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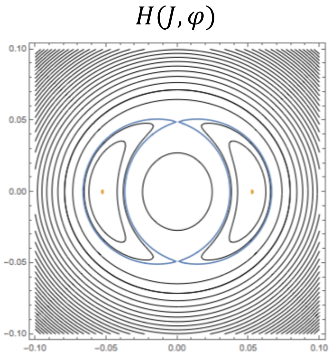
Particle loss near the half integer resonance: transition from frozen space charge to self-consistent models



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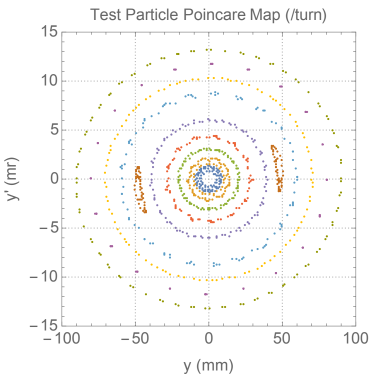

Space Charge 17, GSI Darmstadt, 4-6 October 2017

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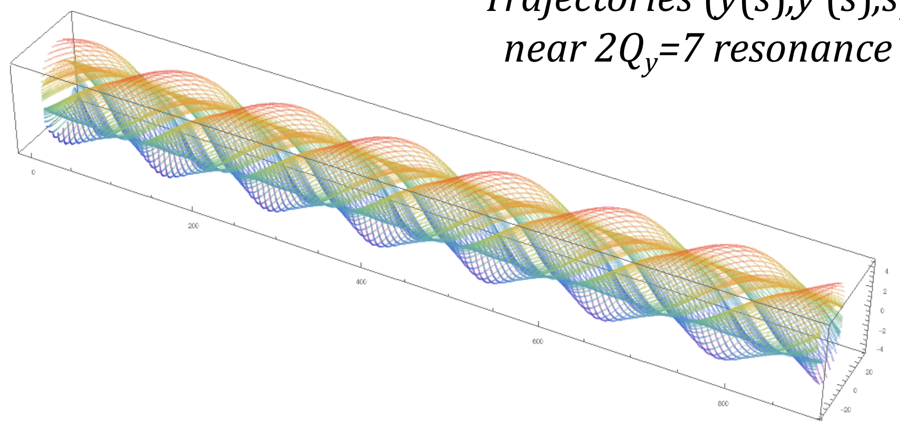
Trajectories $(y(s), y'(s), s)$ near $2Q_y=7$ resonance



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1. Introduction

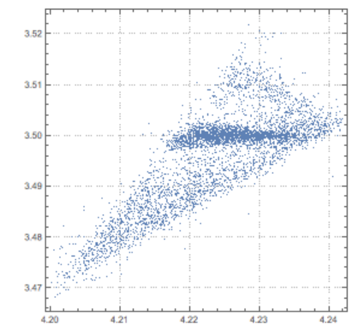
- Why does the half integer resonance matter?
  The high intensity limit ~ how near coherent limit?
  Can we understand and minimise its effects?
  Run at higher space charge levels – lower linac costs?

- ISIS RCS: half integer important loss mechanism
  Early cycle loss:
  1 ~ increase Q_y → loss due to head-tail instability
  2 ~ decrease Q_y → loss due to half integer ($\Delta Q_{incoh} \approx 0.5$)
  Whilst can see 1 directly, 2 is inferred from loss … how real?

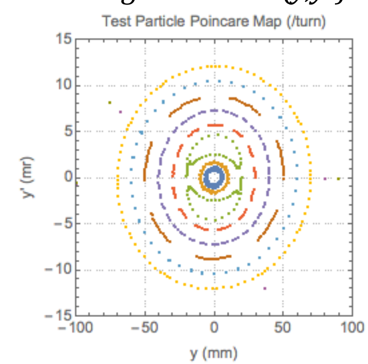
- Undertook experiments to observe more directly
  Simplify: RCS → SRM, 2D coasting beam; measure loss, profiles
  Can we explain what we really see, beyond beam loss?
  Explain evolution of beam distributions …

(*SRM=Storage Ring Mode*)

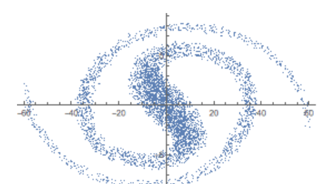
Incoherent Tunes (Q_x,Q_y)



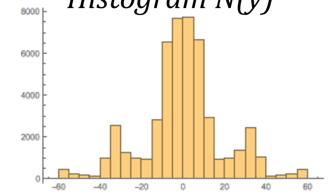
Single Particle (*y*,*y′*)



Single Particle (*y*,*y′*)

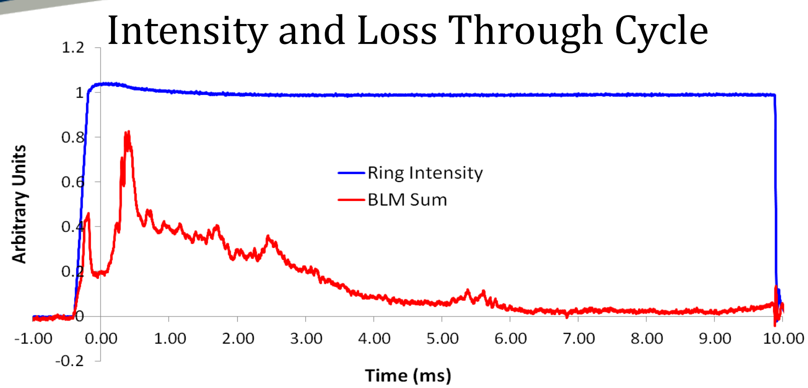


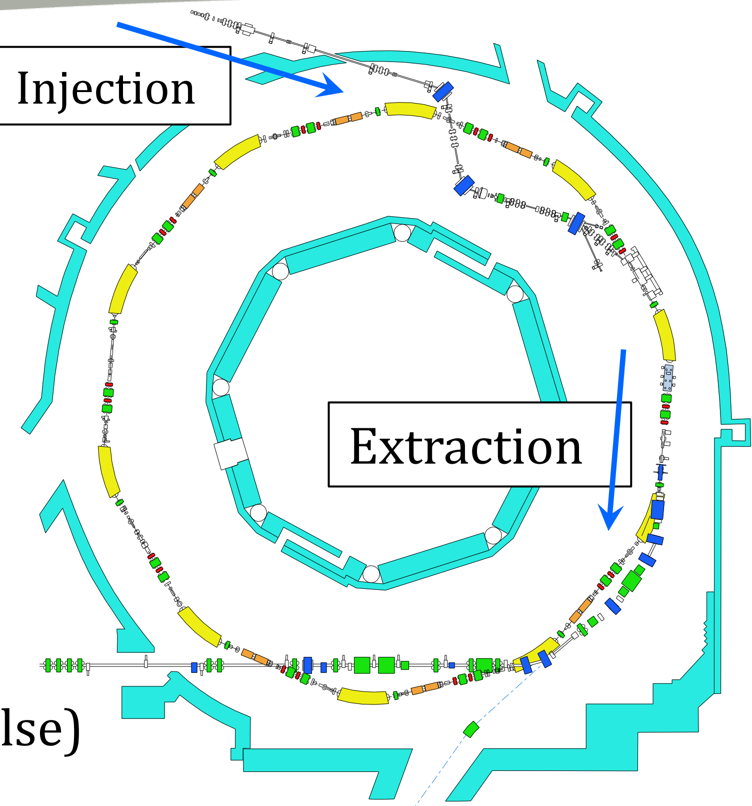
Histogram N(y)



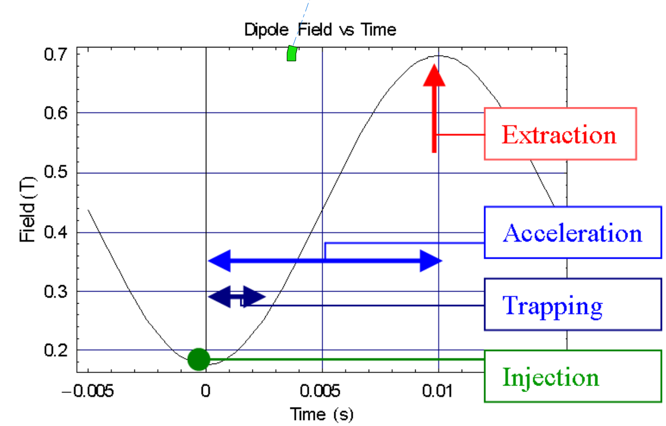
1. The ISIS Synchrotron

2.8×10^{13} ppp



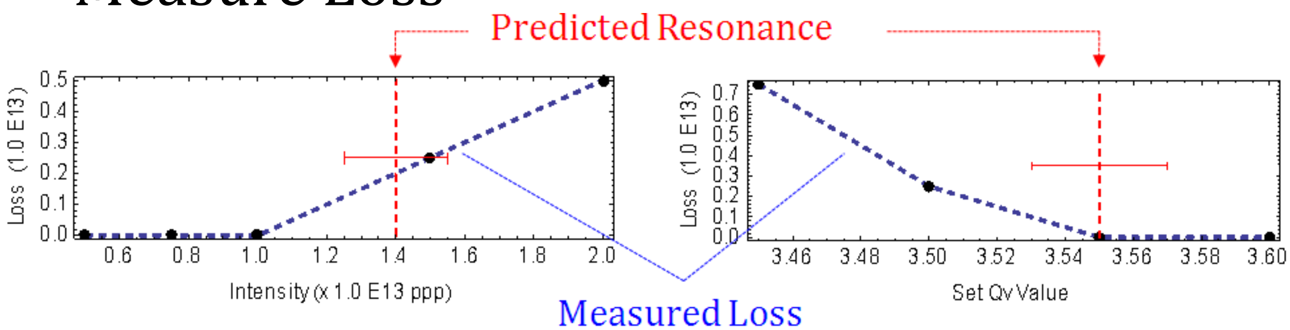


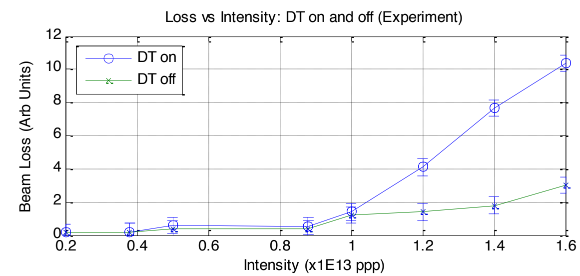
Circumference:	163 m
Energy Range:	70-800 MeV
Rep Rate:	50 Hz
Intensity:	2.5-3.0 $\times 10^{13}$ ppp (protons per pulse)
Beam Power:	160-200 kW
Losses:	Inj: 2%, Trap: <3%, Acc/Ext <0.5%
Injection:	130 turn, H^- charge-exchange
Acceptances:	Collimated ~350 π mm mr
RF System:	h=2, f_2=1.3-3.1 MHz, V_2 ~160kV/turn
(*2 bunches*)	h=4, f_4=2.6-6.2 MHz, V_4 ~80 kV/turn
Extraction:	Single turn, vertical
Tunes:	(Q_x, Q_y)=(4.31, 3.83) (*programmable*)



2. Review of Experiments

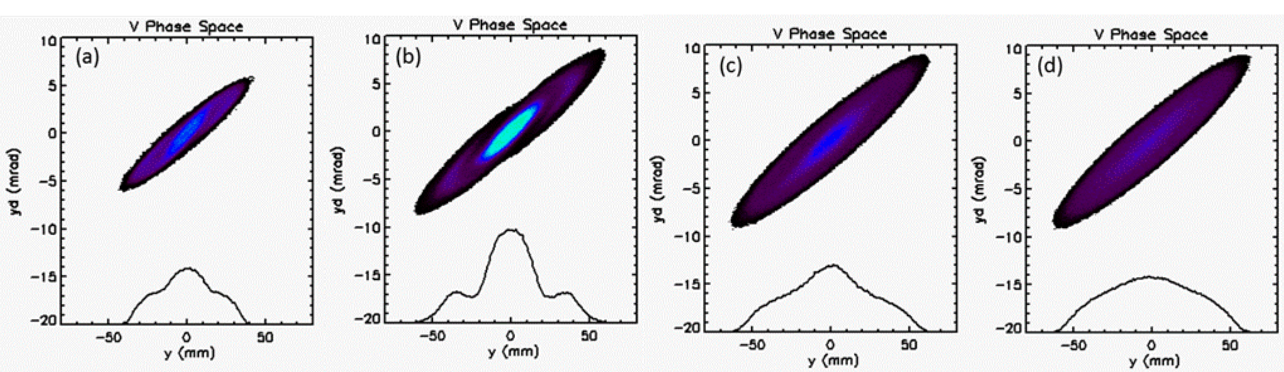
- Experimental studies 2D coasting beam (Storage Ring Mode)
  RF off, DC magnet field, inject small beam $\varepsilon_x = \varepsilon_y$, multi-turn injection
  $\varepsilon_{rms} \approx 20\ \pi$ mm mr, $2Q_y$=7 quadrupole driving term ($\Delta k_7 = 0.05$), Q_y=3.6
  Ramp intensity (N_p~1E13 ppp), ramp tune: push onto coherent resonance
- Measure Loss





