High Order Image Terms and Harmonic Closed Orbits at the ISIS Synchrotron

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With thanks to Dr Chris Warsop, ISIS Synchrotron Group and Operations Team

ISIS, Rutherford Appleton Laboratory, UK
Science and Technology Facilities Council

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Origin of the idea

- At ISIS, 3rd, 4th and 5th harmonics of the closed orbit are corrected at highest intensities
- Rees and Prior [1]suggested this due to higher order image terms driven by the closed orbit
- Term they looked at quadrupole term proportional to square of closed orbit
- Suggested vertical closed orbit could excite horizontal resonance

[1]GH Rees and CR Prior, Image Effects on Crossing an Integer Resonance, Particle Accelerators 1995, Vol 48.

Origin of the idea (2)

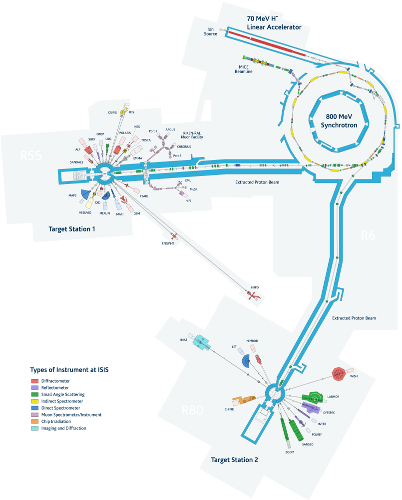
- Idea examined again by Baartman [2]
- Expanded Laslett's image term calculation to include more terms
- Suggested term Rees and Prior had looked at was an envelope resonance

[2] R Baartman, Betatron Resonances with Space Charge, Proc. Workshop on High Intensity Hadron Rings, 1998

Overview

- ISIS facility
- Image terms from pencil beams in parallel plate geometry
- Numerical results from round beams in rectangular geometry
- Resonance theory for high order image terms
- Simulation results for beams with harmonic closed orbits

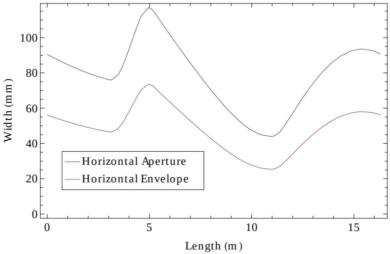
ISIS



- ISIS is the spallation neutron source at RAL
- 50 Hz 800 MeV RCS
- H^- injection at 70 MeV over ~ 200 turns
- High intensity, up to 3×10^{13} ppp accelerated
- Beam loss is the main limit to intensity
- Beam loss is controlled at low energy on collimators

Beam loss

- Beam loss takes many forms e.g. longitudinal/transverse, injection/extraction
- My work focuses on losses resulting from transverse space charge
- In particular from image forces
- ISIS has a unique conformal vacuum vessel, which follows the profile of the design beam envelopes
- Limits the range over which the tunes can be changed
- Makes image forces much more complicated



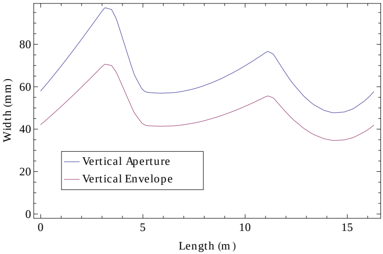


Image terms: Laslett and Baartman

Solution for a potential of pencil beam between parallel plate boundary

$$U = -\frac{\lambda}{2\pi\varepsilon_0} \ln \left| \frac{\sin(\pi y/2h) - \sin(\pi \bar{y}/2h)}{1 + \cos(\pi(y + \bar{y})/2h))} \right| .$$

This leads to the usual Laslett coefficients of $\epsilon_1 = \frac{\pi^2}{48}$, and $\xi_1 = \frac{\pi^2}{16}$

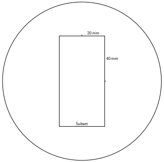
$$E_y \simeq -\frac{\lambda}{\pi\varepsilon_0 h^2} \left(\epsilon_1 \hat{y} + \xi_1 \bar{y} \right) .$$

