NN) attractive interaction ⟹

NN-Cooper pair (boson-like), which condense at low temperature (superfluid)

Neutron star: equation of state, cooling, gliches … …



Finite nuclei: single particle spectra(Bohr et.al.), rotational spectra(Migdal) … ...



Introduction and Motivations

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BCS-BEC crossover and asymmetric pairing in nuclear matter?

Yes!

Density change triggers a BCS(high density)-BEC(low density) like crossover

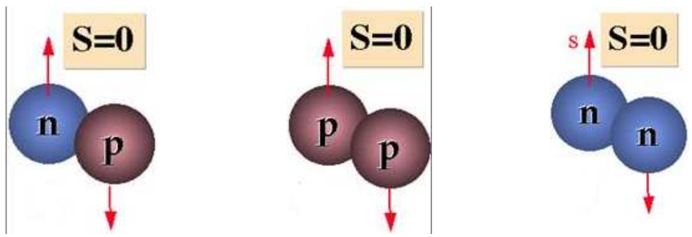
np pairing: Baldo, Lombardo & Schuck, 1995, Lombardo, Nozieres, Schuck, Schulze, Sedrakian 2001, … …

nn pairing: Matsuo 2007, Isayev 2008, Margueron, Sagawa & Hagino 2007, … …

Asymmetric nucleon superfluid: Sedrakian & Lombardo, 2000, Muther & Sedrakian 2003, Akhiezer 2001, Jin, He & Zhuang 2006, … …

More general and systematic: consider nn,pp and np pairings simultaneously: the competition between I=0 amd I=1 pairings

New



Mean Field Formalism

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Lagrangian with Isospin I=0 and I=1 two-body potential

$$\hat{\mathcal{L}} = \sum_{\sigma=\uparrow,\downarrow}\left[\hat{p}^\dagger_\sigma(\mathbf{x})\left(-\frac{\partial}{\partial\tau}+\frac{\nabla^2}{2m}+\mu_p\right)\hat{p}_\sigma(\mathbf{x})\right.$$
$$\left.+\hat{n}^\dagger_\sigma(\mathbf{x})\left(-\frac{\partial}{\partial\tau}+\frac{\nabla^2}{2m}+\mu_n\right)\hat{n}_\sigma(\mathbf{x})\right]$$
$$-\int d^3\mathbf{x}' V_1(\mathbf{x}-\mathbf{x}')[\hat{n}^\dagger_\uparrow(\mathbf{x})\hat{n}^\dagger_\downarrow(\mathbf{x}')\hat{n}_\downarrow(\mathbf{x}')\hat{n}_\uparrow(\mathbf{x})$$
$$+\hat{p}^\dagger_\uparrow(\mathbf{x})\hat{p}^\dagger_\downarrow(\mathbf{x}')\hat{p}_\downarrow(\mathbf{x}')\hat{p}_\uparrow(\mathbf{x})]$$
$$-\frac{1}{2}\int d^3\mathbf{x}' V_0(\mathbf{x}-\mathbf{x}')[\hat{n}^\dagger_\uparrow(\mathbf{x})\hat{p}^\dagger_\downarrow(\mathbf{x}')-\hat{p}^\dagger_\uparrow(\mathbf{x})\hat{n}^\dagger_\downarrow(\mathbf{x}')]$$
$$\times[\hat{p}_\downarrow(\mathbf{x}')\hat{n}_\uparrow(\mathbf{x})-\hat{n}_\downarrow(\mathbf{x}')\hat{p}_\uparrow(\mathbf{x})]$$

Adopt a density-dependent contact interaction(DDCI)

Garrido et.al. 1999,2001

$$V_I(\mathbf{x}-\mathbf{x}') = g_I\delta(\mathbf{x}-\mathbf{x}')$$
$$g_I = v_I[1-\eta_I(\rho/\rho_0)^{\gamma_I}]$$

I=1 channel:

$\eta = 0.45, \gamma = 0.47, v = -481$ MeV fm^3

I=0 channel:

$\eta = 0, v = -530$ MeV fm^3

m=m(ρ)

This potential well produces the pairing gaps given by realistic potentials

Mean Field Formalism

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Partition function

$$Z = \Pi_\sigma \int [d\hat{n}_\sigma][d\hat{p}_\sigma][d\hat{n}_\sigma^\dagger][d\hat{p}_\sigma^\dagger] \exp\left(\int_0^\beta d\tau \int d^3\mathbf{x}\, \hat{\mathcal{L}}\right)$$

Thermodynamic potential at mean field approx.

$$\Omega = -\frac{T}{V}\ln Z = -\frac{2\Delta_{np}^2}{g_0} - \frac{\Delta_{nn}^2 + \Delta_{pp}^2}{g_1} - T\sum_\nu \int \frac{d^3\mathbf{k}}{(2\pi)^3} \mathrm{Tr}\ln G^{-1}(i\omega_\nu, \mathbf{k})$$