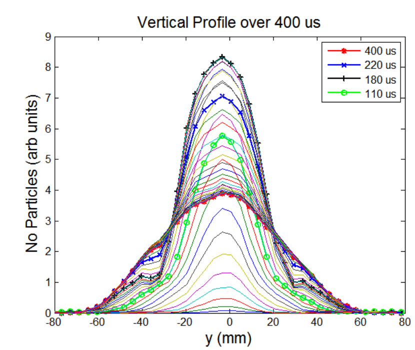
- Measure Profiles: Observations agree with ORBIT

ORBIT results



Transverse profile

Measured over 400 μs



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2. Review of Experiments

- Experiments characterized profile evolution

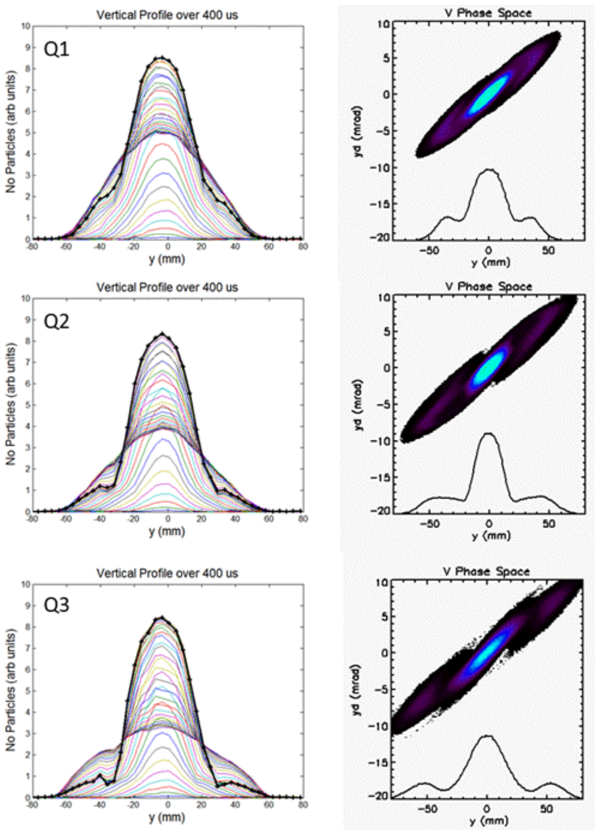
Dependence on tune

Measured ORBIT

Q_1=3.71

Q_2=3.67

Q_3=3.63



Dependence on driving term

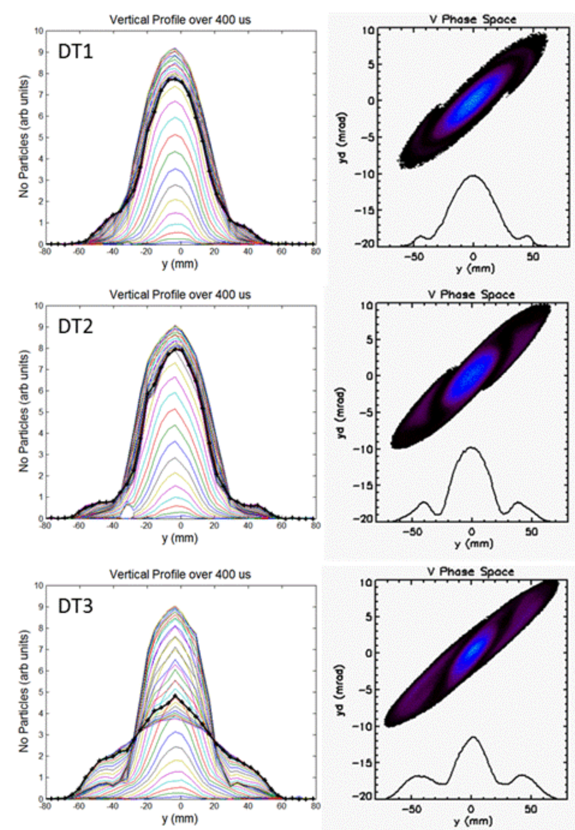
Measured ORBIT

$\Delta k_7 = 0.02$

$\Delta k_7 = 0.03$

$\Delta k_7 = 0.05$

$\left(\Delta k_7 = \frac{k_7}{k}\right)$



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2. Review of Experiments

- Summary of Experiments

- Loss “brick wall” corresponding to coherent limit
    - But see slow build up of loss on approach

- Complicated beam profile evolution
    - Verified, controllable, half integer “lobes”
    - Characterized behaviour as function of tune, driving term, intensity

- Key questions:
    - What models can we use to understand the observations?
    - Can we understand the time evolution of profiles?
    - How accurate are the profile measurements? [1]

[1] C C Wilcox, R E Williamson *et al.*: Proc. IBIC2016, WEPG68, also Ionisation Profile Monitor Workshop GSI, May 2017

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3. Frozen Space Charge Model

Phase Averaged Frozen WB Hamiltonian (1)

- Want to understand evolution of beam distributions
  Need understanding of single particle motion ~ start with simple model

- Assume space charge of frozen water bag (WB) distribution
  Essentially half integer resonance in non-linear field of WB beam
  Frozen WB: no coherent motion, not self consistent
  Ignore the fact WB is not stationary: look at "*short-term invariants*"
  Follows Venturini *et al.* [1] (but here there is no invariant KV distribution)
  Gives idea of particle motion, as a function of tune, intensity, driving term
  Here calculations summarised (details in [2])

- Distribution for 4D WB beam, with radius *a* is

$$n(r) = n_0(1 - \frac{r^2}{a^2}) \text{ for } r \leq a, \text{ zero otherwise) with } r^2 = x^2 + y^2$$

Consider motion in one plane only: y=0, r=x
(emittance of particle in orthogonal plane zero)

[1] Venturini & Gluckstern, PRSTAB V3 034203 (2000)

[2] Warsop, Proc. HB2016

3. Frozen Space Charge Model

Phase Averaged Frozen WB Hamiltonian (2)

- Smooth focusing Hamiltonian is:

$$H(x, P_x, s) = \tfrac{1}{2}P_x^2 + \tfrac{1}{2}\omega^2 x^2 + K_d(s)x^2 + V(x)$$

- Piecewise potential:

$$V(x) = \begin{cases} V_i = -k(\frac{x^2}{a^2} - \frac{x^4}{4a^4}), & \text{if } x \leq a \\ V_o = -k(\frac{3}{4} + \log\frac{|x|}{a}), & \text{if } x > a \end{cases}$$

where $k = \frac{q^2 N}{2\pi\epsilon_0 m c^2 \beta^2 \gamma^3}$

- Use action-angle variables: $x = \sqrt{\frac{2J}{\omega}}\sin\phi; \quad P_x = \sqrt{2J\omega}\cos\phi$

- And evaluate: $\overline{H(\phi, J, s)} = \frac{1}{2\pi}\int_0^{2\pi} H(\phi, J, s)d\phi$

- Assume system near half integer resonance:

$$2Q = l\theta; \quad \theta \equiv \theta(s) = \frac{s}{R}; \quad K_d(s) = k_l \cos l\theta$$

- Do integrals and a canonical transformation …

- Is complicated by piecewise potential of WB
- Use phase averaging to derive "smoothed" motion

q and m the particle charge and mass
N the number line density,
c the speed of light and β and γ relativistic parameters.

3. Frozen Space Charge Model

Phase Averaged Frozen WB Hamiltonian (3)

- The result is $$\overline{H} = \overline{H(J,\bar{\phi})} = \delta J + SJ\cos 2\bar{\phi} + \overline{V(J)}$$
- with $$\delta = (\omega - \frac{l}{2R}) = \frac{1}{R}(Q - \frac{l}{2}), \qquad S = \frac{k_l}{2\omega}$$

$$\overline{V(J)} = \begin{cases} V_c = I_0 & \text{if } J \le J_a \quad (core) \\ V_h = I_1 + I_2 & \text{if } J > J_a \quad (halo) \end{cases}$$

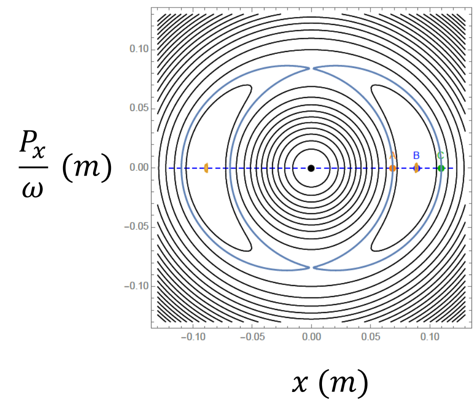
$$I_0 = k\left[-\frac{1}{a^2\omega}J + \frac{3}{8}\frac{1}{a^4\omega^2}J^2\right]$$

and with $\sin\phi_1 = 1/\sigma$ *and* $\sigma = x/a = \sqrt{2J/wa^2}$

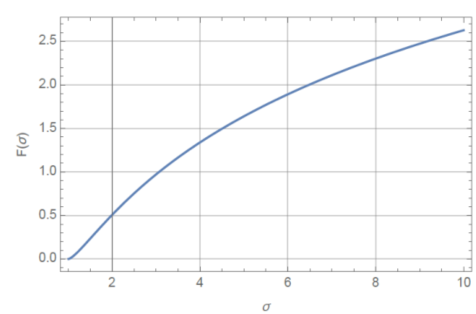
$$I_1 = -\frac{k}{a^2}\left[\frac{J}{\omega}(\phi_1 - \frac{1}{2}\sin 2\phi_1) - \frac{J^2}{w^2a^2}(\frac{3}{8}\phi_1 - \frac{1}{4}\sin 2\phi_1 + \frac{1}{32}\sin 4\phi_1)\right]$$

$$I_2 = -k\left[\frac{3}{4}(\frac{\pi}{2} - \phi_1) + F(\sigma)\right] \quad \textit{with} \quad F(\sigma) = \int_{\phi_1}^{\pi/2}\log(\sigma|\sin\phi|)d\phi,$$

$H(J,\phi)$



The Function F(σ)



3. Frozen Space Charge Model

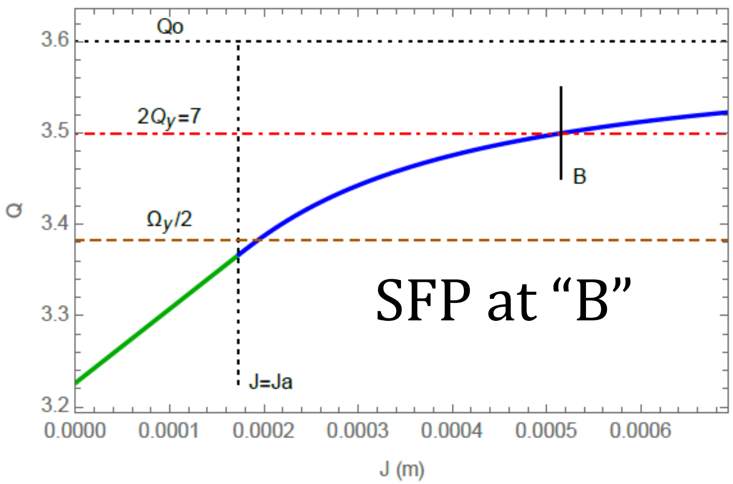
Essentials of Particle Motion

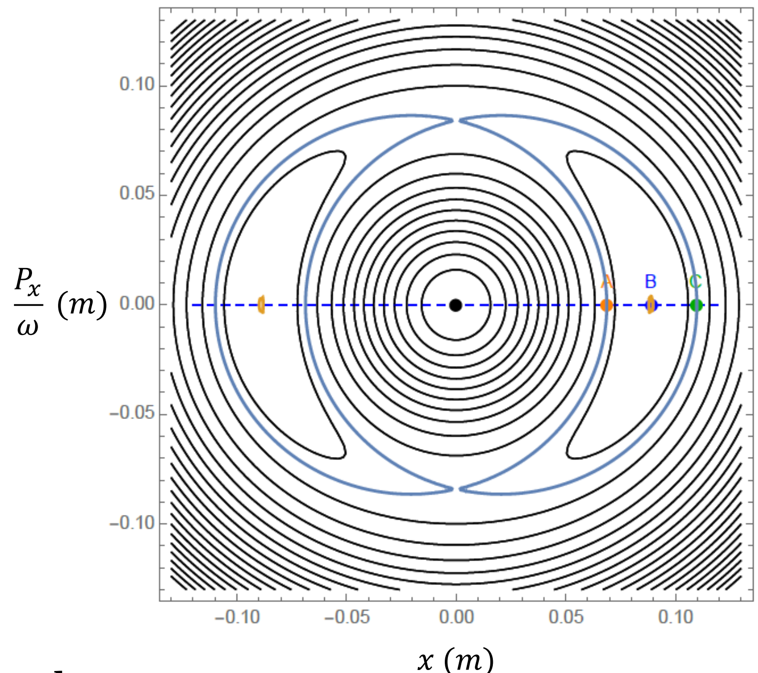
- Example of coasting beam on ISIS $2Q_y = 7$

Parameters are: $Q_y = 3.60$, $R = 26\,\mathrm{m}$, $a = 0.05\,\mathrm{m}$, $l = 7$, $\Delta k_7 = 0.005$ ($\Delta k_7 = k_7/k = 2k_7/\omega^2$), and $N_p = 4.4 \times 10^{13}$ ppp (protons per pulse, where $N = N_p/(2\pi R)$).

