New developments on adaptive SC methods

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Subject

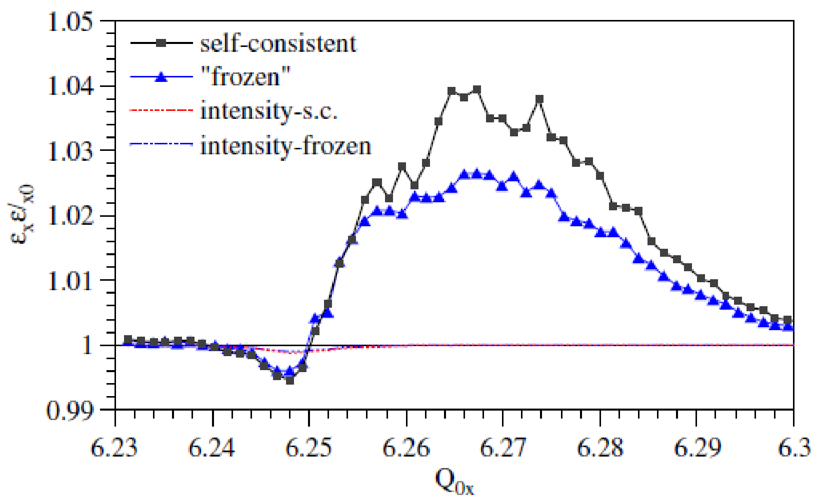
Methods:

- Truly self-consistent: SC field by solving Poisson eq. for actual distribution of tracking particles (PIC etc.)
- "Express": Bunch density distribution is approximated by a predefined form(s) ("templates"), e.g. Gaussian:

- "Frozen" : the distribution does not change geometrically (but may change in time proportionally to the number of surviving particles)
- "Adaptive" : geometrical parameters of the template (sizes, but may be c.o.m. position as well) are updated based on the ensemble evolution during tracking. (It is not fully self-consistent and therefore sometimes also called "frozen")

My concern is adaptive simulations in the above sense, but let me start with some results by PIC codes to make the point.

Motivation



PIC simulations of 2D Gaussian beam. (I. Hoffmann, G. Franchetti, NIMA 561, 2006)

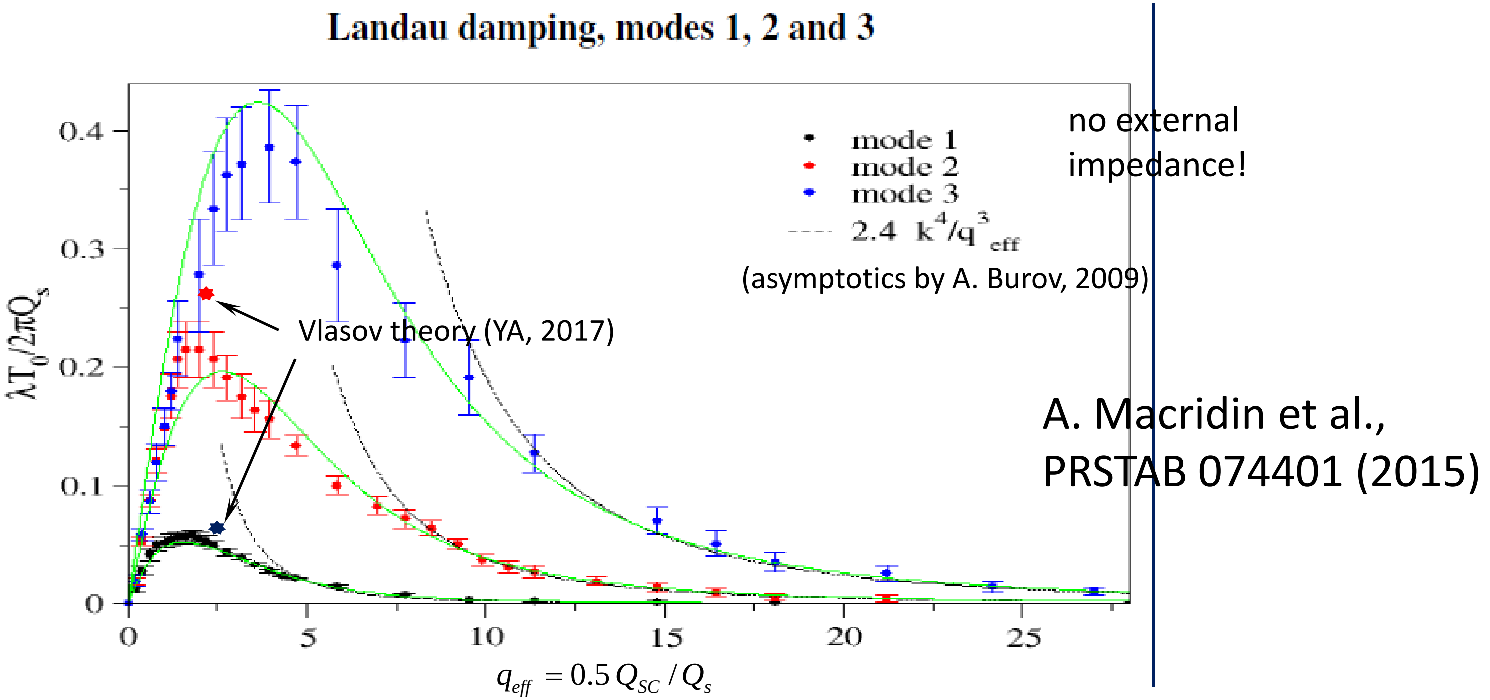
- "Frozen" space charge model predictions differ quantitatively from self-consistent (and "adaptive") simulations, especially for resonance crossing.
- "Frozen" model misses collective phenomena (e.g. beam envelope resonances)
- Self-consistent (or at least "adaptive") approach is especially important for large tuneshifts when the beam footprint overlaps half-integer:

