

# Effect of space charge on the CERN LHC and SPS transverse instabilities: simulations vs. measurements

Eliás Métral

BE/ABP-HSC (Collective/Coherent Effects)

<https://espace.cern.ch/be-dep-workspace/abp/HSC/SitePages/Home.aspx>

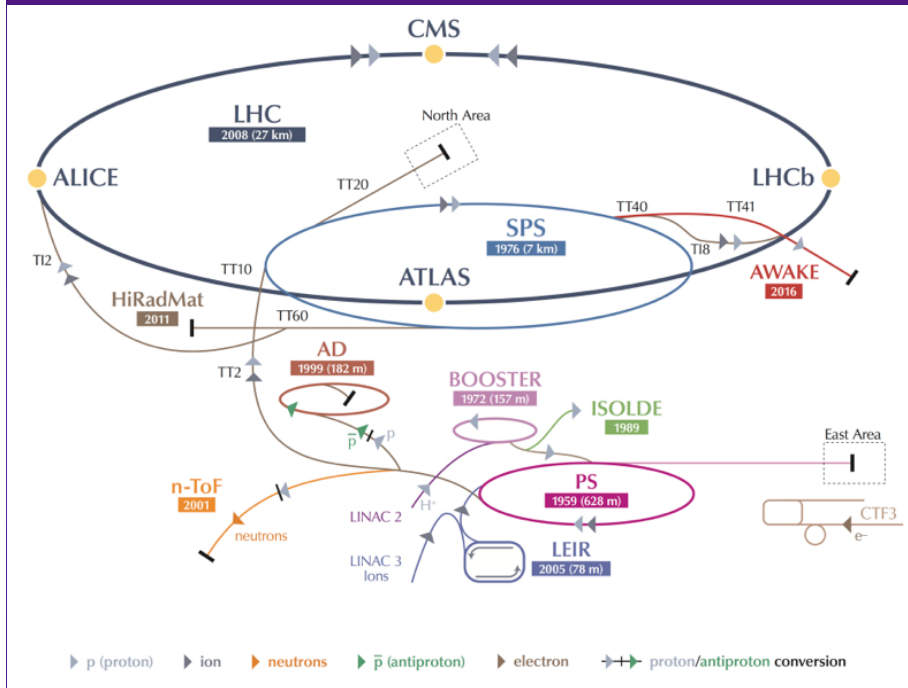
[Elias.Metral@cern.ch](mailto:Elias.Metral@cern.ch)

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<http://emetral.web.cern.ch/emetral/>

# MAIN MESSAGE

At CERN, it seems that only the LHC (highest energy machine) sees the (beneficial) effect of Space Charge (SC)

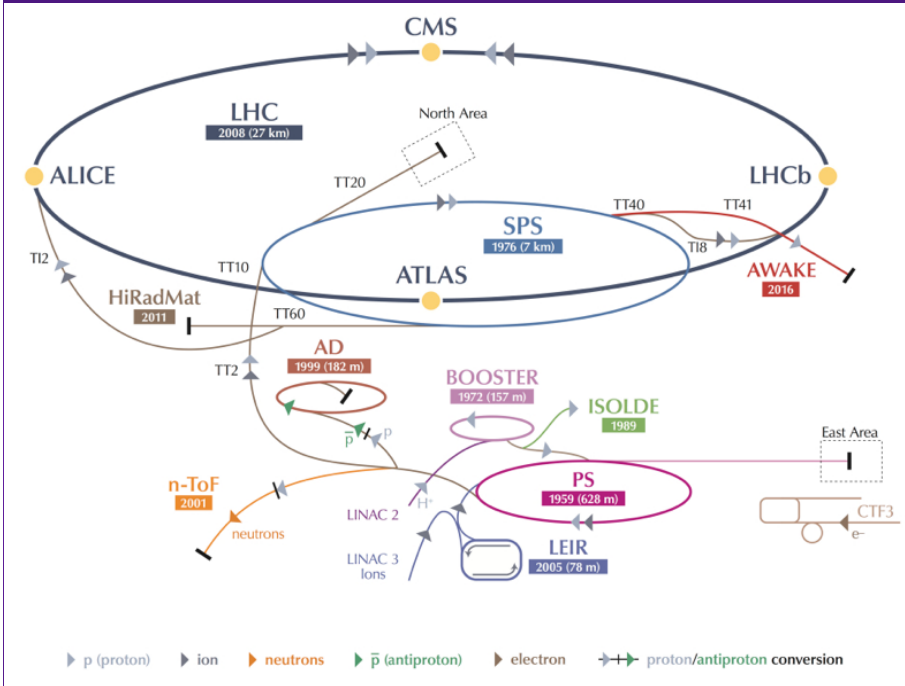


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**SPS** 25 => 450 GeV  
**LHC** 450 => 7 (6.5) TeV

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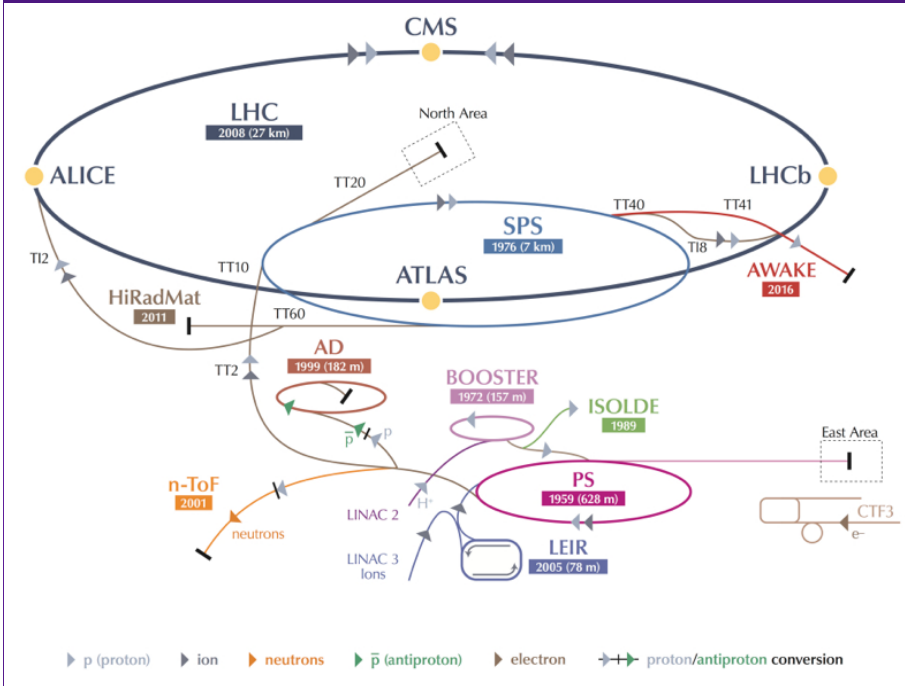


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  - TMCI between modes - 2 and - 3 at injection ( $Q' \sim 0$ )

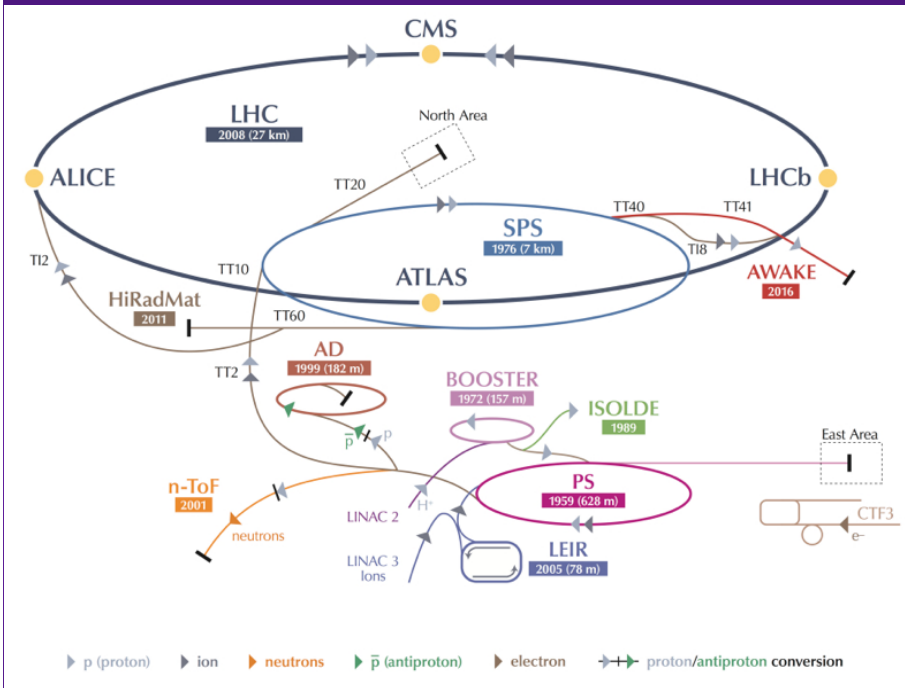


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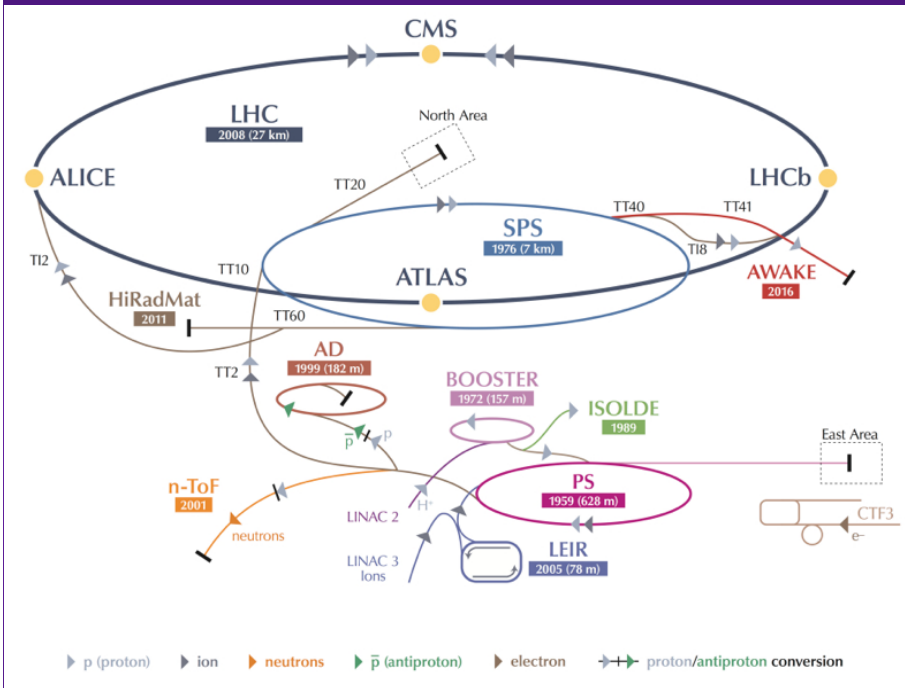
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  - Head-Tail instability with 1 node ( $Q' \sim 5$ ) => Stabilized by SC below a certain energy
  - Predicted threshold for TMCI (modes - 1 and 0) at injection ( $Q' \sim 0$ ) increased by SC

=> Detailed studies still ongoing

# CONTENTS

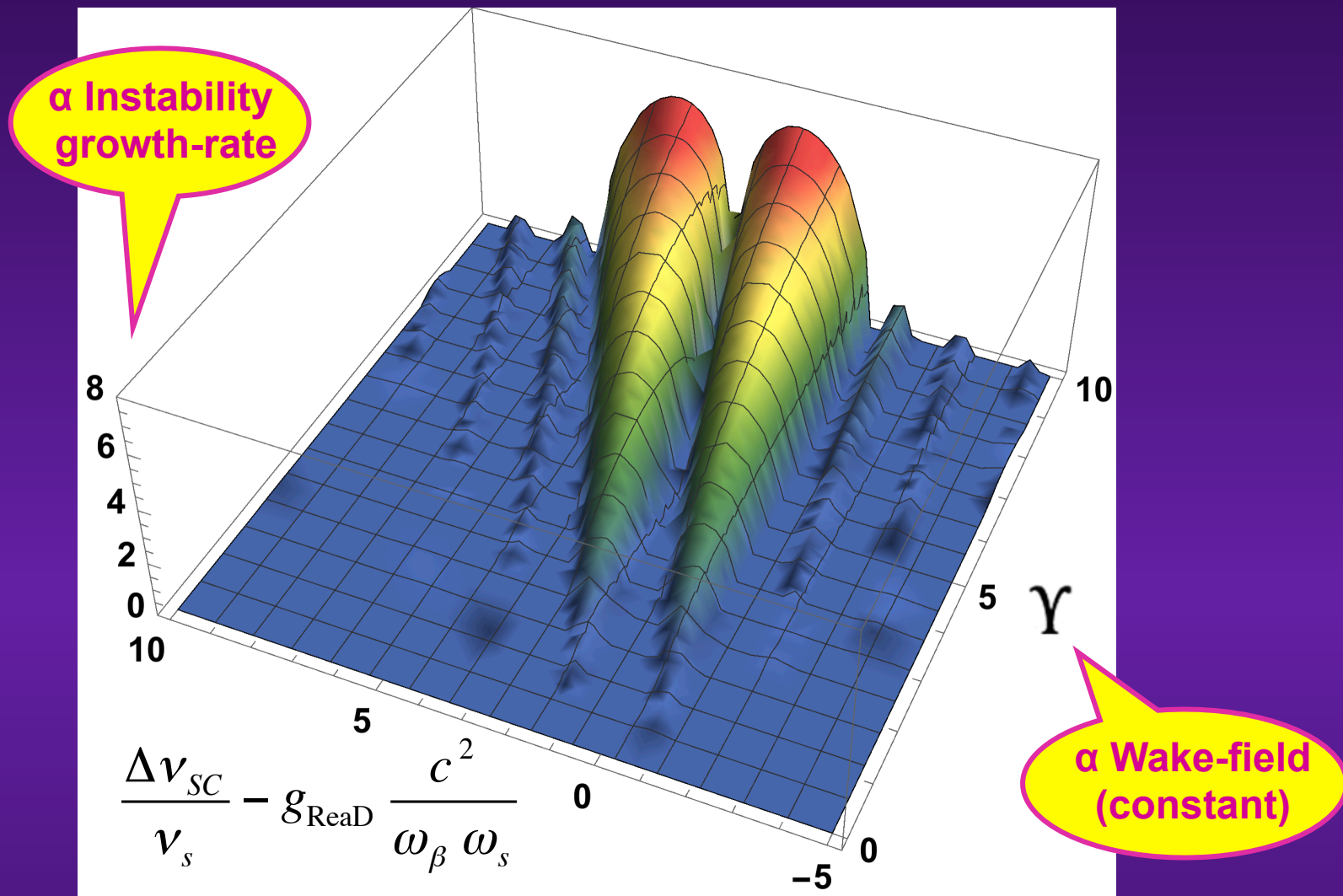
Reactive  
transverse damper

- ◆ **2 – particle model for TMCI ( $Q' = 0$ ) with SC and/or ReaD**
- ◆ **2 – mode model for TMCI ( $Q' = 0$ ) with SC and/or ReaD**
  - “Short-bunch” regime => (HL-) LHC case
  - “Long-bunch” regime => SPS case
- ◆ **Vlasov solver for TMCI ( $Q' = 0$ )**
  - “Short-bunch” regime
  - “Long-bunch” regime
- ◆ **pyHEADTAIL simulations with SC**
  - (HL-) LHC TMCI ( $Q' = 0$ )
  - SPS TMCI ( $Q' = 0$ )
  - (HL-) LHC Head-Tail instability ( $Q' = 5$ )
- ◆ **Conclusions**
- ◆ **Appendix: General formula for the space charge tune spread from a QPU**

From Adrian Oeftiger  
with his new (2.5D)  
PIC SC module

# 2 – PARTICLE MODEL FOR TMCI ( $Q' = 0$ ) WITH SC AND/OR ReaD

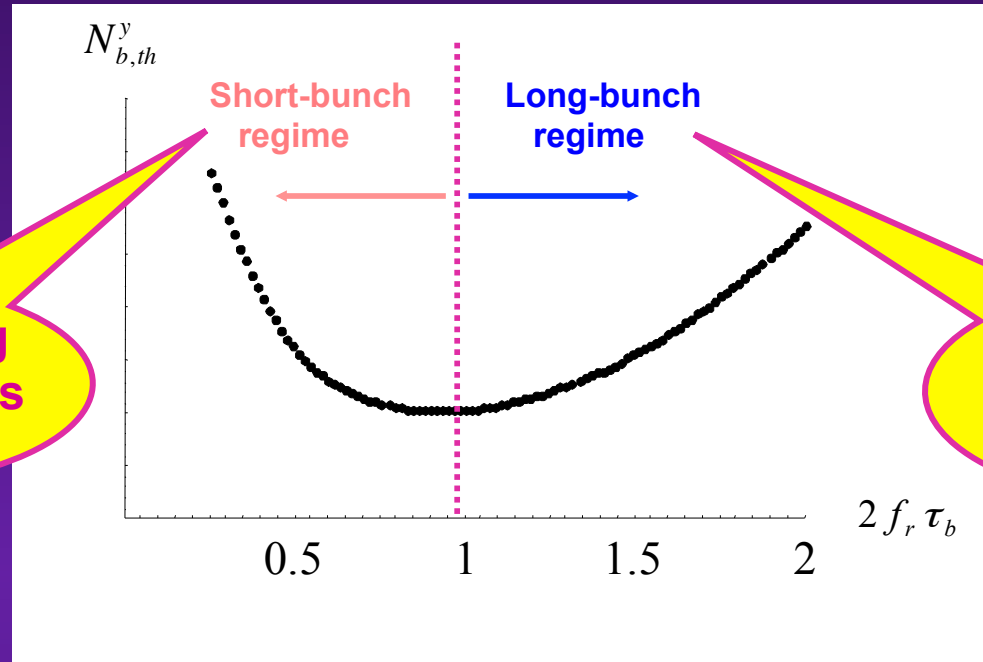




- ◆ Results from Burov\_2016 (using a ReaD only) and Chao-Chin-Blaskiewicz\_2016 (using SC only) have been recovered and combined
- ◆ Both SC and ReaD affect TMCI in a similar way and can suppress it

