Bayes-Tracking
A novel Approach to Gamma-Ray Tracking

P. Napiralla, H. Egger, P. John, N. Pietralla, M. Reese, C. Stahl

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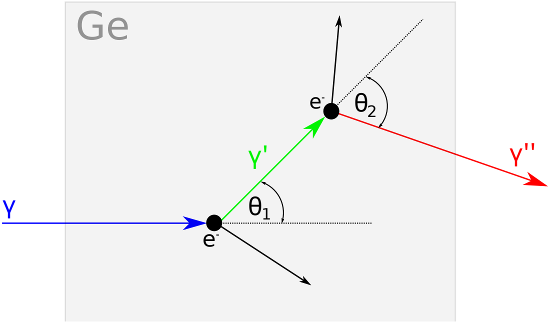

Figure: Compton-Escape Event in a Germanium detector

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05P15RDFN1 - TP9 (Experiment)

Progress of AGATA

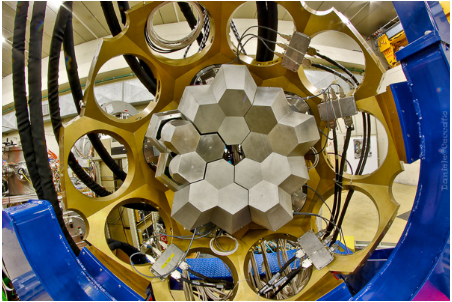

Figure: AGATA at LNL (15 crystals)
(D. Ceccato. Agata demonstrator.
http://agata.lnl.infn.it/Inaugurazione/foto/official/content/AD_7_large.html)

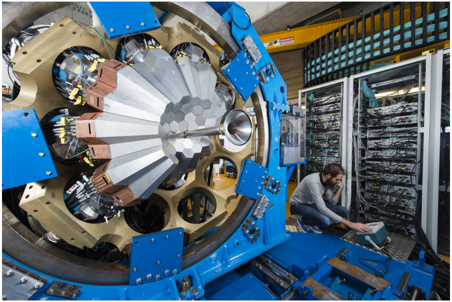

Figure: AGATA at GANIL (now 35 crystals)
(Photo by E. Clément. https://www.agata.org/sites/default/files/_STR7915.jpg)

Major Gamma-Ray Tracking Algorithms

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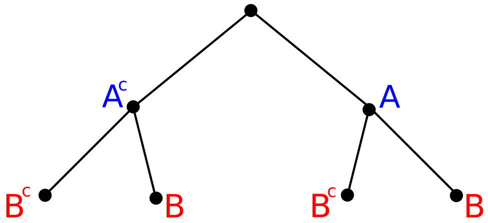
	Major Tracking algorithms	
	Forward Tracking	*Back-Tracking*
Starting point	First interaction in cluster	Assumed photo-absorption
E_γ *identification*	$\sum E_{\text{dep}}$ in cluster	$\sum E_{\text{dep}}$ in cluster

Problem: Compton-Escaped photons “useless” for energy reconstruction

⇒ New algorithm: ***Bayes-Tracking***

Bayes-Tracking

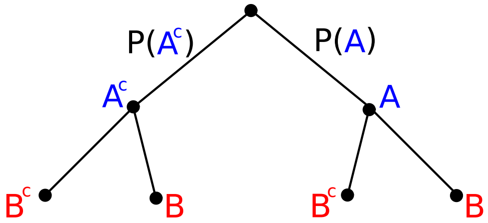
Bayes' Theorem



Let *A* and *B* be two events.

Bayes-Tracking

Bayes' Theorem



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Bayes-Tracking

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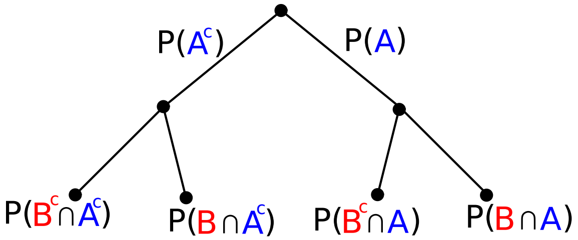

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Bayes-Tracking

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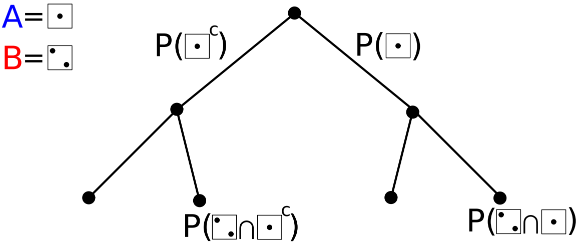