FNAL Booster: now PIP+

$\Delta Q_x / \Delta Q_y$ = (-) 0.23/0.31 → 0.3/0.4

PIC codes

PIC codes (MICROMAP, Synergia etc) provide truly self-consistent modelling



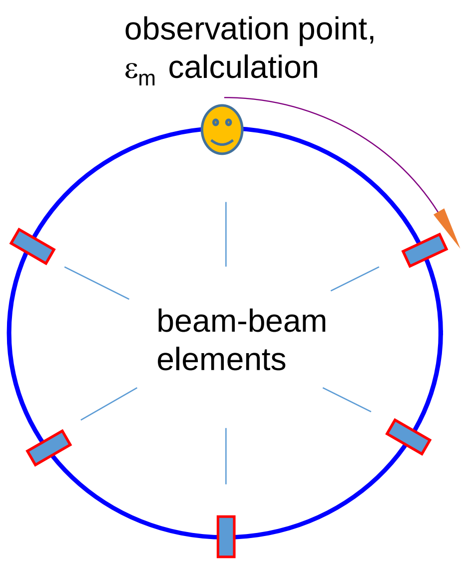
Synergia: Hi-Fi tool used both for accelerator physics research (see above) and detailed simulations of real machines.
Drawback - very time consuming. One point above takes ~24 hours on 1000 nodes cluster. (2000 turns, 10^8 macroparticles, but only one SC)

For practical purposes a simpler adaptive approach can be used, like the one being developed with MADX

Adaptive MADX-SC

The crucial issue: the emittance evaluation method which would suppress the halo contribution but give the exact result for a Gaussian distribution.

Presently a simple algorithm is used for exponential fitting of 1-dimensional distributions in the transverse action variables (requires optics functions):



$$\sigma_u^2 = \sum_m \beta_{um}\varepsilon_m, \quad u = x, y, z$$

1. The action values J (half the Courant-Snyder invariants) in the transverse planes are calculated for each particle using stored Twiss parameters (can be periodically updated).

