PATH INTEGRAL MONTE CARLO SIMULATIONS FOR THE QUARK – GLUON PLASMA – FIRST RESULTS

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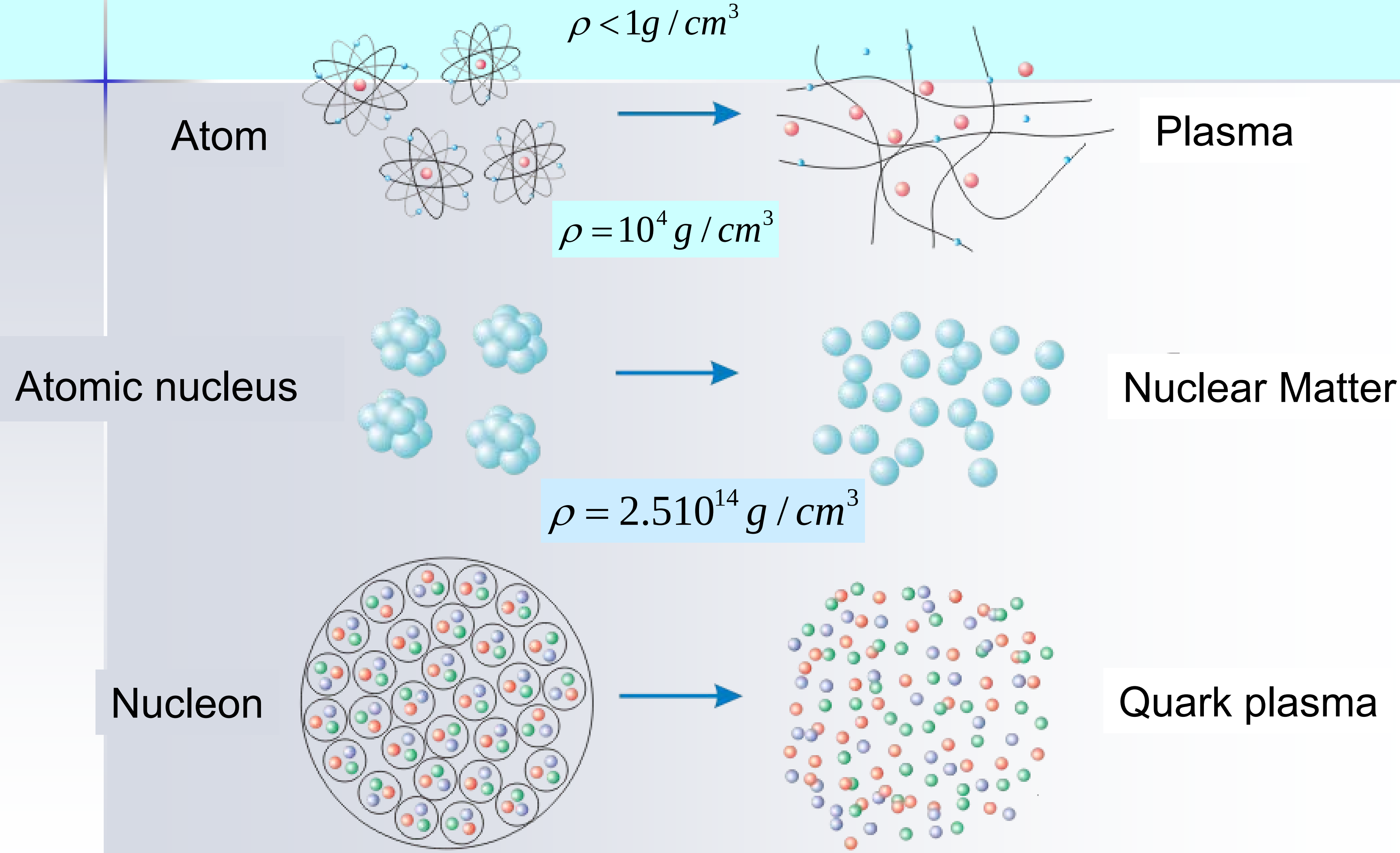
OUTLINE

- States of matter at high energy concentration
- Formation of quark-gluon plasma
- Conclusions from Lattice QCD simulations
- Simulation of quantum many-particle systems
- Path integral Monte Carlo method
- Applications to hydrogen plasmas
- Applications to the models of quark-gluon plasma

Matter transformation at high energy concentration



APPROACHES TO QUANTUM THERMODYNAMICS

- Thomas-Fermi method
- Hartree-Fock(-Slater) method
- Density Functional Theory
- Monte Carlo methods (variational, diffusion, **path-ingegral**)

Interaction and quantum effects in dense 3D and 2D plasma media with different mass ratio of charges.

Classical one-component plasma - COCP

Quantum one-component plasma - QOCP

Classical two-component plasma - CTCP

Quantum two-component plasma - QTCP

Coulomb interaction: $U_{ab}(r) = e_a e_b / r$

$T \sim 10^{13} K, n \sim 10^{45} cm^{-3}$

— Nonideality boundary:

$$< U_{Coul} > = < E_{Kin} >$$

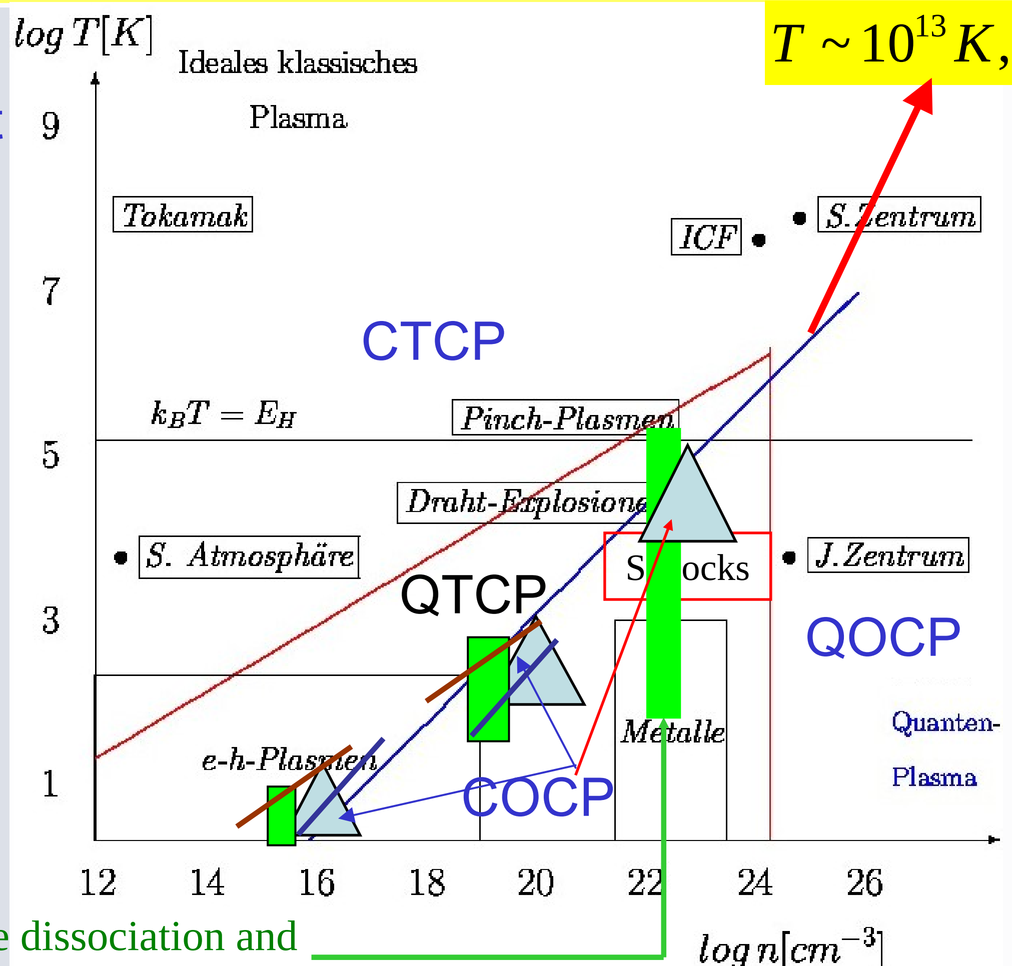
Inside: Strong Coulomb interaction, Many-body effects

