Few-body Physics with Ultracold Fermi Gases

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August 7, 2009

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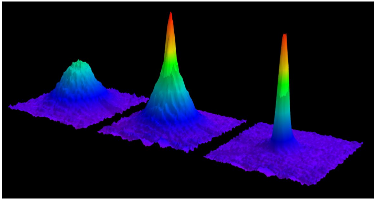
Motivation

- Research on ultracold atoms addresses many- and few-body phenomena



Ketterle group, MIT, 1995

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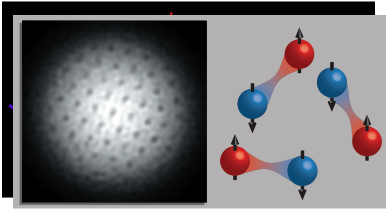

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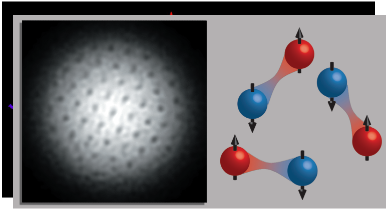

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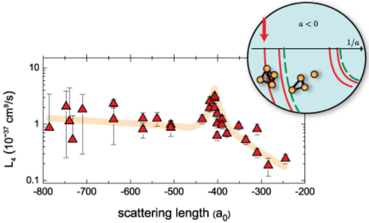

Grimm group, Innsbruck, 2008/09

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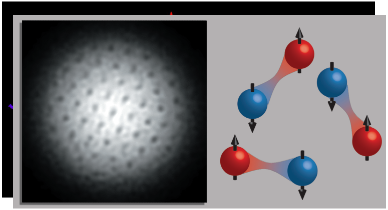

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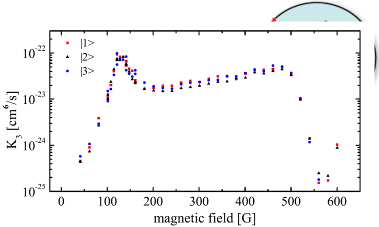

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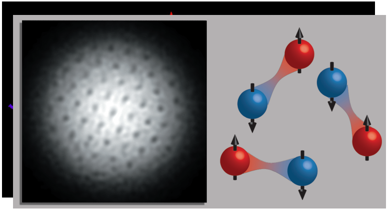

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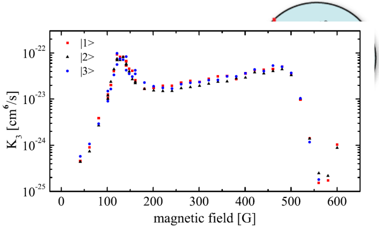

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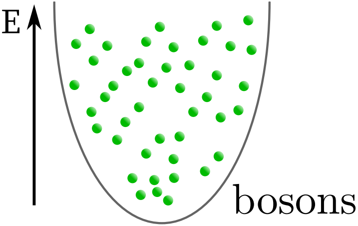
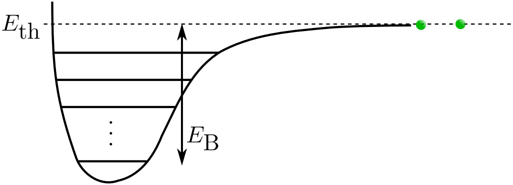
- Few-body physics: One important issue is the stability of ultracold atomic gases (losses → recombination collisions)

Recombination Processes





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$$\dot{n} = -K_3 n^3$$

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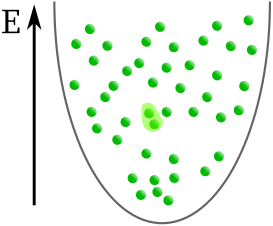

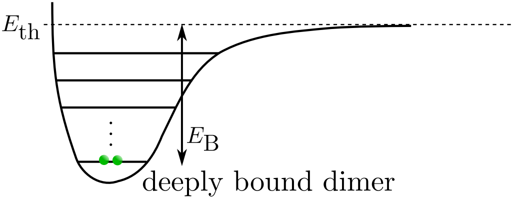

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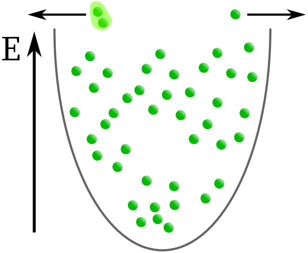

