Modeling chiral criticality and its consequences for heavy-ion collisions

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HIC for FAIR
Helmholtz International Center

Exploring the phase diagram

I. Heavy-ion collisions

- Experimental data on the freeze-out line

II. Lattice QCD

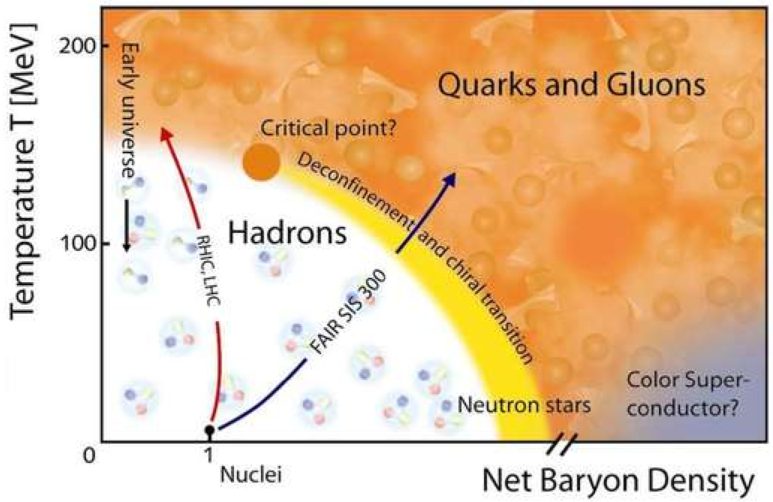
- First principle calculations
- Sign problem: difficult to explore $\mu \neq 0$

III. Effective models

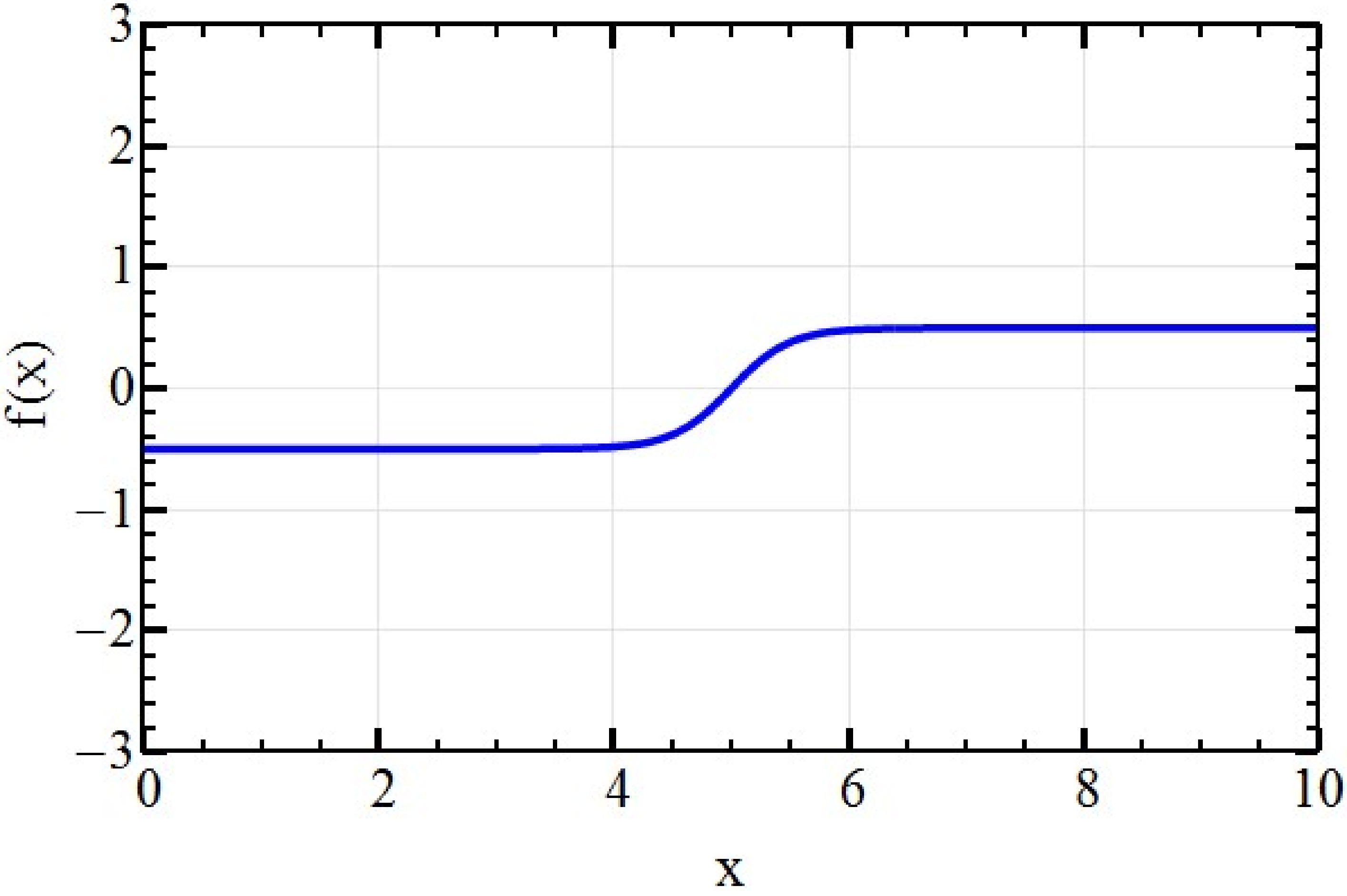
- Same universality class as QCD
- Hard to make qualitative predictions

IV. Functional methods

- Apply methods developed for effective models to QCD

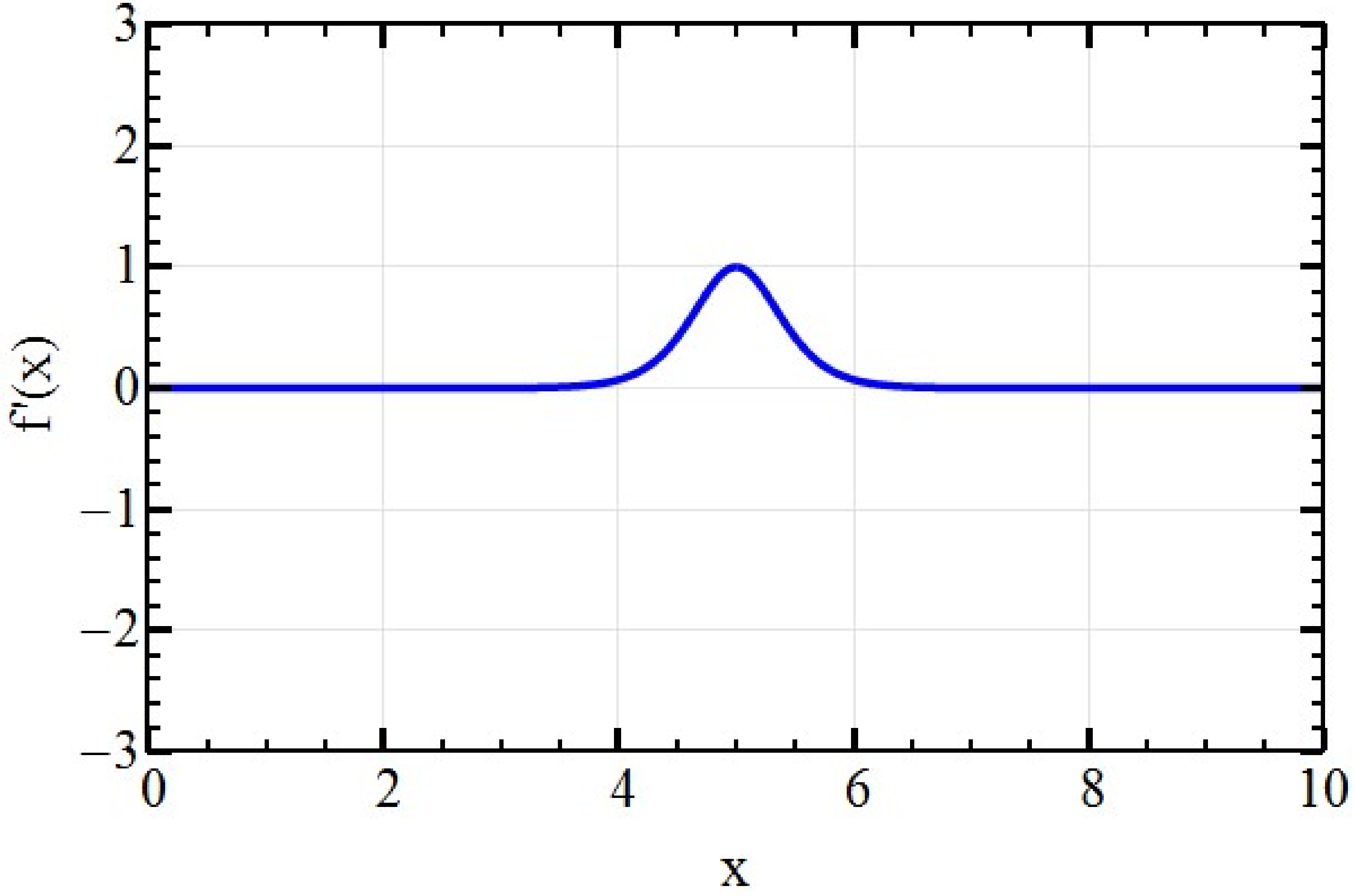


The power of derivatives



Higher derivatives – stronger signal at the transition

The power of derivatives



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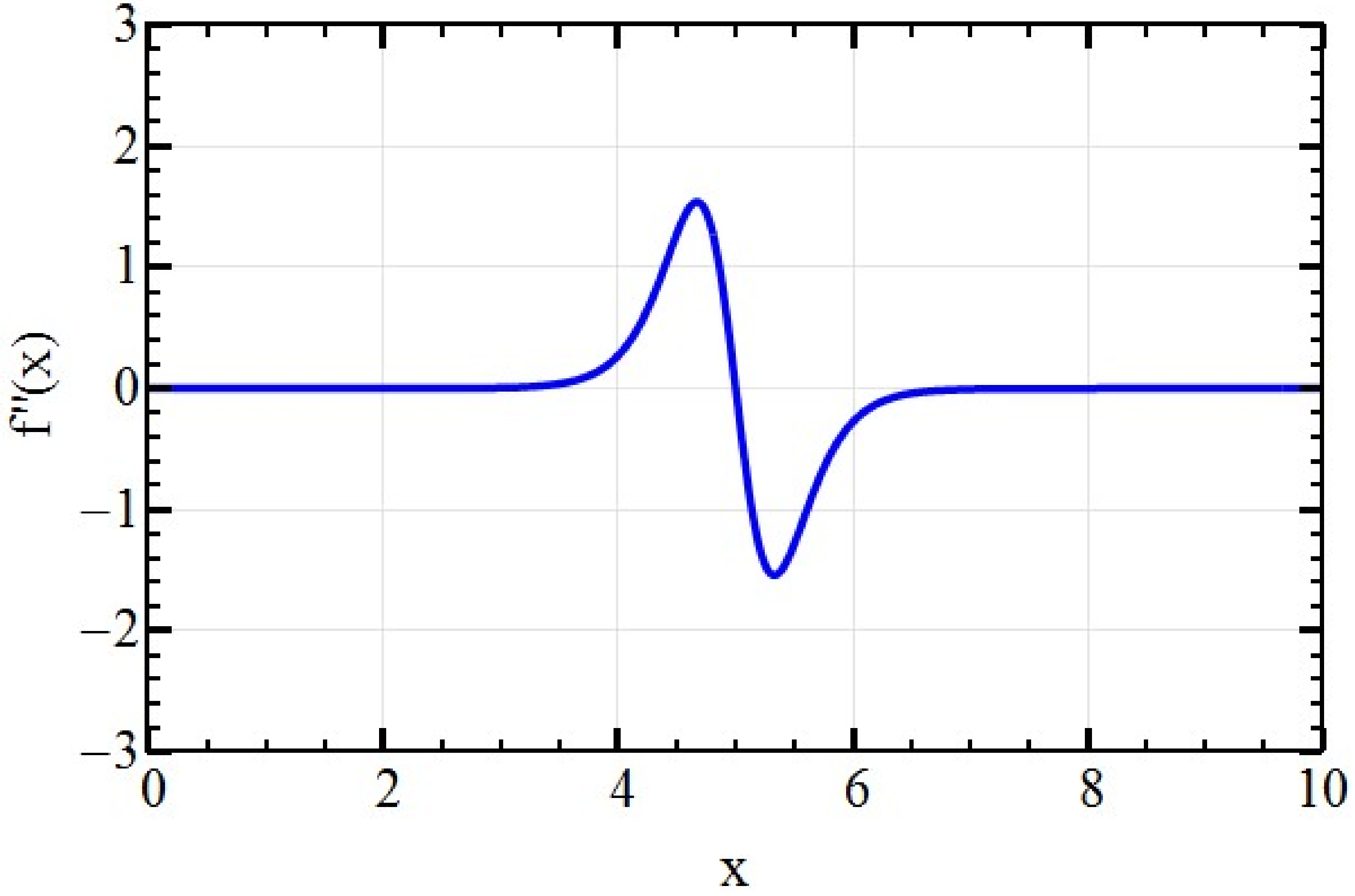

