The 4th Dimension of the Proton

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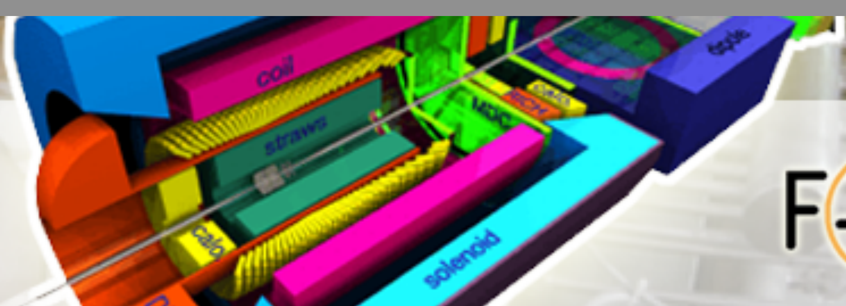
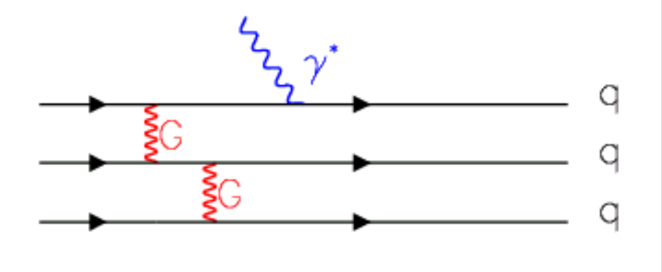