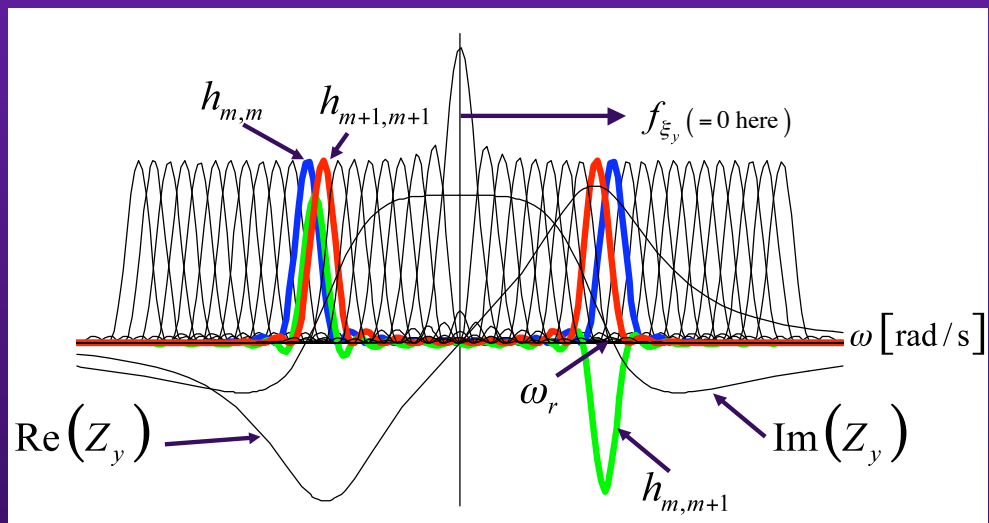
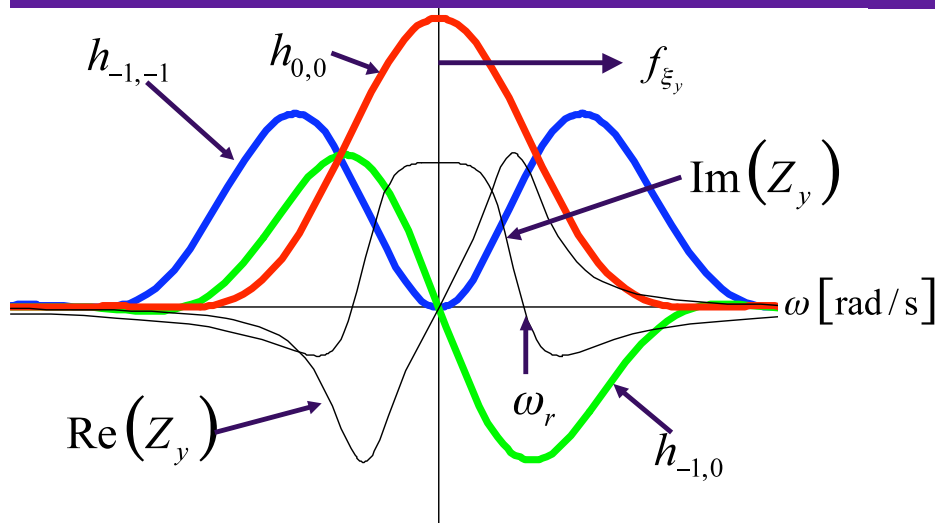
# **2 – MODE MODEL FOR TMCI ( $Q' = 0$ ) WITH SC AND/OR ReaD**

# BROAD-BAND IMPEDANCE WITH NEITHER SC NOR ReaD

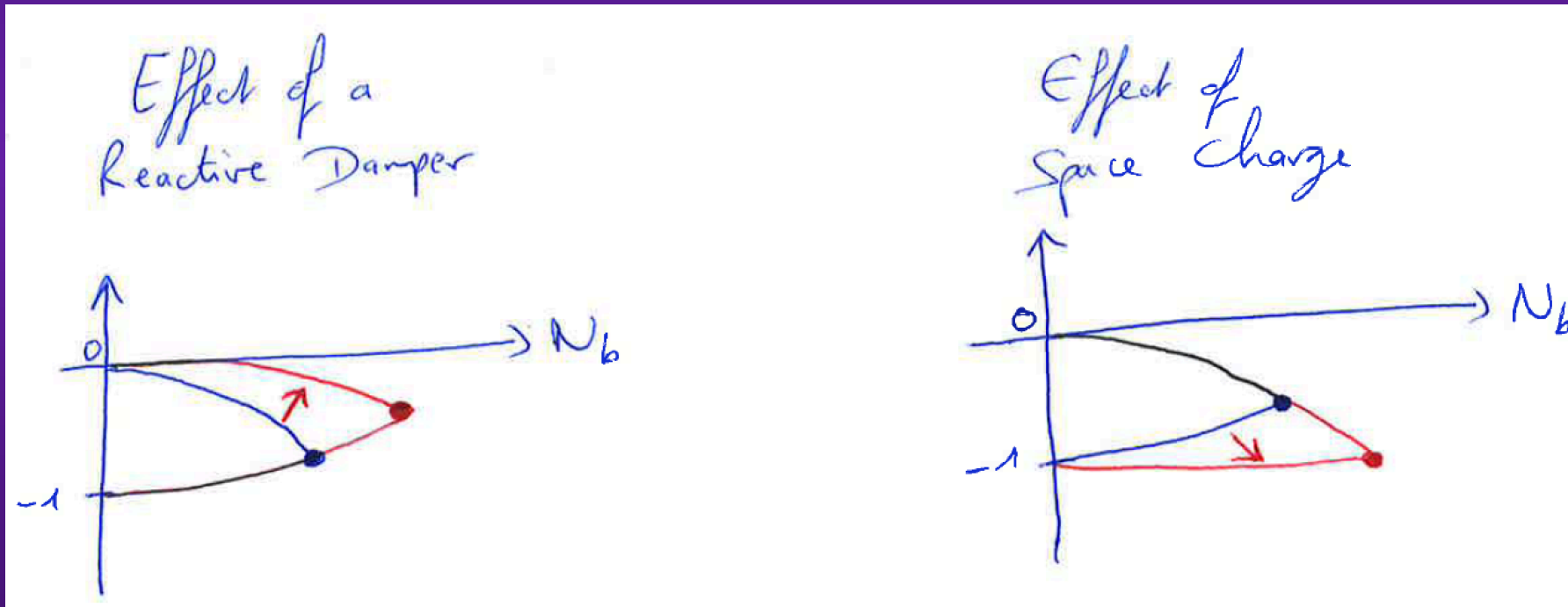


Mode-coupling between modes - 1 and 0

Mode-coupling between higher-order modes



- ◆ **“Short-bunch” regime** => (HL-) LHC case
  - Both ReaD & SC are expected to be beneficial
    - ReaD => Shifts mode 0 up
    - SC => Shifts mode - 1 down



Still under discussion

- ◆ **“Long-bunch” regime** => SPS case
  - Both ReaD & SC *“are expected”* to have no / small effect
    - ReaD => Modifies only (main) mode 0 and not the others (where the mode-coupling occurs)
    - SC => Modifies all the modes (except 0) similarly

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WITHOUT SC

$$\left| Q_s + \Delta Q_{m+1}^{S,y} - \Delta Q_m^{S,y} \right| = 2 \left| \Delta Q_{m,m+1}^{S,y} \right|$$

~ 0  
(see slide 7)

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result with peak values

$$\Rightarrow N_{b,th}^y \propto |\eta| Q_y \varepsilon_L$$



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Coasting-beam  
result with peak values

And SC should have no  
effect for coasting beams

$$\Rightarrow N_{b,th}^y \propto |\eta| Q_y \varepsilon_L$$

# **VLASOV SOLVER: GALACTIC**

**=> Application to LHC and SPS  
assuming a Broad-Band impedance**

## ◆ **GALACTIC: GARNIER-LACLARE Coherent Transverse Instabilities Code**

- Uses a decomposition on the low-intensity eigenvectors (as proposed by Garnier-Laclare in 1987) => “Water-bag” longitudinal distribution (for now)
- Effect of transverse damper recently added (to study the destabilizing effect of the resistive transverse damper)

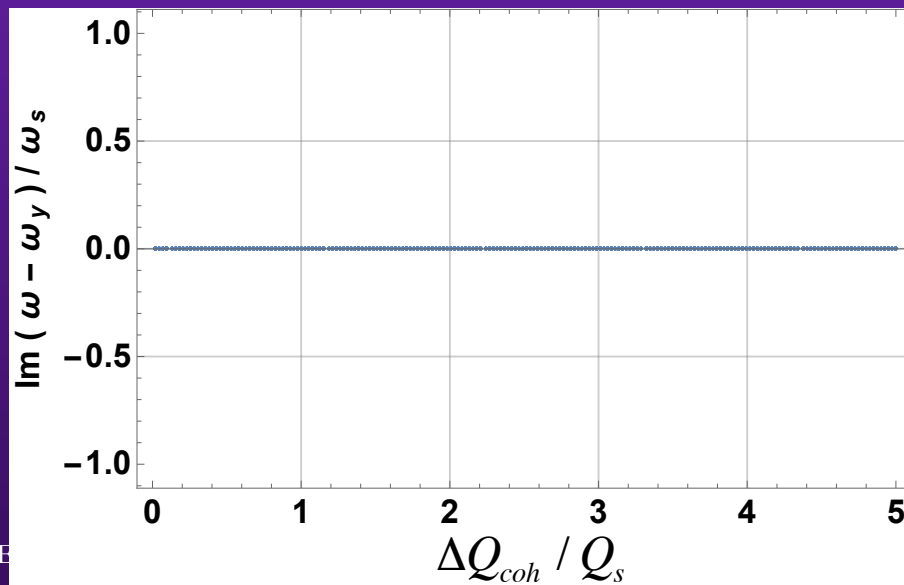
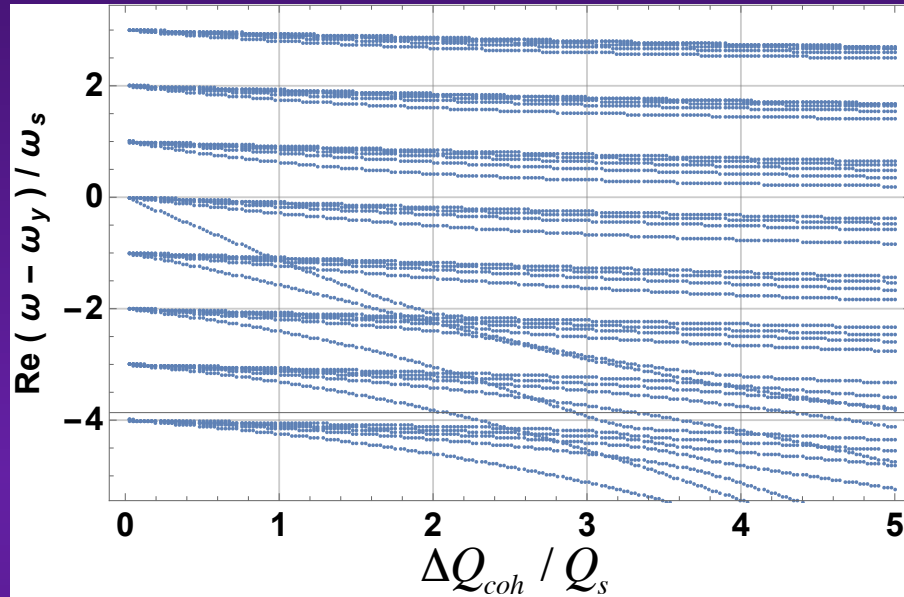
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- Effect of transverse damper recently added (to study the destabilizing effect of the resistive transverse damper)
- Remark: 2 other codes (Vlasov solvers) including the transverse damper were developed in the recent years
  - A. Burov developed a Nested Head-Tail Vlasov Solver (NHTVS) with transverse damper in 2014
  - N. Mounet solved Sacherer integral equation with transverse damper, using a decomposition over Laguerre polynomials of the radial functions (DELPHI code, 2015)

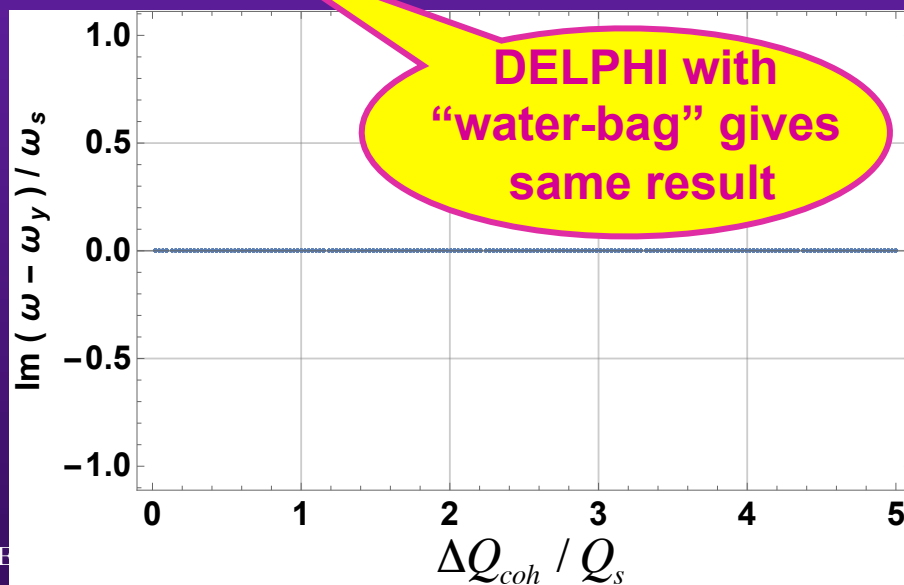
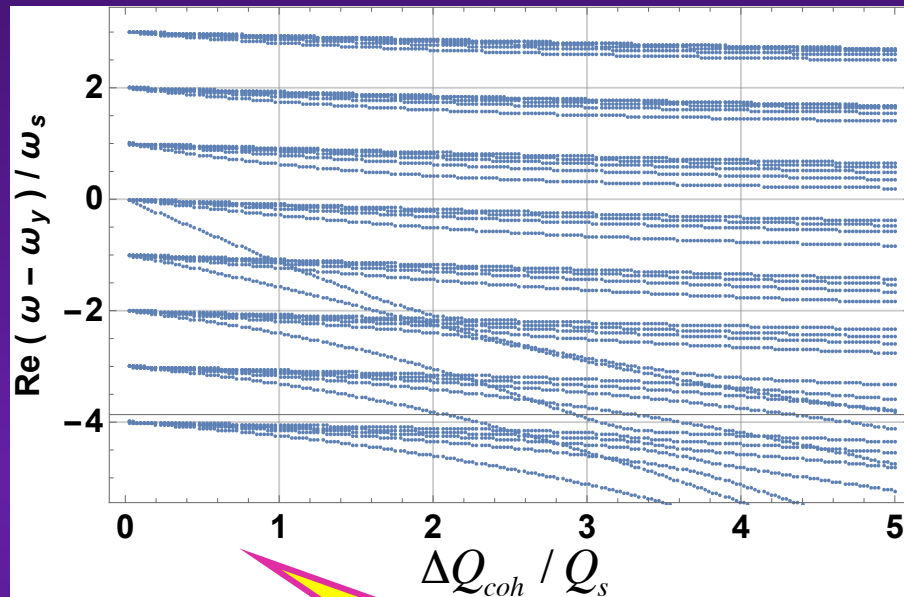
\* *Sacherer integral equation was also solved using a decomposition over Laguerre polynomials of the radial functions by Besnier in 1974 and Y.H. Chin in 1985 in the code MOSES*

*Without transverse damper*

# Constant inductive impedance

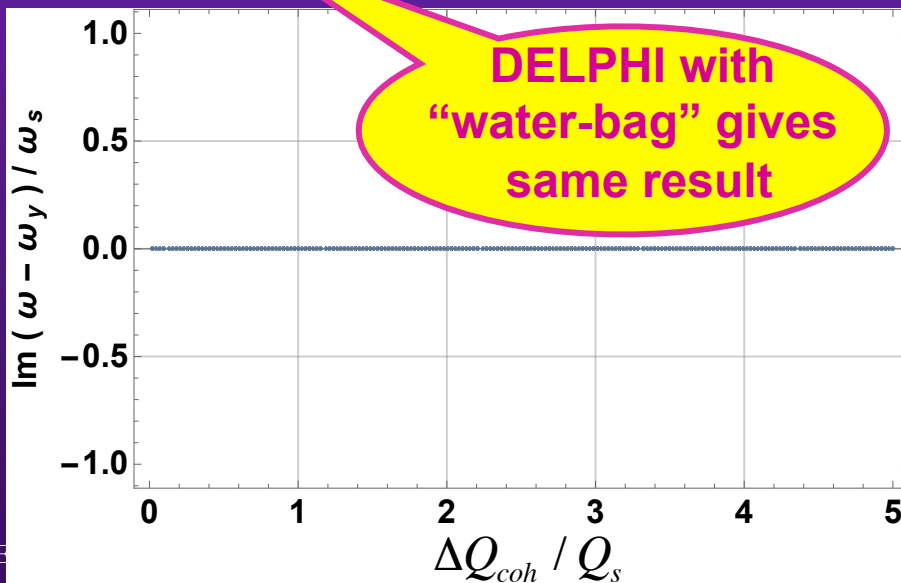
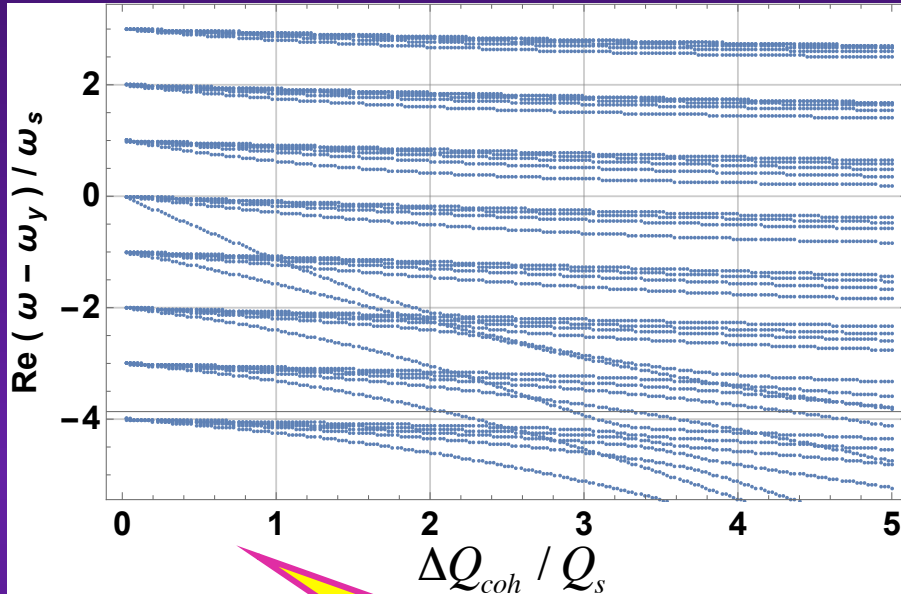