Condensates

$$\Delta_{nn} e^{i2\mathbf{q}\cdot\mathbf{x}} = -g_1 \langle \hat{n}_\downarrow(\mathbf{x}) \hat{n}_\uparrow(\mathbf{x}) \rangle$$

$$\Delta_{pp} e^{i2\mathbf{q}\cdot\mathbf{x}} = -g_1 \langle \hat{p}_\downarrow(\mathbf{x}) \hat{p}_\uparrow(\mathbf{x}) \rangle$$

$$\Delta_{np} e^{i2\mathbf{q}\cdot\mathbf{x}} = -(g_0/2) \langle \hat{p}_\downarrow(\mathbf{x}) \hat{n}_\uparrow(\mathbf{x}) - \hat{n}_\downarrow(\mathbf{x}) \hat{p}_\uparrow(\mathbf{x}) \rangle$$

q: FFLO momentum

G: Gorkov Propagator

$$G^{-1} = \begin{pmatrix} i\omega_\nu - \epsilon_n^+ & 0 & \Delta_{np} & \Delta_{nn} \\ 0 & i\omega_\nu - \epsilon_p^+ & \Delta_{pp} & -\Delta_{np} \\ \Delta_{np} & \Delta_{pp} & i\omega_\nu + \epsilon_p^- & 0 \\ \Delta_{nn} & -\Delta_{np} & 0 & i\omega_\nu + \epsilon_n^- \end{pmatrix}$$

$$\epsilon_{n,p}^\pm = (\mathbf{k} \pm \mathbf{q})^2/(2m) - \mu_{n,p}$$

Mean Field Formalism

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Condensates and FFLO momentum are determined by the gap equations

$$\frac{\partial\Omega}{\partial\Delta_{np}}=0,\quad \frac{\partial\Omega}{\partial\Delta_{nn}}=0,\quad \frac{\partial\Omega}{\partial\Delta_{pp}}=0,\quad \frac{\partial\Omega}{\partial\mathbf{q}}=0$$

Chemical potentials are determined by the number equations

$$\rho_n=-\frac{\partial\Omega}{\partial\mu_n},\quad \rho_p=-\frac{\partial\Omega}{\partial\mu_p}$$

or

$$\rho=-\frac{\partial\Omega}{\partial\mu},\quad \delta\rho=-\frac{\partial\Omega}{\partial\delta\mu}$$

$$\delta\rho=\rho_n-\rho_p \quad \mu=(\mu_n+\mu_p)/2$$

The ground states are specified by the solutions globally minimizing the free energy

$$\mathcal{F}=\Omega+\mu_n\rho_n+\mu_p\rho_p=\Omega+\mu\rho+\delta\mu\delta\rho$$

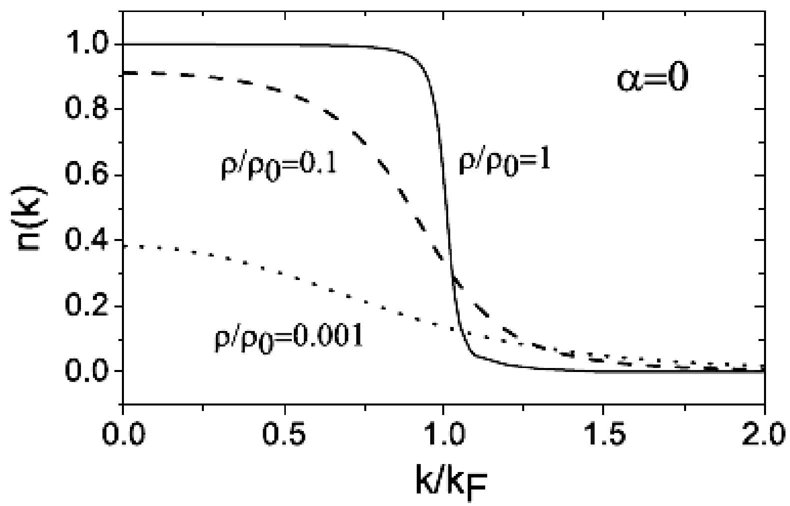
No pair fluctuations are included in mean field approx., so it can work only at weak coupling or low temperature

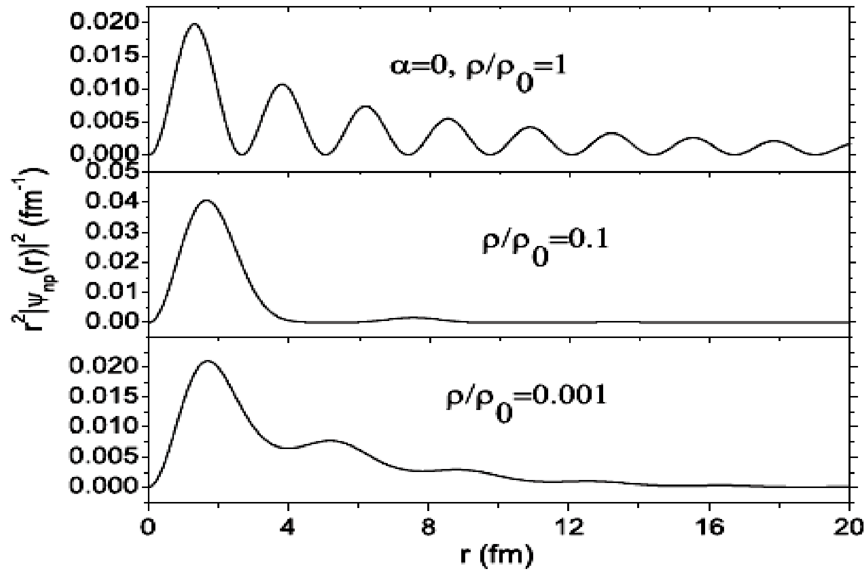
BCS-BEC crossover in symmetric nuclear matter

The ground state is always corresponding to the solution:

$$\Delta_{np} \neq 0, \Delta_{nn} = \Delta_{pp} = 0$$

Do not consider the FFLO state since which is unstable in BEC regime





Asymmetry param.: $\alpha = \delta\rho/\rho.$

Pair wave function:

$$\psi_{ij}(\mathbf{r}) = C\langle \mathrm{BCS}|\hat{a}^{\dagger}_{i\uparrow}(\mathbf{x})\hat{a}^{\dagger}_{j\downarrow}(\mathbf{x}+\mathbf{r})|\mathrm{BCS}\rangle$$

