BRIDGING THE GAP BETWEEN THEORY PREDICTIONS AND FLUCTUATION MEASUREMENTS FOR RELATIVISTIC NUCLEAR COLLISIONS

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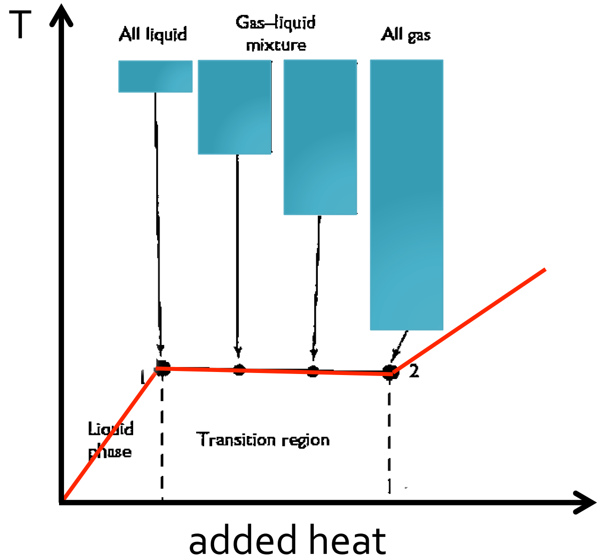
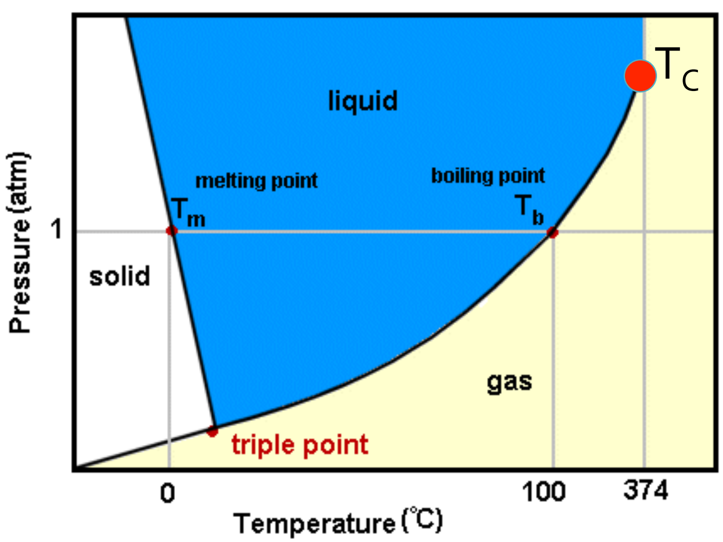
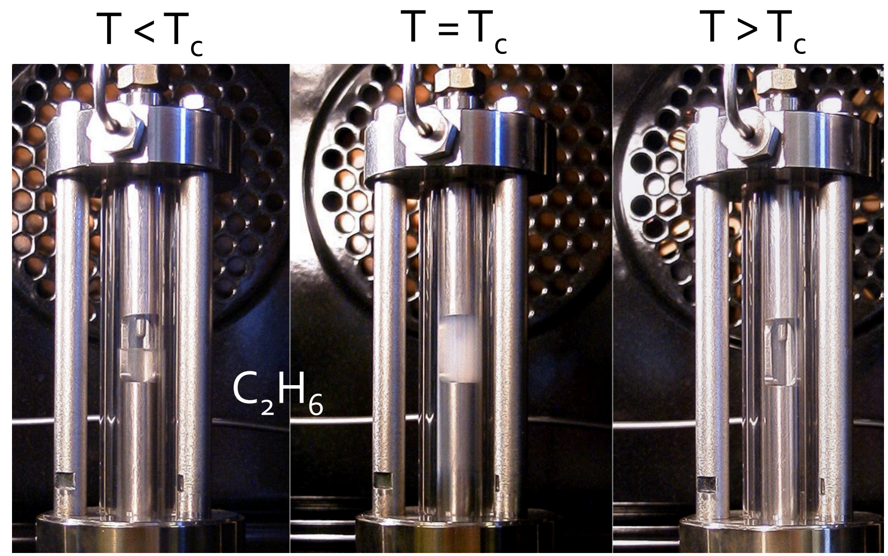
- Motivation
- Cumulants of probability distributions
- Particle number fluctuations
- Bridging the gap
- Summary

Critical phenomena







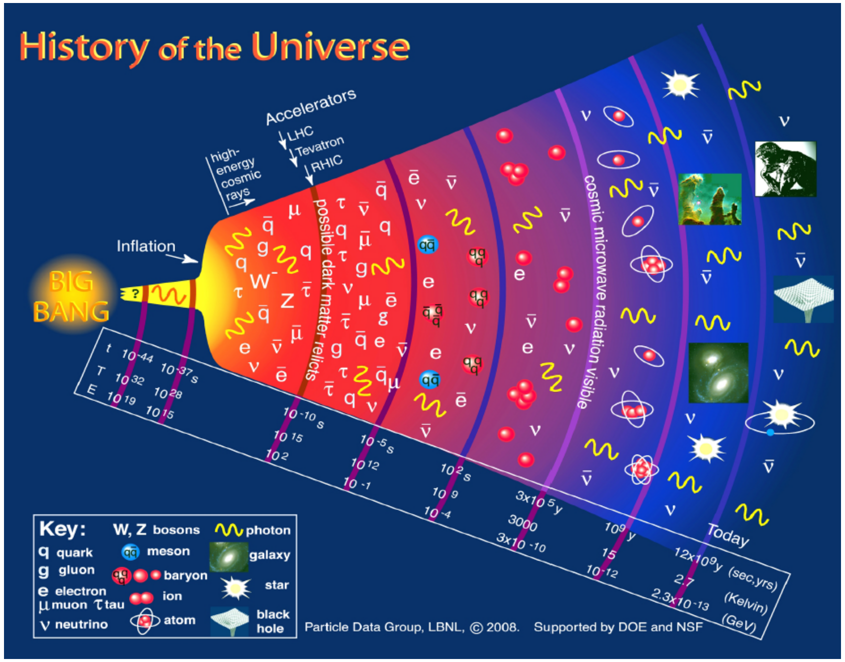
$$\frac{\langle\rho^2\rangle - \langle\rho\rangle^2}{\langle\rho\rangle^2} = \frac{T\chi}{V}, \quad \chi = -\frac{1}{V}\frac{\partial V}{\partial P}$$

Einstein, 1910

$$Rayleigh\, Ratio \propto \chi$$

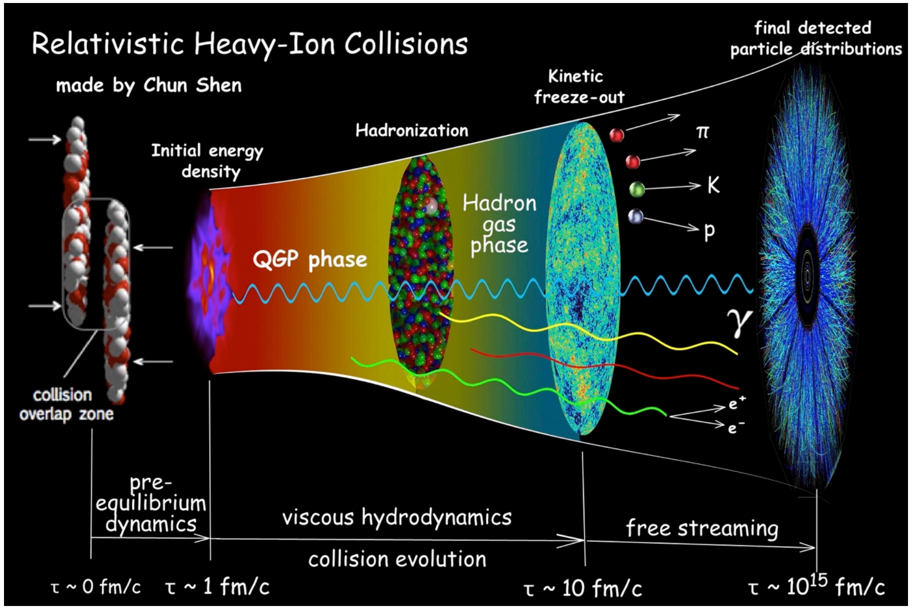
probing phase transitions
with fluctuations