atomes, molecules, clusters

Degeneracy boundary

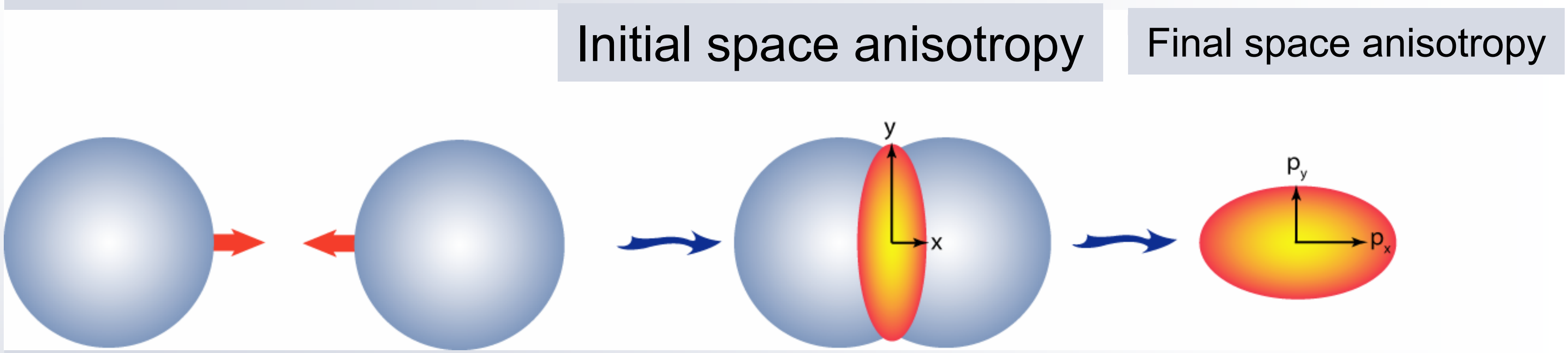
$$\lambda_e = \bar{r}$$

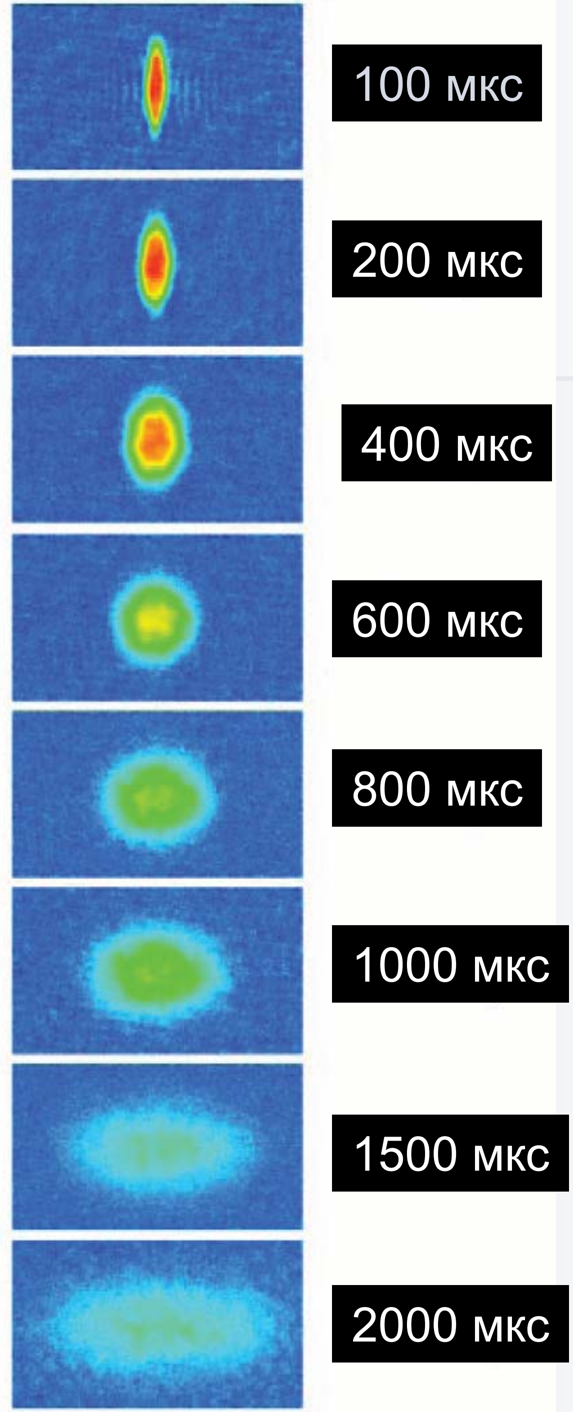
Below: overlapping electron Wave functions, Quantum and spin effects