- Calculate *Q(J)* $w(J) = Q(J)/R = \frac{\partial H(J)}{\partial J}$.
- "Short term invariant" *H*

Q vs *J*



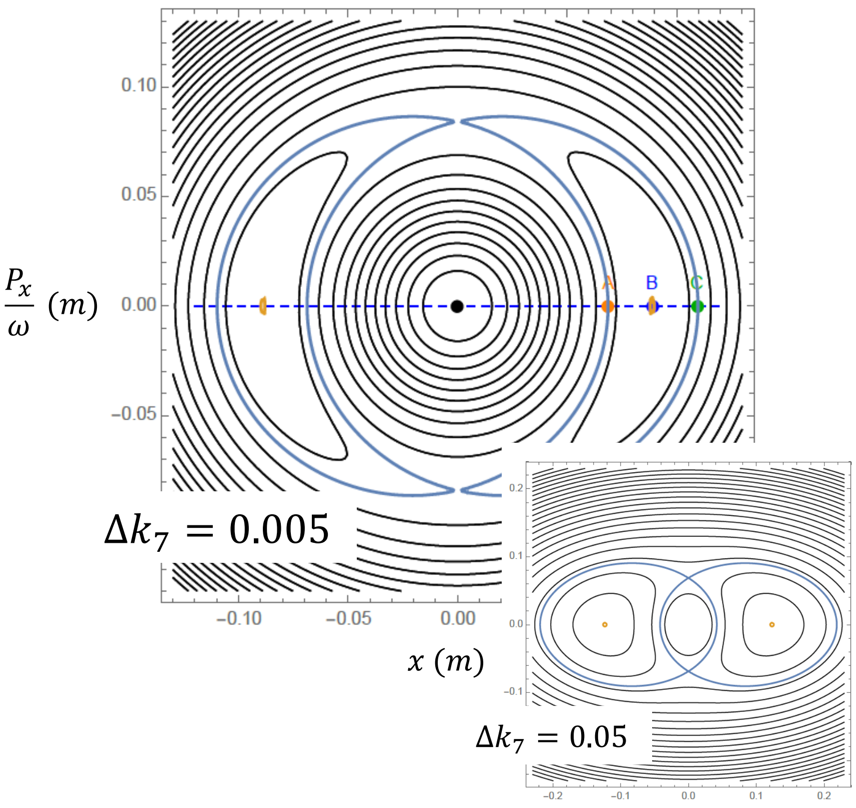


$\frac{\Omega_y}{2}$ = coherent frequency of KV equivalent beam

3. Frozen Space Charge Model

Predictions of model

- Driving strength
- *H* for increased driving strength



- Variation of SFP & island limits with driving strength

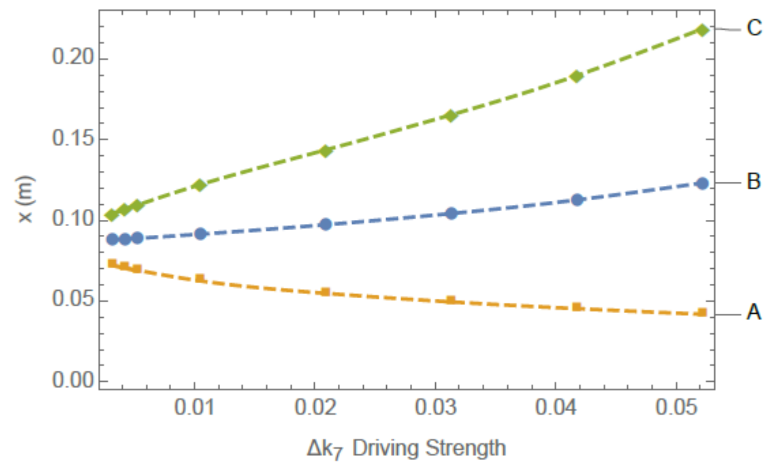


Figure 5: Dependance of island location and size on driving term strength.

$$N_p = 4.4 \times 10^{13} ppp$$

- Results confirmed with simple, direct 1D tracking

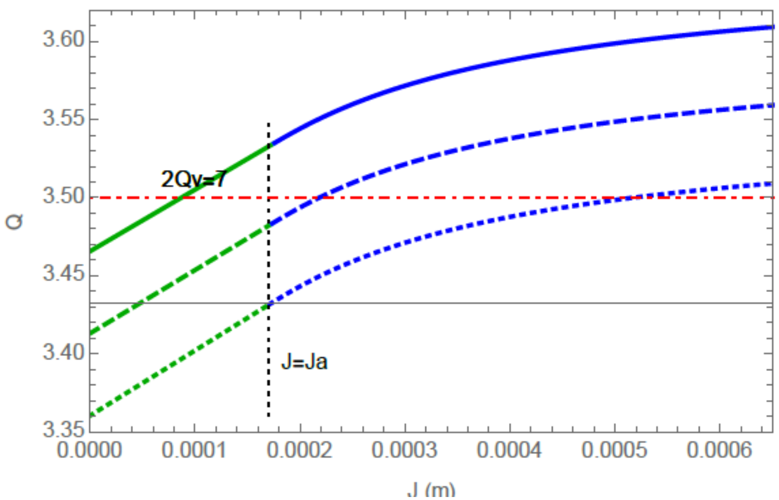
(*SFP=Stable Fixed Point*)

3. Frozen Space Charge Model

Predictions of model

- Variation of *Q(J)* with tune

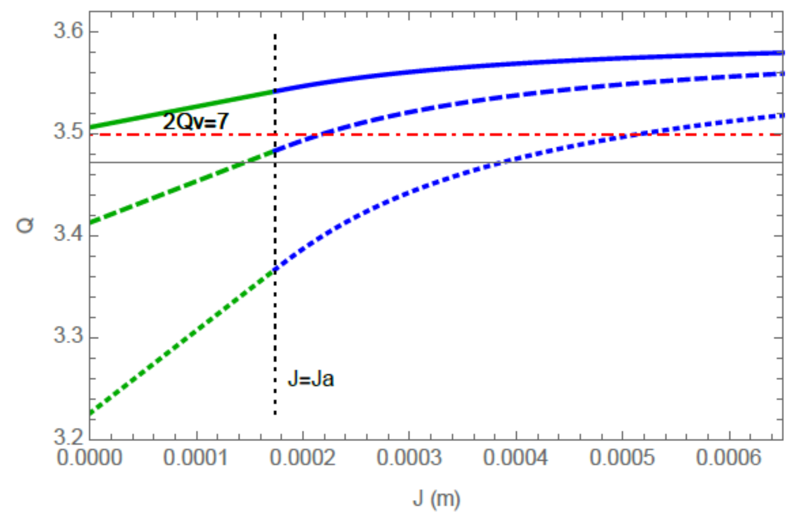
$$R = 26\,\mathrm{m},\ a = 0.05\,\mathrm{m},\ l = 7,\ N_p = 2.2\times 10^{13} ppp.$$



$$Q = 3.65,\ 3.60,\ 3.55$$
(top to bottom)

SFP moves out with decreasing Q

- Variation of *Q(J)* with intensity



$$N_p = 1.1,\ 2.2,\ 4.4\times 10^{13} ppp.$$
(top to bottom)

SFP moves out with increasing N_p

4. Comparison with Simulations

4. Self-consistent, 2D PIC code simulations, Set*

- See how theory compares to simulation

- Nominal ISIS like parameters:
  70 MeV, N_p=1.1-4.4E13 protons per pulse
  (Q_x, Q_y)=(4.31, 3.60)
  $\varepsilon_{rmsx} \approx \varepsilon_{rmsy} \approx 70\ \pi\ mm\ mr$, adjusted so a_x=a_y=0.05 m
  $2Q_y$=7 driving term, $\Delta k_7 = \frac{k_7}{k} = 0.01$
  Smooth focusing approximation
  100x100 binning of 5E5 macro particles
  WB distribution, ~ RMS matched, tracked 100 turns

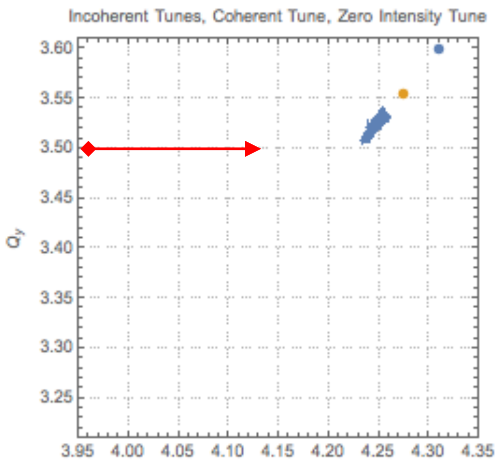
- Run a series of simulations as approach resonance
  No ramp of N_p or Q_y during run; repeat for different constant values
  Monitor trajectories of *test particles* inside and outside core

