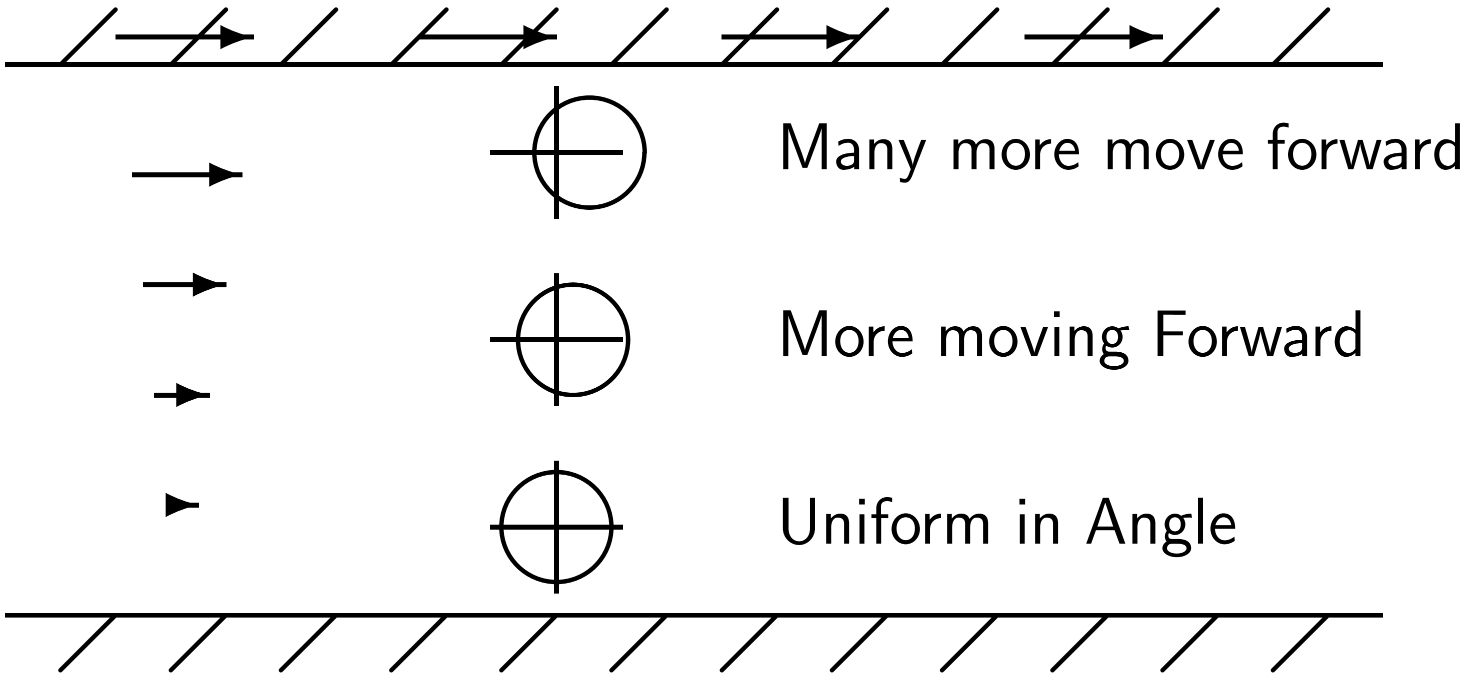
Shear Viscosity in QCD

Guy Moore, TU Darmstadt: proposed SFB-TR 221

- What is shear viscosity?
- Is QCD shear viscosity big? Is it small?
- Why it is hard: theory extraction from experiments
- Why it is hard: direct theory calculations