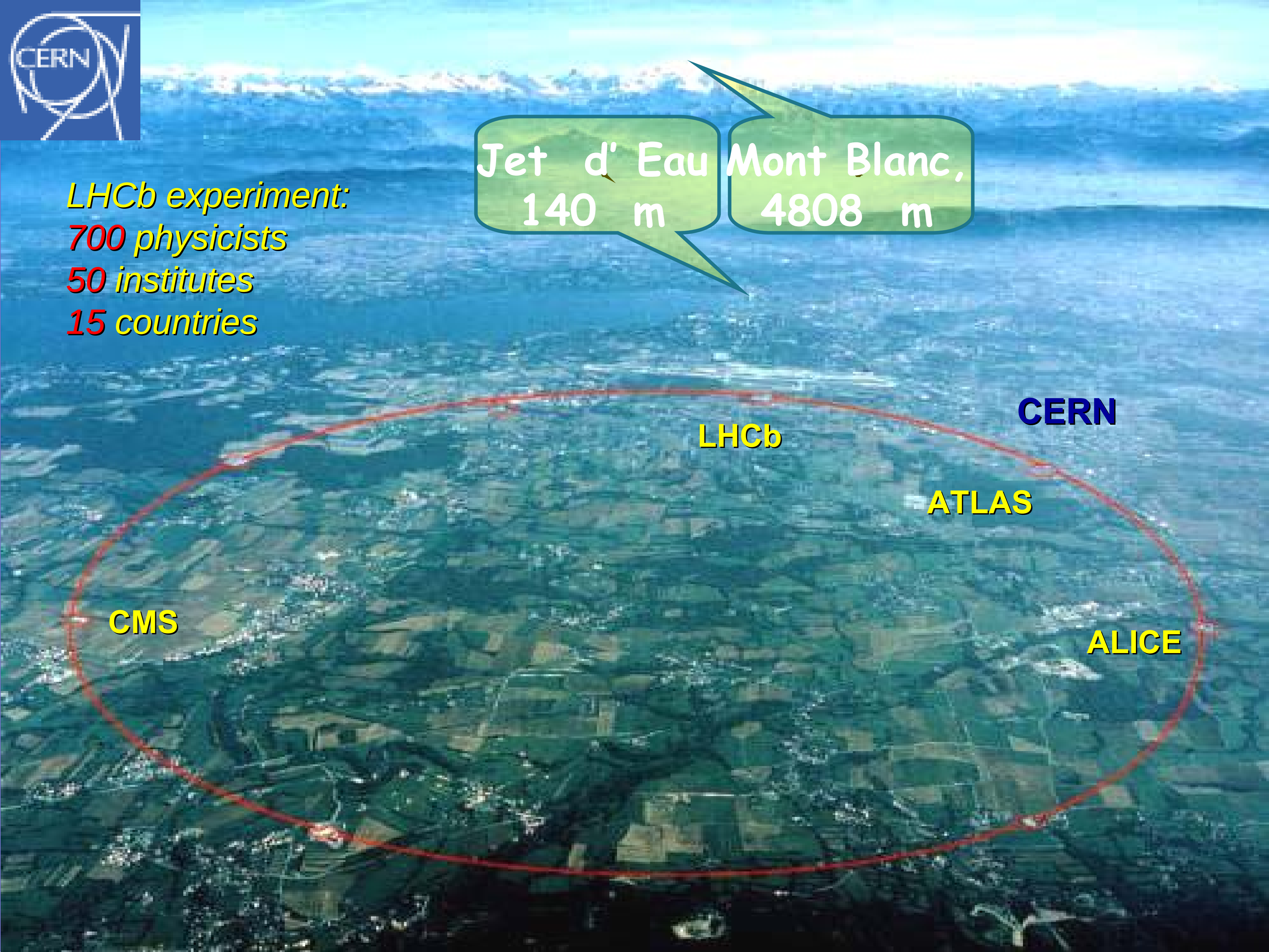


Pressure dissociation and ionization, Mott effect

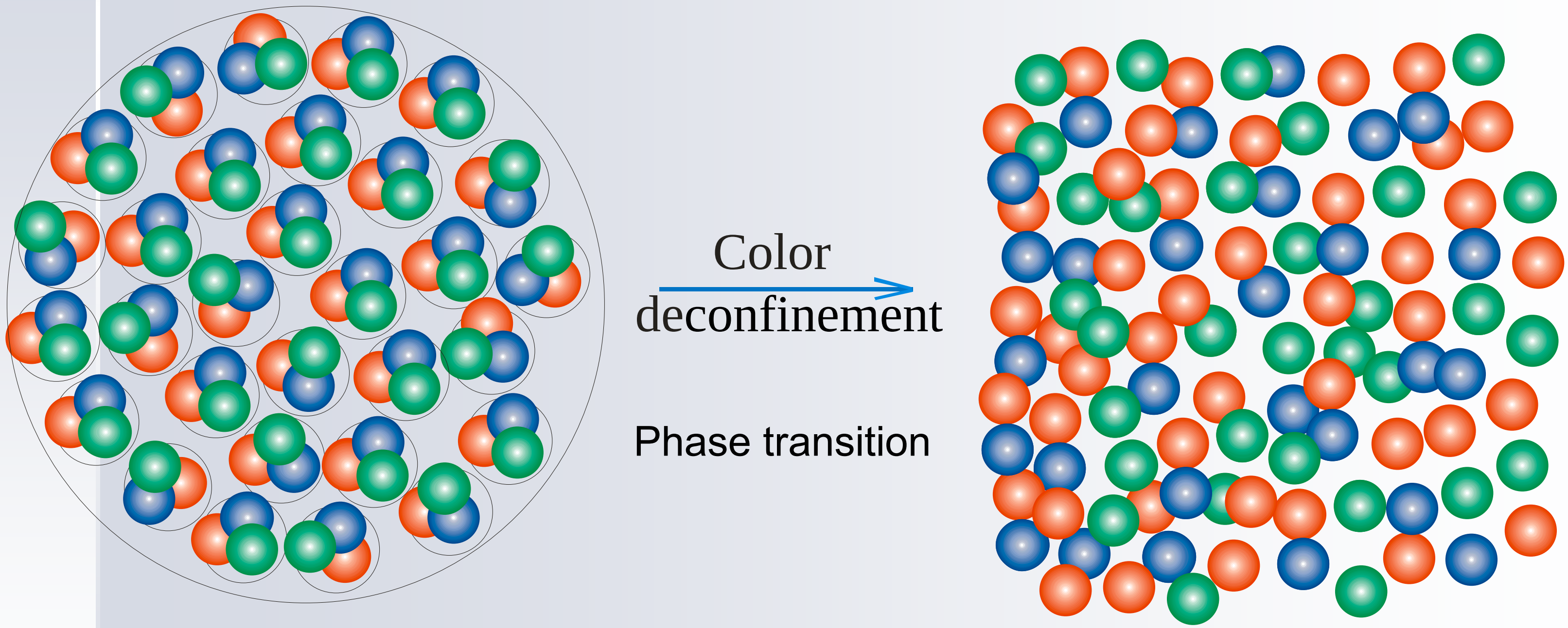
Relativistic collisions heavy ions, elliptic flows



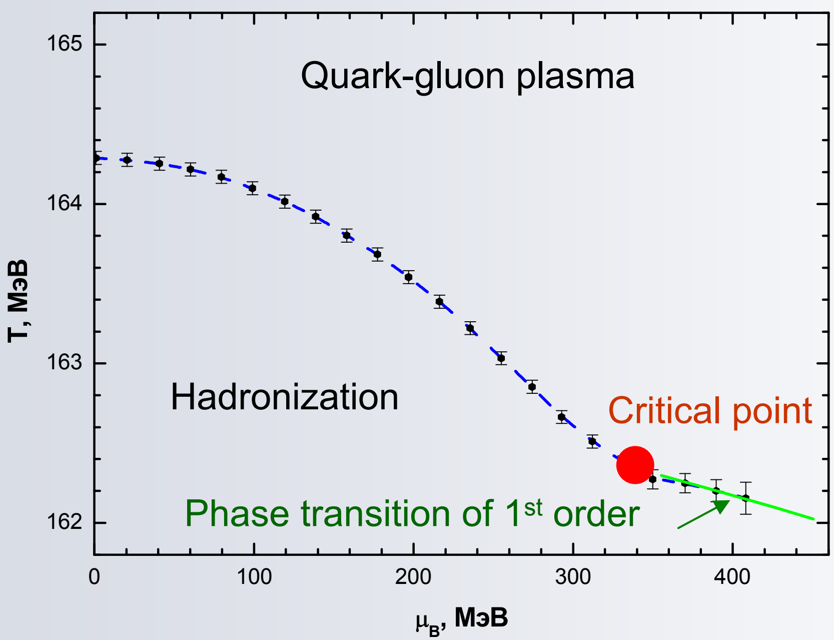

100 мкс
200 мкс
400 мкс
600 мкс
800 мкс
1000 мкс
1500 мкс
2000 мкс


LHCb experiment:
700 physicists
50 institutes
15 countries
Jet d' Eau 140 m
Mont Blanc, 4808 m
CERN
LHCb
ATLAS
CMS
ALICE

