Quadrupolar Pick-ups to Measure Space Charge Tune Spreads of Bunched Beams

First MD Results from the PS

Adrian Oeftiger, Malte Titze

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Goals of Study

Motivation

In the context of **strong space charge regime** with **LHC Injectors Upgrade** beam parameters: determine beam brightness (or incoherent space charge (SC) tune shift) **directly** via coherent quadrupolar modes

Basic principle:

- coherent dipolar (centroid) oscillation:
  no influence from SC (Newton's third law, actio = reactio)
- coherent quadrupolar (envelope) oscillation:
  transverse SC **reduces** frequency

$\Longrightarrow$ quantify SC from envelope mode shift in quadrupolar spectrum

Content of this talk:

- peculiarities of CERN proton synchrotrons w.r.t. earlier experiences:
  1. tunes close to coupling resonance, no "far off coupling approximation"
  2. bunched beams, large incoherent tune spreads from strong SC
- first observations from PS set-up with a quadrupolar RF kicker

Schematic Quadrupolar Pick-up

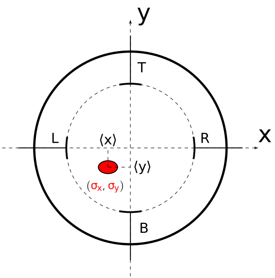


image taken from [1]

In the PS, re-cabling the Long Pick-up 72 as

$$(L+R)-(T+B)$$

results in the time signal

$$S_{\mathrm{QPU}}(t) \propto \langle x^2 \rangle - \langle y^2 \rangle = \sigma_x^2(t) - \sigma_y^2(t) + \langle x \rangle^2(t) - \langle y \rangle^2(t) \quad . \qquad (1)$$

Some Historical Perspective

QPU in **time domain** for emittance measurements:

- 1983, R. H. Miller et al. at SLAC [2]
- 2002, A. Jansson at CERN in PS [3]
- 2007, C.Y. Tang at Fermilab [4]

QPU in **frequency domain** for space charge measurements:

- 1996, M. Chanel at CERN in LEAR [5]
- 1999, T. Uesugi et al. at NIRS in HIMAC [6]
- 2000, R. Bär at GSI in SIS-18 [7]
- 2014, R. Sing et al. at GSI in SIS-18 [1]

$\Longrightarrow$ all far off coupling and coasting beams

CERN's proton synchrotrons:

$\longrightarrow$ close to coupling $\Longrightarrow$ quadrupolar mode frequencies change

$\longrightarrow$ bunched beam

QPU measurements at GSI by R. Singh, M. Gasior et al. [1]

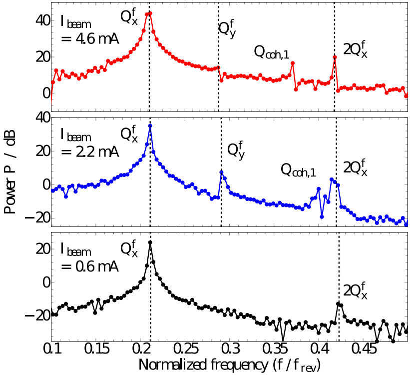


Figure 6: Shift of coherent quadrupole mode $Q_{coh,1}$ with beam current.

particle type	N^{7+}
E_{kin} (MeV/u)	11.45
I_{beam} (mA)	0.6 – 6
ϵ_x, ϵ_y (mm-mrad)	8, 12.75
Q_{x0}, Q_{y0}	4.21, 3.3

$$Q_x^f \hat{=} Q_x$$

$$Q_y^f \hat{=} Q_y$$

$$Q_{coh} \hat{=} Q_\pm$$

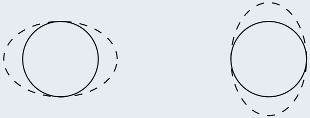
⟶ far off coupling resonance

⟶ coasting beam ⟹ sharp envelope peak

Far Away vs. On the Coupling Resonance

2 eigenmodes for coherent quadrupolar oscillation:

far away from coupling

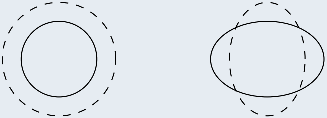


(a) horizontal mode (b) vertical mode

Relation of mode frequencies to incoherent KV tune shift:

$$Q_\pm = 2Q_{x,y} - \left|\Delta Q^{\mathrm{KV}}_{x,y}\right| \left(3 - \frac{\sigma_{x,y}}{\sigma_x + \sigma_y}\right) / 2 \qquad (2)$$

full coupling



(a) breathing mode (b) antisym. mode

Relation of mode frequencies to incoherent KV tune shift:

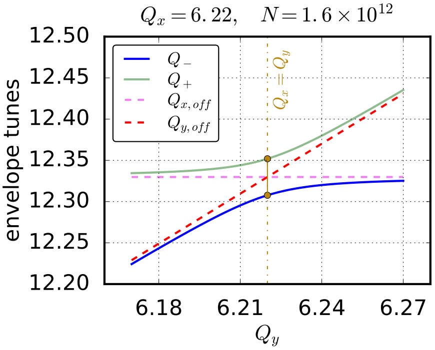
$$Q_+ = 2Q_0 - \left|\Delta Q^{\mathrm{KV}}_{x,y}\right| \qquad (3a)$$

$$Q_- = 2Q_0 - \frac{3}{2}\left|\Delta Q^{\mathrm{KV}}_{x,y}\right| \qquad (3b)$$

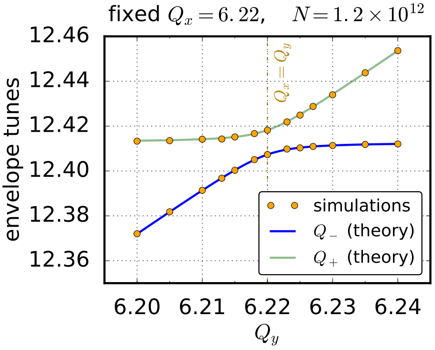
(assuming round beams, $Q_{x,y} \equiv Q_0$)

At vanishing lattice coupling, keep constant incoherent SC tune shift and fixed Q_x. Vary Q_y for a coasting round beam:

analytic expression



simulation results



(N refers to intensity of *coasting* beam within total bunch length $B_L = 180\,\text{ns}$)

Assumption:

- synchrotron motion much slower than betatron motion, $Q_s \ll Q_{x0,y0}$

$\longrightarrow$ 3D RMS envelope equation (Sacherer) decouples to 2D + 1D

$\Longrightarrow$ for a given longitudinal bunch slice, the coherent transverse quadrupolar oscillation depends on local line charge density $\lambda(z)$, longitudinal motion is quasi-stationary and independent (see e.g. [6])

$\Longrightarrow$ envelope tune spread with longitudinal bunch shape imprinted

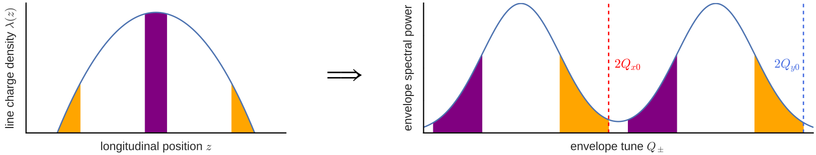


Figure: sketch of envelope detuning scaling with local line charge density

Expectation:

- lower end of envelope tune spread $\leftrightarrow$ strong SC at bunch centre

$\Longrightarrow$ RMS-equivalent (maximal) KV tune shift $\Delta Q^{\mathrm{KV}}_{x,y}$ from envelope spread

Incoherent KV Tune Shift

The uniform (Kapchinskij-Vladimirskij / KV) beam distribution has all particles at same incoherent space charge tune shift:

$$\Delta Q_{x,y}^{\mathsf{KV}} \doteq -\frac{K^{\mathsf{SC}} R^2}{4\sigma_{x,y}(\sigma_x + \sigma_y) Q_{x,y}} \tag{4a}$$

$$\doteq \frac{1 + \sigma_{x,y}/\sigma_{y,x}}{2Q_{x,y}} \Lambda \tag{4b}$$

Connect Λ quantity to general envelope mode expressions in terms of **observables**:

$$\Lambda = \frac{Q_+^2 + Q_-^2 - 4(Q_x^2 + Q_y^2)}{4 + 3(\sigma_x/\sigma_y + \sigma_y/\sigma_x)} \tag{5}$$

(Gaussian tune spread = 2x the RMS-equivalent KV tune shift!)

space charge perveance $K^{\mathsf{SC}} \doteq \dfrac{q\lambda}{2\pi\epsilon_0 \beta\gamma^2 p_0 c}$

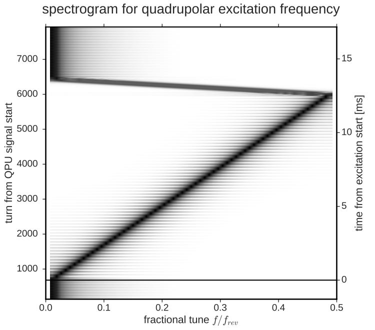
Experimental Set-up

Ingredients:

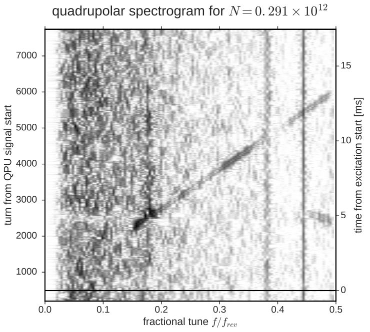
- small time window of 15 ms with decoupled optics to single out space charge influence on envelope tune separation
- chirped excitation of beam via transverse feedback: external waveform generator appropriately connected to kicker plates
  - 12 ms long frequency sweep with 1 ms return
  - harmonic $h = 5$ with frequency range 2.19 MHz to 2.4 MHz
- single bunch in PS with a factor 5 smaller incoherent SC tune shift compared to currently operational LHC beams, off coupling

intensity	$N \approx 0.3 - 0.4 \times 10^{12}$ ppb
transverse emittance	$\epsilon_{x,y} \approx 2.3$ mm mrad
average betatron function	$\beta_x \approx \beta_y \approx 16$ m
average dispersion	$D_x \approx 3$ m
momentum deviation spread	$\sigma_\delta \approx 1 \times 10^{-3}$
bunch length	$B_L \approx 180$ ns
synchrotron tune	$Q_s = 1/600 = 1.67 \times 10^{-3}$
KV space charge tune shift	$\Delta Q_{x,y}^{\mathrm{KV}} \approx 0.02$

excitation signal

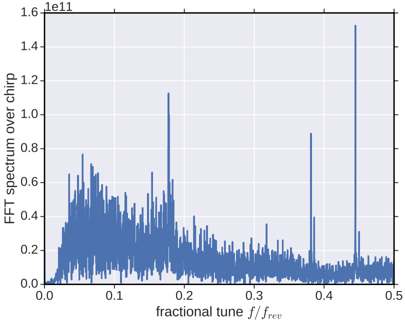


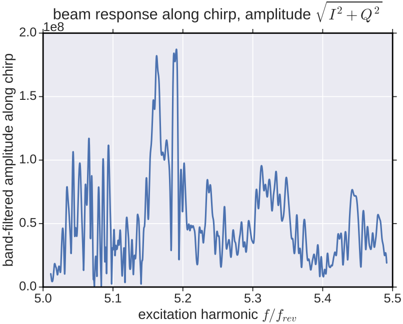
beam response (via QPU)



- dipolar excitation due to feed-down from quadrupolar excitation, seen in beam response at lower frequencies $f < 0.25 f_{\text{rev}}$

(a) FFT across up-chirp time is not such a useful idea...





(b) ... instead project and band filter along local excitation frequency

Approach: In-phase and Quadrature Components

Take

a) QPU time signal $S_{\mathsf{QPU}}(t)$

b) excitation signal $S_{\mathsf{exc}}(t)$ (sine wave with increasing frequency)

c) 90 deg shifted excitation signal $C_{\mathsf{exc}}(t) = S_{\mathsf{exc}}(t)|_{\phi\to\phi+90\,\mathrm{deg}}$

Assume immediate beam response to chirp:

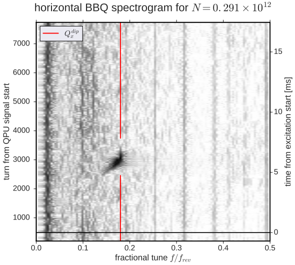
1. **correlation**: find excitation start in $S_{\mathsf{QPU}}(t)$ by correlation with $S_{\mathsf{exc}}(t)$
2. **demodulation** of measured QPU time signal into

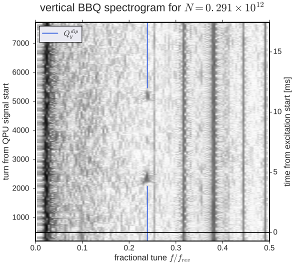
$$I(t) = S_{\mathsf{QPU}}(t) \cdot S_{\mathsf{exc}}(t) \qquad \text{(in-phase component)}$$

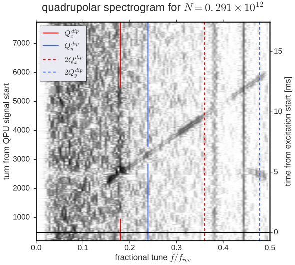
$$\text{and} \quad Q(t) = S_{\mathsf{QPU}}(t) \cdot C_{\mathsf{exc}}(t) \qquad \text{(quadrature component)}$$

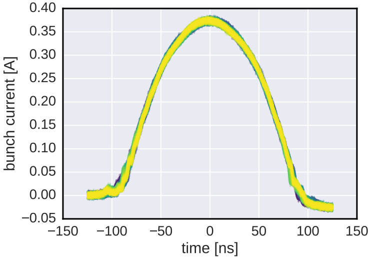
3. **band filter** original $S_{\mathsf{QPU}}(t)$ around time-varying excitation frequency by low pass filtering $I(t)$ and $Q(t)$
4. **amplitude** of beam response along chirp amounts to $\sqrt{I^2(t) + Q^2(t)}$