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Bayes-Tracking

Bayes' Theorem

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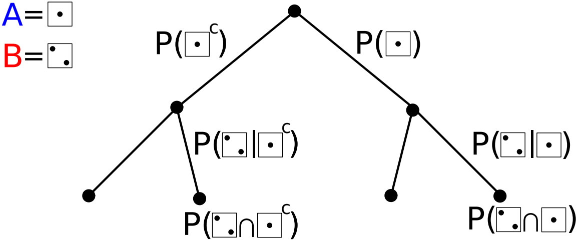
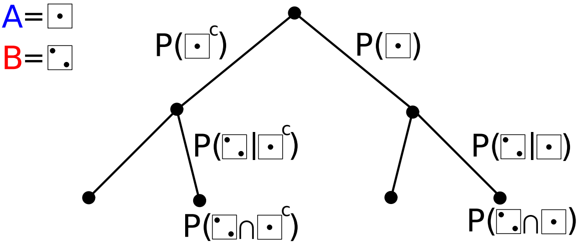


Let *A* and *B* be two events. Conditional probability of *B*, given *A* is true:

$$P\left(B|A\right) = \frac{P\left(A \cap B\right)}{P\left(A\right)}, \qquad P\left(A \cap B\right) = P\left(B \cap A\right)$$

Bayes-Tracking

Bayes' Theorem



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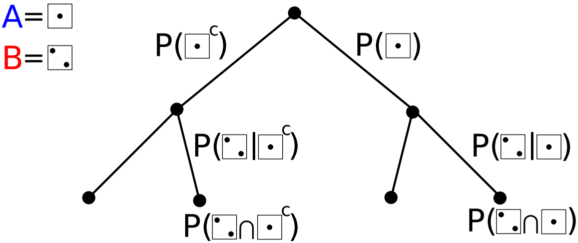
From this *Bayes' theorem* follows

$$P\left(B|A\right) = \frac{P\left(A|B\right) \cdot P\left(B\right)}{P\left(A\right)}.$$

Bayes-Tracking

Bayes' Theorem



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From this *Bayes' theorem* follows

$$P\left(hypothesis|data\right) = \frac{P\left(data|hypothesis\right) \cdot P\left(hypothesis\right)}{P\left(data\right)}.$$

Bayes-Tracking

Bayes' Theorem

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$$P\left(hypothesis \middle| data\right) = \frac{P\left(data \middle| hypothesis\right) \cdot P\left(hypothesis\right)}{P\left(data\right)}$$

	data	*hypothesis*
data		
hypo-thesis	Posterior $P\left(hypothesis \middle| data\right)$ Probability of *hypothesis*, given *data* is true	

Bayes-Tracking

Bayes' Theorem

$$P\left(hypothesis \middle| data\right) = \frac{P\left(data \middle| hypothesis\right) \cdot P\left(hypothesis\right)}{P\left(data\right)}$$

	data	*hypothesis*
data	*Evidence* $P(data)$ Knowledge about *data*	
hypo-thesis	*Posterior* $P\left(hypothesis \middle| data\right)$ Probability of *hypothesis*, given *data* is true	

Bayes-Tracking

Bayes' Theorem

$$P\left(\textit{hypothesis} \middle| \textit{data}\right) = \frac{P\left(\textit{data} \middle| \textit{hypothesis}\right) \cdot P\left(\textit{hypothesis}\right)}{P\left(\textit{data}\right)}$$

	data	*hypothesis*
data	Evidence $P(\textit{data})$ Knowledge about *data*	
hypo-thesis	Posterior $P\left(\textit{hypothesis} \middle| \textit{data}\right)$ Probability of *hypothesis*, given *data* is true	Prior $P(\textit{hypothesis})$ Knowledge about *hypothesis*

Bayes-Tracking

Bayes' Theorem

$$P\left(\textit{hypothesis} \middle| \textit{data}\right) = \frac{P\left(\textit{data} \middle| \textit{hypothesis}\right) \cdot P\left(\textit{hypothesis}\right)}{P\left(\textit{data}\right)}$$

	data	*hypothesis*
data	*Evidence* $P(\textit{data})$ Knowledge about *data*	**Likelihood Fct.** $P\left(\textit{data} \middle| \textit{hypothesis}\right)$ Plausibility of *data*, given *hypothesis* is true
hypo-thesis	*Posterior* $P\left(\textit{hypothesis} \middle| \textit{data}\right)$ Probability of *hypothesis*, given *data* is true	*Prior* $P(\textit{hypothesis})$ Knowledge about *hypothesis*

Bayes-Tracking

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Requirements on *Bayes-Tracking*

- **Goal**: Identify incident photon energy E_γ
- **Data**: Deposited energies $\left\{E_{\text{dep}_1}, \dots, E_{\text{dep}_N}\right\}$ at interaction points $\{\vec{x}_1, \dots, \vec{x}_N\}$

$\Rightarrow$ Using Bayes' theorem, calculate $P\left(e_0 | \left\{\{E_{\text{dep}_1}, \vec{x}_1\}, \dots, \{E_{\text{dep}_N}, \vec{x}_N\}\right\}\right)$