The Time-like Region

$$e^+ + e^- \to \bar{p} + p\,,$$
$$\bar{p} + p \to e^+ + e^-$$

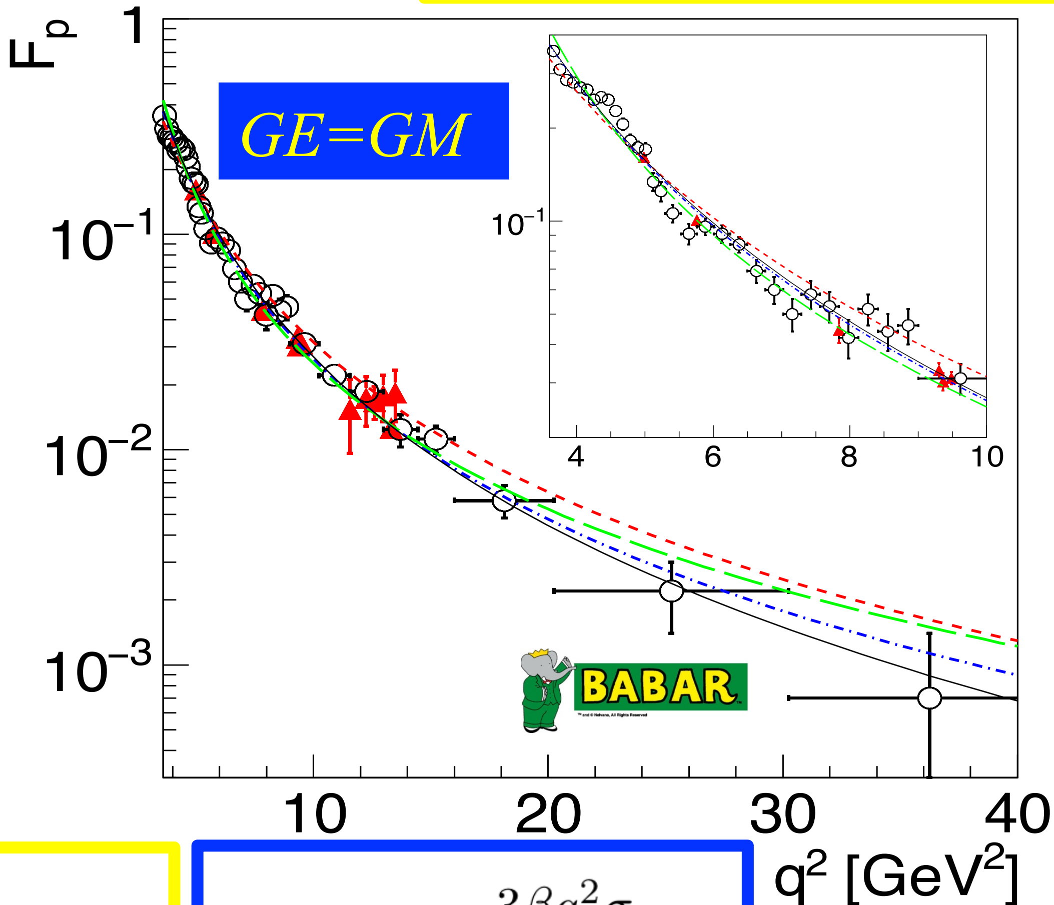


Expected QCD scaling $(q^2)^2$

$$\frac{\mathcal{A}}{(q^2)^2\left[\log^2(q^2/\Lambda^2)+\pi^2\right]}.$$

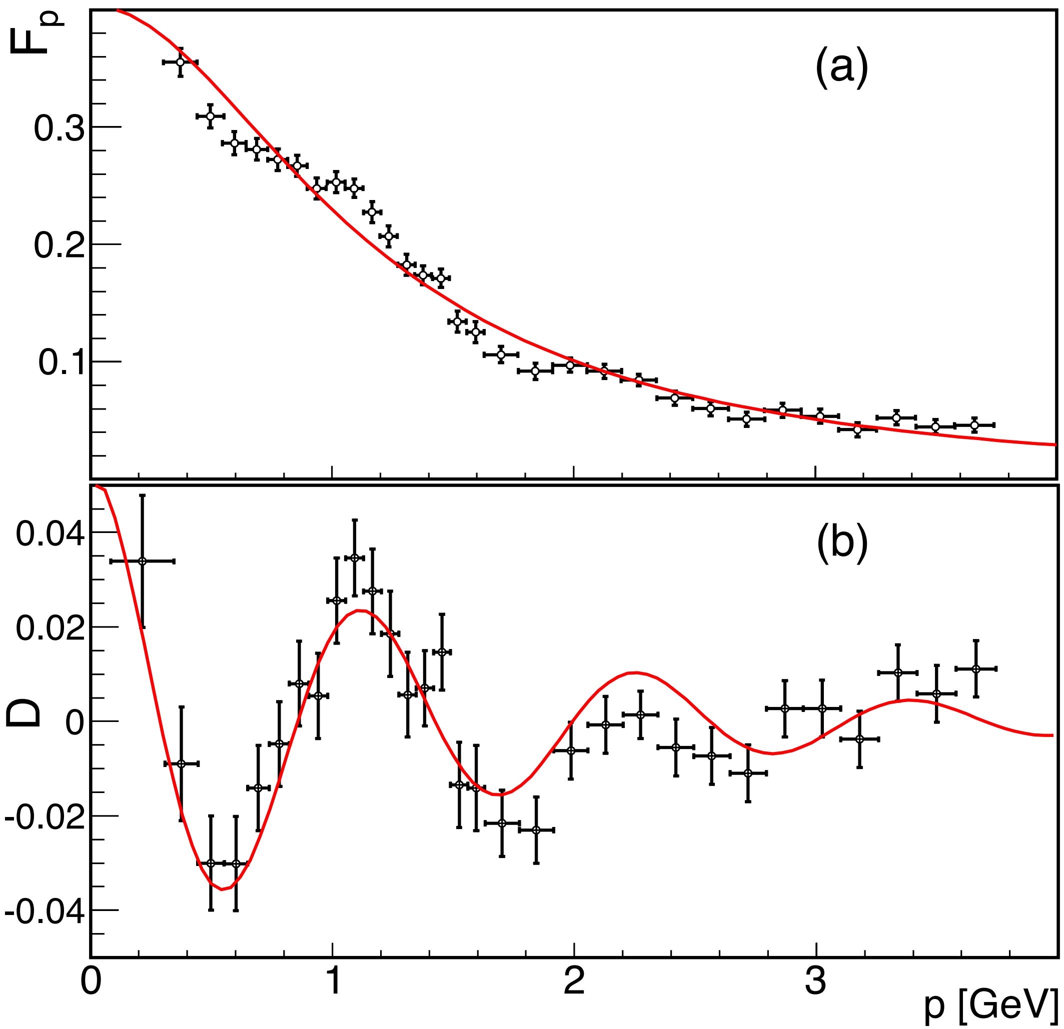
$$\frac{\mathcal{A}}{(1+q^2/m_a^2)\left[1-q^2/0.71\right]^2},$$

$$|F_{T3}(q^2)| = \frac{\mathcal{A}}{(1-q^2/m_1^2)(2-q^2/m_2^2)}.$$



$$|F_p|^2 = \frac{3\beta q^2 \sigma}{2\pi\alpha^2\left(2+\frac{1}{\tau}\right)};$$

