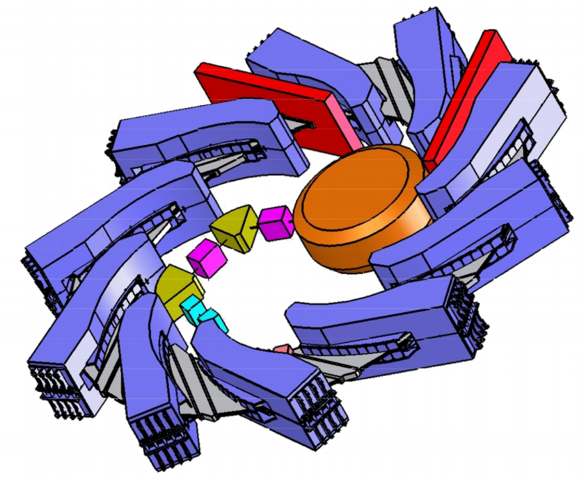
Space Charge Modelling in OPAL for FFAGs



C. Rogers,
ISIS Intense Beams Group
Rutherford Appleton Laboratory

FFAG Rings

- Fixed field alternating gradient (FFAG) rings invented more than 50 years ago
  - Special combined function magnets
  - Constant tune with energy
- Recently redeveloped in Japan and elsewhere for proton beam acceleration
- Seek now to develop FFAG for proton driver
  - Need to demonstrate operation in the presence of space charge
  - Need theoretical tools to support that operation

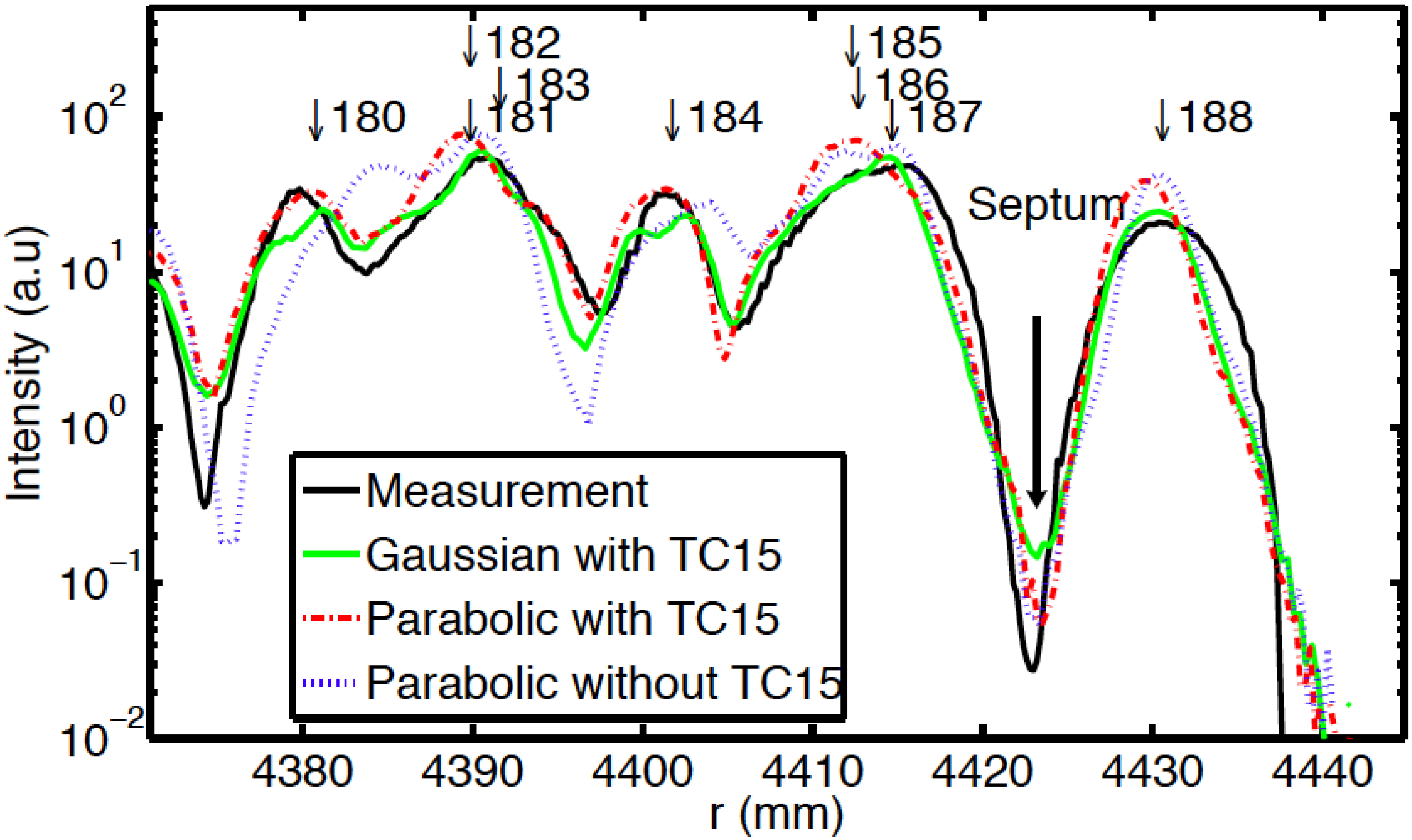
OPAL and FFAGs

- In 2013, S-Code (Machida) was the only code that could track FFAGs and model space charge effects
- OPAL-CYCL code developed originally for simulation of cyclotrons
  - Benchmarked against e.g. PSI cyclotron
- Leverage similarities of cyclotrons and FFAGs
  - Use OPAL-CYCL space-charge solving together with new FFAG field map routines
- Focus of the effort has been on the routines for FFAG magnets, rather than space charge solver itself

A Little Bit of History...

- OPAL originated from MAD9p
  - MAD9 (CERN)
  - Pooma (LANL)
- MAD9 & Pooma died
  - Andreas Adelmann refactored, improved and started OPAL
- Changed integration variable from s to t
  - OPAL-T
- Cyclotron tracking – OPAL-CYCL
  - J.Y. Yang
- Envelope propagation – OPAL-ENVELOPE
- D. Winklehner (MIT/PSI) generalised OPAL-CYCL in 2014
- FFAG capabilities added 2014-2017

PSI 590 MeV Ring - last 8 turns @ 2.2 mA



[Y. Bi, A. Adelmann, et al., PR-STAB **14**(5) (2011)]