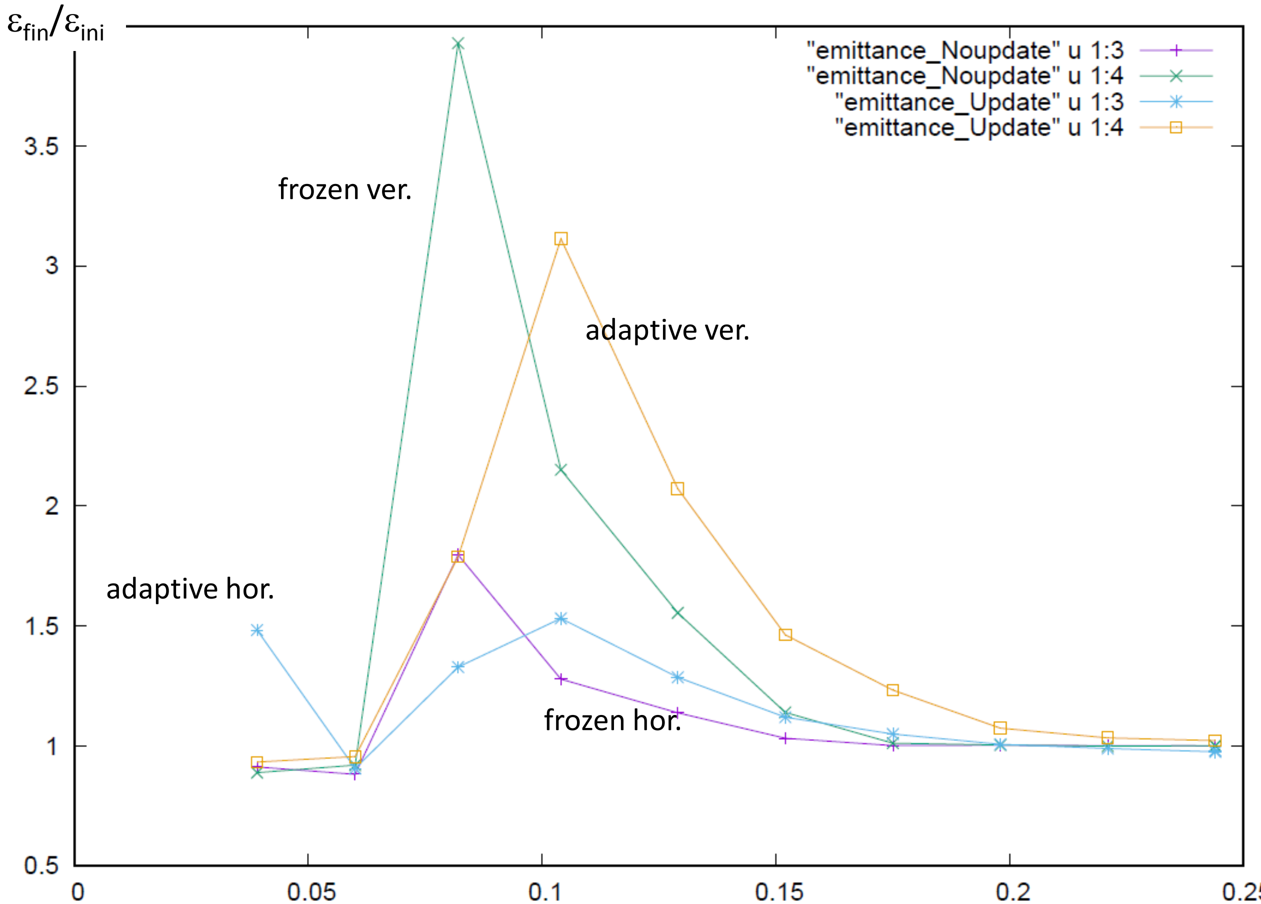
2. The particles are ordered so that $J_k \geq J_{k-1}$.

3. The emittances are calculated as

$$\frac{1}{\varepsilon} = -\sum_{k=1}^{N} w(J_k) J_k \log[1 - \frac{k-1}{N}] / \sum_{k=1}^{N} w(J_k) J_k^2$$

The weight function was chosen as $w(J) = 1/(J^2 + J_0^2)$ with some small J_0. It provides a moderate suppression of the halo contribution .