Proton TL EM Form Factors



Periodic structures recently discovered in TL region

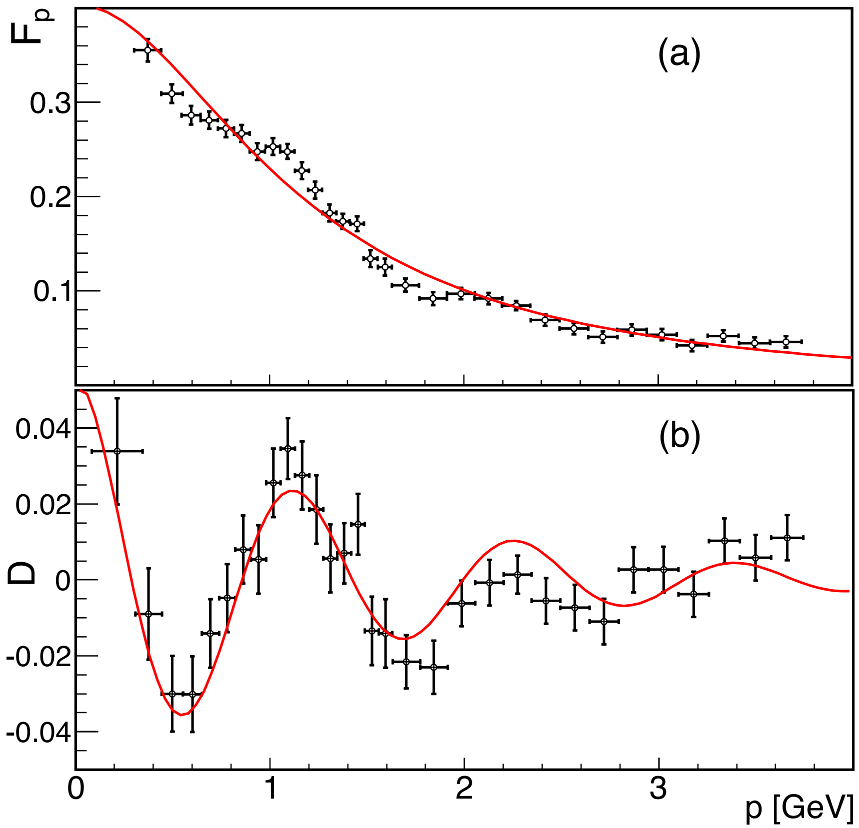
- *Hadron creation from vacuum*

(Resonances?)

A. Bianconi, E. T-G. Phys. Rev. Lett. 114,232301 (2015), PRC 93, 035201 (2016)

Oscillations : regular pattern in P_{Lab}

The relevant variable is p_{Lab} associated to the relative motion of the final hadrons.



$$\boxed{F_{osc}(p) \equiv A \exp(-Bp) \cos(Cp + D).}$$

$A \pm \Delta A$	$B \pm \Delta B$ $[GeV]^{-1}$	$C \pm \Delta C$ $[GeV]^{-1}$	$D \pm \Delta D$	$\chi^2/n.d.f$
0.05 ± 0.01	0.7 ± 0.2	5.5 ± 0.2	0.03 ± 0.3	1.2

A: Small perturbation B: damping
C: r < 1fm D=0: maximum at p=0

Simple oscillatory behaviour
Small number of coherent sources

A. Bianconi, E. T-G. Phys. Rev. Lett. 114,232301 (2015), PRC 93, 035201 (2016)

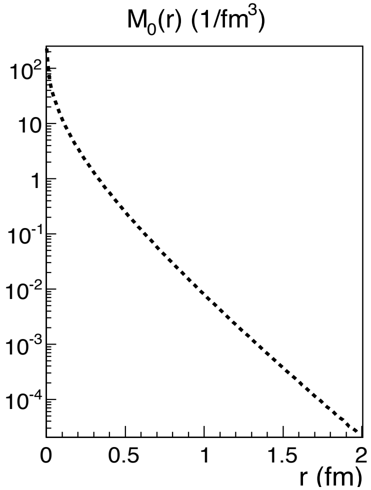
Fourier Transform

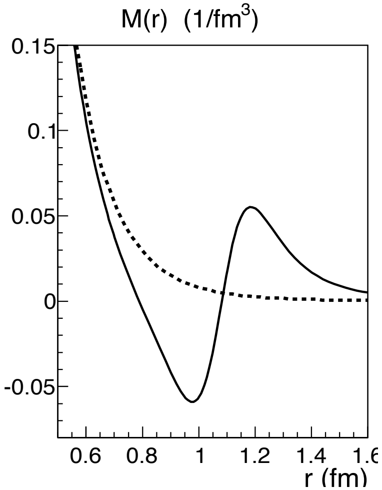
$$F_0(p) \equiv \int d^3\vec{r}\ exp(i\vec{p}\cdot\vec{r})\ M_0(r)$$

$$F(p) = F_0(p) + F_{osc}(p) \equiv \int d^3\vec{r}\ exp(i\vec{p}\cdot\vec{r}) M(r).$$

$$F_0 = \frac{\mathcal{A}}{(1 + q^2/m_a^2)\left[1 - q^2/0.71\right]^2},$$

$$F_{osc}(p) \ \equiv\ A\ \exp(-Bp)\ \cos(Cp\ +\ D).$$





- *Rescattering processes*
- *Large imaginary part*
- *Related to the time evolution of the charge density?*