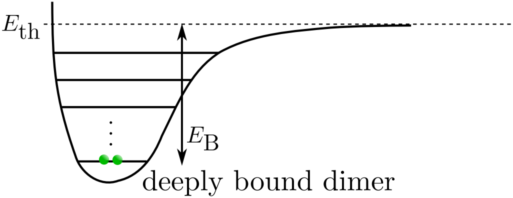

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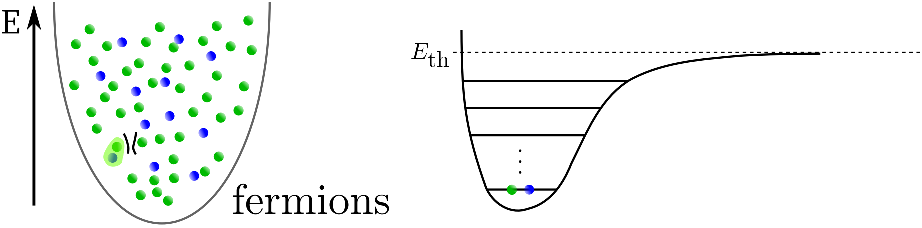

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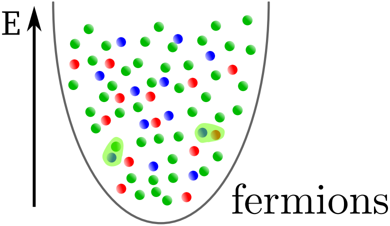

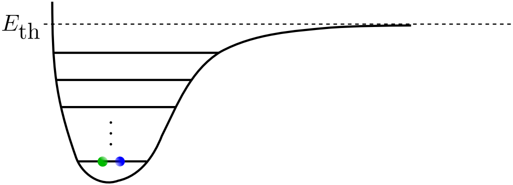

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- Recombination process is suppressed for two-component Fermi gases due to Pauli blocking
- This talk: *What is the stability of a three-component Fermi gas?*

Theoretical Background

- (Quantum) statistical physics: The partition function

$$Z_{qst} = \mathrm{Tr}\ e^{\beta H[\hat{\varphi},\mu]} = \int \mathcal{D}\varphi e^{-S[\varphi,\mu,\beta]} = \int \mathcal{D}\varphi e^{-\int_0^\beta d\tau \mathcal{L}[\varphi,\mu]}$$

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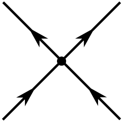

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Z_{qst}, Z_{qft}: two sides of the same coin

In the "vacuum limit" ($n = 0,\ T = 0$) Z_{qst} has to be equal to Z_{qft}.
Many-body calculations have to recover the few-body limit correctly

The Functional Renormalization Group (Talk by S. Floerchinger)

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- The computation is difficult.

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$$Z_k[J] = \int_\Lambda \mathcal{D}\varphi \, e^{-S[\varphi]+\int J\varphi - \frac{1}{2}\int \varphi R_k \varphi}$$

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- Legendre transformation: *effective flowing action*

$$\Gamma_k[\phi] \sim -\ln Z_k[J] + \int J\phi$$

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The Wetterich Equation

$$\partial_k \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2)} + R_k}\partial_k R_k\right]$$

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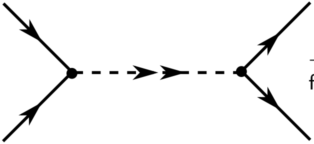

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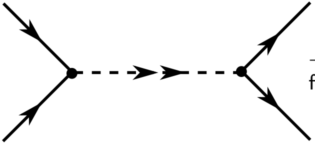

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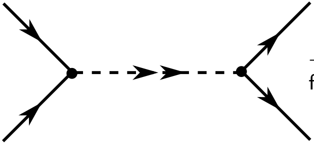

$\longrightarrow$ resonance if incoming particle energy is fits bound state energy

- Losses may be enhanced due to decay through these bound states
- Of special interest in the three-component gas: Existence of a three-body bound state (not prohibited by Pauli blocking)
  $\longrightarrow$ Efimov physics
  $\longrightarrow$ Phase of trions **(S. Floerchinger, RS, S. Moroz, C. Wetterich, PRA 79, 013603 (2009))**