*Set code~ *thanks to Ben Pine*

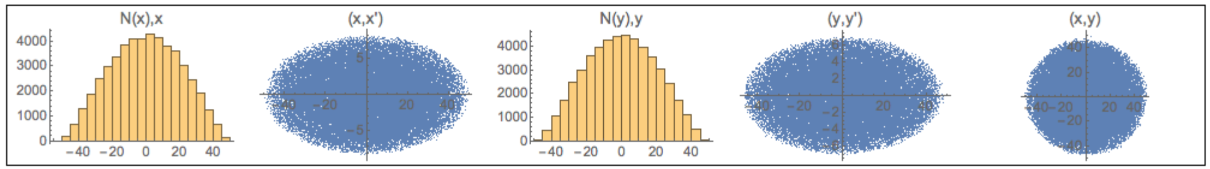
4. Comparison with Simulations

N_p=1.10E13 ppp, Q_y=3.60, $\Delta k_7 = 0.01$

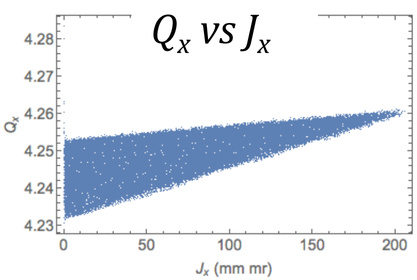
Tunes: incoherent, coherent

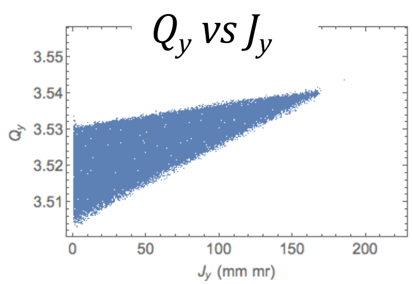


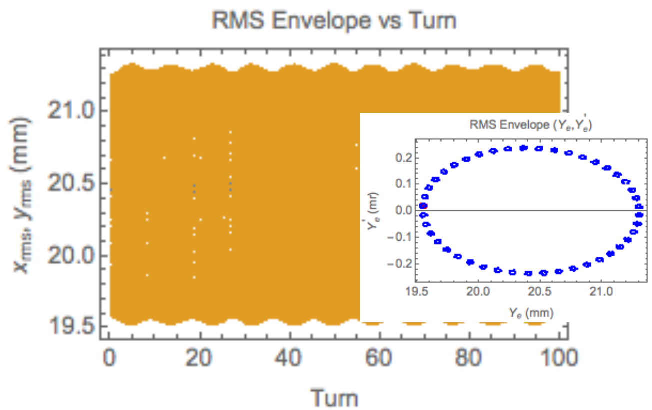
Distributions on 100th turn

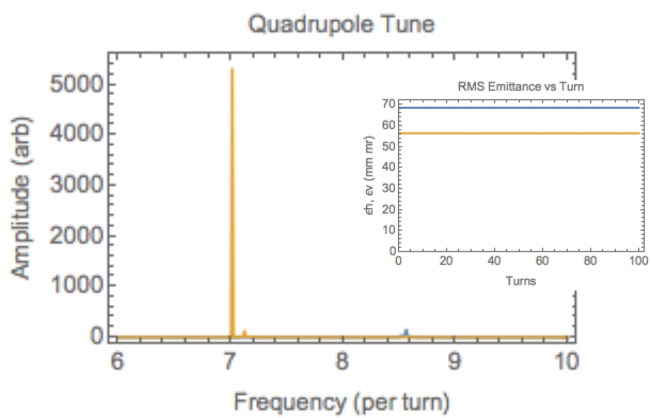


Tune vs initial action



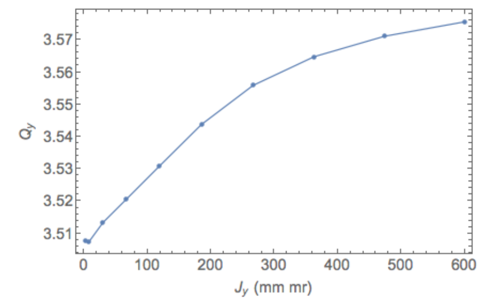


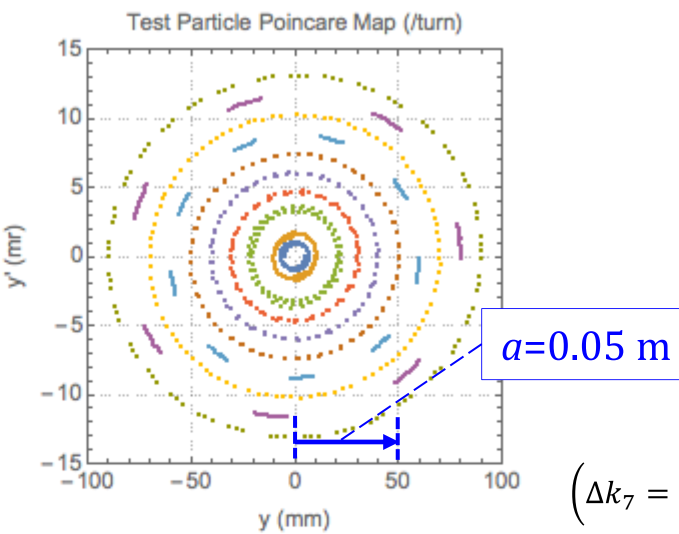




Test particles Q_y vs J_y

J_x=0, J_y=0-600 π μ m r

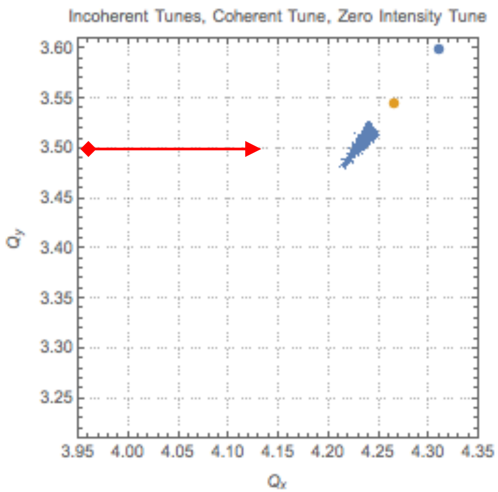


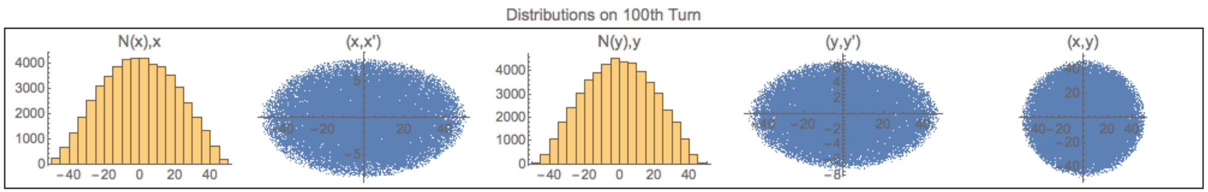


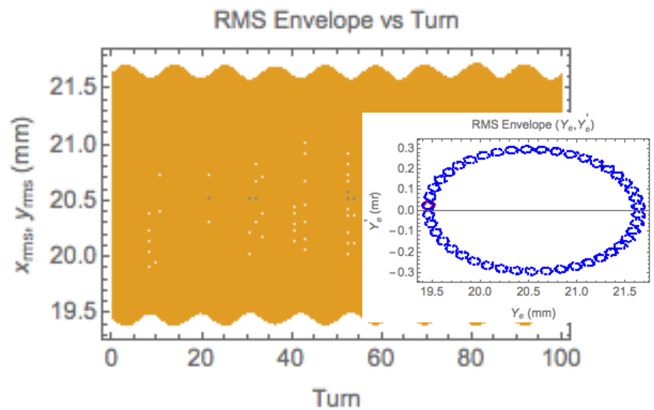
$\left(\Delta k_7 = \frac{k_7}{k}\right)$

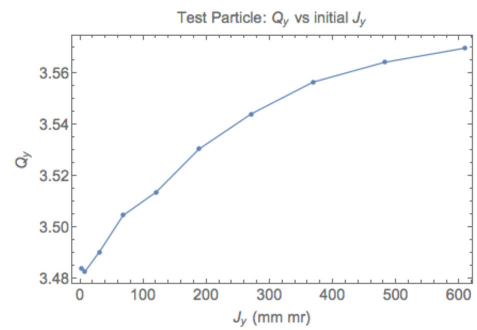
4. Comparison with Simulations

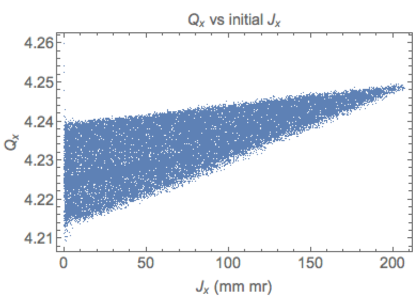
N_p=1.38E13 ppp, Q_y=3.60, $\Delta k_7 = 0.01$

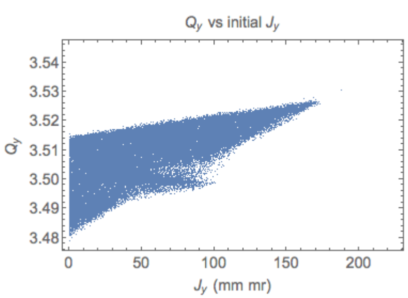


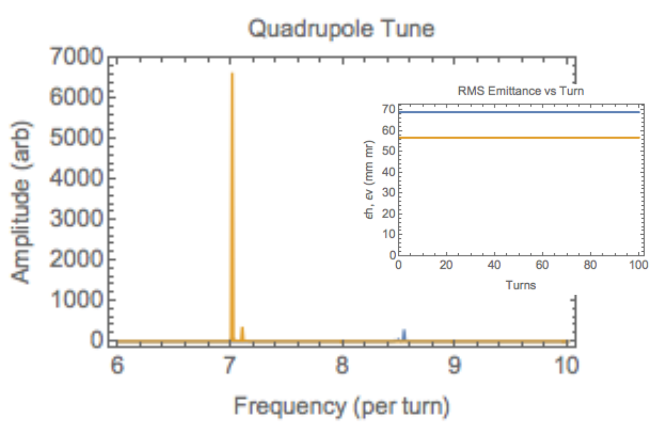


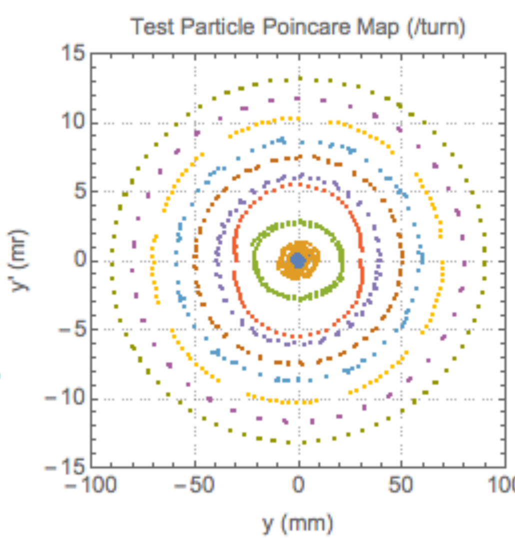






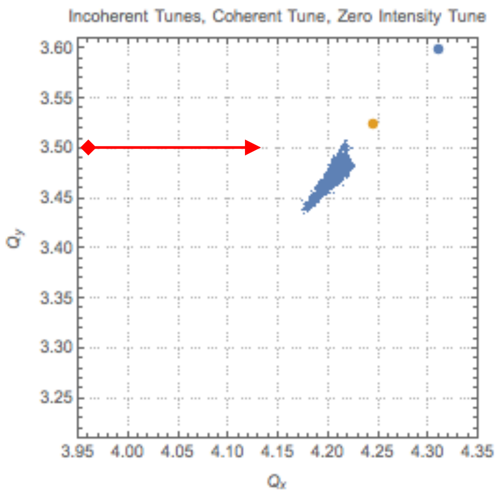


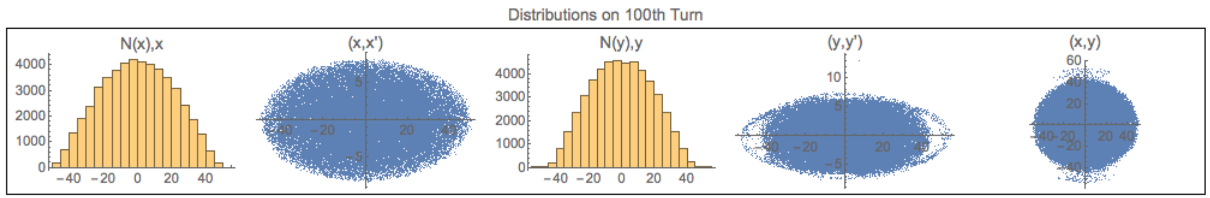


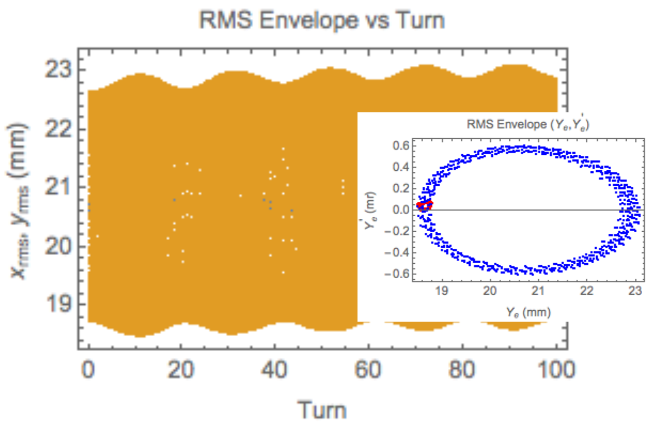


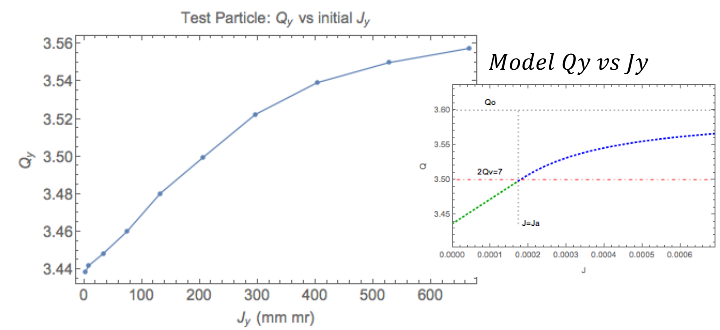
4. Comparison with Simulations

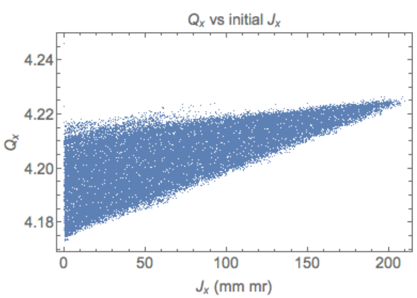
N_p=1.93E13 ppp, Q_y=3.60, $\Delta k_7 = 0.01$