MADX-SC Adaptive vs Frozen Mode

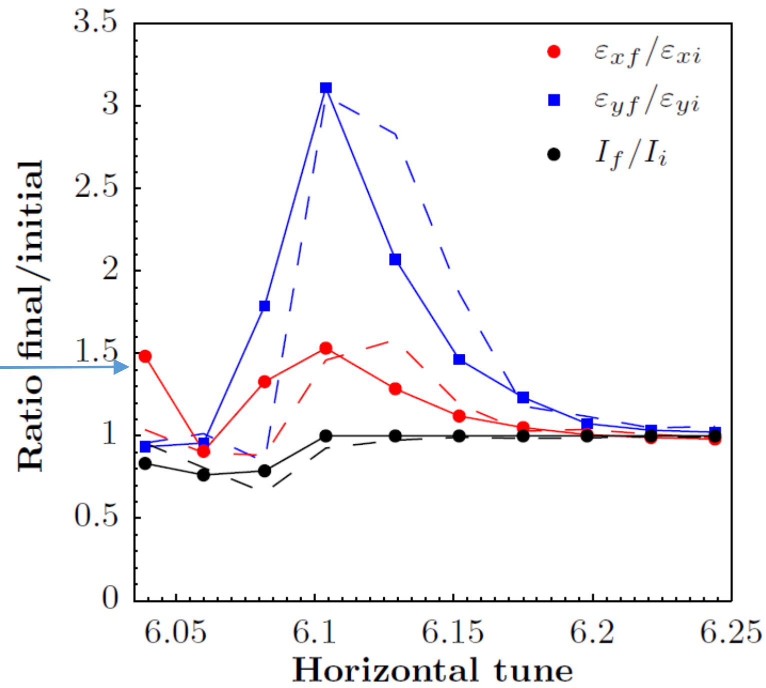


PS beam emittance evolution over $5 \cdot 10^5$ turns at 2GeV vs. Q_{x0} computed with MADX-SC in adaptive and frozen modes (Q_{y0} = 6.476, SC tuneshifts: $\Delta Q_x \approx$-0.05, $\Delta Q_y \approx$-0.07).

The adaptive mode better describe the measurements data overall but…

MADX-SC vs PS

Blowup at Q_{x0} = 6.035 is absent in both experiment and frozen mode simulations



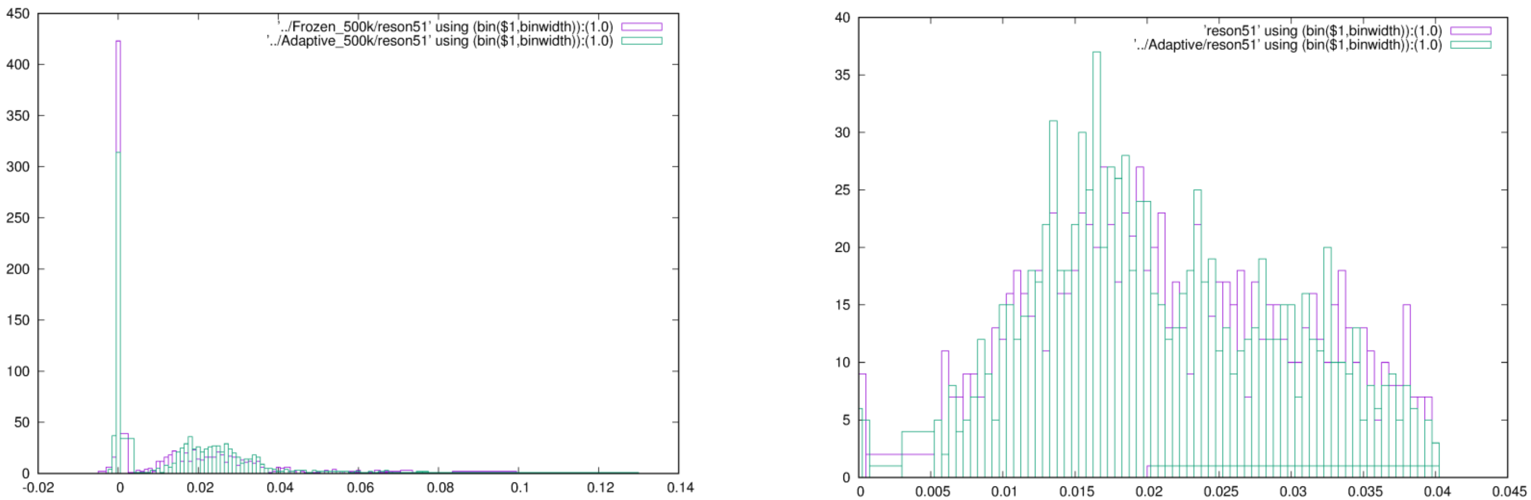
Q_{y0} = 6.476,
SC tuneshifts :
$\Delta Q_x \approx$-0.05, $\Delta Q_y \approx$-0.07.

PS beam emittance evolution over $5 \cdot 10^5$ turns at 2GeV vs. Q_{x0}. Dashed lines present experimental results, solid lines with dots present MADX simulations with adaptive SC.

One of the reasons for discrepancy was too aggressive cut (at 2sigmas) when calculating the r.m.s. bunch length & momentum spread (used in the SC kick formula) - longitudinal dimensions should be either obtained by a fitting algorithm (like the transverse) or not updated at all.

But probably this was not the main reason.

Statistical noise



Fourier spectra of emittance oscillations: over $5{\cdot}10^5$ turns (left) and over 1000 turns (right).

With small number of particles there are large beam size fluctuations ("Schottky noise") which spectrum coincides with twice the incoherent tunespread and may lead to emittance growth - especially close to (half) integer resonance.