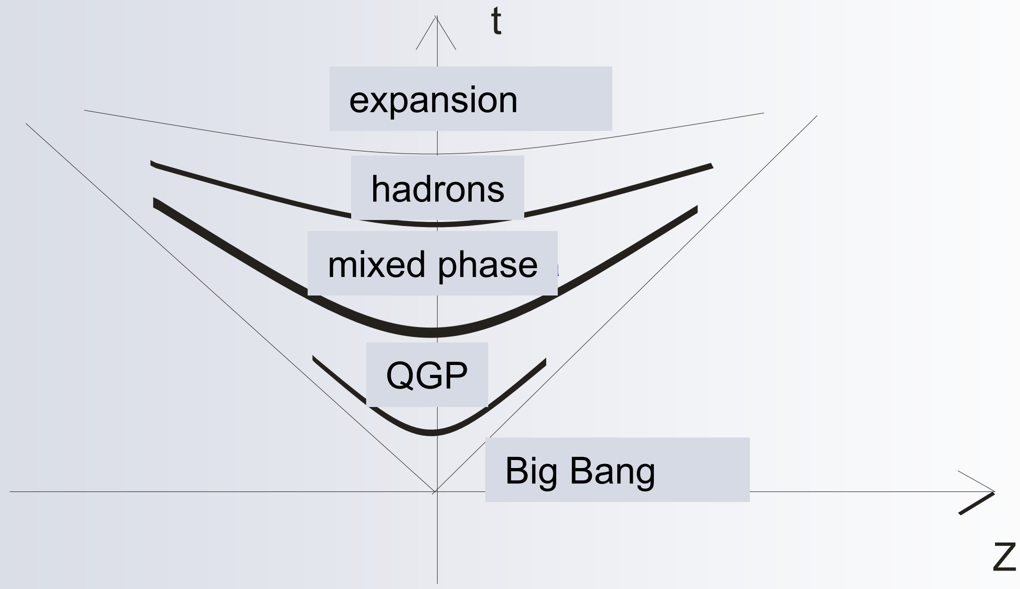
Formation of quark-gluon plasma



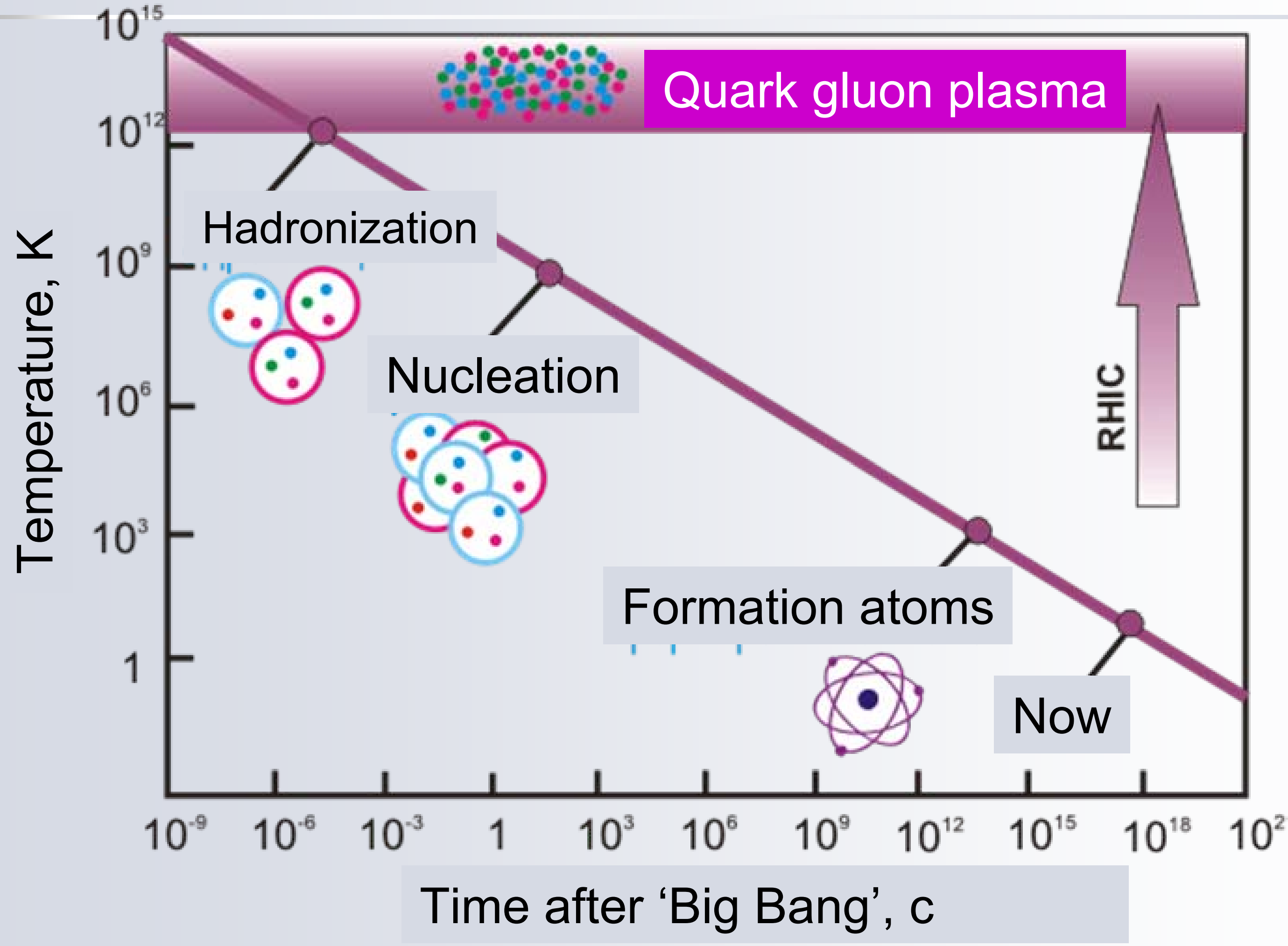
Phase diagram of quark-gluon plasma



Space-time evolution of matter after collision or 'Big Bang'



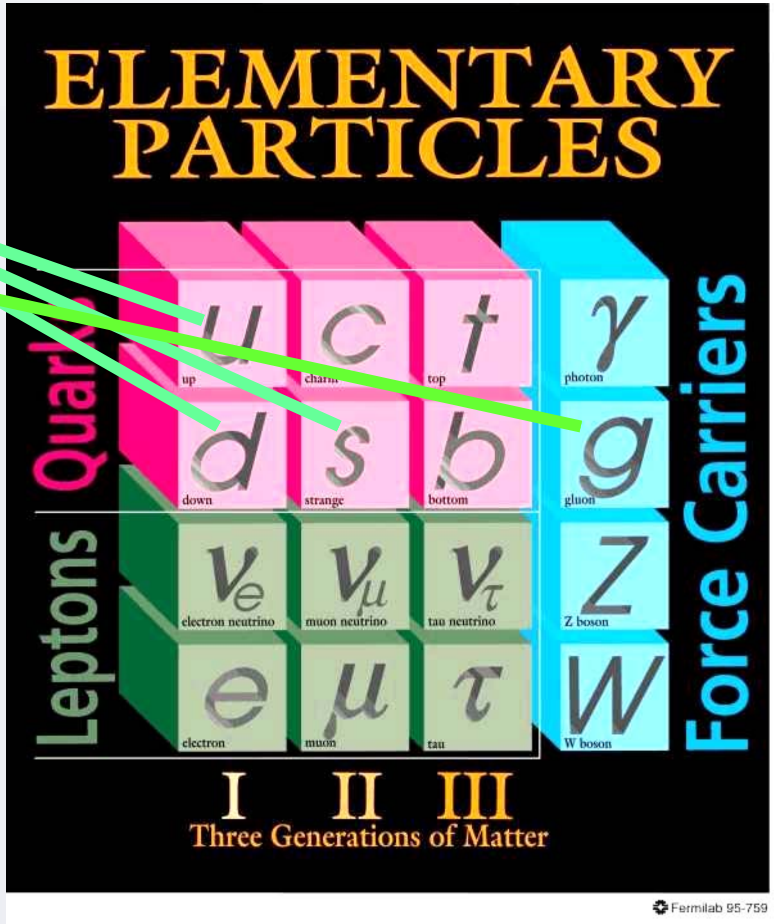
Evolution of matter after 'Big Bang'



INTRODUCTION

Main Problems: starting from Lagrangian

$$L = -\frac{1}{g^2} \mathrm{Tr}\, F_{\mu\nu}^2 + \sum_f \bar{\psi}_f (D+m) \psi_f$$



(1) obtain hadron spectrum,
(2) calculate various matrix elements,
(3) describe phase transitions, and phase diagram
(4) explain confinement of color

http://www.claymath.org/millennium/

INTRODUCTION

Methods

- Imaginary time $t \rightarrow it$

$$Z = \int D\varphi \exp\{i\, S[\varphi]\} \longrightarrow Z = \int D\varphi \exp\{-S[\varphi]\}$$

- Space-time discretization

$$D\varphi(x) \Rightarrow \prod_x d\,\varphi_x$$

$$Z = \int \prod_x d\varphi_x \exp\{-S[\varphi]\}$$

- Thus we get from functional integral the partition function for statistical theory in four dimensions

http://www.ihed.ras.ru/fortov/polikarpov.ppt

For lattice L^4 $(L=48,\ L^4=5{,}308{,}416)$

Integrals of dimension $32L^4$ $(L=48,\ 32L^4=169{,}869{,}312)$

Work with matrices $12L^4 \times 12L^4$

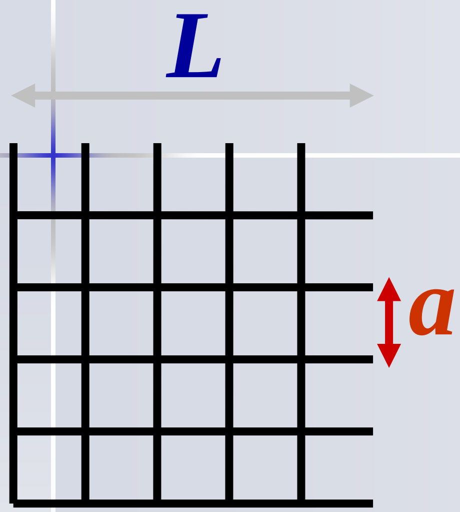
$(L=48,\ 12L^4=63{,}700{,}992)$

$$\boxed{Z=\int d\varphi\, d\bar{\varphi}\, \exp\{\bar{\varphi} M \varphi\}}$$

http://www.ihed.ras.ru/fortov/polikarpov.ppt

INTRODUCTION

Three limits

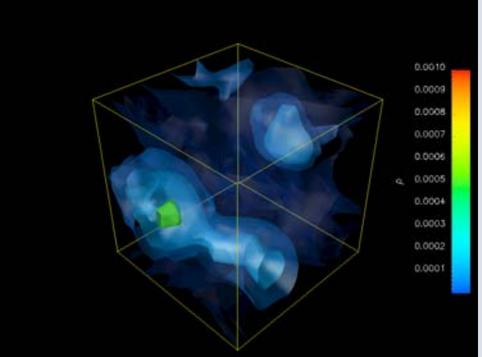


Lattice spacing	$a \to 0$
Lattice size	$L \to \infty$
Discrete quark mass	$m_q \to 0$

