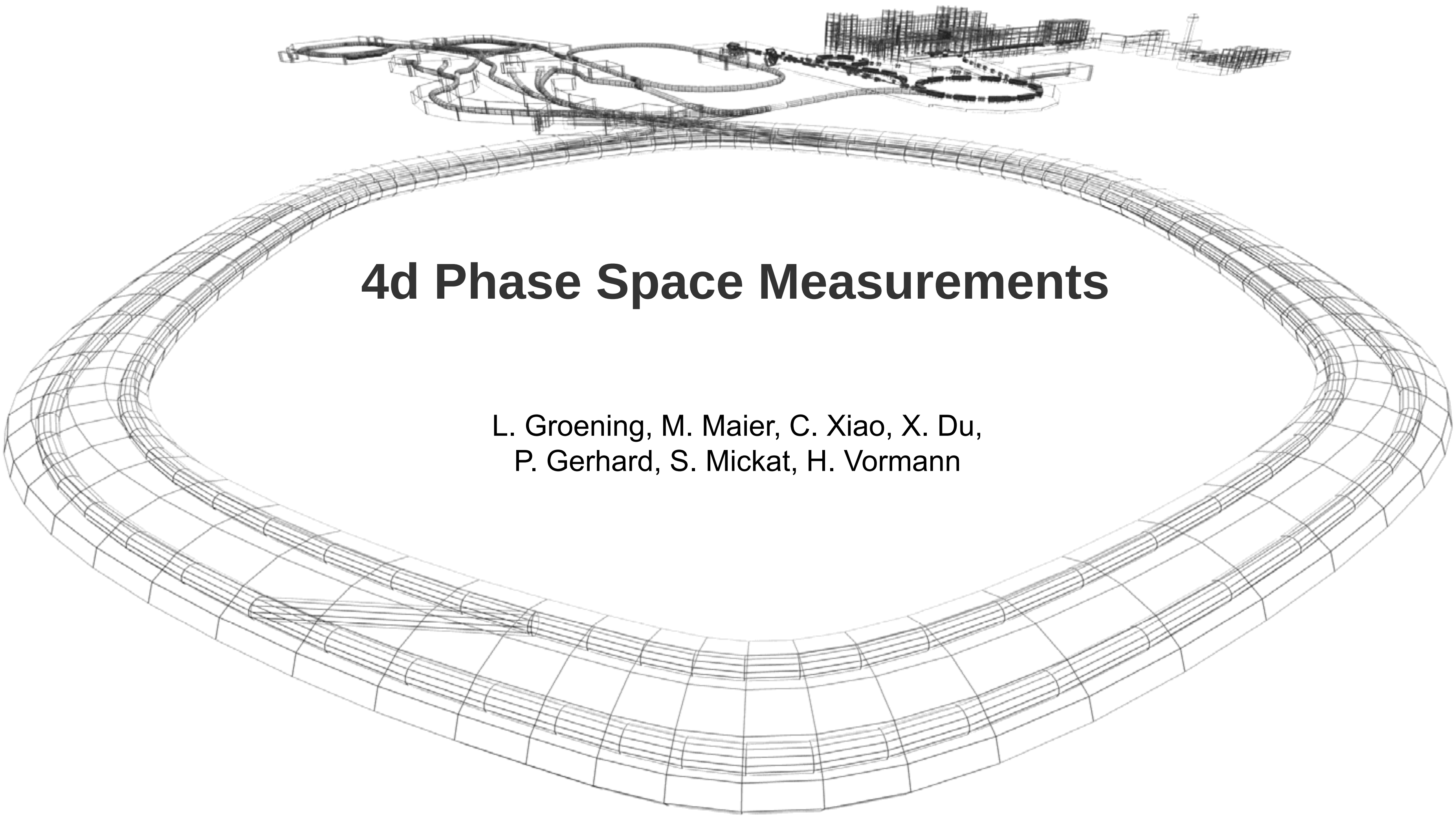


4d Phase Space Measurements

L. Groening, M. Maier, C. Xiao, X. Du,
P. Gerhard, S. Mickat, H. Vormann

Outline

- 4d rms emittance and eigen-emittances
- Motivation
- ROSE principle
- Error mitigation
- Results and error analysis
- Summary and outlook

Eigen-Emittances

- linear (4d), Hamiltonian beam line elements preserve :
  - 4d rms emittance $E_{4d}^2 = det \begin{bmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{bmatrix}$

  - the two eigen-emittances

$$\varepsilon_1 = \frac{1}{2}\sqrt{-tr[(CJ)^2] + \sqrt{tr^2[(CJ)^2] - 16det(C)}}$$

$$E_{4d} = \varepsilon_1 \cdot \varepsilon_2$$

$$\varepsilon_2 = \frac{1}{2}\sqrt{-tr[(CJ)^2] - \sqrt{tr^2[(CJ)^2] - 16det(C)}}$$

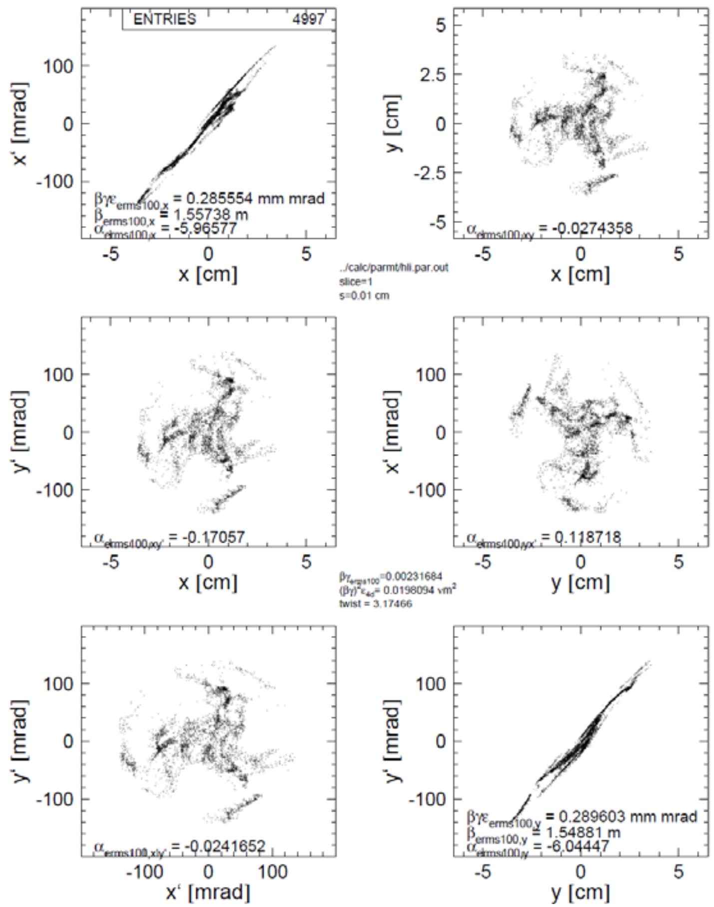
$$C = \begin{bmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{bmatrix} \qquad J := \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Eigen-Emittances

- if, and only if there is no x↔y correlation, i.e. $C = \begin{bmatrix} <xx> & <xx'> & 0 & 0 \\ <x'x> & <x'x'> & 0 & 0 \\ 0 & 0 & <yy> & <yy'> \\ 0 & 0 & <y'y> & <y'y'> \end{bmatrix}$
  - rms emittances = eigen-emittances
- if there is any coupling
  - rms emittances ≠ eigen-emittances
  - coupling parameter $t = \frac{\varepsilon_x \varepsilon_y}{\varepsilon_1 \varepsilon_2} - 1 \geq 0$

- term „eigen-emittance“ is quite unknown, since generally beams are considered as uncoupled
- measured moments are just reliable, if they deliver reasonable eigen-emittances !

Motivation for 4d Diagnostics:
ECR Source Performance Evaluation



4d distribution behind ECR source

- $\varepsilon_{rms,x}$ = 123 mm mrad
- $\varepsilon_{rms,y}$ = 125 mm mrad

- E_1 = 17 mm mrad
- E_2 = 231 mm mrad

- $\varepsilon_{4d} = E_1 \cdot E_2$ = 3927 (mm mrad)^2
- $\varepsilon_{rms,x} \cdot \varepsilon_{rms,y}$ = 15375 (mm mrad)^2
- $\varepsilon_{rms,x} \cdot \varepsilon_{rms,y} = 3.9\ \varepsilon_{4d}$

- to quantify the source performance, knowledge of $\varepsilon_{rms,x}$ and $\varepsilon_{rms,x}$ is not sufficient
- knowledge of E_1 and E_2 is required
- the correlation moments <xy>... etc are needed
- they give access to E_1 and E_2

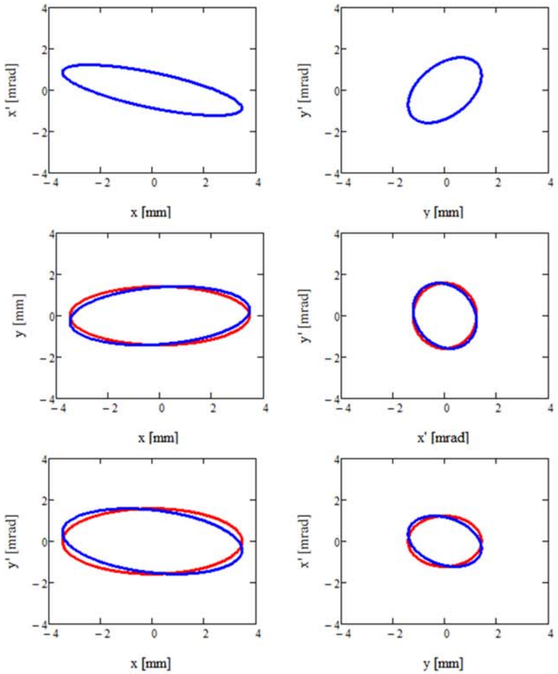
Motivation:
Envelopes along Solenoid Channel