But if you expand the answer to obtain more terms:

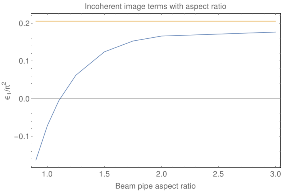
$$\frac{E_{yimage}}{4\lambda} = \frac{1}{4\pi\varepsilon_0} \left(\epsilon_1 \frac{\hat{y}}{h^2} + \xi_1 \frac{\bar{y}}{h^2} + \kappa_{30} \frac{\bar{y}^3}{h^4} + \kappa_{21} \frac{\hat{y}\bar{y}^2}{h^4} + \kappa_{12} \frac{\hat{y}^2\bar{y}}{h^4} + \kappa_{03} \frac{\hat{y}^3}{h^4} + \ldots \right)$$

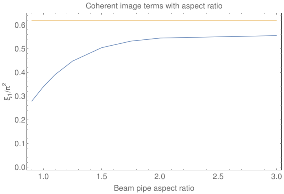
This expression was taken as starting point for numerical work on two dimensional round beams in rectangular apertures.

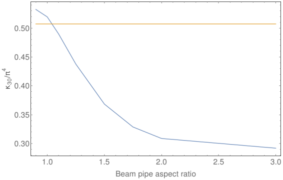
Numerical results - off-centred beams



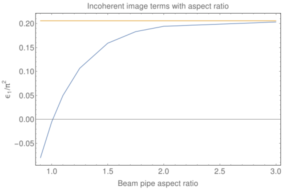
Whole beam results

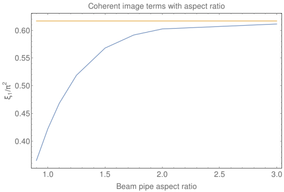


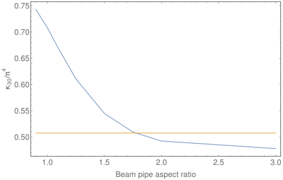




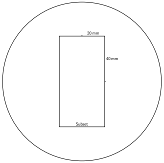
Section of beam results



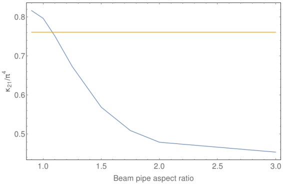


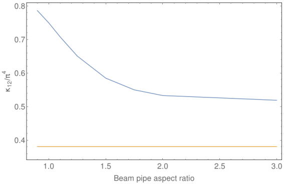


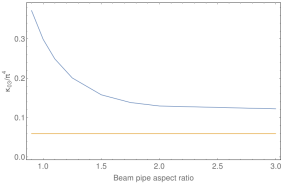
Numerical results - off-centred beams



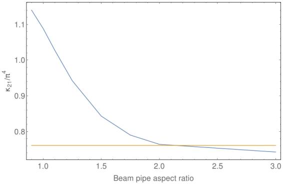
Whole beam results

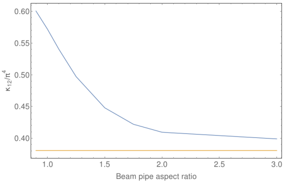


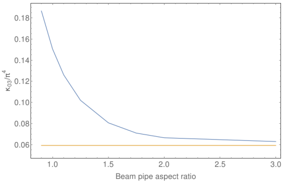




Section of beam results







Closed orbits at low and high intensity

Starting with the equation for transverse betatron motion

$$\frac{d^2y}{dt^2} + (Q\Omega)^2 y = \frac{F_y}{\gamma m_0}.$$

Changing to the longitudinal coordinate from time and expressing F_y as the Lorentz Force Law, $F_y = e\left[\bar{E} + \bar{v} \times \bar{B}\right]_y$,

$$\frac{d^2y}{ds^2} + \left(\frac{Q}{\rho_0}\right)^2 y = \frac{1}{B_0 \rho_0 v_s}\left[\bar{E} + \bar{v} \times \bar{B}\right]_y.$$