Reminder of Stefan's talk

- Truncation for the SU(3) symmetric three-component Fermi gas:

$$\begin{aligned}\Gamma_k &= \int_x \psi^\dagger(\partial_\tau - \Delta + E_\psi)\psi + \phi^\dagger(A_\phi(\partial_\tau - \Delta/2) + m_\phi^2)\phi \\ &+\chi^*(A_\chi(\partial_\tau - \Delta/3) + m_\chi^2)\chi + \frac{h}{2}\epsilon_{ijk}(\phi_i^*\psi_j\psi_k + \phi_i\psi_k^*\psi_j^*) \\ &+g(\chi^*\psi_i\phi_i - \chi\psi_i^*\phi_i^*) + \lambda_{\phi\psi}(\phi_i^*\psi_i^*\phi_j\psi_j)\end{aligned}$$

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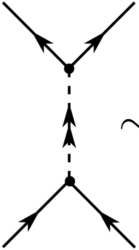

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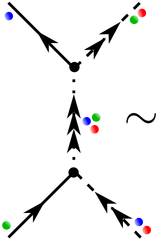

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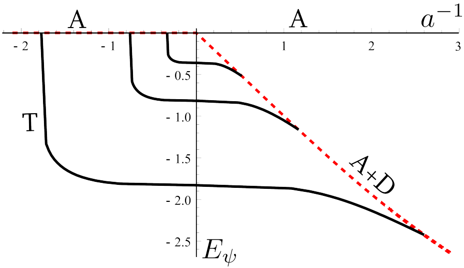

S. Floerchinger, RS, S. Moroz, C. Wetterich, PRA 79, 013603 (2009)

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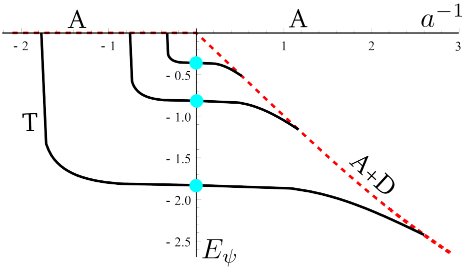

S. Floerchinger, RS, S. Moroz, C. Wetterich, PRA 79, 013603 (2009)

- *Universal* ratio between trimer levels

$$\frac{E_T^{(n+1)}}{E_T^{(n)}} = e^{-\frac{2\pi}{s_0}}$$

$$s_0 = 1.006\,(\text{exact})$$

Moroz, Floerchinger, RS, Wetterich PRA 79, 042705 (2009)

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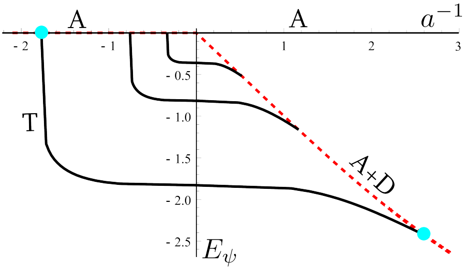

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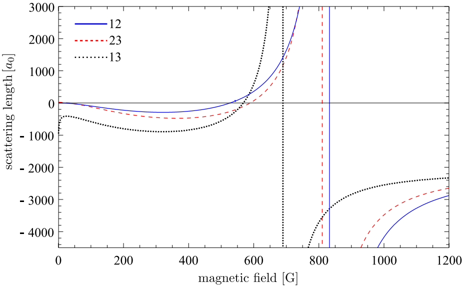
- overall energy degeneracy position depends on three body parameter: cutoff scale Λ (Braaten, Hammer: Λ_*)

The case of the three-component $^6\mathrm{Li}$ Fermi Gas

- Pairwise scattering length are not equal (SU(3) symmetry broken)
- Quite large and negative scattering lengths



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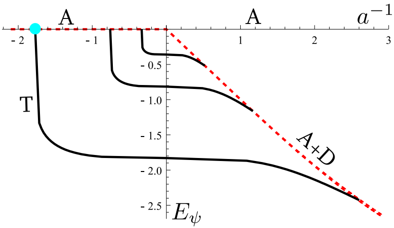
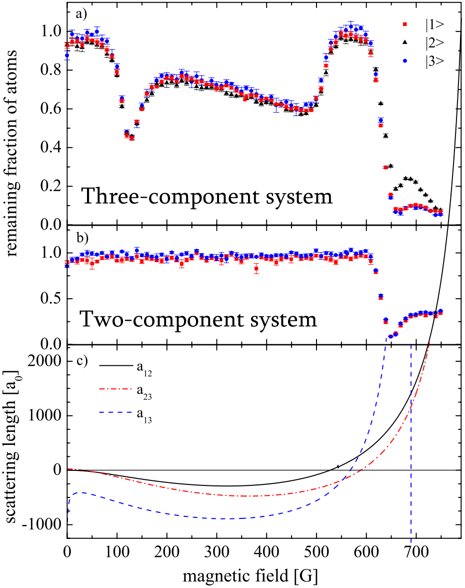
- Quite large and negative scattering lengths
- No losses in two-component system
- Loss features in three-component system
- Three-body effect. Efimov states crossing the atom threshold?