  (E.A. Kuraev, E. T.-G., A. Dbeyssi, PLB712 (2012) 240)
- *Consequences for the SL region?*
- *Data from BESIII confirm the structure*
- *Expected from PANDA*

Definition of TL-SL Form Factors

$$F(q^2) = \int_{\mathcal{D}} d^4 x e^{iq_\mu x^\mu} \rho(x), \; q_\mu x^\mu = q_0 t - \vec{q} \cdot \vec{x}$$

In SL- Breit frame (zero energy transfer):

$$F(q^2) = \delta(q_0) F(Q^2), \; Q^2 = -(q_0^2 - \vec{q}^2) > 0.$$

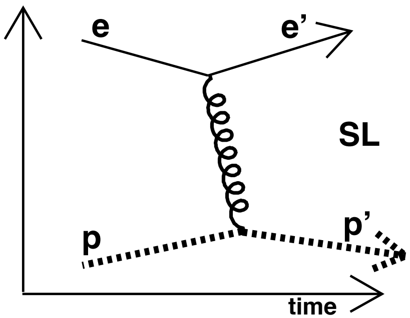
In TL-(CMS): $F(q^2) = \int_{\mathcal{D}} dt e^{i\sqrt{q^2}t} \int d^3\vec{r} \rho(\vec{r}, t) = \int_{\mathcal{D}} dt e^{i\sqrt{q^2}t} Q(t),$

$Q(t)$ *distribution in time of the qqbar pair formation*

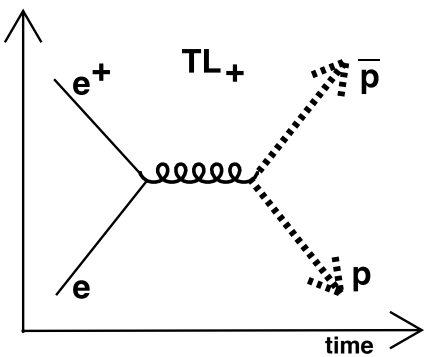
E.A. Kuraev, A. Dbeyssi, E. T-G., PLB 712, 240 (2012)

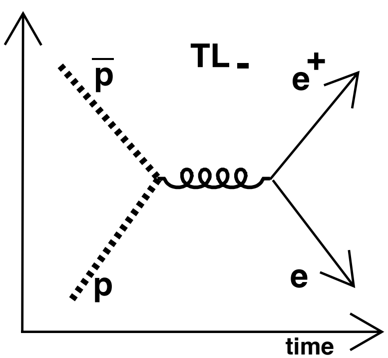
Definition of TL-SL Form Factors

$$F(q^2) = \int_{\mathcal{D}} d^4x\, e^{iq_\mu x^\mu}\rho(x),\quad q_\mu x^\mu = q_0 t - \vec{q}\cdot\vec{x}$$



$\rho(x) = \rho(\vec{x}, t)$ space-time distribution of the electric charge in the space-time volume $\mathcal{D}$.





SL photon 'sees' a charge density

TL photon can NOT test a space distribution.

How to connect and understand the amplitudes?

Definition of TL-SL Form Factors

$$\boxed{F(q) \;=\; \int d^4x e^{iqx} F(x).}$$

$$F_{SL,Breit}(q) \;=\; \int d^3\vec{x}\; e^{-i\vec{q}\cdot\vec{x}} \int dt F(t,\vec{x}) \;\equiv\; \int d^3\vec{x}\; e^{-i\vec{q}\cdot\vec{x}} \rho(|\vec{x}|),$$

$$\rho(|\vec{x}|) \;=\; \int dt F(t,\vec{x}).$$

$$F_{TL,CM}(q) \;=\; \int dt\; e^{iqt} \int d^3\vec{x} F(t,\vec{x}) \;\equiv\; \int dt\; e^{iqt} R(t),$$

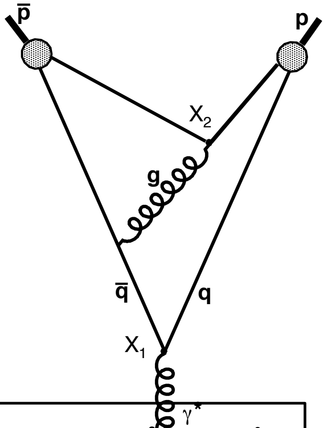
$$R(t) \;=\; \int d^3\vec{x} F(t,\vec{x}).$$