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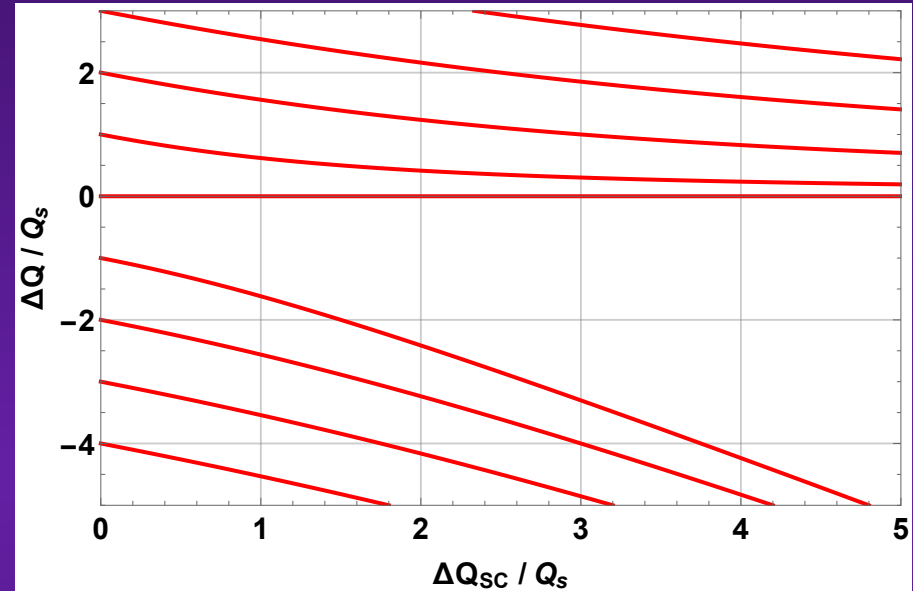
DELPHI with  
"water-bag" gives  
same result

# Constant inductive impedance



# SC

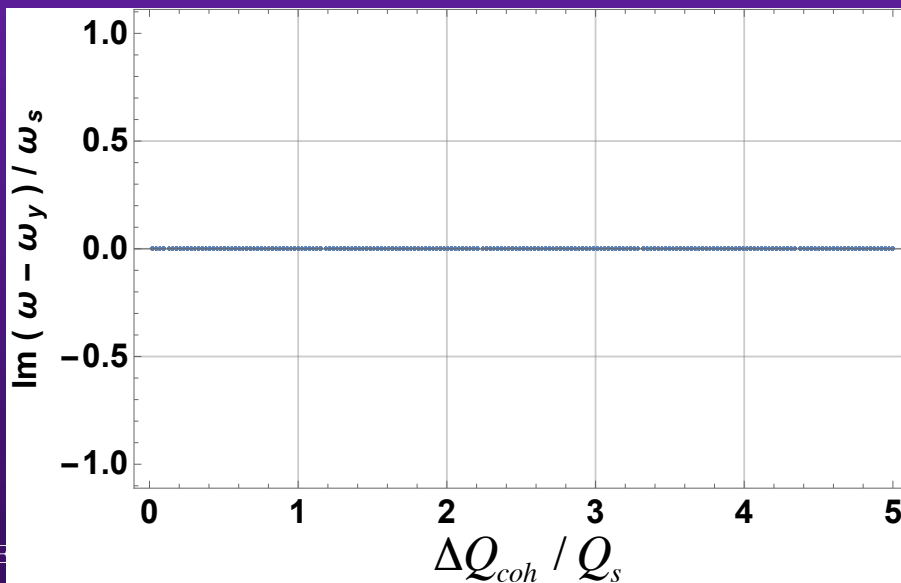
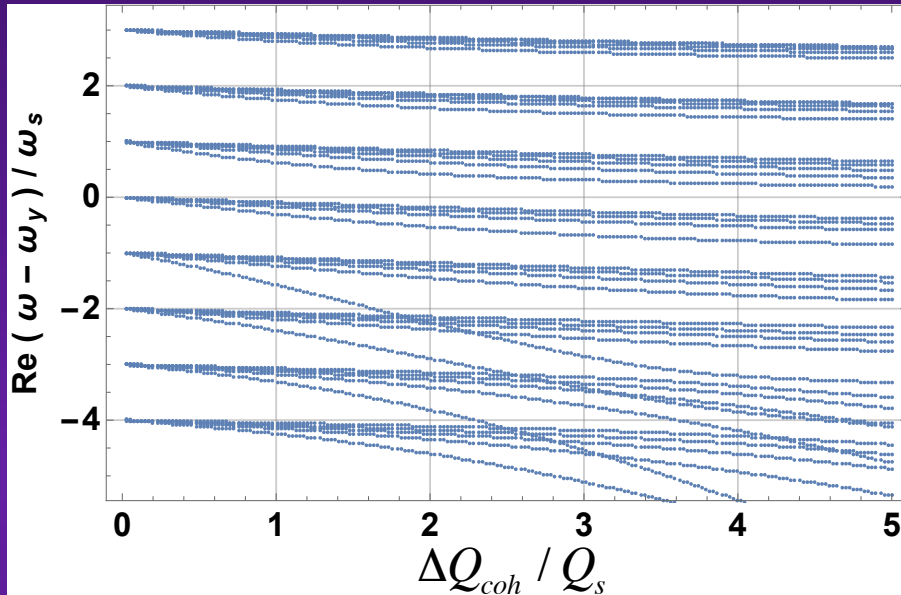
(square-well air-bag, Blaskiewicz1998)



$$\Delta Q_{m \geq 0}^y = -\frac{\Delta Q_{SC}}{2} + \sqrt{\left(\frac{\Delta Q_{SC}}{2}\right)^2 + (m Q_s)^2}$$

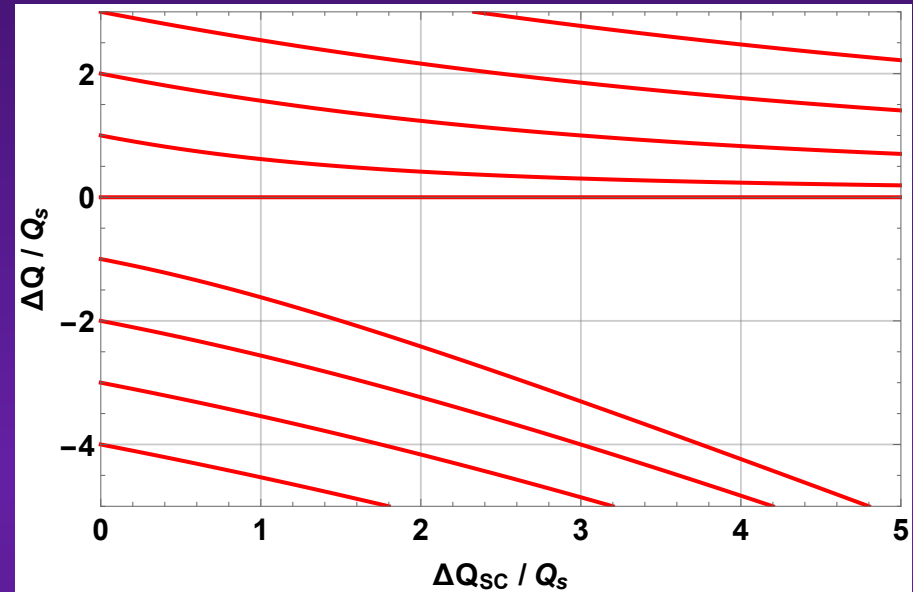
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# Constant inductive impedance with ReaD (1 turn)



# SC

(square-well air-bag, Blaskiewicz1998)



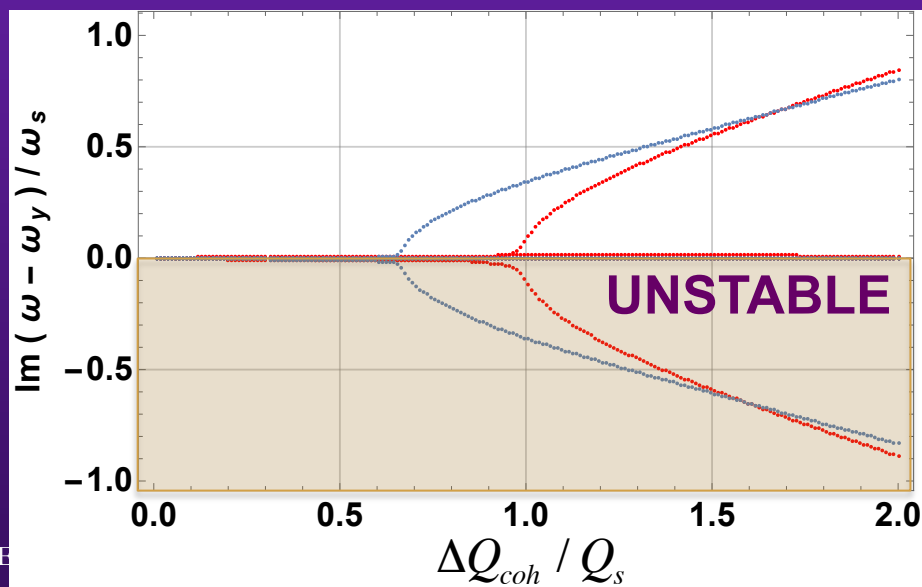
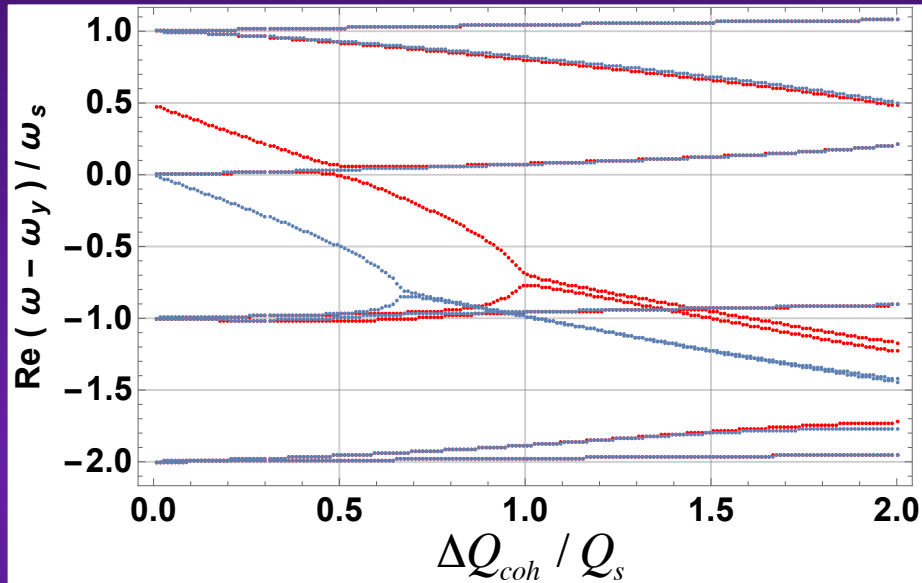
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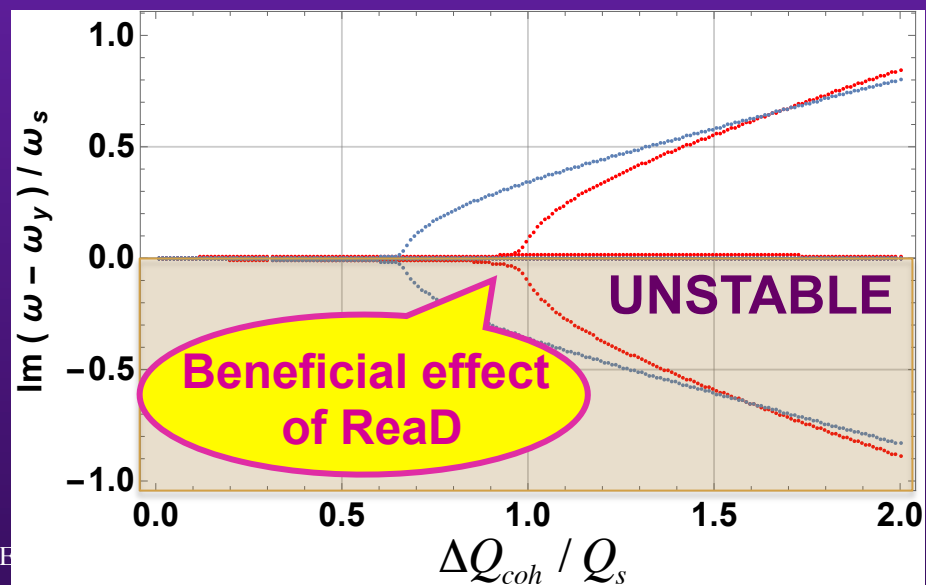
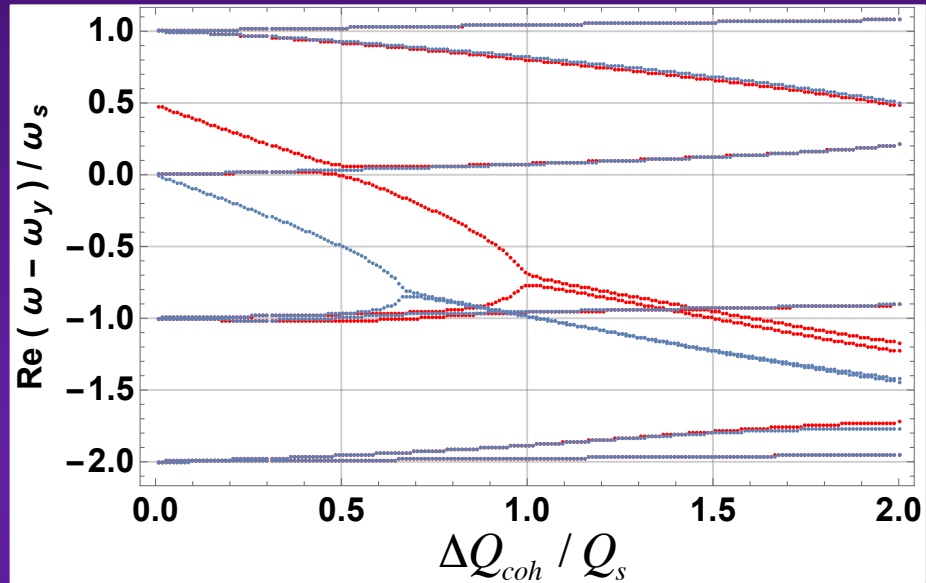
# ~ LHC case ( $Q' = 0$ ) – No SC