Friedel oscillation at high density: appearance of Fermi surface

Indication of BEC-BCS crossover, more quantitatively

BCS-BEC crossover in symmetric nuclear matter

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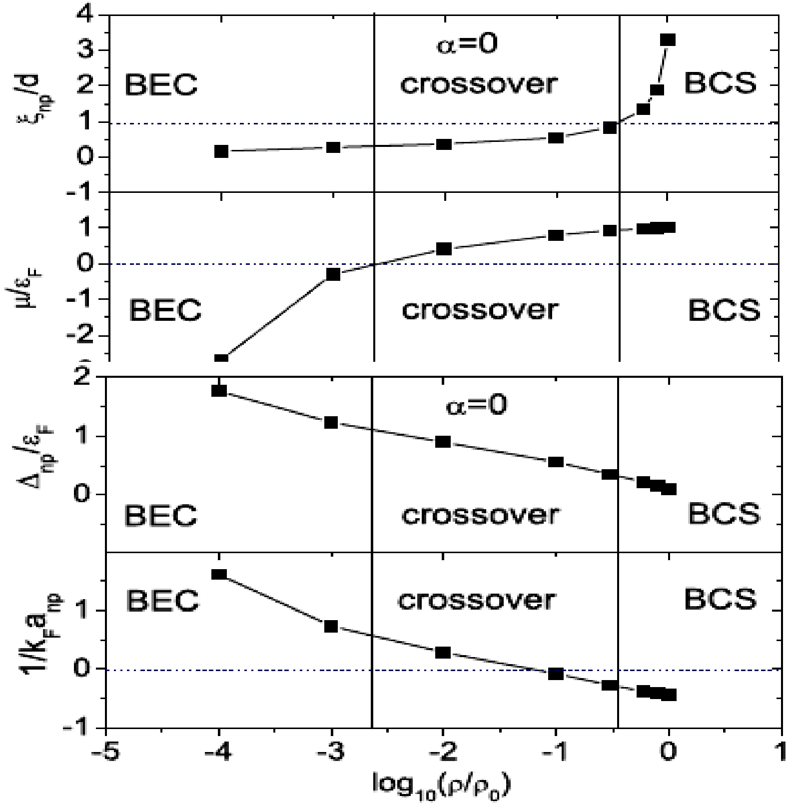


Average distance:

$$d = (\rho/2)^{1/3}$$

Size of pair:

$$\xi_{ij} = \sqrt{\langle r^2 \rangle_{ij}} \text{ with } \langle r^2 \rangle_{ij} = \int d^3\mathbf{r}\, r^2 |\psi_{ij}(\mathbf{r})|^2$$

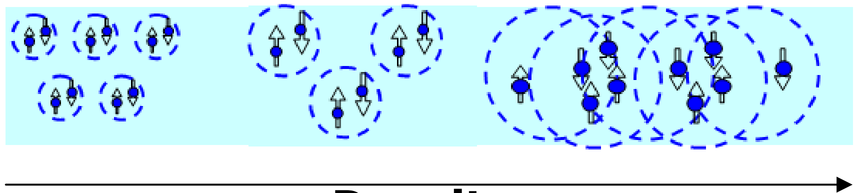
S-wave scattering length:

$$\frac{m}{4\pi a_{ij}} = \frac{1}{g_{ij}} + \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{2\epsilon_\mathbf{k}}$$

$$\frac{\mathbf{k}^2}{m}\psi_{np}(\mathbf{k}) + (1 - 2n_\mathbf{k})g_0 \int \frac{d^3\mathbf{k}'}{(2\pi)^3}\psi_{np}(\mathbf{k}') = 2\mu\psi_{np}(\mathbf{k})$$