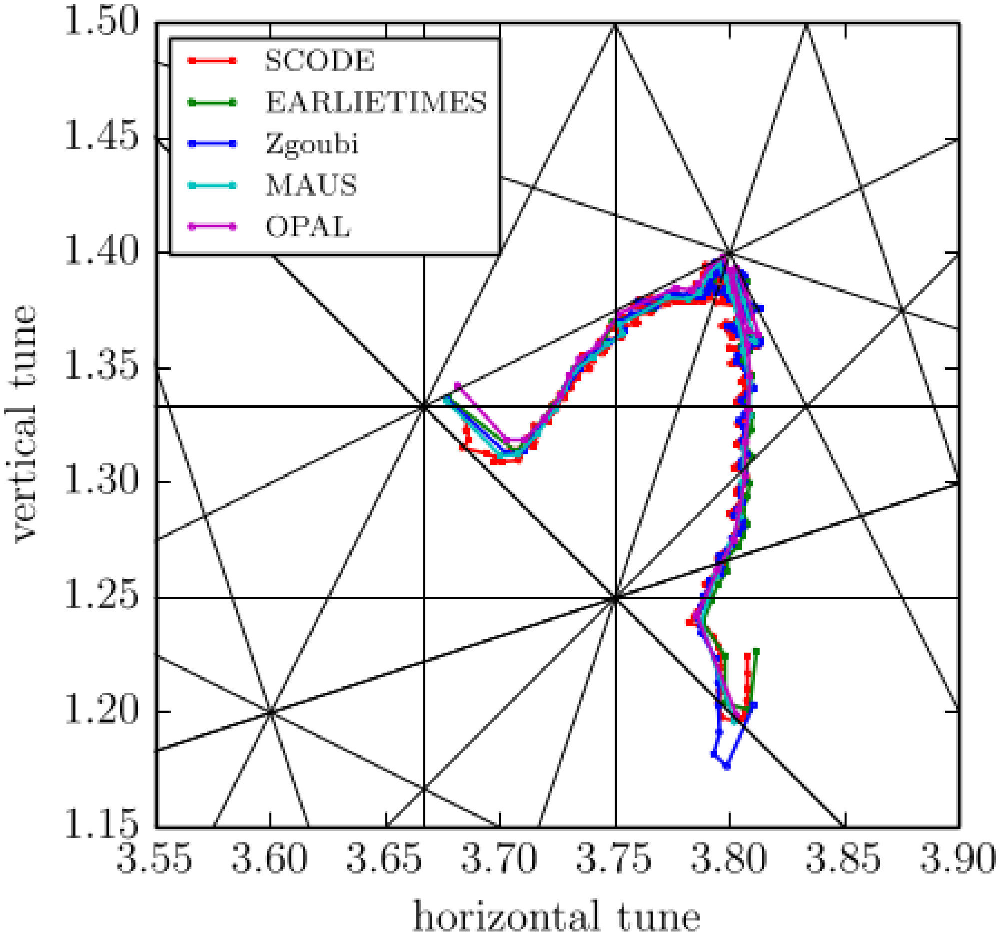
FFAG Modelling

- Various iterations of FFAG modelling implemented
  - Full 3D field map for the ring and fixed-frequency RF (S. Sheehy et alia, 2013)
  - Individual 3D field map for each cell/ring element (C. Rogers, 2014)
  - Variable frequency RF and improved placement routines (C. Rogers, 2015)
  - Analytical model for FFAG fields (C. Rogers, 2017)
- Working on multipole-based storage rings (T. Dascalu, 2017)

Validation of single particle tracking

- 3D field map routines with field maps for each ring element
- Generate field maps using OPERA3D
- Arbitrary order interpolation and smoothing

Simulation of KURRI Main Booster FFAG



Generalised field map

- For optics/design work, implemented analytical field map
  - Follow scaling law in the midplane
  - Arbitrary end-field shape (E.g. enge, tanh function, etc)

Derivation of field expansion (1)

- Consider a midplane field of the form

$$\begin{aligned} B_r(r,\phi,z=0) &= 0 \\ B_\phi(r,\phi,z=0) &= 0 \\ B_z(r,\phi,z=0) &= f_0(\psi)h(r) \end{aligned}$$

- With

$$\begin{aligned} \psi &= \phi - g(r), \\ g &= \tan(\delta)\ln(r/r_0), \\ h(r) &= B_0\left(\frac{r}{r_0}\right)^k \end{aligned}$$

Derivation of field expansion (2)

- Assume there exists a solution as a power law expansion in vertical axis (*z*)

$$B_z = \sum_{n=0} f_{2n}(\psi) h(r) \left(\frac{z}{r}\right)^{2n}$$

$$B_\phi = \sum_{n=0} f_{2n+1}(\psi) h(r) \left(\frac{z}{r}\right)^{2n+1}$$

- Use Maxwell $\nabla \times \vec{B} = 0$
- ***z*** direction gives

$$f_{2n+1} \quad = \quad \frac{\partial_\psi f_{2n}}{2n+1}.$$

- **Φ** direction gives

$$B_r = \sum_{n=0} \left[\frac{k-2n}{2n+1} f_{2n} - \tan(\delta) f_{2n+1}\right] h(r) \left(\frac{z}{r}\right)^{2n+1}$$

Derivation of field expansion (3)

- Use Maxwell $\nabla \vec{B} = 0$

$$f_{2n+2} = \frac{\frac{(z-2n)^2}{2n+1} f_{2n} - 2(k-2n)tan(\delta) f_{2n+1} + (1+\tan^2(\delta))\partial_\psi f_{2n+1}}{(2n+2)}.$$

- Assume f_n has a form like

$$f_n = \sum_{i=0} a_{i,n} \partial_\psi^i f_0$$

- Then we get recursion relations for odd and even *n* like

$$a_{i,2n+1} = \frac{a_{i-1,2n}}{2n+1}.$$

$$a_{i,2n+2} = \frac{1}{2n+2} \left[a_{i,2n+1} 2(k-2n) \tan(\delta) - \frac{(k-2n)^2}{2n+1} a_{i,2n} - (1+\tan^2(\delta)) a_{i-1,2n+1} \right].$$

Implementation

- We have a set of recursion relations for off-midplane fields that only depend on f_0 and its derivatives
- Use any function for f_0 that has continuous derivatives
  - Use C++ inheritance (function pointers) to determine at run time which fringe field to use
  - End field “plug in”
- Implemented Enge
  - Good for fitting an existing field map
  - Easy to do horrible things with poor choice of parameters
  - Recursion relation for arbitrary derivative

$$f_0(\psi) = E\left(\frac{\psi - \psi_0}{\lambda}\right) + E\left(\frac{-\psi - \psi_0}{\lambda}\right) - 1$$