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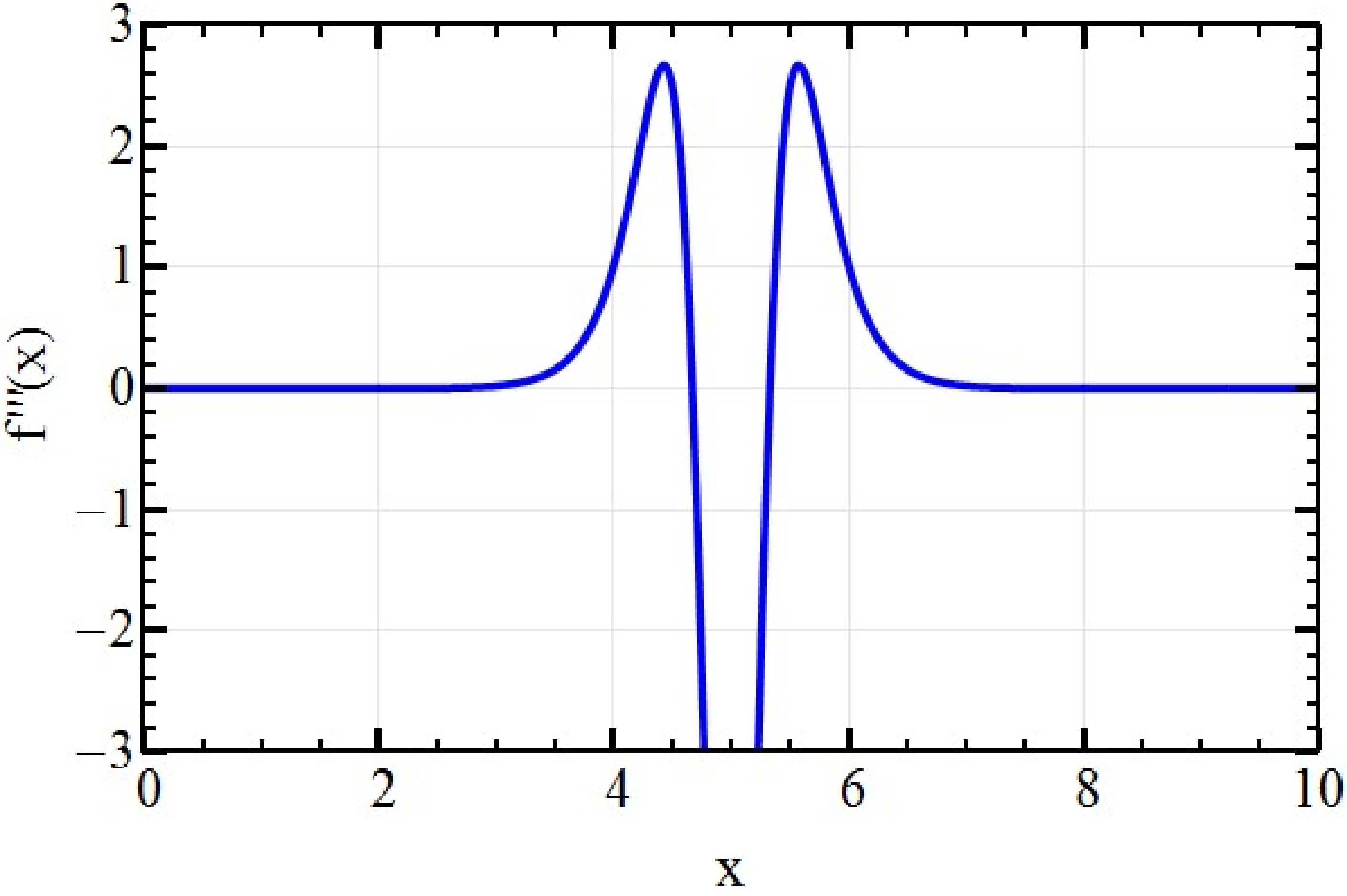

Higher derivatives – stronger signal at the transition

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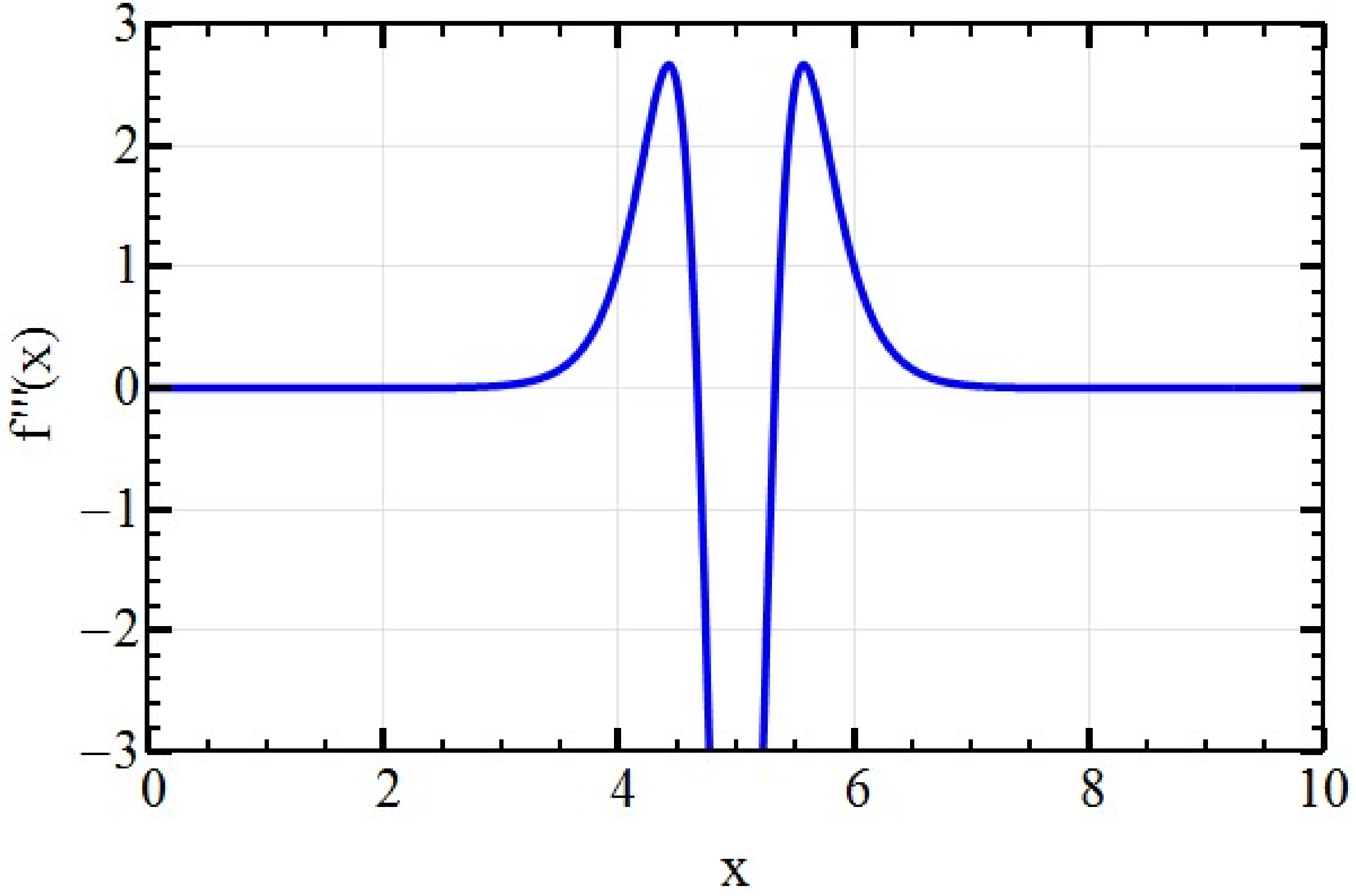


Higher derivatives – stronger signal at the transition

Helps to suppress background

Baryon number cumulants

Theory:

Experiment:

Calculation of susceptibilities of net baryon number:

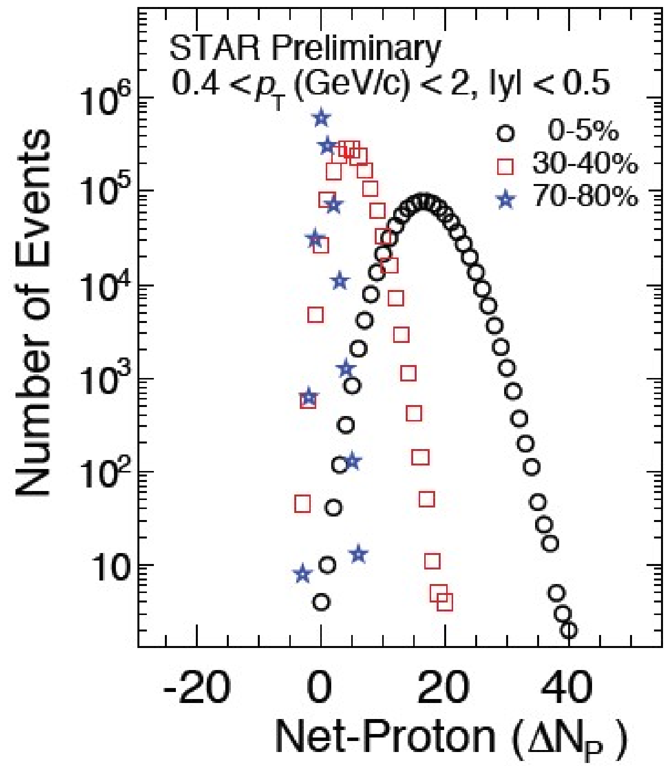
$$\chi_B^n = T^{n-4} \frac{\partial^n P(\mu_B, T)}{\partial \mu_B^n}$$

P: pressure

T: temperature

μ_B: baryon chemical potential

Measurement of net proton distribution:



QM 2015 talk by Jochen Thäder , STAR Coll. PRL112 (2014)

Translation between theory and experiment

Cumulants in the function of moments:

$$\chi^1 = \frac{1}{VT^3}\langle N\rangle \qquad \chi^2 = \frac{1}{VT^3}\left\langle(\Delta N)^2\right\rangle$$

$$\chi^3 = \frac{1}{VT^3}\left\langle(\Delta N)^3\right\rangle \qquad \chi^4 = \frac{1}{VT^3}\left(\left\langle(\Delta N)^4\right\rangle\text{-}3\left\langle(\Delta N)^2\right\rangle\right)$$

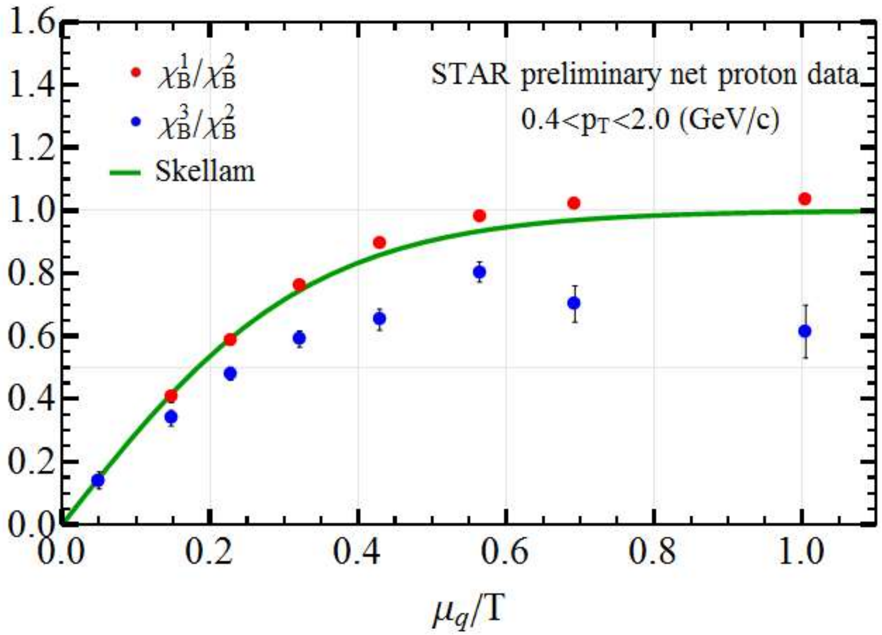
To cancel the volume dependence:

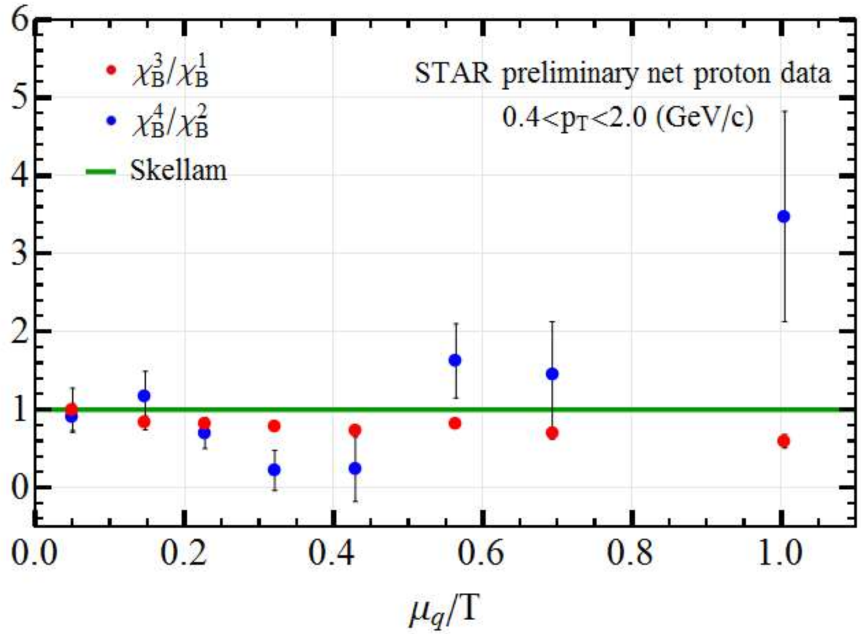
$$\chi^1/\chi^2 = \frac{M}{\sigma^2} \qquad \chi^3/\chi^2 = S\sigma$$

$$\chi^4/\chi^2 = \kappa\sigma^2 \qquad \chi^3/\chi^1 = S\sigma^3/M$$

M: Mean σ: Variance S: Skewness κ: Kurtosis

STAR Beam Energy Scan (BES)





Baseline: Skellam distribution

$\chi^{2k+1}/\chi^{2l} = \tanh(\mu_B/T)$

$\chi^{2k}/\chi^{2l} = \chi^{2k+1}/\chi^{2l+1} = 1$

STAR Beam Energy Scan (BES)

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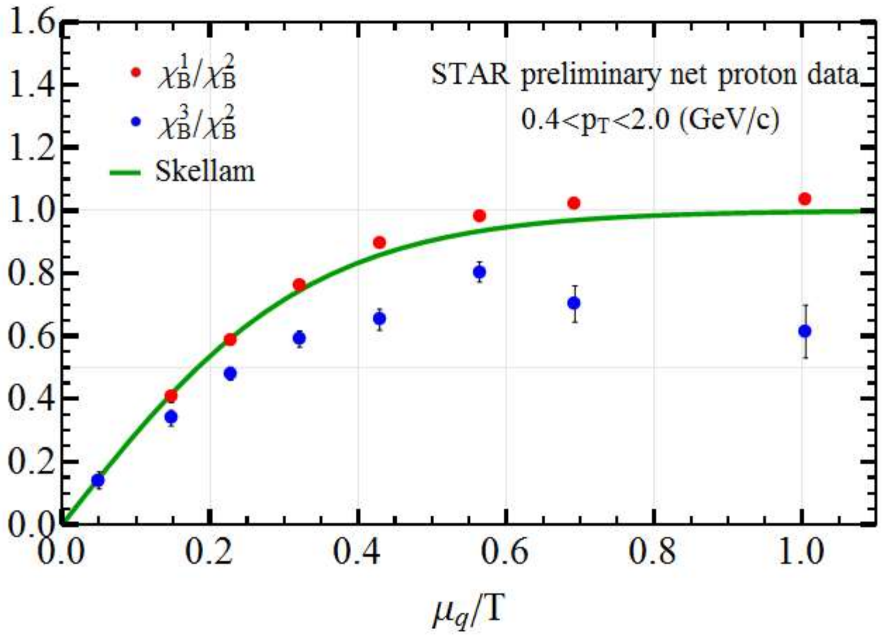

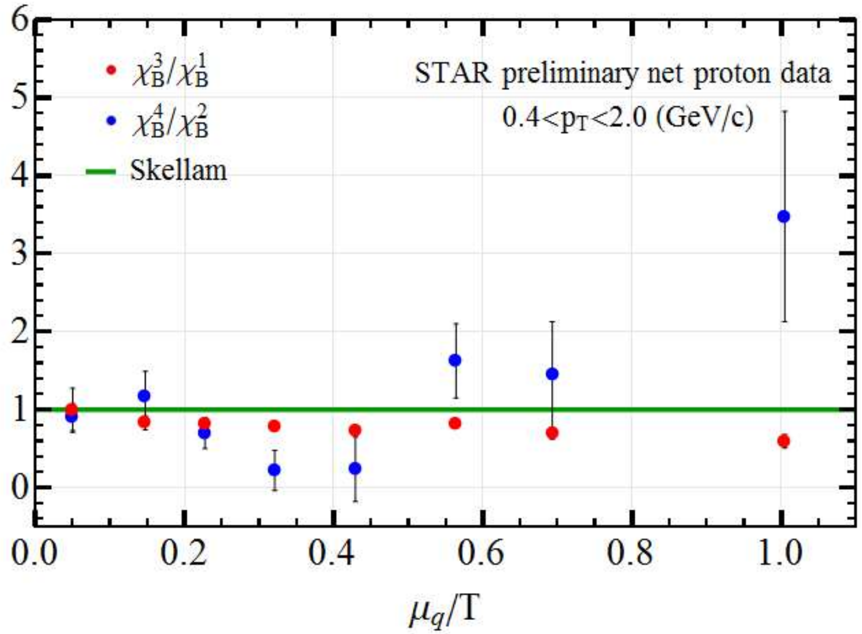

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Deviations → Critical endpoint?