Without / with ReaD  
(50 turns) in blue / red



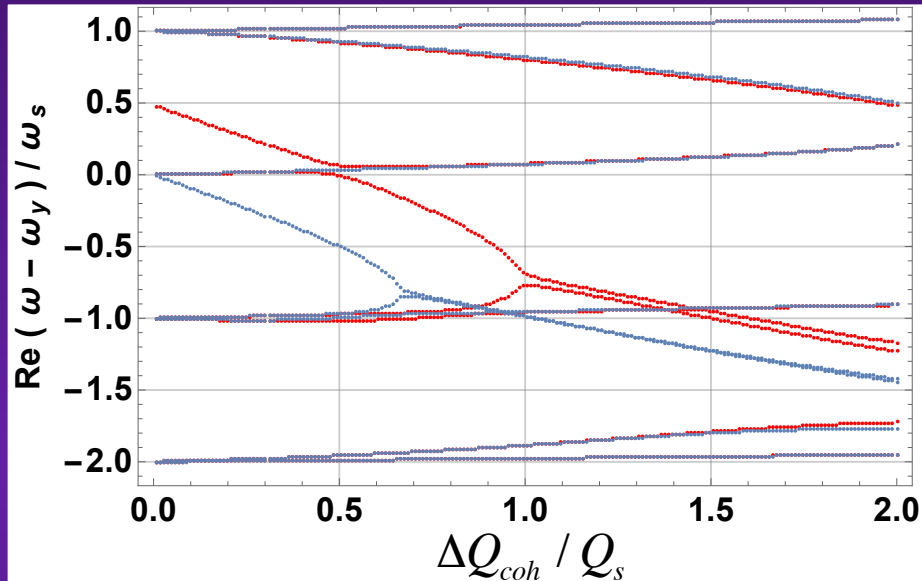
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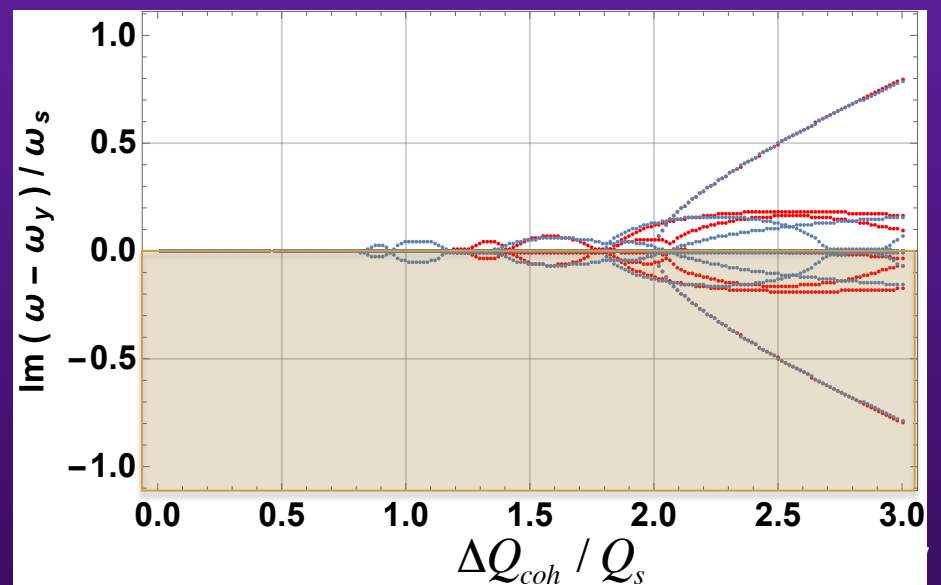
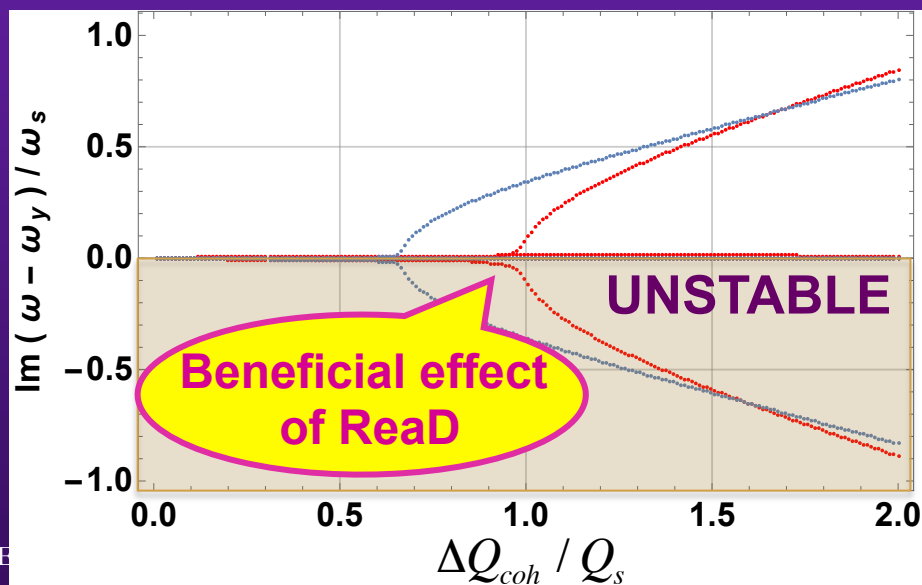
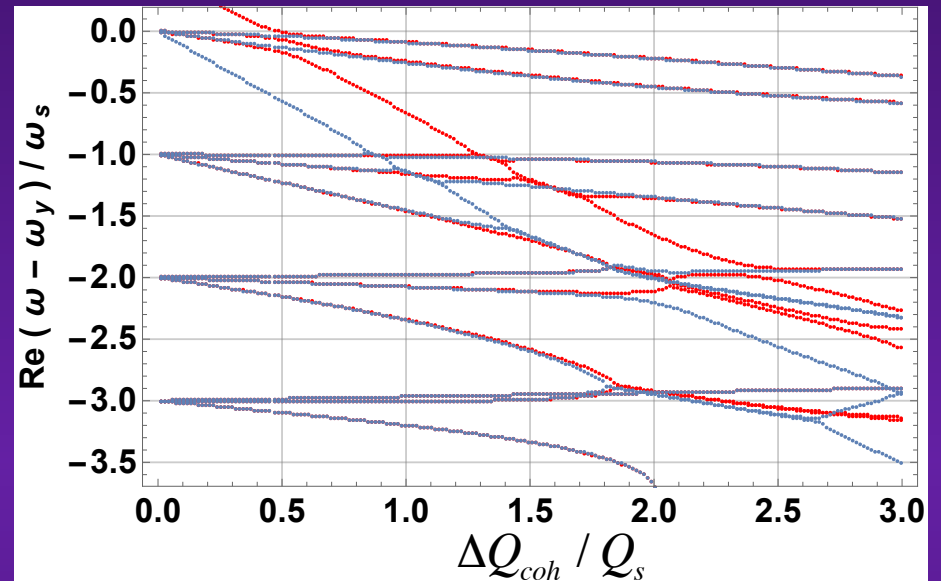
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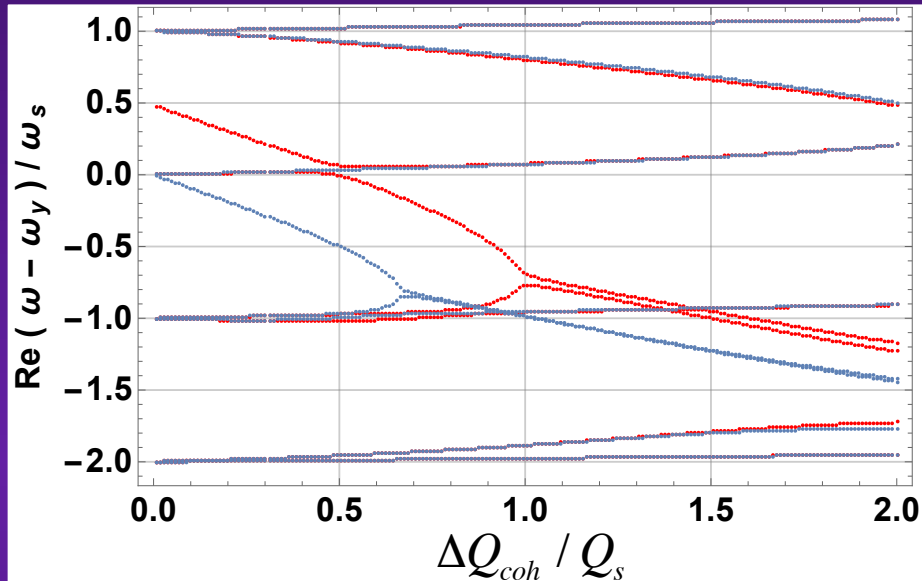
~ SPS case ( $Q' = 0$ ) – No SC

Without / with ReaD  
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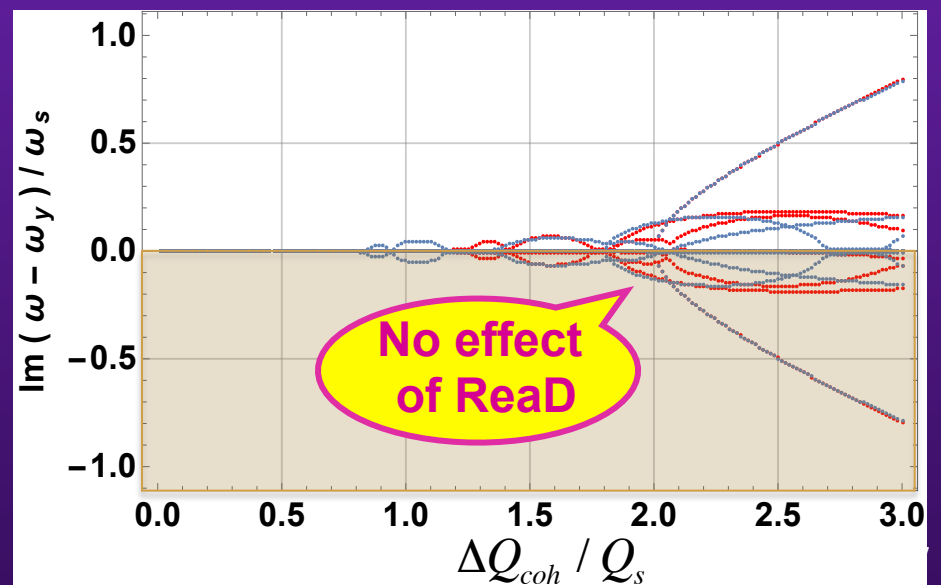
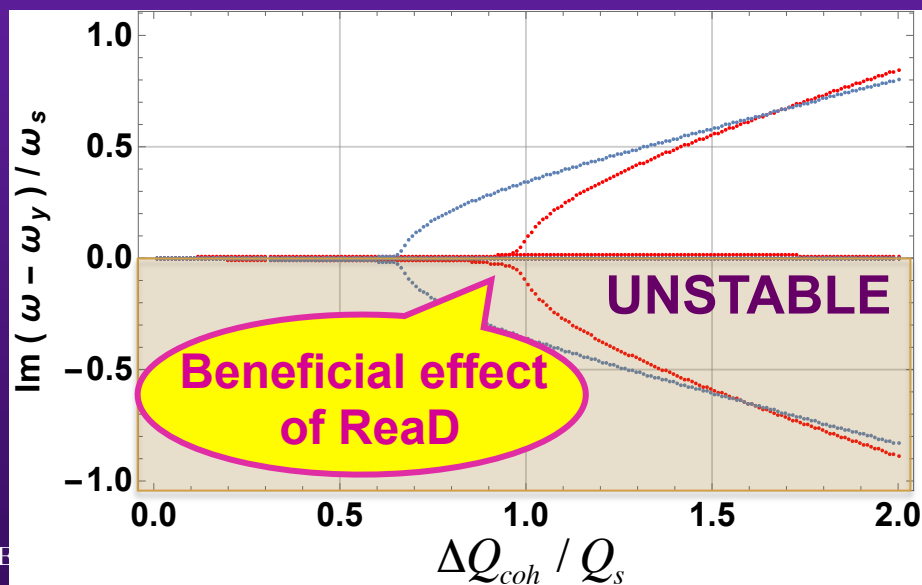
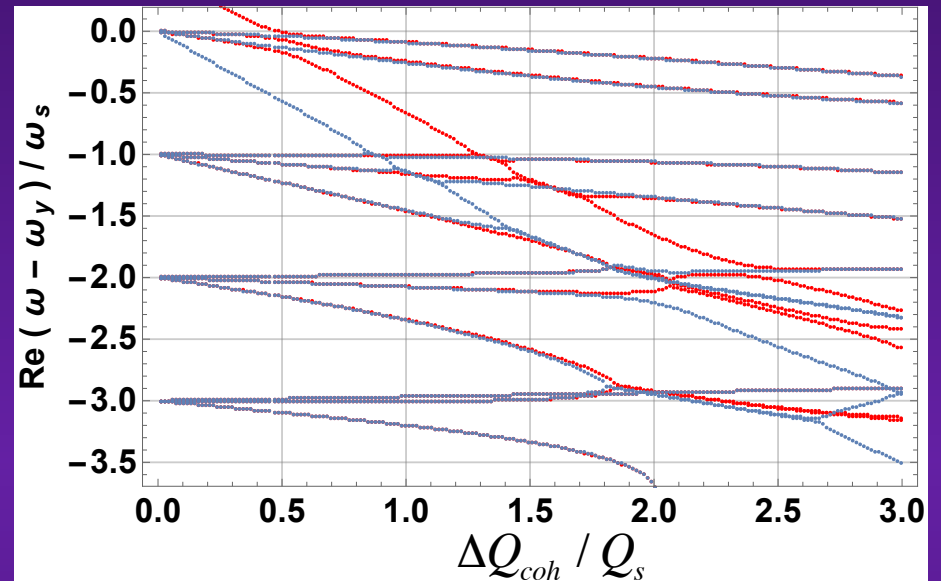
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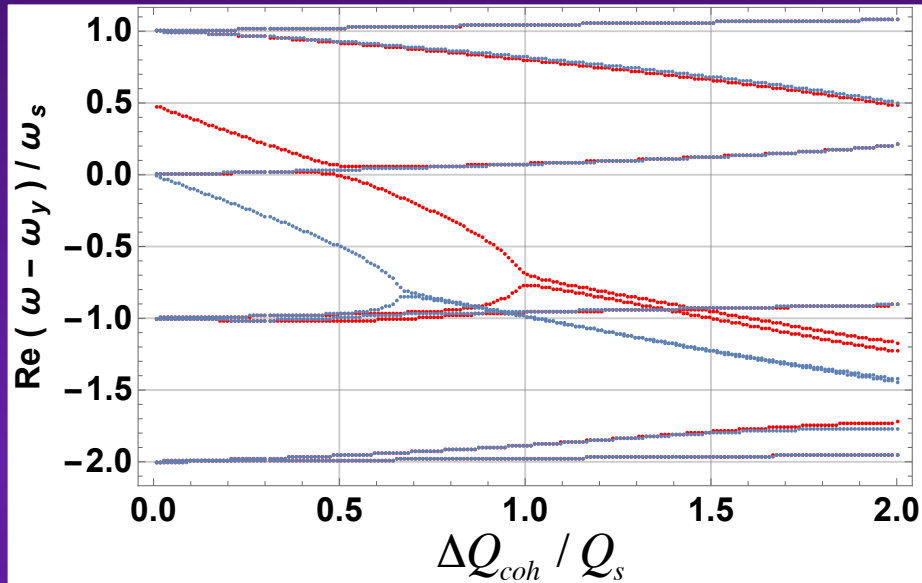
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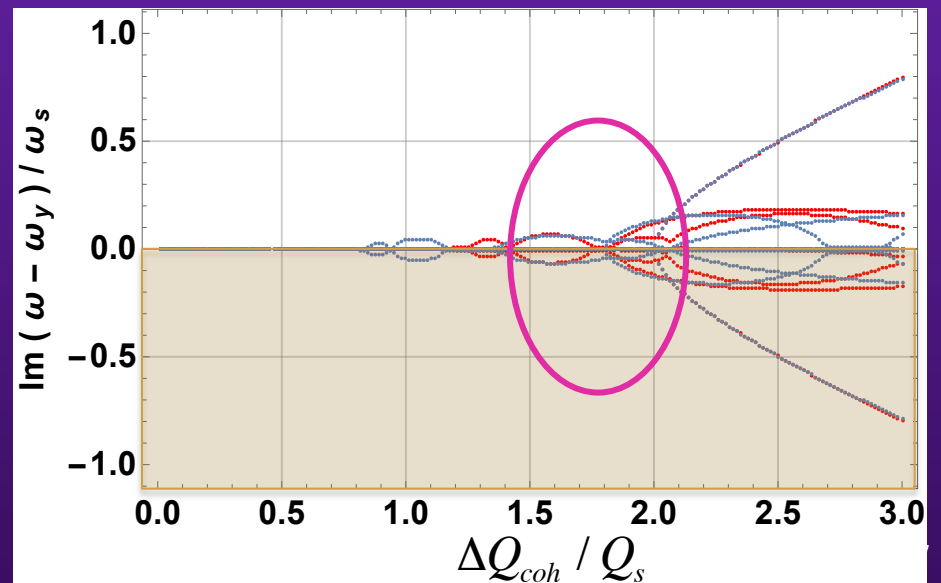
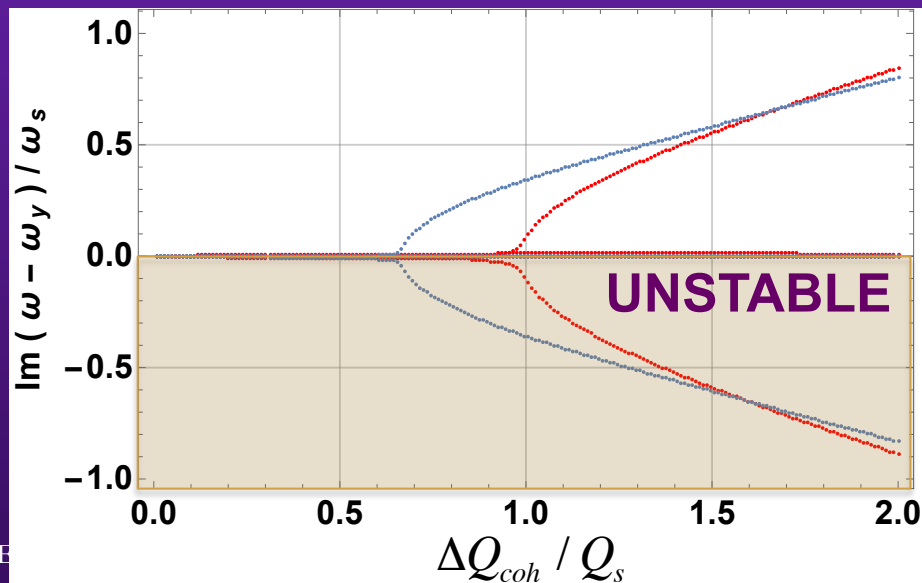
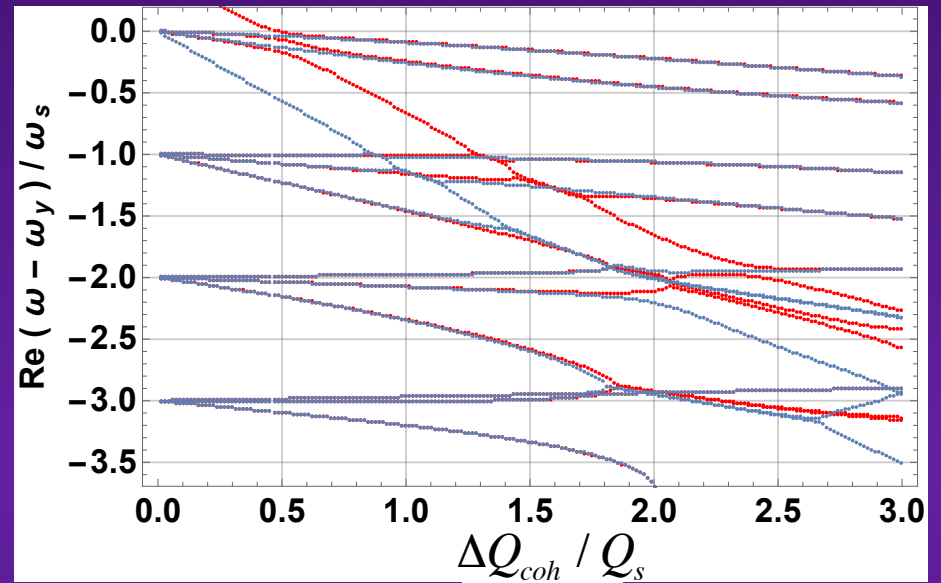
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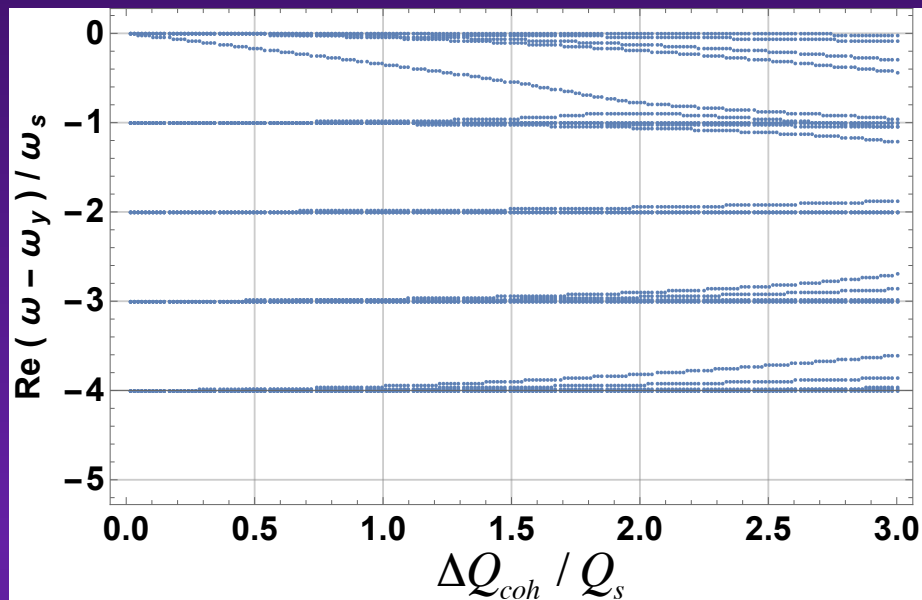
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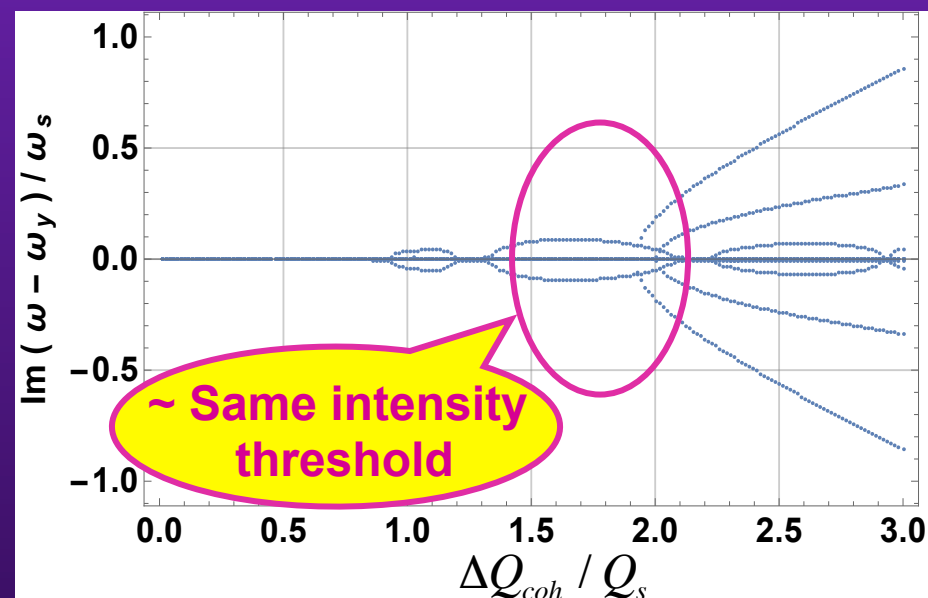
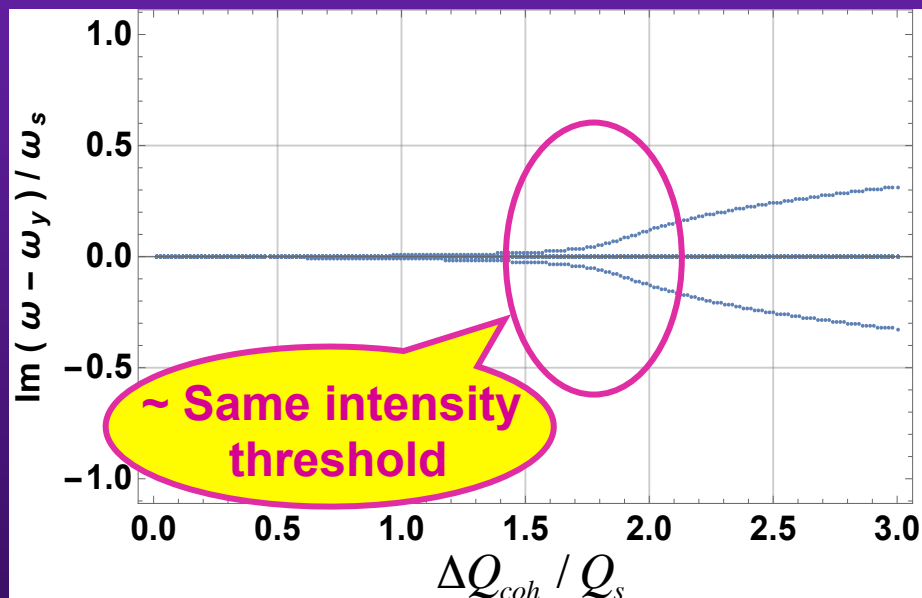
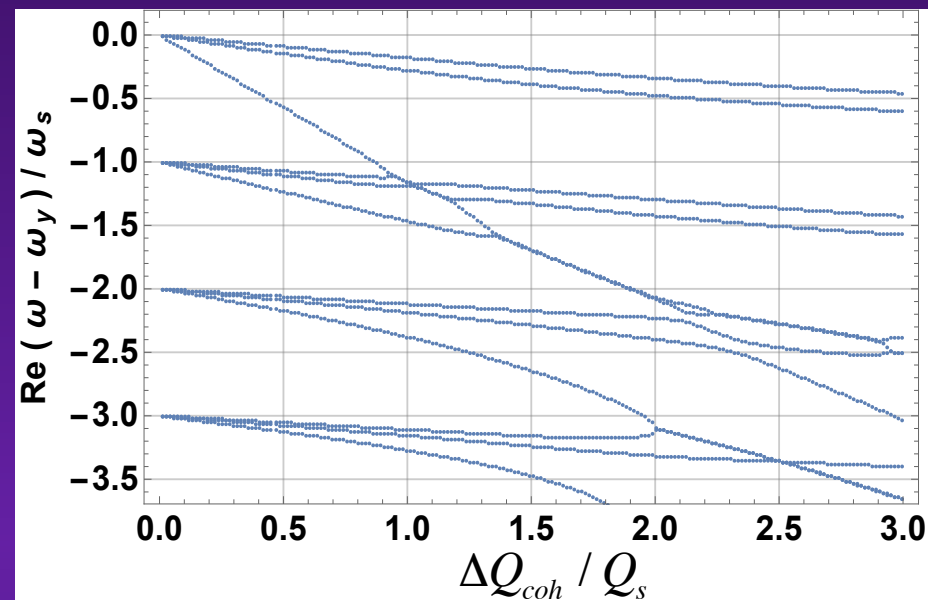