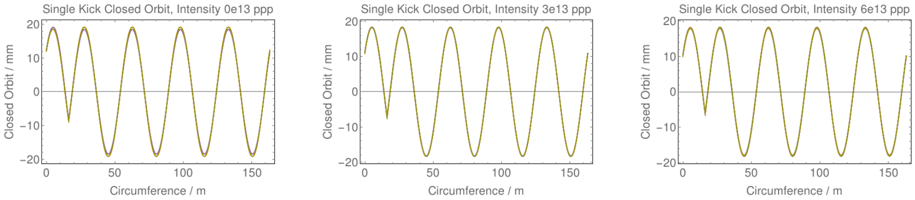
At low intensity the E term is zero. Considering just dipole errors, with a single kick ΔB

$$\frac{d^2y}{ds^2} + \left(\frac{Q}{\rho_0}\right)^2 y = \frac{\Delta B}{B_0 \rho_0}.$$

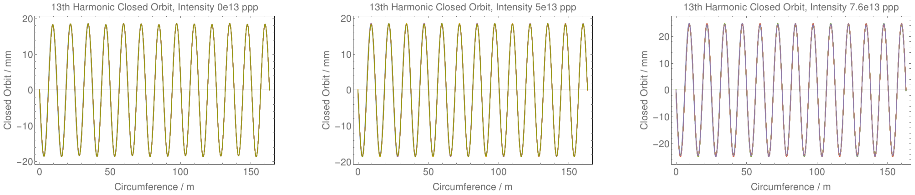
This equation is solved by the particular integral

$$y = \left(\frac{\beta_s}{2|\sin \pi Q|}\frac{\Delta B}{B_0 \rho_0}\right)\cos Q\theta$$

Closed orbits at low and high intensity



Simulations were with a single angular kick at the beginning of the second superperiod.



Simulations were with a distributed angular kick to produce a 13$^{\text{th}}$ harmonic closed orbit.

Image driving terms

The equation of motion with respect to the established closed orbit is

$$y'' + ky = F_D + F_I$$

The one dimensional single particle Hamiltonian can be written

$$H(y, P_y, s) = \frac{1}{2}P_y^2 + \frac{1}{2}ky^2 + V_D + V_I$$

Writing the indirect space charge forces in Baartman's general form

$$V_I = \frac{1}{\gamma m_0 \beta^2 c^2}\frac{\lambda}{\pi\varepsilon_0}\left[\epsilon_1\frac{y^2}{2h^2} + \xi_1\frac{y\bar{y}}{h^2} + \kappa_{30}\frac{y\bar{y}^3}{h^4} + \right.$$
$$\left. \kappa_{21}\frac{y^2\bar{y}^2}{2h^4} + \kappa_{12}\frac{y^3\bar{y}}{3h^4} + \kappa_{03}\frac{y^4}{4h^4} + \dots\right]$$

Hamiltonian including κ_{12}

κ_{12} image term inserted into Hamiltonian as $T_{12} y^3 \bar{y}$ where T_{12} is constant absorbing κ_{12} and other constant terms. Changing y to action-angle variables and substituting for $\bar{y}$:

$$H = \omega J + T_{12} \left(\frac{2J}{\omega} \right)^{\frac{3}{2}} \sin^3 \phi \; a_n \cos n\theta + V_0(J)$$

$V_0(J)$ is non-linear term in J due to direct space charge and other image terms.

$$H = \omega J + a_n T_{12} \left(\frac{2J}{\omega} \right)^{\frac{3}{2}} \left(\frac{3}{8} \left(\sin(\phi - n\theta) + \sin(\phi + n\theta) \right) \right.$$
$$\left. - \frac{1}{8} \left(\sin(3\phi - n\theta) + \sin(3\phi + n\theta) \right) \right) + V_0(J)$$

κ_{12} term has resonances at $Q = n$ and $3Q = n$.