True BCS-BEC crossover for np pairs in symmetric nuclear matter

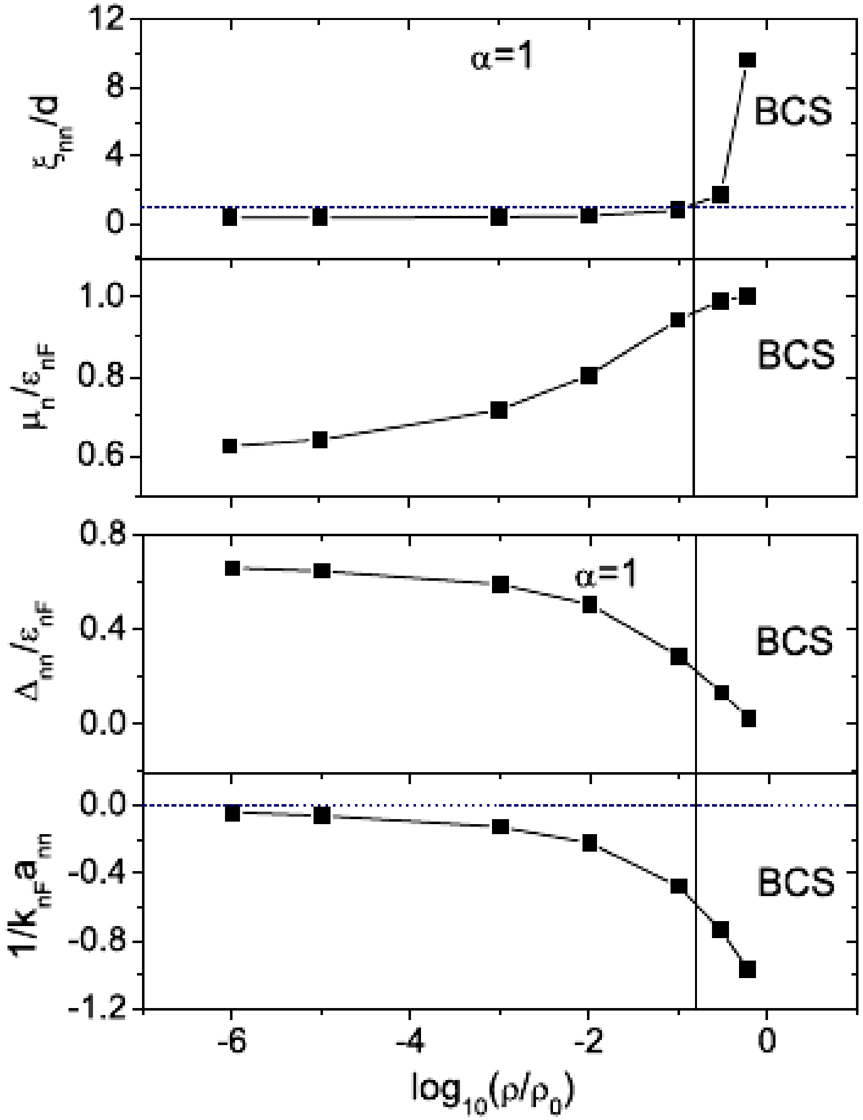
Deuteron



np Cooper pair

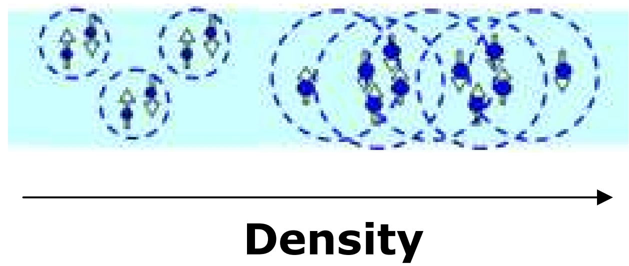
Density

BCS-BEC crossover in pure neutron matter



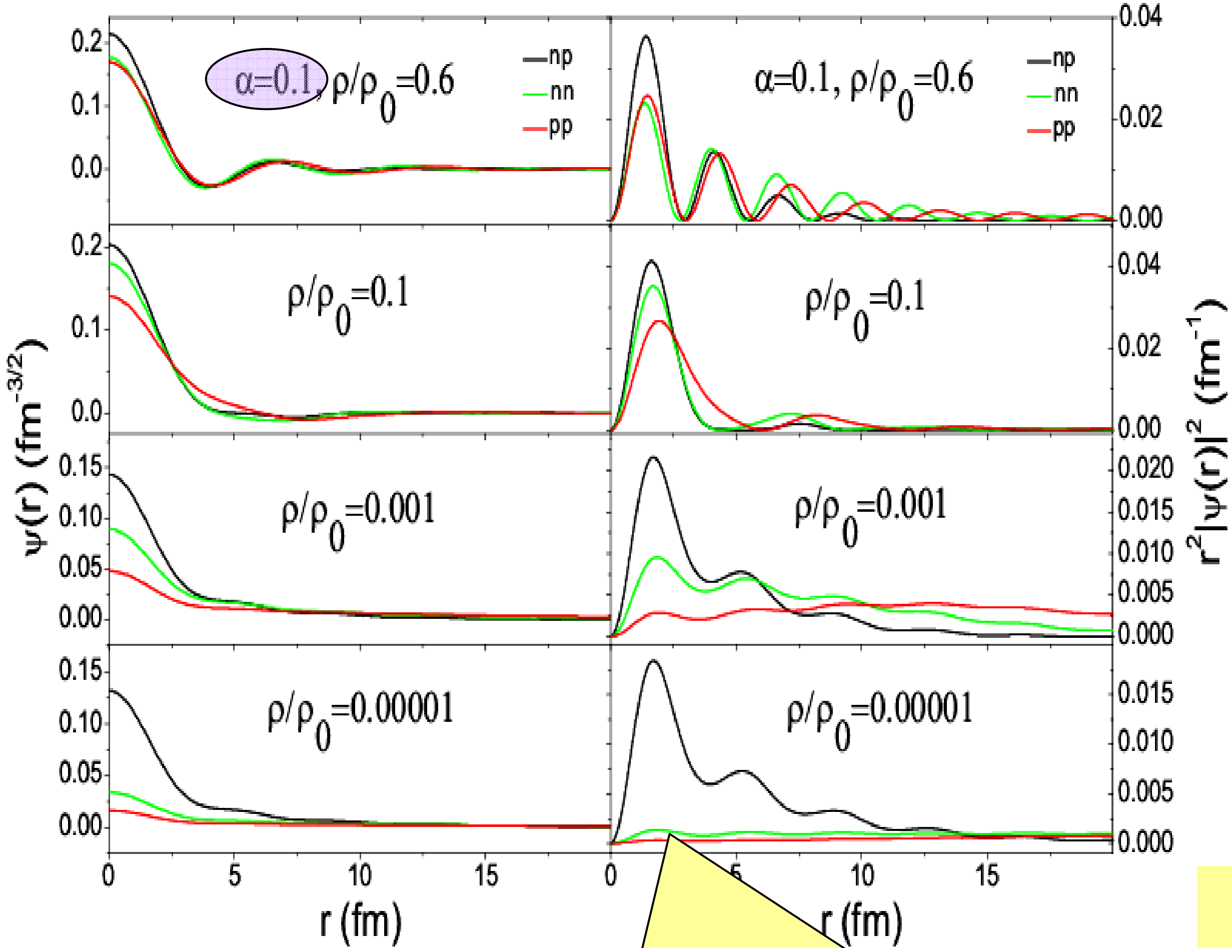
Chemical potential is always positive, scattering length is always negative: BEC is not reached