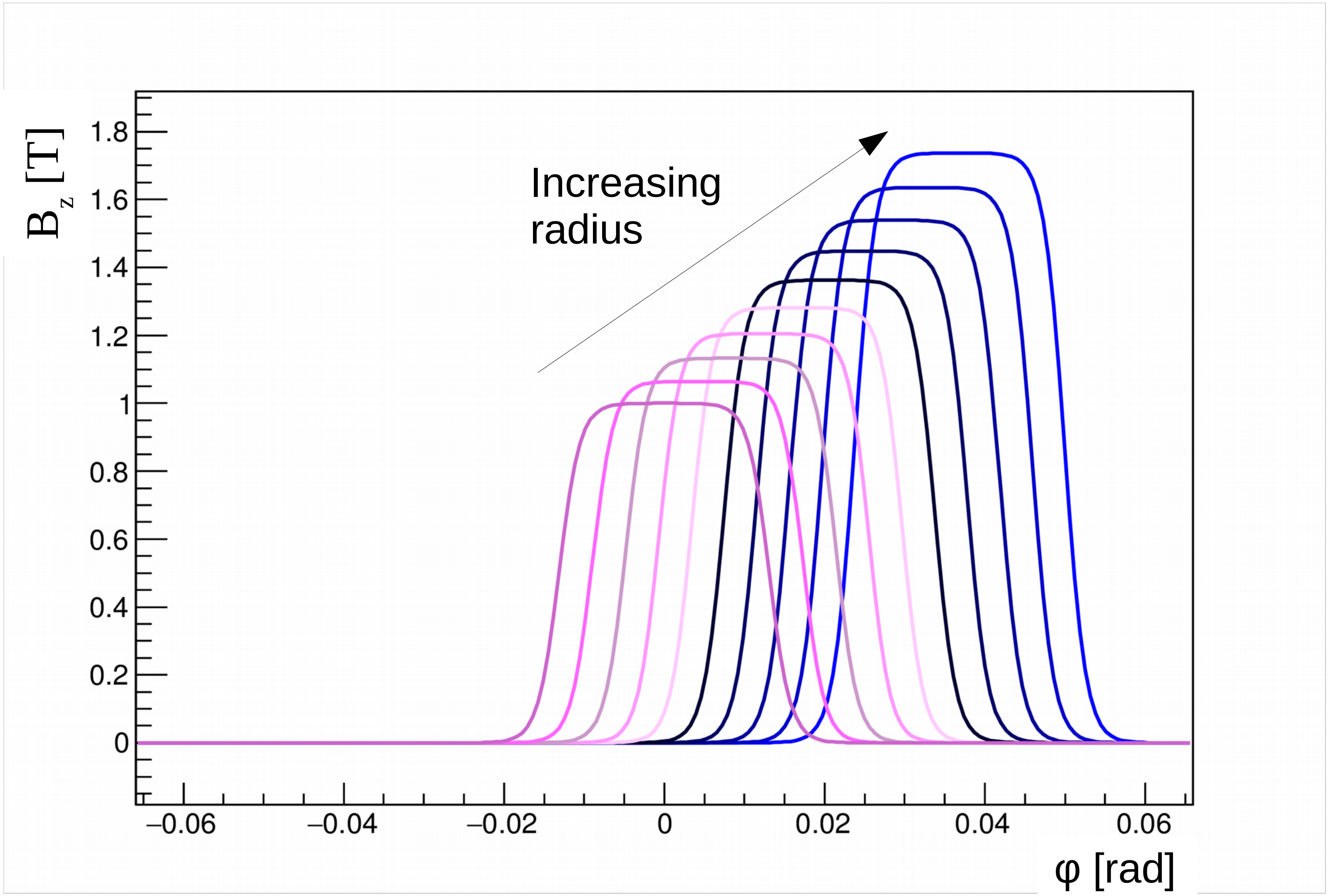
$$E(s) = \frac{B_0}{R_0^n} \frac{1}{1 + \exp(C_1 + C_2 s + C_3 s^2 + ...)}$$

Implementation

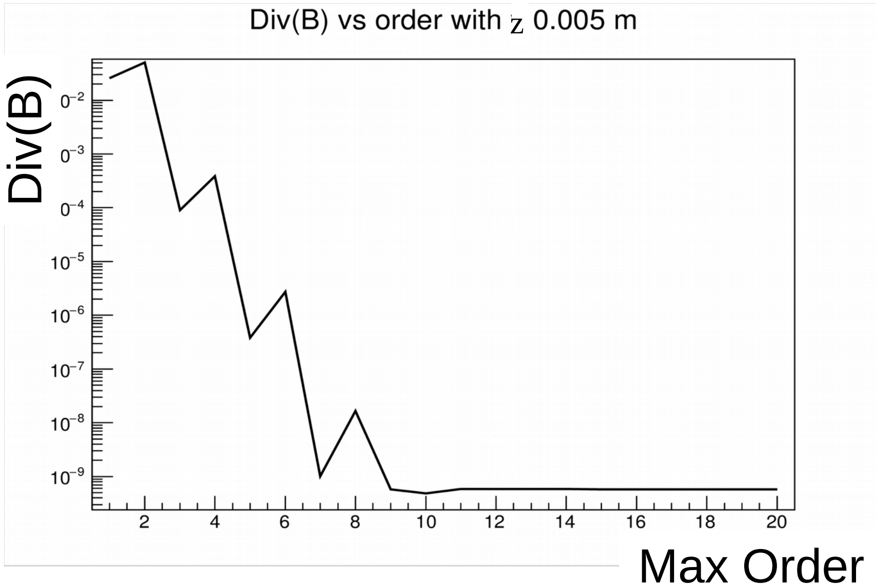
- Implemented Tanh
  - Clear parameterisation with two parameters:-
  - Centre length (ψ_0)
  - Fringe field length (λ)
  - Recursion relation for arbitrary derivative
- Most functions can be implemented
  - Need to have known derivatives

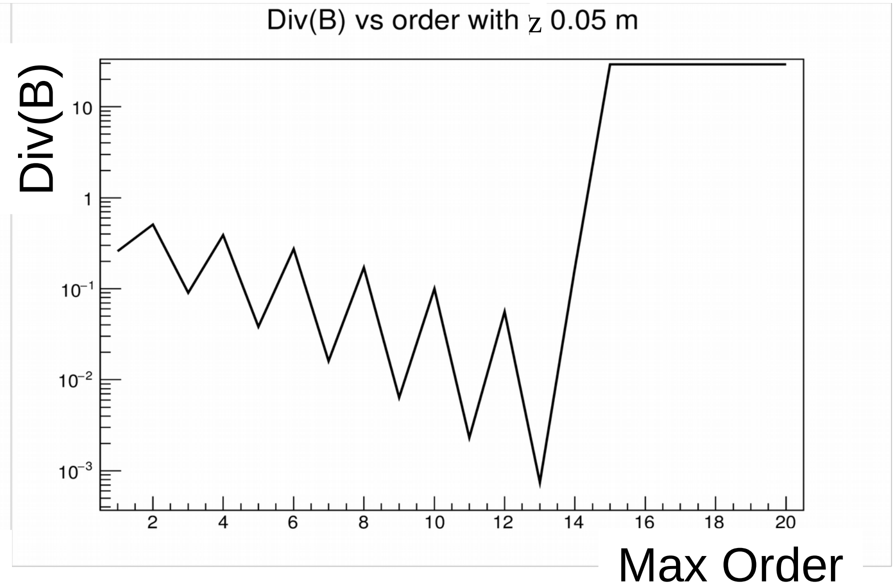
$$f_0(\psi) = \frac{1}{2}\left[\tanh\left(\frac{\psi + \psi_0}{\lambda}\right) - \tanh\left(\frac{\psi - \psi_0}{\lambda}\right)\right]$$