Hamiltonian including κ_{21}

κ_{21} image term inserted as $T_{21} y \bar{y}^2$ where T_{21} is constant absorbing κ_{21} and other constant terms. Changing y to action-angle variables and substituting for $\bar{y}$:

$$H = \omega J + T_{21} \frac{2J}{\omega} \sin^2 \phi \; a_n^2 \cos^2 n\theta + V_0(J)$$

$$H = \omega J + T_{21} a_n^2 \frac{J}{2\omega} (1 + \cos 2n\theta - \cos 2\phi - \cos 2\phi \cos 2n\theta) + V_0(J)$$

Of terms in bracket, 1st is tune shift and 2nd and 3rd average to zero. Last term is dominant.

$$\cos 2\phi \cos 2n\theta = \frac{1}{2} (\cos(2\phi - 2n\theta) + \cos(2\phi + 2n\theta))$$

Resonance due to κ_{21} term at $2Q = 2n$.

Coherent response of the beam

- Single particle analysis incomplete
- Beam responds coherently to excitation
- Resonant frequencies are modified
- Behaviour can be described with an altered resonance formula

$$\omega = nQ_H + mQ_V + \Delta\omega = N$$

- In simulations to follow resonances were located by scanning intensity and so shifting the coherent frequencies

Selected simulation results

Overview

- Self-consistent PIC simulations
- Smooth focusing or alternating gradient
- Simulations run for 100 turns
- RMS orbit and envelope matched beam distributions
- Coherent moments, single particle tunes, phase space distributions and losses recorded
- Dipole driving terms applied singly or harmonically

Selected simulation results

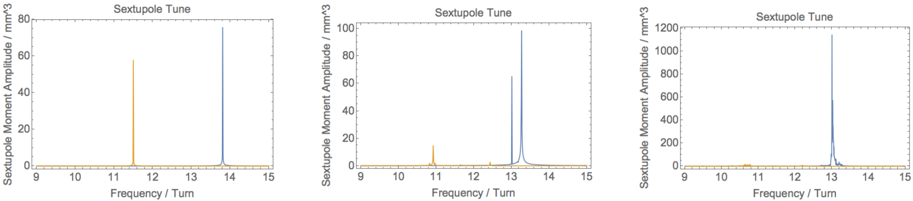
Overview

For each higher order term that was investigated the same procedure was followed

1. Start with a zero intensity beam
2. Move the tune of the lattice to a suitable point
3. Kick the beam to create the required closed orbit harmonic
4. Match the new closed orbit
5. Increase the beam intensity until the coherent beam frequency interacted with the image driving term
6. With each step in intensity, re-match the RMS orbit and envelope

Selected simulation results

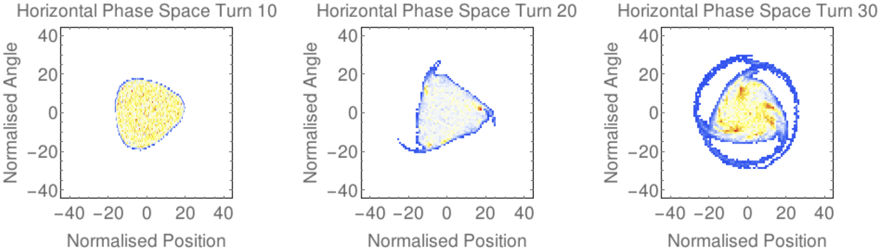
KV beam, smooth focusing lattice, κ_{12} image term, driven with 13th harmonic closed orbit



Quadrupole moment spectra with closed orbit excited by single kick driving κ_{21} image term at 9th harmonic, at intensities of 0, 3 and 5.5×10^{13} ppp. Blue: horizontal, yellow: vertical.

Selected simulation results

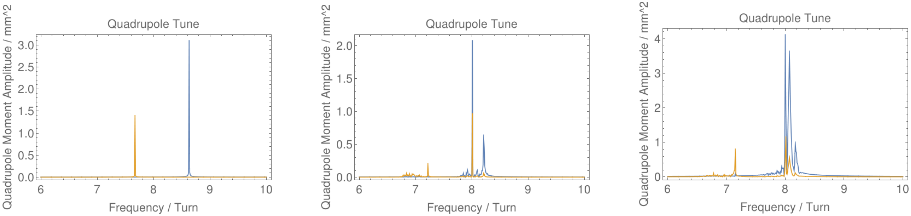
KV beam, smooth focusing lattice, κ_{12} image term, driven with 13$^{\text{th}}$ harmonic closed orbit



Horizontal phase space on turns 50, 60 and 70 for beam excited by single kick driving κ_{21} image term at 9$^{\text{th}}$ harmonic, at intensity of 5.5×10^{13} ppp.