Phase transitions, then and now



1/100.000 seconds after the Big Bang quarks and gluons recombine to hadrons

- recreating that stage in laboratories
- exploring phase transitions



$m_p \approx 937$ MeV
$2m_u + m_d \approx 10$ MeV

off by a factor of about 100!

no isolated quarks seen thus far

probing phase transitions with fluctuations

Cumulants of distributions

In each event count particle type A →Generate P(A)

$$n^{th} \text{ moment} = \sum_{A=0}^{\infty} A^n P(A) = \frac{1}{N_{event}} \sum_{i=1}^{N_{event}} A_i^n$$

$$k_1 = \langle A \rangle$$

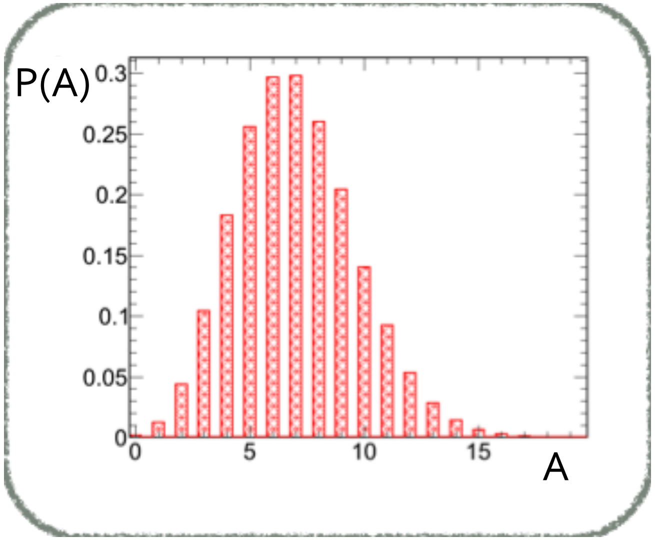
$$k_2 = \left\langle \left(A - \langle A \rangle\right)^2 \right\rangle = \langle A^2 \rangle - \langle A \rangle^2 = \sigma^2$$

$$k_3 = \left\langle \left(A - \langle A \rangle\right)^3 \right\rangle$$

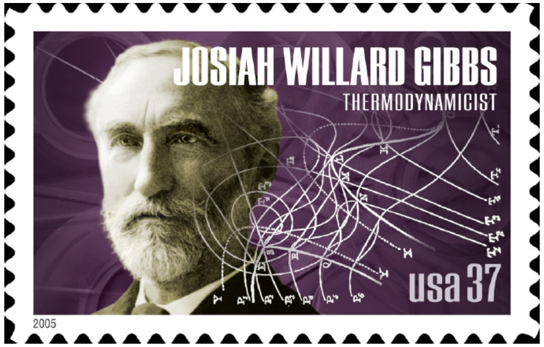
$$k_4 = \left\langle \left(A - \langle A \rangle\right)^4 \right\rangle - 3k_2^2$$

$$\text{fort he Poisson distribution: } \mathrm{p}(A;\lambda) = \frac{\lambda^A e^{-\lambda}}{A!}$$

$$k_n = \langle \lambda \rangle$$



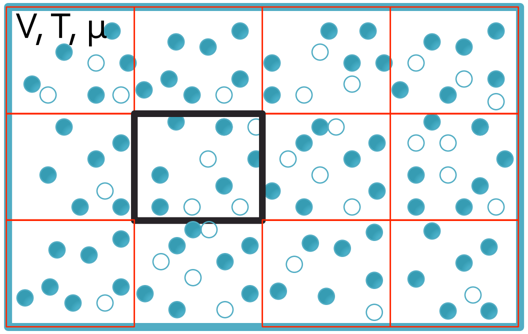
Ensemble averaging



Ergodicity hypothesis: Averaging over time is equivalent to the averaging over ensembles.

Ensemble is an idealisation consisting of a large number of mental copies of a system, considered all at once, each represents a possible state that the real system!

Grand Canonical Ensemble



probability of a given state with E_j and N_j

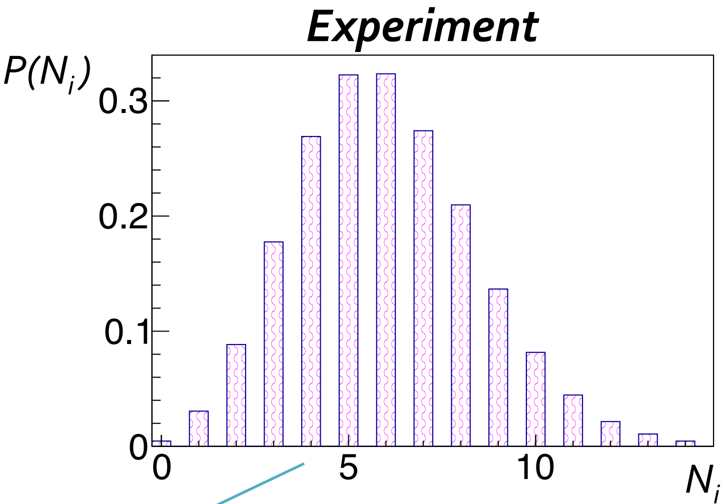
$$p_j = \frac{\exp\left[-\left(E_j - \mu N_j\right)/T\right]}{Z_{GCE}}$$

$$Z_{CGE}\left(T,V,\mu\right) = \sum_j \exp\left[-\frac{E_j - \mu N_j}{T}\right]$$ partition function

$$\left\langle N \right\rangle = \sum_j N_j p_j = T \left.\frac{\partial \ln Z_{CGE}}{\partial \mu}\right|_V$$

$$k_2\left\langle N \right\rangle = \left\langle N^2 \right\rangle - \left\langle N \right\rangle^2 = \sum_j N_j^2 p_j = T^2 \left.\frac{\partial^2 \ln Z_{CGE}}{\partial \mu^2}\right|_V$$

Hadron-Resonance Gas Model

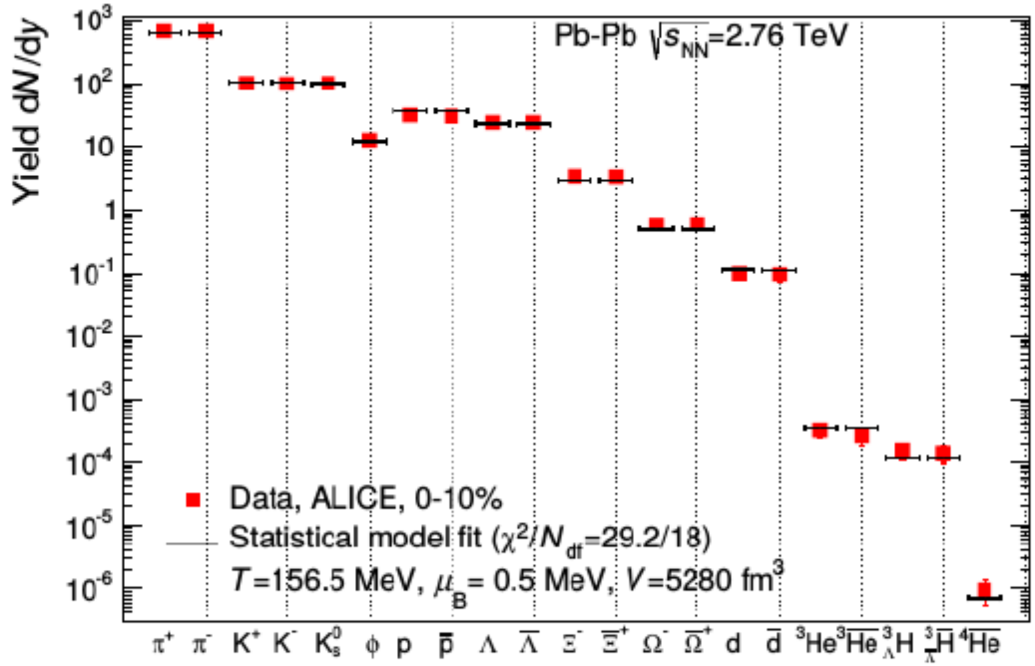


$$\langle N_i \rangle = V \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp\left[\left(E_i - \mu_i\right)/T\right] \pm 1}$$

$$\mu_i = \mu_B B_i + \mu_s S_i + \mu_I I_i$$

ensures conservation of B, S, I_3 on average

$$\chi^2 = \sum_{k=1}^{n} \frac{\left(\langle N_k^{\exp} \rangle - \langle N_k^{HRG} \rangle\right)^2}{\sigma_k^2}$$