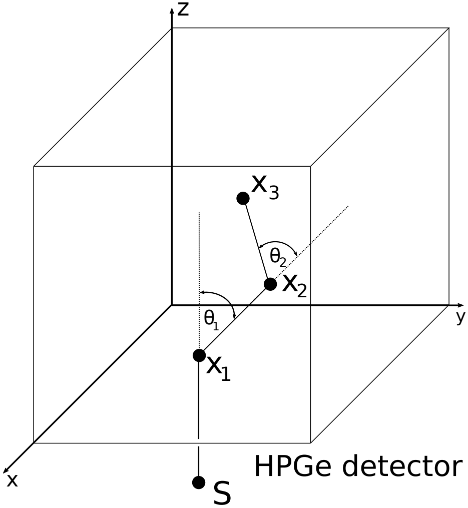
$$\Rightarrow P\left(e_0 | \dots, \{E_{\text{dep}_i}, \vec{x}_i\}, \dots\right) \propto P\left(\dots, \{E_{\text{dep}_i}, \vec{x}_i\}, \dots | e_0\right)$$

e_0: hypothetical incident photon energies

Bayes-Tracking

<u>Sources of Information:</u>

- Compton-scattering for $i = 1, \ldots, N-1$
- Mean-Free-Path λ
- Last interaction: Compton- or Photoelectric effect
- Measurement uncertainties



Bayes-Tracking

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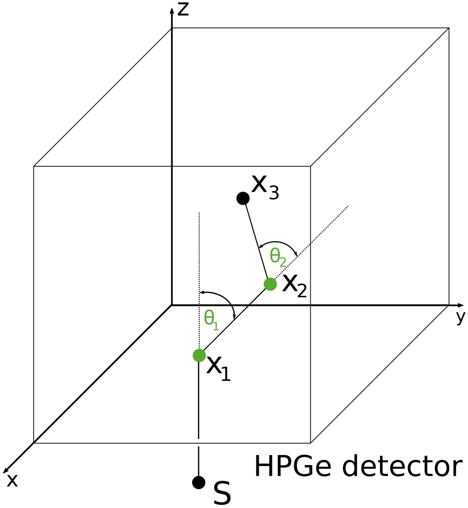

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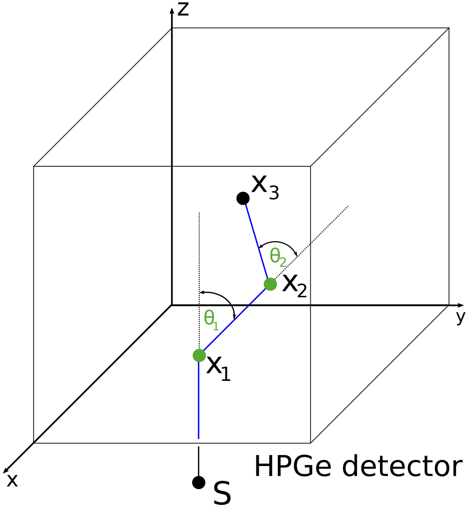

Bayes-Tracking

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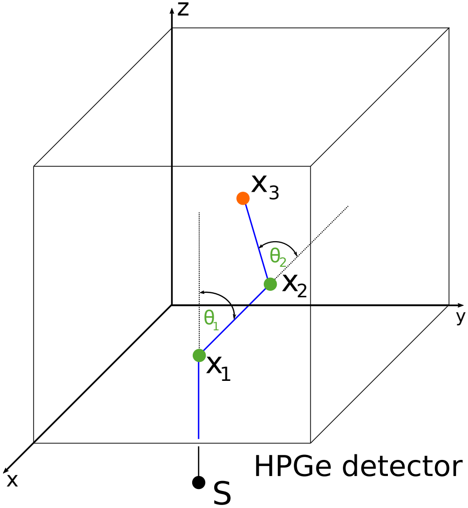

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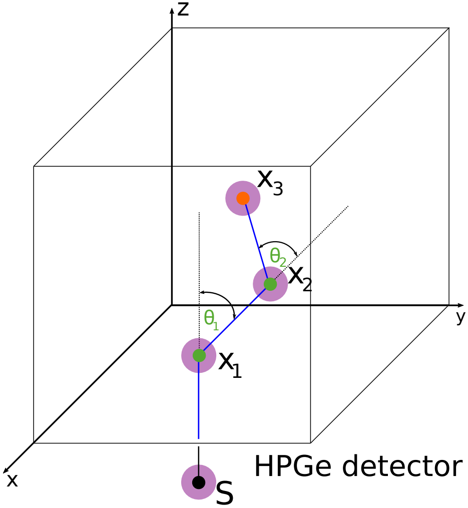


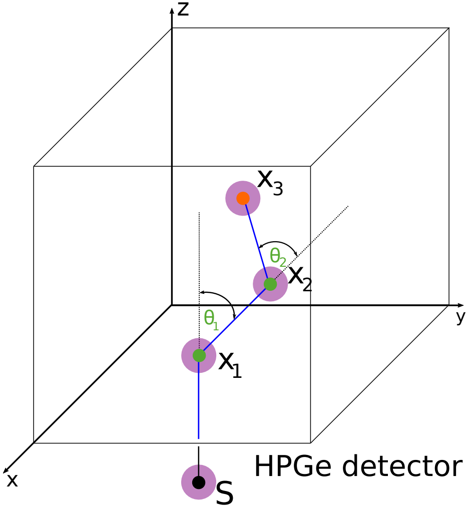
Bayes-Tracking

Sources of Information:

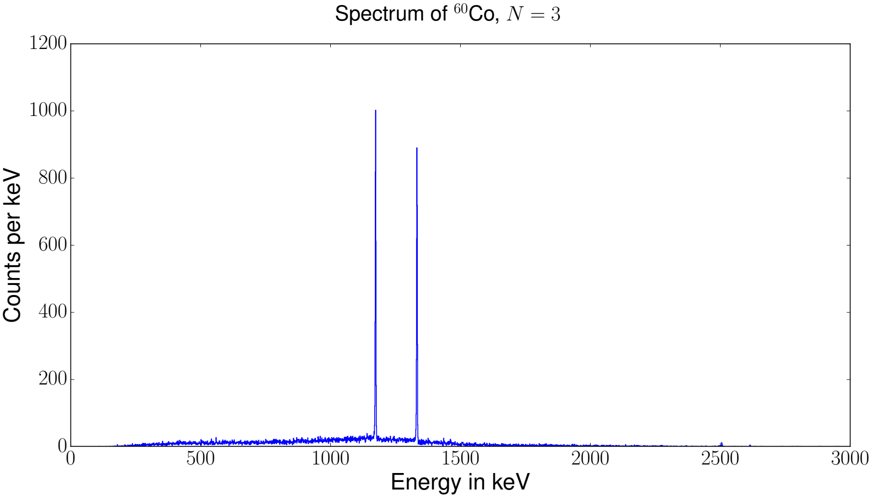
- Compton-scattering for $i = 1, \ldots, N-1$
- Mean-Free-Path λ
- Last interaction: Compton- or Photoelectric effect
- Measurement uncertainties

$$\Rightarrow P\left(\ldots, \{\vec{x}_i, E_{\mathrm{dep}_i}\}, \ldots \middle| e_0\right) = $$

$$= \int_\Omega \prod_{i=1}^{N-1} \left[P_{\mathrm{int},i} \cdot P_{\lambda,i} \cdot \mathcal{G}_{\vec{x},i} \cdot \mathcal{G}_{E,i}\right] \cdot P_{\mathrm{last}}\, \mathrm{d}\vec{\mu}_0 \cdots \mathrm{d}\vec{\mu}_N\,.$$



Bayes-Tracking – Tracking Performance



Bayes-Tracking – Tracking Performance

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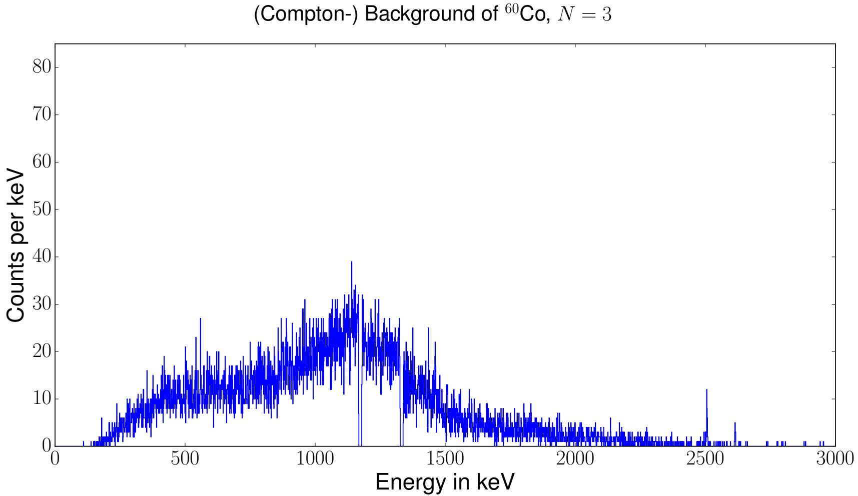

Bayes-Tracking – Tracking Performance

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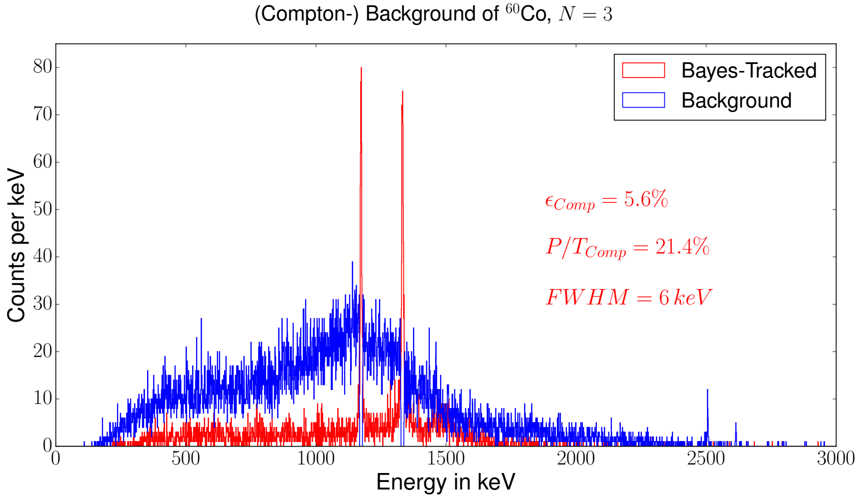