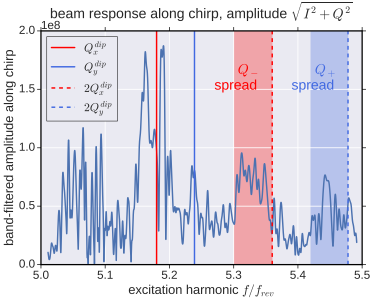
First Results: Parabolic Bunch



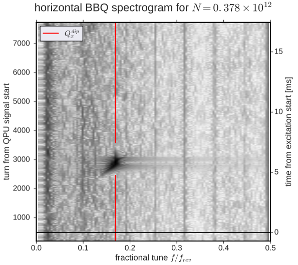


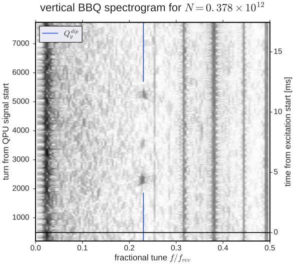


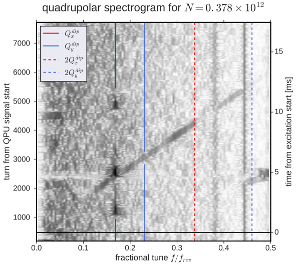


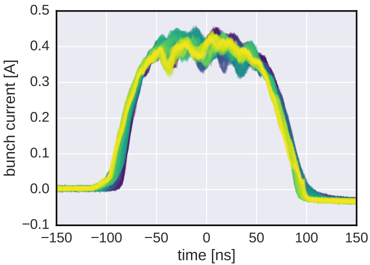


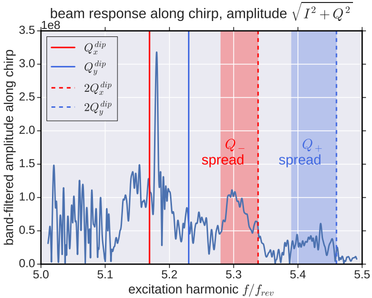
First Results: Flat Bunch











Summary and Outlook

In conclusion:

- transverse feedback (TFB) strong enough for quadrupolar excitation
- observed envelope tune spread in SC depressed bunched beam

$\Longrightarrow$ Exciter + QPU = potentially very powerful diagnostic tool for SC

Next steps:

- QPU hardware: planned to have all H, V, Q channels simultaneously after this coming technical stop (for PS as well as SPS!)
- TFB: generate synchronous and clean signals for quadrupolar excitation with internal waveform generator
- improve QPU set-up and spectral analysis (e.g. subtract dipolar contribution from quadrupolar signal → 3 channels required!)
- dedicated space charge experiments (e.g. resonance studies)
- more realistic space charge simulations with bunched distributions

Thank you for your attention!

Appendix

References I

[1] R Singh et al. “Observations of the quadrupolar oscillations at GSI SIS-18”. In: (2014).

[2] R H Miller et al. *Nonintercepting emittance monitor*. Tech. rep. Stanford Linear Accelerator Center, 1983.

[3] Andreas Jansson. “Noninvasive single-bunch matching and emittance monitor”. In: *Physical Review Special Topics-Accelerators and Beams* 5.7 (2002), p. 072803.

[4] Cheng-Yang Tan. *Using the quadrupole moment frequency response of bunched beam to measure its transverse emittance*. Tech. rep. Fermi National Accelerator Laboratory (FNAL), Batavia, IL, 2007.

[5] Michel Chanel. *Study of beam envelope oscillations by measuring the beam transfer function with quadrupolar pick-up and kicker*. Tech. rep. 1996.

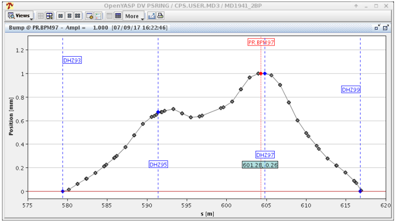
[6] T Uesugi et al. “Observation Of Quadrupole Mode Frequency And Its Connection With Beam Loss”. In: KEK-99-98 (1999). URL: http://cds.cern.ch/record/472700.

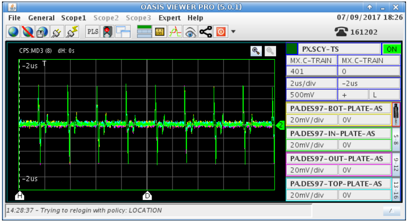
References II

[7] R C Baer. “Untersuchung der quadrupolaren BTF-Methode zur Diagnose intensiver Ionenstrahlen”. Universitaet Frankfurt, Germany, 2000.