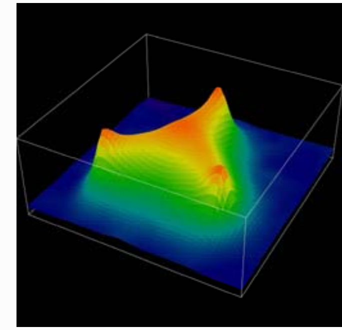
For typical lattice ***$L=48$, $L^4=5,308,416$***

We calculate integrals of dimension ***$32L^4 = 169,869,312$***

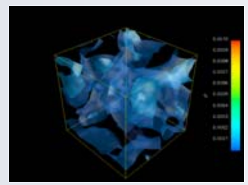
http://www.ihed.ras.ru/fortov/polikarpov.ppt



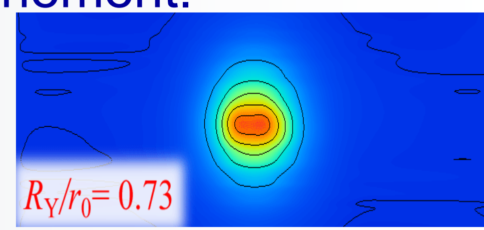
Conclusions from Lattice simulations

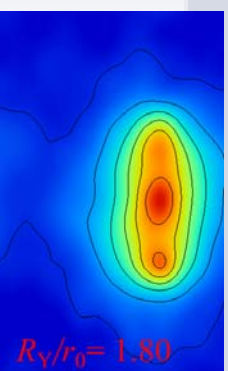


- Computer simulations a) reproduce well known hadron properties b) predict new phenomena c) help to create new theoretical ideas.



- Low dimensional objects (regions or quasiparticles - dressed quarks and gluons) are responsible for most interesting nonperturbative effects: chiral symmetry breaking, topological susceptibility and confinement.



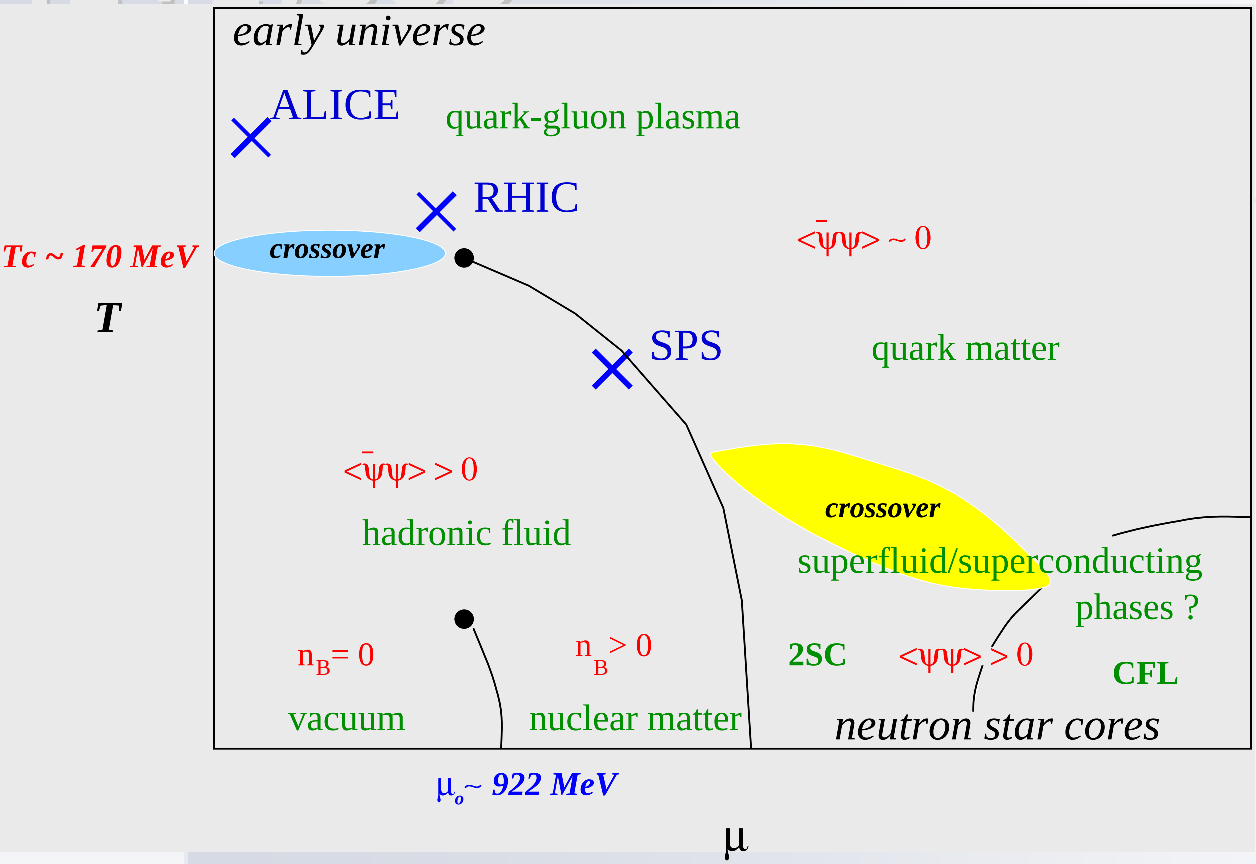


- The era of traditional quantum field theory (Feynman graphs, perturbation theory) is over, nonperturbative field theory is close in spirit to solid state theory; we have to study dislocations, fractals, phase transitions etc.

http://www.ihed.ras.ru/fortov/polikarpov.ppt

SU(2) glue SU(3) glue **2qQCD** (2+1)QCD

Phase diagram
(F.Karsch)



Talks at Lattice 2006!

Mark G. Alford hep-lat/0610046; C. Schmidt hep-lat/0610116; Urs M. Heller hep-lat/061011; M.A. Stephanov hep-lat/0701002

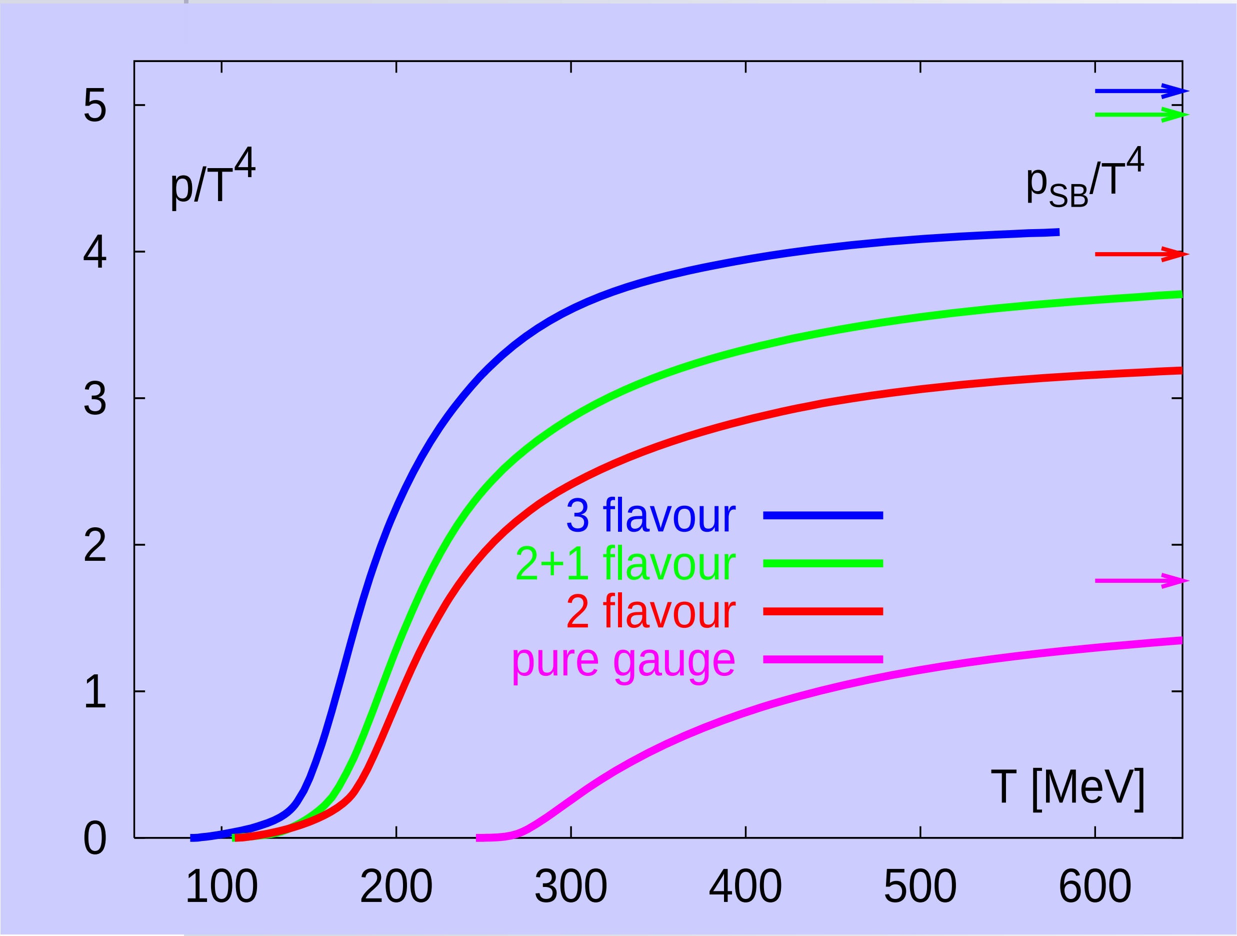
Talks at Lattice 2007!