Possible cures:

- Filtering of fluctuations – may suppress real physics as well
- Larger number of particles – requires faster SC kick computation (ongoing work with F. Schmidt and H. Bartosik)

New algorithm

Existing algorithm drawbacks:
- requires stable linear optics
- longitudinal beam size computed as truncated RMS (too agressivly!)
- transverse beam sizes considered as equilibrium ones on each turn - no envelope resonances
- uncoupled optics assumed

New algorithm is based on Gaussian fit of the sigma matrix (of any rank)
- does not require stable optics
- allows for nonstationary distribution - envelope resonances!
- stronger suppression of the halo contribution (less noise?)
- provides number of particles in the core

The Σ matrix (with all its cross-correlations) can be propagated from point 1 to point 2 using linear transport matrix T (again, no stable optics required)

$$\Sigma^{(2)} = \mathrm{T} \cdot \Sigma^{(1)} \cdot \mathrm{T}^{t}$$

Gaussian Fit

The outer loop is on the fraction of particles in the core η (if also fitted).

With η fixed the following equation for Σ- matrix is solved (by iterations)

$$\Sigma_{ij} = \frac{1}{N}\sum_{k=1}^{N}\zeta_i^{(k)}\zeta_j^{(k)}\exp[-\frac{1}{2}(\underline{\zeta}^{(k)},\Sigma^{-1}\underline{\zeta}^{(k)})]/\left(\frac{1}{N}\sum_{k=1}^{N}\exp[-\frac{1}{2}(\underline{\zeta}^{(k)},\Sigma^{-1}\underline{\zeta}^{(k)})]-\frac{\eta}{2^{n/2+1}}\right)$$

where *n* is the dimensionality of the problem (any, e.g. 4 or 6)

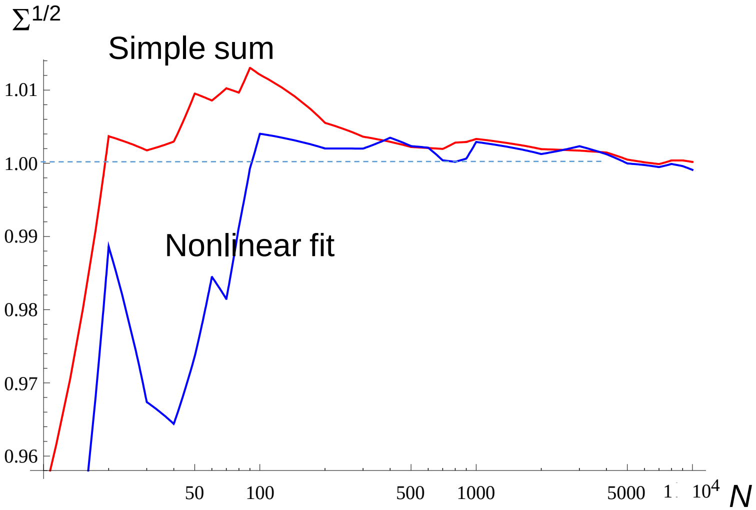
At each iteration the bunch center coordinates are updated as

$$\bar{z}_i = \sum_{k=1}^{N} z_i^{(k)}\exp[-\frac{1}{2}(\underline{\zeta}^{(k)},\Sigma^{-1}\underline{\zeta}^{(k)})]/\sum_{k=1}^{N}\exp[-\frac{1}{2}(\underline{\zeta}^{(k)},\Sigma^{-1}\underline{\zeta}^{(k)})], \quad \zeta_i^{(k)} = z_i^{(k)} - \bar{z}_i$$

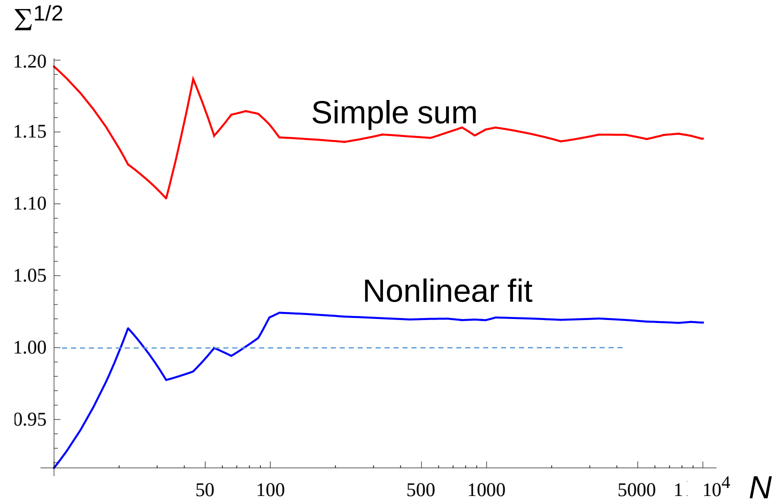
Then new value of η is found for the next step in the outer loop

$$\eta = \frac{2^{n/2}}{N}\sum_{k=1}^{N}\exp[-\frac{1}{2}(\underline{\zeta}^{(k)},\Sigma^{-1}\underline{\zeta}^{(k)})]$$