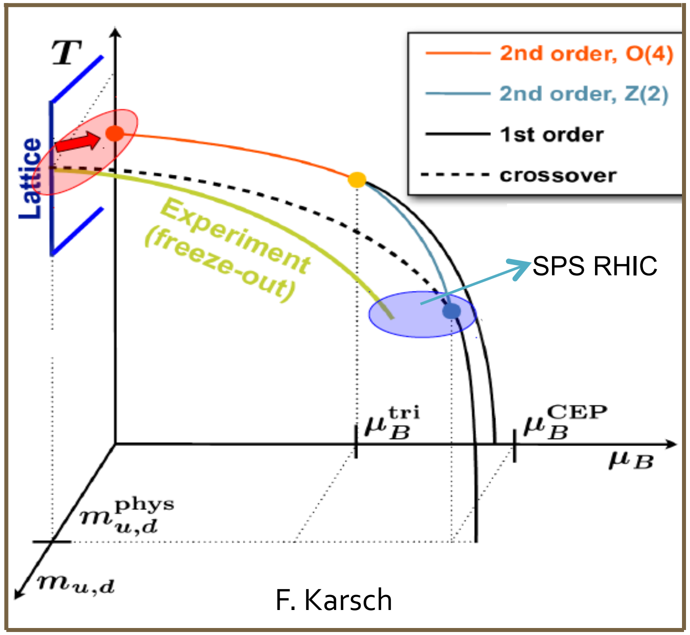


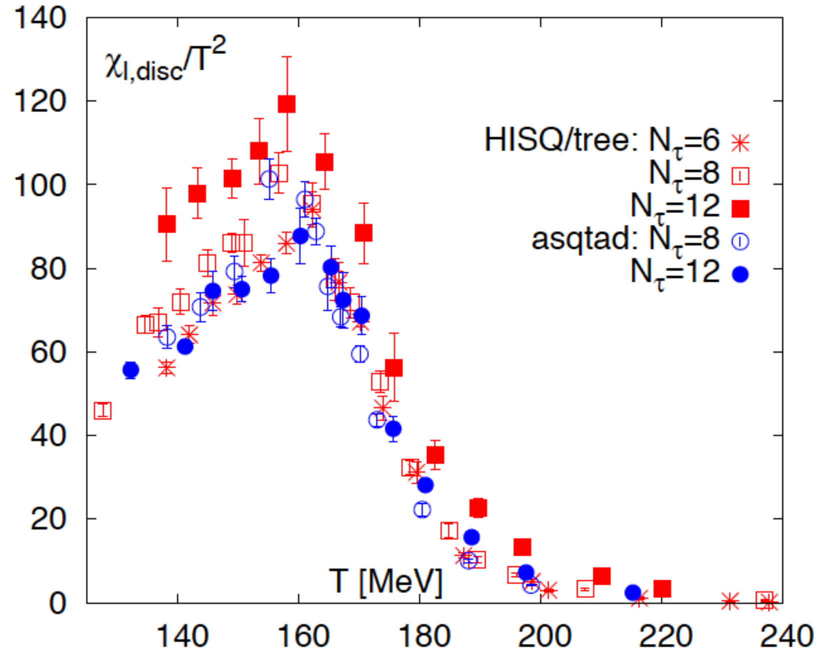
ALICE, PLB 726 (2013) 610

J. Stachel, A. Andronic, P. Braun-Munzinger and K. Redlich
J. Phys. Conf. Ser. 509 (2014) 012019

works in the energy range spanning by 3 orders of magnitude! y axis: 9 orders of magnitude!

Criticality at crossover?





A. Bazavov et al., Phys.Rev. D85 (2012) 054503

freeze-out at the phase boundary!

$$T_c^{lattice} = 154 \pm 9\, MeV, \quad T_{fo}^{ALICE} = 156 \pm 3 MeV$$

- ***E-by-E fluctuations:***
  - To study dynamics of the phase transitions
  - To locate phase boundaries

Exploring the structure of matter

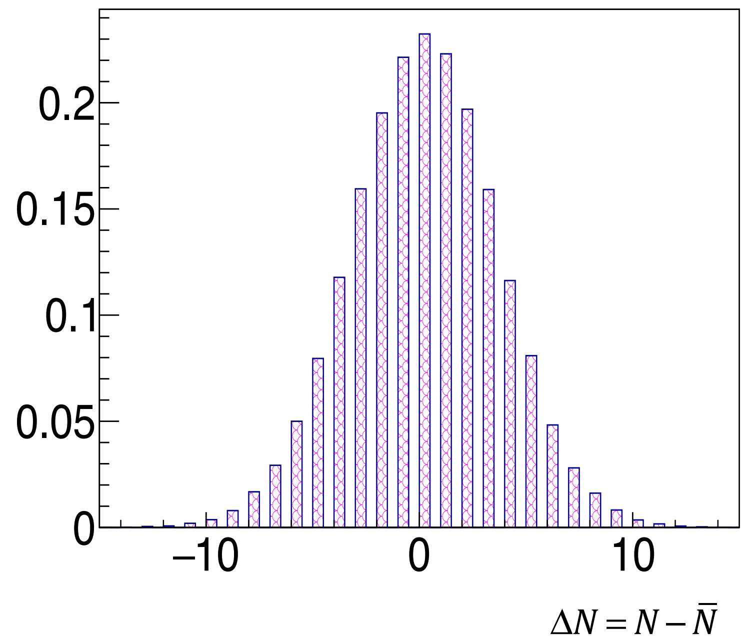
$$\hat{\chi}_n^{N=B,S,Q} = \frac{\partial^n P/T^4}{\partial(\mu_N/T)^n} \qquad \frac{P}{T^4} = \frac{1}{VT^3}\ln Z(V,T,\mu_{B,Q,S})$$

$$VT^3\,\hat{\chi}_2^N = \langle \Delta N^2\rangle - \langle \Delta N\rangle^2 \equiv k_2(N-\bar{N})$$

Do ratios eliminate volume dependencies?

$$\frac{k_4(N-\bar{N})}{k_2(N-\bar{N})} \equiv \gamma_2\sigma^2 = \frac{\hat{\chi}_4^N}{\hat{\chi}_2^N} \qquad \frac{k_3(N-\bar{N})}{k_2(N-\bar{N})} \equiv \gamma_1\sigma = \frac{\hat{\chi}_3^N}{\hat{\chi}_2^N}$$

- ***Assumptions:***
  - *Volume is fixed in each event*
  - *Conservations are imposed on the averages*
- ***Reality:***
  - *Volume fluctuates from E-to-E*
  - *Conservations depend on the acceptance (p_T, y)*