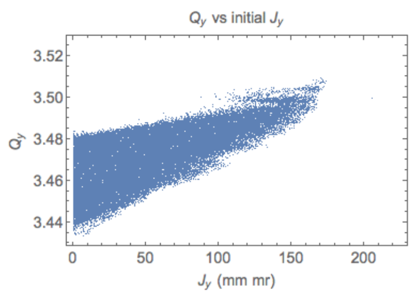


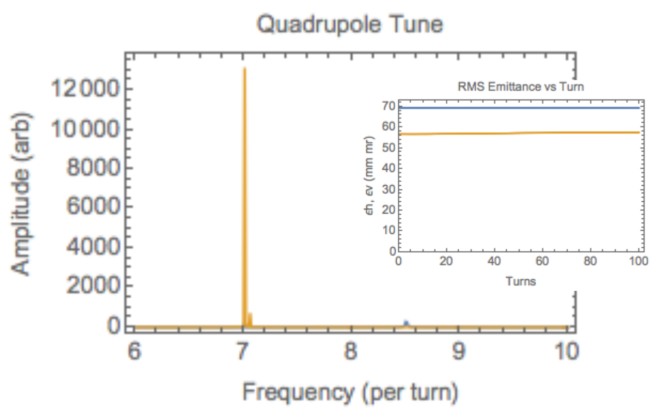


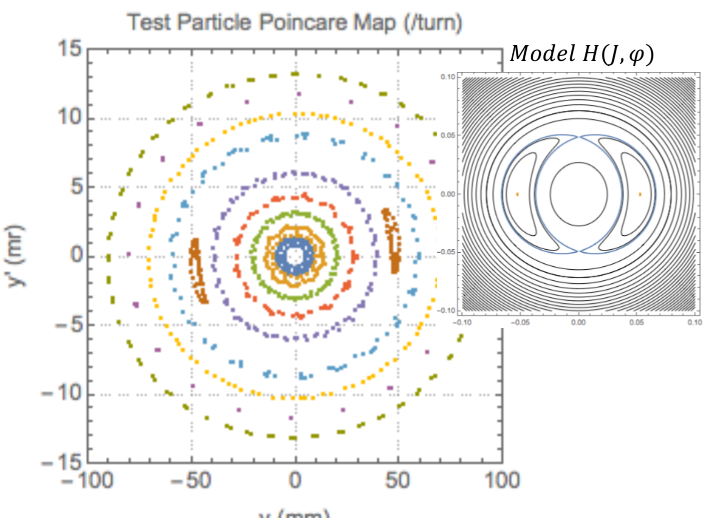












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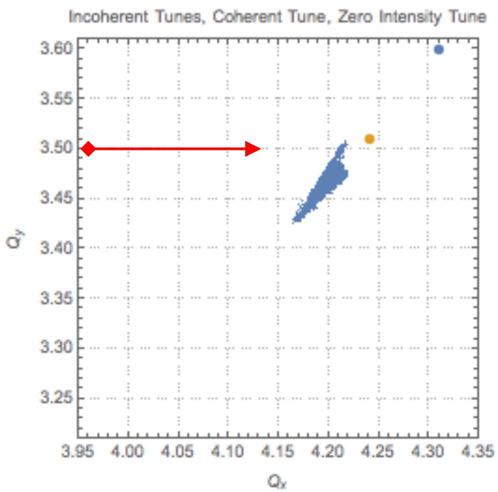
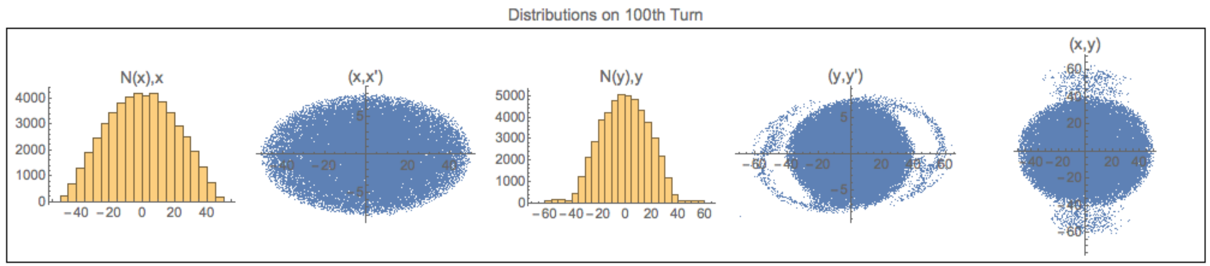
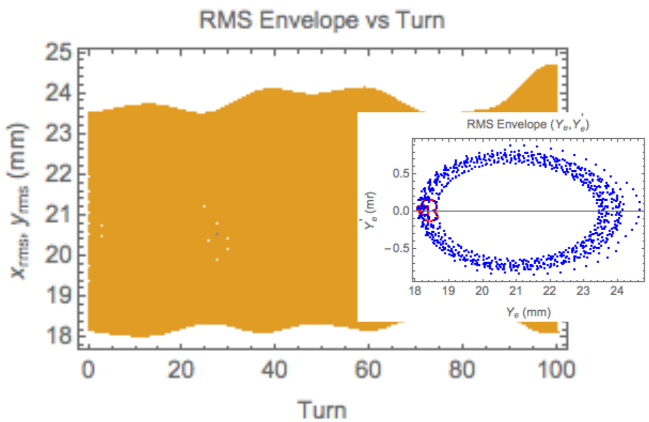
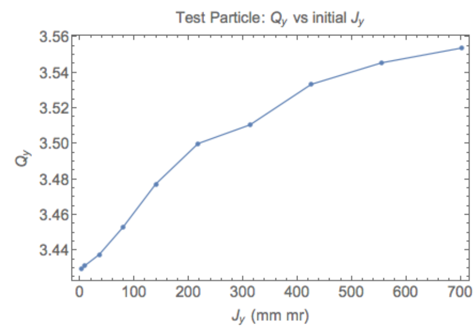
4. Comparison with Simulations

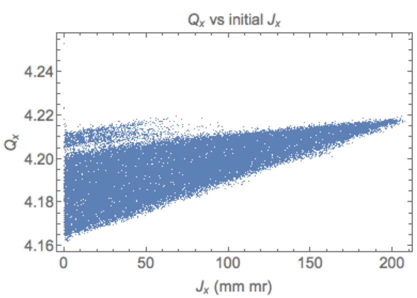
N_p=2.09E13 ppp, Q_y=3.60, $\Delta k_7 = 0.01$

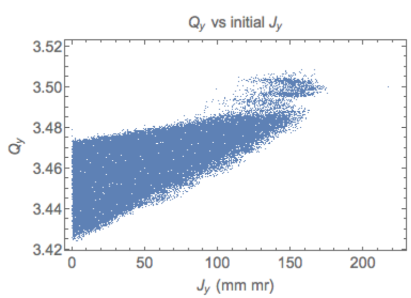


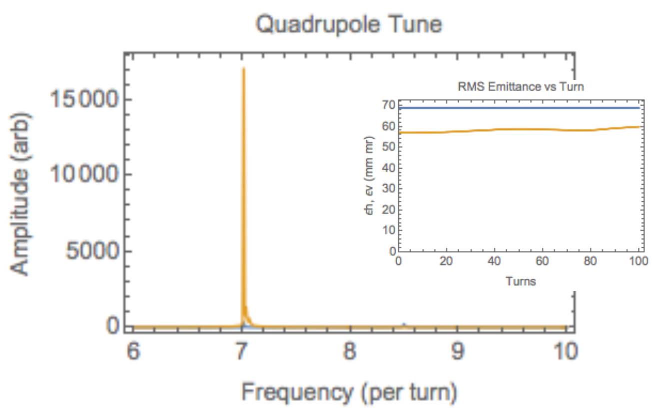


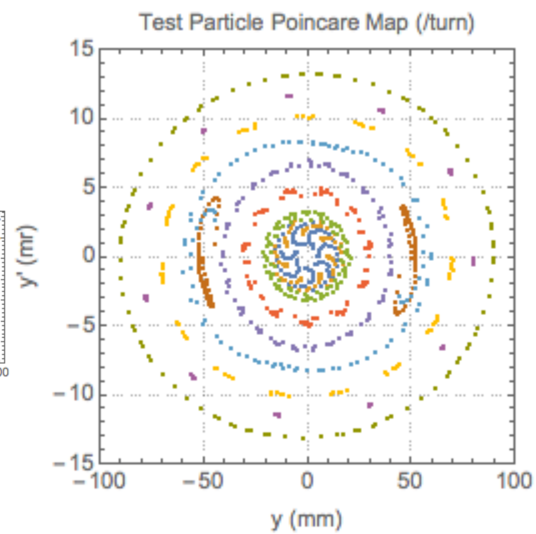






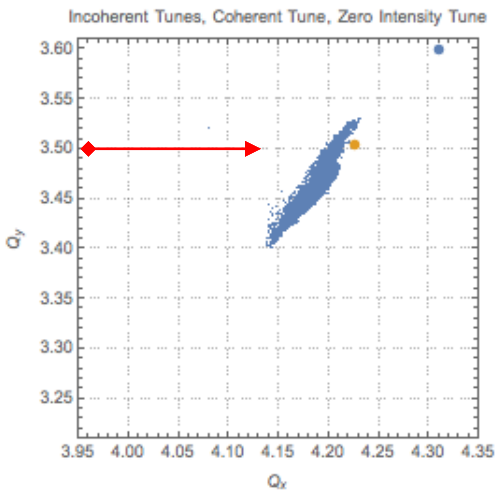


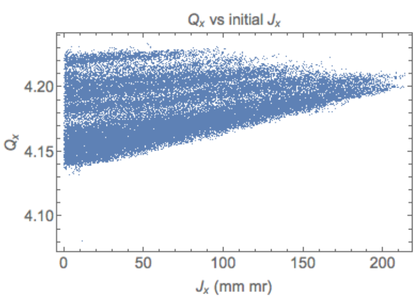


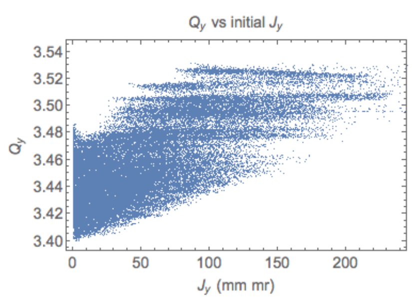


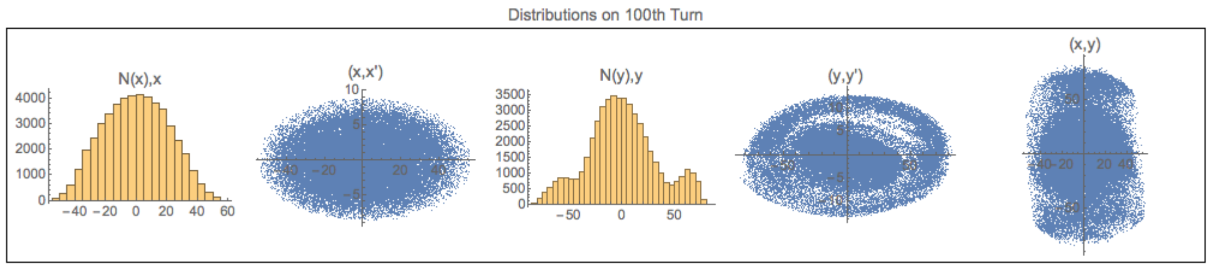
4. Comparison with Simulations

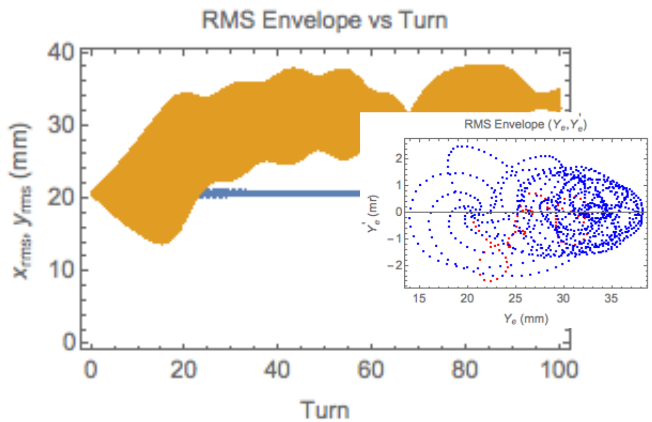
N_p=2.75E13 ppp, Q_y=3.60, $\Delta k_7 = 0.01$