# ~ SPS case ( $Q' = 0$ ): Broad-Band impedance for Real part

0 imaginary impedance

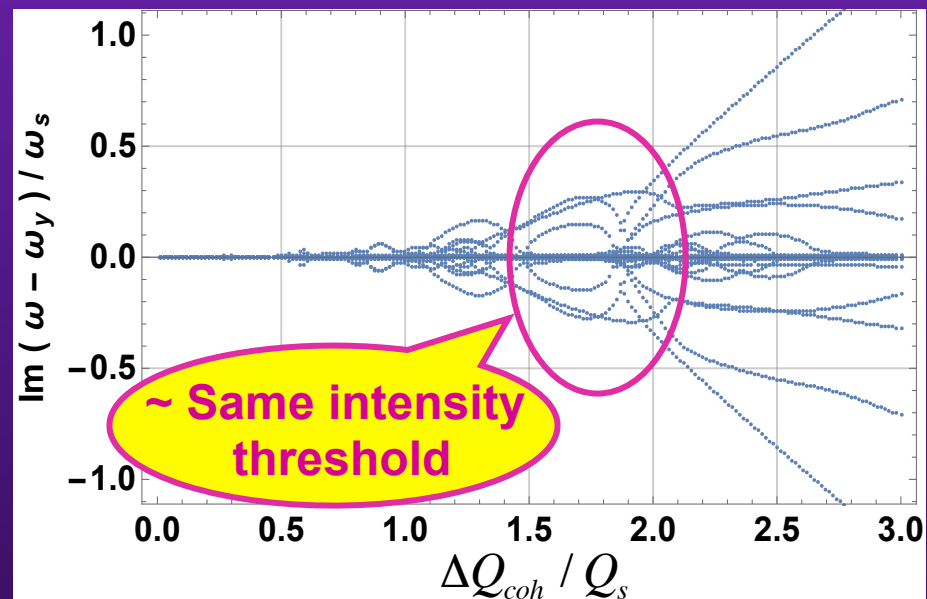
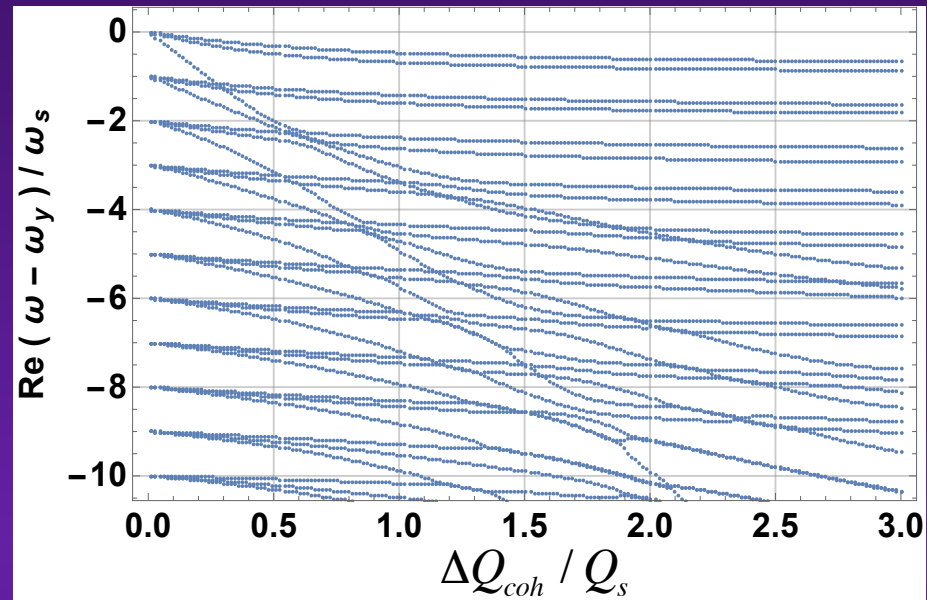


Constant imaginary impedance



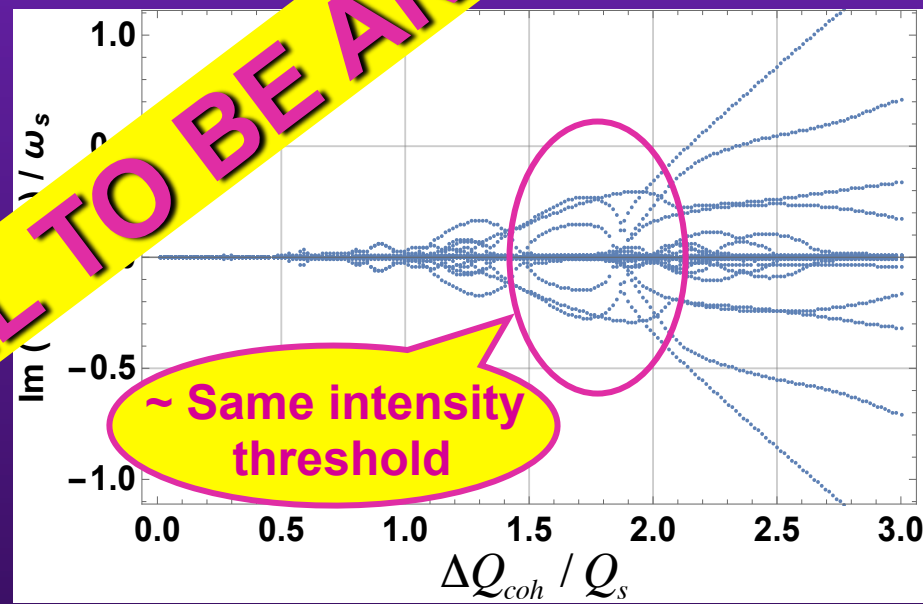
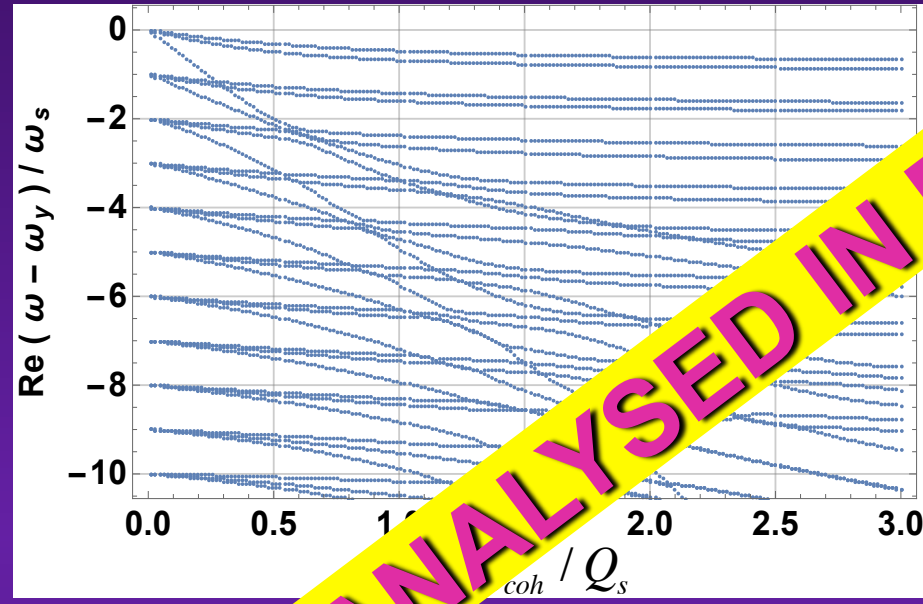
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Constant imaginary impedance  $\times 5$



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Constant imaginary impedance  $\times 5$

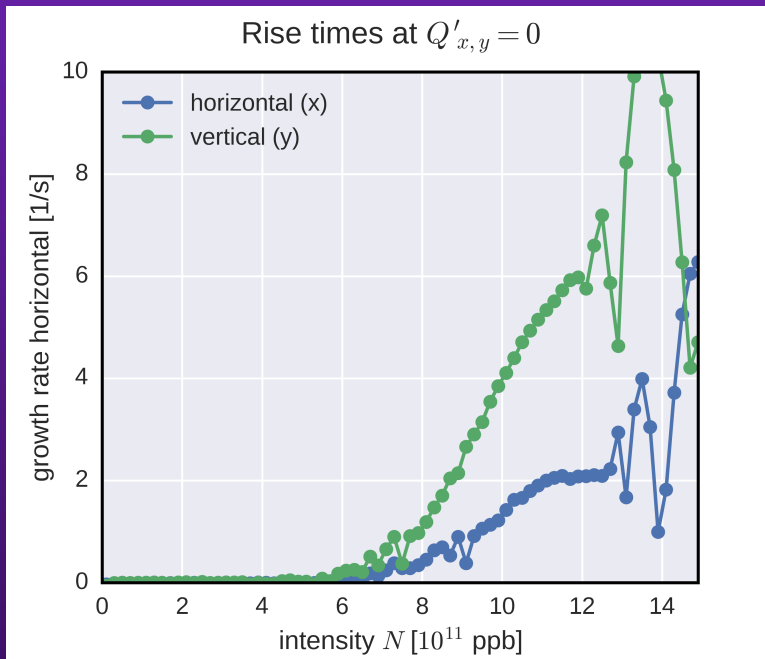
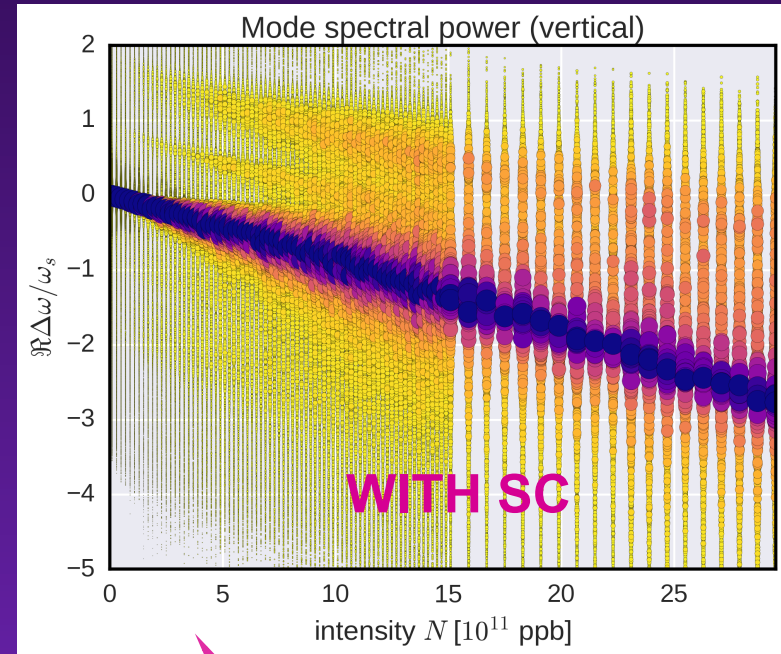
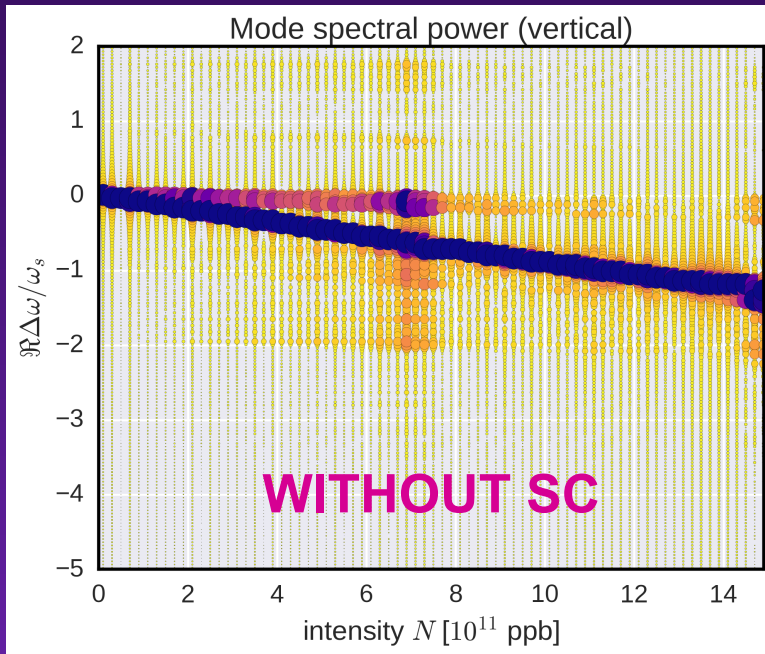


