Universal three-body physics in ultracold quantum gases

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Stephan Häfner,
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and Matthias Weidemüller

http://physi.uni-heidelberg.de/Forschung/QD

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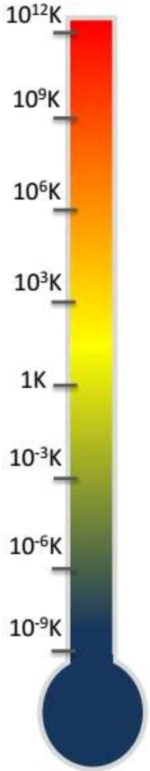
University of Excellence

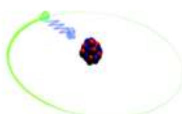
Physikalisches Institut

Center for
Quantum
Dynamics

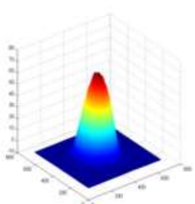
Universal quantum systems in extreme conditions



Heavy-ion collisions (QCD)

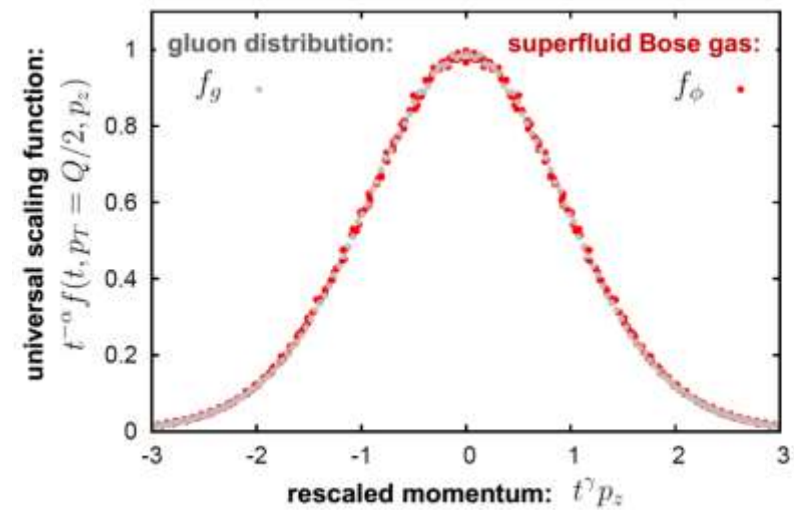


Highly charged ions (QED)



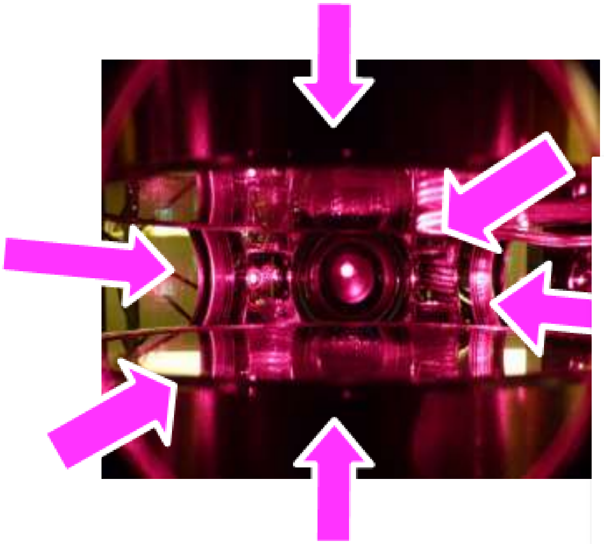
Ultracold quantum gases
(tunable model systems)