Pairing gap/Fermi energy is larger at low density; pair size/ average distance is smaller at low density: strongly correlated nn-pair at low density



BCS-BEC crossover in asymmetric matter

Wave functions and probabilities



High density:

nn,pp,np

Large spatial extensions

Strong oscillations

Typical BCS behaviors

Low density:

Wave functions shrink

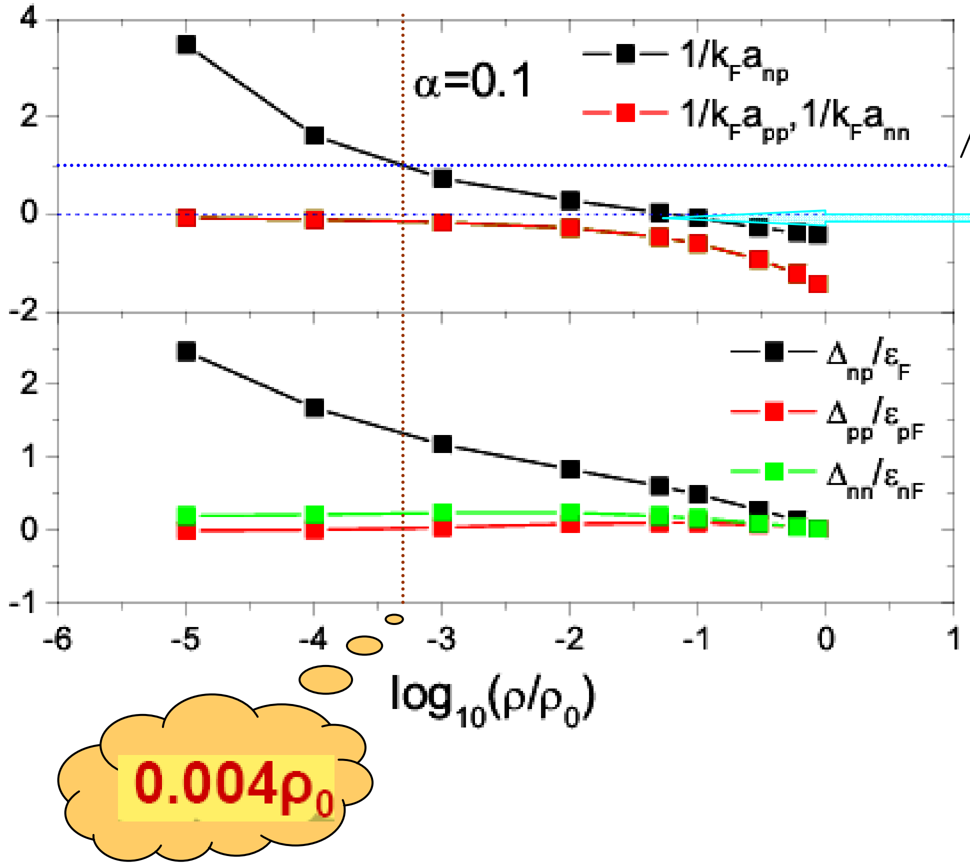
Oscillations weacken

Indication of BEC

np is more strongly correlated than nn, pp pairings

BCS-BEC crossover in asymmetric matter

More quantitative



Define the BEC boundary:

$$\frac{1}{k_F a_{ii}} = 1$$

Unitary limit for np pairing

Density below BEC boundary:

np scattering length >1, gap larger than fermi energy: BEC

nn,pp scattering lengthes keep negative but close to zero, gaps much smaller than fermi energy: strongly correlated but no BEC

BCS-BEC crossover in asymmetric matter

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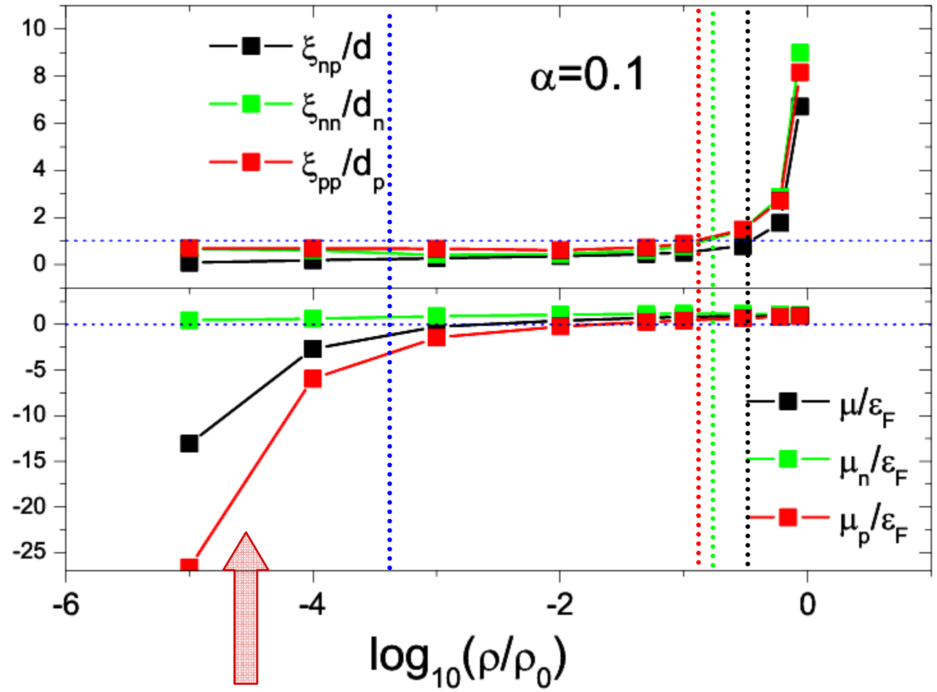