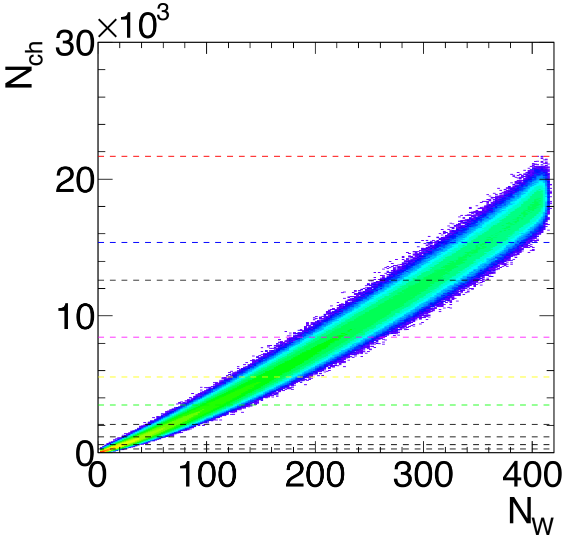


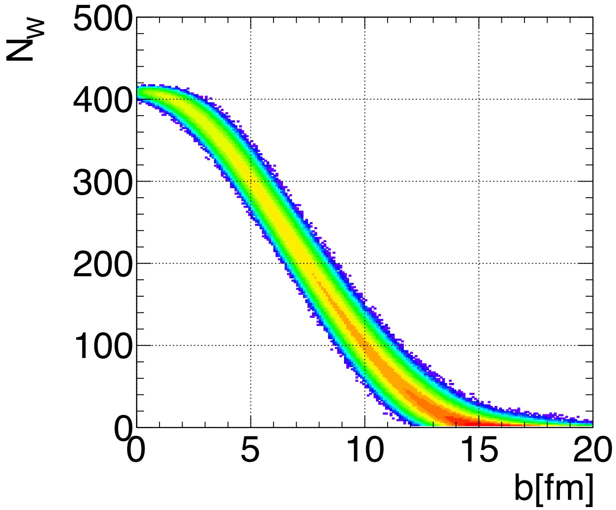
$$\frac{k_4(N-\bar{N})}{k_2(N-\bar{N})} \neq \frac{\hat{\chi}_4^N}{\hat{\chi}_2^N}$$

experiment　　　　theory

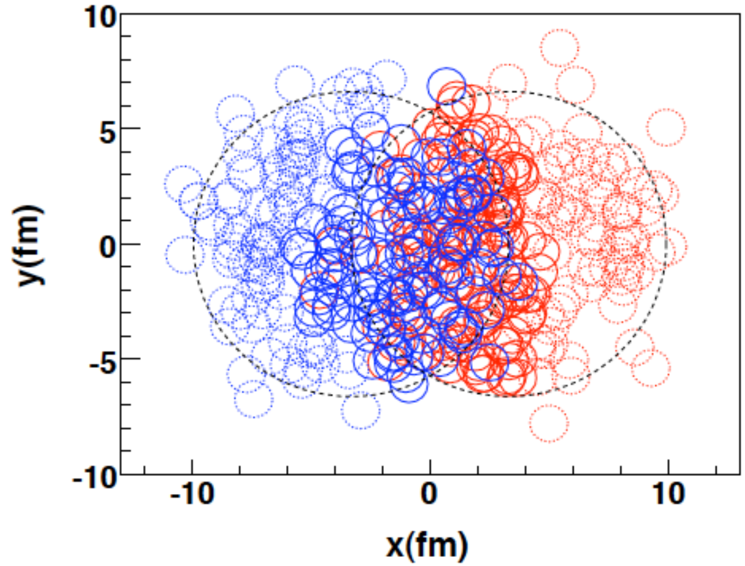
Link to the volume, different approaches

Experimental approaches:





Glauber Model



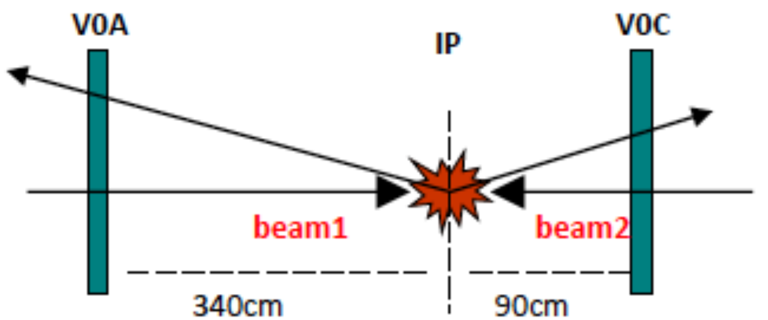
- **Each approach gives:**
  - Similar $\langle N_w \rangle$
  - Very different $\langle N_W^n \rangle$

For higher moments centrality selection is crucial!

Centrality selection in ALICE

One of the different methods

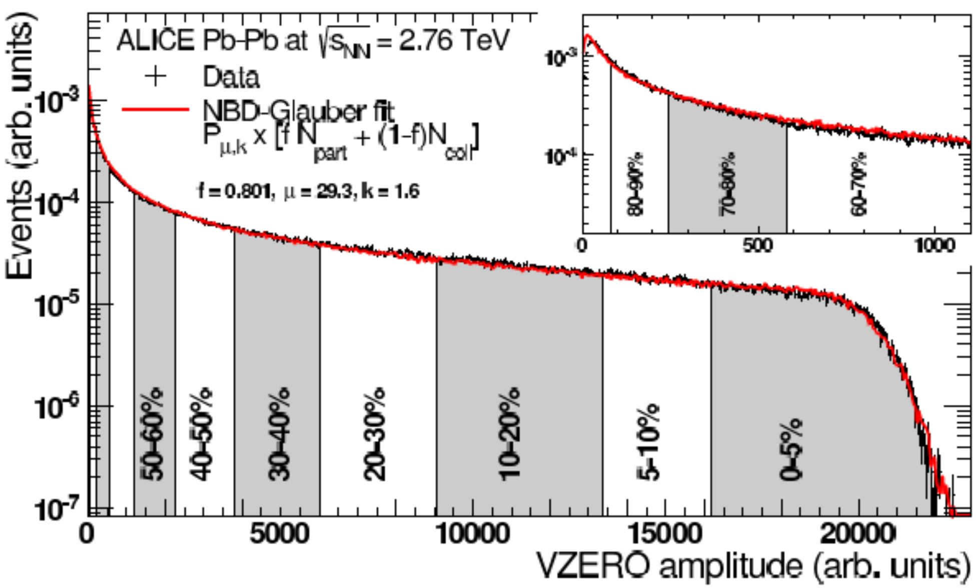
ALICE Phys.Rev. C88 (2013) no.4, 044909

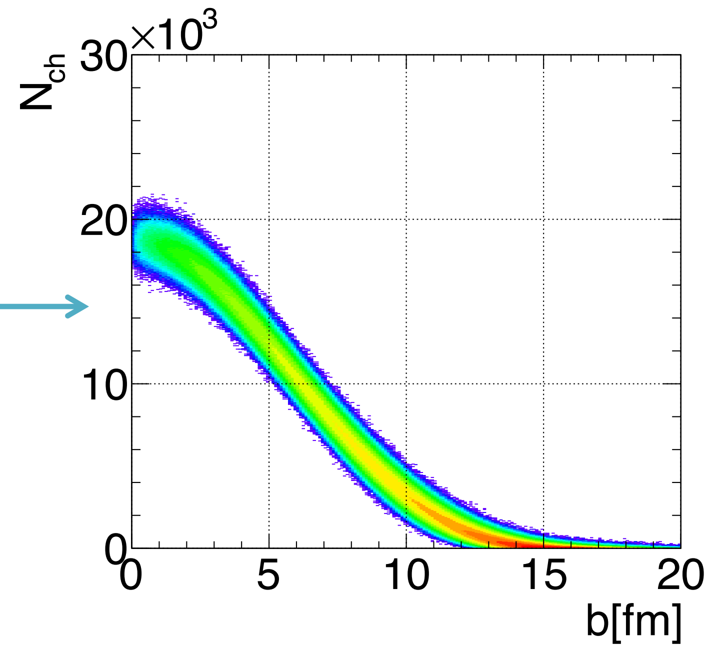


$2.8 < \eta < 5.1$ $-3.7 < \eta < -1.7$

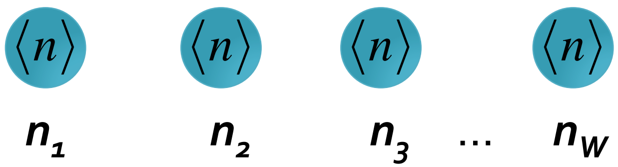
$$P_{\mu,k}(n) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)} \frac{\left(\mu/k\right)^n}{\left(\mu/k + 1\right)^{n+k}}$$