$$\frac{\lambda \cdot \langle \varphi^2 \rangle}{Q^2} \sim 1$$



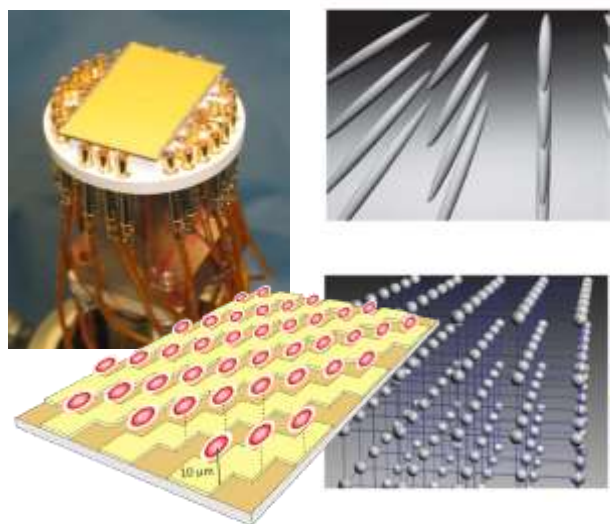
Ultracold atoms as model systems

Atomic motion



Magneto-optical trap: $v \sim 1$ cm/s

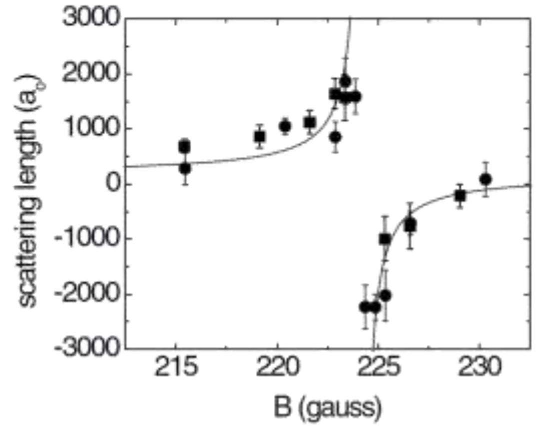
Geometry / dimensionality



Atom chip

Optical lattice

Interactions

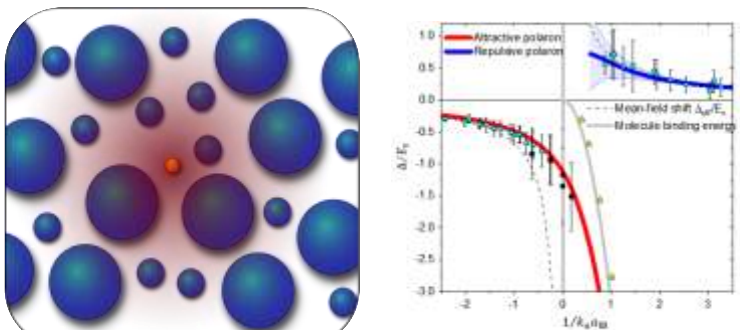


Regal *et al.*, PRL (2003)

Feshbach resonances

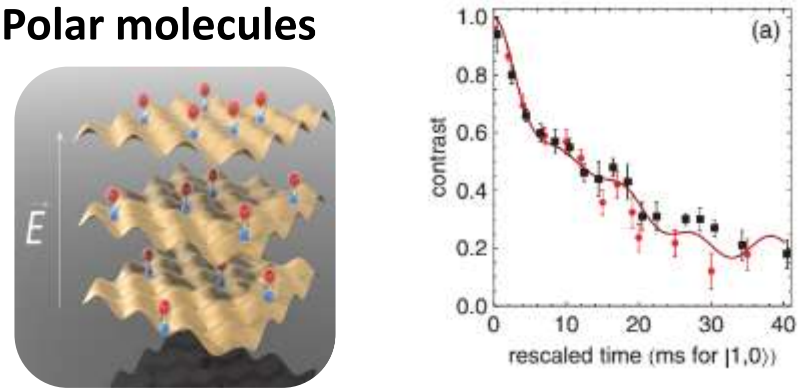
Strongly interacting ultracold systems

Polarons



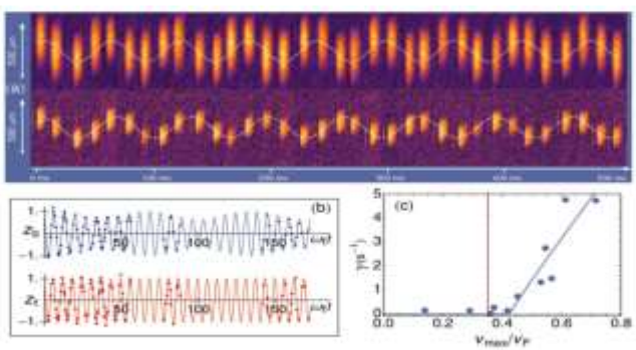
Hu *et al.*, PRL **117**, 055301 (2016)

Polar molecules



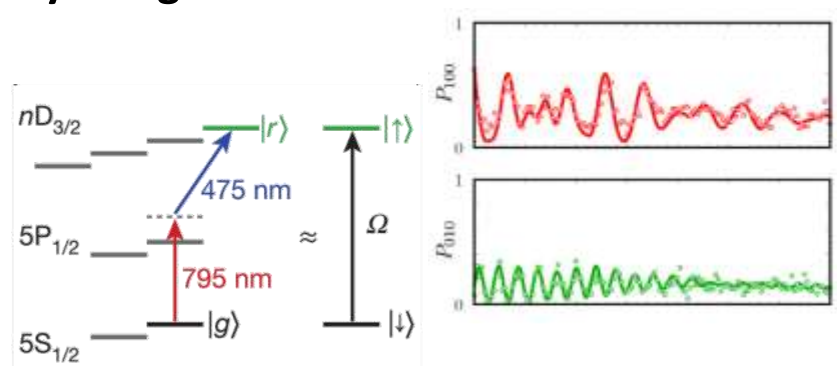
Hazzard *et al.*, PRL **113**, 195302 (2014)

Bose-Fermi superfluids



Ferrier-Barbut *et al.*, Science **345**, 1035 (2014)

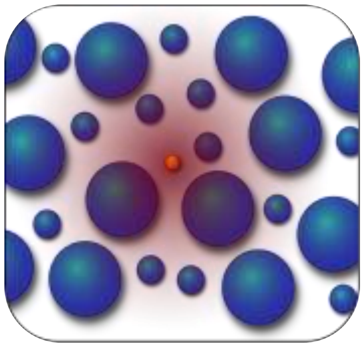
Rydberg atoms

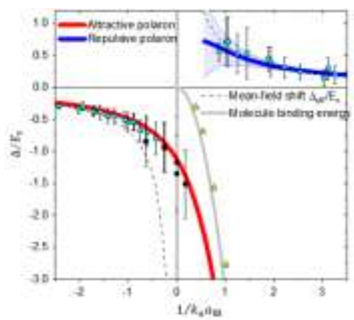


Barredo *et al.*, PRL **114**, 113002 (2015)