Define the BCS boundary:

$$\xi_{ij} / d_{ij} = 1,$$

$$(\xi_{ij} = \sqrt{\int_0^\infty r^4 \psi_{ij}^2 dr}\ )$$

I=0, $\rho > 0.5\rho_0 \rightarrow BCS$

I=1, $nn, \rho > 0.17\rho_0 \rightarrow BCS$

$pp, \rho > 0.14\rho_0 \rightarrow BCS$

I=0, ρ <0.004 ρ_0 , *BEC*

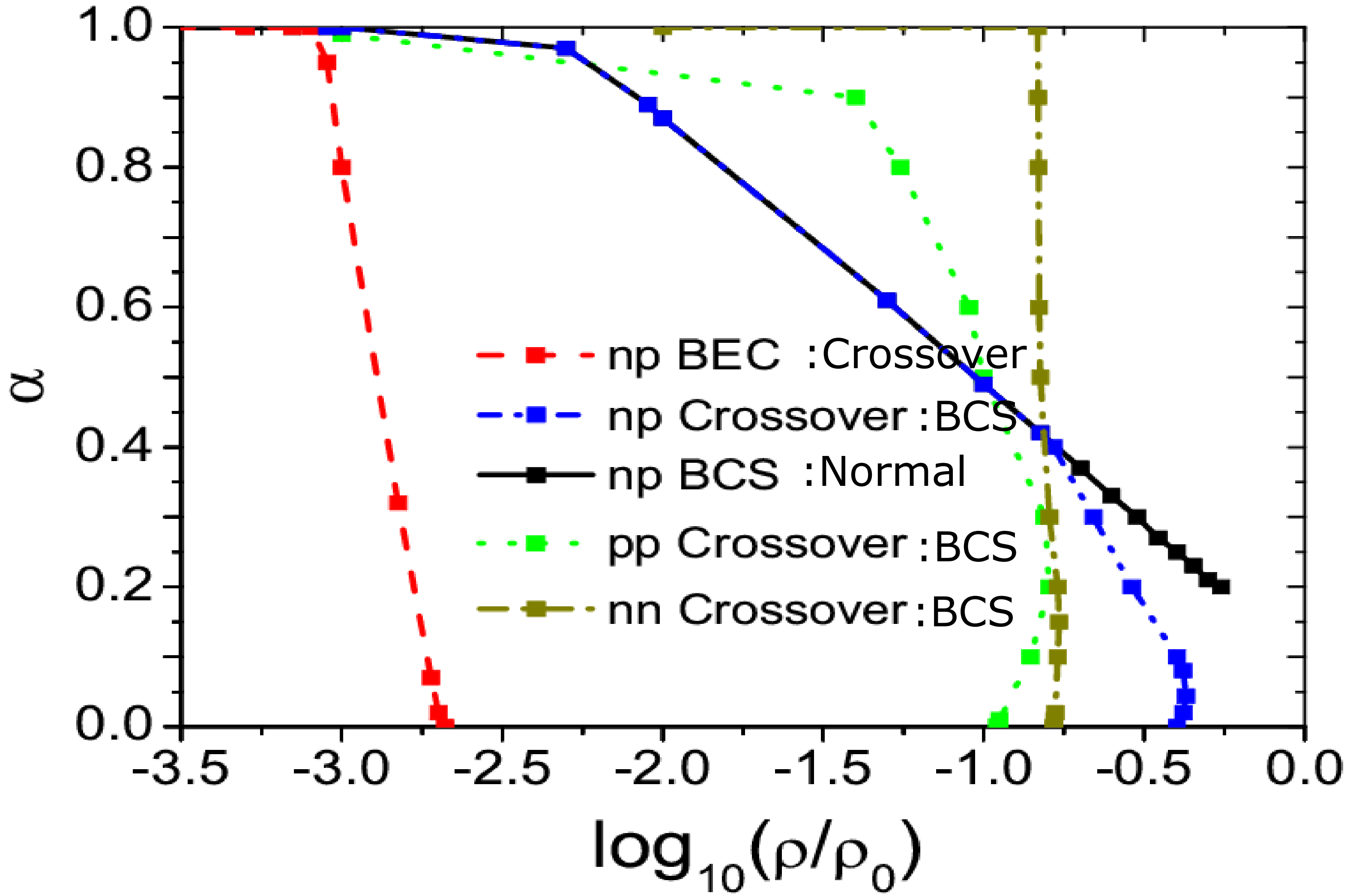
Chemical potential is no longer a good character for BCS-BEC crossover in asymmetric case