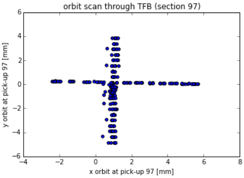
TFB: Impact of Orbit

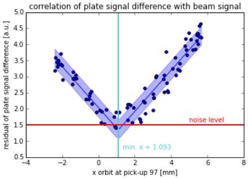
Set up a local bump through the TFB and measure the induced beam signal on the plates (effectively a BPM):

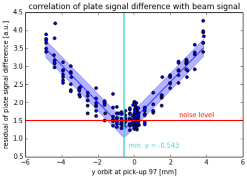




By scanning the orbit location one can minimise the difference signal:

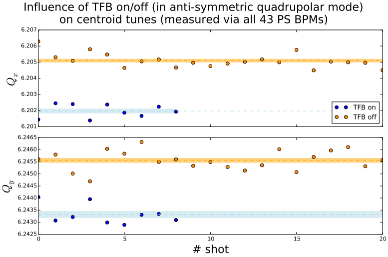


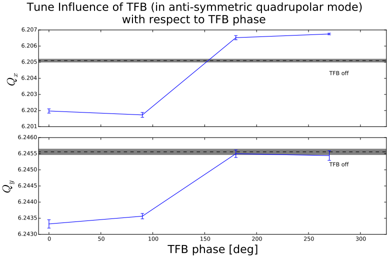




TFB: Static Quadrupole on $h = 1$

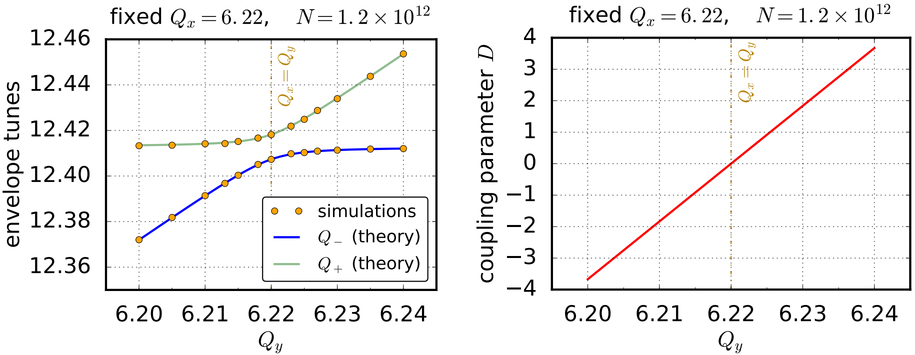
- TFB pulsing at f_{rev} becomes a static quadrupole to the beam
- varying the phase of the pulsing RF quadrupole changes the tune impact





Simulations for Tune Scan

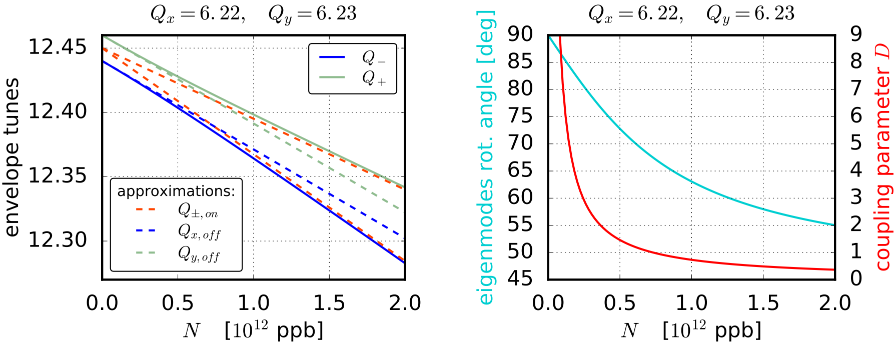
Simulations with KV beams for $N = 1.2 \times 10^{12}$ confirm theory:



→ r.m.s. equivalent Gaussian beams (with same $\sigma_{x,y}$ like KV beams) exhibit same quadrupolar tunes as KV

⚠ Gaussian spectra broaden quickly

With slightly split tunes, approach full coupling by increasing bunch intensity:



$\Longrightarrow$ scan space charge tune shift $\Delta Q^{\mathrm{KV}}_{x,y}$ and verify theory

Envelope Equations

Envelope equations of motion (e.o.m.)

$$r_x'' + K_x(s)\,r_x - \frac{\epsilon_{x,\mathrm{geo}}^2}{r_x^3} - \frac{K^{\mathrm{SC}}}{2(r_x + r_y)} = 0 \quad , \tag{6a}$$

$$r_y'' + K_y(s)\,r_y - \frac{\epsilon_{y,\mathrm{geo}}^2}{r_y^3} - \frac{K^{\mathrm{SC}}}{2(r_x + r_y)} = 0 \tag{6b}$$

for transverse r.m.s. beam widths $r_{x,y} = \sigma_{x,y}$ have equilibrium

$$\frac{Q_x^2}{R^2}\,r_{x,\mathrm{m}} - \frac{\epsilon_{x,\mathrm{geo}}^2}{r_{x,\mathrm{m}}^3} - \frac{K^{\mathrm{SC}}}{2(r_{x,\mathrm{m}} + r_{y,\mathrm{m}})} = 0 \quad , \tag{7a}$$

$$\frac{Q_y^2}{R^2}\,r_{y,\mathrm{m}} - \frac{\epsilon_{y,\mathrm{geo}}^2}{r_{y,\mathrm{m}}^3} - \frac{K^{\mathrm{SC}}}{2(r_{x,\mathrm{m}} + r_{y,\mathrm{m}})} = 0 \tag{7b}$$