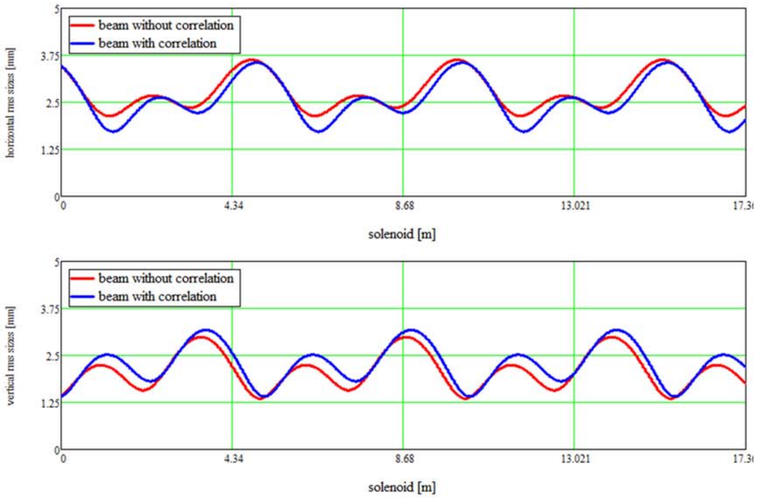
initial beam:

$$C_1=\begin{bmatrix} 12.00 & -3.00 & 0.00 & 0.00 \\ -3.00 & 1.50 & 0.00 & 0.00 \\ 0.00 & 0.00 & 2.00 & 1.00 \\ 0.00 & 0.00 & 1.00 & 2.50 \end{bmatrix}$$ uncorrelated

$$C_2=\begin{bmatrix} 12.00 & -3.00 & 1 & -1.5 \\ -3.00 & 1.50 & -0.5 & -0.35 \\ 1.00 & -0.50 & 2.00 & 1.00 \\ -1.50 & -0.35 & 1.00 & 2.50 \end{bmatrix}$$ correlated

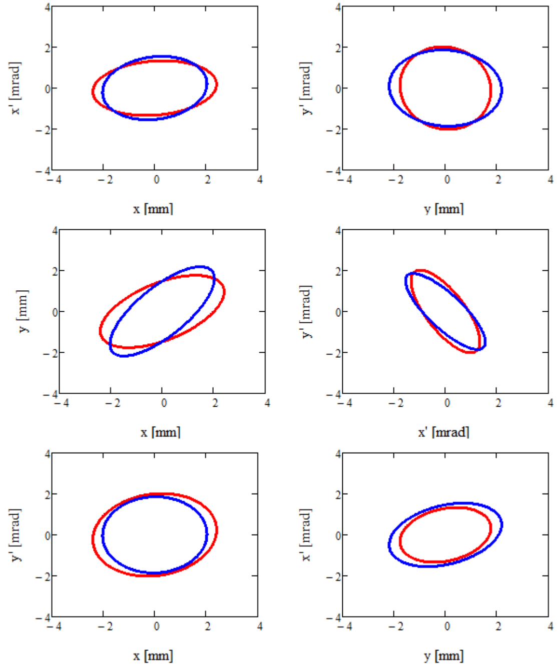
$$M_{sol} = \begin{bmatrix} C^2 & \frac{SC}{K} & SC & \frac{S^2}{K} \\ -KSC & C^2 & -KS^2 & CS \\ -SC & -\frac{S^2}{K} & C^2 & \frac{SC}{K} \\ KS^2 & -SC & -KSC & C^2 \end{bmatrix}$$



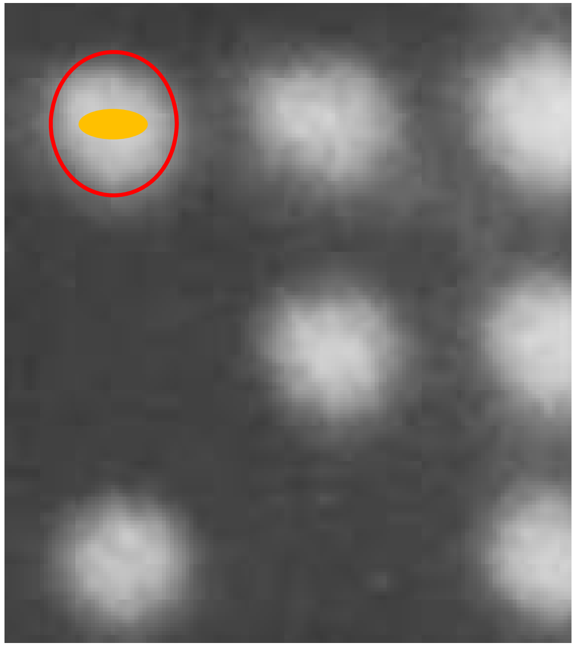


red and blue envelopes differ

final beam:



Emittance Measurements: Pepper Pot (Praxis)



screen image size

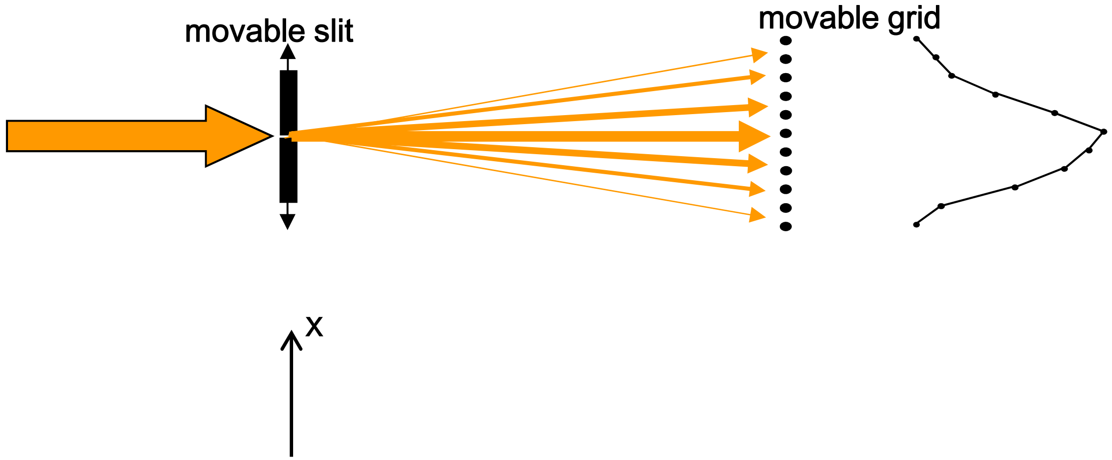
beamlet size

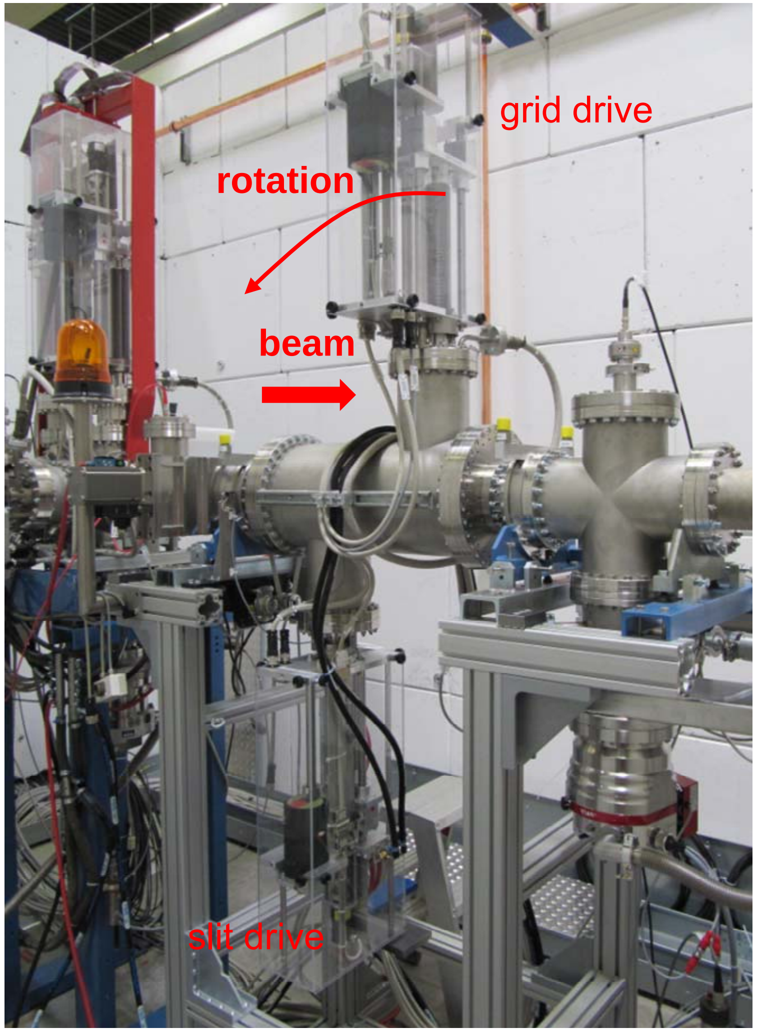
- to our knowledge, pepper pots do not work for ions with energies beyond ≈ 150 keV/u
- size of the beamlet image is dominated by screen resolution
- resolution limited by re-fluorescence
- excessive material research without success so far
- beamlet angles are always over-estimated
- GSI‘s energies are in MeV/u range
- pepper pots measure just few percent of the beam, as the mask dumps the main part

It seems that ion beam community misses a reliable tool for complete 4d transverse diagnostics

ROSE: Concept

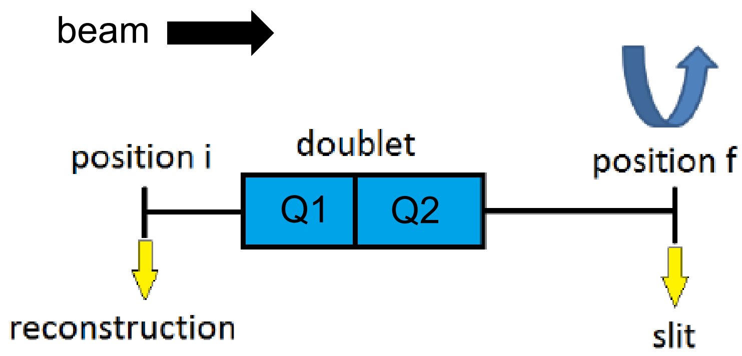
ROtating System for Emittance Measurements





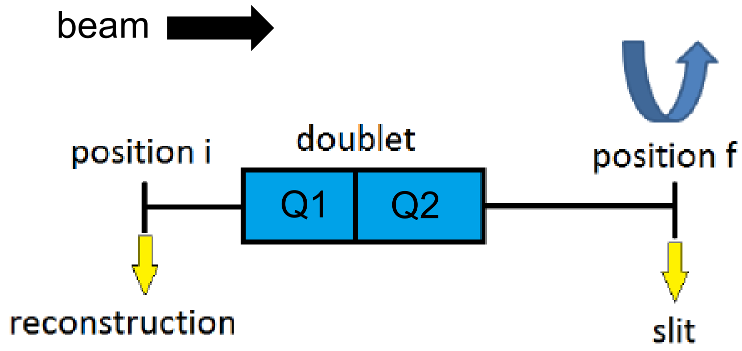
- slit & grid being mounted inside a rotatable chamber
- rotation gives access to <xy>, <xy'>, <x'y>, <x'y'>
- chamber is fixed during measurement
- chamber does not rotate during measurement