$$N = fN_W + (1-f)N_{coll}$$

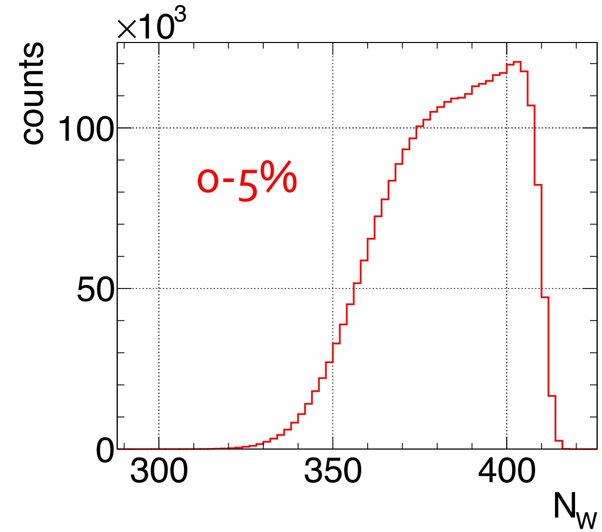


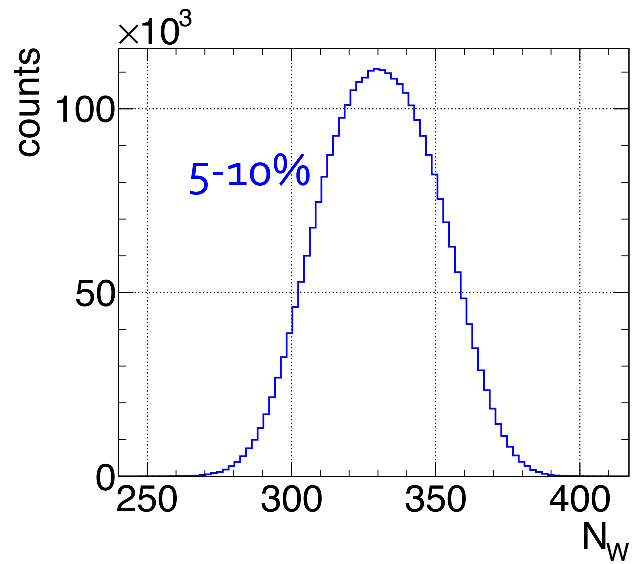


Building the model, Particle production

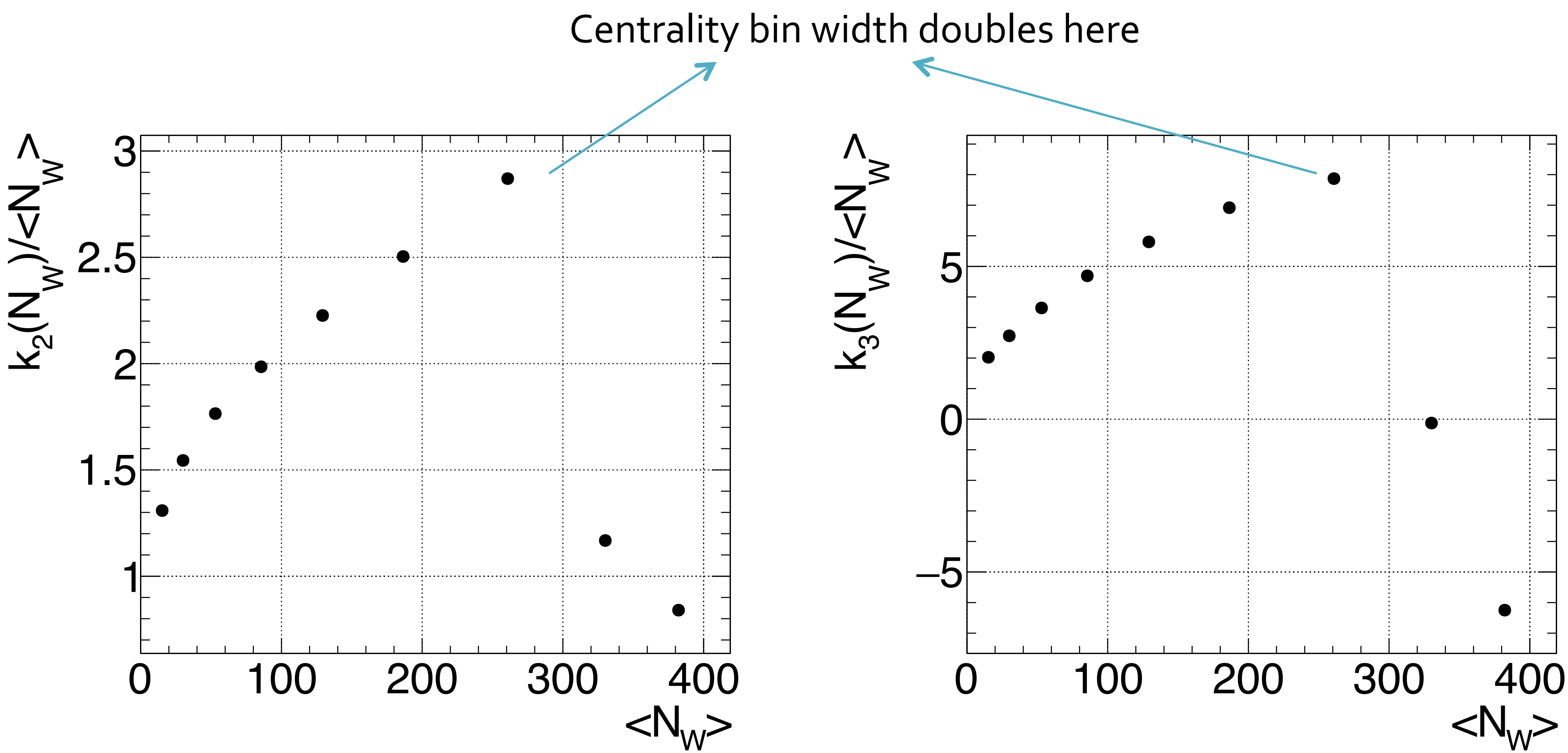


- ***N_W fluctuates with the Glauber initial conditions***
- ***Each source is treated Grand Canonically***
- ***Mean proton multiplicities $\langle p \rangle$, $\langle \bar{p} \rangle$ from ALICE Pb+Pb@2.76 TeV***
- ***Expected results without volume fluctuations:***
  - particles: $k_n = N_w \langle n \rangle = \langle p \rangle = \langle \bar{p} \rangle$
    $k_n / k_2 = 1$
  - net-particles: $k_n = \langle p \rangle + (-1)^n \langle \bar{p} \rangle$
    $k_3 / k_2 = 0, \quad k_4 / k_2 = 1$