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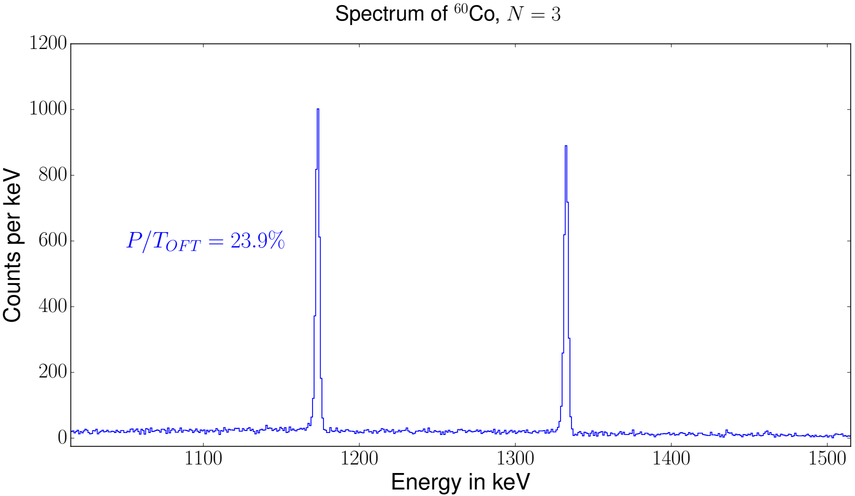

Bayes-Tracking – Tracking Performance

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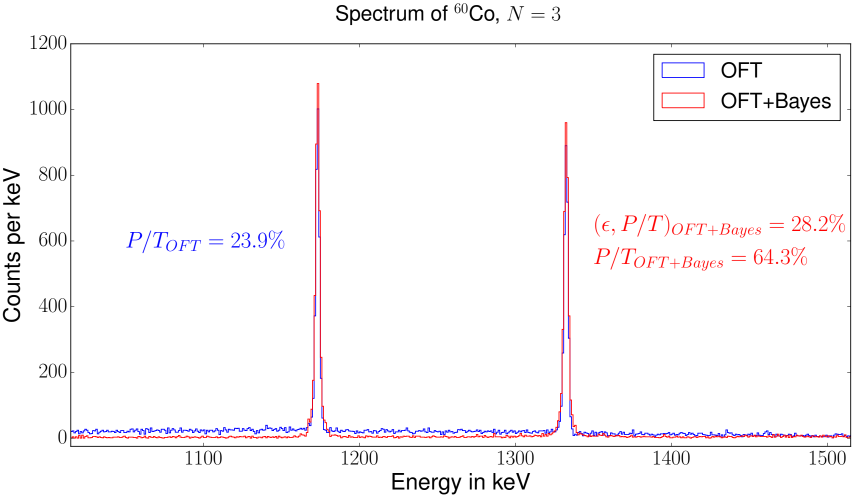

Conclusion & Outlook

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Conclusion:

- Bayes-Tracking as new Tracking algorithm
- Energy reconstruction using Compton-Escape-Events (FWHM $\approx$ 6 keV)

Outlook:

Conclusion & Outlook

Conclusion:

- Bayes-Tracking as new Tracking algorithm
- Energy reconstruction using Compton-Escape-Events (FWHM $\approx$ 6 keV)

Outlook:

- Implementation of Clustering algorithm
- Higher reconstruction/tracking efficiency
- Pair production and photon polarization
- Embed Bayes-Tracking into *Femul*

Additional plots

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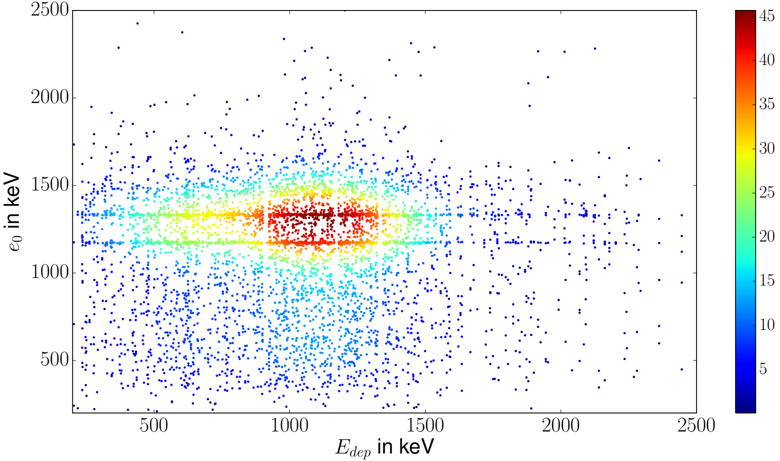