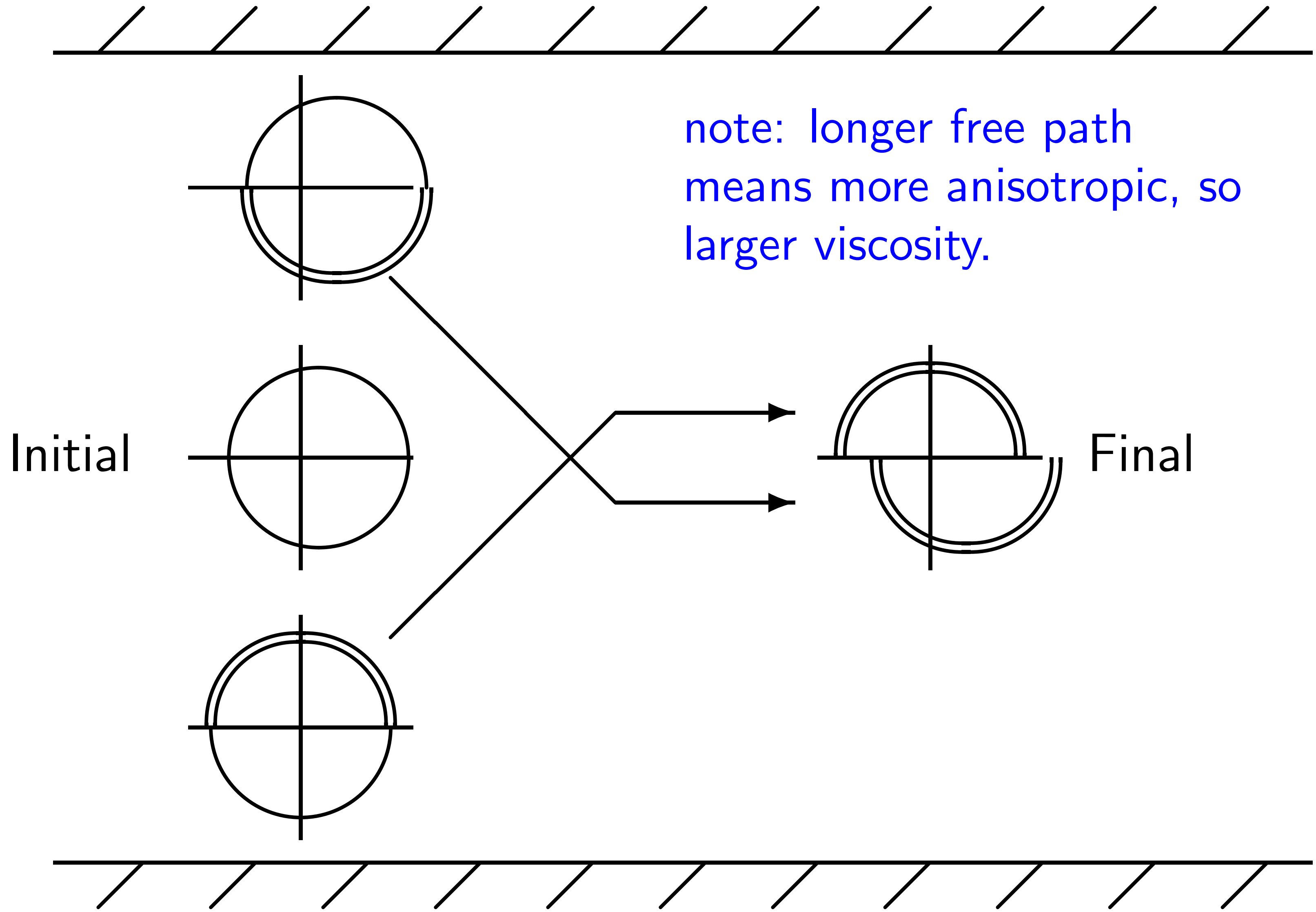
What is shear viscosity? Version 1

Fluid between two plates: move top plate



Fluid in between has varying velocity

Fluid momentum distribution becomes anisotropic:



Skewed momentum distribution.

Force-per-area on bottom surface:

$$\frac{F}{A} = \eta \frac{\Delta v}{L}$$

with Δv velocity difference, L height difference.

Coefficient η defines shear viscosity.

Technically a near-equilibrium quantity: will change if velocities get very large! Defined as small $\Delta v/L$ limit with $L \gg \lambda_{\rm mfp}$ (or other scales)

What is shear viscosity: Hydrodynamics

Consider a fluid system. Assume near equilibrium!

Described only by conserved number densities: ε, π, n_B, ρ_e

Locally in equilibrium: conservation laws

$$\partial_\mu T^{\mu\nu} = 0\,, \quad \partial_\mu J^\mu = 0 \qquad \text{(always true!)}$$

PLUS equation of state:

$$T^{\mu\nu} = T^{\mu\nu}(\varepsilon, \pi, n_B, \rho_e)\,, \qquad J^\mu = J^\mu(\varepsilon, \pi, n_B, \rho_e)\,.$$

Near equilibrium: corrections in gradient expansion

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} - \eta \left(\partial^\mu v^\nu + \partial^\nu v^\mu - \frac{2}{3} \Delta^{\mu\nu} \partial \cdot v \right) - \zeta \Delta^{\mu\nu} \partial \cdot v$$

Insert in $\partial_\mu T^{\mu\nu} = 0$: hydrodynamics. η=shear, ζ=bulk

The shear viscosity of hot QCD matter is enormous!!!