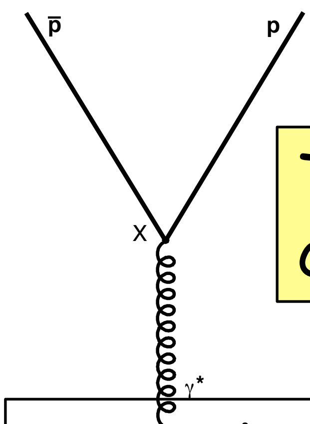
$\boxed{\rho(\vec{x}) \text{ and } R(t),}$ represent projections of the same distribution in orthogonal subspaces

Charge: photon-charge coupling

$\boxed{\rho(\vec{x})}$ Fourier transform of a stationary charge and current distribution

$\boxed{R(t)}$ Amplitude for creating charge-anticharge pairs at time t. Charge distribution => distribution in time of $\boxed{\gamma^* \to charge - anticharge \text{ vertexes}}$

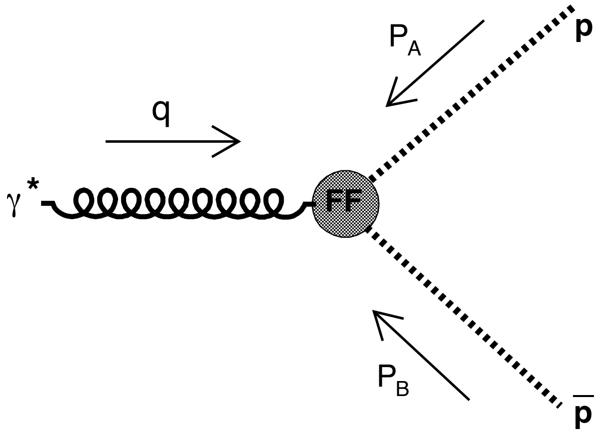




The simplest picture: qq pair + compact di-quark

Resolved Unresolved representation

Amplitudes



$$
\begin{aligned}
SL &: \quad \gamma^*(q_\mu) \;+\; p(p_\mu) \to p(p'_\mu) \\
TL_+ &: \quad \gamma^*(q_\mu) \;\to p(p'_\mu) \;+\; \bar{p}(\bar{p_\mu}') \\
TL_- &: \quad p(p_\mu) \;+\; \bar{p}(\bar{p}_\mu) \;\to \gamma^*(q'_\mu)
\end{aligned}
$$

$$A_{charge}(q, P_A, P_B) \equiv \; \delta^4(q + P_A + P_B)\, T_{point\ charge}(q, P_A, P_B)\; F(q).$$

$$\vec{x}^2 \;>\; t^2 : \; F_{outLC}(x_\mu) \;=\; f(x^\mu x_\mu), \; t \to -t \;\; \textit{forbidden}$$

$$t^2 \;>\; \vec{x}^2 : \; F_{inLC}(x_\mu) \;=\; f_+(x^\mu x_\mu)\theta(t) \;+\; f_-(x^\mu x_\mu)\theta(-t),$$

$$F_{inLC}(x_\mu) \;=\; 1/2\,[f_+ + f_-] \;+\; 1/2\,[f_+ - f_-][\theta(t) - \theta(-t)].$$

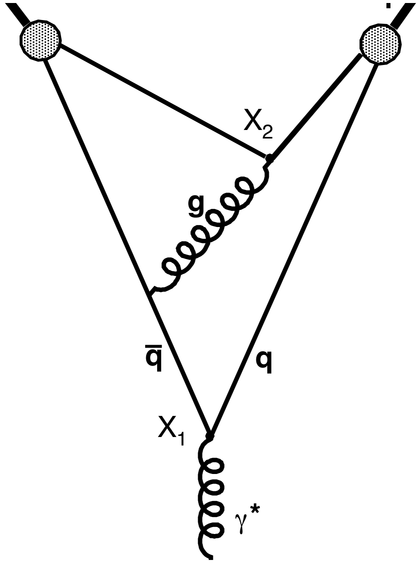
leads to imaginary part of F(q) even if F(x) is real
affects only phases (T-conservation)