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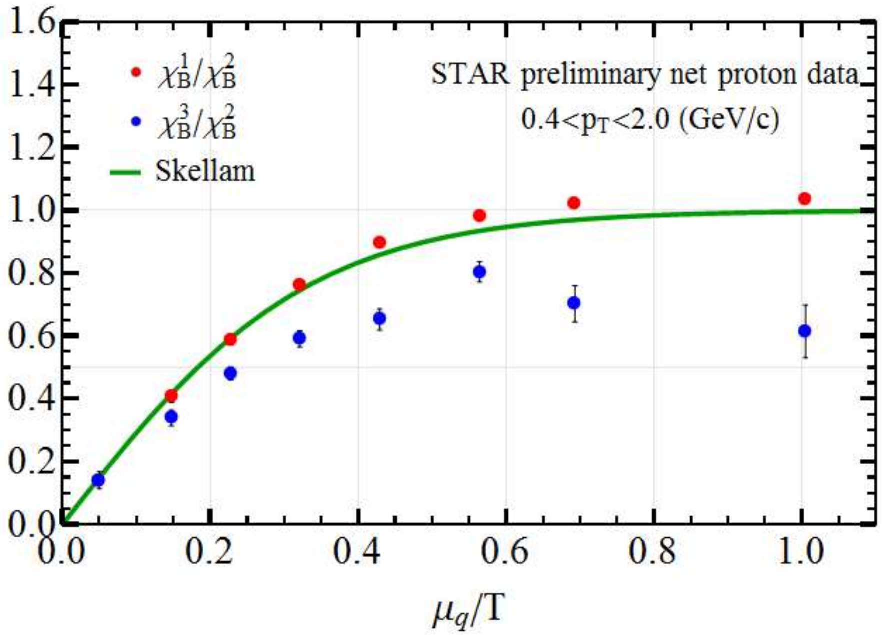

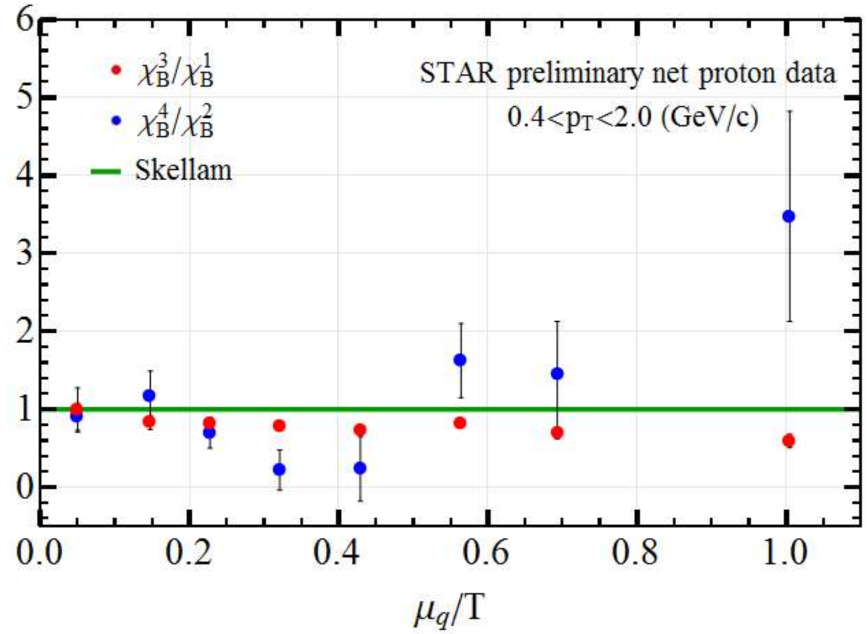

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Deviations → Critical endpoint?

Check: effective models

Polyakov-quark-meson (PQM) model

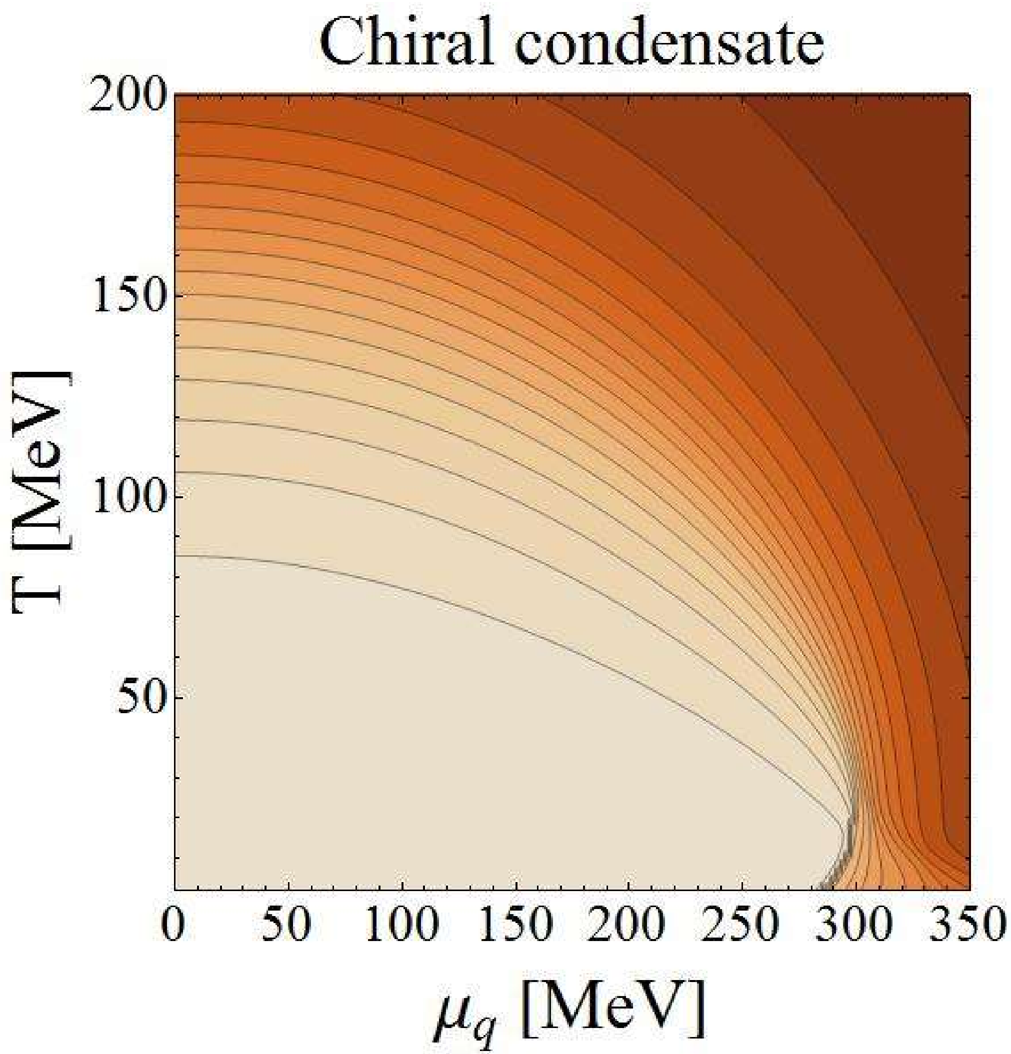
$$\mathcal{L} = \overline{q}\left[iD_\mu\gamma^\mu - g(\sigma + i\gamma_5\vec{\tau}\,\vec{\pi})\right]q + \frac{1}{2}\left(\partial_\mu\sigma\right)^2 + \frac{1}{2}\left(\partial_\mu\pi\right)^2 - U(\sigma,\vec{\pi}) - U_P\left(T,\ell,\overline{\ell}\right)$$

with the mesonic potential

$$U(\sigma,\vec{\pi}) = \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - v^2\right)^2 - H\sigma$$

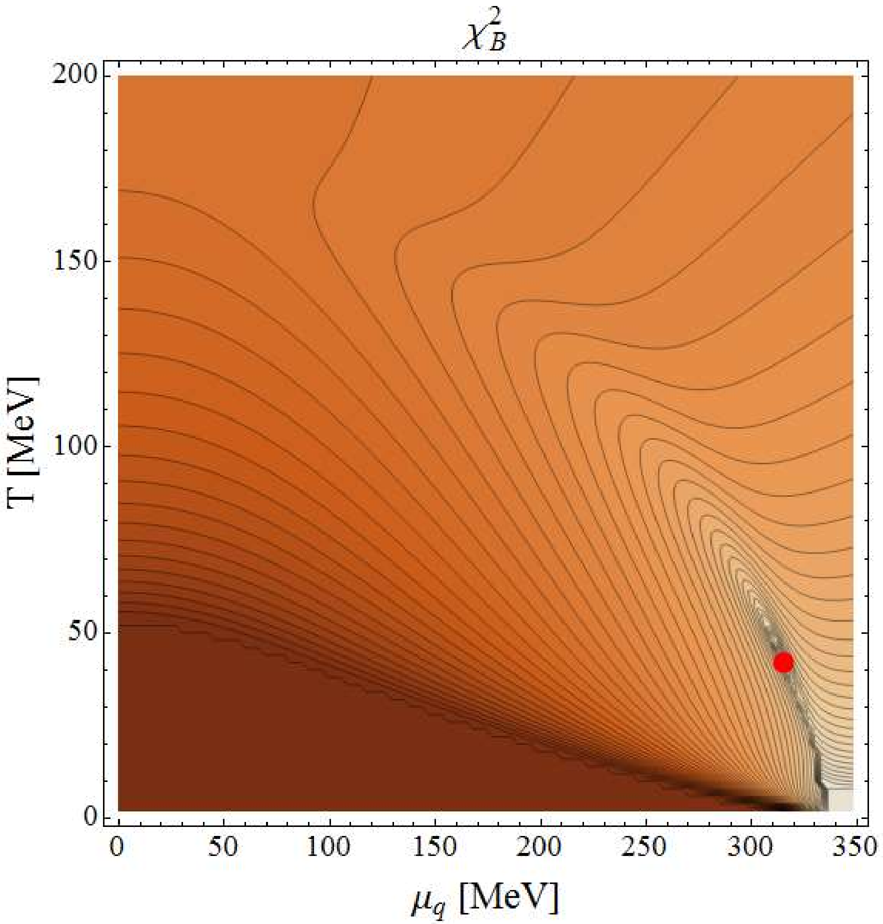
- Low energy effective theory of QCD
- Degrees of freedom: light quarks, pions, sigma meson
- Describes chiral symmetry breaking
- Polyakov-loop: suppression of single quark fluctuations at low temperatures
- Same universality class as QCD
- Solution needs approximation: MF, FRG…

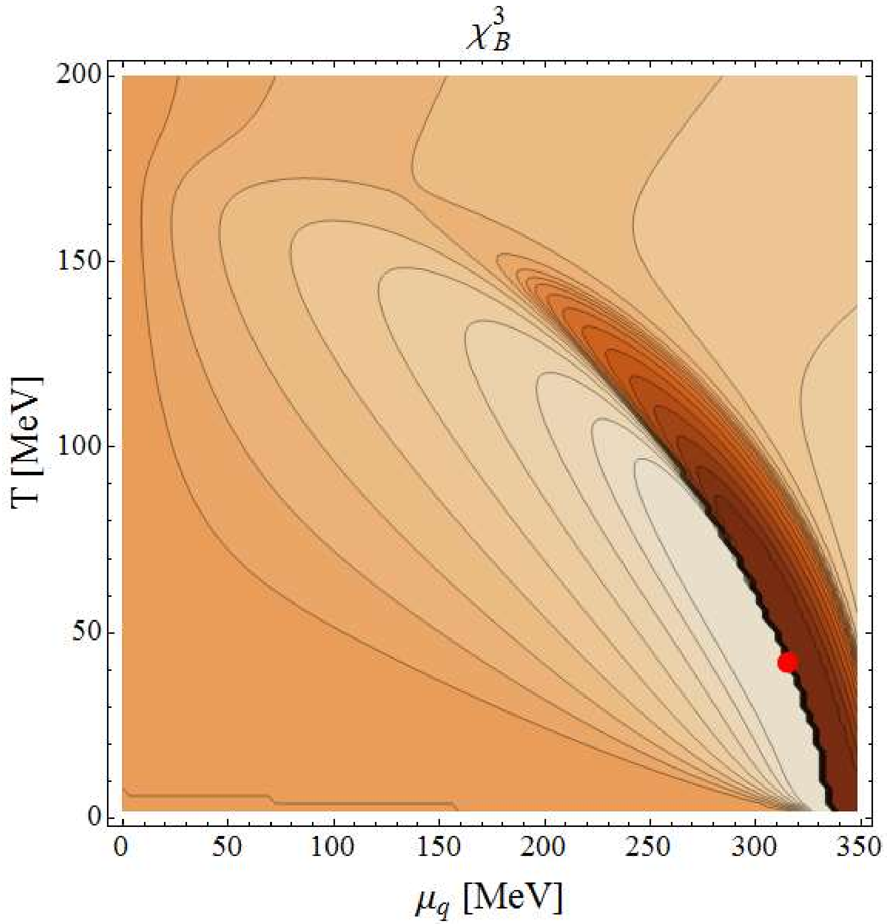
Phase diagram in PQM-FRG



- Spontaneous chiral symmetry breaking ✓
- Crossover-transition ✓
- 1st order transition ✓
- CEP ✓