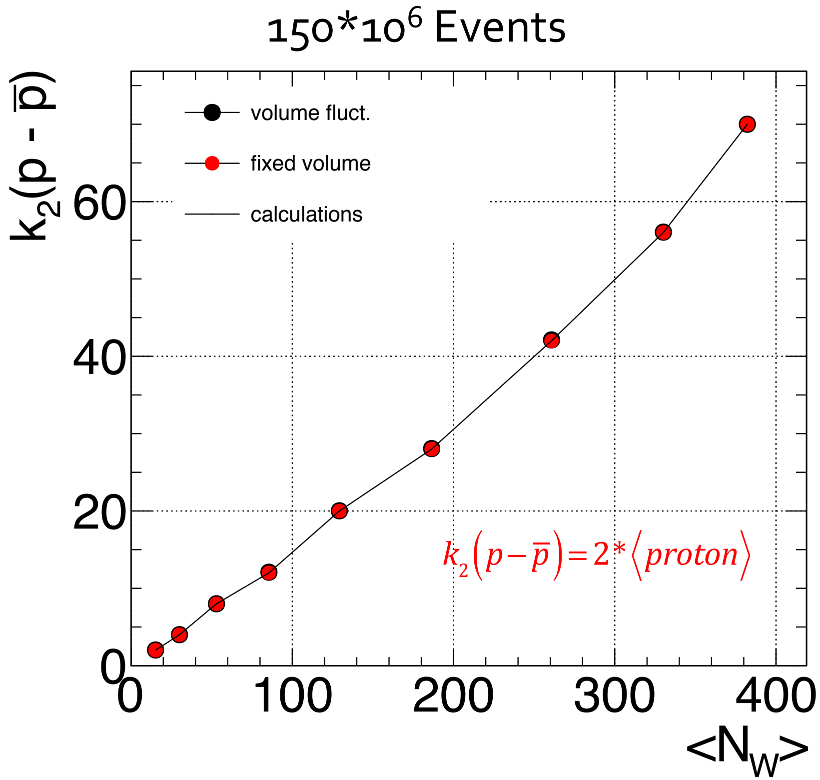


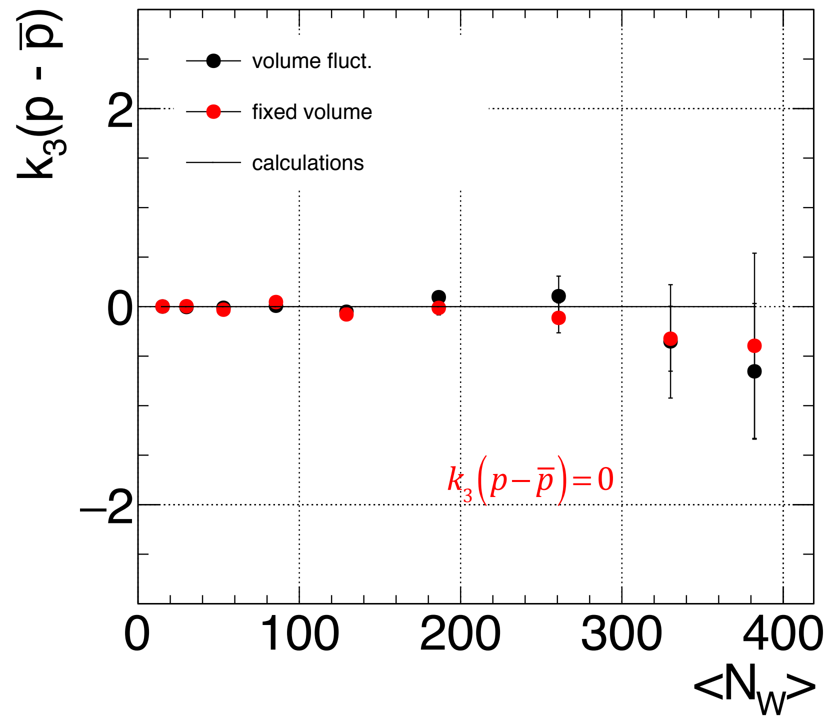


Fluctuations of wounded nucleons



Second and third cumulants





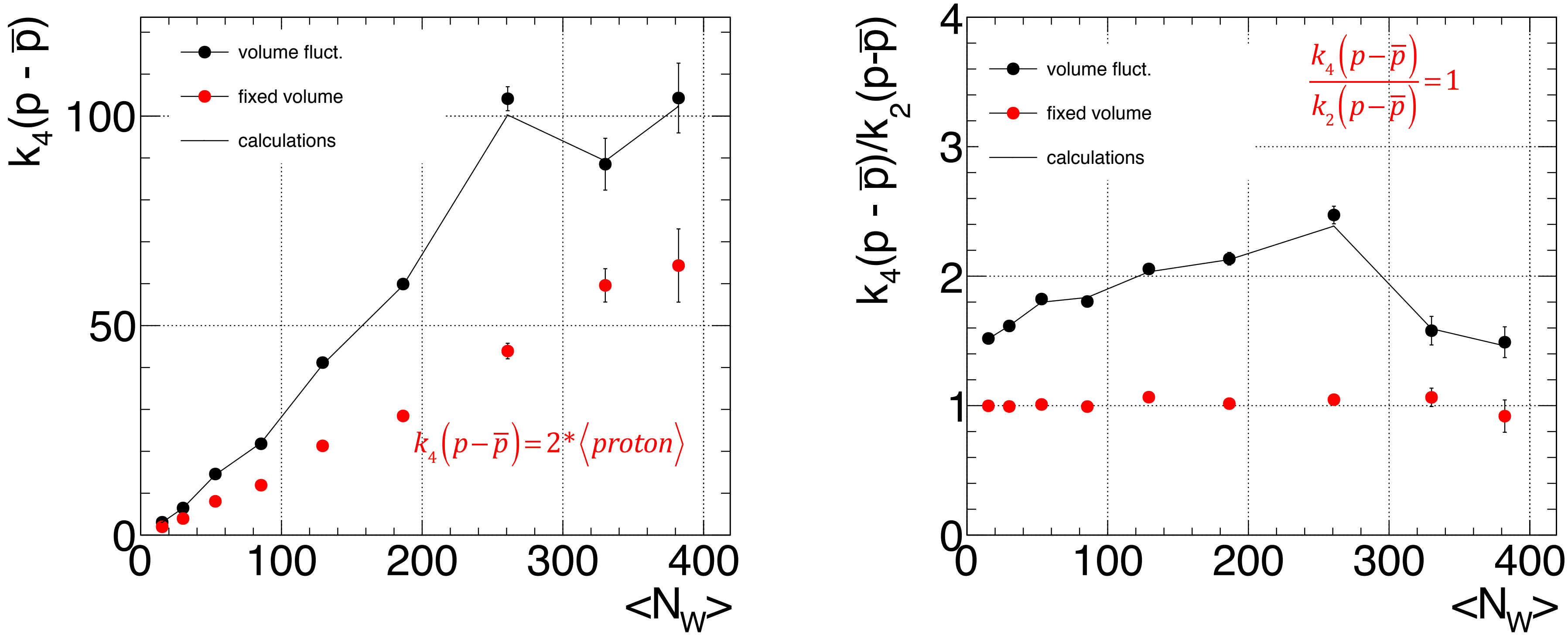
$$k_2(p-\bar{p}) = \langle N_w \rangle k_2(n-\bar{n}) + \langle n-\bar{n} \rangle^2 k_2(N_w)$$

$$k_3(p-\bar{p}) = \langle N_w \rangle k_3(n-\bar{n}) + \langle n-\bar{n} \rangle (\ldots)$$