G. Aarts hep-lat/0710.0739; Z. Fodor hep-lat/0710.0739; F. Karsch hep-lat/0711/0661

SU(2) glue SU(3) glue 2qQCD (2+1)QCD

Quark-Gluon Plasma at Finite Temperature

Plasma thermodynamics, example: pressure



F. Karsch (2001-2005)

Quantum Monte Carlo method
Canonical ensemble

- Binary mixture of *Ne* electrons and *Ni protons*
- Partition function:

$$Z(N_e,N_i,V,\beta) = Q(N_e,N_i,\beta)/N_e!N_i!$$

$$Q(N_e,N_i,\beta) = \sum_\sigma \boxed{\int_V dr\,\rho(r,\sigma;\beta)}$$

- N-particle density matrix:

$$\rho = \exp(-\boxed{\beta}H) = \underbrace{\exp(-\boxed{\Delta\beta}H)\times\ldots\times\exp(-\Delta\beta H)}_{n+1}$$

$$\beta = 1/kT \qquad \Delta\beta = \beta/(n+1)$$

ACCURACY OF DIRECT PIMC

$$\hat{\rho} = e^{-\beta \hat{H}} = \underbrace{e^{-\Delta\beta\hat{H}} \times \ldots \times e^{-\Delta\beta\hat{H}}}_{n+1}$$

$\beta = 1/k_B T$ $\quad \hat{H} = \hat{K} + \hat{U}$ $\quad \Delta\beta = \beta/(n+1)$

$$\hat{\rho}_{\Delta\beta} = e^{-\Delta\beta\hat{H}} = e^{-\Delta\beta\hat{K}} e^{-\Delta\beta\hat{U}} e^{-\frac{(\Delta\beta)^2}{2}[\hat{K},\hat{U}]}$$

$1+O(1/n)^2$ for every multiplier at $n \to \infty$

Error ~ $(\beta / n)^2$ for every multiplier

Total error $\Delta\rho \sim \beta^2 / n \to 0$ at $n \to \infty$

$$\rho_{\Delta\beta} = \rho_{free}\rho_{pot}(\Delta\beta)$$

High-temperature pseudopotential

Quark-gluon plasma in canonical ensemble

- High temperature partition function [Phys. Rev. C, **74**, 044909, (2006)]
- NEW: Mixture of quasiparticles: N_q quarks, $N_{q'}$ antiquarks, and N_g *gluons,*
- Q – *color coordinates of each quasiparticle - vector on unit 3D sphere*

$$\hat{H} = \sum \frac{\hat{p}_\alpha^{\,2}}{2m_\alpha} + \sum \frac{g^2 < \vec{Q}_\alpha | \vec{Q}_\beta >}{4\pi | r_\alpha - r_\beta |}$$

$$Z\left(N_q, N_{q'} N_g, V, \beta\right) = Q\left(N_q, N_{q'} N_g, \beta\right) \Big/ N_q! N_{q'}! N_g!$$

$$Q\left(N_q, N_{q'}, N_g, \beta\right) = \sum_\sigma \int_V dr dQ \, \rho\left(r, Q, \sigma; \beta\right)$$

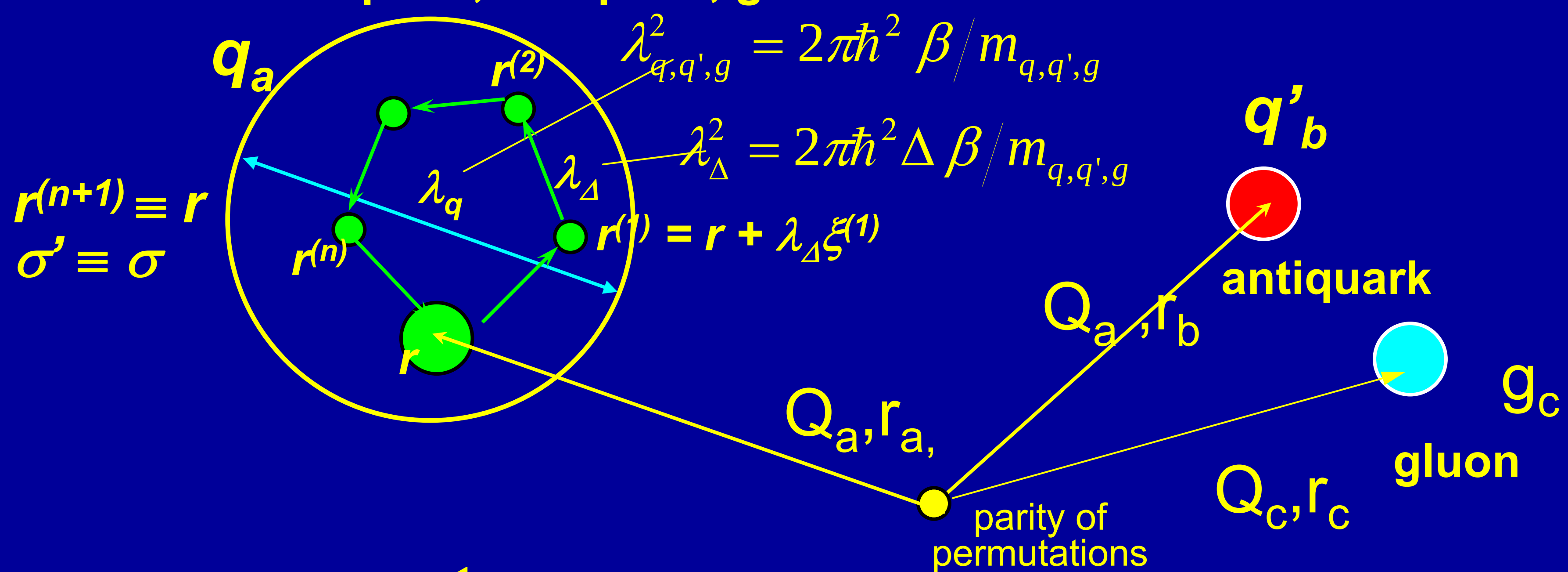
- Our modification of [Phys.Rev. C, **74**, 044909, (2006)] :
  temperature decomposition of N-particle density matrix:

$$\rho = \exp\left(-\beta\hat{H}\right) = \underbrace{\exp\left(-\Delta\beta\hat{H}\right) \times \ldots \times \exp\left(-\Delta\beta\hat{H}\right)}_{n+1}$$

$$\beta = 1/kT \qquad \Delta\beta = \beta/(n+1)$$

PATH INTEGRALS
MONTE-CARLO METHOD

quark, antiquark, gluon



$$\rho(r,Q,\sigma;\beta) = \frac{1}{\lambda_{\Delta q}^{3N_q}\lambda_{\Delta q'}^{3N_{q'}}\lambda_{\Delta g}^{3N_g}} \sum_{P=P_q,P_{q'},P_g} (\pm 1)^{\kappa_P} \int_V dr^{(1)}\ldots dr^{(n)} dQ^{(1)}\ldots dQ^{(n)} \times$$

$$\rho\left(r,Q;r^{(1)},Q^{(1)};\Delta\beta\right)\ldots\rho\left(r^{(n)},Q^{(n)};\hat{P}r^{(n+1)},\hat{P}Q^{(n+1)};\Delta\beta\right)\boxed{S\left(\sigma,\hat{P}\sigma'\right)}$$

spin matrix

$$\rho(r^{(l)},Q^{(l)};r^{(l+1)},Q^{(l+1)}) \approx \delta(Q^{(l)}-Q^{(l+1)})\rho(r^{(l)},Q^{(l)};r^{(l+1)},Q^{(l+1)})$$