np BEC **np crossover** **np BCS**

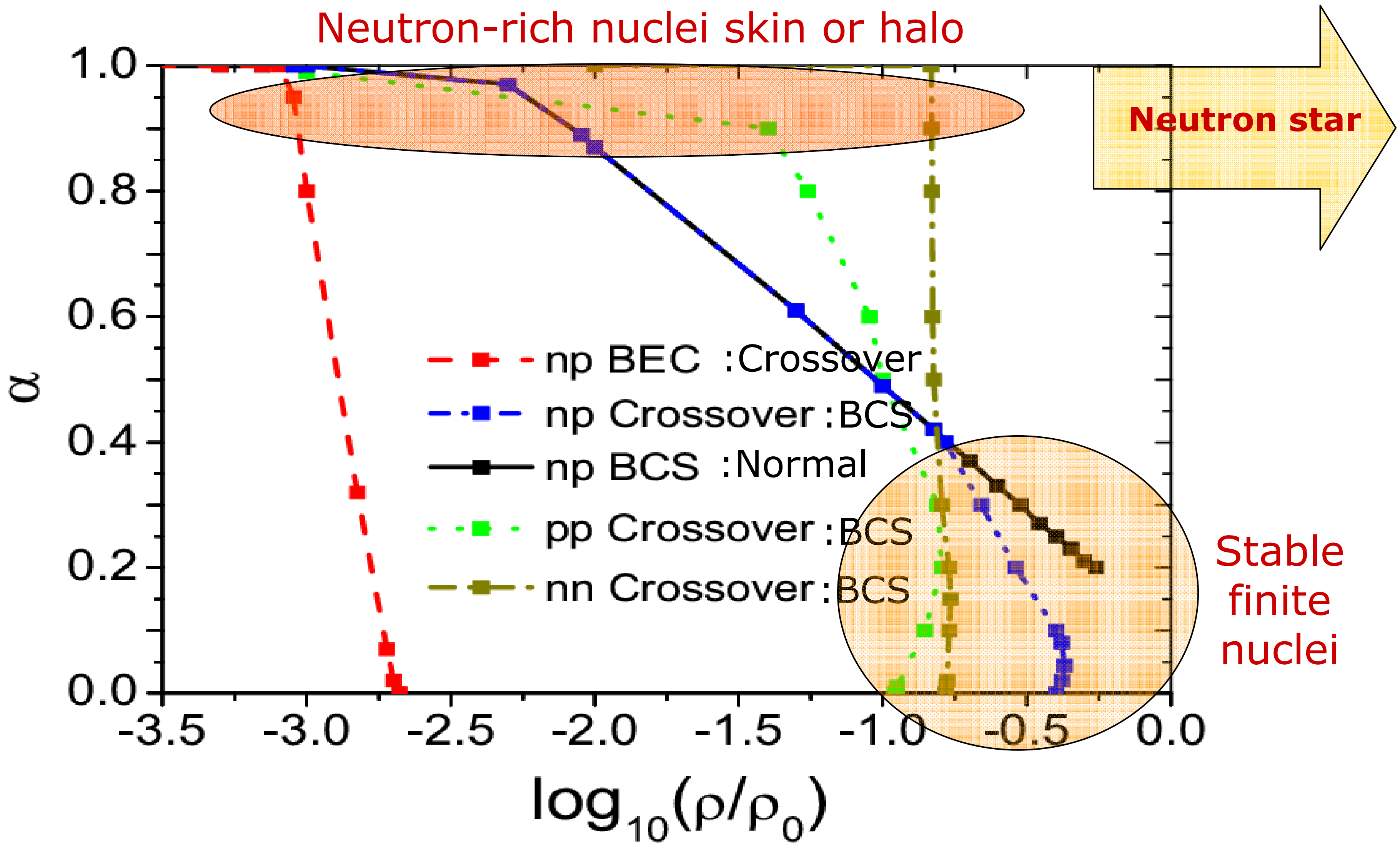
0 → ρ

nn pp strongly correlated **nn, pp BCS**

Alpha-rho phase diagram



Alpha-rho phase diagram



Phase diagram at high density

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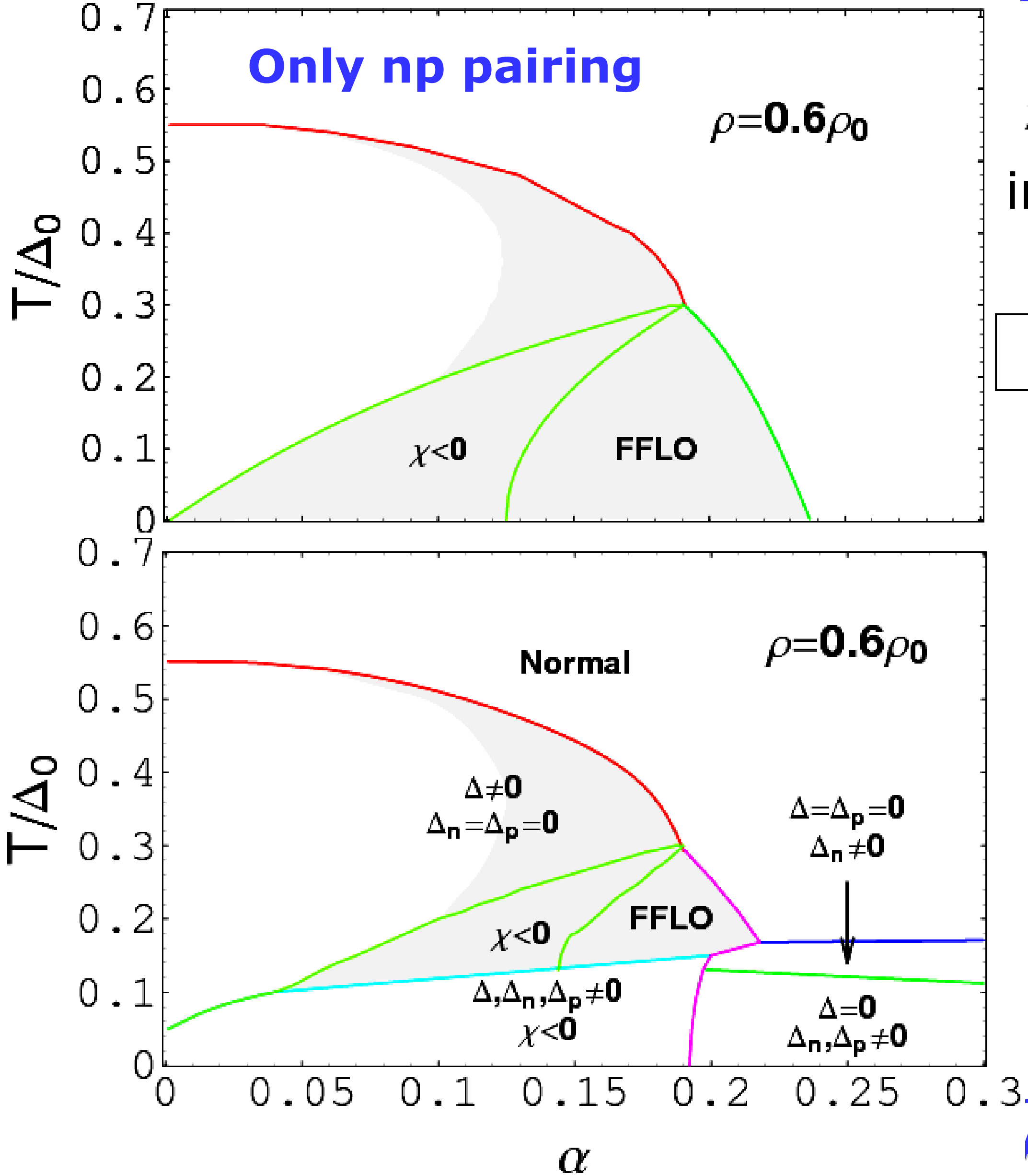