**STILL TO BE ANALYSED IN DETAIL**



# pyHEADTAIL SIMULATION WITH SC FOR (HL-) LHC TMCI ( $Q' = 0$ )

**=> Using the real impedance model**

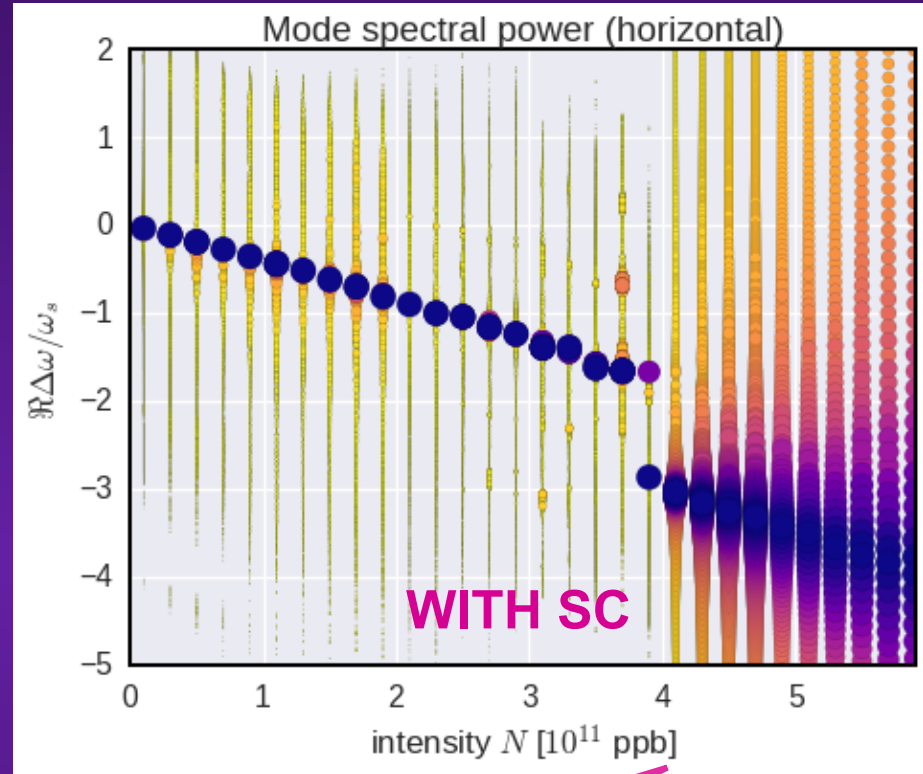
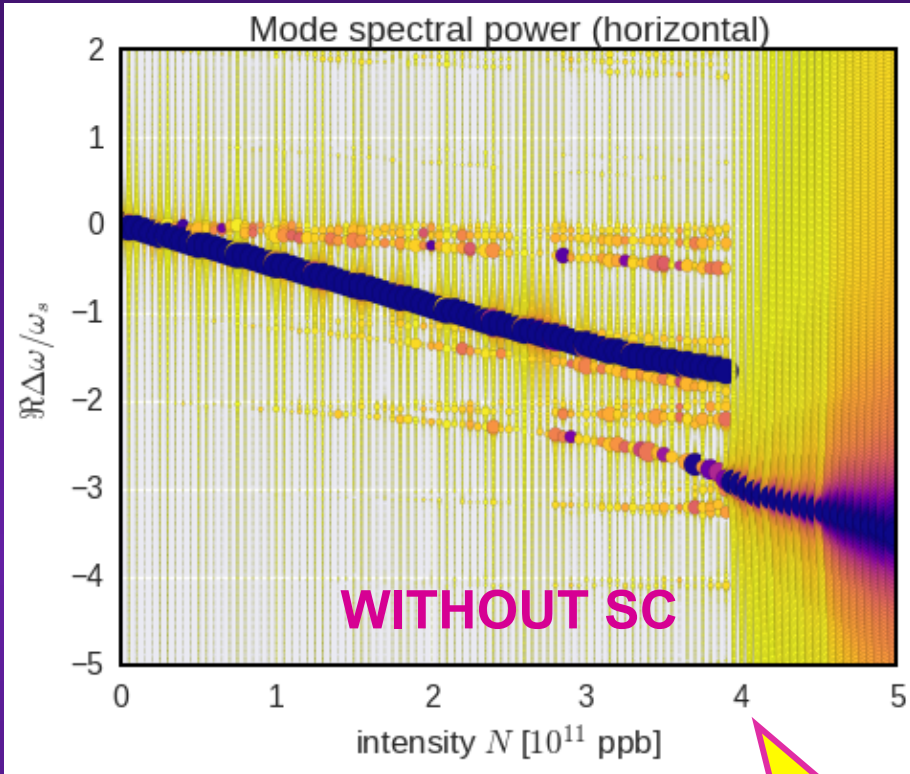


**No instability anymore with SC**

**A. Oeftiger**

# pyHEADTAIL SIMULATION WITH SC FOR SPS TMCI ( $Q' = 0$ )

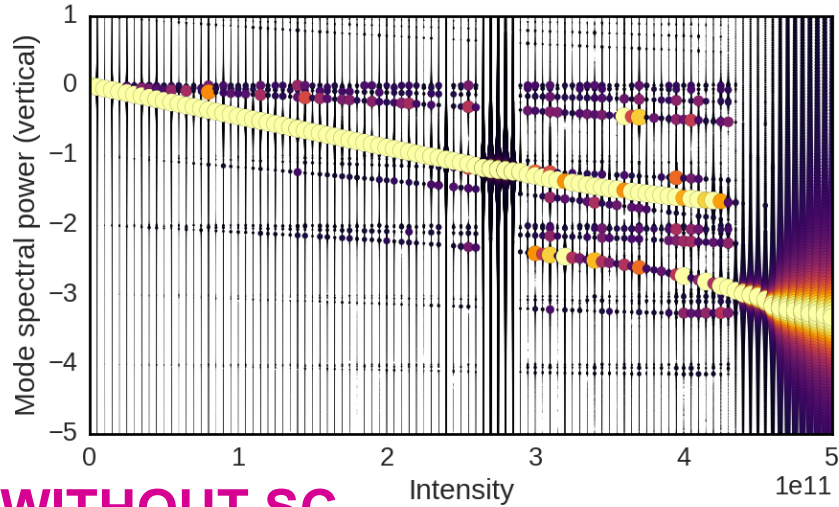
**=> Using an approximate Broad-Band impedance model**



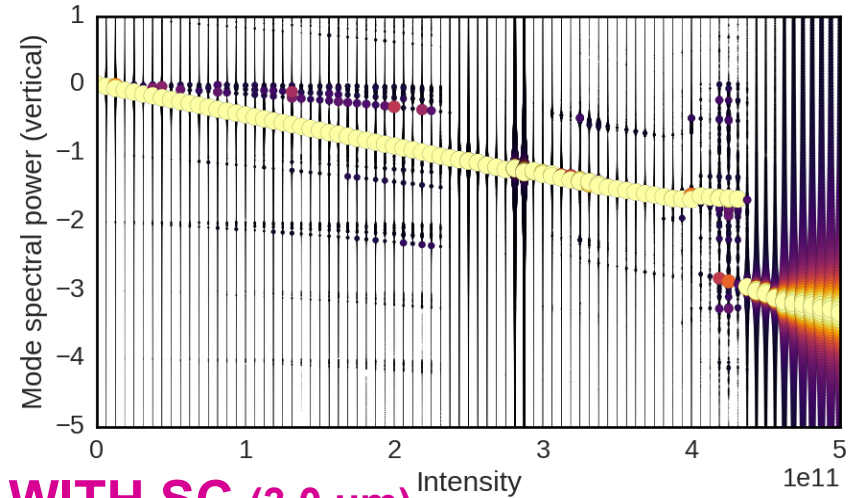
Instability at  
~ same intensity  
threshold without  
(left) and with  
(right) SC

*A. Oeftiger*

- ◆ Good agreement with past pyHEADTAIL simulations using a frozen SC model

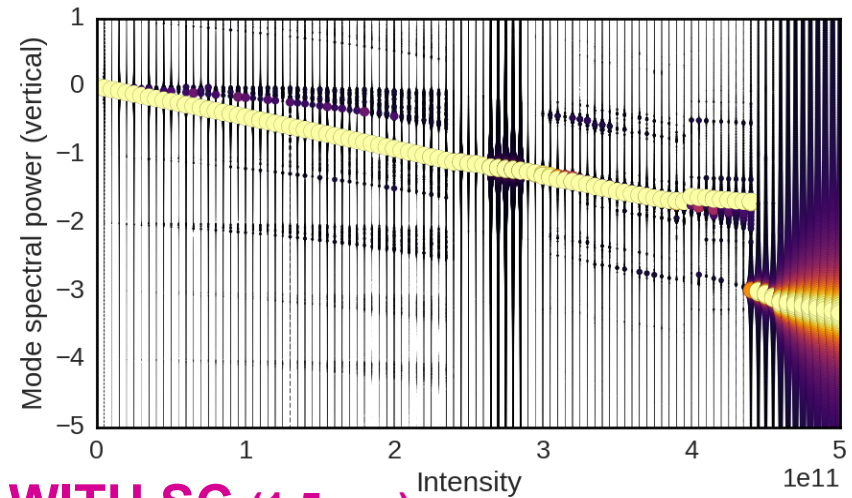


WITHOUT SC



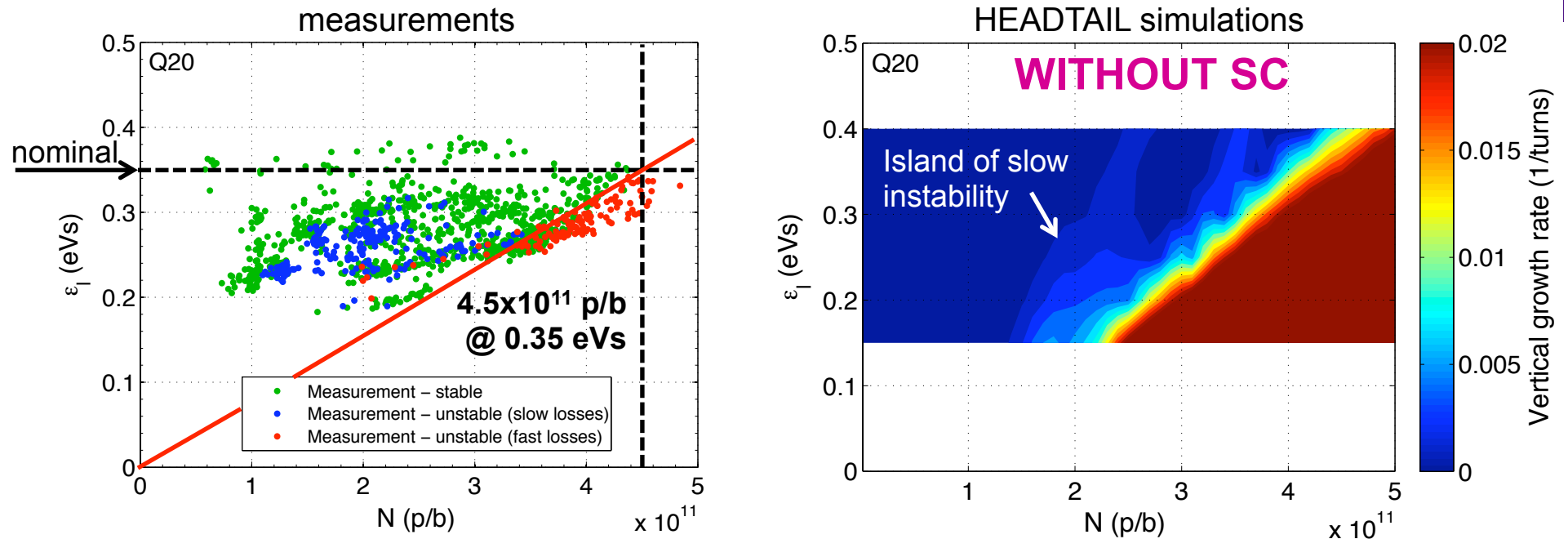
WITH SC (3.0  $\mu\text{m}$ )

*K. Li*



WITH SC (1.5  $\mu\text{m}$ )

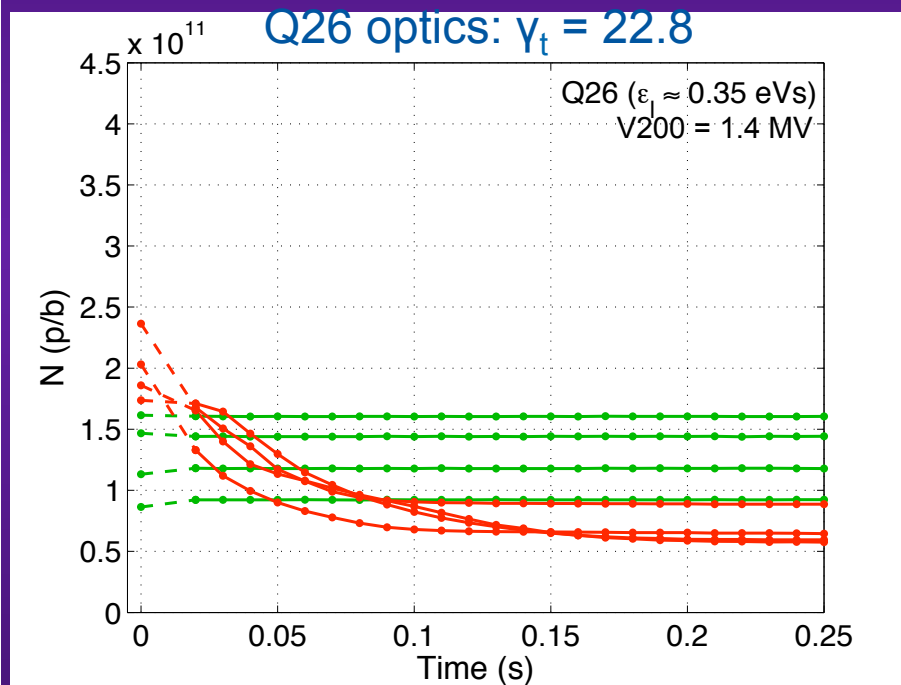
- ◆ Seems to be in very good agreement with measurements as a very good agreement between predictions (simulations and simple formula) and measurements, WITHOUT taking into account SC, was already reached



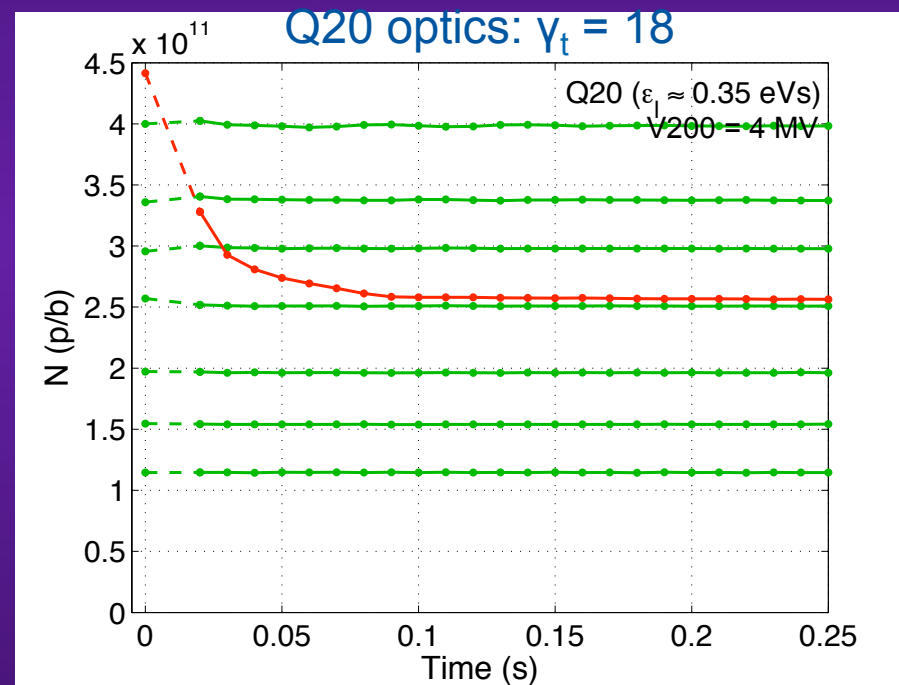
Courtesy of Hannes Bartosik et al.