$n, \bar{n}$ from single wounded nucleon

Fourth cumulants

150*10^6 Events



$$k_4(p-\bar{p}) = \langle N_w \rangle k_4(n-\bar{n}) + 3k_2(n-\bar{n})^2 k_2(N_w) + \langle n-\bar{n} \rangle (\ldots)$$

vanishes for ALICE

$n, \bar{n}$ from single wounded nucleon

Au+Au data at 7.7 GeV

- ***Mean proton multiplicities from:***

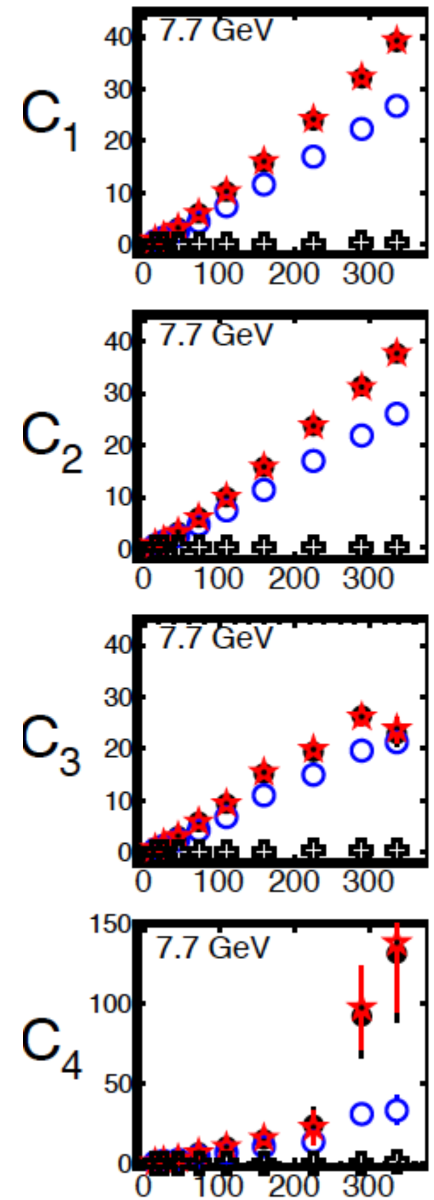
  X. Luo, PoS(CPOD2014)019 [arXiv: 1503.02558]
  X. Luo, Private communications

- ***Expected results without volume fluctuations:***

  - particles: $k_n = N_w \langle n \rangle = \langle p \rangle = \langle \bar{p} \rangle$

    $k_n / k_2 = 1$

  - net-particles ≅particles



Centrality bin width correction

STAR data are corrected for the Centrality Bin Width Correction

$$k_n = \sum_r w_r k_n^r, \quad w_r = \frac{n_r}{\sum_r n_r}$$

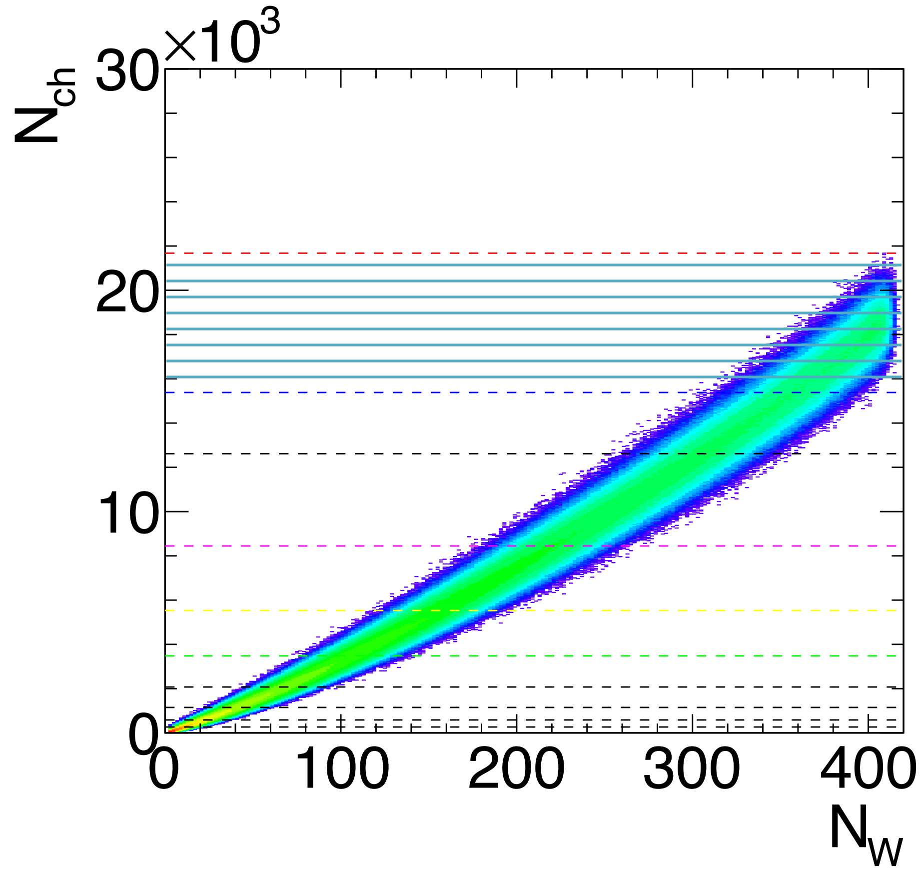
X. Luo, Ji Xu, B. Mohanty and Nu Xu, J.Phys. G40 (2013) 105104

n_r number of events in r^{th} sub-centrality

k^r_n cumulant value in r^{th} sub-centrality

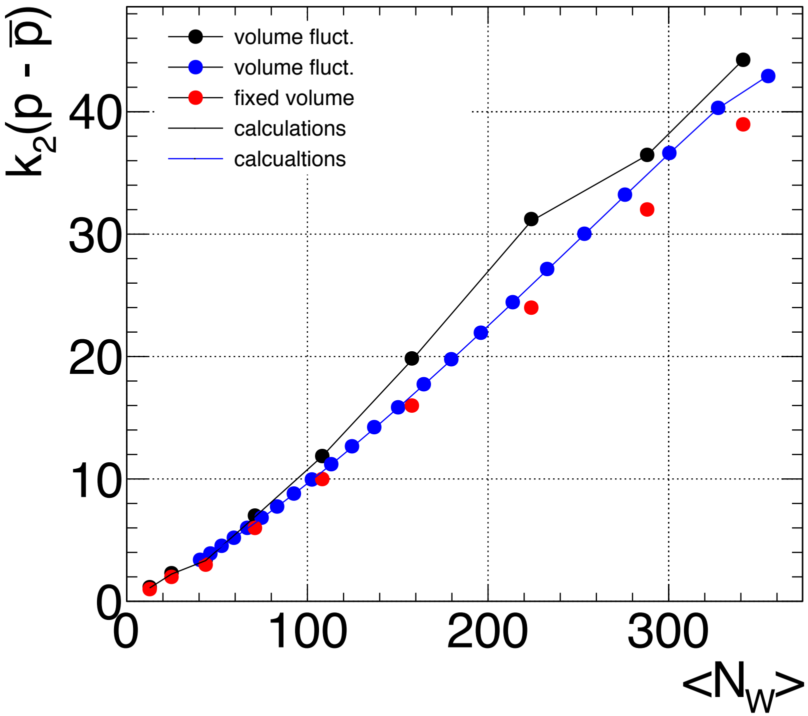
summation is performed incoherently!

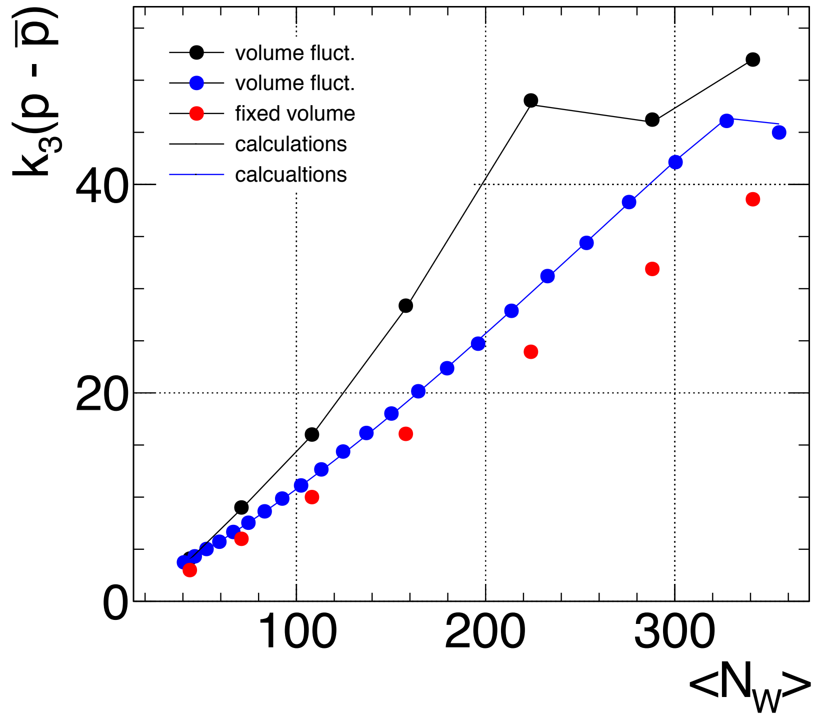
The procedure cannot eliminate N_W fluctuations