Concept



- beam moments to be determined at „reconstruction point“
- „reconstruction point“ and ROSE are separated by adjustable non-coupling element
- slit & grid of ROSE can be rotated simultaneously

Concept

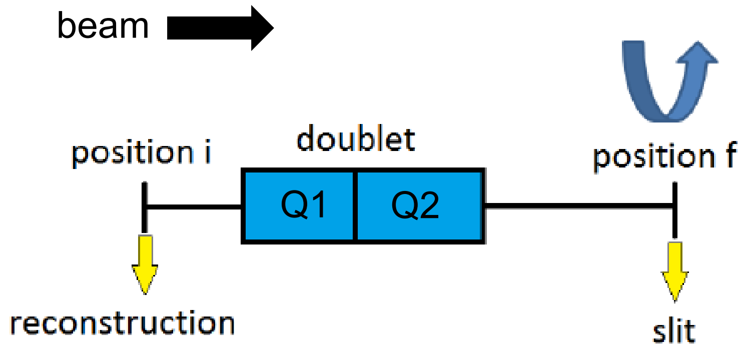


four measurements are needed to determine the correlated moments:

1. at 0° with setting $Q1^a$, $Q2^a$
2. at θ with setting $Q1^a$, $Q2^a$
3. at 90° with setting $Q1^a$, $Q2^a$
4. at θ with setting $Q1^b$, $Q2^b$

doublet settings a & b deliver different envelopes

Algorithm



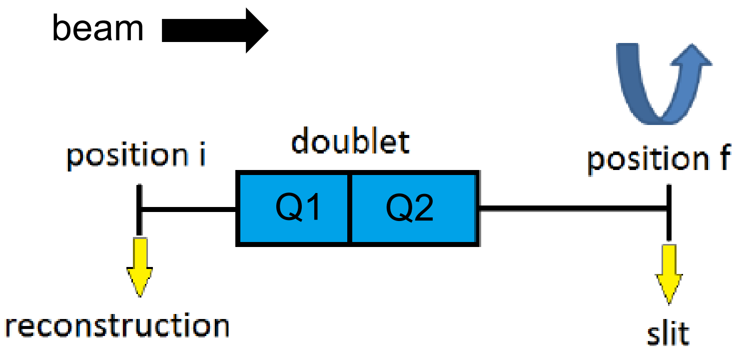
- beam transport to ROSE

$$\begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_f^{a,b} = \begin{bmatrix} m_{11}^{a,b} & m_{12}^{a,b} & 0 & 0 \\ m_{21}^{a,b} & m_{22}^{a,b} & 0 & 0 \\ 0 & 0 & m_{33}^{a,b} & m_{34}^{a,b} \\ 0 & 0 & m_{43}^{a,b} & m_{44}^{a,b}. \end{bmatrix} \begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_i$$

- rotation of ROSE by θ is equivalent to rotation of beam by -θ

$$\begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_\theta^{a,b} = \begin{bmatrix} cos\theta & 0 & sin\theta & 0 \\ 0 & cos\theta & 0 & sin\theta \\ -sin\theta & 0 & cos\theta & 0 \\ 0 & -sin\theta & 0 & cos\theta \end{bmatrix} \begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_f^{a,b}$$

Algorithm



- solving all equations finally gives an over-determined system of equations

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} \\ \Gamma_{21} & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & \Gamma_{34} \\ \Gamma_{41} & \Gamma_{42} & \Gamma_{43} & \Gamma_{44} \\ \Gamma_{51} & \Gamma_{52} & \Gamma_{53} & \Gamma_{54} \\ \Gamma_{61} & \Gamma_{62} & \Gamma_{63} & \Gamma_{64} \end{bmatrix} \begin{bmatrix} \langle xy \rangle_i \\ \langle xy' \rangle_i \\ \langle x'y \rangle_i \\ \langle x'y' \rangle_i \end{bmatrix} = \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \\ \Lambda_3 \\ \Lambda_4 \\ \Lambda_5 \\ \Lambda_6 \end{bmatrix}$$

- Γ-matrix includes quad strengths $Q1^{a,b}$ and $Q2^{a,b}$

- Λ-vector includes individual measurements of $\langle xx\rangle^{a,b,\theta}$, $\langle xx'\rangle^{a,b,\theta}$, and $\langle x'x'\rangle^{a,b,\theta}$

- $\langle ... \rangle_i$ are the correlated beam moments to be determined

- final solution is

$$\begin{bmatrix} \langle xy \rangle_i \\ \langle xy' \rangle_i \\ \langle x'y \rangle_i \\ \langle x'y' \rangle_i \end{bmatrix} = (\Gamma^T \Gamma)^{-1} \Gamma^T \Lambda$$

Measurement Error Mitigation

- individually measured <xx>$^{a,b,\theta}$, <xx'>$^{a,b,\theta}$, and <x'x'>$^{a,b,\theta}$ inhabit measurement errors
- errors will enter into the final result for <...>$_i$
- final errors are minimized if condition number *K* of the matrix Γ is minimized

$$\begin{bmatrix} \langle xy \rangle_i \\ \langle xy' \rangle_i \\ \langle x'y \rangle_i \\ \langle x'y' \rangle_i \end{bmatrix} = (\Gamma^T\Gamma)^{-1}\Gamma^T\Lambda \qquad \Gamma^\dagger = (\Gamma^T\Gamma)^{-1}\Gamma^T \qquad \|\Gamma\|_2 := \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{k}(\Gamma_{i,j})^2}, \quad \|\Gamma^\dagger\|_2 := \sqrt{\sum_{i=1}^{k}\sum_{j=1}^{n}(\Gamma^\dagger_{i,j})^2}. \qquad \kappa(\Gamma) := \|\Gamma\|_2\|\Gamma^\dagger\|_2$$

- *K* quantifies linear dependency of the matrix rows
- low linear dependency → low *K* → errors contrubute few to final result
- examples:

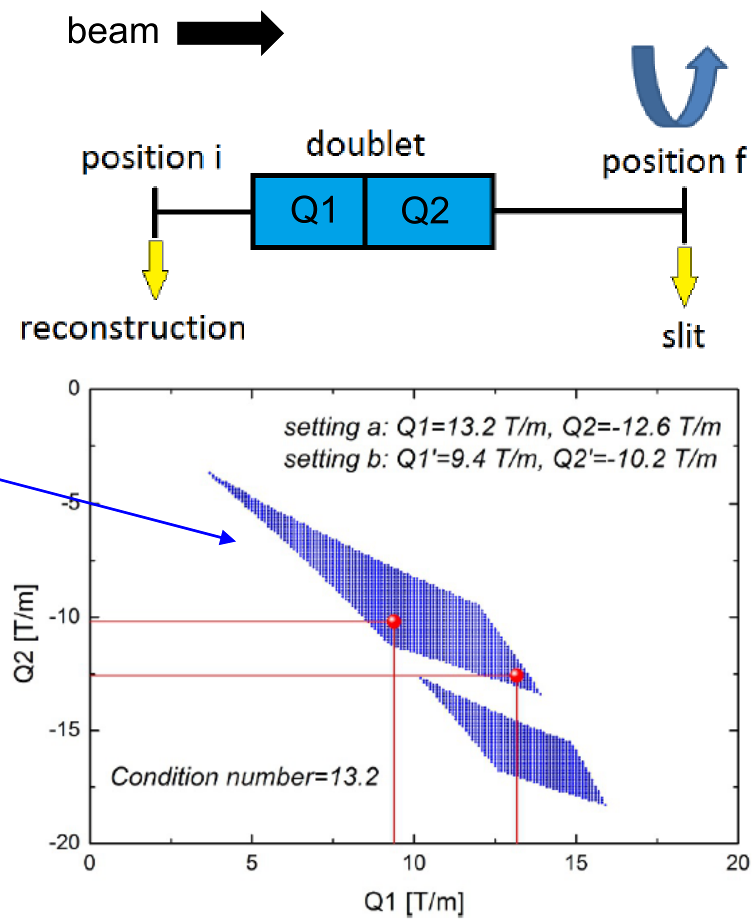
$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow K = 5$$

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 2.1 & 5.9 & 10.2 & 14.1 \\ 0.55 & 1.49 & 2.55 & 3.45 \\ 0.11 & 0.28 & 0.51 & 0.69 \end{bmatrix} \rightarrow K = 1.2\text{x}10^{17}$$

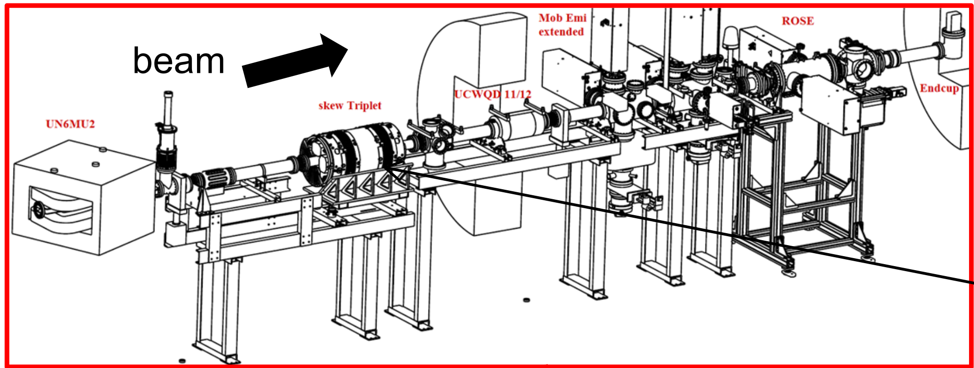
Measurement Error Mitigation

- do initial emittance measurements at 0° and 90°, i.e. in ver. and hor. plane
- backtransform to quad doublet entrance
- vary in brute force way two quad strengths Q1, Q2 and check for (analytically!) :
  - 100% transmission
  - reasonable beam size at slit
  - reasonable beam size at grid
- store settings Q1, Q2 that passed this test
- build all possible pairs of settings, check their *K*
- pick pair with lowest *K* for measurements:

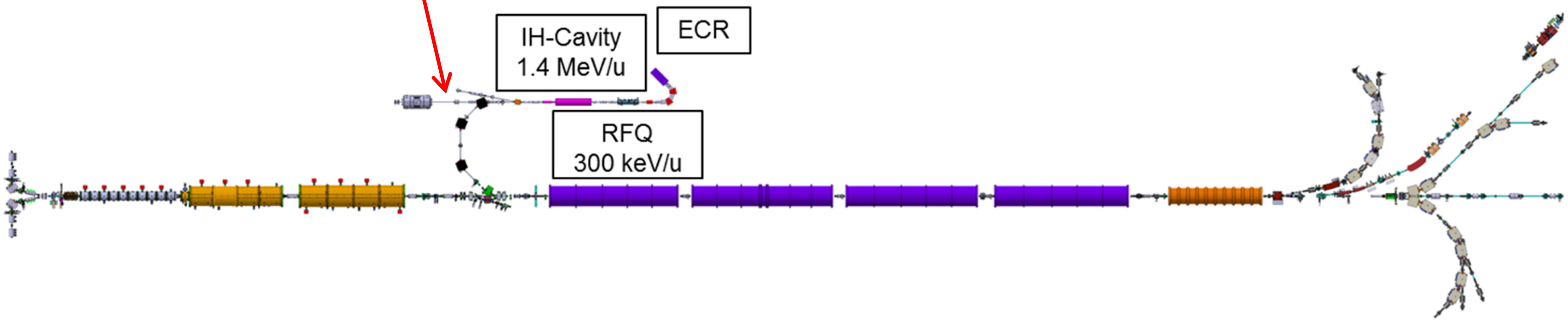
1. slit at 0° with setting $Q1^a$, $Q2^a$
2. slit at θ with setting $Q1^a$, $Q2^a$
3. slit at 90° with setting $Q1^a$, $Q2^a$
4. slit at θ with setting $Q1^b$, $Q2^b$



Measurements



- measurements at exit of GSI's HLI
- 1.4 MeV/u
- low current of $^{83}Kr^{13+}$
- skew triplet to create x↔y correlations
- full transmission

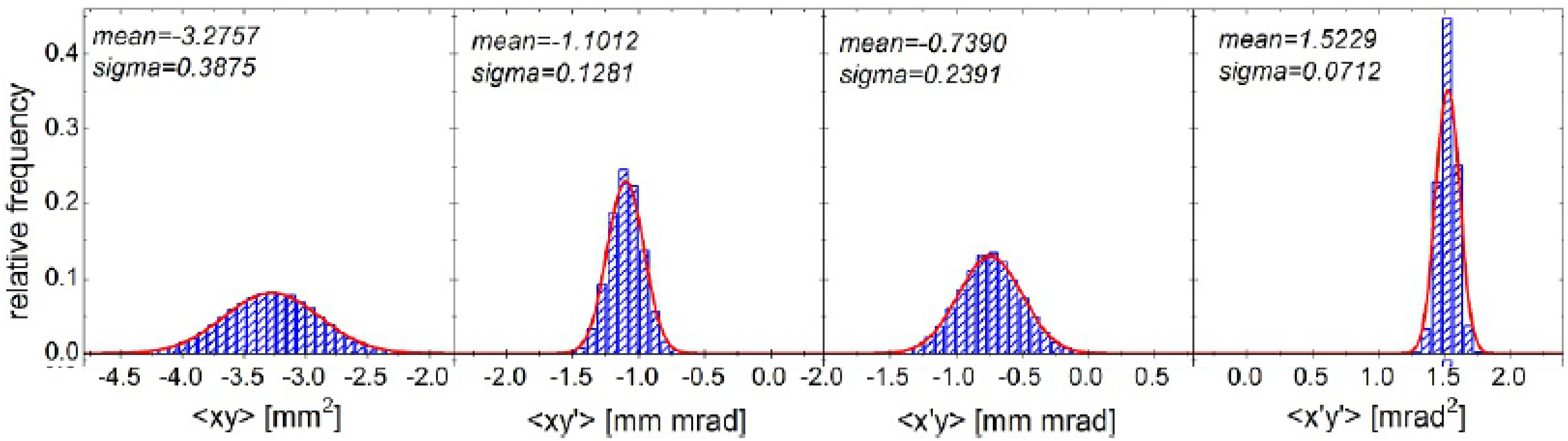


Results and Error Analysis

$$\begin{bmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{bmatrix} = \begin{bmatrix} 8.57 & -4.34 & -3.28 & -1.10 \\ -4.34 & 3.35 & -0.74 & 1.52 \\ -3.28 & -0.74 & 11.20 & -3.05 \\ -1.10 & 1.52 & -3.05 & 1.87 \end{bmatrix}$$

in mm, mrad

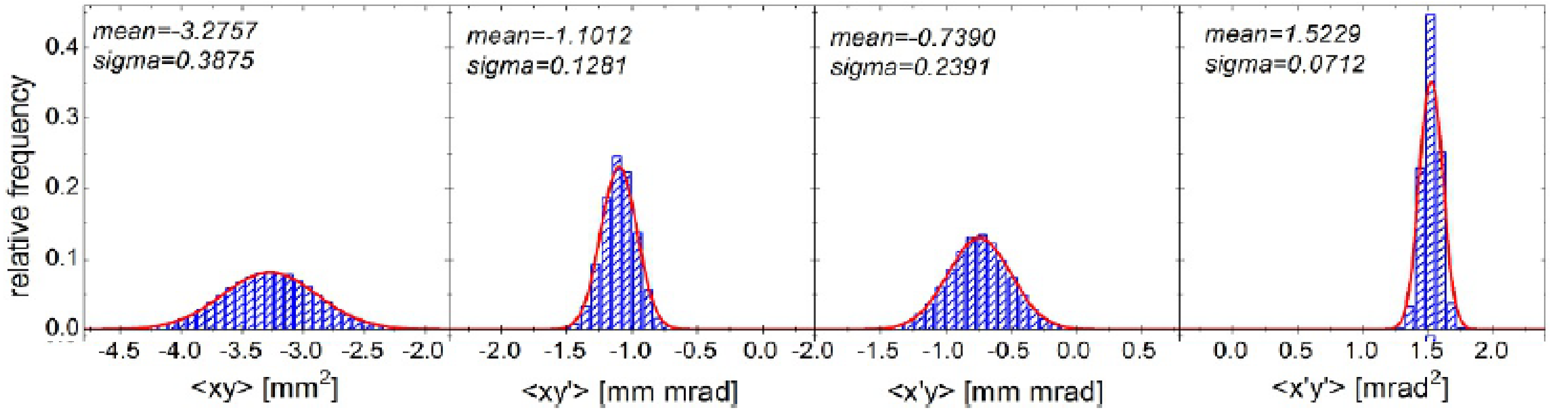
- each measured moment entering into the evaluation was varied randomly following a Gaussian distribution centered around its measured value (3*rms = 10%)
- many sets of errors were used
- error bars for initial correlated moments were obtained

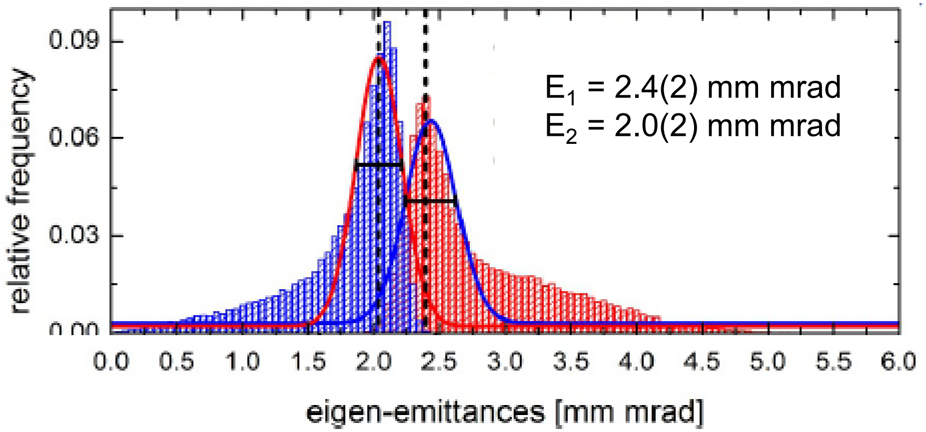


4d Emittance Measurements: Eigen-Emittances

$$\begin{bmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{bmatrix} = \begin{bmatrix} 8.57 & -4.34 & -3.28 & -1.10 \\ -4.34 & 3.35 & -0.74 & 1.52 \\ -3.28 & -0.74 & 11.20 & -3.05 \\ -1.10 & 1.52 & -3.05 & 1.87 \end{bmatrix}$$