T. Ottenstein et al. PRL 101, 203202 (2008)

Adjusting the model

- We generalize the SU(3) symmetric model

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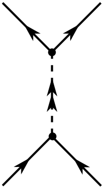
- We are able to implement all different scattering lengths a_{ij} exactly by using auxiliary boson exchange

$$a_{ij} = -\frac{h_{\phi k}^2}{8\pi m_{\phi k}^2(k=0, E_\psi = 0)} \sim$$

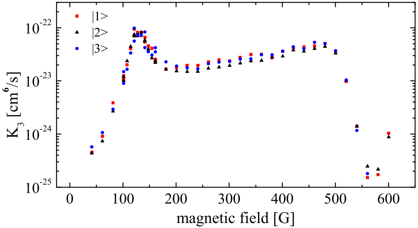


- The bosons (ϕ) do **not** represent the close by Feshbach molecules.

Experimental Findings II

- The quantitative measure of the three-body loss is given by

$$\dot{n} = -K_3 n^3$$



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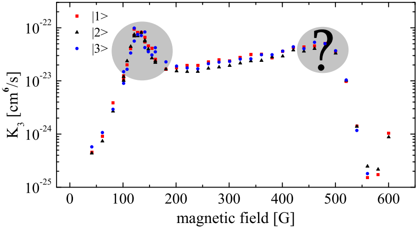

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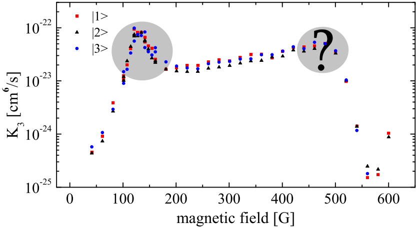
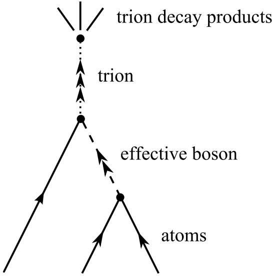
- K_3 loss features are due to a decay to deeply bound dimers through trimer-exchange process





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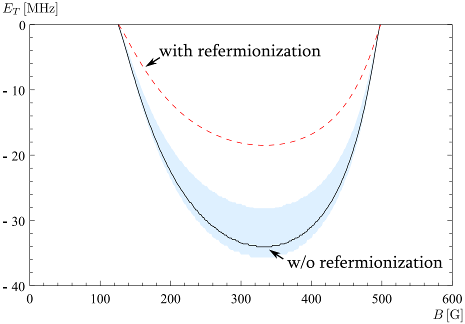

S. Floerchinger, RS, C. Wetterich PRA 79, 053633 (2009)
similar calculation: P. Naidon, M. Ueda, arXiv:0811.4086

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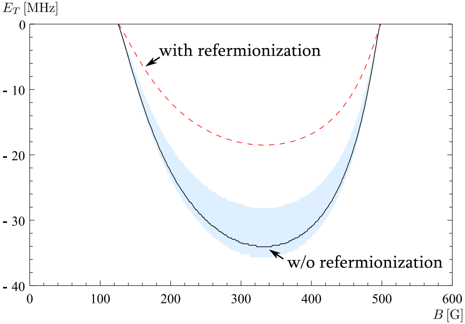
- Trimer energy level $E_T = 3E_\psi$ is given by condition $m^2_\chi(E_\psi) = 0$
- The degeneracy position $m^2_\chi(E_\psi = 0) = 0$ can be tuned by adjusting the three-body parameter Λ.

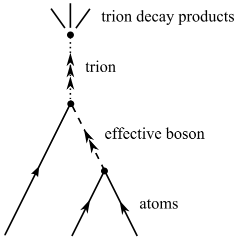


S. Floerchinger, RS, C. Wetterich PRA 79, 053633 (2009)
similar calculation: P. Naidon, M. Ueda, arXiv:0811.4086

- Fitting the first resonance position we find the second at $B = 500\,\mathrm{G}$