Additional plots

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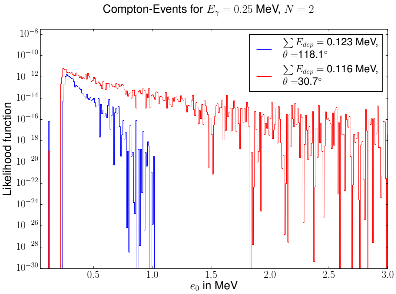
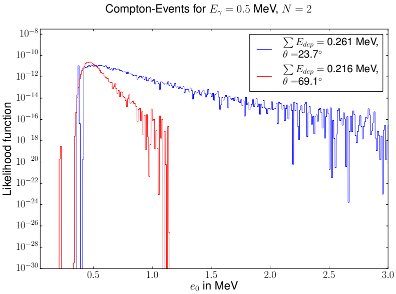


(a) $E_\gamma = 0.25$ MeV



(b) $E_\gamma = 0.5$ MeV

Figure: Results of the Bayes-Tracking for Compton-Events with ingoing photon energies $E_\gamma = 0.25$ MeV (a) and 0.5 MeV (b) with $N = 2$.

Additional plots

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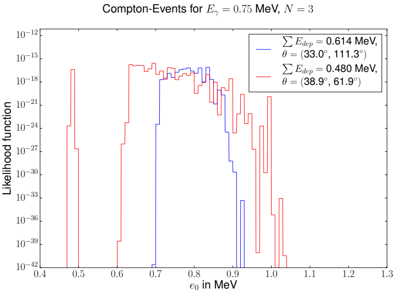
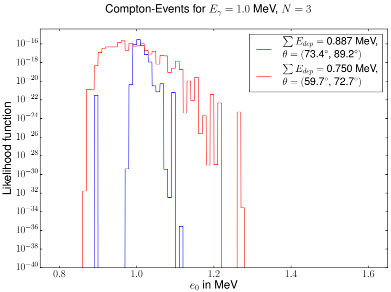


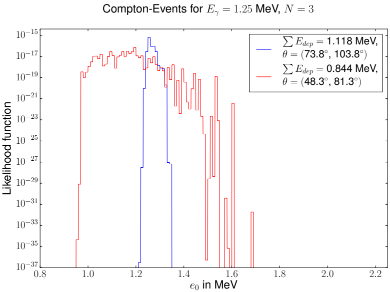
(a) E_γ = 0.75 MeV



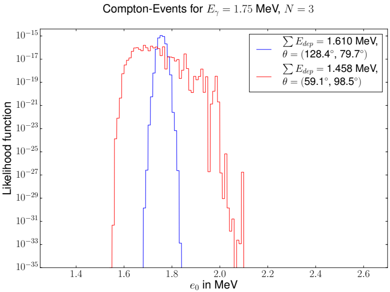
(b) E_γ = 1.0 MeV

Figure: Results of the Bayes-Tracking for Compton-Events with ingoing photon energies E_γ = 0.75 MeV (a) and 1.0 MeV (b) with N = 3.

Additional plots



(a) $E_\gamma = 1.25$ MeV



(b) $E_\gamma = 1.75$ MeV

Figure: Results of the Bayes-Tracking for Compton-Events with ingoing photon energies $E_\gamma = 1.25$ MeV (a) and 1.75 MeV (b) with $N = 3$.

Additional plots

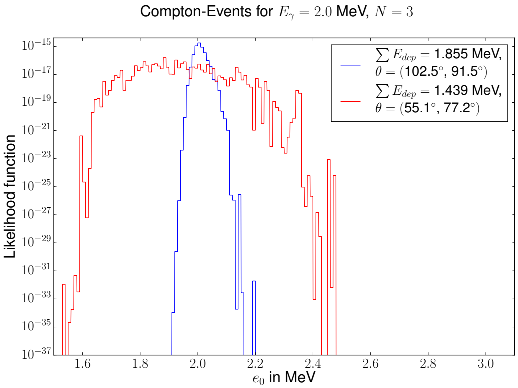


Figure: Results of the Bayes-Tracking for Compton-Events with ingoing photon energies $E_\gamma = 2.0$ MeV with $N = 3$.