Test Field Map



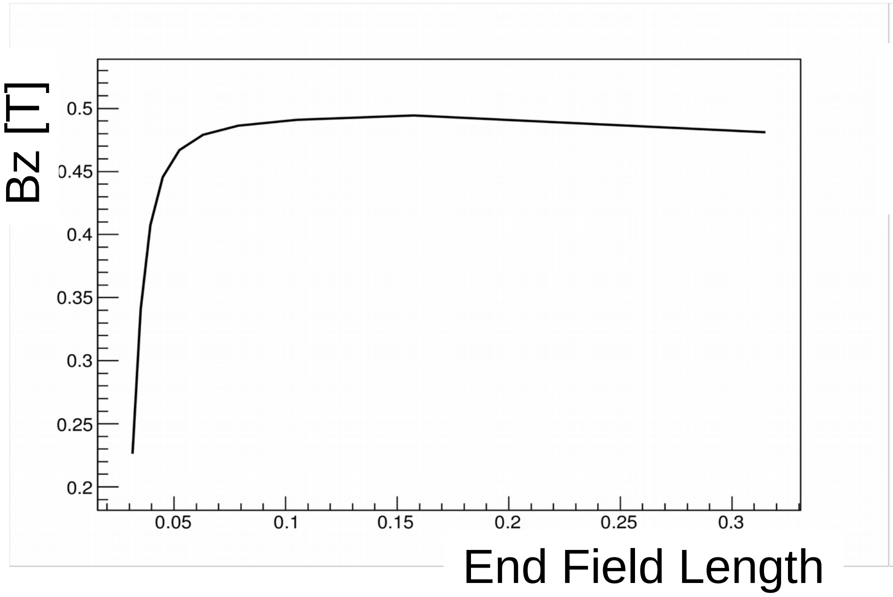
Convergence vs Expansion Power

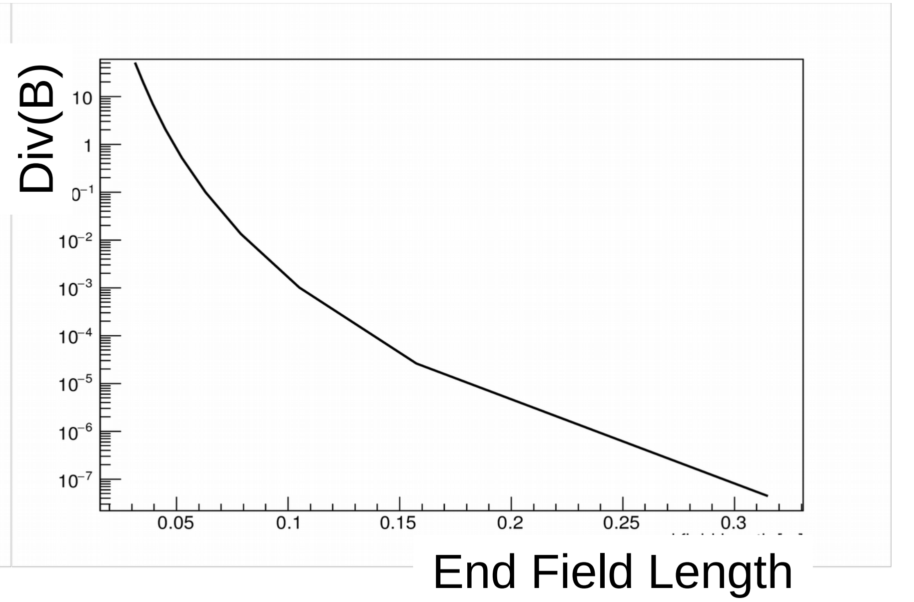




- Near midplane, more Maxwellian as higher powers of z are included
- Further off midplane, convergence is poor
  - Floating point precision limit at >= 14th order