Calculation of K_3

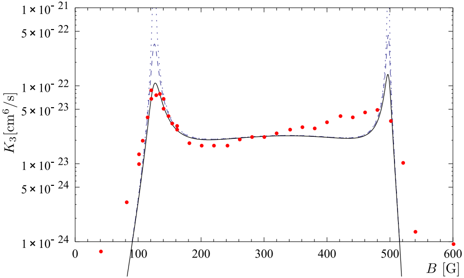
- For the evaluation of K_3 we calculate



$$K_3 \propto \left| \sum_{c=1}^{3} \frac{\bar{h}_c \bar{g}_c}{\bar{m}_{\phi c}^2} \frac{1}{\left(\bar{m}_\chi^2 - i\frac{\Gamma_\chi}{2}\right)} \right|^2$$

- We do not explicitly include the deeply bound states
- For a rough estimate we introduce a decay width Γ_χ assumed to be constant throughout the whole region, Γ_χ is fitted to the width of the first resonance

Calculation of K_3



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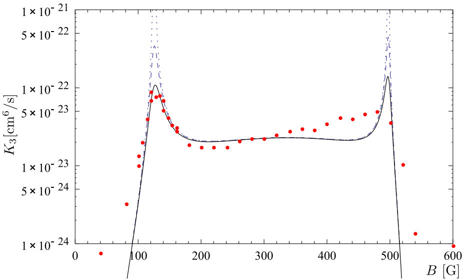

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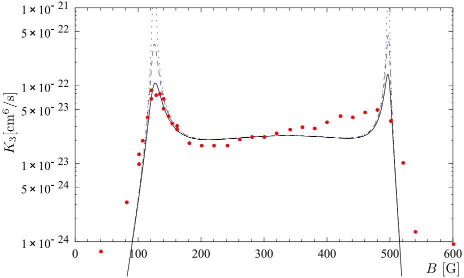

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Conclusion

By the investigation of the $^6\mathrm{Li}$ system we were able to fix all necessary microscopic couplings (two-body parameter: a_{ij}, three-body parameter: Λ)

Good starting point for many-body calculations exploring the (physical) phase diagram of three-component Fermi gases.

Effective field theory for four-body physics

- Consider a system of identical bosons
- Are there not only three-body bound Efimov states, but also four-body bound states?

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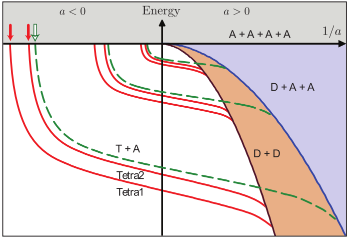

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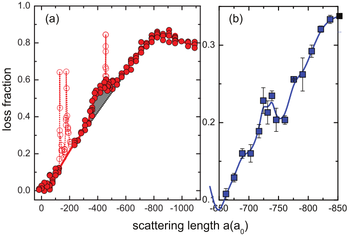

Ferlaino, Knoop, et al. PRL 102, 140401 (2009)

- Stecher et al.: Calculation of lowest five sets of Trimer/Tetramers:

$$\frac{E_{T1}}{E_{Tr}} = 5.88\ldots4.48 \quad \frac{E_{T2}}{E_{Tr}} = 1.01$$
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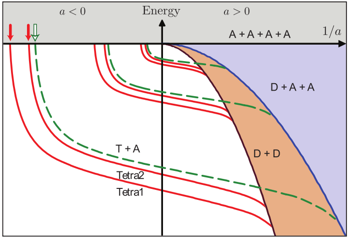

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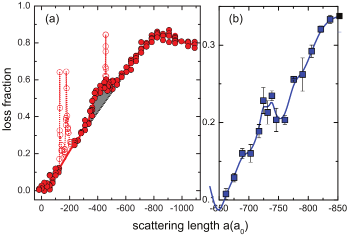

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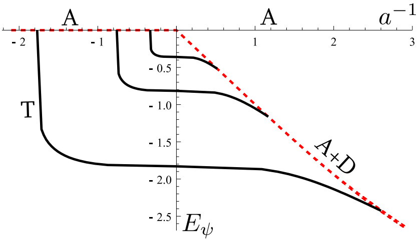
With the FRG: Calculation of the energy spectrum and especially:
Investigation of universality in the 'unitarity limit' ($a \rightarrow \infty$, $E_\psi = 0$)

Idea

Three-body physics



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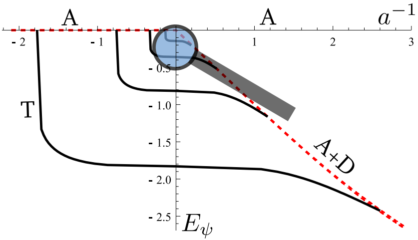