Additional plots

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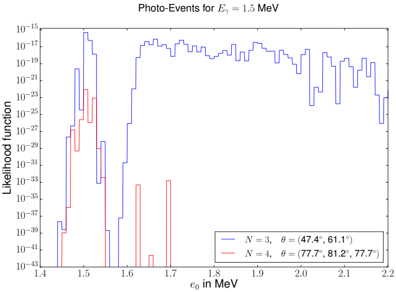
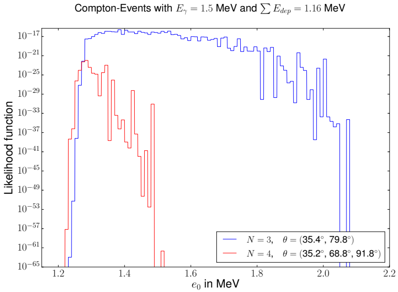


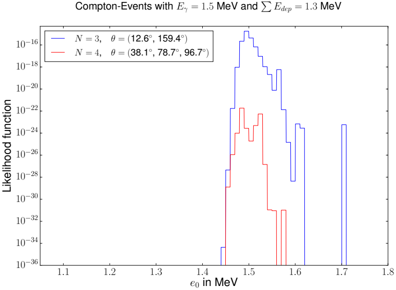
(a) $\sum E_{\text{dep}} = 1.5\,\text{MeV}$



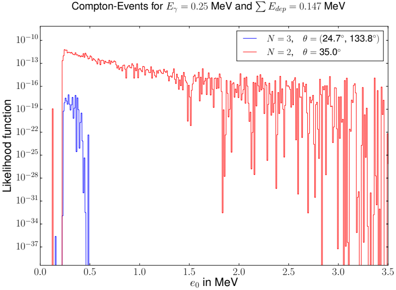
(b) $\sum E_{\text{dep}} = 1.16\,\text{MeV}$

Figure: Comparison of photons with three and four interactions inside the detector that either deposited their whole energy (a), or 1.16 MeV (b).

Additional plots



(a) $\sum E_{\text{dep}} = 1.3\,\text{MeV}$, $N = 3$ and $N = 4$



(b) $\sum E_{\text{dep}} = 0.147\,\text{MeV}$, $N = 2$ and $N = 3$

Figure: Comparison of photons that deposited 1.3 MeV inside the detector in three and four interactions (a). In addtion, the influence of a smaller amount of interactions is shown in (b) for $E_\gamma = 0.25\,\text{MeV}$ and $\sum E_{\text{dep}} = 0.147\,\text{MeV}$ for three and two interactions.

Additional plots

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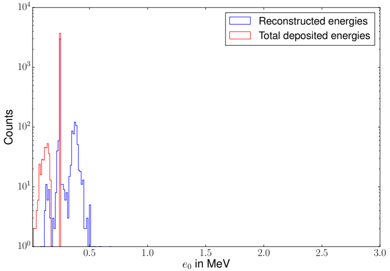
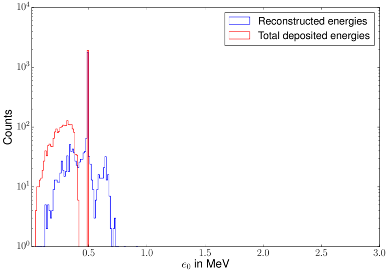


(a) $E_\mu = (0.25 \pm 0.005)\,\text{MeV}$, $N_P/N_C = 1$



(b) $E_\mu = (0.5 \pm 0.005)\,\text{MeV}$, $N_P/N_C = 0.96$

Figure: Energy reconstruction for $N = 3$ with $E_\gamma = 0.25\,\text{MeV}$ and $0.5\,\text{MeV}$ compared to the respective total deposited energy.

Additional plots

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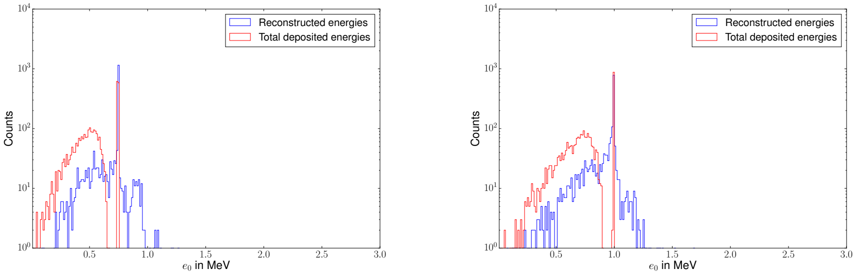
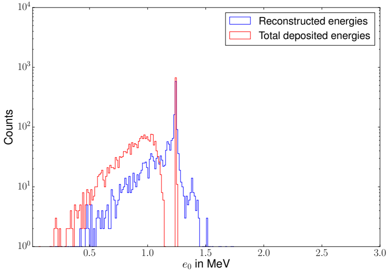


(a) $E_\mu = (0.75 \pm 0.005)$ MeV, $N_P/N_C = 0.6$

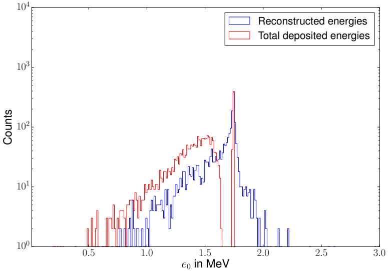
(b) $E_\mu = (1.0 \pm 0.005)$ MeV, $N_P/N_C = 0.44$

Figure: Energy reconstruction for $N = 3$ with $E_\gamma = 0.75$ MeV and 1.0 MeV compared to the respective total deposited energy.