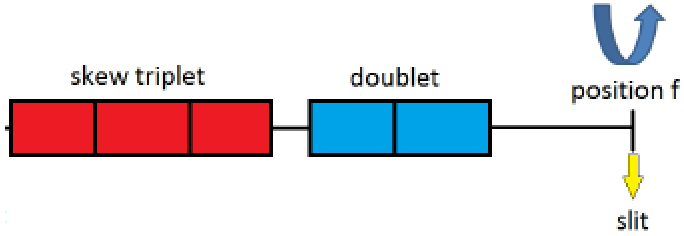
in mm, mrad





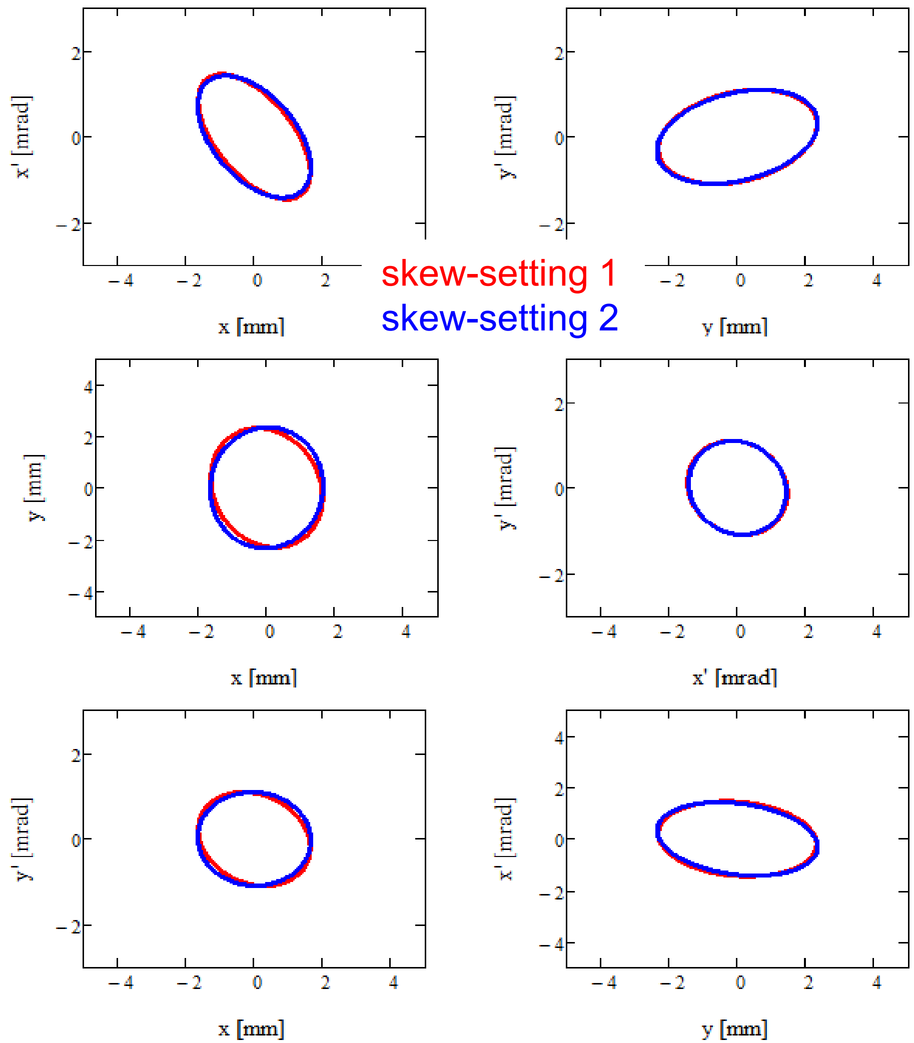
$$t = \frac{\varepsilon_x \varepsilon_y}{\varepsilon_1 \varepsilon_2} - 1 = 1.2(2)$$

4d Emittance Measurements: Check Reliability



- measurements were done for two different skew triplet settings
- both measurements were back-transformed to entrance of skew triplet
- back-transformations delivered very similar results
- the two measurements are consistent
- ROSE seems to be reliable

at skew triplet entrance:



Summary and Outlook

- a new reliable device for full 4d transverse beam moments matrix measurements has been designed, built, and tested

- to our knowledge, it is the only reliable device working at ion energies beyond about 150 keV/u

- it was tested at the HLI of GSI at 1.4 MeV/u with $^{86}Kr^{13+}$

- device has a patent „*Deutsche Patentanmeldung Nr. 102015118017.0 eingereicht am 22.10.2015 beim Deutschen Patent- und Markenamt Titel der Patentanmeldung: Drehmodul für eine Beschleunigeranlage*"

- it will be installed:
  - behind the ECR test stand for source diagnostics
  - in the transfer channel to the synchrotron to detect/remove beam correlations
  - ...

- Literature:
  - Phys Rev. Accel. & Beams **19,** 072802 (2016)
  - Nucl. Instrum. & Meth. A 820 14, (2016)

rms - Emittance

rms-emittances defined through beam's second moments :

- a_i, b_i : two coordinates of particle i
- <ab>: mean of product $a_i b_i$
- C is moment matrix (symmetric)

$$E_x^2 = \langle x\, x\rangle\langle x'x'\rangle - \langle x\, x'\rangle^2$$

$$C_x = \begin{bmatrix} <xx> & <xx'> \\ <x'x> & <x'x'> \end{bmatrix}, \quad E_x^2 = det\, C_x$$

$$C_y = \begin{bmatrix} <yy> & <yy'> \\ <y'y> & <y'y'> \end{bmatrix}, \quad E_y^2 = det\, C_y$$

Transport of Moments

linear transport from point_1 → point_2 through matrices:

$$\begin{bmatrix} x \\ x' \end{bmatrix}_2 = M_x \begin{bmatrix} x \\ x' \end{bmatrix}_1 \qquad M_x = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}, \ det\, M_x = 1$$

beam moments transport by matrix equation:

$$C_{x2} = M_x \, C_{x1} \, M_x^T$$

analogue in y

4d Linear Beam Dynamics

$$E_{4d}^2 = det \begin{bmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{bmatrix}$$

transport of moments from 1 → 2 as usual :

$$\begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_2 = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_1 , \quad det\, M = 1$$

$$C_2 = M\, C_1\, M^T$$

<u>if x & y planes are not coupled</u>

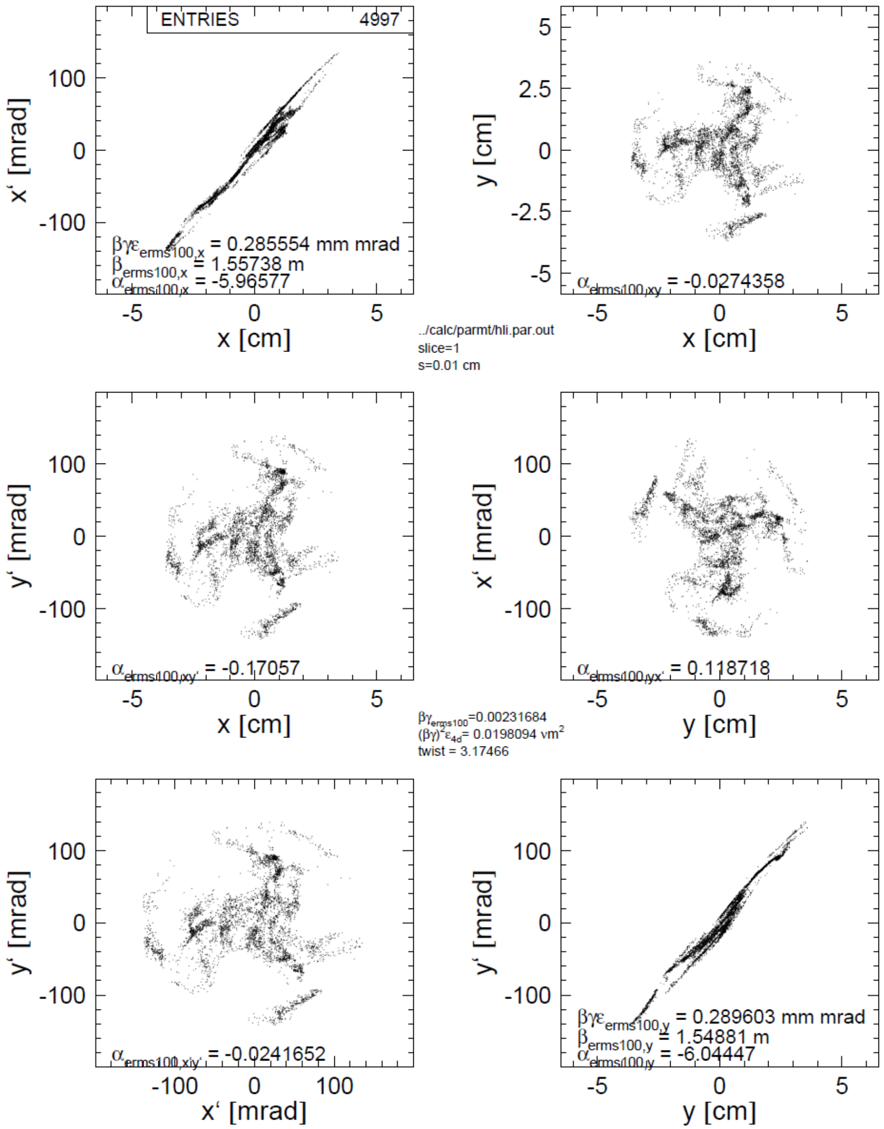
$$E_{4d}^2 = det \begin{bmatrix} <xx> & <xx'> & 0 & 0 \\ <x'x> & <x'x'> & 0 & 0 \\ 0 & 0 & <yy> & <yy'> \\ 0 & 0 & <y'y> & <y'y'> \end{bmatrix} = (E_x \cdot E_y)^2$$

transport of moments from 1 → 2 as usual :

$$M = \begin{bmatrix} m_{11} & m_{12} & 0 & 0 \\ m_{21} & m_{22} & 0 & 0 \\ 0 & 0 & m_{33} & m_{34} \\ 0 & 0 & m_{43} & m_{44} \end{bmatrix} , \quad det\, M = det\, M_x \cdot det\, M_y = 1 \cdot 1 = 1$$

$$C_2 = M\, C_1\, M^T$$

rms- vs Eigen-Emittances: Example



4d-distribution behind ECR source

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- $\varepsilon_{rms,x} \cdot \varepsilon_{rms,y} = 3.9\ \varepsilon_{rms,4d}$

Motivation:
Envelopes along non-Coupling Focusing Elements

- if beam line comprises just elements that do not couple as $M = \begin{bmatrix} m_{11} & m_{12} & 0 & 0 \\ m_{21} & m_{22} & 0 & 0 \\ 0 & 0 & m_{33} & m_{34} \\ 0 & 0 & m_{43} & m_{44} \end{bmatrix}$,

  the beam envelopes <xx> and <yy> do not depend on the initial correlated moments

- *M* has simply no access to <xy>, <xy'> ... when applying $C_2 = M\,C_1\,M^T$ to $C = \begin{bmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{bmatrix}$

- consequence for non-coupling lattices:

  - initial correlated moments can be ignored, even if they are non-zero
  - hor. / ver. envelopes can be calculated, if just the non-correlated moments are known, as <xx>, <xx'>, ..., <y'y'>

- conventional emittance scanners are sufficient for beam dynamics layouts for non-coupling lattices

Motivation:
Envelopes along Coupling Focusing Elements

- if beam line comprises elements that do couple as $\begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}$,

  the beam enevlopes <xx> and <yy> do depend on the initial correlated moments

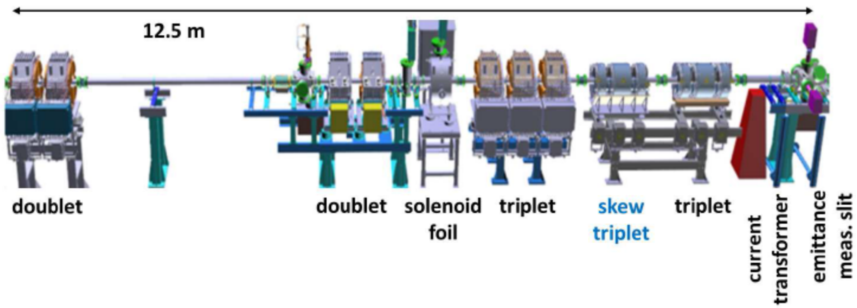
- *M* has access on <xy>, <xy'> ... when applying $C_2 = M\,C_1\,M^T$ to $C = \begin{bmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{bmatrix}$