Cumulants in effective models (PQM-MF)





$\mu_q = \mu_B/3$ quark chemical potential

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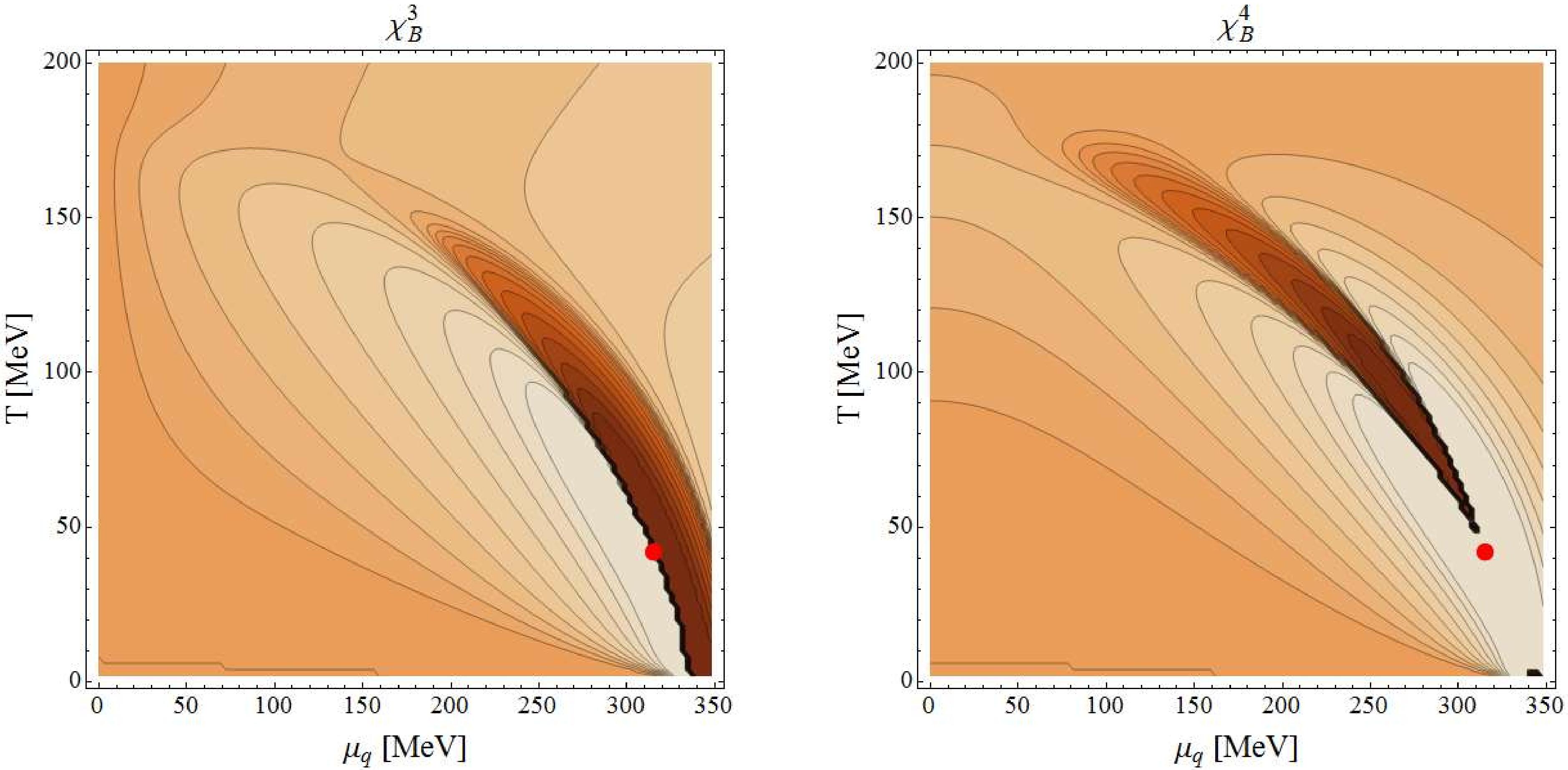

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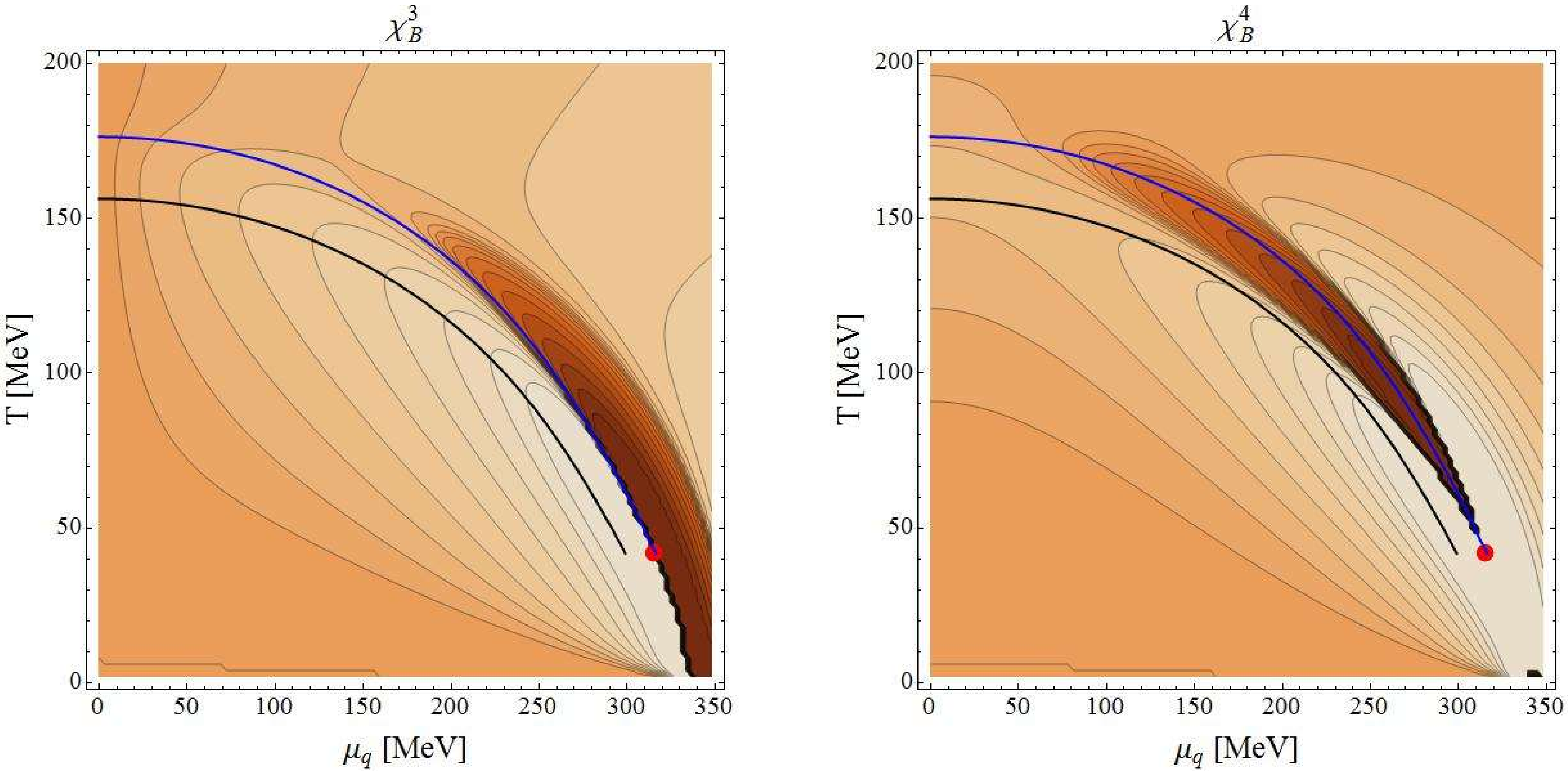

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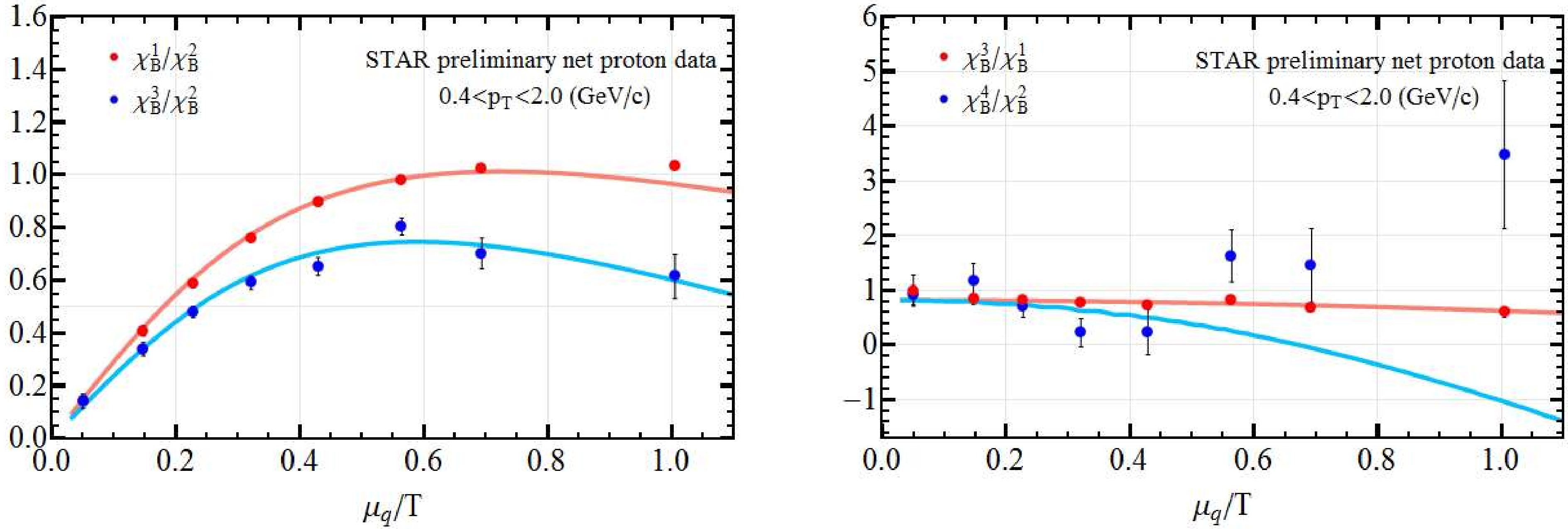
Consistency of the data



- Freeze-out line fitted to reproduce χ_B^3/χ_B^1
- All other cumulant ratios are calculated

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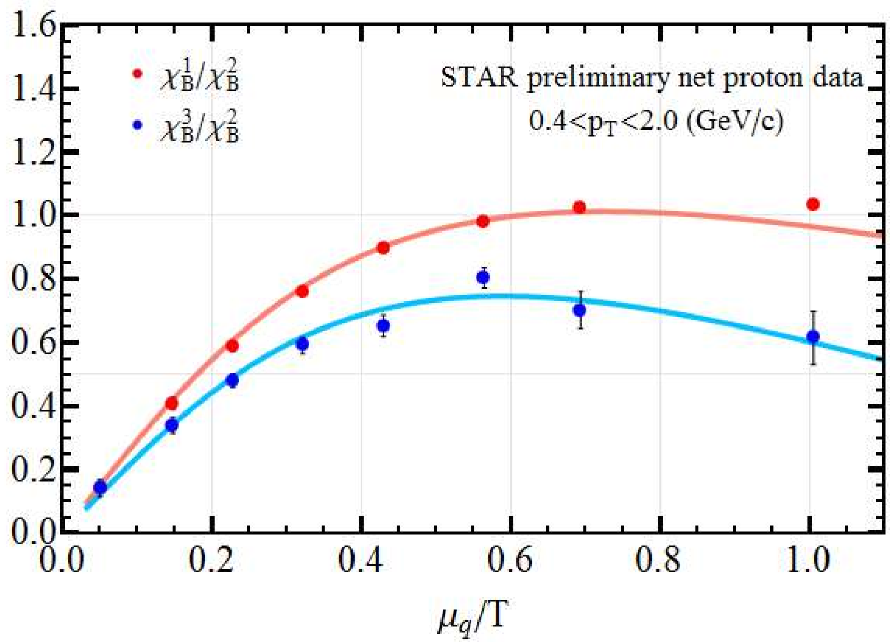
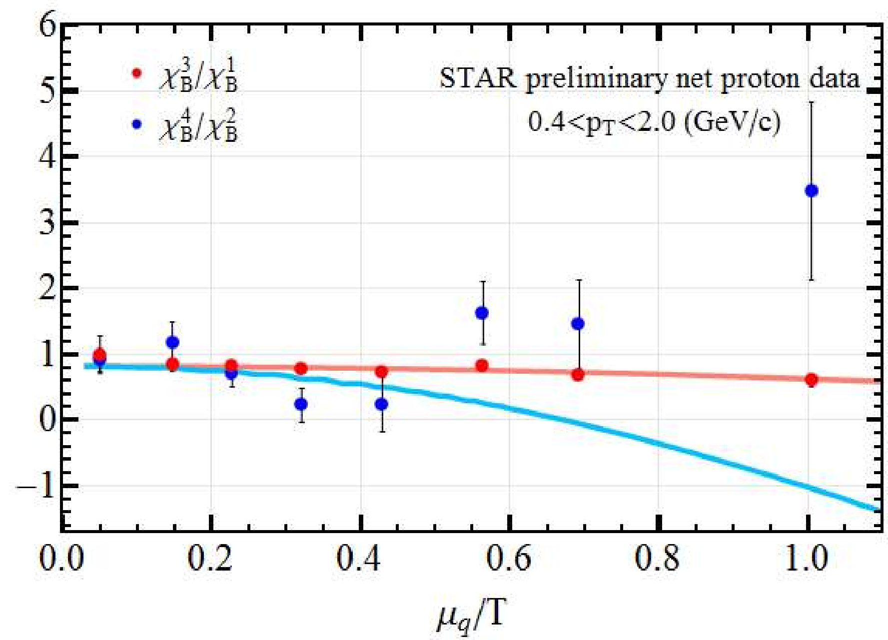
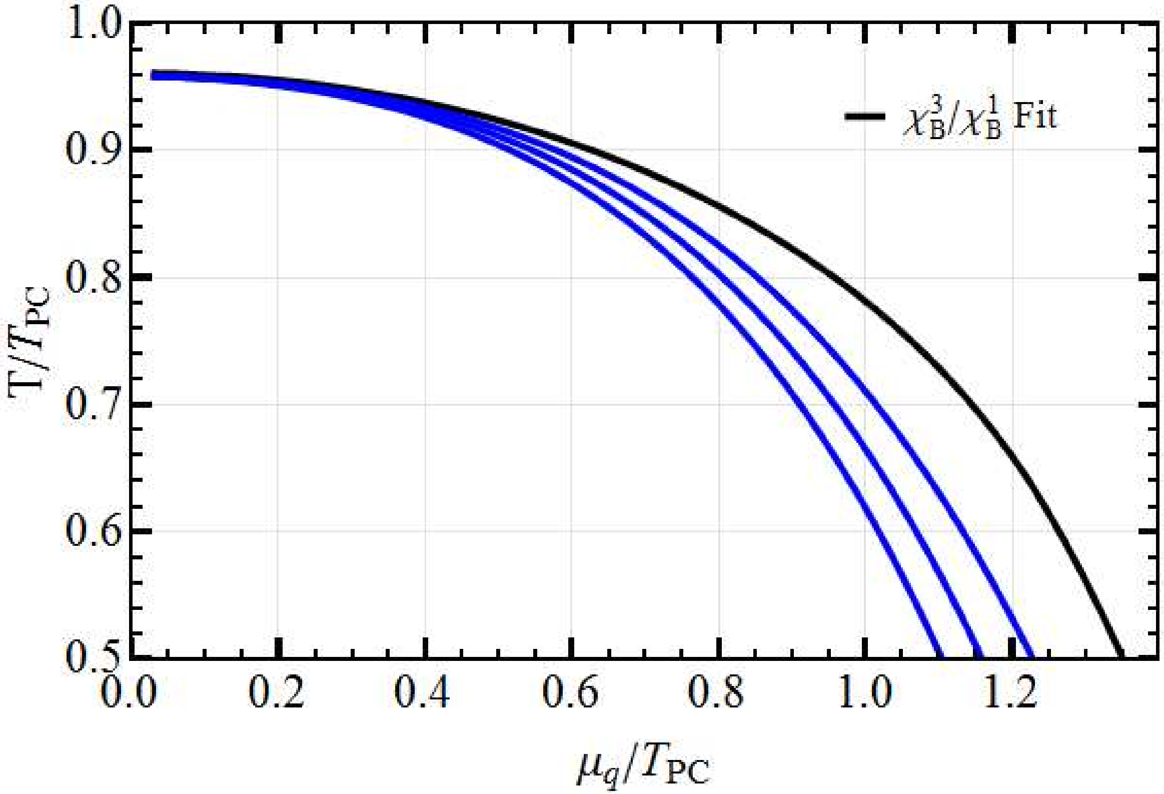