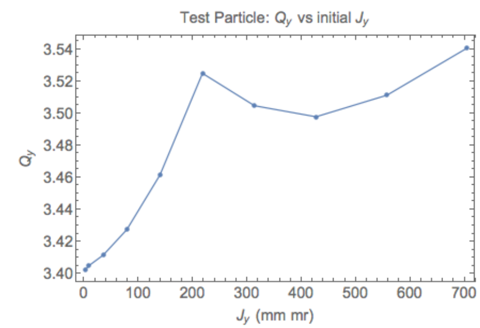


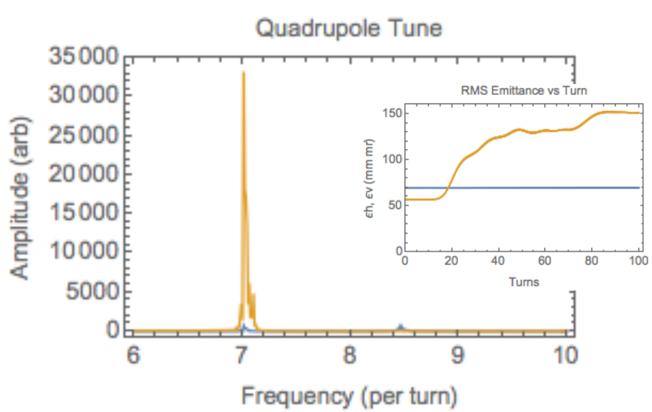


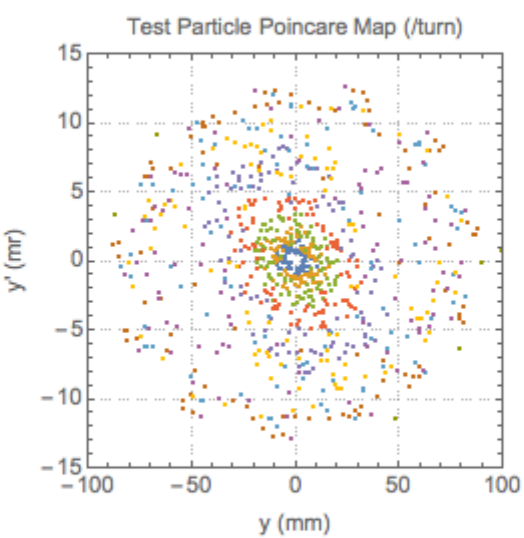






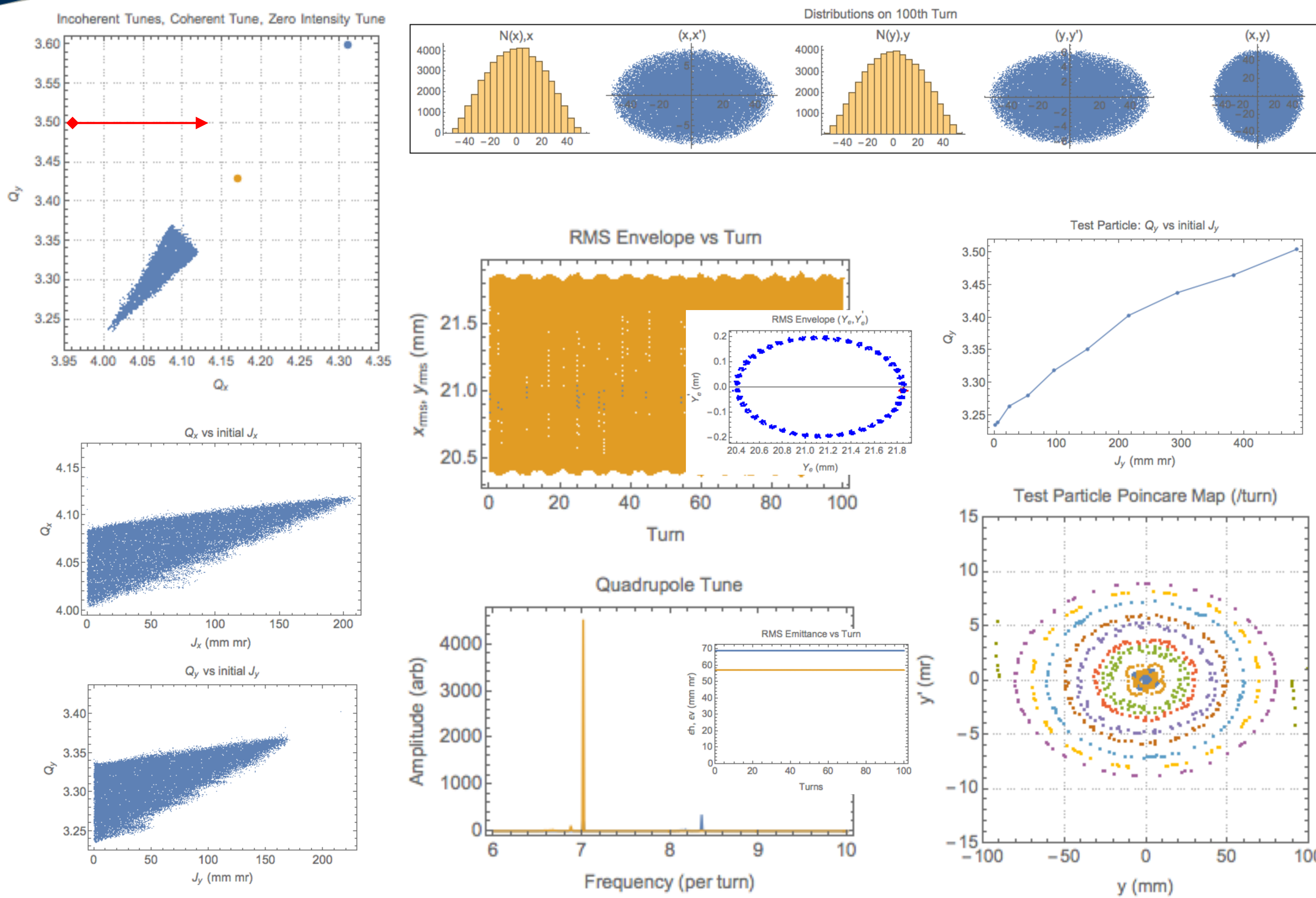






4. Comparison with Simulations

N_p=4.40E13 ppp, Q_y=3.60, $\Delta k_7 = 0.01$



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4. Comparison with Simulations

Summary of results and interpretation

- The model seems to qualitatively describe motion:
  - A "reasonable distance" from coherent resonance
  - At larger *J* ("halo"), *w*ith moderate envelope oscillations
  - SFP are roughly where expect (reduced by coherent motion?)
  - *Possible incoherent loss: resonant islands pulling particles from core?*
- Intuitively expect in a self consistent WB beam:
  - ~ central core: coherent cancellation of driving term (like KV)
  - ~ outer beam: less coherent effect, more incoherent behaviour
- The model is unhelpful at, or very near, coherent resonance
  - But once beam has redistributed, stabilised, it may be useful

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4. Ideas for a better model

Future ideas for next level of approximation a "less-frozen" model

- Effect of envelope oscillation [$a \to a(1 + \delta a \sin l\theta)$]
  Quadrupole field terms oppose driving term (cancel for KV)
  ~ varying "cancellation effect" with J (core→"halo")
  ~ explore effect for oscillation of RMS equivalent beam
  Modulation of octupole (and other) driving terms

- Effect of more realistic beam distributions
  Properties of different distributions (e.g. width, type)
  Time dependence: do quasi-static models predict evolution?

- Plus
  Motion in orthogonal plane; coupling and 2D effects …
  Other relevant coherent, incoherent resonances, instabilities (AG, 3D …)
  Identify mechanisms for ε_{rms} growth

5. Application to Measurements

Can the frozen model tell us anything about the experiments?

- Experiment is more complicated than the simulations above

  Machine parameters constant, but …

  Intensity ramped with multi-turn injection

  Beam distribution accumulates and evolves

- Contours of "short-term invariant" give useful guide

  Interpretation is work in progress, presently *a little speculative!*

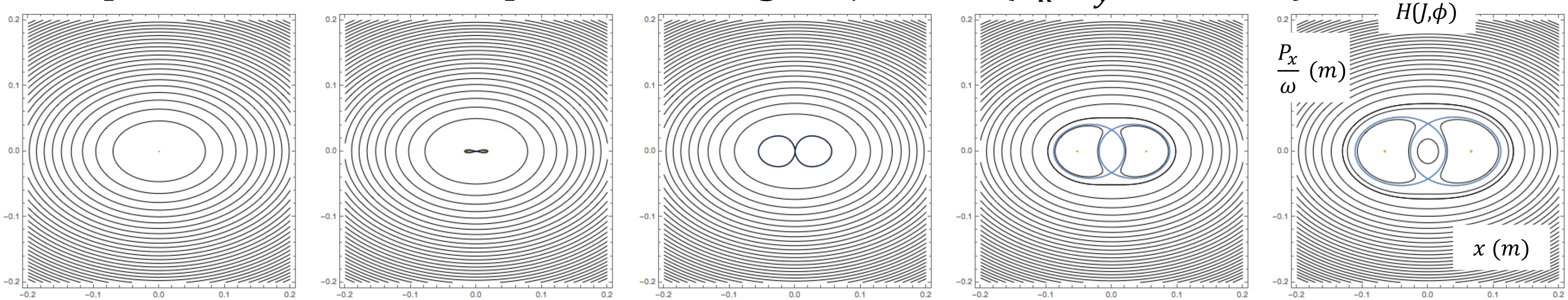
5. Application to Measurements

- Calculate "invariant" for experimental values

  Q_y=3.60, $\varepsilon_{rmsx} = \varepsilon_{rmsy}$ =20 π mm mr, $\Delta k_7 = 0.05$, a_x=a_y=28 mm

  For increasing intensity: N_p=0.10, 0.25, 0.50, 1.0, 1.5 E13 ppp

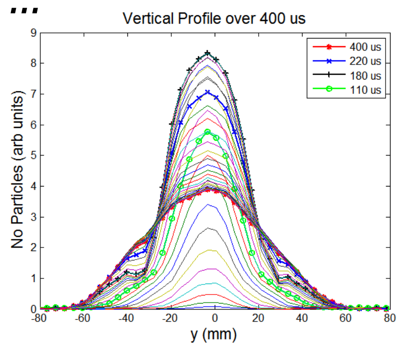
  Equivalent to time snapshots through injection (a_x, a_y constant)



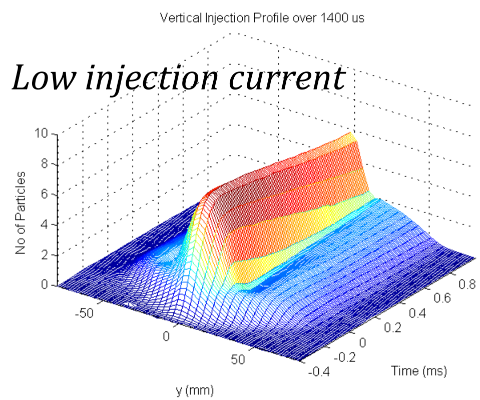
- *Helps* explain measured profiles …

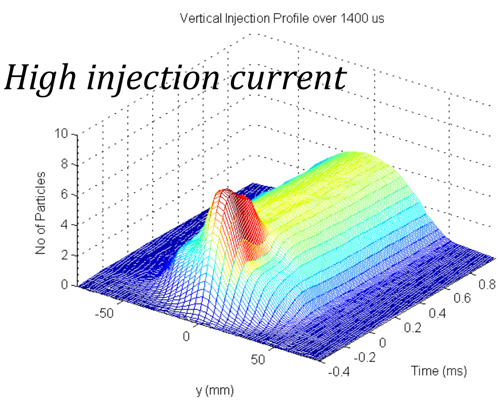
  Consistent with broadening, core and lobes:

Measured profiles …



Evolution of profile with time





Plus effects of coherent resonance, beam redistribution …

5. Application to Measurements

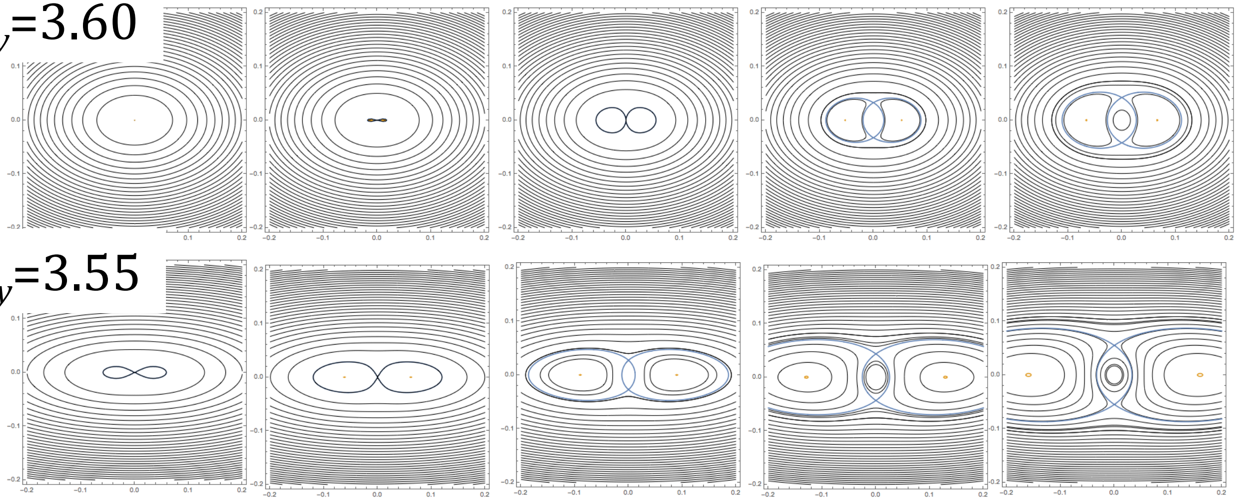
- Calculate "invariant" for experiment with Q_y=3.60, 3.55

  For intensities: N_p=0.10, 0.25, 0.50, 1.00, 1.50 E13 ppp

  Other parameters as above

Q_y=3.60

Q_y=3.55

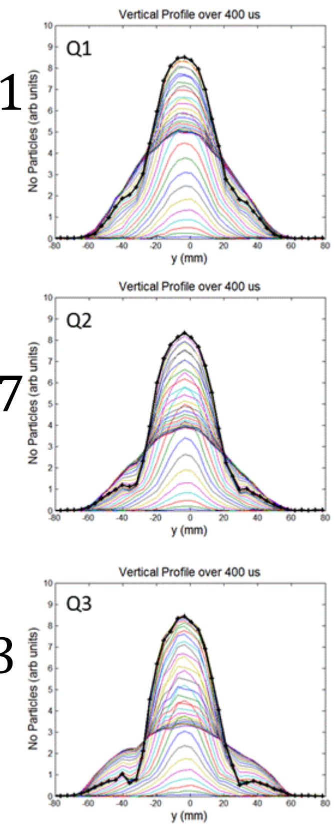


Measurements

Q_1=3.71

Q_2=3.67

Q_3=3.63



- *Helps* explain profile variation with *Q*

  Lobes move away from centre as *Q* drops

Plus effects of coherent resonance, beam redistribution …

6. Summary and Next Steps

- The frozen model is useful guide to help understand observations, but needs more development
- Next some simple approximations will be added to include *effects of coherent motion* and look at *variations with beam distribution*
- These ideas will be tested against simulation and observation
- Development of beam experiments continues
  - More detailed exploration of time dependence
  - Exploit improved understanding of profile monitors
  - Next look at bunched storage ring mode, then accelerated beams (RCS)
- Thus establish a fuller understanding of beam loss on ISIS …