Around 2×10^{11} Pascal-sec! (roughly $\hbar/(1\ \mathrm{fm})^3$)

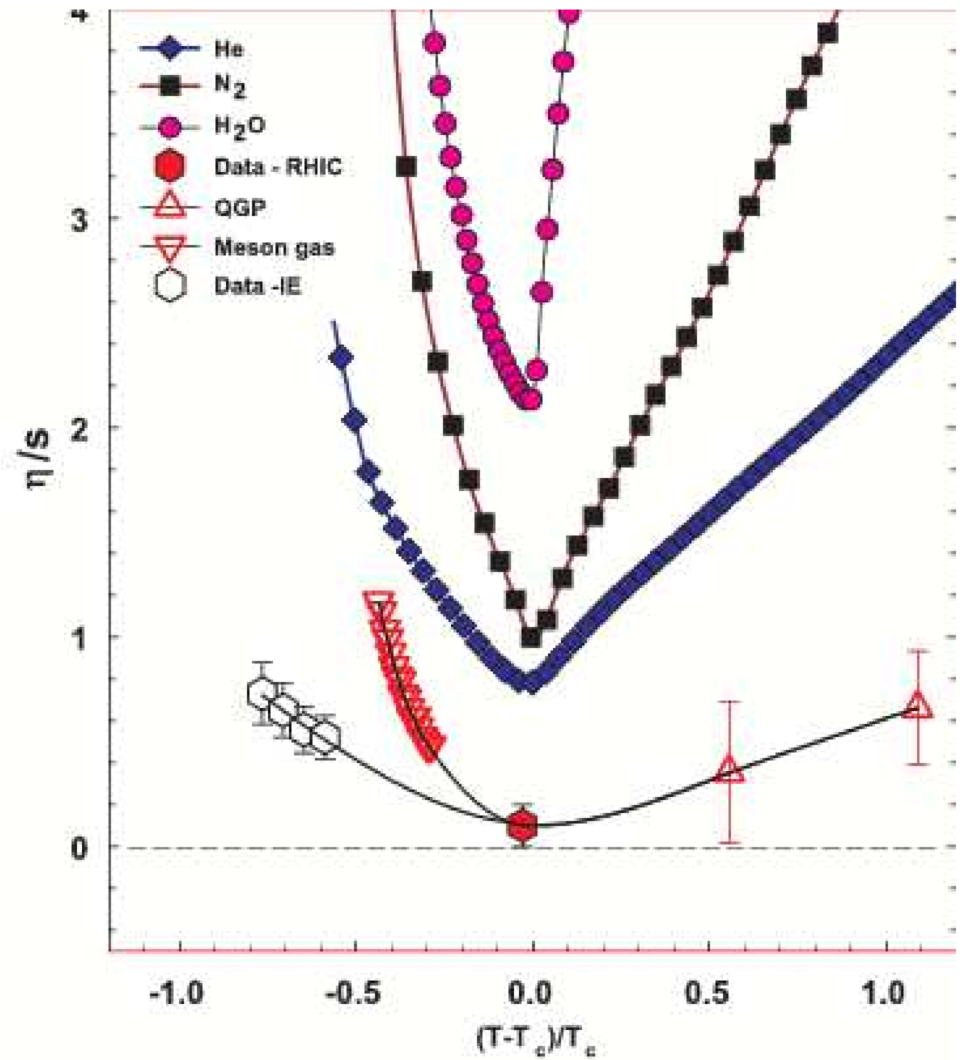
The shear viscosity of hot QCD matter is tiny!!!

Units: same as $\hbar$ per unit volume.
Entropy: "density of stuff:" number / volume. Normalize:

$$\eta \sim (0.1 - 0.2)\hbar s$$

much smaller coefficient on $\hbar s$ than other materials.