Linear Perturbation in Smooth Approximation

Constant focusing channel

$$K_{x,y} = \frac{1}{\beta_{x,y}^2} = \frac{Q_{x,y}^2}{R^2} = \text{const} \tag{8}$$

gives linearised e.o.m. for perturbation around equilibrium $r = r_\mathrm{m} + \delta r$

$$\frac{d^2}{ds^2}\begin{pmatrix}\delta r_x \\ \delta r_y\end{pmatrix} = -\underbrace{\begin{pmatrix}\kappa_x & \kappa_\mathsf{SC} \\ \kappa_\mathsf{SC} & \kappa_y\end{pmatrix}}_{\doteq(\kappa)} \cdot \begin{pmatrix}\delta r_x \\ \delta r_y\end{pmatrix} \tag{9}$$

$$\text{with} \quad \begin{cases} \kappa_{x,y} = 4\frac{Q_{x,y}^2}{R^2} - \frac{2\sigma_{x,y}+3\sigma_{y,x}}{\sigma_{x,y}}\kappa_\mathsf{SC} \\ \kappa_\mathsf{SC} \doteq \frac{K^\mathsf{SC}}{2(\sigma_x+\sigma_y)^2} \end{cases} \tag{10}$$

Definitions

Coupling Parameter

$$D \doteq \frac{\kappa_y - \kappa_x}{2\kappa_{\mathsf{SC}}} = 4\frac{Q_y^2 - Q_x^2}{K^{\mathsf{SC}} R^2}(\sigma_x + \sigma_y)^2 + \frac{3}{2}\left(\frac{\sigma_y}{\sigma_x} - \frac{\sigma_x}{\sigma_y}\right) \tag{11}$$

Rotation Into Decoupled Eigensystem

$$\begin{aligned} \tan(\alpha) &= \frac{1}{2\kappa_{\mathsf{SC}}}\left[\kappa_y - \kappa_x + \sqrt{4\kappa_{\mathsf{SC}}^2 + (\kappa_y - \kappa_x)^2}\right] \\ &= D + \sqrt{1 + D^2} \end{aligned} \tag{12}$$

Incoherent Tune Shifts

KV Space Charge Tune Shift

$$\Delta Q_{x,y}^{\mathsf{KV}} = -\frac{K^{\mathsf{SC}} R^2}{4\sigma_{x,y}(\sigma_x + \sigma_y) Q_{x,y}} \tag{13}$$

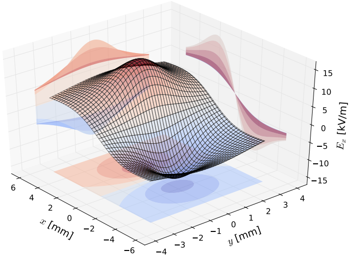
$$\text{with} \quad K^{\mathsf{SC}} \doteq \frac{q\lambda}{2\pi\epsilon_0 \beta\gamma^2 p_0 c} \tag{14}$$

R.m.s. Equivalent Gaussian Space Charge Tune Spread

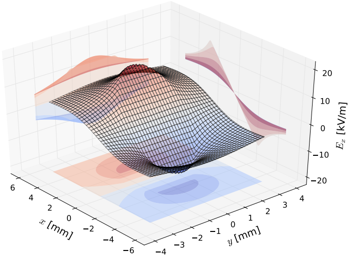
linearised Gaussian e-field = twice r.m.s. equivalent KV e-field

$$\Longrightarrow \qquad \max\left\{\Delta Q_{x,y}^{\mathsf{spread}}\right\} = 2\,\Delta Q_{x,y}^{\mathsf{KV}} \tag{15}$$

Gaussian vs. R.m.s. Equivalent KV



(a) Gaussian beam

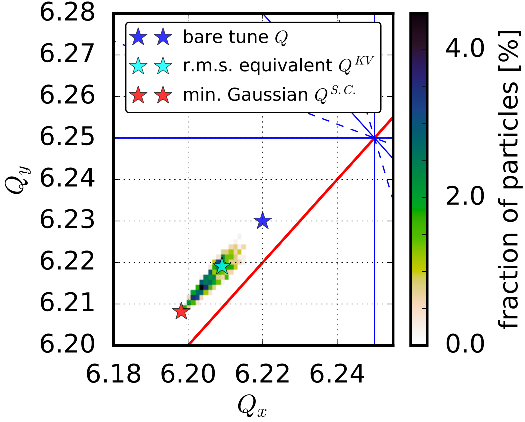


(b) r.m.s. equivalent KV beam

Figure: Electric fields in r.m.s. equivalent distributions with same $\sigma_{x,y}$