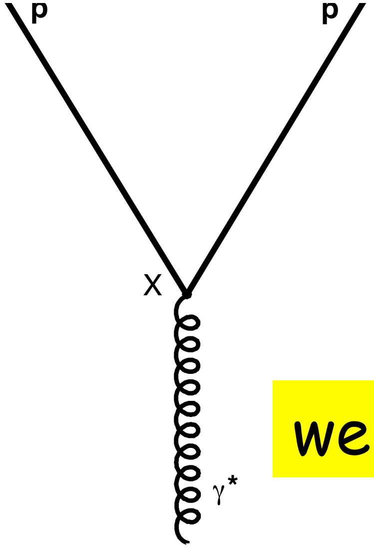
Implementing causality

X (TL) => q(TL)

Fock state of N constituents

$$\psi(X_1, X_2, ...X_N) \equiv e^{ipX}\Phi(x_1, x_2, ...x_N),$$





$$X = \sum w_i X_i, \; i = 1, ...N,$$
$$x_i \equiv X_i - X,$$
$$\sum w_i x_i = 0,$$

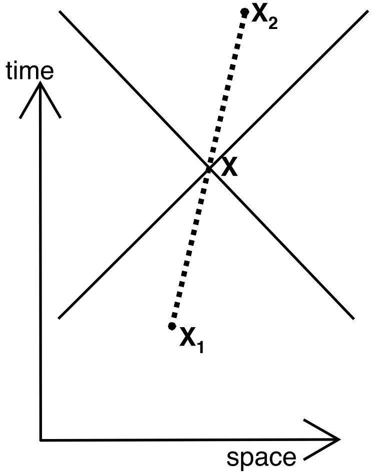
weights related to the masses

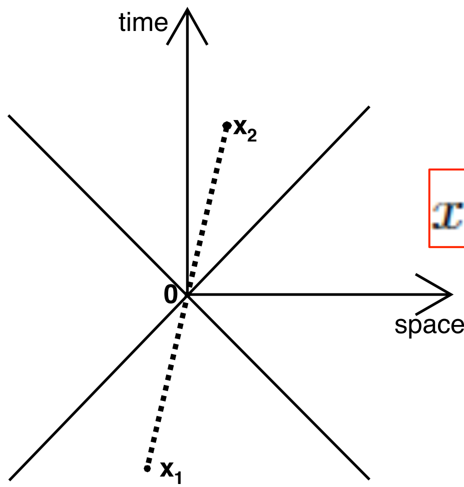
$$A_{TL,charge} = R_{point,charge}(q,p,\bar{p}) \; e_1 \int dX_1 dX_2 ....e^{iiqX_1}\psi^+(X_1, X_2, ...) \; \psi'(X_1, X_2, ...) =$$

$$= R_{point,charge}(q,p,\bar{p}) \; e_1 \int dX e^{[i(q-p-\bar{p})X]} \int dx_1 e^{iqx_1} \int dx_2 ... \delta^4(\sum w_i x_i)\Phi^+(x_1, x_2, ...) \; \Phi'(x_1, x_2, ...) \equiv$$

$$\equiv R_{point,charge}(q,p,\bar{p}) \; \delta^4(q-p-\bar{p}) \int d^4x_1 e^{iqx_1} F(x_1), \qquad x \equiv x_1.$$

Examples



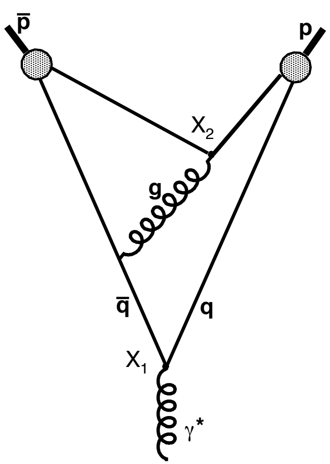


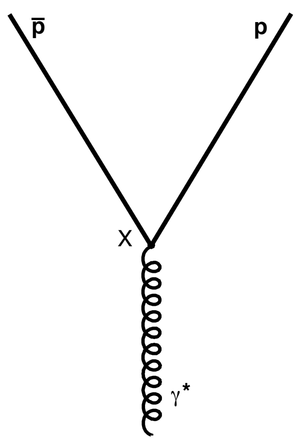
$$e^+e^- \to \bar{p}p$$

$$x \equiv x_1 = (x_1 - x_2)w, \quad w > 0.$$

Causality *implies* $t_1 < t_2$

Unresolved pair created
at $t_1=0$, *implies* $t<0$
Assuming independent probability
of creating a *(anti)proton* (in LC):





$$R(t) = \theta(-t),$$

$$F(q) = \int e^{iqt}\theta(-t) = \frac{\pi}{\epsilon - iq},$$

Examples

- Homogeneous distribution for positive times:

$$R(t) \ = \ \theta(-t),$$

$$F(q) \ = \ \int e^{iqt}\theta(-t) \ = \ \frac{\pi}{\epsilon - iq},$$

- Exponential damping (a is finite):

$$R(t) \ = \ \theta(-t)e^{-a|t|},$$

$$F(q) \ = \ \frac{\pi}{a - iq} \ = \ \frac{a\pi}{a^2 + q^2} + i\frac{q\pi}{a^2 + q^2},$$