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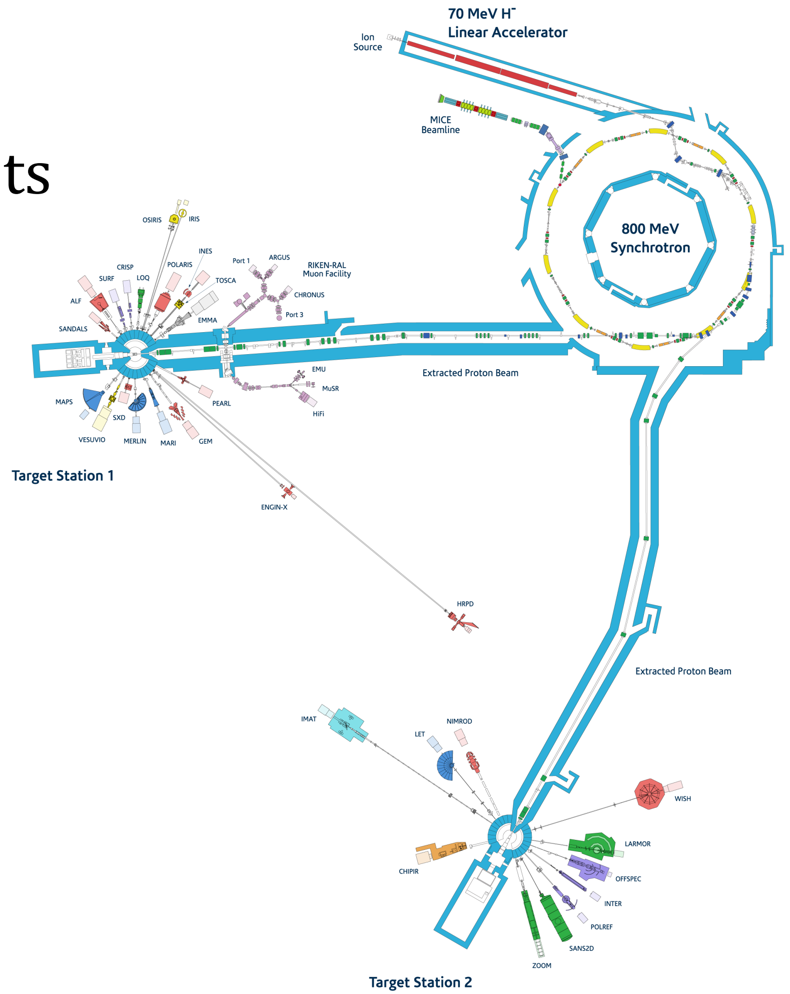
7. Acknowledgements

Many thanks to …

- ISIS Synchrotron Machine Physicists
- ISIS Diagnostics
- ISIS Operations Crew
- ISIS Intense Beams Group



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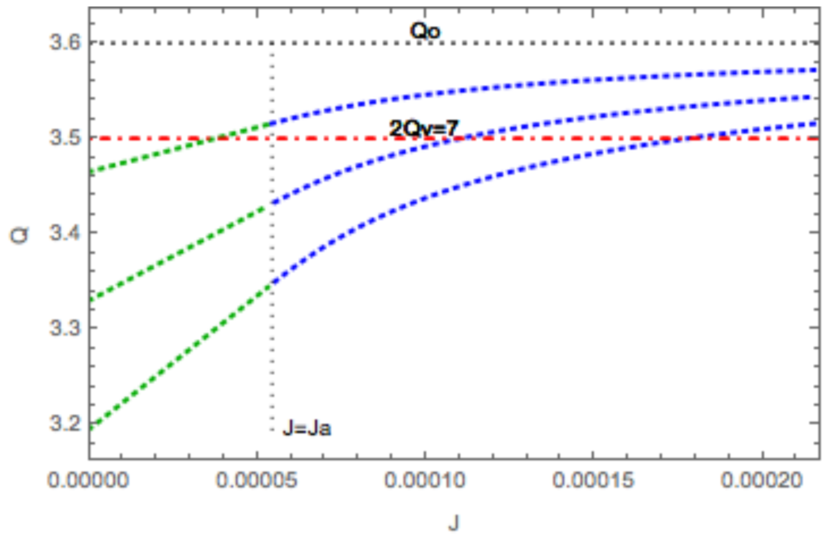
Additional Material

5a. Application to Measurements

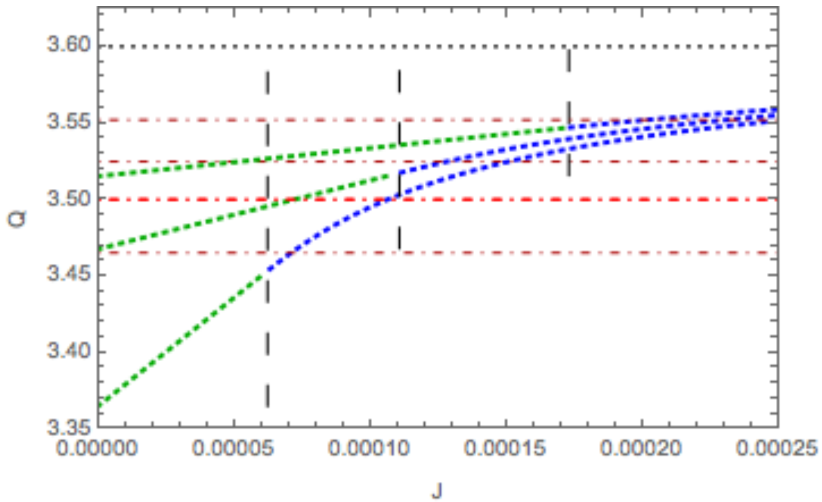
- Frozen model calculation of Q for the experiment (slide 24)
- Calculate Q vs J for the a=0.028 m beam at 0.5, 1.0, 1.5 E13 ppp



(*a=0.028,N_p=0.5,1.0,1.5E13 ppp, Q_y=3.60*)

5a. Application to Measurements

- Recalculating the frozen model for different beam sizes at the same intensity can also give an idea of how a growing beam might behave …
- From the model: *Q* vs *J* for same intensity but different beam size



(*a=0.03 m,0.04 m,0.05 m ,N_p=1E13 ppp,Q_y=3.6*)

- Would have a similar curve for most "sensible" distributions

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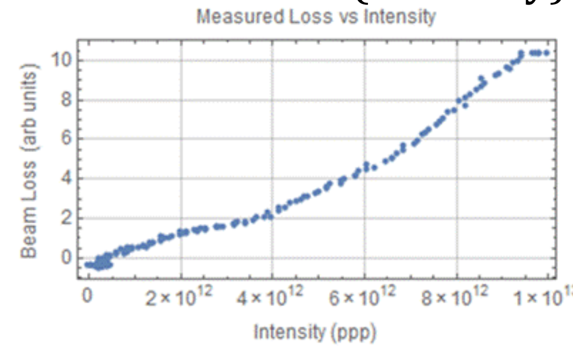
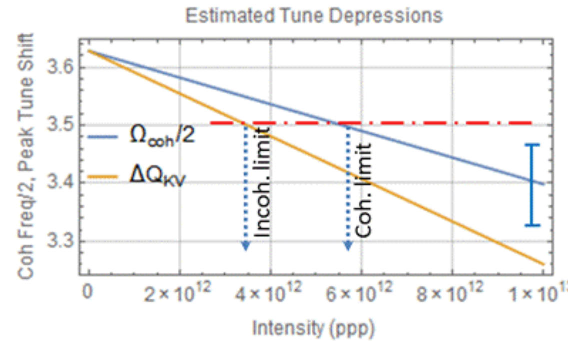
ISIS Space Charge R&D

Half integer resonance with space charge

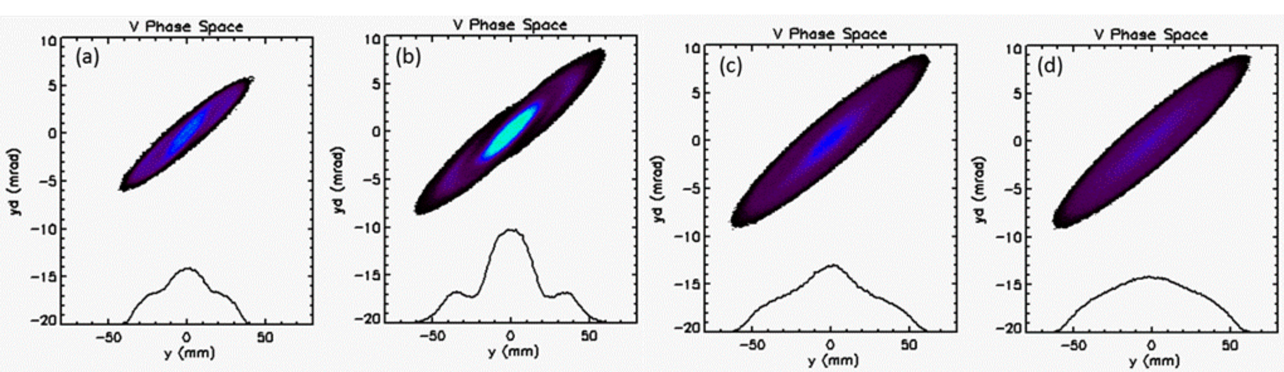
- Key loss mechanism
  - Can we understand, predict evolution of halo, loss?
- Experimental studies 2D coasting beam
  - RF off, DC field, inject small beam $\varepsilon_x = \varepsilon_y$
  - $\varepsilon_{rms} \approx 20\ \pi$ mm mr, $2Q_y$=7 driving term, Q_y=3.6
  - Ramp intensity (1E13 ppp), push onto resonance
- Study evolution of profile
  - Observations agree with ORBIT models
  - Clear formation of core and lobes

Loss & Tune vs time (intensity)



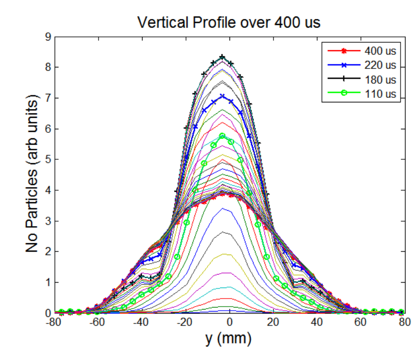


ORBIT results



Transverse profile

Measured over 400 μs



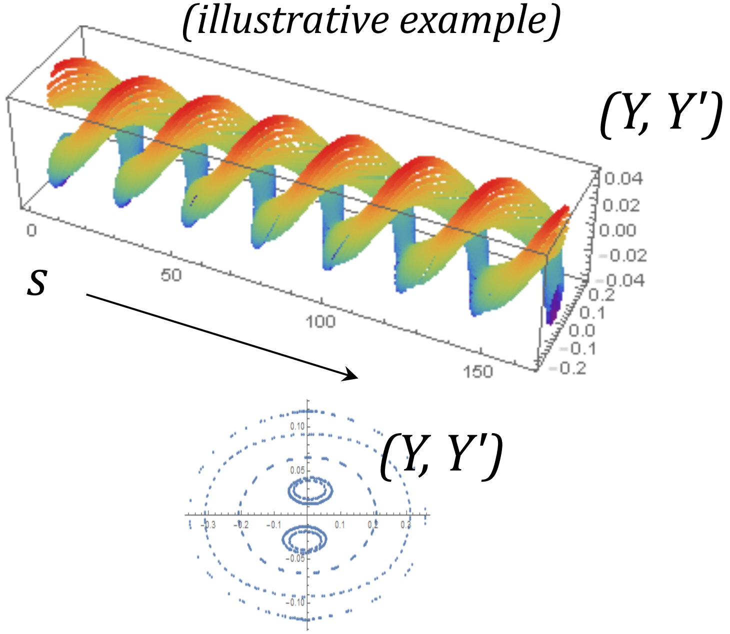
ISIS Space Charge R&D

Half integer resonance

- Previous work: agreement measurement-simulation
- Rotation of half integer "lobes"

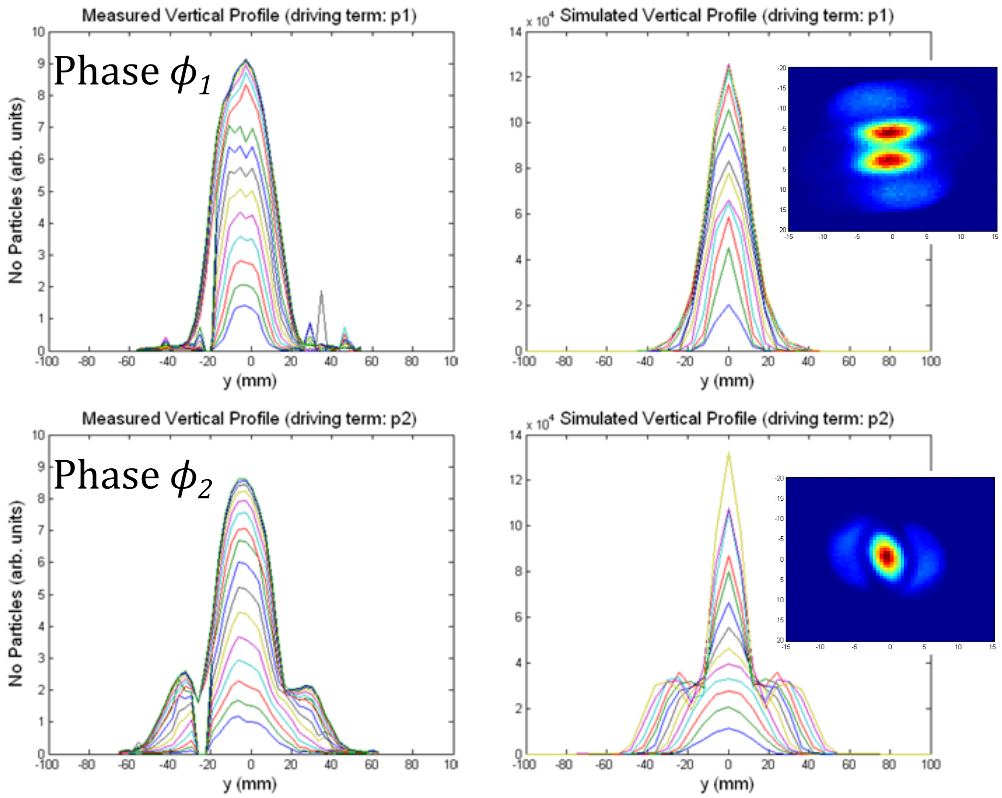
Control with driving term
$\Delta k(\theta)=k_0 \cos(2Q_y\,\theta+\phi)$