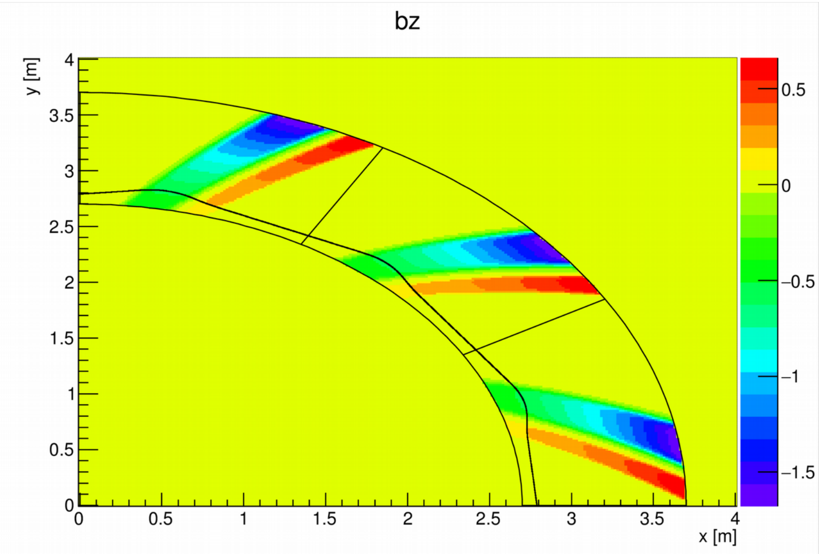
Dependence on End Field





- Shorter fringe fields have smaller convergence radius
  - Expect lower DA

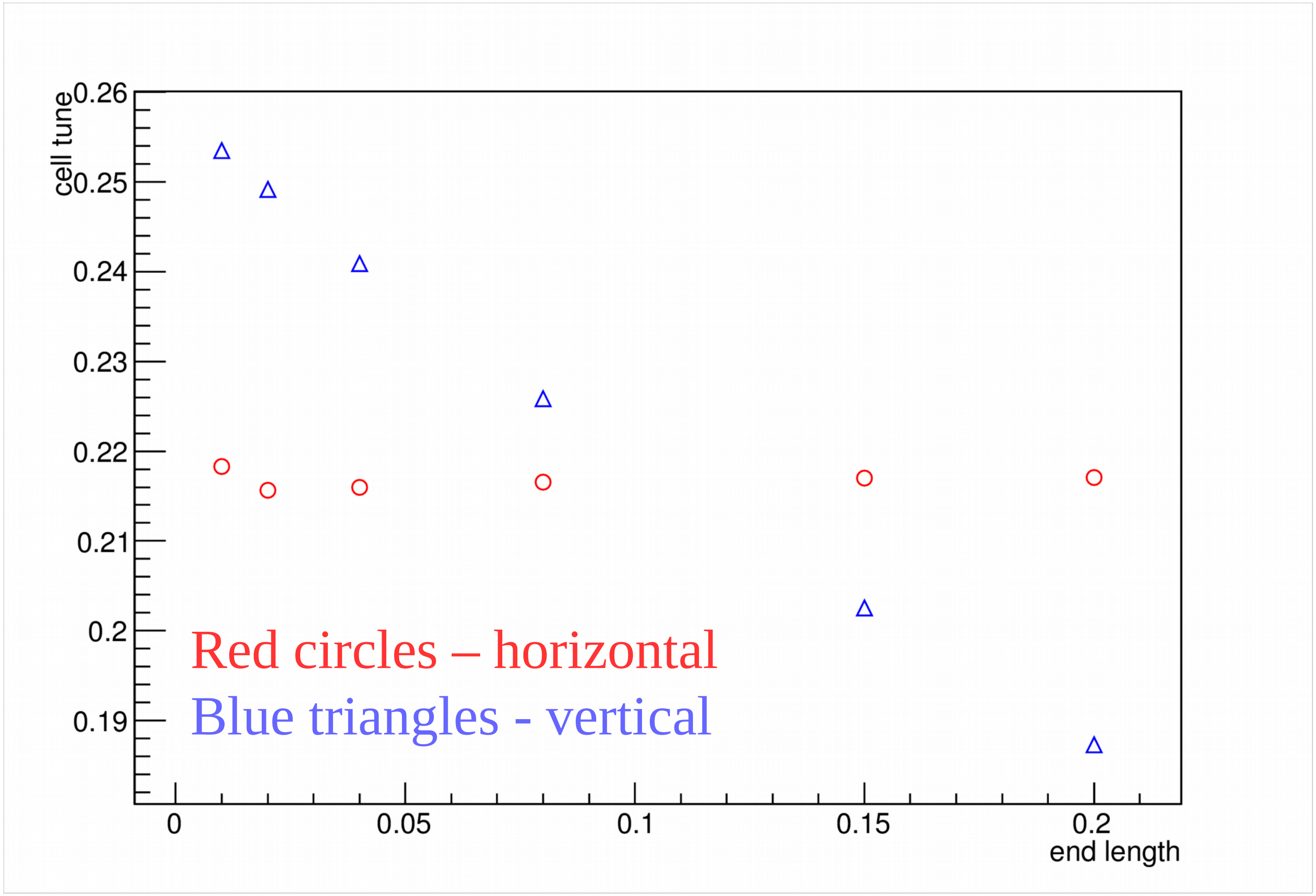
Test Ring – 12 cells



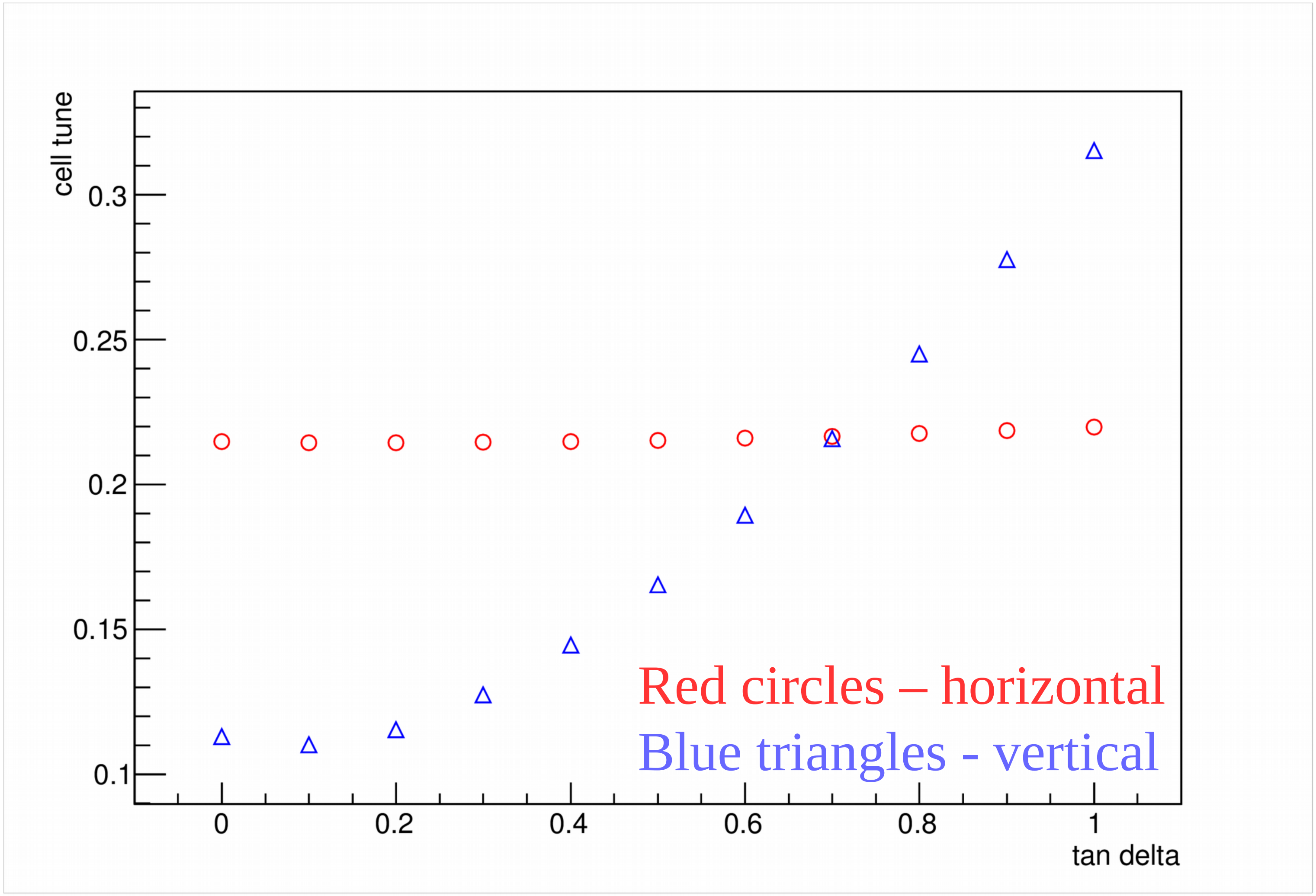
Vertical field in a quadrant of the ring

Kinetic Energy	3 – 30 MeV
Number of cells	12
Radius	2.8 – 3.5 m
Magnet Field	1.2, 0.6 T
Drift length	0.8 m
Magnet Length	0.16, 0.08 m
K index	4.48
End field length	Magnet length*0.1
tan(delta)	0.711
H cell tune	0.2167
V cell tune	0.2188