Examples: Monopole-like shape

F(x) is non zero in past and future LC.
Annihilation and creation processes are time symmetric.
Differ by a phase.
Summing two terms with the same phase:

$$\boxed{R(t) \;=\; \theta(t)e^{-at} \;+\; \theta(-t)e^{at} \;=\; e^{-a|t|},}$$

$$\boxed{F(q)/\pi \;=\; \frac{1}{a-iq} + \frac{1}{a+iq} \;=\; \frac{2a}{a^2+q^2}.}$$

1/a has the meaning of a formation time.
For large t, R(t) is very small.
Either the second pair is formed within 1/a or the system evolves differently.

=> zero mass resonance of width a

Examples: Lorentzian resonance

Replacing q->q-M : one obtains poles

$$F(q)/\pi \; = \; i\Big(\frac{1}{q-M+ia} - \frac{1}{q-M-ia}\Big) \; \propto \; \frac{1}{(q-M)^2+a^2}.$$

By Fourier transform:

$$R(t) \; \propto \; e^{iMt} \; e^{-a|t|}.$$

Response of a classical damped oscillator to an instantenous external force $\delta(t)$

Negative energy states are allowed by particle-antiparticle symmetry.
To each pole q_0 =M+ia corresponds a pole q_0 =-(M +ia)
Positive poles => creation process
Negative poles=> annihilation process

Examples: Breit-Wigner

A Breit-Wigner probability contains all four poles:

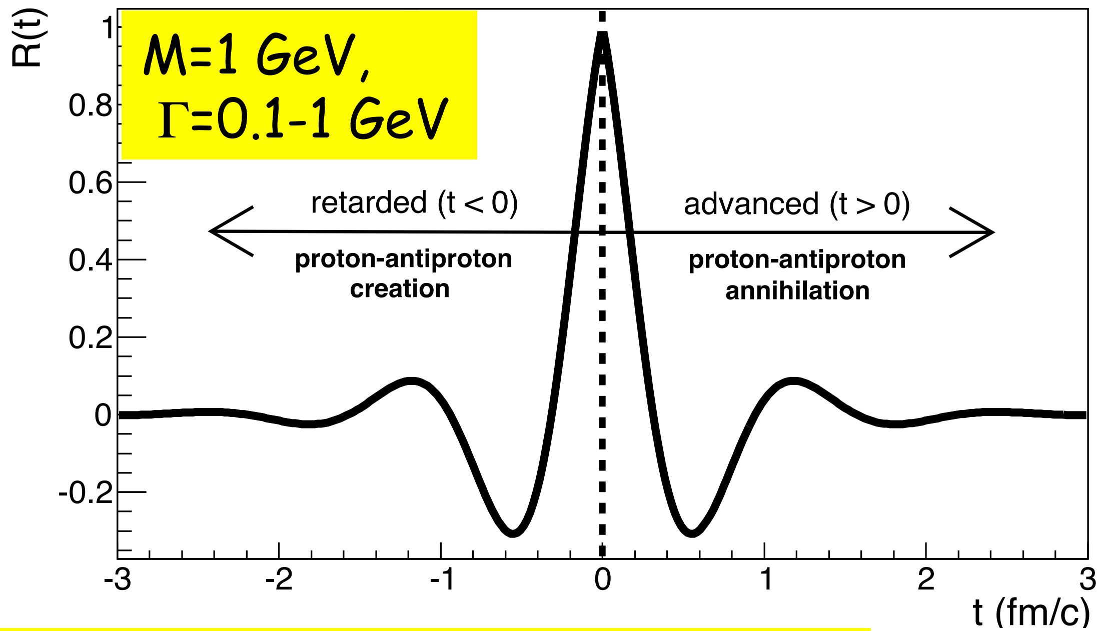
$$F_{\pm}(q) \;\propto\; \frac{1}{(q^2 - M^2) \pm iMa},$$