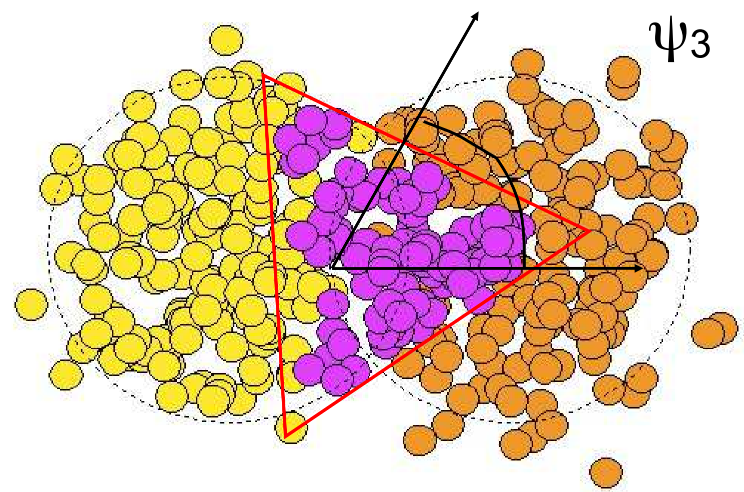
What do we (think we) know about $\eta(\mathrm{QCD})$?



(from Roy Lacey) right: lattice
middle: hydro
right: χPT
Normalized with s, value much smaller than “normal” substances

Intermediate temperatures: fitting data

Colliding ions form irregular-shaped object: collision geometry + random location of nucleons



Need some model for how nuclei + impact parameter turn into initial lumpy anisotropic hot matter.

Then need physical description for how lumpy initial condition turns into final hadronic states.

Hydrodynamics

Assume system becomes nearly-thermal in fairly short time.
Apply equations

$$\partial_\mu T^{\mu\nu} = 0\,,$$
$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq}(v^\mu, \varepsilon) - \eta(\varepsilon)\left(\partial^\mu v^\nu + \partial^\nu v^\mu - \frac{2}{3}\Delta^{\mu\nu}\partial\cdot u\right) - \zeta\Delta^{\mu\nu}\partial\cdot u\,,$$

EoS $P(\varepsilon)$ (lattice), guess for (T-dependent?) η/s

Assume freeze-out at some $T_{\rm fo}$ ($+$ hadronic final scattering?)

Predict final state hadron spectra, esp. angular patterns

Predictive if more observables than (η, ζ, init.condit.) parameters H. Petersen, N. Borghini, etc

Theory: high temperatures

Asymptotic freedom: coupling should be small.

Perturbation theory: long-lived quasiparticles, $\lambda_{\rm mfp} \sim 1/\alpha_{\rm s} T$

Particles described by phase-space density $f(x,p)$:

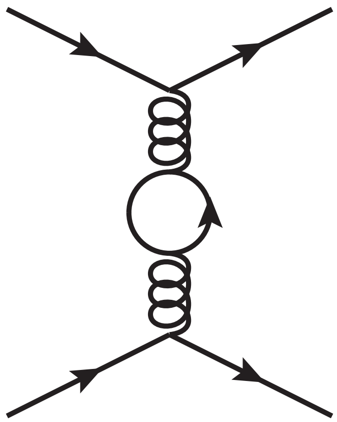
$$p^0 \partial_t f(x,p) = \vec{p} \cdot \partial_x f(x,p) + \mathcal{C}[f]$$

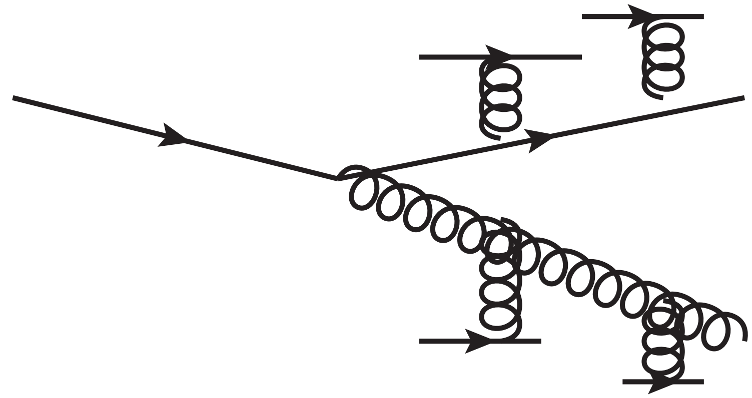
with $\mathcal{C}[f]$ representing how scattering change f.

$$T^{\mu\nu}(x) = \sum_a \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{p^0} f[p,x]$$

Need to compute scattering processes for $\mathcal{C}[f]$.

Naively $2 \leftrightarrow 2$ processes. Actually not even leading-order!





$2 \leftrightarrow 2$ processes are screened and screening important.