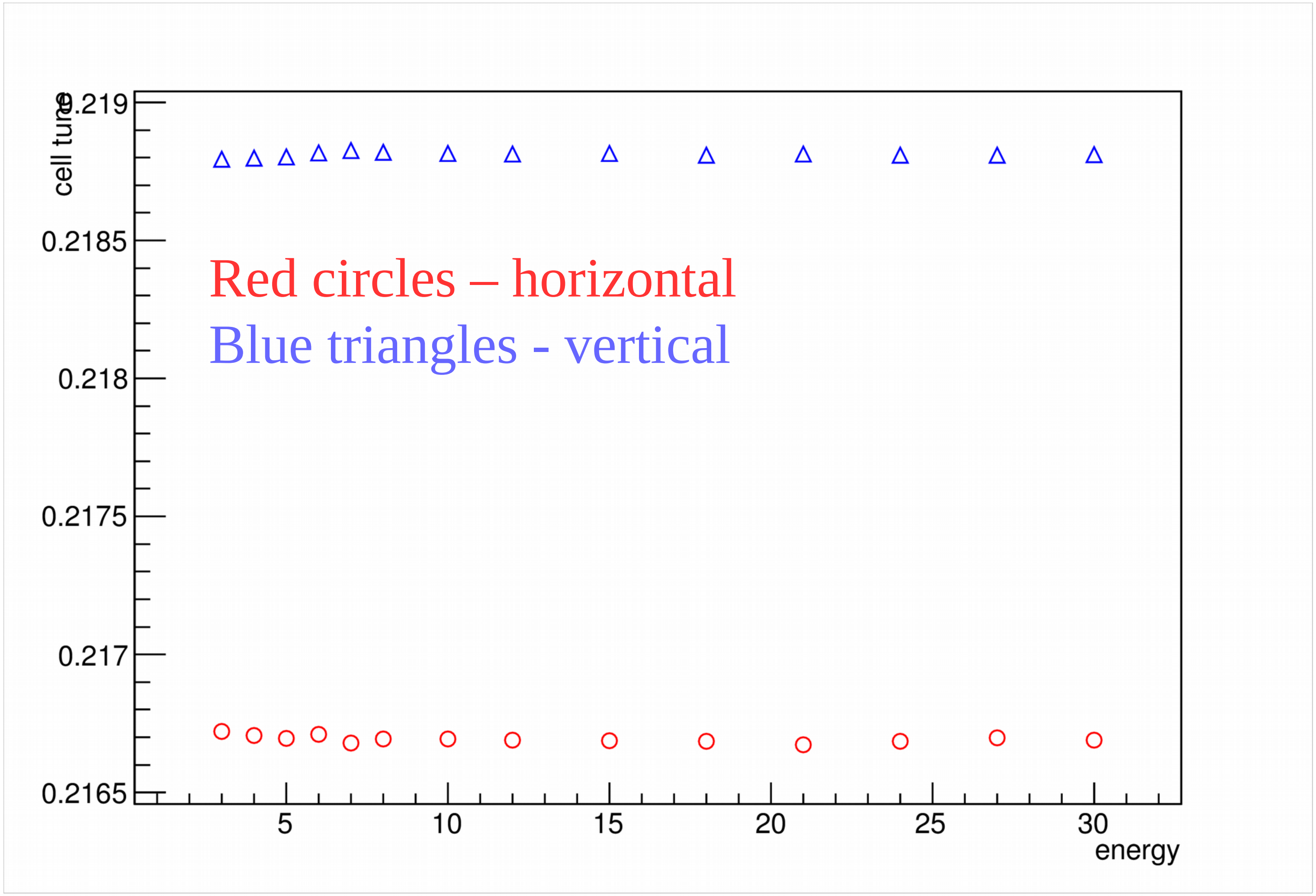
Tune vs end length



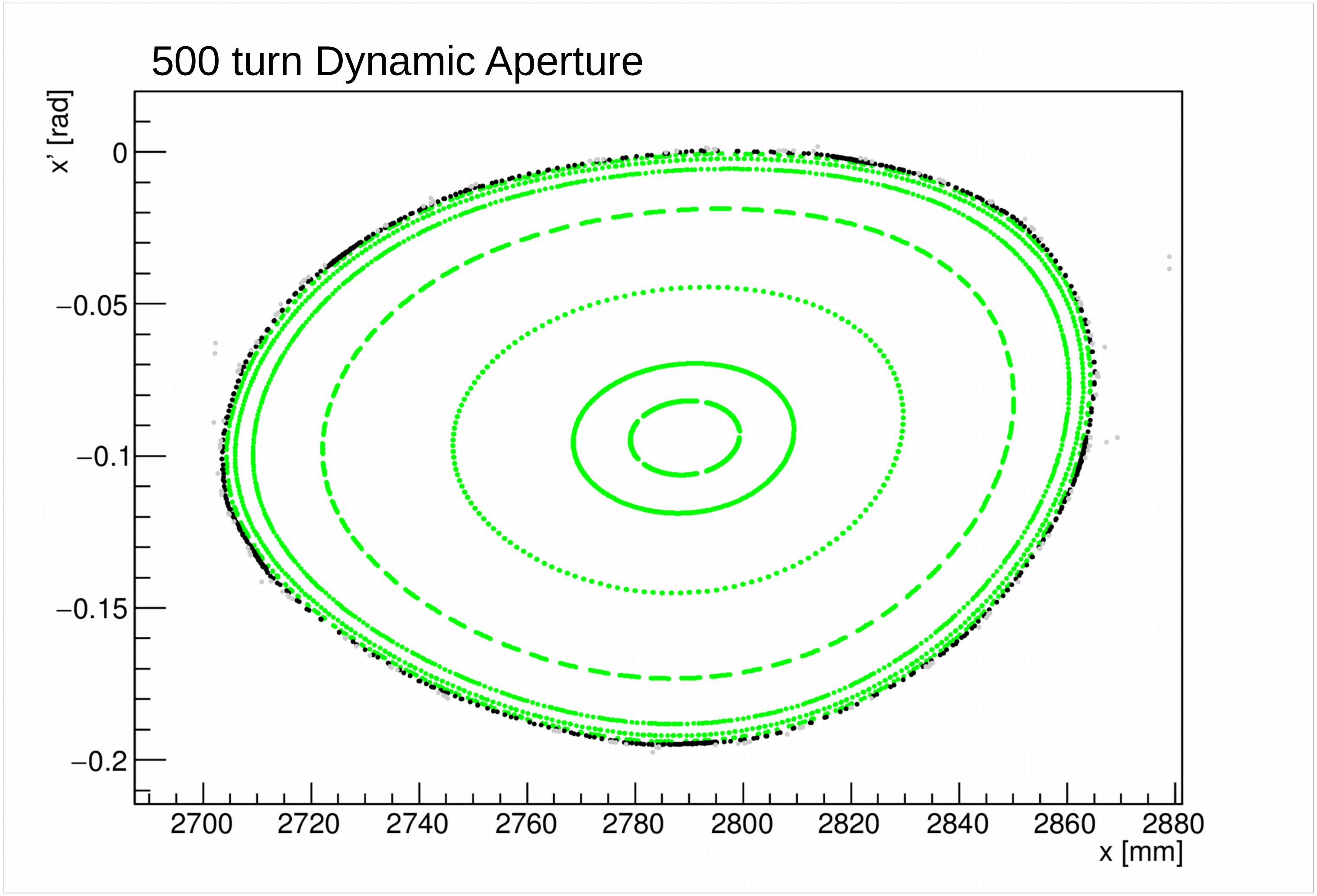
Tune vs tan delta



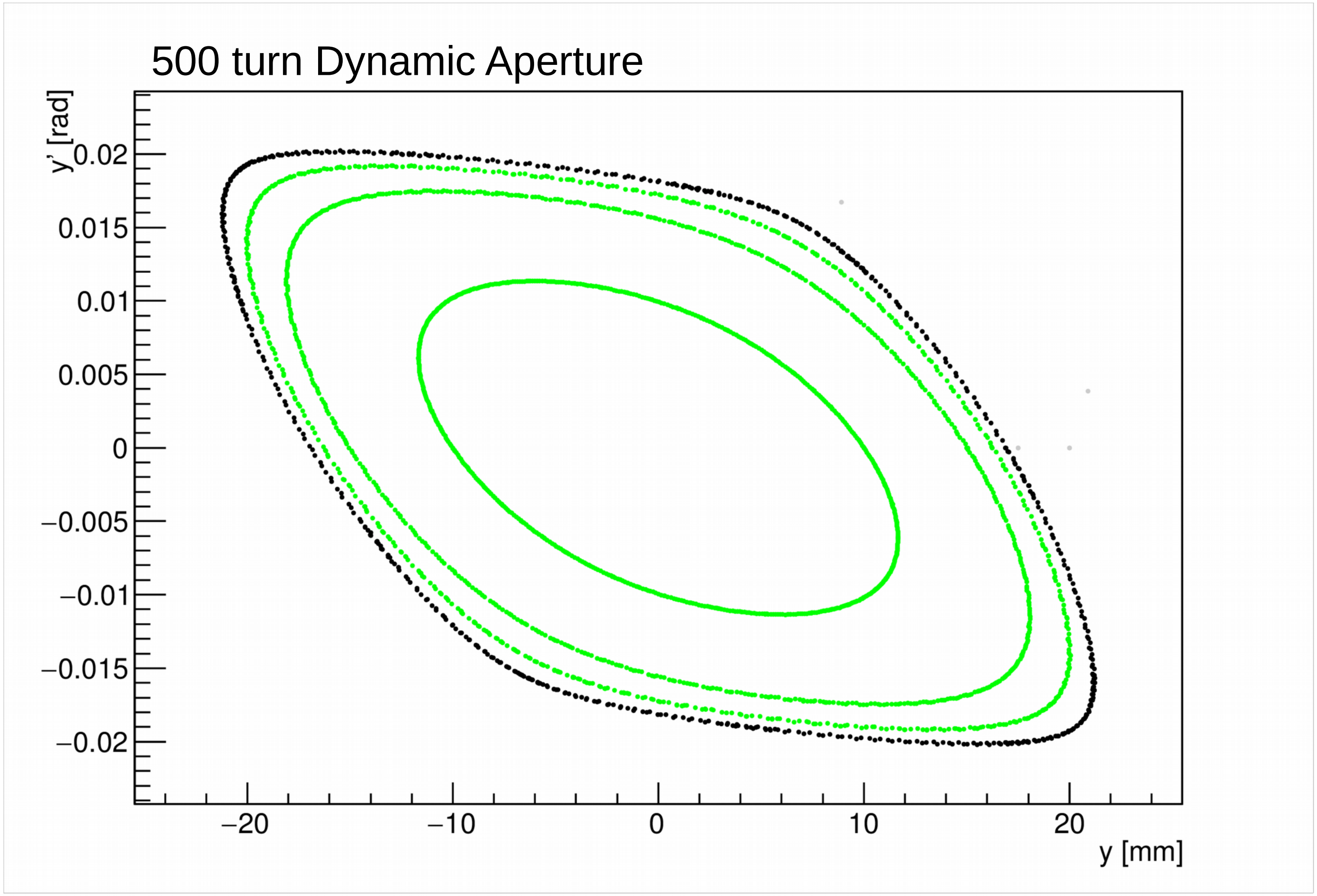
Tune vs energy



Horizontal DA

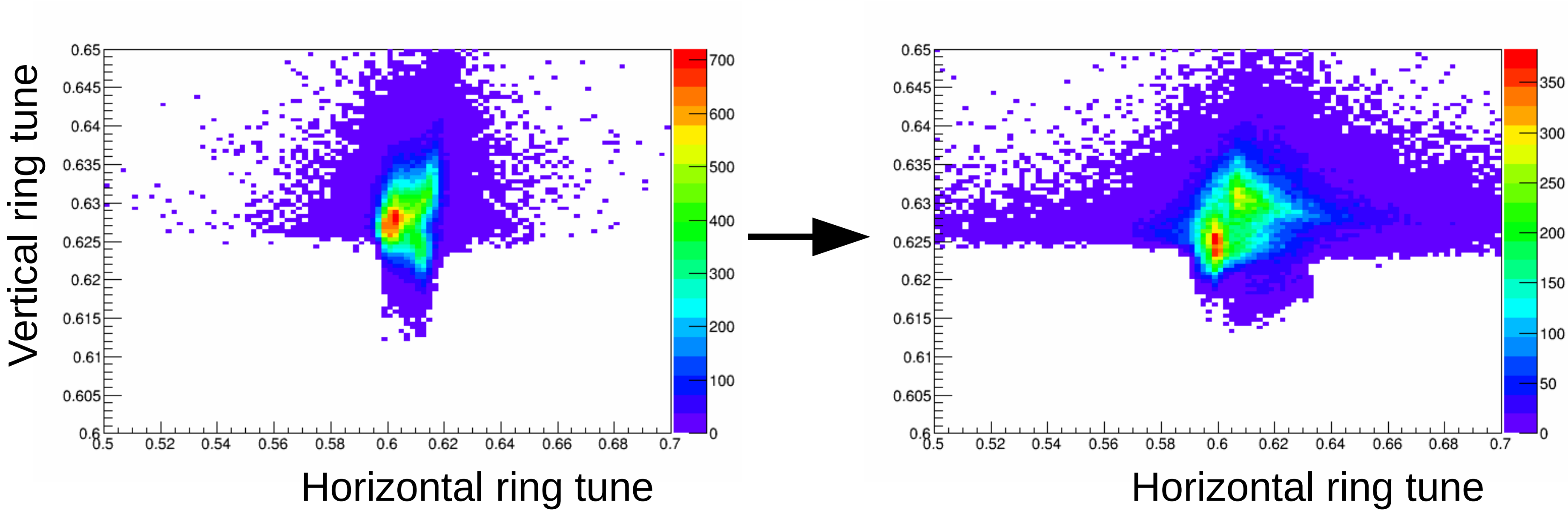


Vertical DA

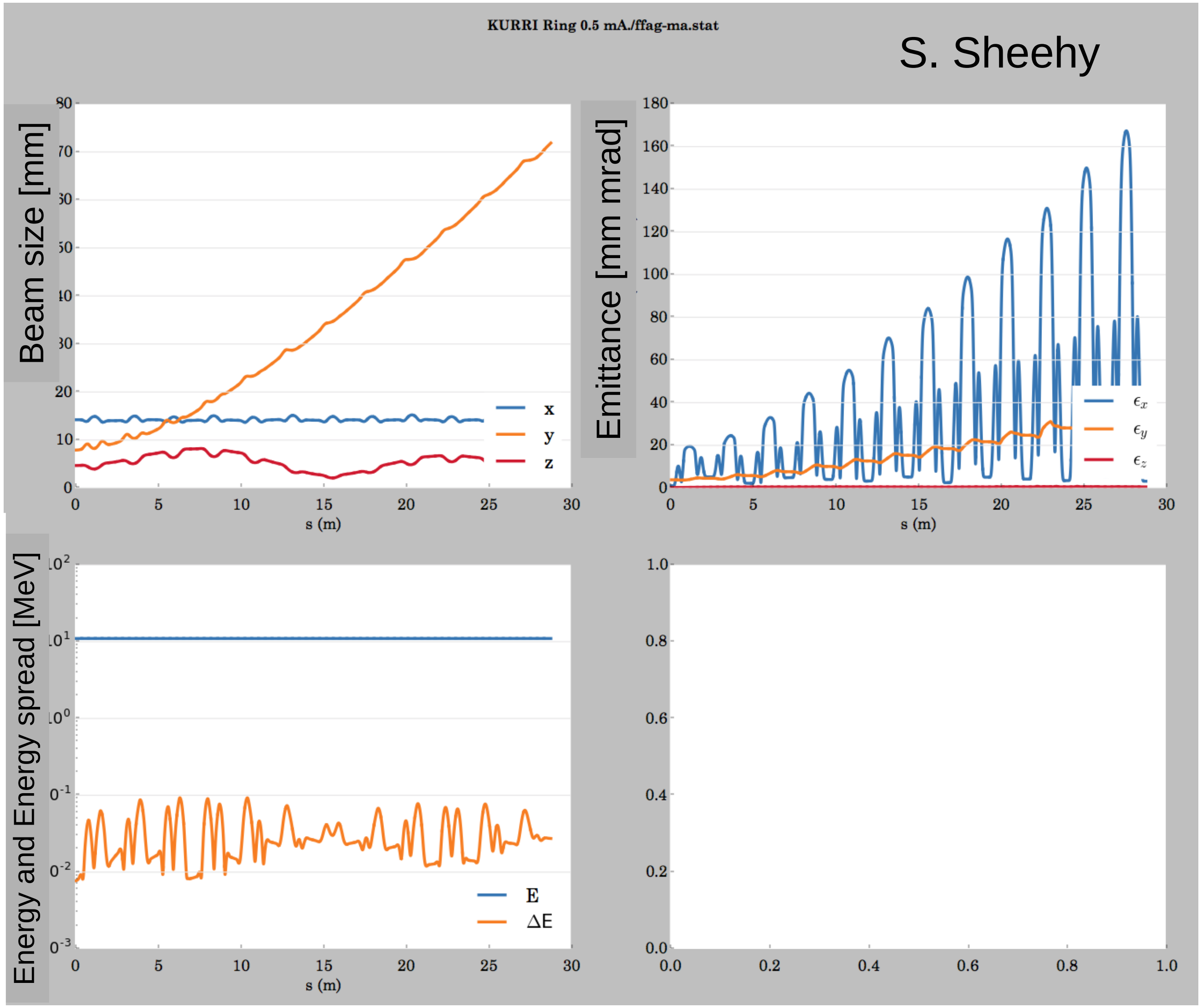

500 turn Dynamic Aperture
y' [rad]
0.02
0.015
0.01
0.005
0
−0.005
−0.01
−0.015
−0.02
−20
−10
0
10
20
y [mm]

OPAL Space Charge Solver

- OPAL space charge solver