- Freeze-out line fitted to reproduce χ_B^3/χ_B^1
- All other cumulant ratios are calculated

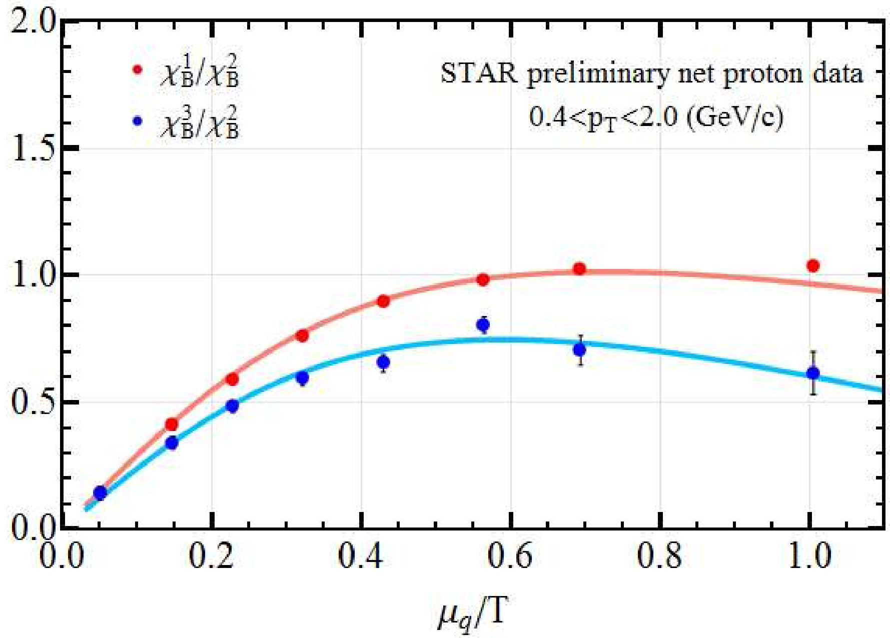
Critical endpoint? χ_B^4/χ_B^2 data not understood

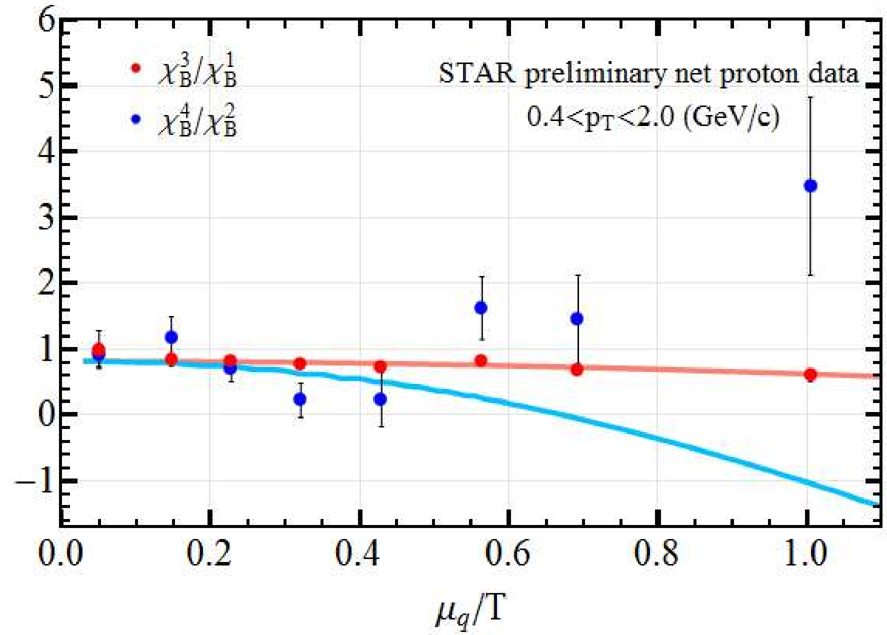
Reproducing χ_B^4/χ_B^2?



- Can we reproduce the χ_B^4/χ_B^2 data on some line in the phase diagram?

Consistency of data II.





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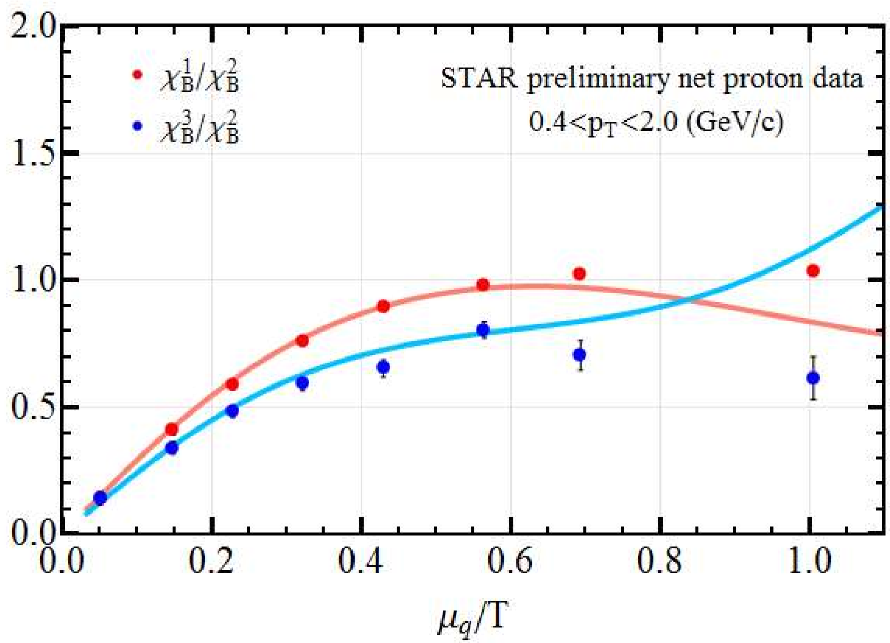

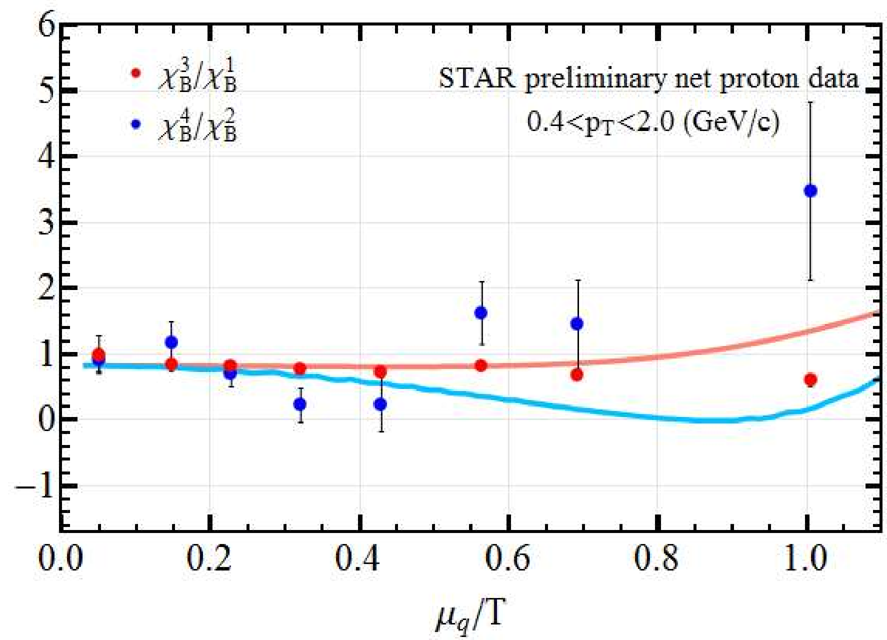

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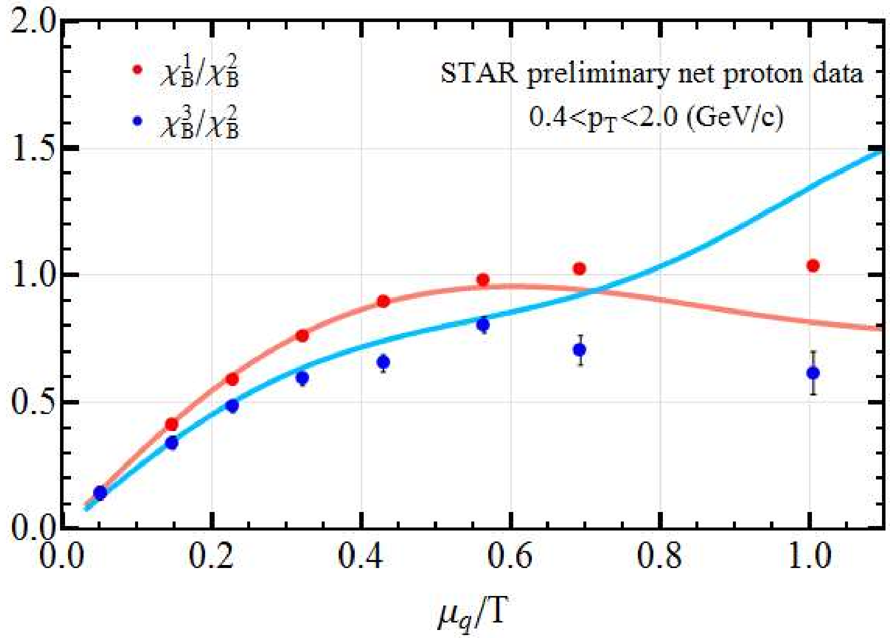