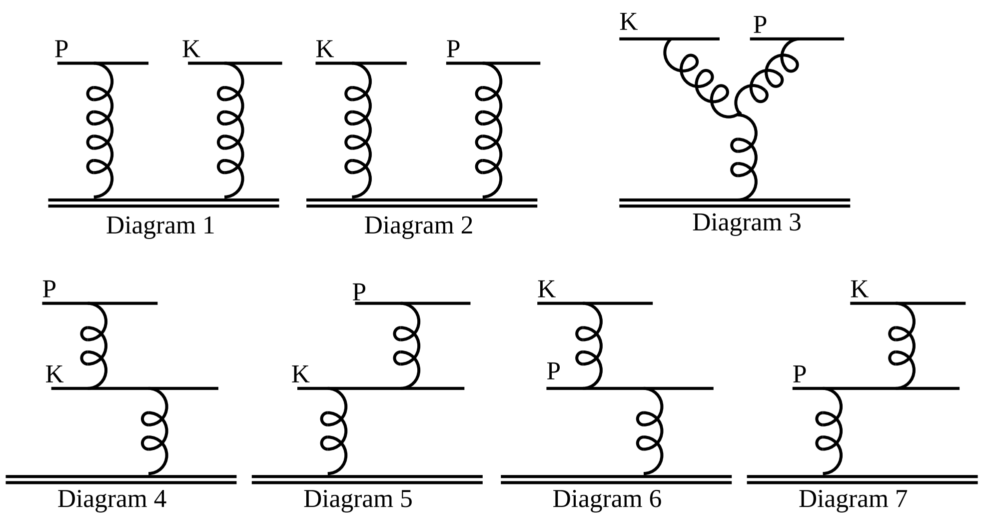
Number-changing small-angle processes important at leading order.

Require certain resummations... Arnold GM Yaffe 2003

Quality of theory?

Theory useless without error bars.

Perturbation theory: usually must go to NLO, use convergence



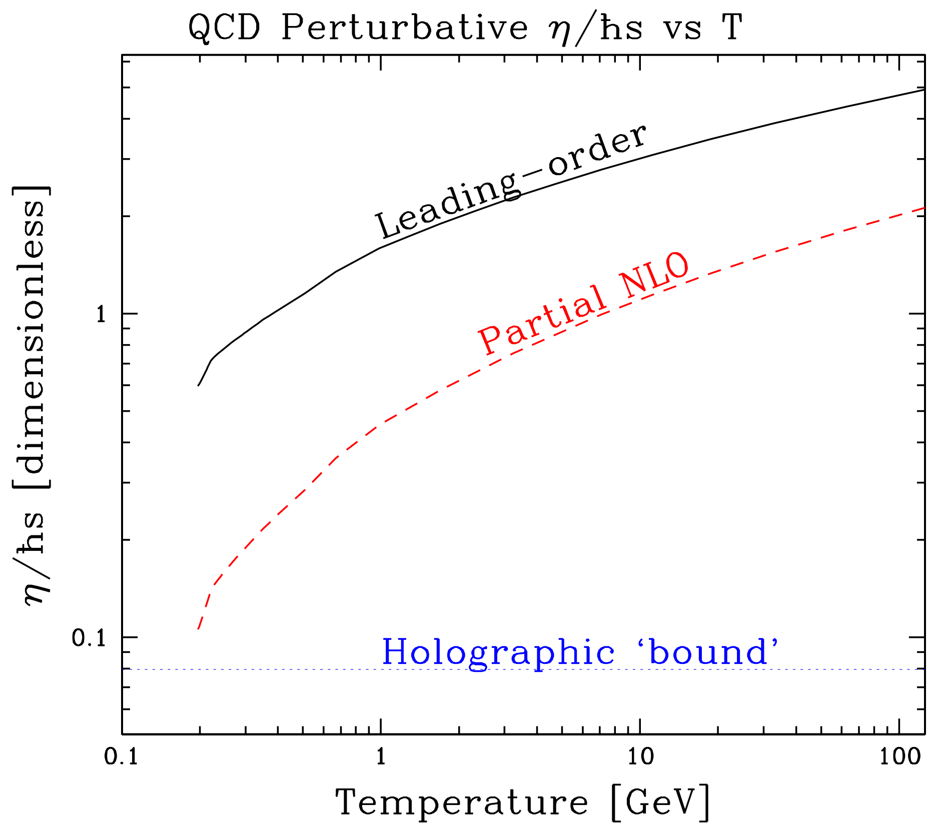
Complex interference effects arise at NLO.