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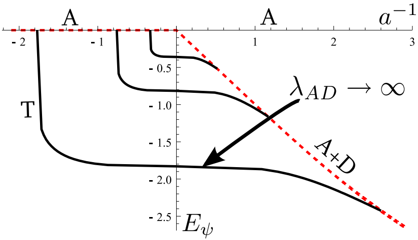

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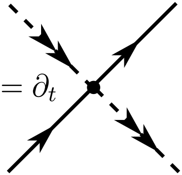

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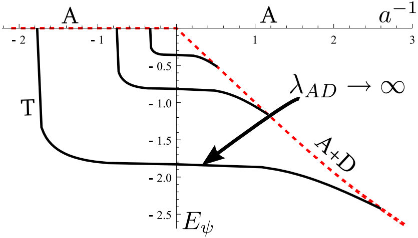

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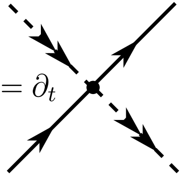
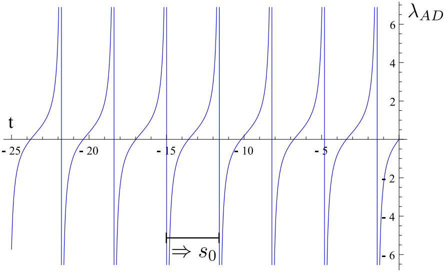
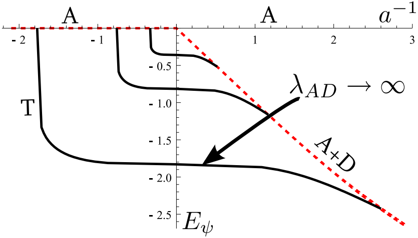




$\to$ RG limit cylce, universal number s_0

Idea

Three-body physics



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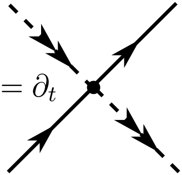
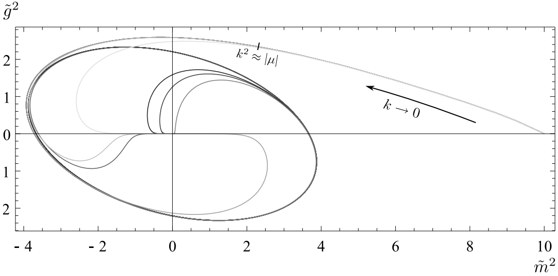


Why cycle?



Idea

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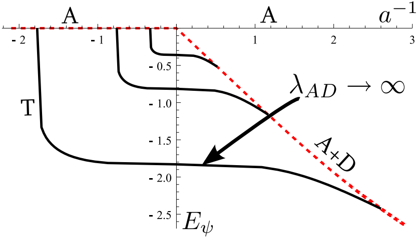

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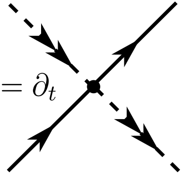
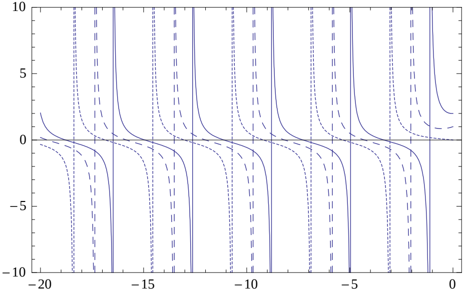


Dependence microscopic $\lambda_{AD}(UV)$?



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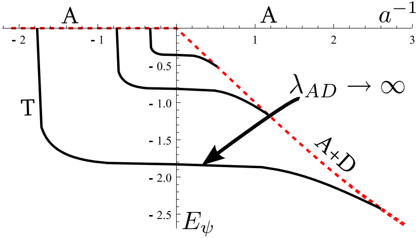

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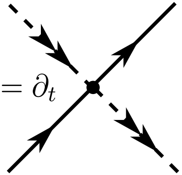

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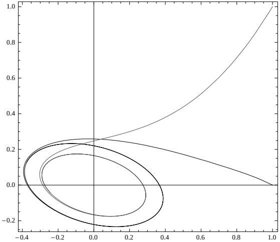


Microscopic details really get 'washed out'

Is there similar behavior in the four-body sector?