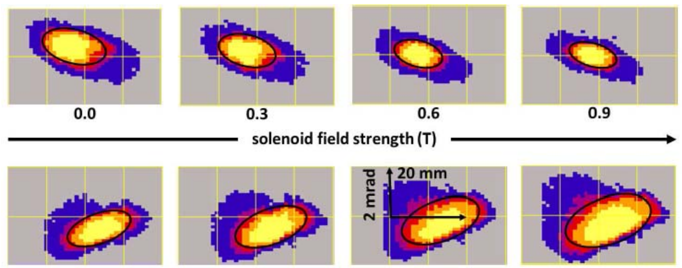
- consequence for coupling beam lines:

  - initial correlated moments cannot be ignored, if they are non-zero
  - hor. / ver. envelopes can only be calculated, if also <u>all</u> correlated moments are known, as <xy>, <xy'>, <x'y>, and <x'y'>

- conventional emittance scanners are <u>not sufficient</u> for beam dynamics layouts for coupling lattices

Motivation:
Diagnostics for Advanced EmTEx

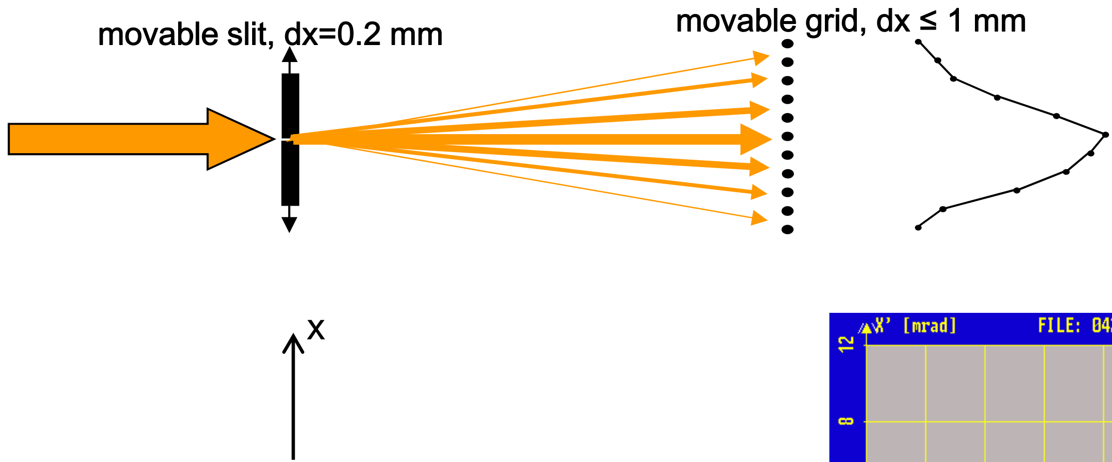


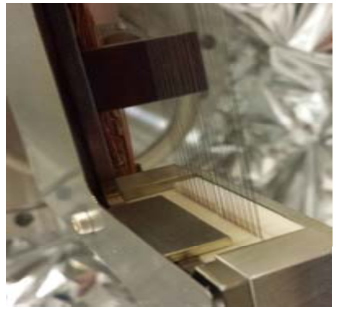


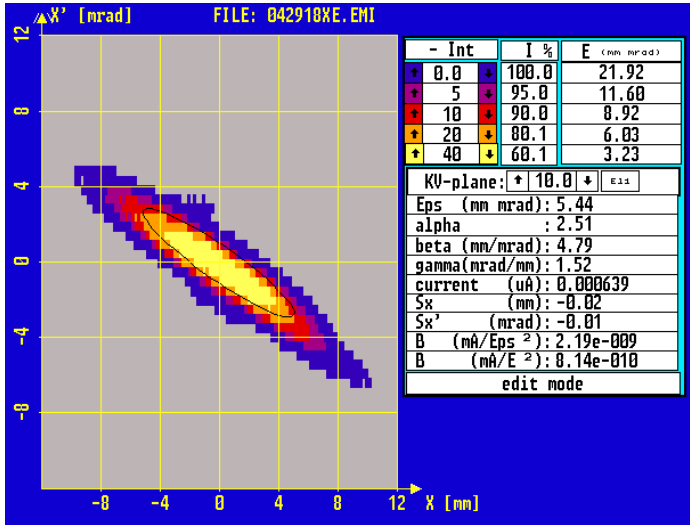
- EmTEX: Emittance Transfer EXperiment with linear dynamics:
  - no space charge, low momentum spread, initially uncorrelated beam
  - was done just by knowing initial 2d-Twiss parameters before solenoid
  - coupling in solenoid, decoupling in skew triplet
  - beam in front of decoupling skew quads from pure linear 4d beam dynamics

- if advanced EmTEX shall be applied to high space charge beams:
  - linear 4d dynamics not applicable to deduce correlations in front of decoupling line
  - full 4d beam moments must be know in front of decoupling skew triplet
  - 4d diagnostics needed

Emittance Measurements:
Slit / Grid

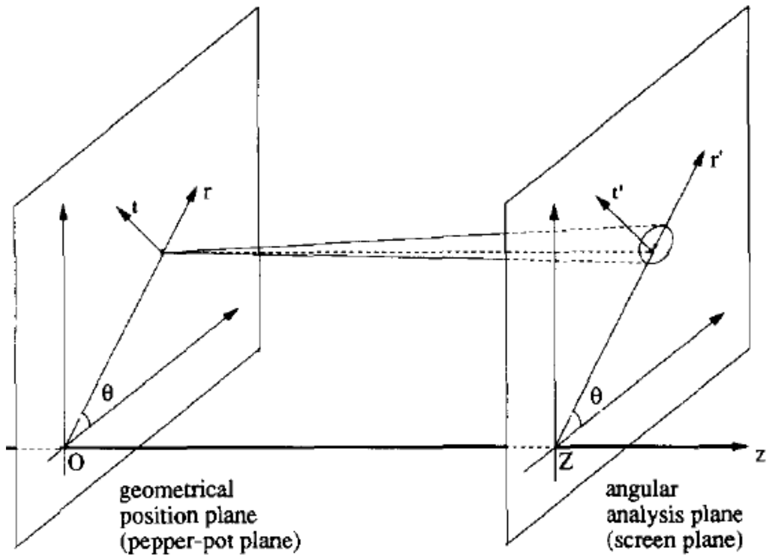




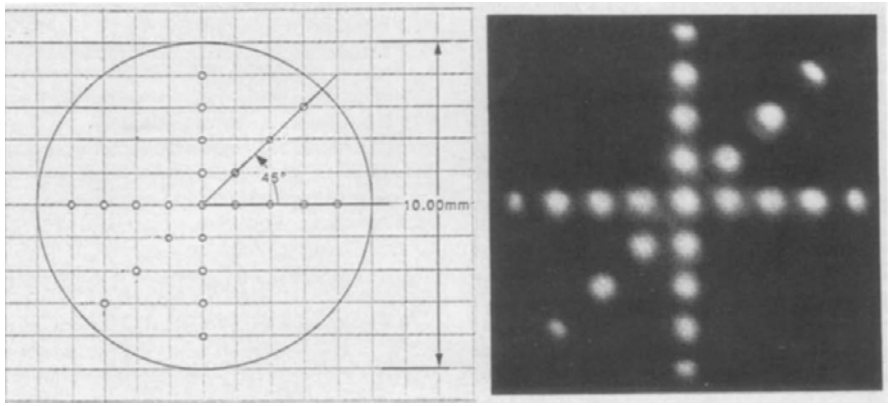


- slit position determines position x
- positions of wire and slit determine x‘
- for each phase space pixel (x,x‘) wire current is recorded
- map of pixel intensities I(x,x‘) is evaluated
- delivers <xx>, <xx‘>, and <x‘x‘>, i.e. the x-Twiss parameters

Emittance Measurements: Pepper-Pot (Concept)



J.G. Wang et al. NIM A 307, p. 190, (1991)



- hole position determines position x,y
- positions of hole and its image determine x‘,y‘
- for each phase space pixel (x,x‘,y,y‘) screen light emission is recorded
- map of pixel intensities I(x,x‘,y,y‘) is evaluated
- delivers 4d-Twiss parameters

Measurement Error Mitigation

- individually measured $\langle xx \rangle^{a,b,\theta}$, $\langle xx' \rangle^{a,b,\theta}$, and $\langle x'x' \rangle^{a,b,\theta}$ inhabit measurement errors
- errors will enter into the final result for $\langle ... \rangle_i$
- final errors are minimized if „condition number *K*" of the matrix Γ is minimized

$$\begin{bmatrix} \langle xy \rangle_i \\ \langle xy' \rangle_i \\ \langle x'y \rangle_i \\ \langle x'y' \rangle_i \end{bmatrix} = (\Gamma^T \Gamma)^{-1} \Gamma^T \Lambda \qquad \Gamma^\dagger = (\Gamma^T \Gamma)^{-1} \Gamma^T \qquad \|\Gamma\|_2 := \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{k} (\Gamma_{i,j})^2}, \quad \|\Gamma^\dagger\|_2 := \sqrt{\sum_{i=1}^{k} \sum_{j=1}^{n} (\Gamma^\dagger_{i,j})^2}. \qquad \boxed{\kappa(\Gamma) := \|\Gamma\|_2 \|\Gamma^\dagger\|_2}$$

- *K* quantifies linear dependency of the matrix rows
- low dependency → low *K* → errors propagate less to final result
- examples:

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow K = 5$$

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 2.1 & 5.9 & 10.2 & 14.1 \\ 0.55 & 1.49 & 2.55 & 3.45 \\ 0.11 & 0.28 & 0.51 & 0.69 \end{bmatrix} \rightarrow K = 1.2 \times 10^{17}$$