  - Solve 3D Poisson equation using FFT method
  - Parallelised

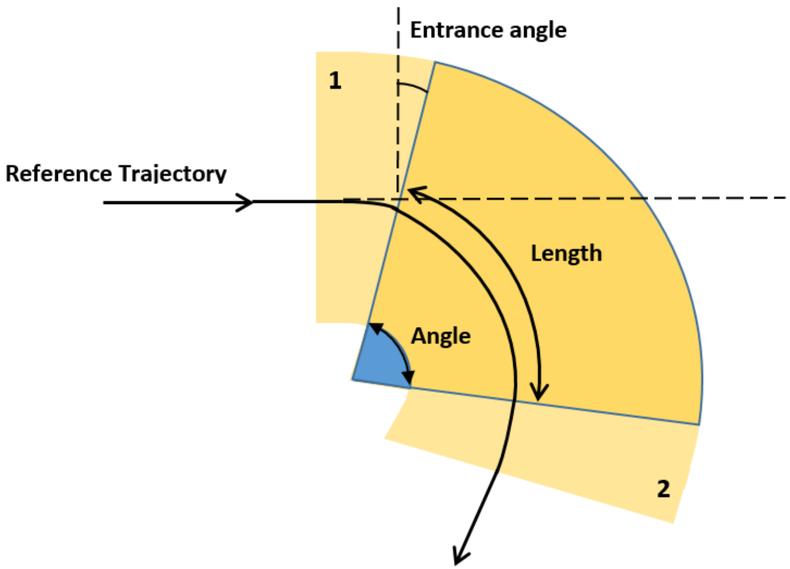


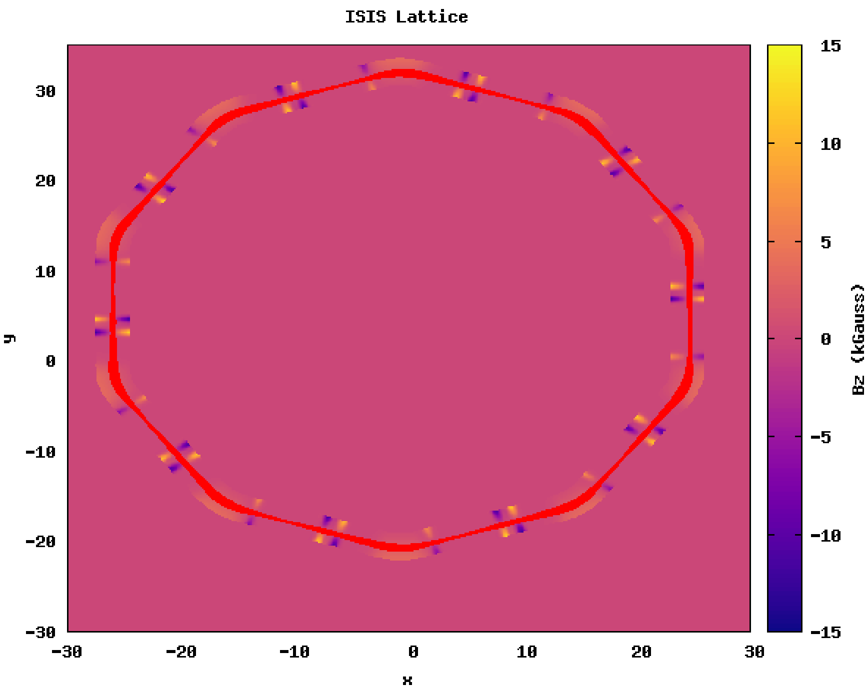
Matching in presence of SC



Using technique developed by C. Baumgarten

OPAL Multipoles (Titus Dascalu)





- Upgraded multipole model coming soon to OPAL
  - Vary entrance and exit fringe fields/angles independently
  - Superpose multipoles of different order for combined function magnets
  - E.g. modelling ISIS in storage ring mode
- Can enable modelling of e.g. mixed FFAG/multipole lattices

Conclusions

- OPAL cyclotron code has been extended to model FFAGs
- Couple of different FFAG magnet models have been included
- Enables modelling of FFAGs in presence of space charge

Thank You!

- OPAL development team

  Andreas Adelmann, Achim Gsell, Valeria Rizzoglio (PSI),
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