Second and third cumulants

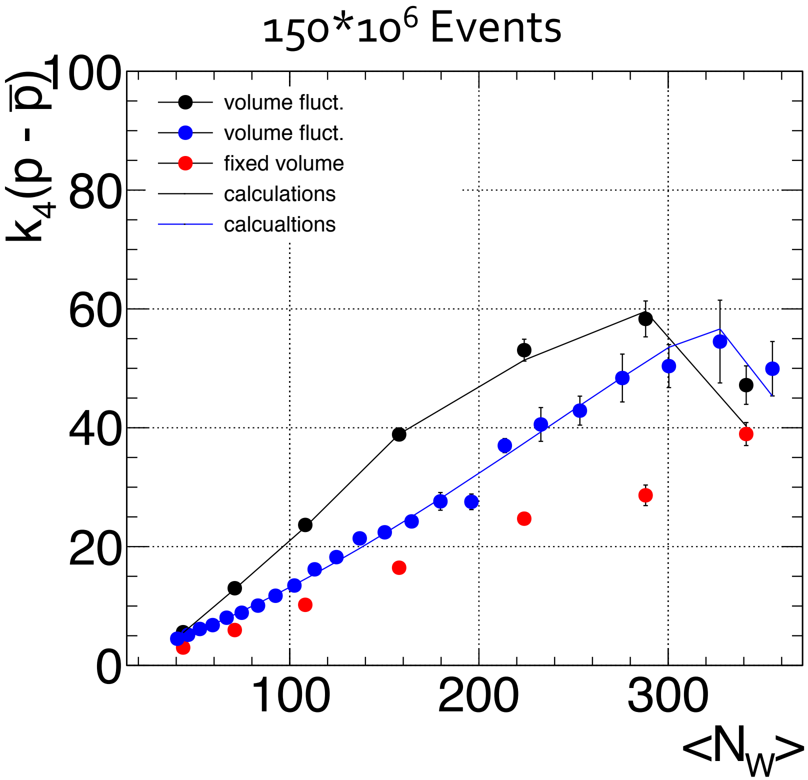
$150*10^6$ Events

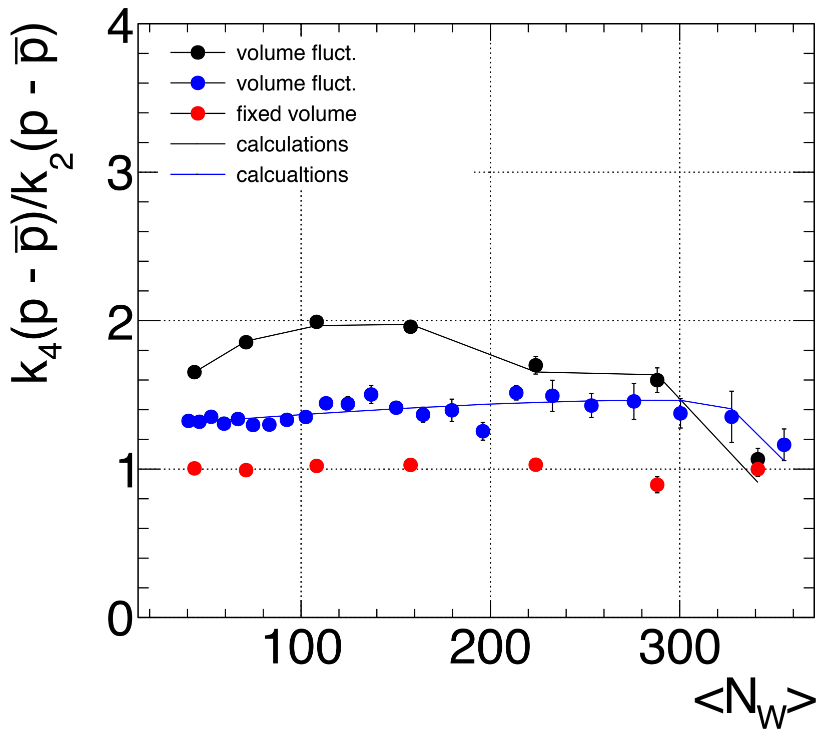




Contribution from N_W fluctuations are visible starting from second cumulants
Non-monotonic structures due to N_W fluctuations!

Fourth cumulants

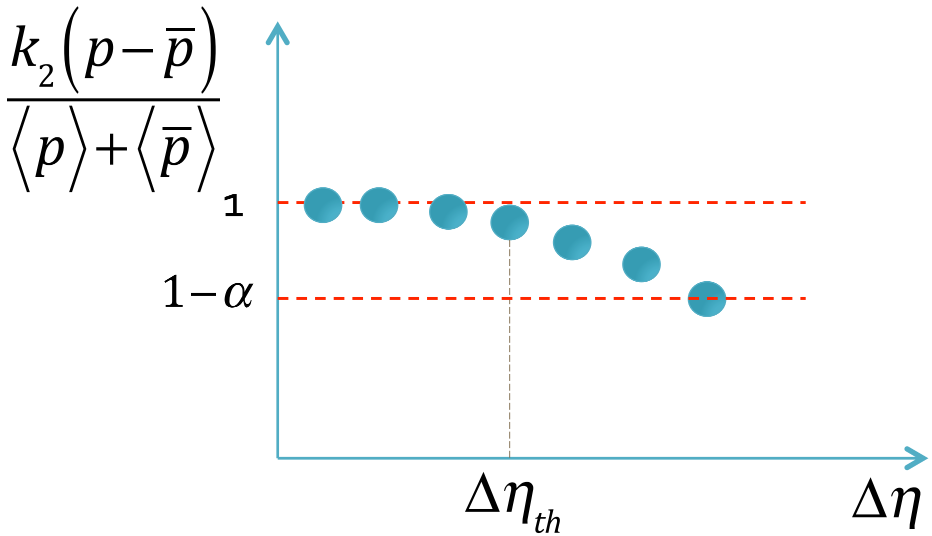




Contributions from N_W fluctuations are larger for higher cumulants

Non-monotonic structures due to N_W fluctuations!

Baryon number conservation



$$\frac{k_2(p-\bar{p})}{\langle p\rangle+\langle\bar{p}\rangle}=1-\alpha$$

$$\alpha=\frac{\langle n_p^{acc}\rangle}{\langle N_B^{4\pi}\rangle}$$

Proposed comparison procedure:

- Eliminate volume dependence
- Perform analysis for $\Delta\eta>\Delta\eta_{th}$
- Calculate acceptance factors based on experimental data $\alpha(\Delta\eta)=\frac{p^{acc}}{B^{4\pi}}$
- Correct the experimental data
- Compare to LQCD

Summary and Outlook

- ***E-by-E fluctuation signals are useful probes of critical phenomena***

- ***For direct comparison between experiment and theory the following have to be considered:***
  - Fluctuations of wounded nucleons have to be understood and subtracted from the data
  - The effect is more important at low energies
  - For the ALICE this does not cause a trouble up to the fourth cumulants

  - Phase space dependence of fluctuation measurements have to be studied
  - Modifications driven by the conservation lows should be corrected

Much more on these and other aspects of fluctuation measurements:

P. Braun-Munzinger, A. R, J. Stachel: Bridging the gap between event-by-event measurements and theory predictions in relativistic nuclear collisions