$$N_{b,th}^y \propto |\eta| Q_y \epsilon_L$$

Measured gain of a factor  
 $4.5 / 1.7 \sim 2.6$  (compared to  $\sim 2.2$   
 predicted with simple formula)



*B. Salvant et al.*



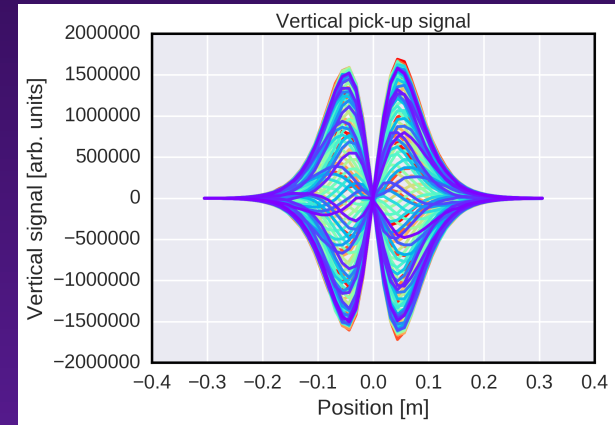
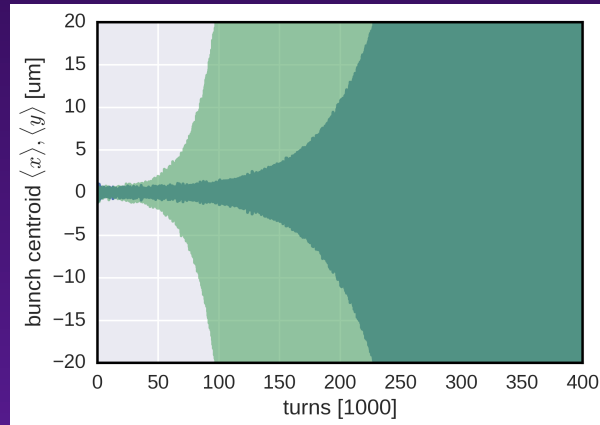
*H. Bartosik et al.*

# pyHEADTAIL SIMULATION WITH SC FOR (HL-) LHC HEAD-TAIL INSTABILITY ( $Q' = 5$ )

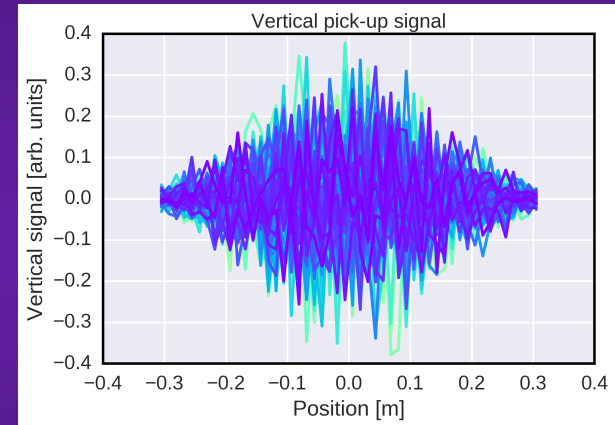
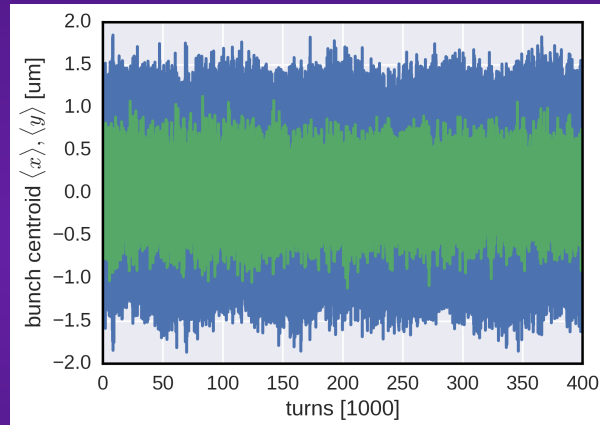
**=> Using the real impedance model**



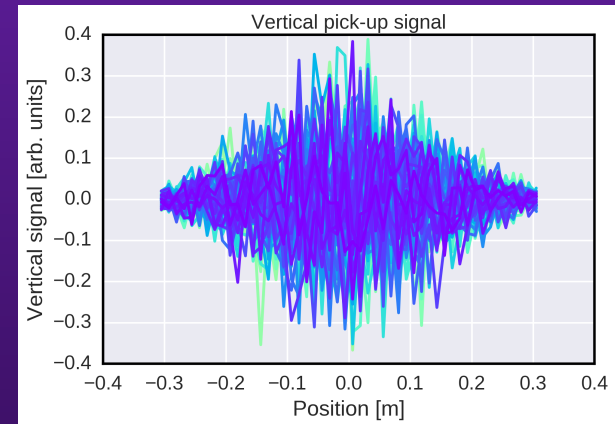
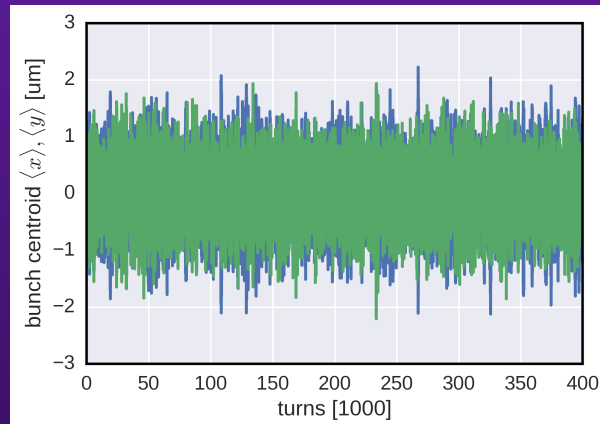
◆ Impedance only



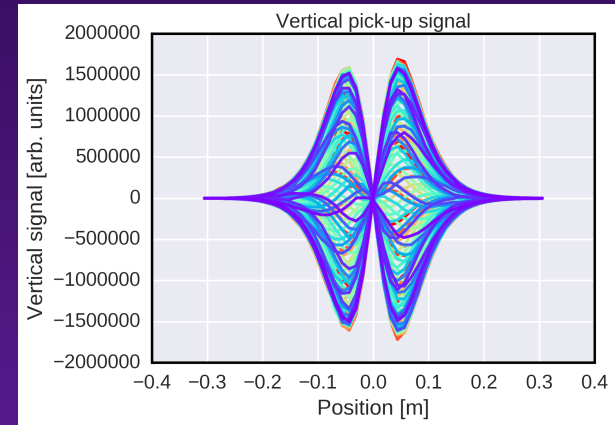
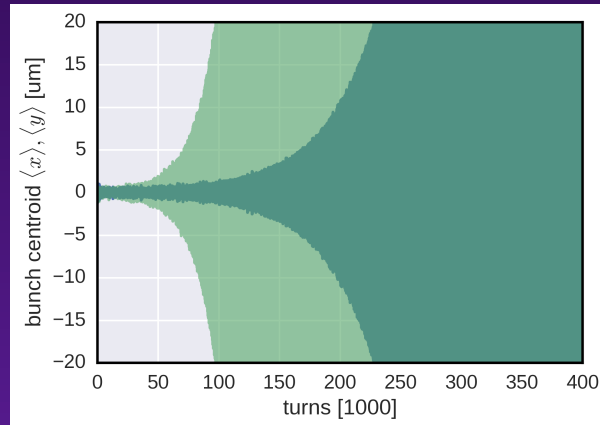
◆ SC only



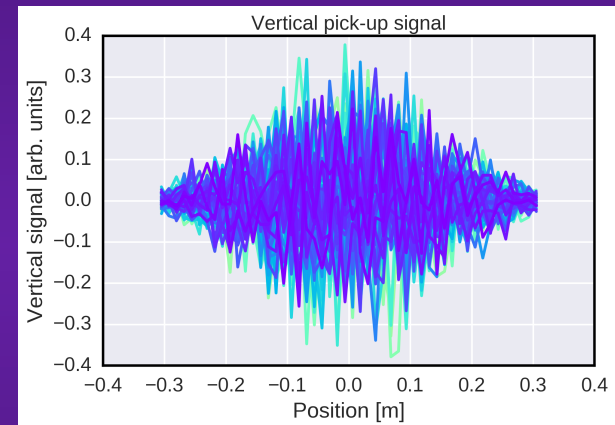
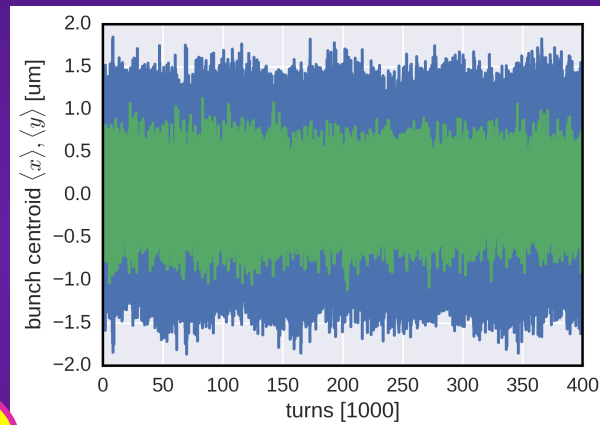
◆ Impedance + SC



◆ Impedance only

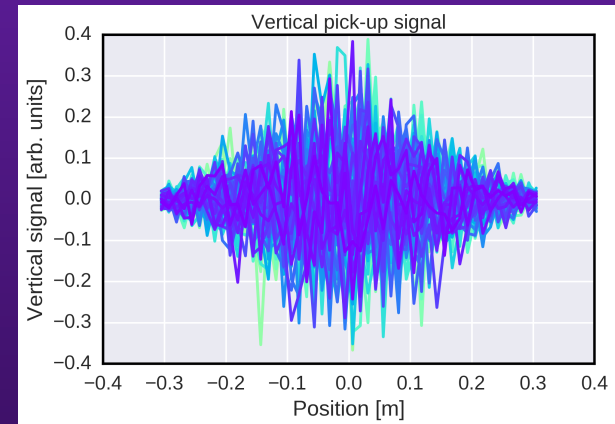
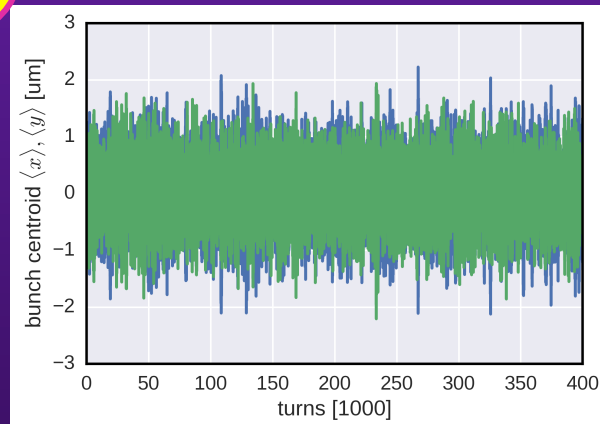


◆ SC only



**SC stabilizes the Head-Tail instability**

◆ Impedance + SC



- ◆ **Studying the effect of energy during the ramp, which reduces the SC tune spread (by increasing the transverse emittances at injection energy), the instability re-appears at  $\sim 2$  TeV...**
- ◆ **... 2 TeV is the energy at which the 1<sup>st</sup> transverse single-bunch instability was observed in the LHC during the 1<sup>st</sup> ramp performed in 2010 with neither Landau octupoles nor transverse damper**

- ◆ **F. Ruggiero was relying on / anticipating the stabilizing effect of SC at injection in the LHC design report (2004)**

these are pushed to the limit. The rigid  $m = 0$  head-tail mode will be damped by the feedback system. In case the lattice nonlinearities need to be reduced to guarantee a sufficient dynamic aperture, the direct space charge tune spread of about  $10^{-3}$  will provide additional Landau damping of the higher-order head tail modes<sup>4</sup>.

- ◆ **I mentioned at that time that this should be checked based on the PS experience...**

<sup>4</sup>This assumption needs experimental validation in the regime of the LHC at injection energy, where the space charge tune spread is somewhat larger than the tune spread associated with the natural nonlinearities of the magnetic lattice. Observations at the CERN PS with very long bunches and space charge detunings much larger than those associated with the lattice nonlinearities indicate that the direct space charge tune spread may not be effective in providing Landau damping of the higher-order transverse head-tail modes [29].

# CONCLUSION

- ◆ **SC simulation with pyHEADTAIL (2.5D PIC code from A. Oeftiger) gives an explanation of 1<sup>st</sup> single-bunch Head-Tail instability observed in LHC during 1<sup>st</sup> ramp in 2010 with neither Landau octupoles nor transverse damper**
  - SC stabilizes the Head-Tail instability below a certain energy ( $\sim 2$  TeV)
  - To be compared in detail with theory from A. Burov et al.

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  - SC stabilizes the Head-Tail instability below a certain energy ( $\sim 2$  TeV)
  - To be compared in detail with theory from A. Burov et al.
- ◆ **SC simulation also predicts that SC increases significantly the TMCI intensity threshold ( $Q' = 0$ ) at (HL-) LHC injection**
  - In agreement with some simplified models
  - To be compared in detail with theory from A. Burov et al.

# CONCLUSION

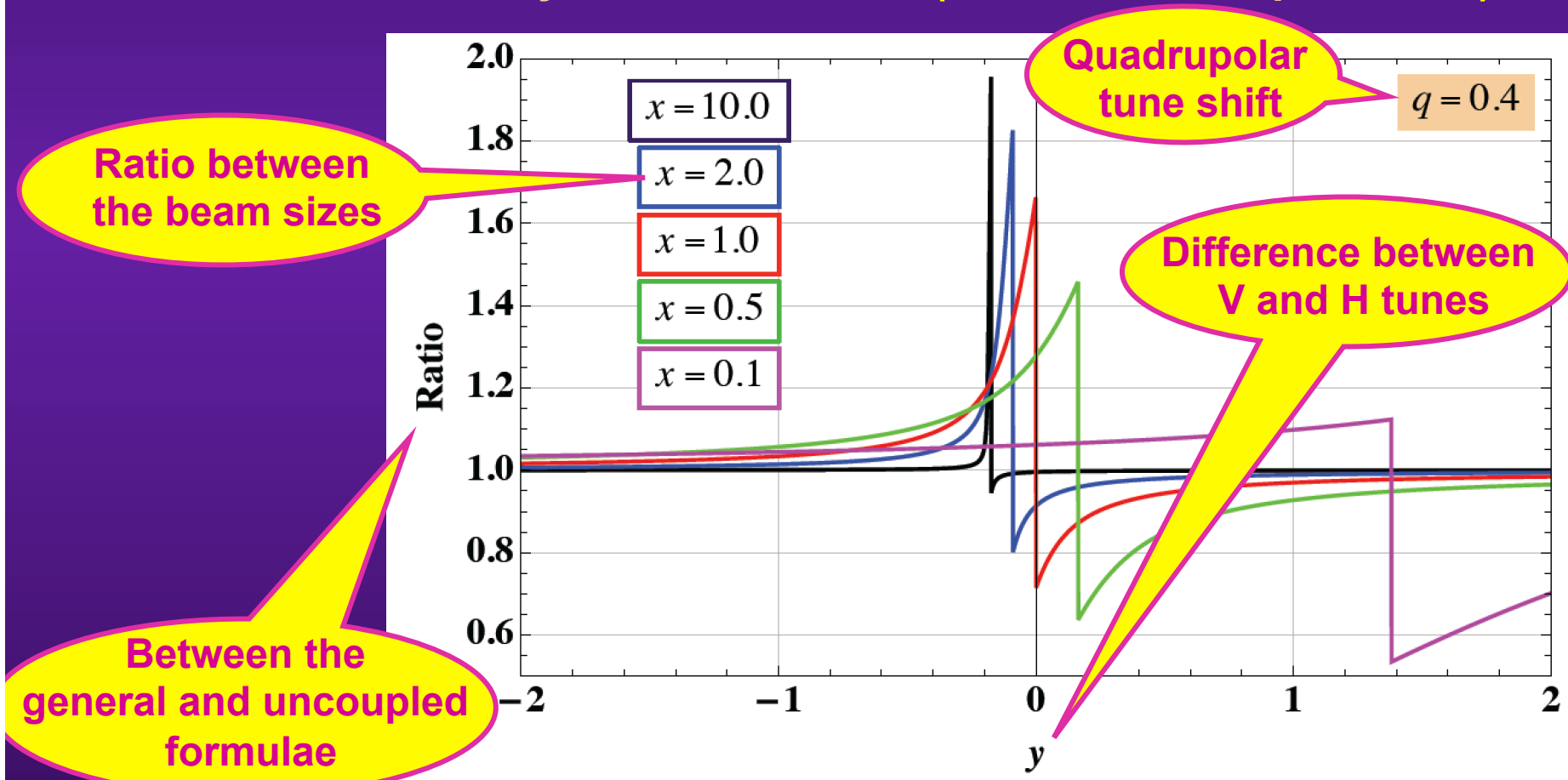
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- ◆ **SC simulation also predicts that SC increases significantly the TMCI intensity threshold ( $Q' = 0$ ) at (HL-) LHC injection**
  - In agreement with some simplified models
  - To be compared in detail with theory from A. Burov et al.
- ◆ **Finally, SC simulation reveals minor effect of SC on SPS TMCI ( $Q' = 0$ )**
  - In agreement with measurements
  - In agreement with some simplified models and past simulations
  - In “apparent disagreement” with theory from A. Burov et al. => To be continued...