The combination:

$$F(q) \;\propto\; F_{+}(q) + F_{-}(q)$$

corresponds to

$$R(t) \;\propto\; cos(Mt)\, e^{-a|t|},$$



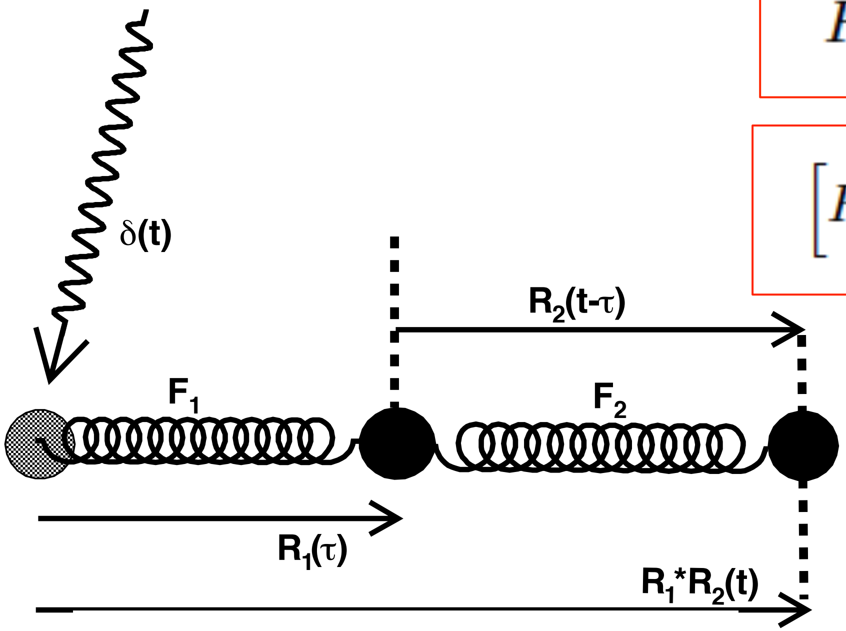
Retarded response of a classical bound and damped oscillator to a $\delta(t)$ external perturbation

$$R(t) \;\equiv\; R_{creation}(t)\theta(-t) \;+\; R_{ann}(t)\theta(t).$$

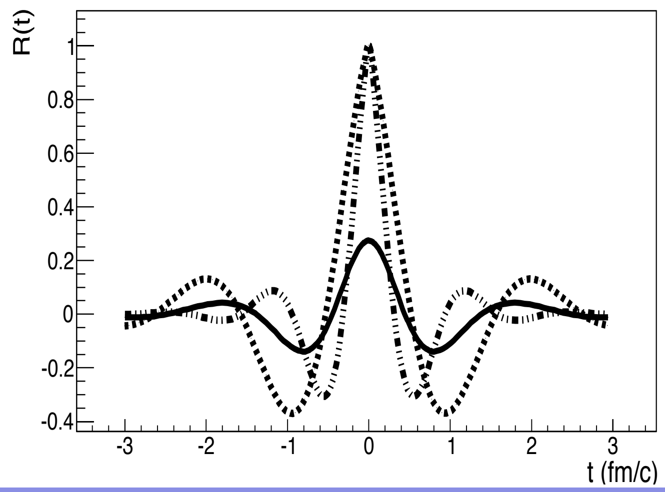
Several spectators: dipole and asymptotics

$$F_1(q)F_2(q) \;=\; F.T.\Big[R_1(t) * R_2(t)\Big],$$

$$\Big[R_1(t) * R_2(t)\Big] \;\equiv\; \int d\tau R_1(\tau)R_2(t-\tau).$$



Use FT properties of convolutions
Chain of two oscillators,
one directly connected to the photon
The second is a decaying correlation
between active quark and spectator



$$R(t) \;=\; \int d\tau\; e^{-a|t-\tau|}\; e^{-b|\tau|}.$$

$$F(q) \;\propto\; \frac{1}{(a^2+q^2)(b^2+q^2)},$$

More complicated examples

$$e^+e^- \rightarrow \bar{p}n\pi^+ \rightarrow \bar{p}p$$

Three quark-antiquarks pair in the intermediate state.

$$R(t) = \Big[[R_1(t) * R_2(t)] * R_3(t)\Big],$$

$$F(q) \propto \frac{1}{(q^2 \pm a^2)(q^2 \pm b^2)(q^2 \pm c^2)},$$

Sum of two contributions of equal shape:

$$\begin{aligned} R(t) &= R_0(t) + aR_0(t-b),\ a \ll 1, \\ F(q) &= F_0(q)[1 + ae^{ibq}], \end{aligned}$$

periodic modulation

Conclusion

- New understanding of Form Factors in the Time-like region: time distribution of quark-antiquark pair creation vertices

$$\boxed{F(q^2) = \int_{\mathcal{D}} d^4x e^{iq_\mu x^\mu} \rho(x), \; q_\mu x^\mu = q_0 t - \vec{q} \cdot \vec{x}}$$

- The distributions tested by the virtual photon are projections in orthogonal 1 and 3-dim spaces of the function F(x): $R(t)$ and $\rho(\vec{x})$

- Simple functions R(t)

- Origin of oscillatory phenomena