Model

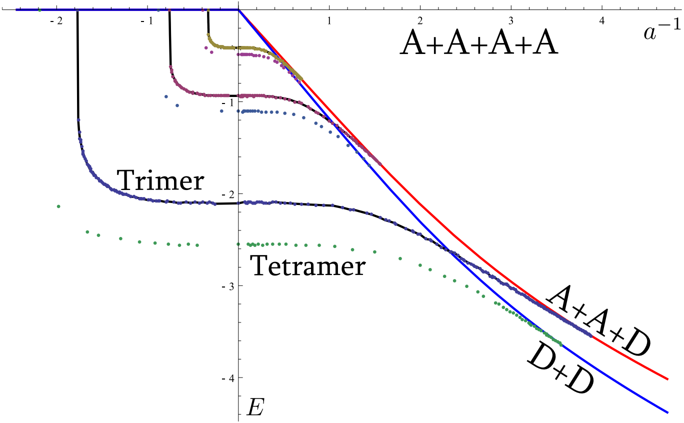
- Our truncation

$$\begin{aligned}\Gamma_k &= \int_p \psi^*(i\omega + \vec{p}^2 - \mu)\psi + \phi^*(A_\phi(i\omega + \frac{\vec{p}^2}{2}) + m_\phi)\phi + h(\phi^*\psi\psi + \phi\psi^*\psi^*) \\ &+ \lambda_{AD}\phi^*\psi^*\phi\psi \\ &+ \lambda_\phi(\phi^*\phi)^2 + \beta(\phi^*\phi^*\phi\psi\psi + \phi\phi\phi^*\psi^*\psi^*) + \gamma\phi^*\psi^*\psi^*\phi\psi\psi\end{aligned}$$

- λ_{AD}, λ_ϕ, β, γ assumed to be momentum-independent.
- All other possible U(1) symmetric coupling terms have vanishing RG flows in vacuum.
- Tetramer (four-body bound) states appear as resonances in the four-body sector couplings.

Results I

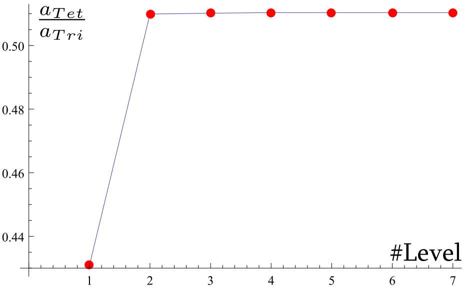
Away from resonance

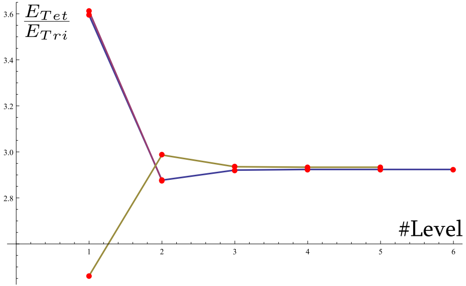


RS, S. Moroz (in preparation)

- We do not find the second tetramer state
  - no problem with the numerical resolution
  - momentum dependencies in three- and four-body sector needed?
  - We tried approximation with dynamical trimer field (Taylor expansion in $i\omega + \vec{p}/3$ to first order)

Results II





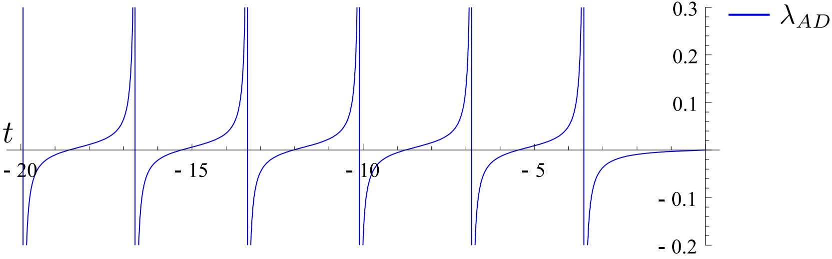
- We find

$$\frac{E_{T1}}{E_{Tr}} = 3.9 \quad (5.88 \ldots 4.48)$$
$$\frac{a_{T1}}{a_{Tr}} = 0.51 \quad (0.43)$$