# APPENDIX



# Quadrupolar Pick-Up (QPU): General formula for the space charge tune spread (HB2016)

- ◆ There is no new physics compared to Hardt\_1966
- ◆ I just expressed explicitly the general formula and compared it to the formula “usually” referred to him (valid for uncoupled case)



## 2 – PARTICLE MODEL

- ◆ **Following the same formalism as Chin-Chao-Blaskiewicz\_2016 (PRAB 19, 014201 (2016): <http://journals.aps.org/prab/pdf/10.1103/PhysRevAccelBeams.19.014201>): “Two particle model for studying the effects of space-charge force on strong head-tail instabilities”**
- ◆ **Adding a reactive transverse damper**

## 2 - PARTICLE MODEL

- ◆ **Chin-Chao-Blaskiewicz\_2016:** WF + SC with constant wake and zero chromaticity

$$y_1'' + \left( \frac{\omega_\beta}{c} \right)^2 y_1 = K (y_1 - y_2) + W y_2$$

$$y_2'' + \left( \frac{\omega_\beta}{c} \right)^2 y_2 = K (y_2 - y_1)$$

Wake Field (WF)

Space Charge (SC)

- ◆ **Discussed here:** WF + SC + TD

$$y_1'' + \left( \frac{\omega_\beta}{c} \right)^2 y_1 = K (y_1 - y_2) + W y_2 + g_{TD} (y_1 + y_2)$$

$$y_2'' + \left( \frac{\omega_\beta}{c} \right)^2 y_2 = K (y_2 - y_1) + g_{TD} (y_1 + y_2)$$

Transverse Damper (TD)

## 2 - PARTICLE MODEL

- ◆ Chin-Chao-Blaskiewicz\_2016  
(with WF + SC)

Dimensionless  
parameter  $\alpha W / T_s$ \*

\*  $T_s$  is the synchrotron period

$g$  = Instability growth rate

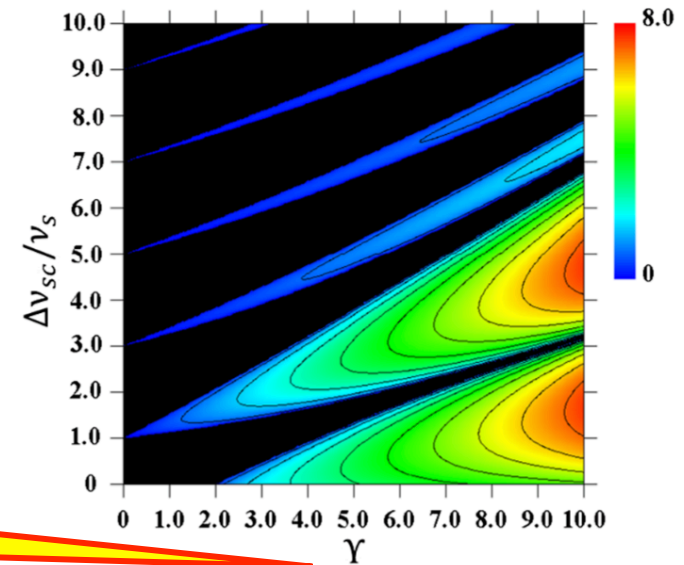


FIG. 4. Flat contour plot for the growth factor  $g \times T_s$  as a function of  $\Upsilon$  and  $\frac{\Delta v_{sc}}{v_s}$ .

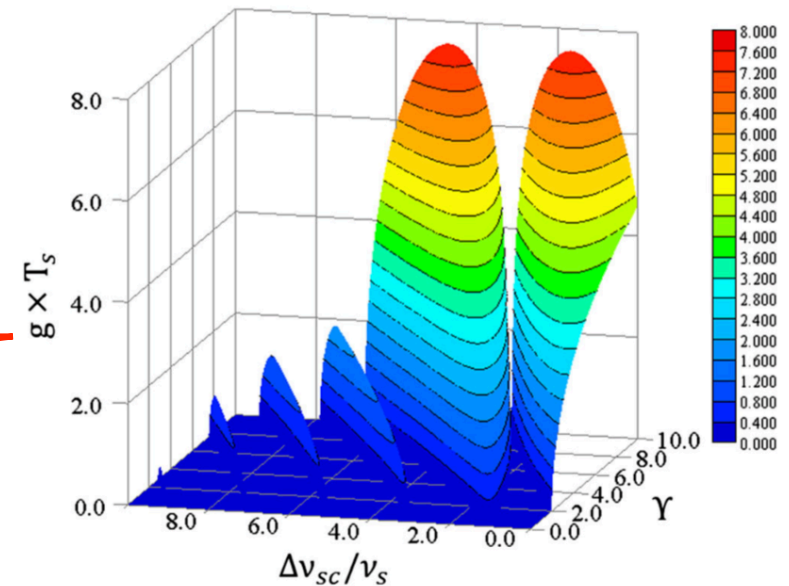


FIG. 5. Three-dimensional contour plot for the growth factor  $g \times T_s$  as a function of  $\Upsilon$  and  $\frac{\Delta v_{sc}}{v_s}$ .

## 2 – PARTICLE MODEL

- ◆ Similar result as Burov\_2016 in his paper “Efficiency of feedbacks for suppression of transverse instabilities of bunched beams” (<https://arxiv.org/abs/1605.06198>), where he considered the case of a reactive damper (on Fig. 1) but without space charge

*Note: Different notations used*

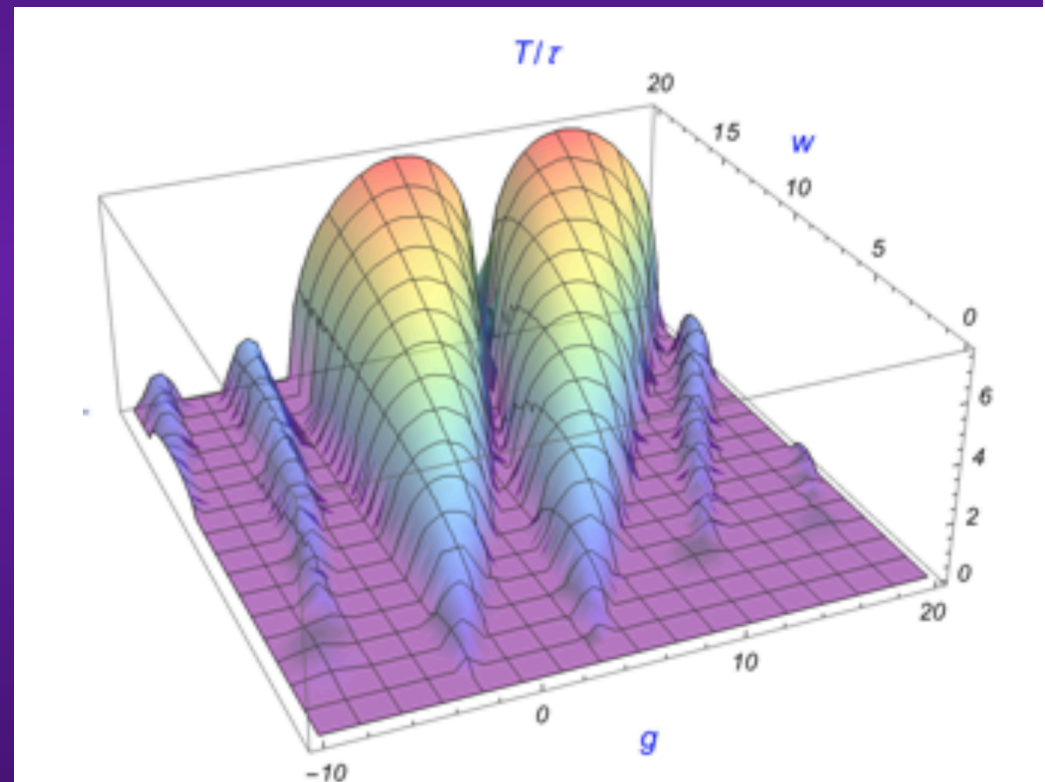


Fig. 1: Two-particle growth rate versus gain  $g$  and constant wake value  $w$  for reactive damper and zero chromaticity. All the values are in the units of the inverse synchrotron period  $1/T$ .

## 2 - PARTICLE MODEL

=> To be able to compare to Burov\_2016, we need to divide by

- 2 the WF axis
- $\pi$  the SC + TD\_reactive axis

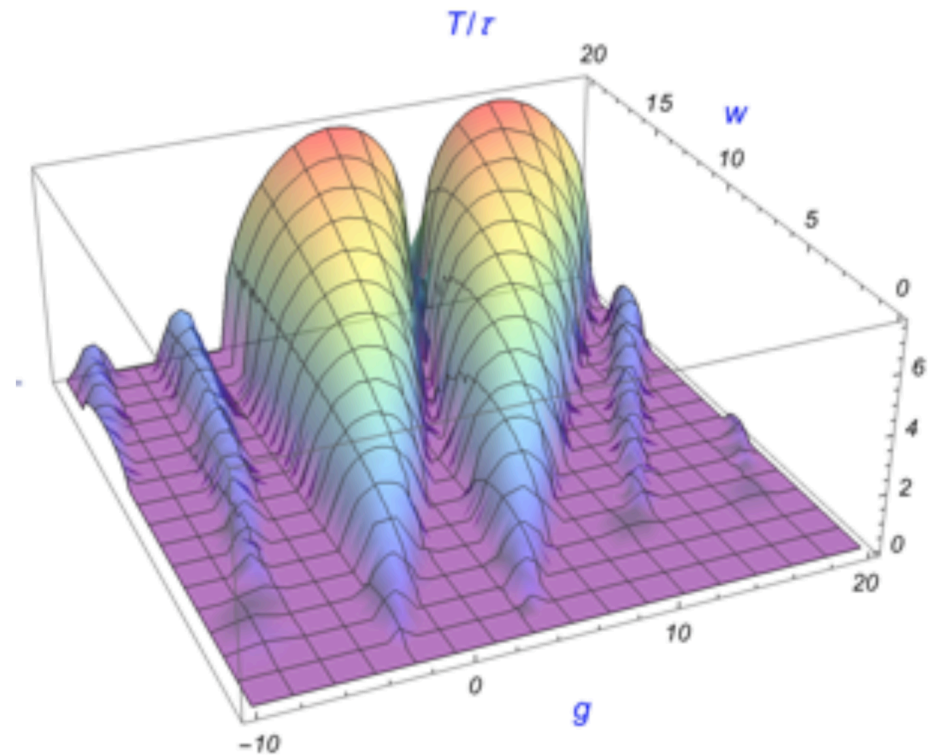
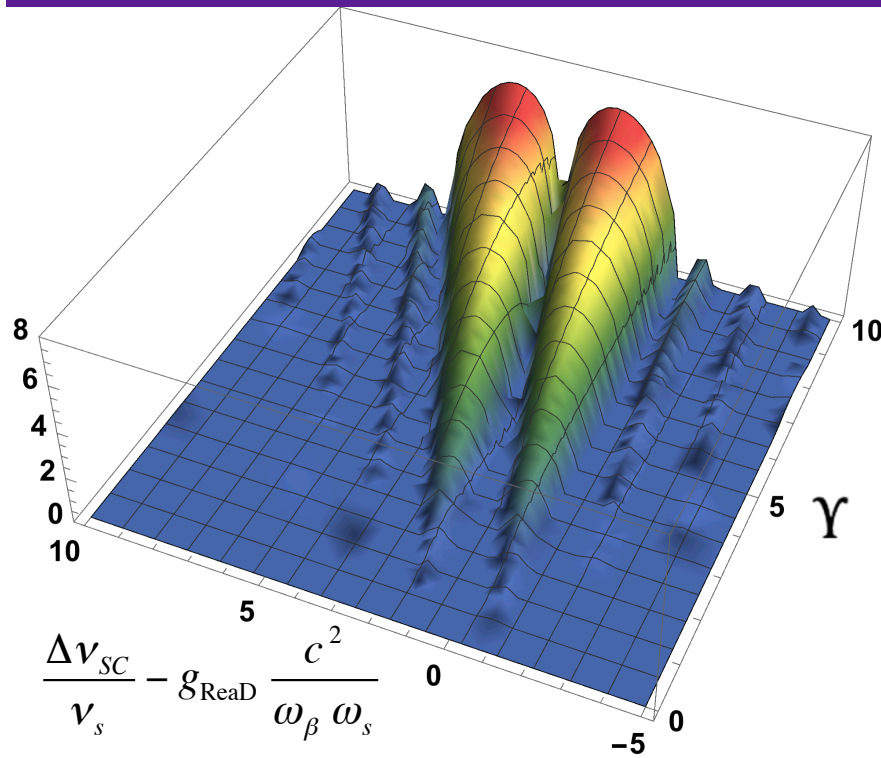


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## Abstract

Previous studies have shown the suppressing effect of direct space charge on impedance-driven head-tail instabilities. We investigate the impact of space charge on the instability thresholds, modes and growth rates for a range of chromaticities and different longitudinal bunch distributions with the aid of the macro-particle simulation tool PyHEADTAIL. The simulated wake fields include dipolar and quadrupolar components. We present predictions on transverse stability for the HL-LHC scenario and compare our simulation results to LHC measurements.

## PyHEADTAIL's Numerical Model

- GPU-accelerated 6D macro-particle tracking with
  - linear betatron tracking in the transverse plane with effective detuning models for chromaticity...
  - non-linear synchrotron tracking in the longitudinal plane
- lumped impedance being applied once per turn as wake field convolution
- space charge nodes distributed around the ring

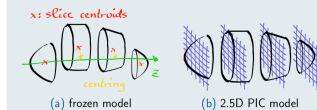
$$\mathcal{M}_{rev} = \exp(\Delta s : \mathcal{H}_{kick}) \exp(\Delta s : \mathcal{H}_{drift}) \dots$$



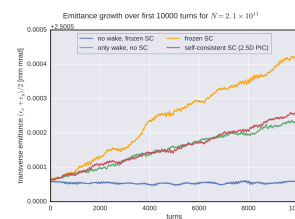
Figure 1: PyHEADTAIL's tracking model

## Employed Space Charge (SC) Models

- frozen field map of bunch**
  - transverse Bassetti-Erskine formula (Gaussian e-field)
  - subtracts each slice centroid
  - fields weighed with local line charge density along slices
  - updates field map if RMS beam size changes by 10% (adaptive)
- self-consistent 2.5D particle-in-cell (PIC) algorithm**
  - self-consistent particle-in-cell (PIC) algorithm
  - solves transverse Poisson equation slice by slice
  - open-space boundary conditions



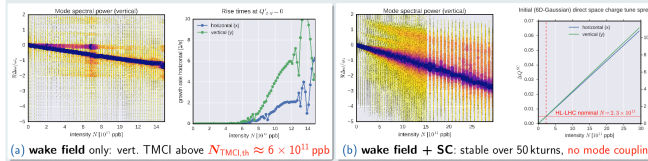
## Impact of SC Models: Self-consistent PIC Better than Frozen!



- frozen model:**
  - symplectic (for smooth Bassetti-Erskine)
  - like a non-linear external magnet
  - violates total momentum conservation (!)
  - incoherent  $\Delta Q_{x,y}^{SC} = 4.5 \times 10^{-3}$  in centroid motion
- 2.5D PIC model:**
  - non-symplectic yet self-consistent
  - conserves global momentum
  - violates local energy conservation (macro-particle → grid conversion) ⇒ unphysical **grid heating**

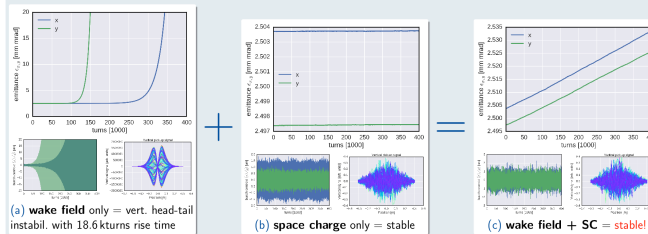
## HL-LHC: Transverse Mode Coupling Instability (TMCI) + Space Charge

HL-LHC injection plateau at  $Q'_{x,y} = 0$ ,  $\sigma_z = 10.4$  cm and  $\epsilon_{x,y} = 2.5$  mmrad:

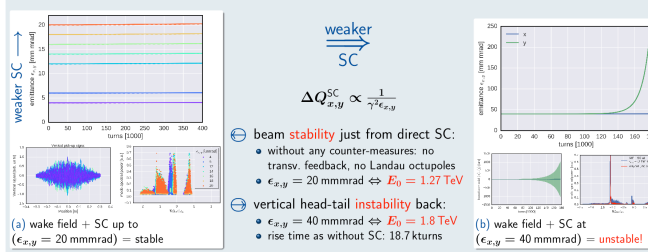


## HL-LHC: Head-Tail Instability + Space Charge

HL-LHC injection plateau at  $Q'_{x,y} = 5$ ,  $\sigma_z = 10.4$  cm and  $N = 4 \times 10^{11}$  ppb <  $N_{TMCI,th}$ :



## Space Charge Limit when Head-Tail Instability Rises



**Measurements at LHC**

- first instability observed in 2010 during the first ramp:
  - both beams affected independently, mode  $m_x = -1$  with 1 node
  - flat-bottom and initial part of the ramp stable until  $E_0 \approx 2$  TeV
- (plots from [1], flat-top measurements of this head-tail instability at  $E_0 = 3.5$  TeV)

## Conclusions

- Including direct space charge (SC) into coherent instability studies...
  - requires self-consistent SC modelling to avoid finite centroid kicks from SC
  - increases (or cancels) the Transverse Mode Coupling Instability threshold for the short-bunch regime with modes 0 and -1 coupling
  - can explain the inherent **single-bunch stability** at LHC at low energies in absence of machine non-linearities and transverse damping

## References

[1] Elias Métral et al. *Measurement and interpretation of transverse beam instabilities in the CERN large hadron collider (LHC) and extrapolations to HL-LHC*. Tech. rep. CERN-ACC-2016-0098. Geneva: CERN, July 2016.