Density matrix

$$\sum_{\sigma} \rho(r, Q, \sigma; \beta) = \frac{1}{\lambda_{\Delta}^{3N_q} \lambda_{\Delta}^{3N_{q'}} \lambda_{\Delta}^{3N_g}} \sum_{s=0}^{N_q} \sum_{s'=0}^{N_{q'}} \sum_{s''=0}^{N_g} \rho_{ss's''}([rQ], \beta)$$

$$\rho_{ss's''}([rQ], \beta) = \frac{C_{N_q}^{s}}{2^{N_q}} \frac{C_{N_{q'}}^{s'}}{2^{N_{q'}}} \frac{C_{N_g}^{s''}}{2^{N_g}} \exp\{-\beta U([rQ], \beta)\} \times$$

$$\times \prod_{l=1}^{n} \prod_{p=1}^{N_e} \phi_{pp}^{l} \det \left| \psi_{ab}^{n,1} \right|_{s} \prod_{p=1}^{N_i} \tilde{\phi}_{pp}^{l} \det \left| \tilde{\psi}_{ab}^{n,1} \right|_{s'} \prod_{p=1}^{N_i} \approx{\phi}_{pp}^{l}\, per \left| \approx{\psi}_{ab}^{n,1} \right|_{s''}$$

$$U([rQ], \beta) = \sum_{l=0}^{n} \frac{U_{l}^{qq'g}([r^{(l)}Q], \beta)}{n+1}$$

Kelbg potentials

Exchange matrix

$$\left\| \psi_{ab}^{n,1} \right\|_{s} \equiv \left\| \exp\left\{ -\frac{\pi}{\lambda_{\Delta}^{2}} \left| (r_a - r_b) + y_a^n \right|^2 \right\} \right\|_{s}$$

KELBG PSEUDOPOTENTIAL

First order perturbation theory solution of two-particle Bloch equation for density matrix in the limit of weak coupling

$$\Phi^{ab}(\mathbf{r}_{ab},\mathbf{r}'_{ab},\Delta\beta) = e_a e_b \int_0^1 \frac{d\alpha}{\boxed{d_{ab}}(\alpha)} \operatorname{erf}\left(\frac{d_{ab}(\alpha)}{2\boxed{\lambda_{ab}}\sqrt{\alpha(1-\alpha)}} \right)$$

$$d_{ab}(\alpha) = \left|\alpha\mathbf{r}_{ab} + (1-\alpha)\mathbf{r}'_{ab}\right|$$

$$\lambda_{ab} = \hbar^2\beta/2\mu_{ab}$$

$$\mu_{ab}^{-1} = m_a^{-1} + m_b^{-1}$$

Diagonal Kelbg potential:

$$\boxed{\Phi^{ab}(\mathbf{r}_{ab},\Delta\beta)} = \frac{e_a e_b}{\lambda_{ab} x_{ab}} \left\{ 1 - e^{-x_{ab}^2} + \sqrt{\pi}\, x_{ab} \left[1 - \operatorname{erf}(\boxed{x_{ab}}) \right] \right\}$$

$$|\mathbf{r}_{ab}| \to 0$$

$$\frac{\sqrt{\pi}\, e_a e_b}{\lambda_{ab}}$$

$$|\mathbf{r}_{ab}| >> \lambda_{ab}$$

$$\frac{e_a e_b}{|r_{ab}|}$$

$$x_{ab} = |\mathbf{r}_{ab}| / \lambda_{ab}$$

Color Coulomb or Color Kelbg potential ?

Richardson, Gelman, Shuryak, Zahed, Harmann, Donko, Leval, Kalman (r=0 ?)

$$x_{ab} = |\mathbf{r}_{ab}|/\lambda_{ab} \qquad \lambda_{ab} = 2\pi\hbar^2\varepsilon/\mu_{ab}$$

Our suggestion: Color Kelbg potential

$$\boxed{\Phi^{ab}(x_{ab},\varepsilon) =} \frac{<\vec{Q}_a \,|\, \vec{Q}_b> g^2}{\lambda_{ab} x_{ab}} \left\{1 - e^{-x_{ab}^2} + \sqrt{\pi}\, x_{ab}\left[1 - erf\left(x_{ab}\right)\right]\right\}$$

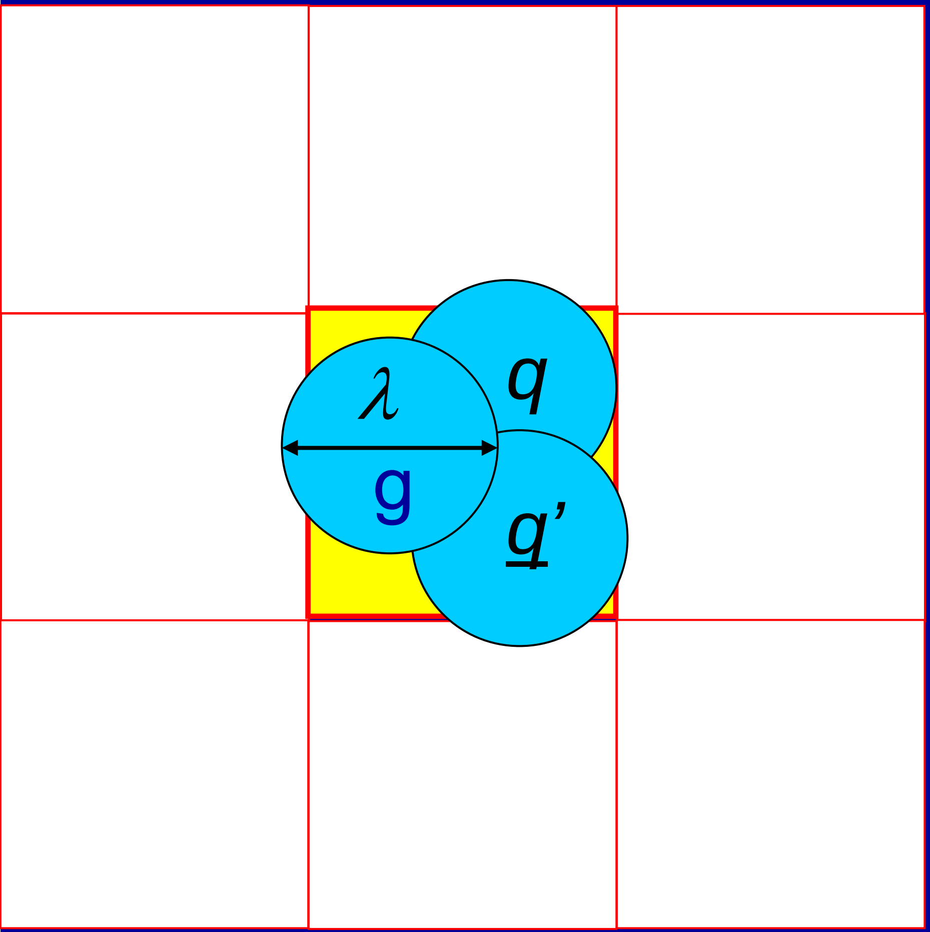
$$\boxed{|\mathbf{r}_{ab}| \to 0} \qquad \boxed{|\mathbf{r}_{ab}| >> \lambda_{ab}}$$

$$\boxed{\frac{\sqrt{\pi}\, e_a e_b}{\lambda_{ab}}} \qquad \boxed{\frac{e_a e_b}{|r_{ab}|}}$$

Objects Q are color coordinates of quarks and gluons
There is no divergence at small interparticle distances and it has a true asymptotics (T, x_{ab})

Ha -> $k_B T_c$, T_c =175 Mev,
$T_c < T$, $m_a \sim 5 k_B T_c/c^2$,
$L_o \sim hc/k_B T_c$, $r_s = \langle r\rangle/L_o < 0.1$,
$L_o \sim 7\ 10^{-15}$ m, $x_{ab} \sim 1$