Measurement Error Mitigation

- accuracy of measurement is high for low *K*

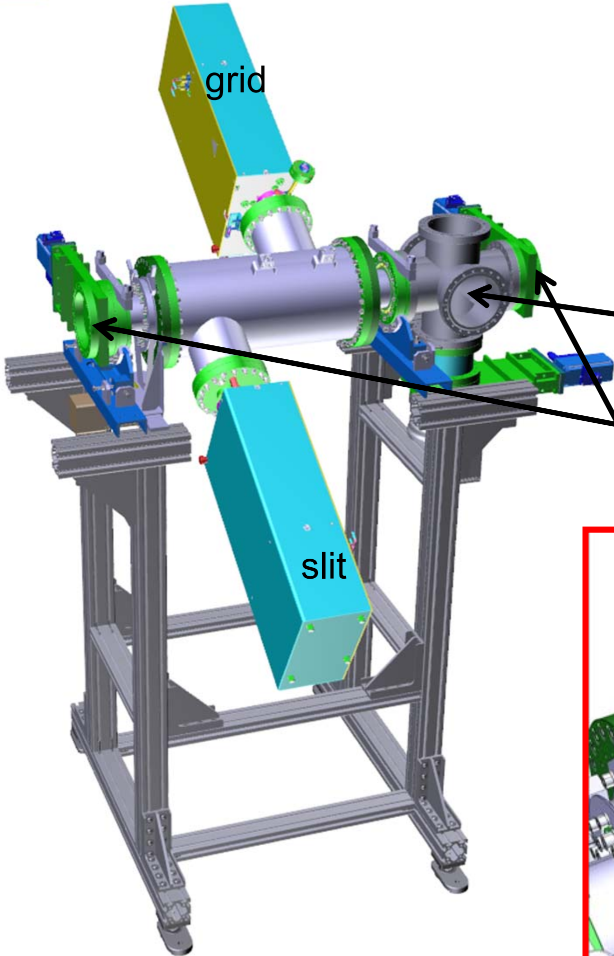
- *K* is pure function of the transport matrix, i.e. of quadrupole settings a & b

- procedure to determine the two quad settings that minimize *K* :

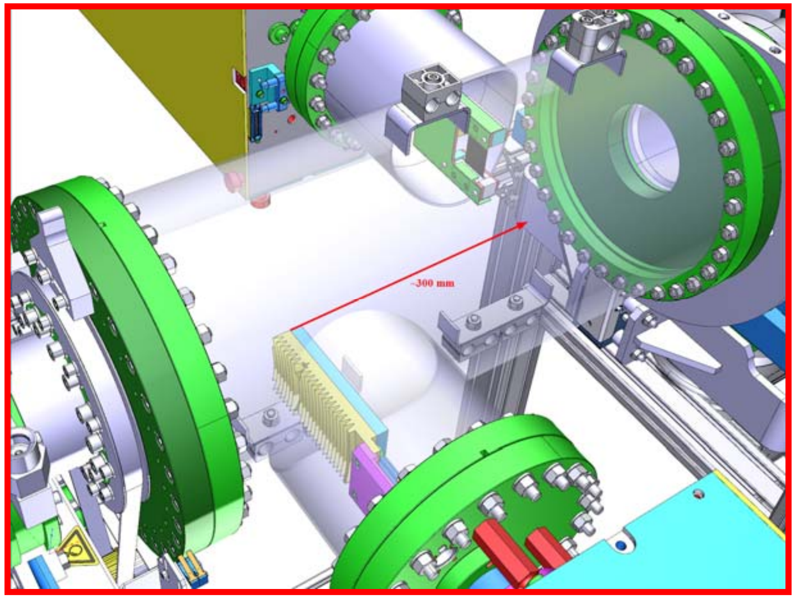
  1. settings should provide 100% transmission up to the grid of ROSE

  2. settings should deliver lowest possible *K*

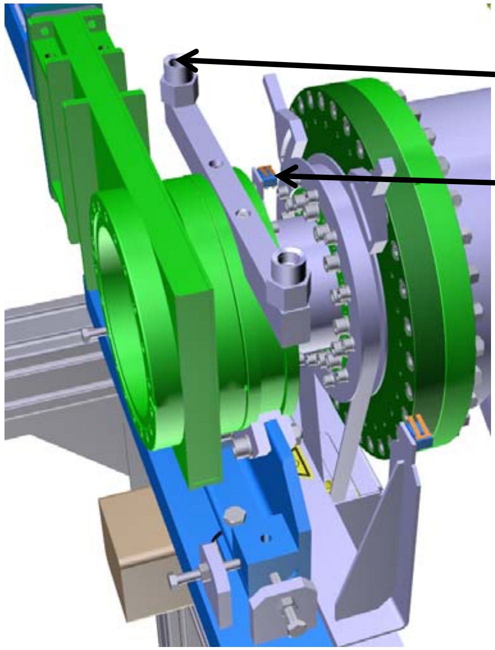
Mechanical Set-up



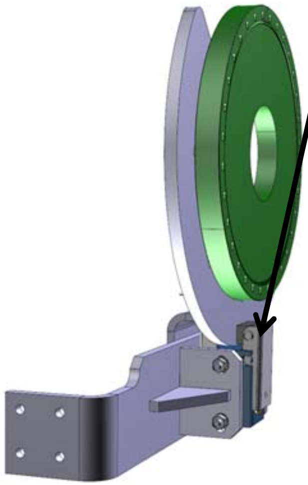
- slit & grid mounted on opposite sides to minimize torque
- cables wrapped around chamber
- fixed pumping chamber
- two gate valves (were not needed during rotation)

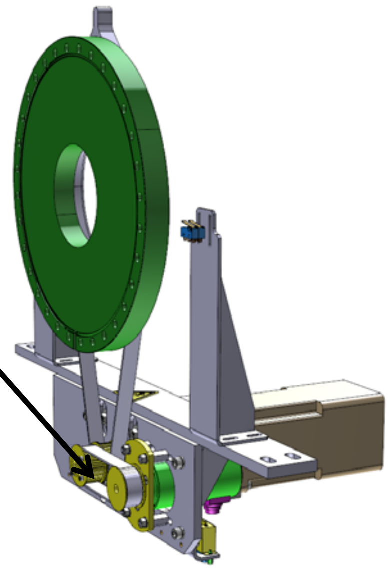


Rotation

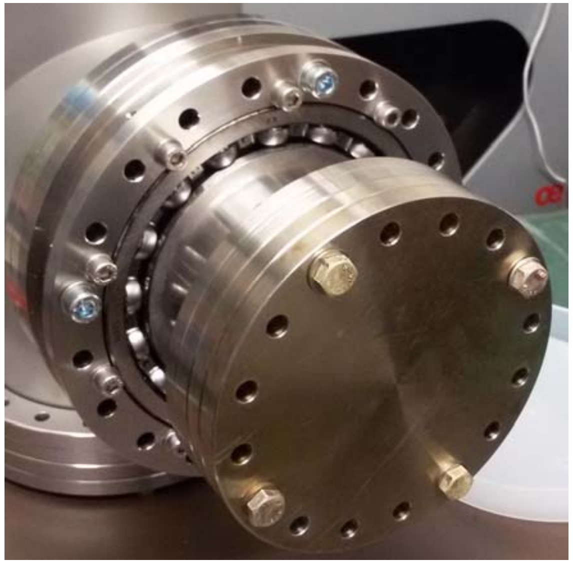


- fiducial points for alignment
- end switches separated by 180°
- disk brake (closed during measurements)
- motor driver with belts ($\delta\theta \leq 0.5°$)
- rotation speed slowed activley down to ≈ 90°/min