Incoherent Tunes and R.m.s. Equivalence

Incoherent tune spread of a coasting, transversely Gaussian distribution:



Quadrupolar Mode Formulae

Quadrupolar Mode Tunes (General Formula)

$$Q_{\pm}^2 = \frac{R^2}{2}\left[\kappa_x + \kappa_y \pm \sqrt{4\kappa_{\mathrm{SC}}^2 + (\kappa_y - \kappa_x)^2}\right] \tag{16}$$

$$= 2(Q_x^2 + Q_y^2) - \frac{K^{\mathrm{SC}} R^2}{(\sigma_x + \sigma_y)^2}\left[1 + \frac{3}{4}\left(\frac{\sigma_y}{\sigma_x} + \frac{\sigma_x}{\sigma_y}\right) \mp \frac{\sqrt{1 + D^2}}{2}\right]$$

Quadrupolar Mode Formulae

Off-resonance $D \gg 1$ With Round Beam

$$Q_+ = 2Q_y - \frac{5}{4}|\Delta Q_y^{\mathsf{KV}}| \quad , \tag{17a}$$

$$Q_- = 2Q_x - \frac{5}{4}|\Delta Q_x^{\mathsf{KV}}| \quad . \tag{17b}$$

for $Q_y > Q_x$ otherwise exchange $x \leftrightarrow y$

Quadrupolar Mode Formulae

On-resonance $D \approx 0$ With Round Beam

$$Q_+ = 2Q_0 - |\Delta Q^{\mathsf{KV}}| \quad , \tag{18a}$$

$$Q_- = 2Q_0 - \frac{3}{2}|\Delta Q^{\mathsf{KV}}| \quad . \tag{18b}$$

for $Q_0 \doteq Q_x = Q_y$ and $\Delta Q^{\mathsf{KV}} \doteq \Delta Q_x^{\mathsf{KV}} = \Delta Q_y^{\mathsf{KV}}$

QPU Simulations in SPS

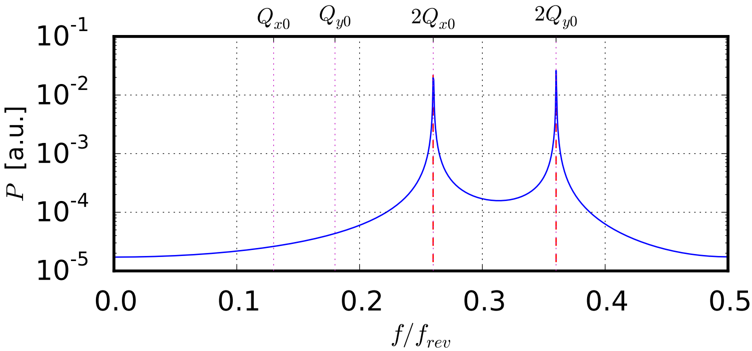
simulation parameters:

- machine: SPS at injection
- $\gamma = 27.7$
- $\epsilon_x = \epsilon_y = 2.5\,\mathrm{mm-mrad}$
- $N_b = 1.25 \times 10^{11}$
- 512 – 2048 turns
- 2.6×10^5 macro-particles
- longitudinally matched Gaussian-type distribution
- betatron mismatch by 10% in both x, y

$\Longrightarrow$ injection oscillations

QPU Spectrum: Only Betatron Mismatch

need beam mismatched to both β_x, β_y to see clear peaks

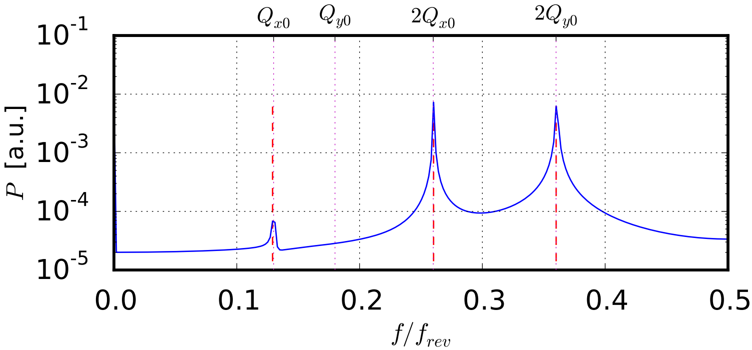


$\Longrightarrow$ $2Q_{x0}$, $2Q_{y0}$ from undepressed envelope oscillations

$\Longrightarrow$ including synchrotron motion: same spectrum (no coupling!)