Selected simulation results

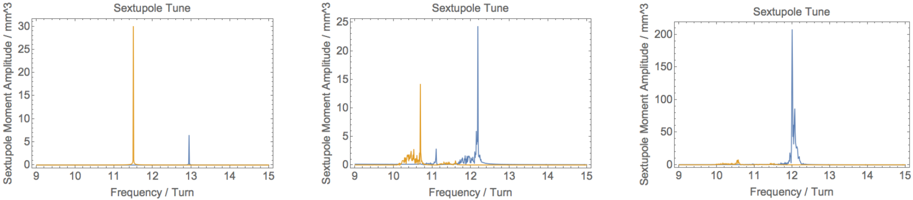
WB beam, smooth focusing lattice, investigation of effects of 4th harmonic closed orbit at ISIS nominal tunes



Quadrupole moment spectra with 4th harmonic closed orbit driving κ_{21} image term at 8th harmonic, at intensities of 0, 5 and 6×10^{13} ppp. Blue: horizontal, yellow: vertical.

Selected simulation results

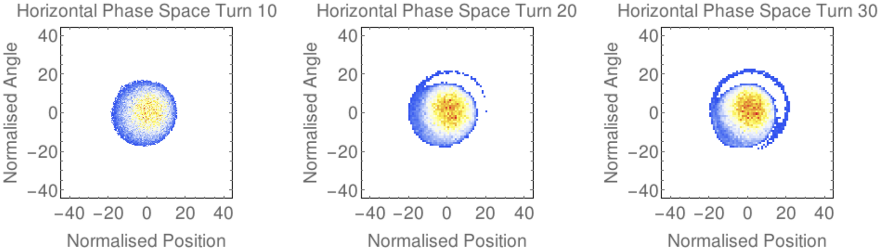
WB beam, smooth focusing lattice, investigation of effects of 4^{th} harmonic closed orbit at ISIS nominal tunes



Sextupole moment spectra with 4^{th} harmonic closed orbit driving κ_{30} image term at 12^{th} harmonic, at intensities of 0, 5 and 6×10^{13} ppp. Blue: horizontal, yellow: vertical.

Selected simulation results

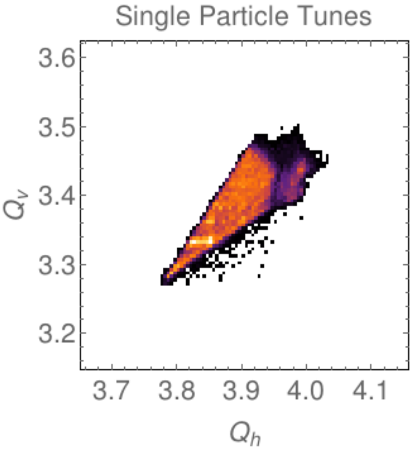
WB beam, smooth focusing lattice, investigation of effects of 4th harmonic closed orbit at ISIS nominal tunes



Horizontal phase space showing integer resonance for waterbag beam driven with 4th harmonic closed orbit at intensity of 6×10^{13} ppp, on turns 10, 20 and 30.

Selected simulation results

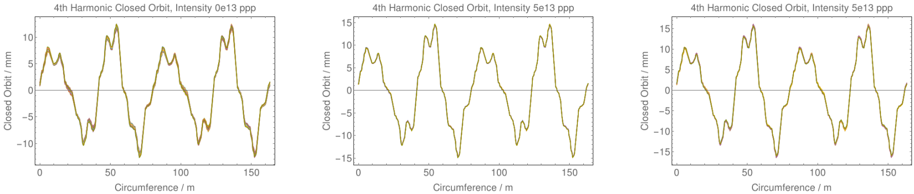
WB beam, smooth focusing lattice, investigation of effects of 4^{th} harmonic closed orbit at ISIS nominal tunes



Single particle tune footprint for ISIS nominal tunes and 4^{th} harmonic closed orbit, at intensity of 6×10^{13} ppp, with a waterbag beam.