New techniques needed: recently completed, Ghiglieri GM Teaney

Shear viscosity at NLO

Recently completed (unpublished) almost-NLO treatment

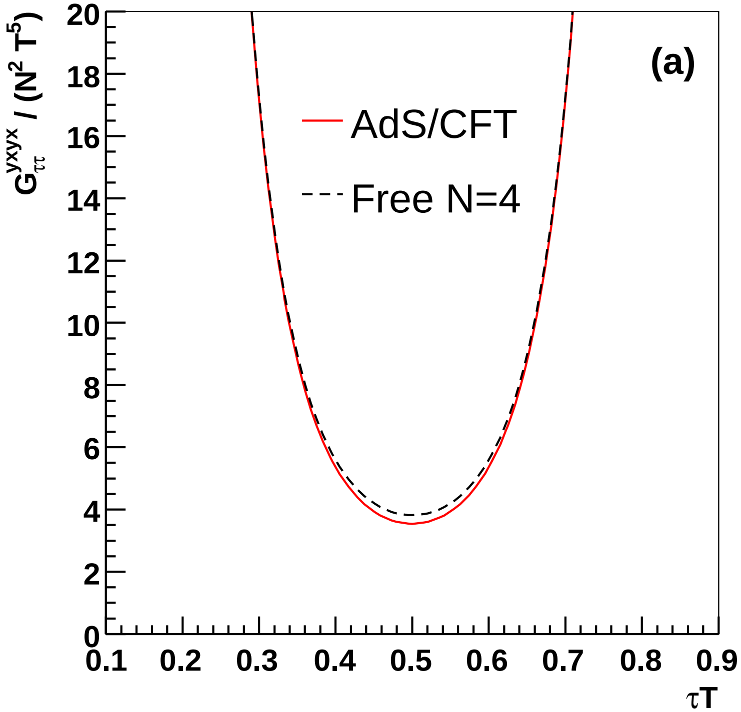
Express as dimensionless ratio η/s: LO vs NLO

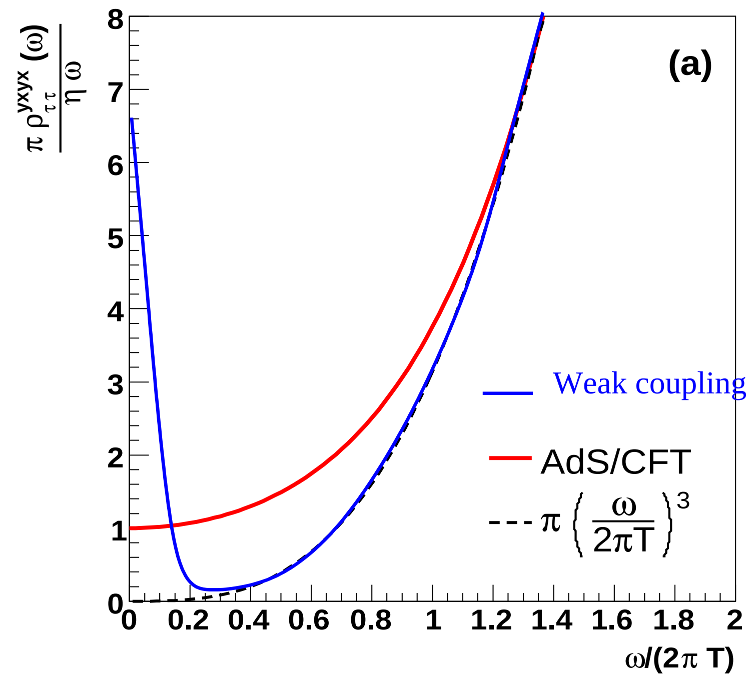


Very large downward NLO corrections. "$\times e^{\pm 2}$" error bars.

Lattice calculation??

Calculate Euclidean quantity (L). Try to learn real-frequency (R)





Tiny differences in Euclidean space become huge changes near $\omega = 0$ ($\omega = 0$ intercept $=$ viscosity) Teaney hep-ph/0602044

Conclusions

- Shear at "middle" temp $150\,\mathrm{MeV} < T < 300$ MeV: Use Hydrodynamics to fit experiment
- Shear at "middle" temp: lattice ??? Big challenges.
- Shear at "high" temp $T > 300$ MeV: perturbation theory?!?
- Shear at "super-high" $T > 10^5$ GeV: perturbation theory!

Hydro$+$Data seems to work. Pure theory not very successful! Lattice: need more work on continuation.