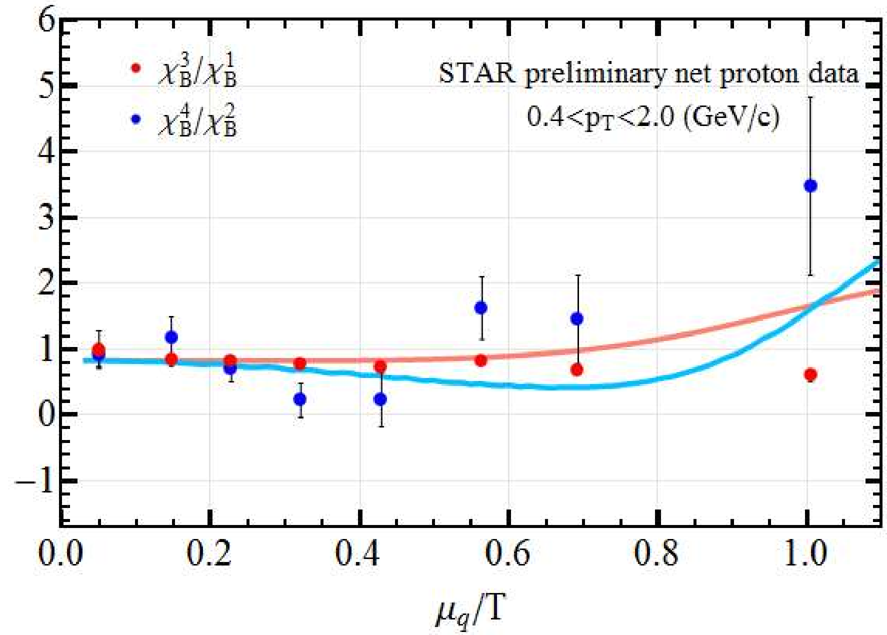

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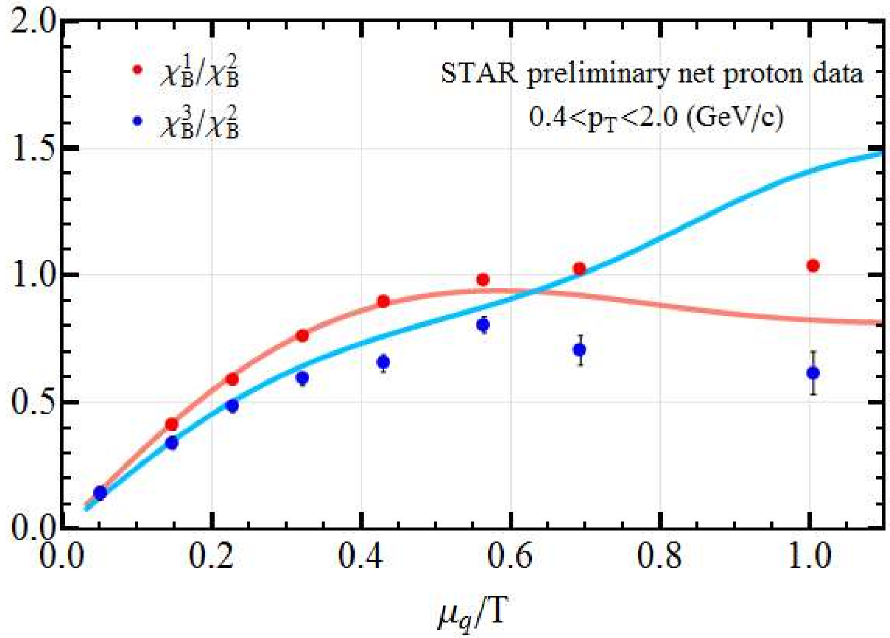
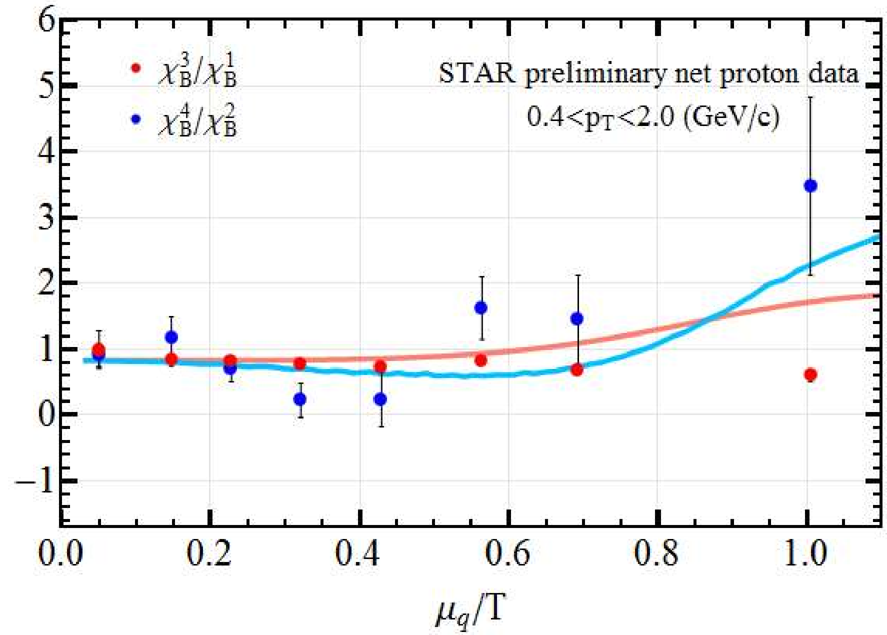
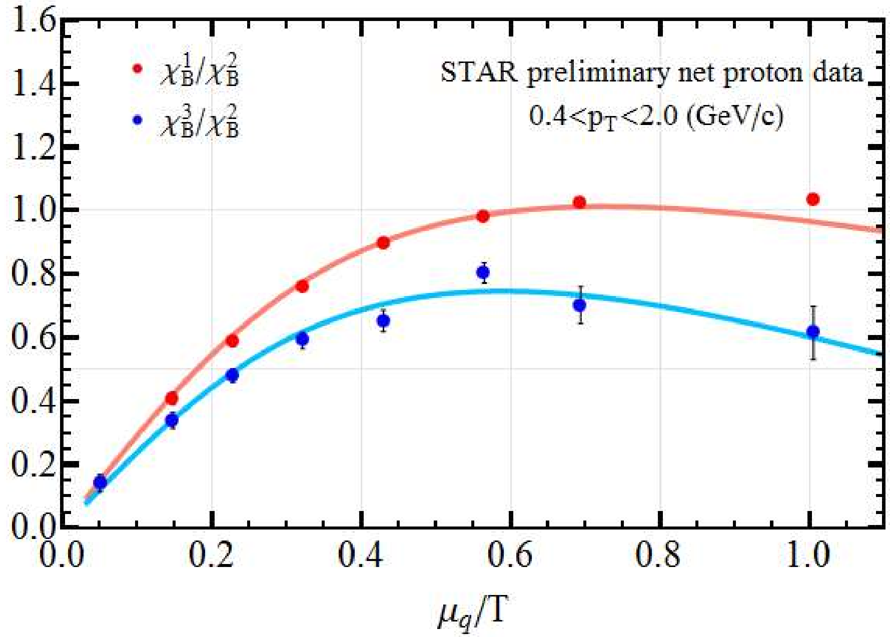


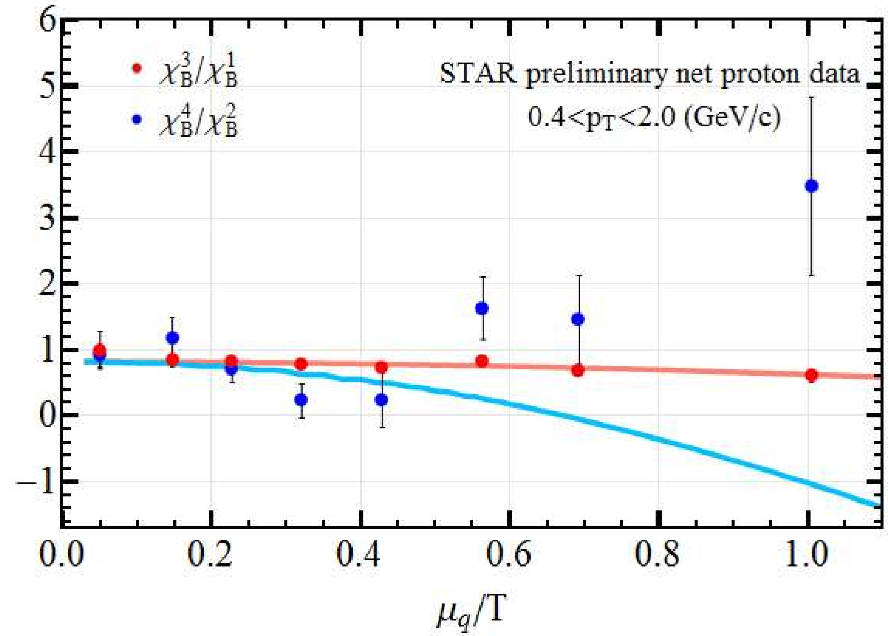


χ_B^4/χ_B^2 can be qualitatively reproduced

Other ratios are inconsistent

How to proceed?





How does finite volume change the picture?

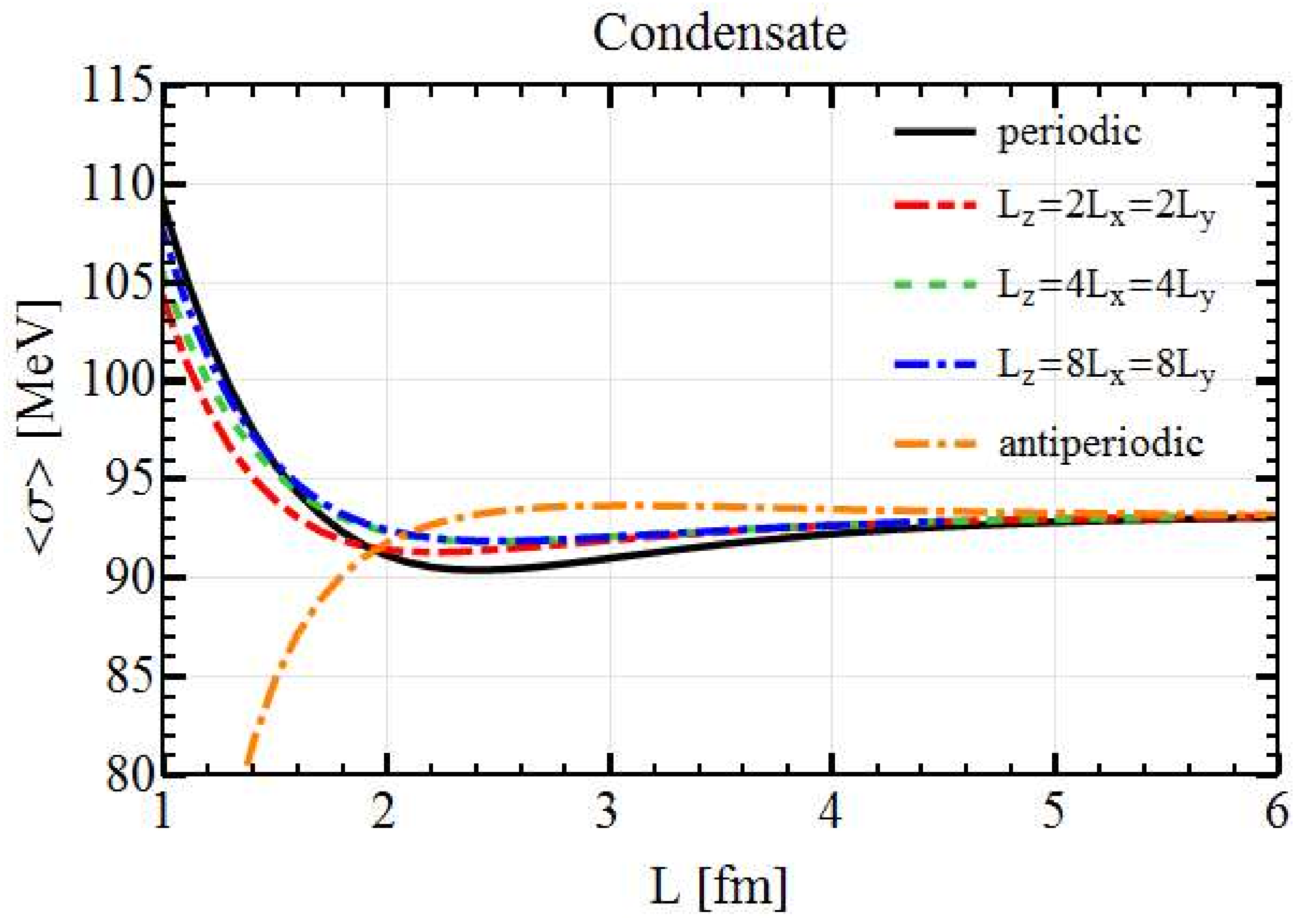
Finite volume – chiral condensate

Finite volume: momentum integrals are replaced by summation

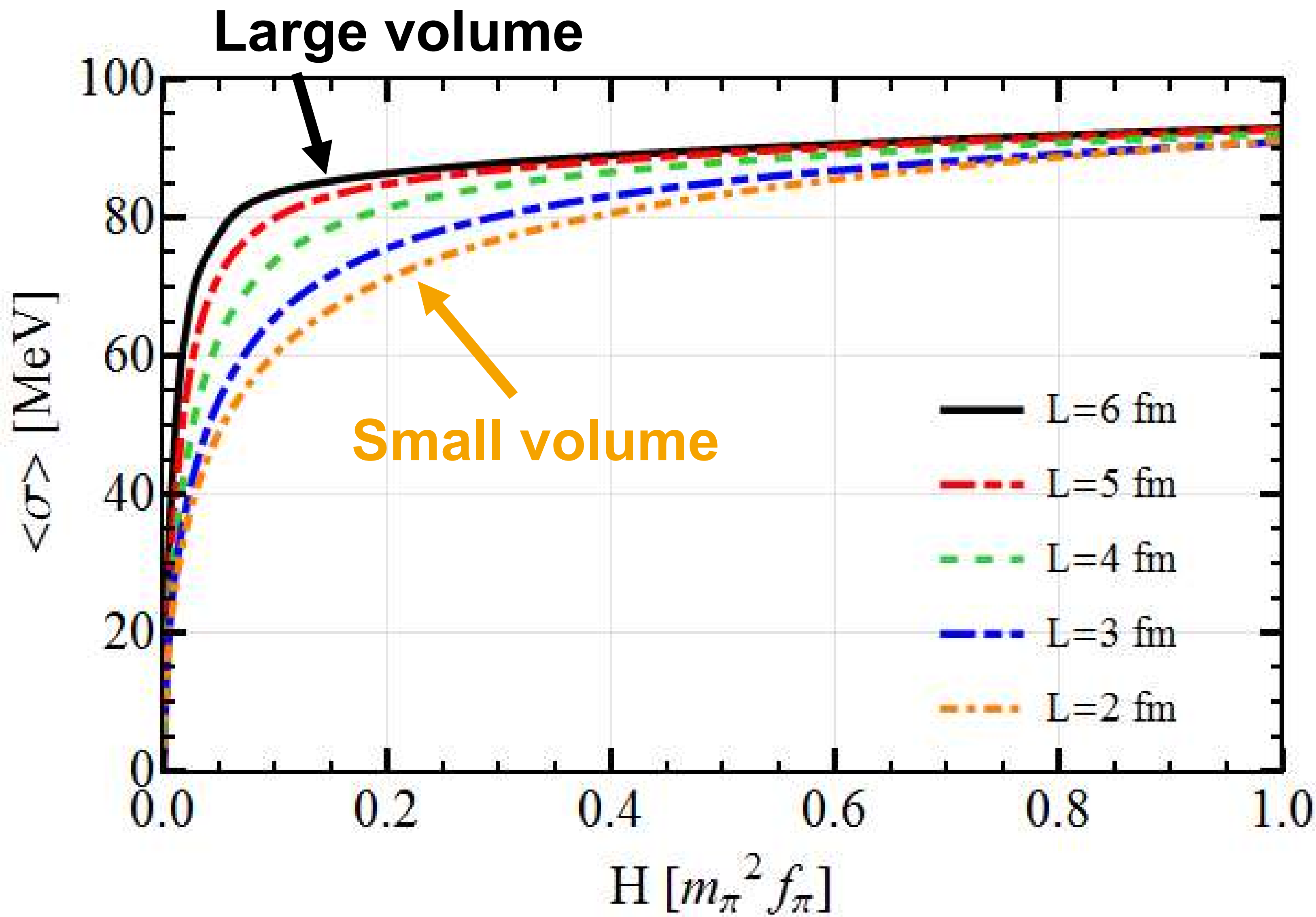
$$\int d^3q \rightarrow \frac{1}{L^3} \sum_{n_x, n_y, n_z}$$