PERIODIC BOUNDAY CONDITIONS TREATMENT OF EXCHANGE EFFECTS



Inside main cell –
exchange matrix (exact)

Accuracy control of sign problem – comparison with ideal degenerate gas

$$n_e \lambda_e^3 \sim 30$$

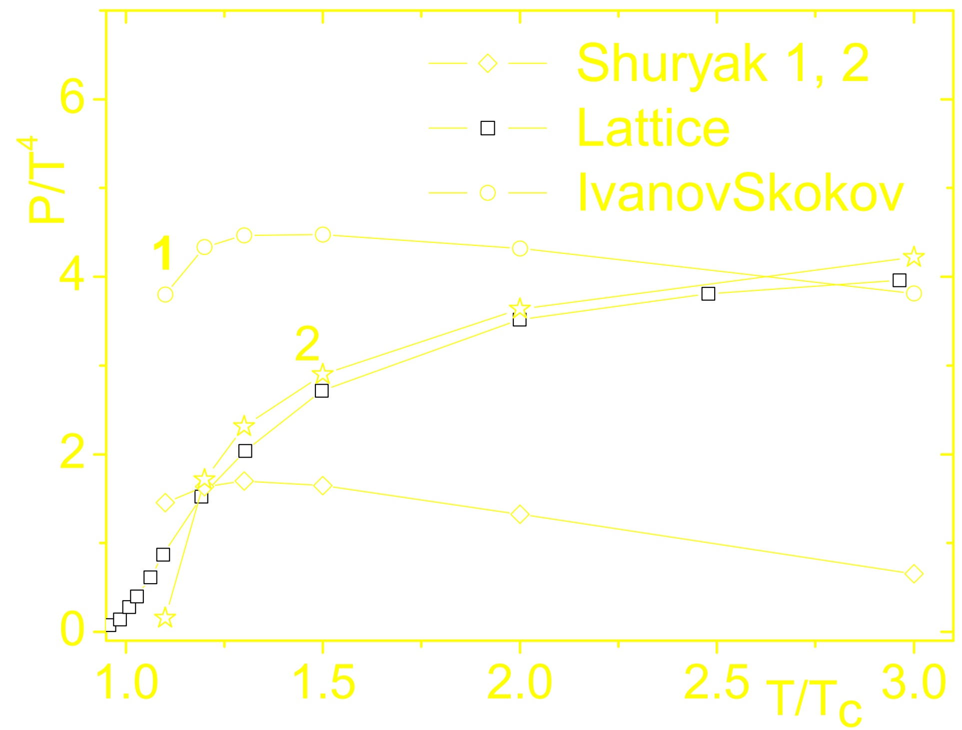
Filinov V.S. // J. Phys. A: Math. Gen. ***34****, 1665 (2001)*
Filinov V.S. et al. // J. Phys. A: Math. Gen. ***36****, 6069 (2003)*

EQUATION OF STATE

Three models for Mall (T), n(T), g2(T):
1) Shuryak,2)Ivanov, Skokov, 3)Modified Shuryak

Shuryak 2 -

<*r*> = 0.42 fm



Pair distribution functions

Color correlation functions

$$\hat{H} = \sum \frac{\hat{p}_\alpha^{\,2}}{2m_\alpha} + \sum \frac{g^2 < \vec{Q}_\alpha | \vec{Q}_\beta >}{4\pi | r_\alpha - r_\beta |}$$

$$Z\left(N_q, N_{q'} N_g, V, \beta\right) = Q\left(N_q, N_{q'} N_g, \beta\right) / N_q! N_{q'}! N_g!$$

$$Q\left(N_q, N_{q'}, N_g, \beta\right) = \sum_\sigma \int_V drdQ \rho\left(r, Q, \sigma; \beta\right)$$

$$g_{ab}(R_1 - R_2) = g_{ab}(R_1, R_2) = \frac{1}{Q(N_q, N_{q'}, N_g)} \times$$

$$\sum_\sigma \int_V drd\, Q\, \delta(R_1 - r^a{}_1) \delta(R_2 - r^b{}_2) \rho(r, Q, \sigma; \beta),$$

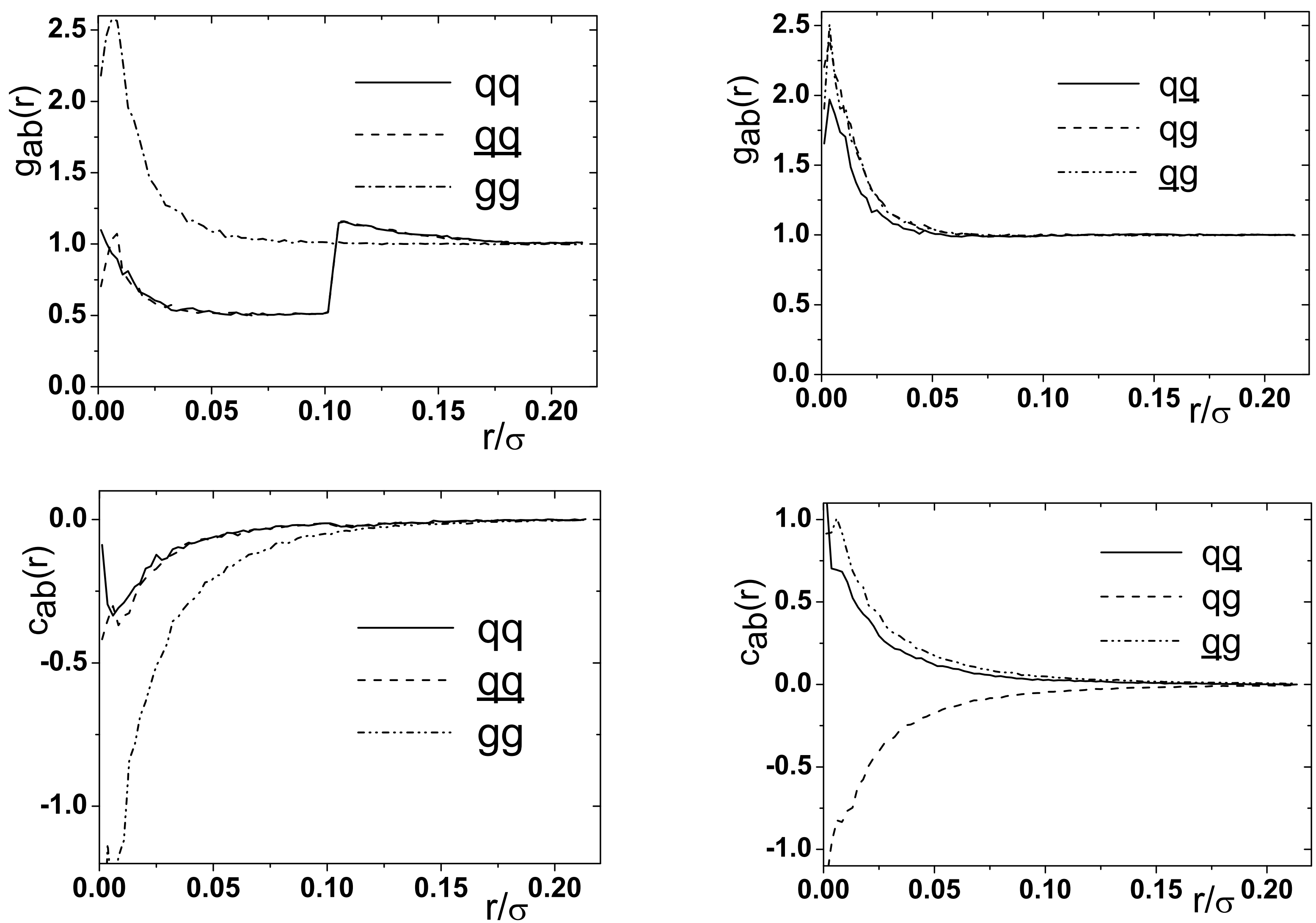
$$c_{ab}(R_1 - R_2)_{Def} = \frac{1}{Q(N_q, N_{q'}, N_g)} \sum_\sigma \int_V drd\, Q \times$$

$$\delta(R_1 - r^a{}_1) \delta(R_2 - r^b{}_2) < Q^1{}_a | Q^2{}_b > \rho(r, Q, \sigma; \beta)$$

PAIR DISTRIBUTION FUNCTIONS

$T = 3\ T_c$

COLOR CORRELATION FUNCTIONS



Thank you for attention.

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