1D precision test



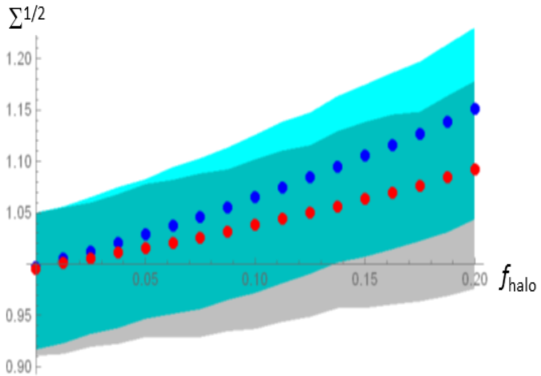
Square root of Σ averaged over 25 realizations of 1D Gaussian distribution with σ=1 as function of the number of particles *N*.

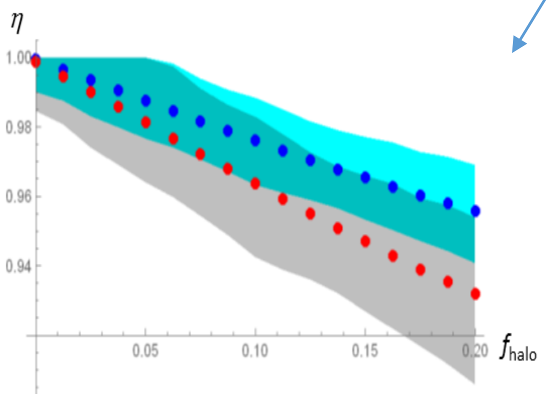


Square root of Σ averaged over 25 realizations of superposition of 1D Gaussian distributions with σ=1(90%) and σ=3(10%)

The fraction of particles in the core η was not fitted $\rightarrow$ the SC kick will be overestimated by a few %% in the case on the right despite larger Σ.

Fitting %% of particles in the core





Top:
Average over 100 realizations beam size ($\Sigma^{1/2}$) vs. fraction of particles in the halo,
Bottom:
Average fraction of particles in the core (η)
- η = 1 fixed during iterations.
- η included in the fit.

The shadowed areas show the spread in plotted values.

With fitted η the SC kick from the core is about right.

We may even try to add the kick from the halo. Its size can be approximated as

$$\Sigma^{(\text{halo})} = \Sigma_0 + \frac{\eta}{1-\eta}\sqrt{\frac{\det \Sigma^{(\text{core})}}{\det \Sigma^{(\text{halo})}}}\left(\Sigma_0 - \Sigma^{(\text{core})}\right)$$

$$(\Sigma_0)_{i,j} = \frac{1}{N}\sum_{k=1}^{N} \zeta_i^{(k)} \zeta_j^{(k)},$$ being traditional Σ-matrix

For comparison at f_{halo}=0.2:

"simple sum" $\Sigma_0^{1/2} = 1.61$

exponential fit $\Sigma_{\text{exp}}^{1/2} = 1.24$

3DoF symplectic kick

Transverse SC kick depends on longitudinal position in bunched beam, to make transformation symplectic it should be complemented with longitudinal kick coming from the same potential

$$\varphi(x,y,z,t) \cong \lambda(z-v_0 t)\cdot\Phi(x,y)$$

There are different representations for Φ, e.g.

$$\Phi(x,y)=\int_0^1\left\{\exp\left(-\frac{x^2t}{2\sigma_x^2}-\frac{y^2r^2t}{2\sigma_y^2[1+(r^2-1)t]}\right)-1\right\}\frac{dt}{t\sqrt{1+(r^2-1)t}},\qquad r=\frac{\sigma_y}{\sigma_x}$$

For $d_{res}=\left(\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2}\right)^{1/2}\le 5-7$ i.e. for the entire beam a power expansion is available that gives better than 5-digit accuracy