Expected motion around ring
at half integer resonance
(illustrative example)



Dependence on driving term phase

Measured ORBIT



Half integer resonance

- Recent work: agreement measurement-simulation
- Measure as a function of tune and driving term

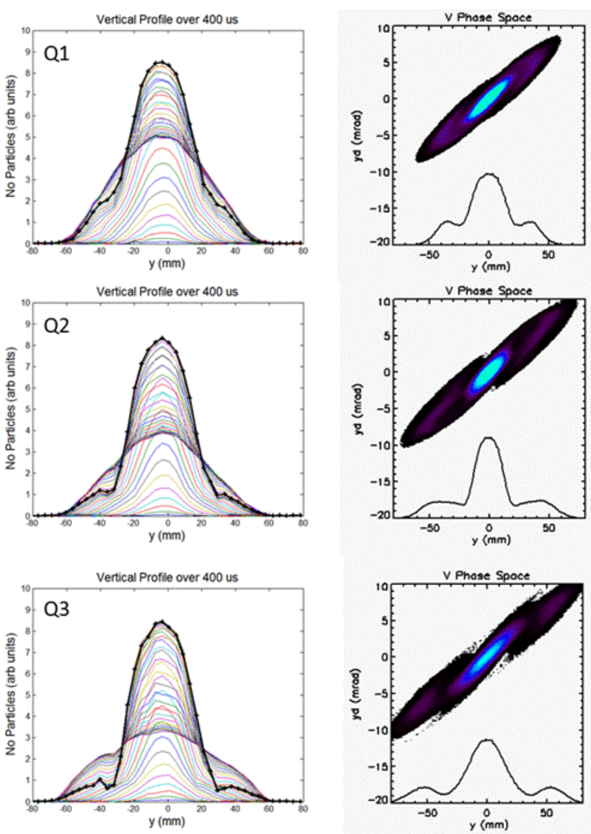
Dependence on tune

Measured ORBIT

Q_1=3.71

Q_2=3.67

Q_3=3.63



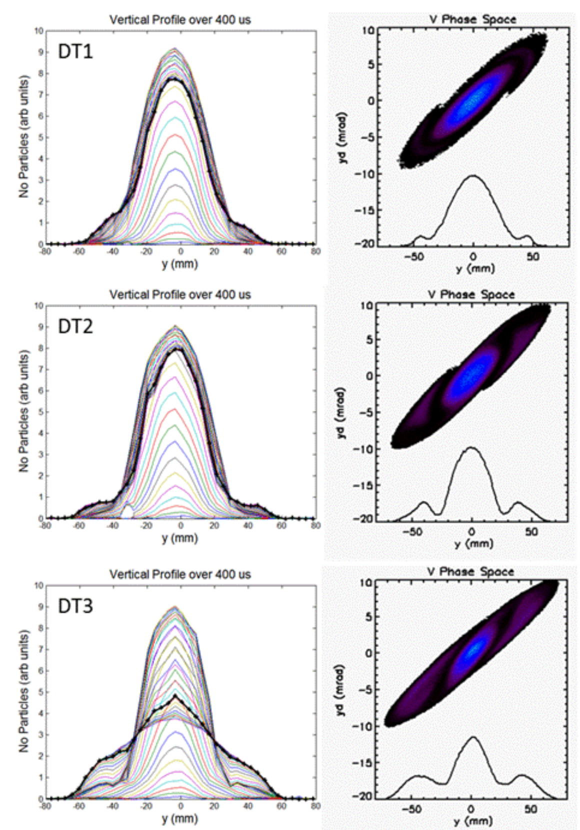
Dependence on driving term

Measured ORBIT

DT_1=0.02

DT_2=0.03

DT_3=0.06



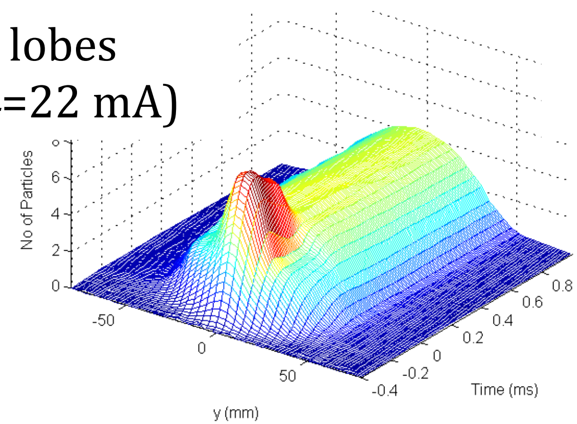
ISIS Space Charge R&D

Half integer resonance

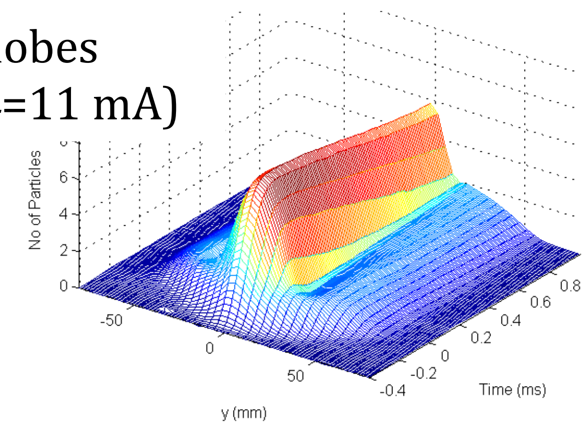
- Recent work: Observation of "stationary" distributions
- Slower accumulation of beam – formation of stable "lobes"

Measured transverse profiles over 1 ms

Short lived lobes
~50 turns (I_{inj}=22 mA)

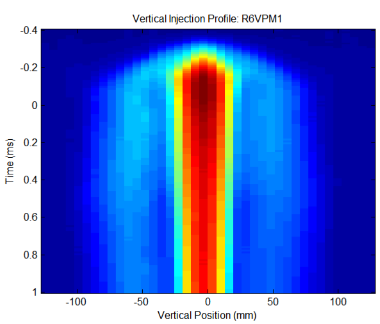


Long lived lobes
~500 turns (I_{inj}=11 mA)

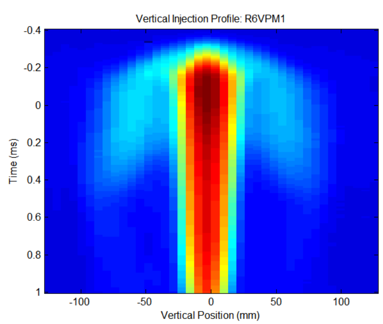


Initial experiments on stable halo (profiles now shown as colour contour)

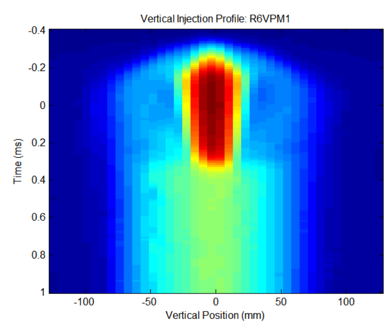
(i) Constant (as above)



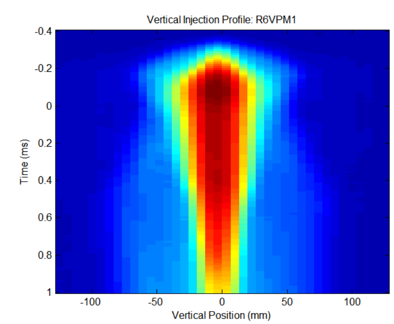
(ii) Ramp Q down



(iii) Ramp Q down/up



(iv) Rotate phase

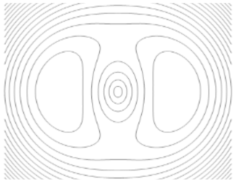


ISIS Space Charge R&D

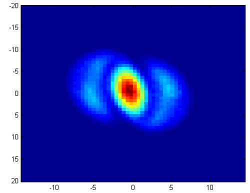
Speculation & work in progress!

- Models to explain observations?
  - Coherent model limited: coherent limit
  - Approach from incoherent direction?
- Simplest 1D single particle model

$H(J,\varphi) = \delta J + G_2 J \cos(2\varphi) + G_4 J^2$ “Observation”

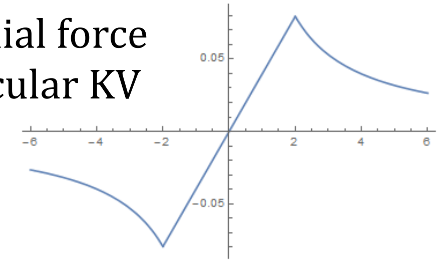


(Y, Y′)

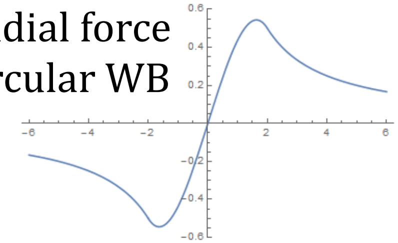


- Here have space charge potential
  - 1st guess usually KV model
  - Linear motion: cannot describe growth
  - 2nd guess WB model (*non-stationary*)
  - Non-linear motion: predict halo?

Radial force circular KV

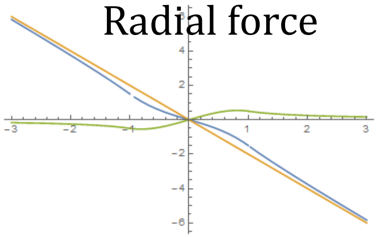


Radial force circular WB

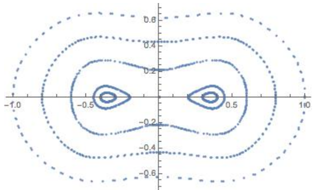


Half integer resonance

- Total radial force
  - Focussing + space charge



(Y, Y′)



Rough example!

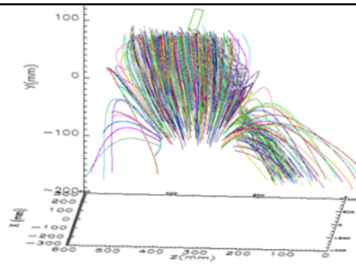
- Simple simulations
  - Driving term, fixed potential
  - Non-linear motion ~ edge of core
  - Complicated ~ *still studying …*
  - “Incoherent” model of halo?
- Next add coherent motion?
  - RMS envelope → modify halo
  - “Coherent” model of halo?
- *May* be a useful idea …
  - Different KV-WB coherent motion?

ISIS Space Charge R&D

R&D for transverse profile measurements

- Good transverse profile measurements essential
  - Detailed models of ISIS residual gas ionisation monitors
  - CST fields solvers and "in-house" code tracks ion trajectories
  - Allow for non-linearities and space charge.
- Recent results checking halo measurements
  - Input distributions predicted by ORBIT
  - Check behaviour as function of drift field and intensity

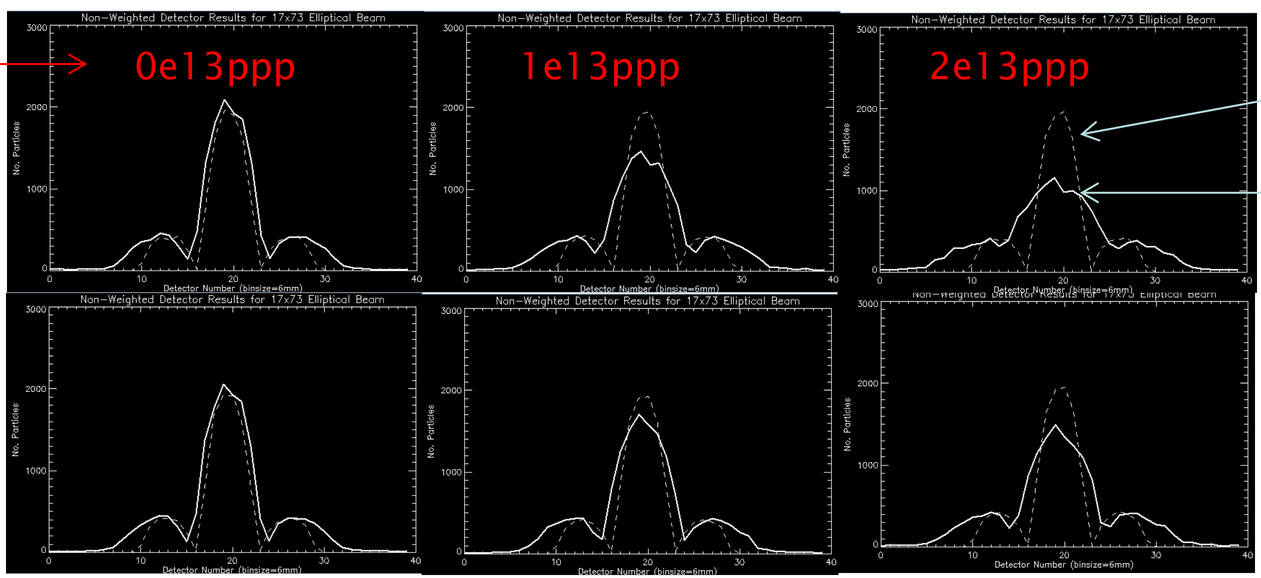
Simulation of Ion tracks



Intensity

Drift field 15 kV

Drift field 30 kV



Input profile

Predicted measured profile

C C Wilcox, R E Williamson, S J Payne, C M Warsop, et al