Selected simulation results

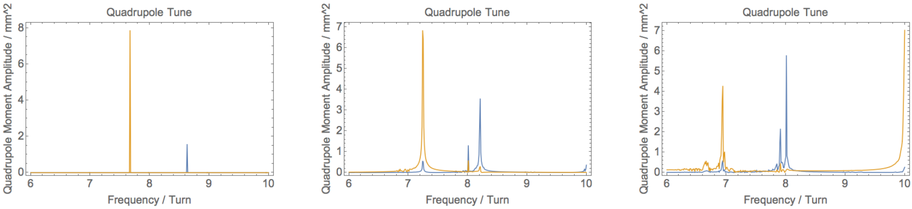
WB beam, AG lattice, conformal vacuum vessel, investigation of effects of 4th harmonic closed orbit at ISIS nominal tunes



Closed orbit plotted at intensities of 0, 5 and 7×10^{13} ppp, overplotted in each case every 10 turns. Simulations with distributed angular kick to produce 4th harmonic closed orbit.

Selected simulation results

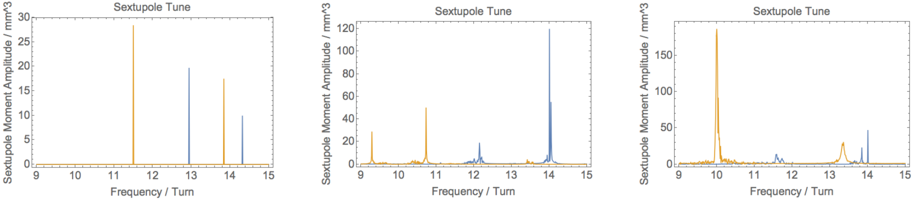
WB beam, AG lattice, conformal vacuum vessel, investigation of effects of 4^{th} harmonic closed orbit at ISIS nominal tunes



Quadrupole moment spectra with 4^{th} harmonic closed orbit driving κ_{21} image term at 8^{th} harmonic, at intensities of 0, 5 and 9×10^{13} ppp. Blue: horizontal, yellow: vertical.

Selected simulation results

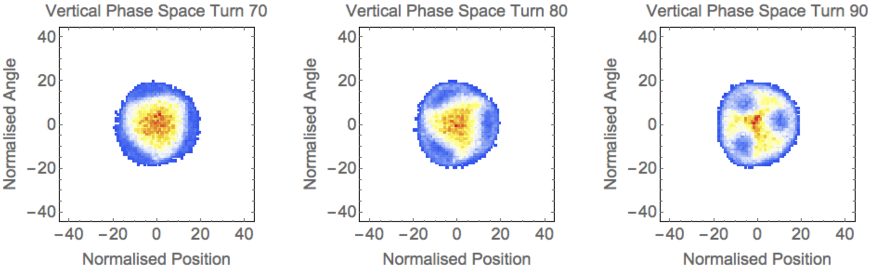
WB beam, AG lattice, conformal vacuum vessel, investigation of effects of 4^{th} harmonic closed orbit at ISIS nominal tunes



Sextupole moment spectra with 4^{th} harmonic closed orbit at intensities of 0, 5 and 9×10^{13} ppp. Blue: horizontal, yellow: vertical.

Selected simulation results

WB beam, AG lattice, conformal vacuum vessel, investigation of effects of 4th harmonic closed orbit at ISIS nominal tunes



Vertical phase space showing sextupole resonance for waterbag beam driven with 4th harmonic closed orbit at intensity of 9×10^{13} ppp, on turns 70, 80 and 90.

Thank you for your attention

Any questions?

Experimental results with low intensity stored beams - NOT IMAGES