Additional plots



(a) $E_\mu = (1.25 \pm 0.005)$ MeV, $N_P/N_C = 0.34$



(b) $E_\mu = (1.75 \pm 0.01)$ MeV, $N_P/N_C = 0.22$

Figure: Energy reconstruction for $N = 3$ and $E_\gamma = 1.25$ MeV and 1.75 MeV compared to the respective total deposited energy.

Additional plots

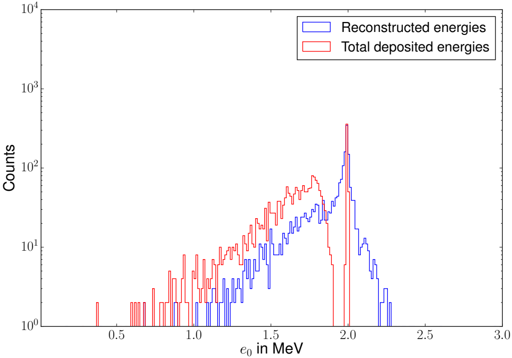
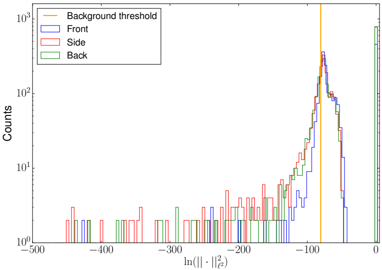
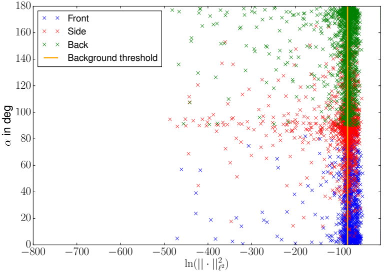


Figure: Energy reconstruction for $N = 3$ with $E_\gamma = (2.0 \pm 0.01)$ MeV ($N_P/N_C = 0.21$) compared to the respective total deposited energy.

Additional plots



(a) $\ln(||(p_n)_n||^2_{\ell^2})$ for general directions



(b) $\ln(||p_n)_n||^2_{\ell^2})$ depending on α

Figure: Histogram of $\ln(||(p_n)_n||^2_{\ell^2})$ for the general ingoing directions of the background photons (front, side, back of detector) (a) and $\ln(||(p_n)_n||^2_{\ell^2})$ depending on the exact angle between the source photon direction and the background photon direction α (b).

Additional plots

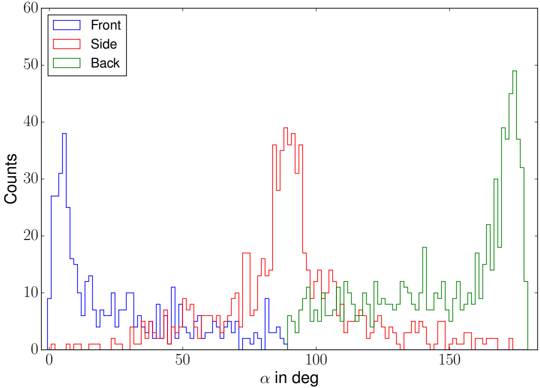


Figure: Histogram for background photons that yielded a likelihood function of zero depending on their angle of incidence α.

Additional plots

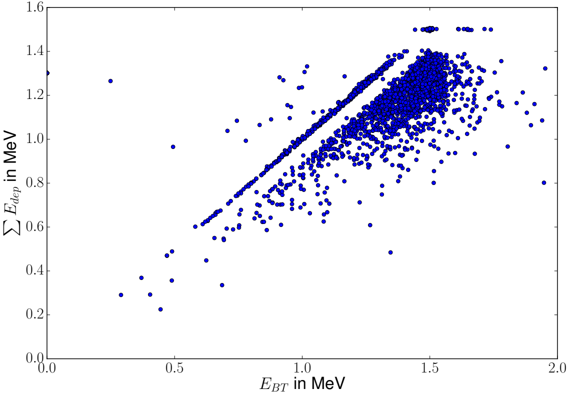


Figure: Influence of the total deposited energy $\sum E_{\text{dep}}$ on the reconstructed energy E_{BT}.

Additional plots

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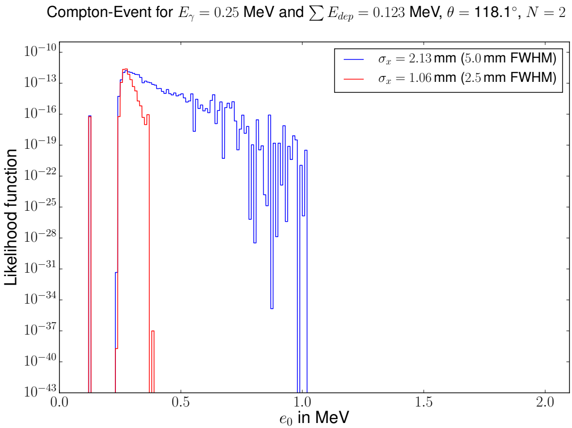

Figure: Influence of the interaction point measurement uncertainty σ_x on the likelihood function.