$\chi_{ij} = -\left.\frac{\partial^2 \mathcal{F}}{\partial\rho_i \partial\rho_j}\right|_\rho$ <0 is a strong indication for phase separation

Shadowed region: gapless

Due to nn pp pairing, at low temperature:

No gapless excitation

No FFLO phase

New kind of "phase separation"

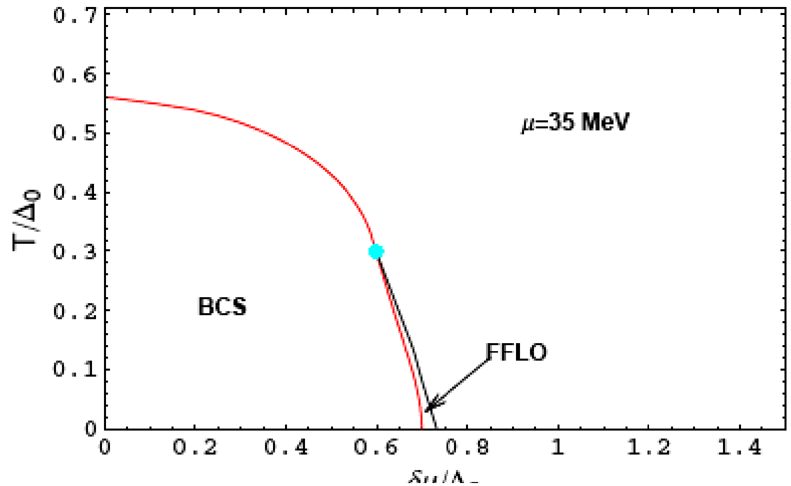
Summary and outlook

- Using a DDCI, we investigated the ***density driven*** BCS-BEC crossover in asymmetric nuclear system at mean field level.
- We obtained the phase diagram on alpha-rho plane at zero temperature and on T-alpha plane at high density.
- True BEC forms in np channel. No BEC reaced in nn, pp channel.
- Abundant phase structure. New kind of phase separation is formsed. Gapless superfluid is washed out at low T.

- Fluctuation effect on BCS-BEC crossover: aim to finite temperature.
- Finite size effect: aim to finite nuclei.

Phase diagram at high density

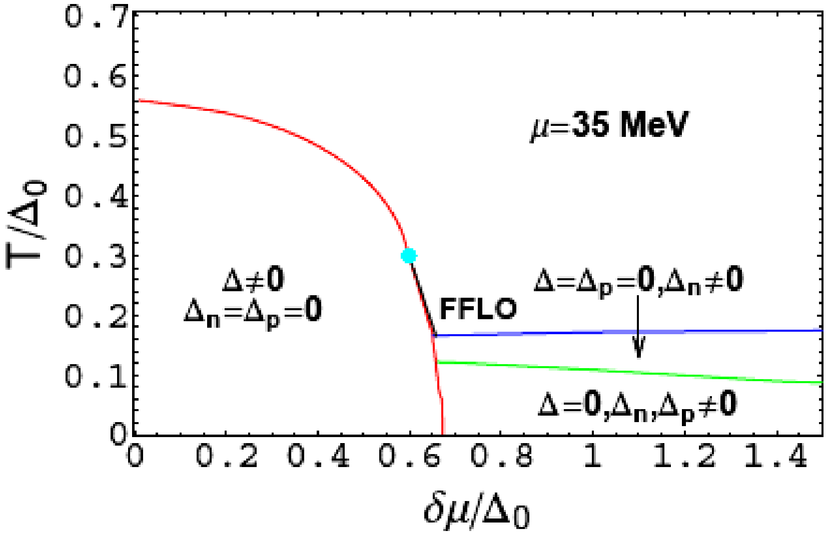
Map into temperature-chemical potential plane



No phase separation

1st order to FFLO

2nd order to Normal



No FFLO at low temperature