Momenta are determined by boundary conditions

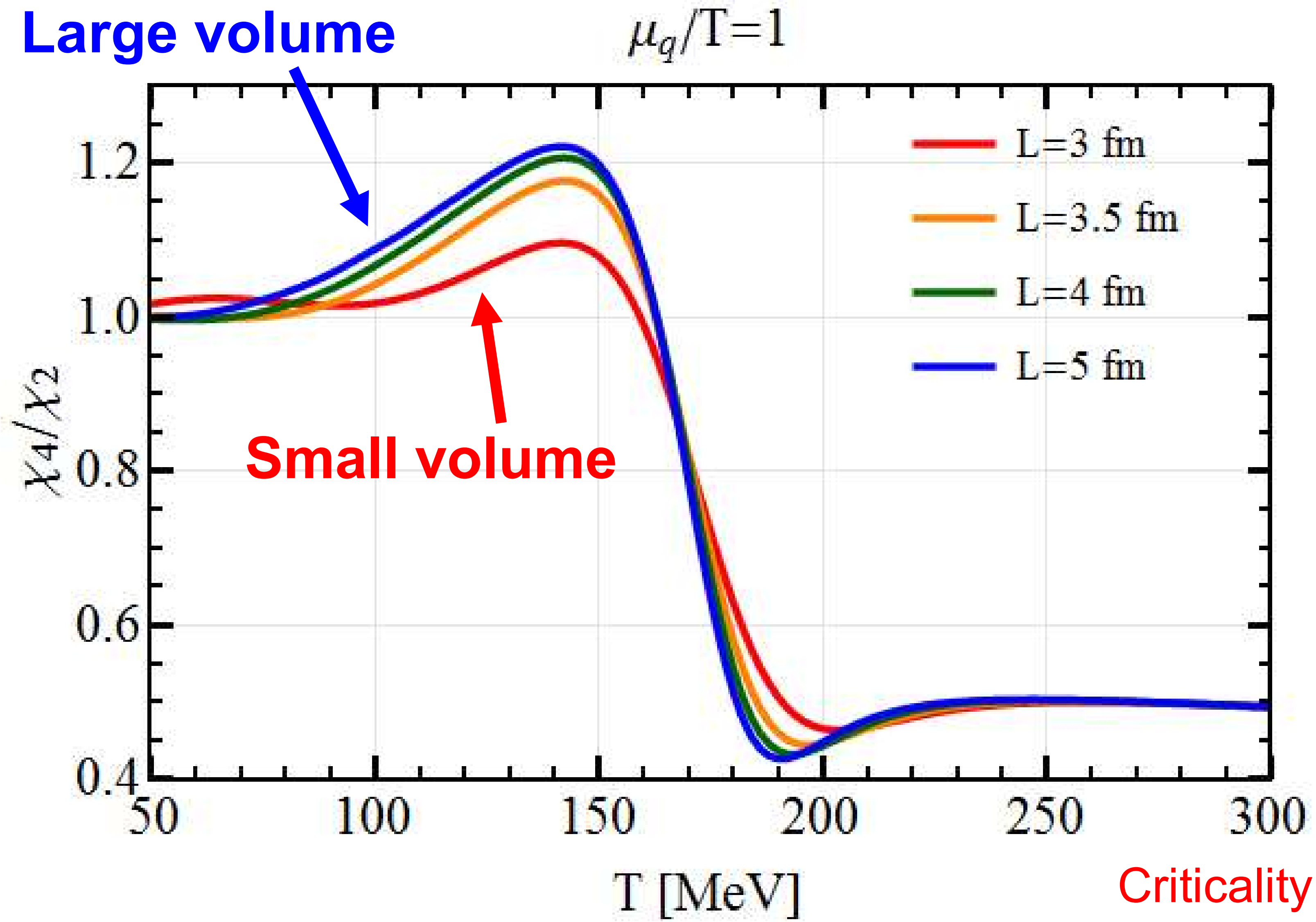
Periodic BC:
No spontaneous SB



Finite volume – external field dependence

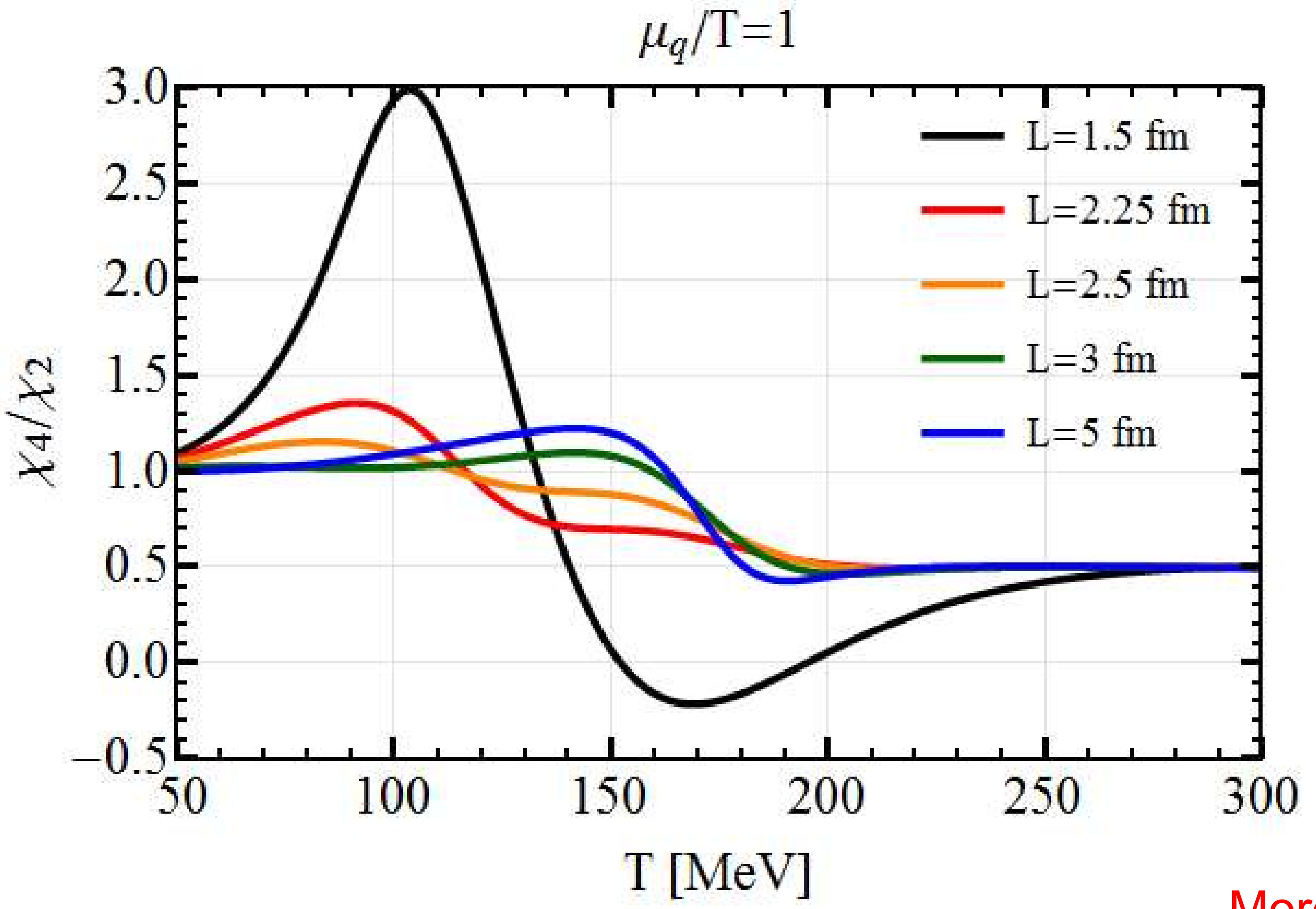


Cumulants in finite volume



Criticality suppressed?

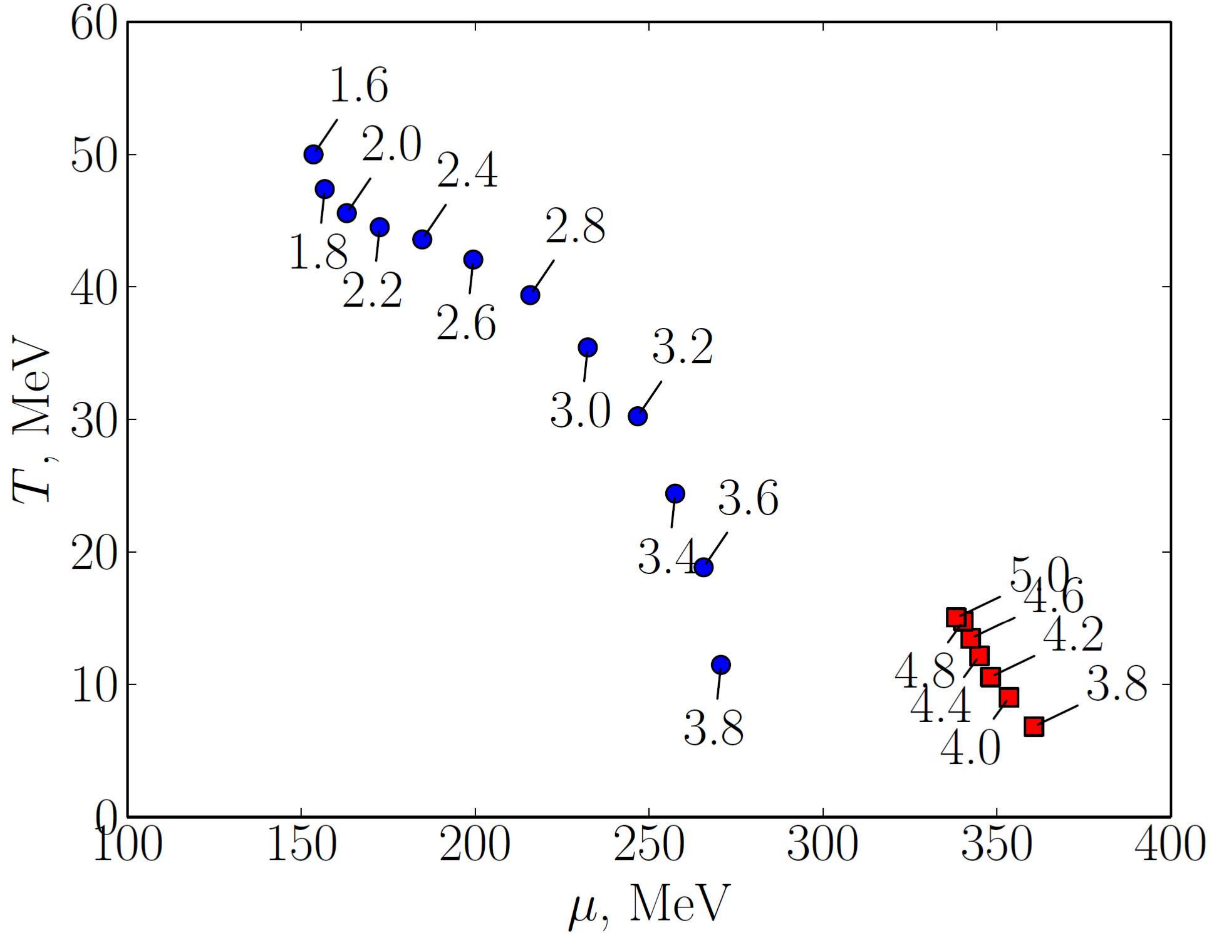
Cumulants in finite volume – small volumes



More complicated!

Apparent critical points (ACPs)

No critical point in finite volume



- ACP: local maximum of chiral susceptibility
- ACP1: close to infinite volume CEP
- ACP2: in very small volumes due to zero mode

Summary

I. **PQM model**
- Calculations are possible anywhere on the phase diagram
- Same universality class as QCD
- Baryon number cumulants calculated

II. **Comparison to experiment**
- 3 cumulant ratios qualitatively unterstood, χ_B^4/χ_B^2 not
- Many effects to consider

III. **Finite volume**
- No spontaneous symmetry breaking
- Behavior of cumulants far from trivial

Backup

Comparing theory to experiment…

Theory

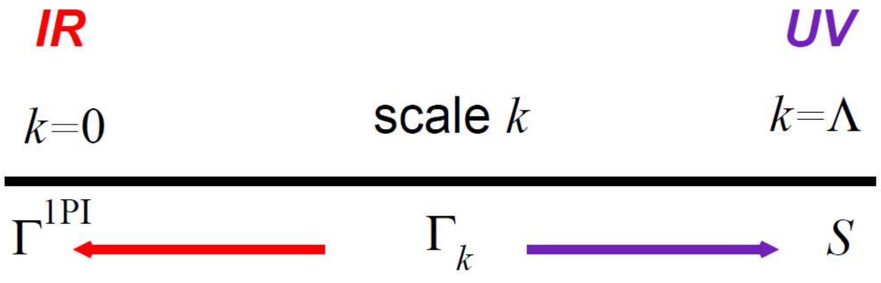
- Homogeneus system
- Infinite matter
- Grand canonical ensemble
- Information about particles of all momenta
- Static

Experiment

- Inhomogenities
- Finite size effects
- Global conservation laws
- Momentum space cuts, finite efficiency
- Rapidly changing

Functional Renormalization Group (FRG)

Scale dependent regulation of modes:



$$Z_k[J] = \int D\Phi\, e^{-S[\Phi]+\int_x \Phi(x)J(x)-\Delta S_k[\Phi]}$$

$$\Delta S_k[\Phi] = \frac{1}{2}\int \frac{d^d q}{(2\pi)^d}\Phi(-q)R_k\,(q)\Phi(q)$$

Effective average action: $\Gamma_k[\phi] = \sup_J \left(\int_x J(x)\phi(x) - \log Z_k[J]\right) - \Delta S_k[\phi]$

Scale evolution governed by the Wetterich equation:

$$\partial_k \Gamma_k[\phi] = \frac{1}{2}\int_x \left(\Gamma_k^{(2)}[\phi] + R_k\right)^{-1} \partial_k R_k$$

Typical regulator (Litim):

$$R_k\,(q) = \left(k^2 - q^2\right)\theta\left(k^2 - q^2\right)$$

FRG applied to PQM model

At finite temperature and chemical potential flow for the grand canonical potential:

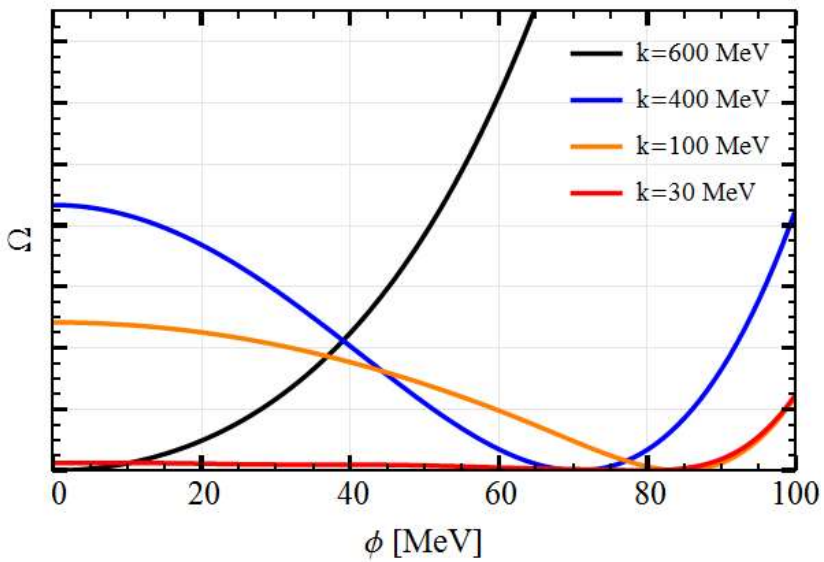
$$\partial_k \Omega_\mathrm{k} = \frac{k^4}{12\pi^2}\left(\frac{3}{E_\pi}\coth\frac{E_\pi}{2T} + \frac{1}{E_\sigma}\coth\frac{E_\sigma}{2T} - \frac{24}{E_q}\left\{1 - N_q\left(T,\mu,\ell,\bar{\ell}\right) - N_{\bar{q}}\left(T,\mu,\ell,\bar{\ell}\right)\right\}\right)$$

$$N_q\left(T,\mu,\ell,\bar{\ell}\right) = N_{\bar{q}}\left(T,-\mu,\bar{\ell},\ell\right) = \frac{1 + 2\bar{\ell}e^{(E_q-\mu)/T} + 2\ell e^{(E_q-\mu)/T}}{1 + 3\bar{\ell}e^{(E_q-\mu)/T} + 3\ell e^{(E_q-\mu)/T} + e^{3(E_q-\mu)/T}}$$

$$E_\pi = \sqrt{k^2 + 2\Omega_k'}$$

$$E_\sigma = \sqrt{k^2 + 2\Omega_k' + 4\phi^2\Omega_k''}$$

$$E_q = \sqrt{k^2 + g^2\phi^2}$$



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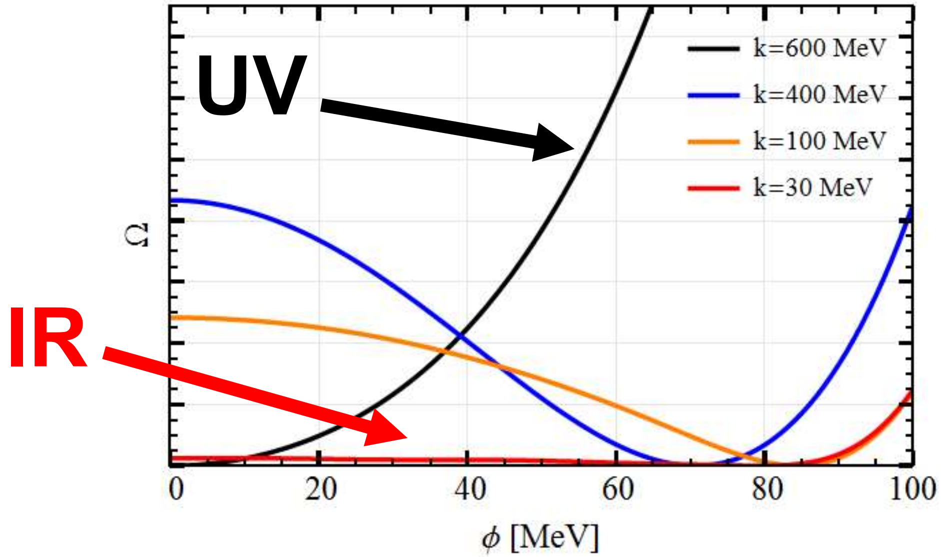


Inclusion of vector interaction

$$\mathcal{L} = \mathcal{L}_{PQM} - g_\omega \bar{q}\omega_\mu \gamma^\mu q - \frac{1}{2} m_\omega^2 \omega^2 + F_{\mu\nu}F^{\mu\nu}$$

Mean field approximation in ω :$\langle \omega_0 \rangle \neq 0$

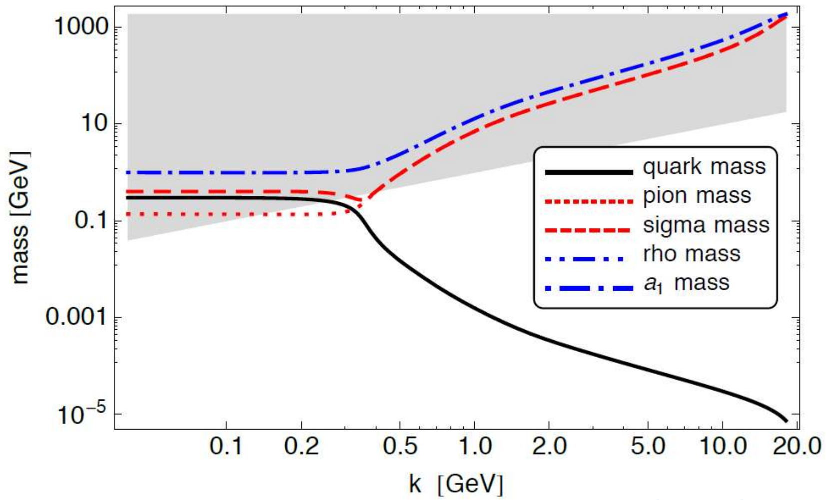
$$P(T,\mu) = P_{PQM}(T,\mu_{eff}) + \frac{g_\omega^2}{2m_\omega^2} n_{PQM}^2(T,\mu_{eff}),$$

$$\mu_{eff} \equiv \mu - g_\omega \langle \omega_0 \rangle$$

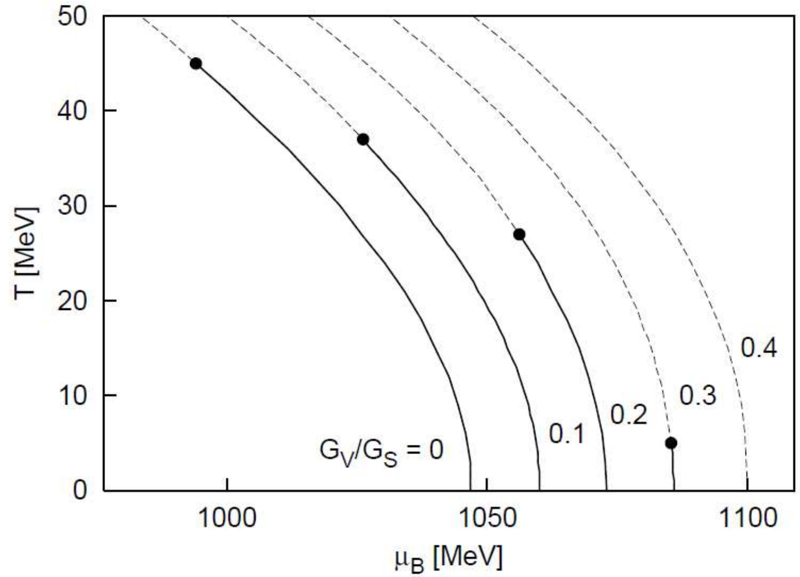
$$\langle \omega_0 \rangle = \frac{g_\omega}{m_\omega^2} n_{PQM}(T,\mu_{eff})$$

Main effects:

- Shift in chemical potential: $\mu \to \mu_{eff}$
- CEP to lower T, higher μ

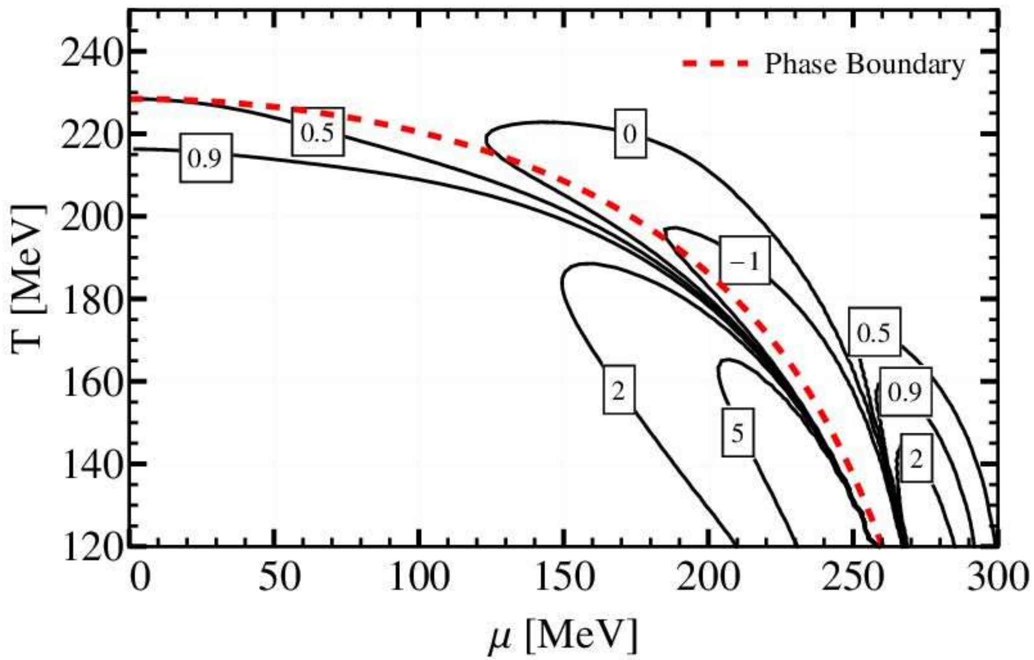


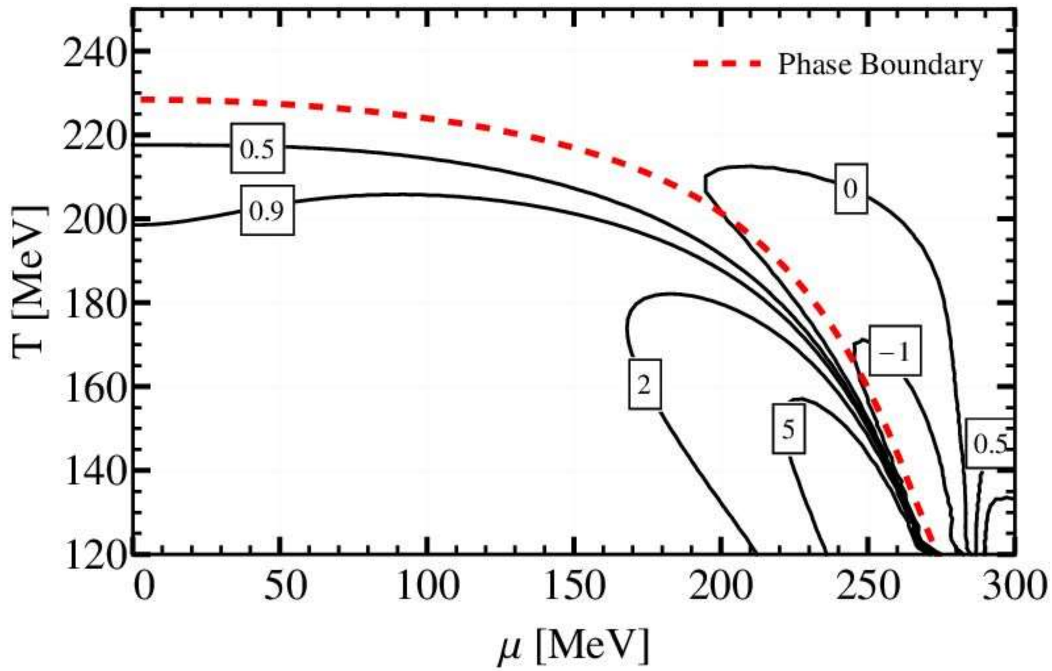
Fabian Rennecke, PRD92 (2014)



M. Kitazawa et al, Prog.Theor.Phys. 108 (2002)

Effect of the repulsive vector interaction





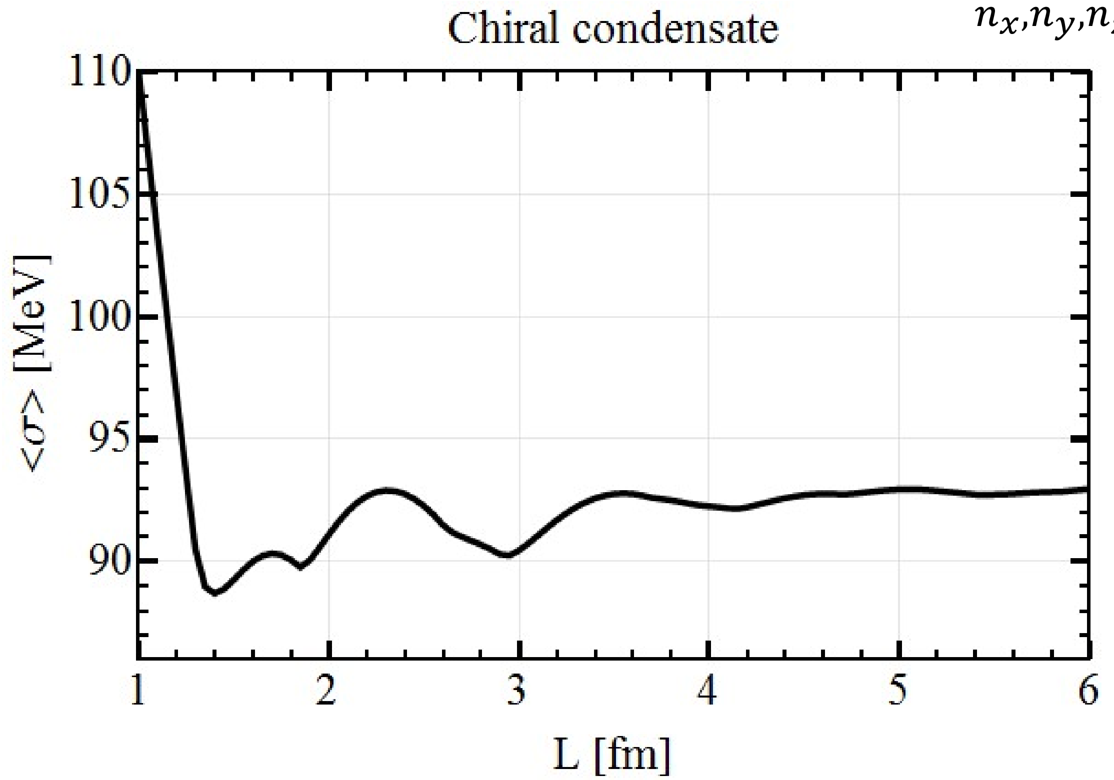
χ_B^4/χ_B^2 No vector interaction

χ_B^4/χ_B^2 Strong vector interaction

Quark Meson model in finite volume

Finite volume: momentum integrals are replaced by summation

$$\int d^3q \rightarrow \frac{1}{L^3} \sum_{n_x, n_y, n_z}$$



Litim regulator

$R_k(q) = (k^2 - q^2)\theta(k^2 - q^2)$

not appropriate for finite volume studies!

Exponential regulator $R_k(q) = \frac{q^2}{\exp(q^2/k^2) - 1}$