- Universal region is reached within a few sets of levels

Instead of a conclusion

The unitarity limit $E_\psi = 0$, $a \to \infty$



3-body sector limit cycle

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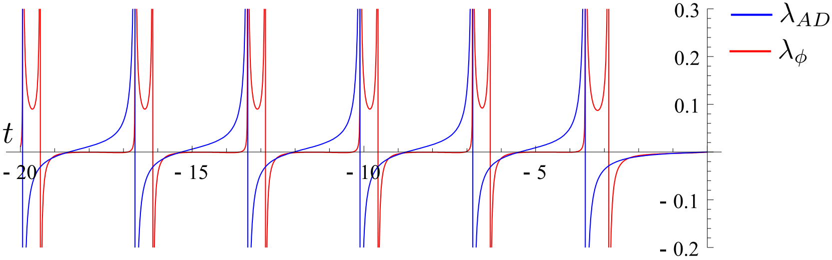

3-body and 4-body sector limit cycle. Shift of resonances determines new universal number

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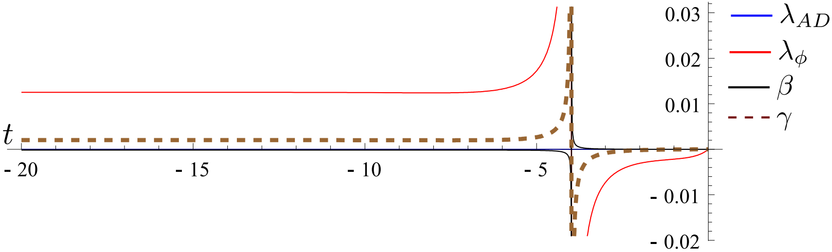

λ_{AD} set to zero. Two-body sector driggers exactly one four-body bound state

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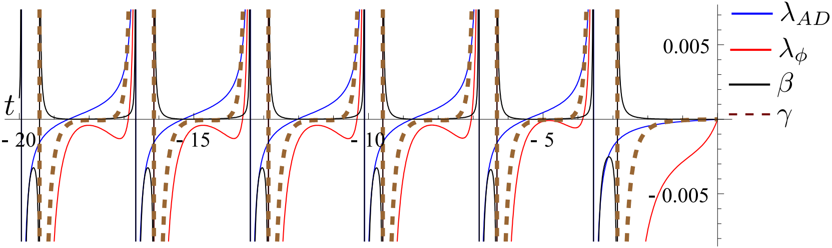

Full RG running: Three-body limit cycle leads to 'self-sustained' 4-body sector limit cycle

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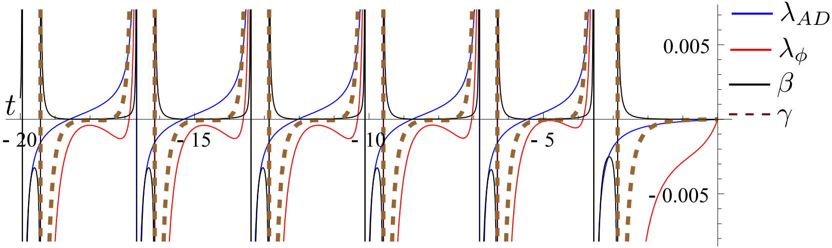

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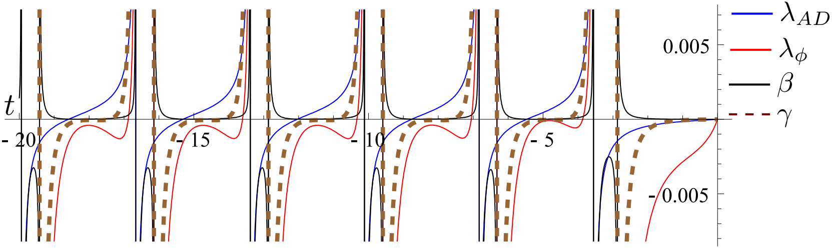

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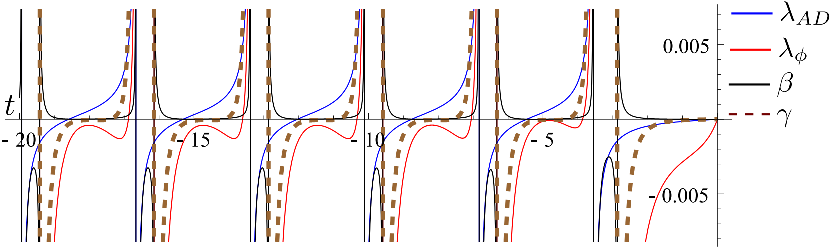

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Conclusion

- Approximation not sufficient for 'detection' of second tetramer state
- Still, we might learn about which momentum dependencies are crucial in n-body physics
- Field theoretical calculation possible in unitarity limit:
  No additional four-body parameter ⇒ Universality