QPU Spectrum: Include Dispersion

smooth approximation: constant $D_x = 2.96$ around the ring



$\Longrightarrow$ peak at Q_{x0} comes from dispersion

Reason for Dispersion Peak

$$\sigma_x(i_{\text{turn}}) = \sqrt{\langle x_i^2 \rangle_{\text{beam}} - \langle x_i \rangle_{\text{beam}}^2}$$

with $$x_i(i_{\text{turn}}) = \sqrt{\beta_x \epsilon_{x,i}^{\text{s.p.}}} \cos(2\pi Q_{x0} i_{\text{turn}} + \Psi_0) + D_x \delta_i$$

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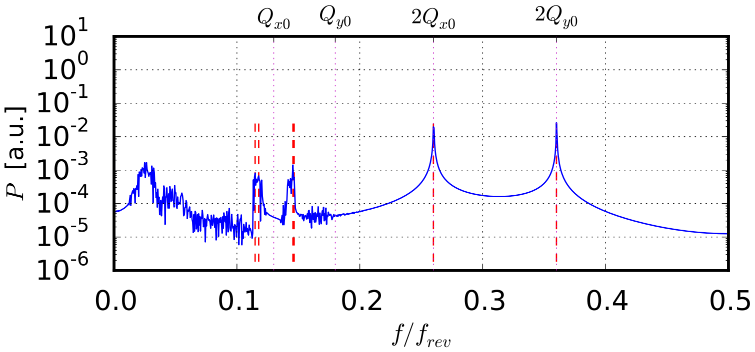
$$\stackrel{\sim}{\Longrightarrow} x_i^2 = \ldots \underbrace{\cos^{2}(2\pi Q_{x0} i_{\text{turn}} + \ldots)}_{\ldots \cos(2\pi 2 Q_{x0} i_{\text{turn}} + \ldots)} + \ldots D_x \delta_i \cdot \cos(2\pi Q_{x0} i_{\text{turn}} + \ldots) + \ldots$$

due to: $\quad 2\cos^2(\alpha) = \cos(2\alpha) + 1$

i.e. only for $D_x \neq 0 \implies$ peak at Q_{x0}

QPU Spectrum: Include Synchrotron Motion

synchrotron motion couples to betatron motion through non-zero $D_x = 29.6\,\mathrm{m}$ (smooth approximation!)

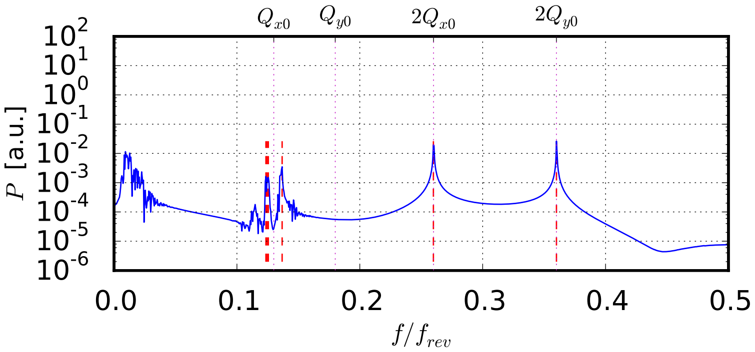


$\Longrightarrow$ peak separation at Q_{x0} from synchrobetatron coupling

- $Q_s = 0.017$ at injection for $V = 5.75\,\mathrm{MV}$

QPU Spectrum: Slower Synchrotron Motion

synchrotron motion couples to betatron motion through non-zero $D_x = 29.6\,\mathrm{m}$ (smooth approximation!)



- $Q_s = 0.007$ changing $\gamma_{tr} = 17.95 \longrightarrow 25$ (while $\gamma = 27.7$)

$\Longrightarrow$ peak separation shrinks

Reason for Peak Separation with Q_s

$$x_i^2 = \ldots + \ldots D_x \delta_i \cdot \cos(2\pi Q_{x0} i_{\text{turn}} + \ldots) + \ldots$$

with $\delta_i(i_{\text{turn}}) = \hat{\delta}_i \cos(2\pi Q_s i_{\text{turn}} + \ldots)$

Reason for Peak Separation with Q_s

$$x_i^2 = \ldots + \ldots D_x \delta_i \cdot \cos(2\pi Q_{x0} i_{\text{turn}} + \ldots) + \ldots$$

with $\quad \delta_i(i_{\text{turn}}) = \hat{\delta}_i \cos(2\pi Q_s i_{\text{turn}} + \ldots)$

$$\overset{\sim}{\Longrightarrow} x_i^2 = \ldots + \ldots \underbrace{\cos(2\pi Q_{x0} i_{\text{turn}} + \ldots)\cos(2\pi Q_s i_{\text{turn}} + \ldots)}_{\cos(2\pi(Q_{x0}-Q_s)i_{\text{turn}}+\ldots)+\cos(2\pi(Q_{x0}+Q_s)i_{\text{turn}}+\ldots)} + \ldots$$

due to: $\quad 2\cos(\alpha)\cos(\beta) = \cos(\alpha-\beta) + \cos(\alpha+\beta)$

i.e. for $D_x \neq 0$ and $Q_s \neq 0$:

one peak at $Q_{x0} \Longrightarrow$ two peaks located at $Q_{x0} \pm Q_s$