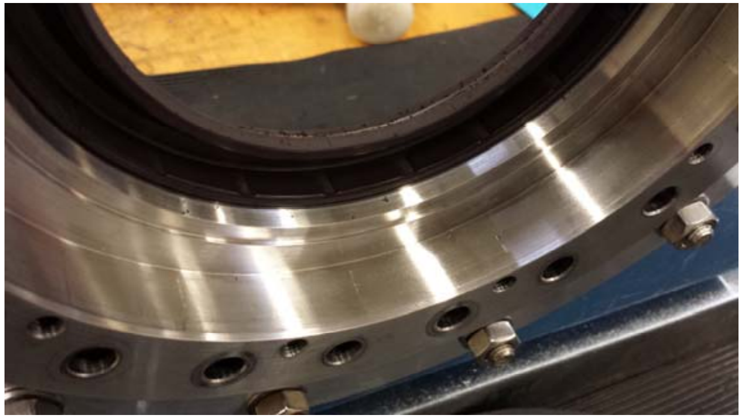


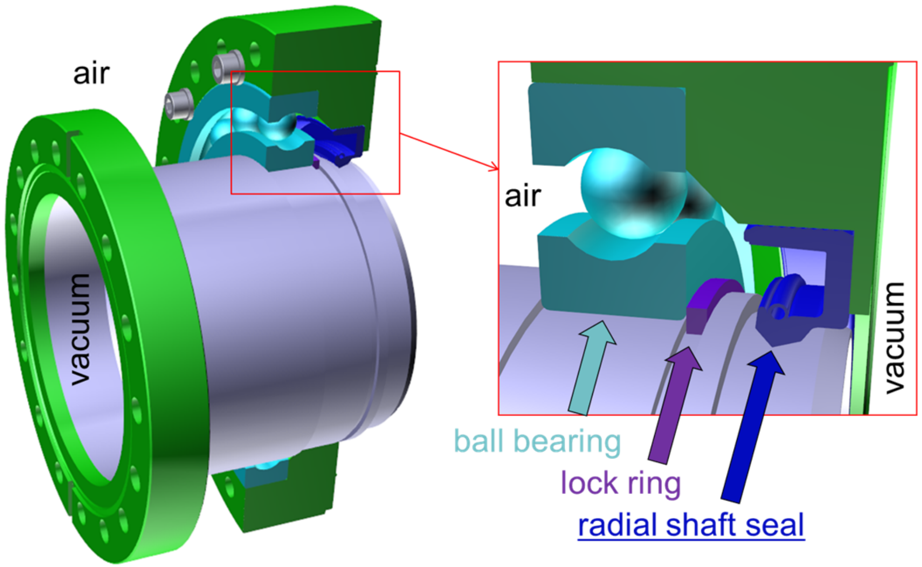


Vacuum Issues



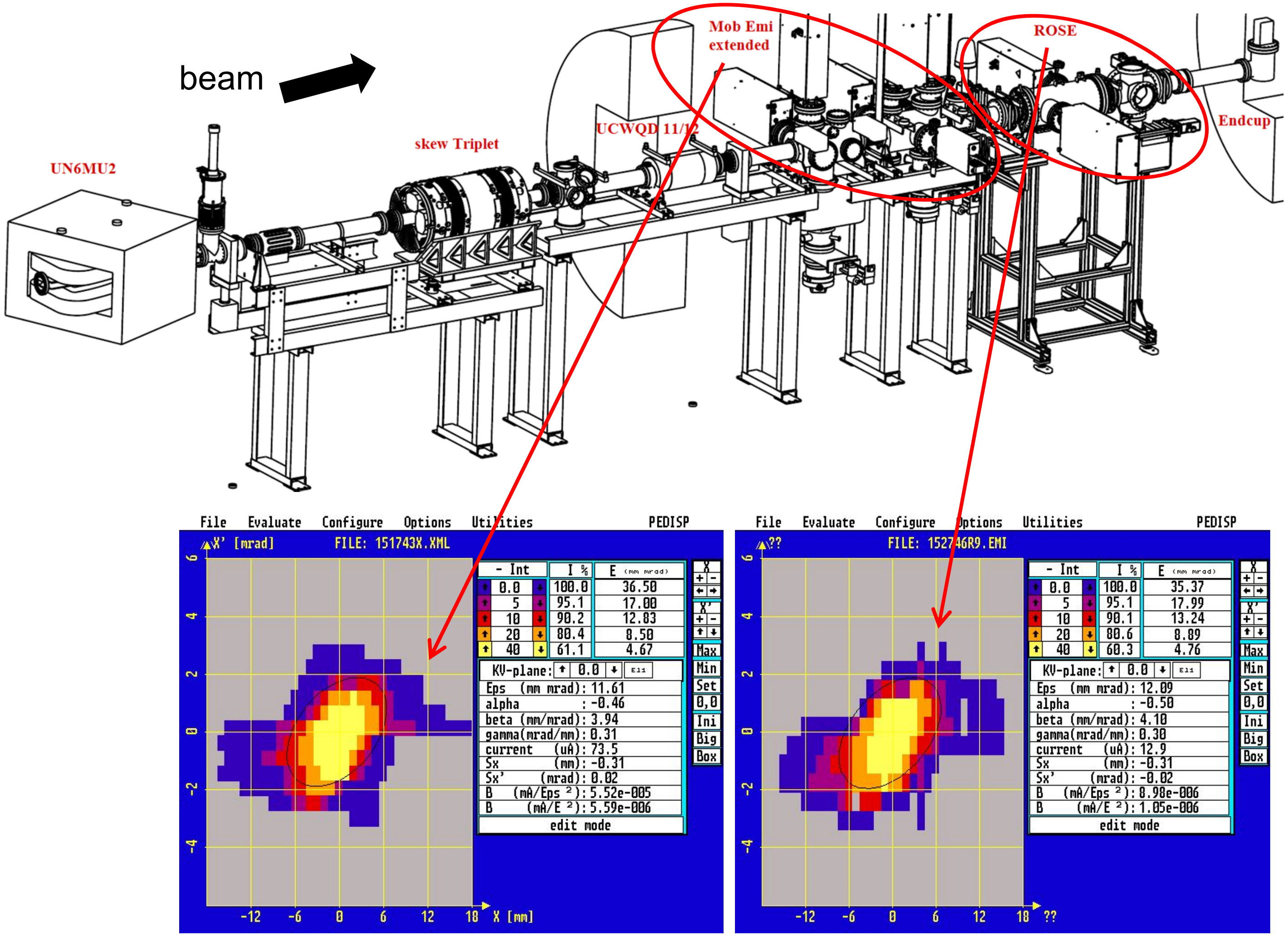
- static pressure ≈ 5×10^{-8} mbar
- max. pressure during rotation ≈ 9×10^{-8} mbar
- recovery time ≈ 1 min





Callibration

ROSE (θ = 0° and 90°) successfully benchmarked to well-tested „MobEmi“ – slit/grid device



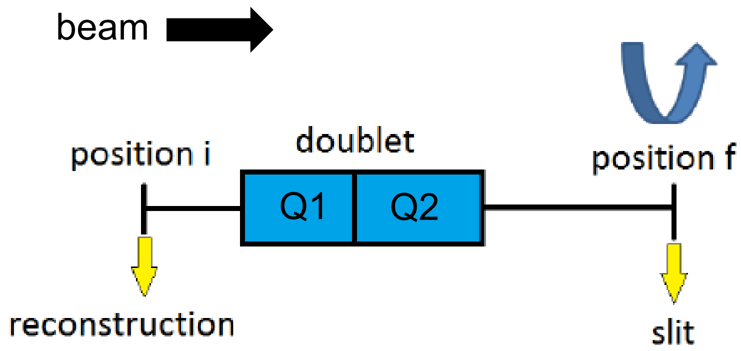
4d Emittance Measurements

- initial measurements at 0° and 90° revealed at ROSE:

θ [deg]	ε_{rms} [mm mrad]	β [m]	α
0	3.1	4.6	-0.1
90	3.4	8.8	-2.4

- optimized quadrupole settings:
  a: 13.2/-12.6 and b: 9.4/-10.2 T/m

- measurements:
  1. at 0° with setting $Q1^a$, $Q2^a$
  2. at 30° with setting $Q1^a$, $Q2^a$
  3. at 90° with setting $Q1^a$, $Q2^a$
  4. at 30° with setting $Q1^b$, $Q2^b$



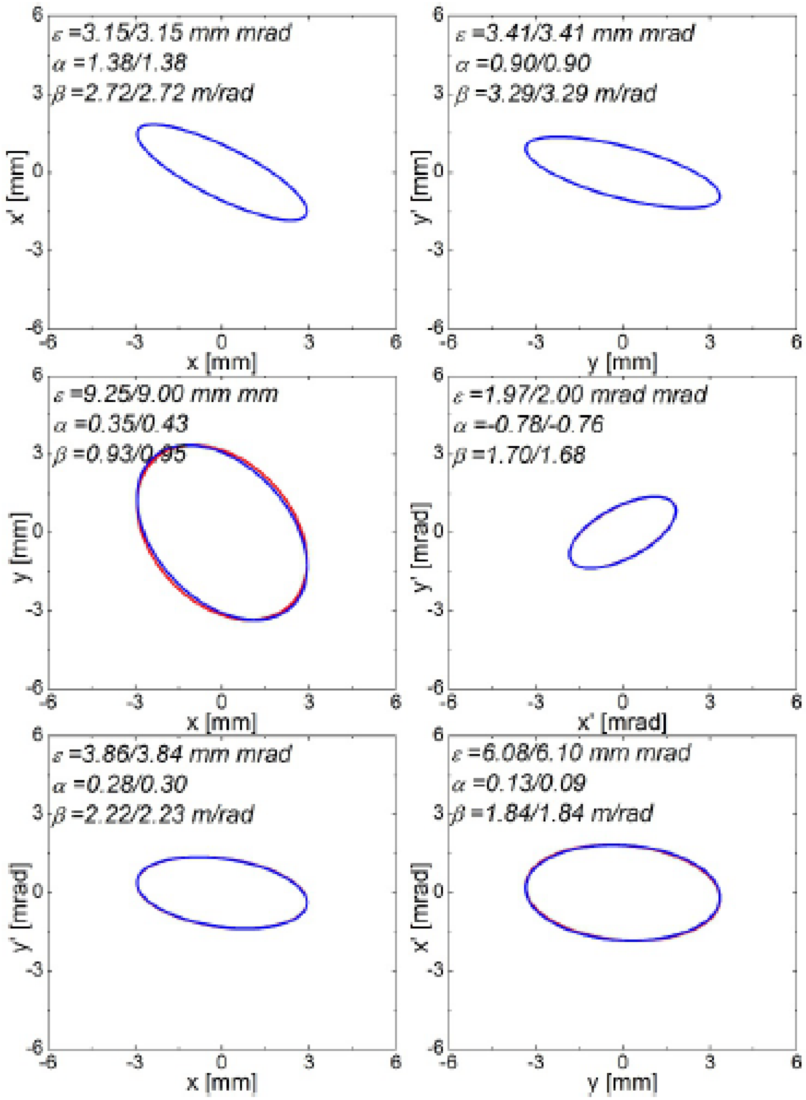
- evaluation:

$$\begin{bmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{bmatrix} = \begin{bmatrix} 8.57 & -4.34 & -3.28 & -1.10 \\ -4.34 & 3.35 & -0.74 & 1.52 \\ -3.28 & -0.74 & 11.20 & -3.05 \\ -1.10 & 1.52 & -3.05 & 1.87 \end{bmatrix}$$

in mm, mrad

E_1 = 2.4 mm mrad
E_2 = 2.0 mm mrad
E_{4d} = 4.8 mm² mrad²

4d Emittance Measurements



check: do results change if six measurements are used instead of four?

- red ellipses from four measurements:
  1. at 0° with setting $Q1^a$, $Q2^a$
  2. at 30° with setting $Q1^a$, $Q2^a$
  3. at 90° with setting $Q1^a$, $Q2^a$
  4. at 30° with setting $Q1^b$, $Q2^b$

- blue ellipses from six measurements:
  1. at 0° with setting $Q1^a$, $Q2^a$
  2. at 30° with setting $Q1^a$, $Q2^a$
  3. at 90° with setting $Q1^a$, $Q2^a$
  4. at 0° with setting $Q1^b$, $Q2^b$
  5. at 30° with setting $Q1^b$, $Q2^b$
  6. at 90° with setting $Q1^b$, $Q2^b$

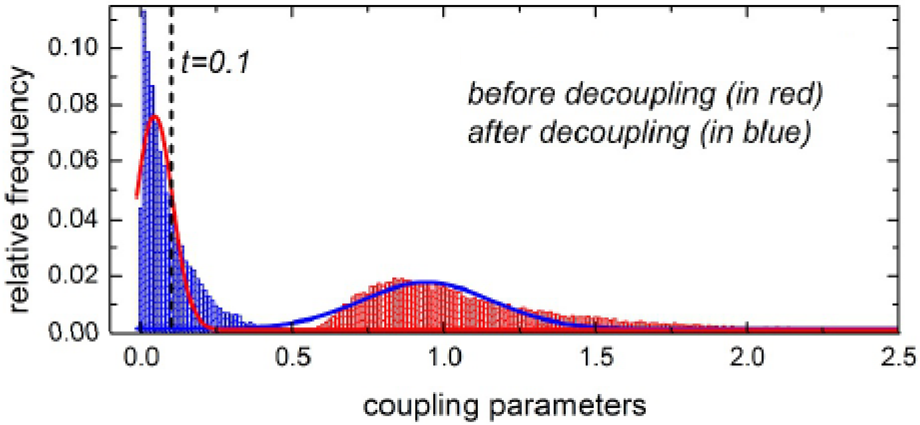
four measurements are sufficient

4d Emittance Measurements: Input for Decoupling

- decoupling reduces projected emittances, i.e. the usual hor. / ver. rms-emittances
- to decouple, the correlated moments must be known (EmTEx with space charge, ...)
- are the ROSE results sufficiently precise to determine the decoupling lattice?

4d Emittance Measurements:
Input for Decoupling

- to answer that question, a virtual decoupling line was constructed (regular triplet and skew triplet for instance)
- the decoupling gradients are calculated from measured moments w/o errors
- moments with errors (from random error study) are transported through decoupling line
- the spectrum of final coupling parameters $t = \frac{\varepsilon_x \varepsilon_y}{\varepsilon_1 \varepsilon_2} - 1$ is evaluated:



ROSE results are sufficiently precise to perform decoupling