Strongly interacting ultracold systems

Polarons from few-body perspective





Hu *et al.*, PRL **117**, 055301 (2016)

Polar molecules

Hazzard *et al.*, PRL **113**, 195302 (2014)

Bose-Fermi superfluids

Ferrier-Barbut *et al.*, Science **345**, 1035 (2014)

Rydberg atoms

Barredo *et al.*, PRL **114**, 113002 (2015)

Efimov's scenario in a nutshell

ENERGY LEVELS ARISING FROM RESONANT TWO-BODY FORCES IN A THREE-BODY SYSTEM

V. EFIMOV

A.F.Ioffe Physico-Technical Institute, Leningrad, USSR

Received 20 October 1970

Resonant two-body forces are shown to give rise to a series of levels in three-particle systems. The number of such levels may be very large. Possibility of the existence of such levels in systems of three α-particles (^{12}C nucleus) and three nucleons (^{3}H) is discussed.

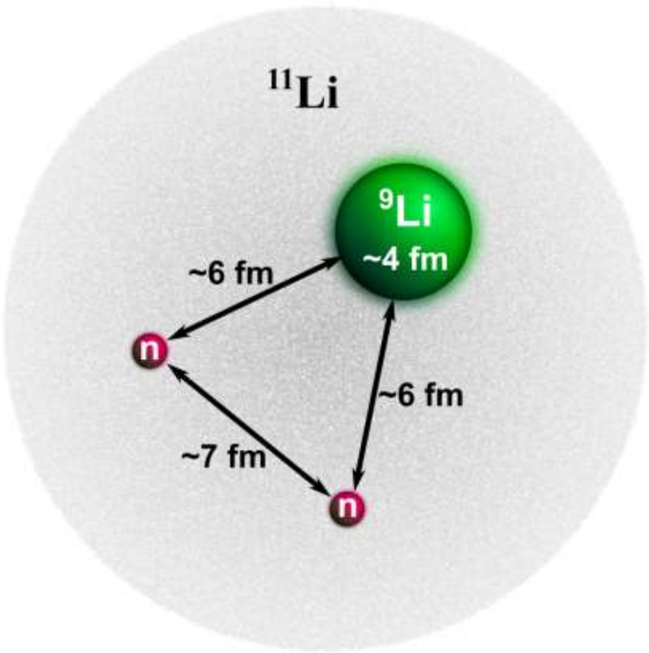
- Take three particles with huge, but finite range two-body forces (resonant)
- Even if the forces are effectively repulsive (no dimer state), there is a bound trimer
- The number of trimer states is very large
- Typical binding lengths are much larger than the range of the two-body potential

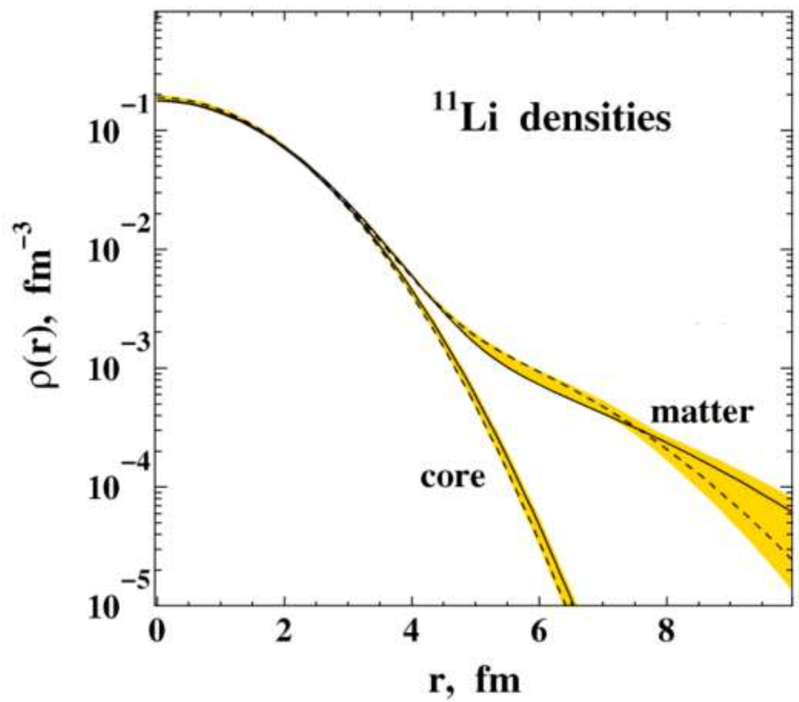
Quantum halos

Halo nuclei

$^{6}\mathrm{He}$ $^{11}\mathrm{Li}$ $^{11}\mathrm{Be}$ $^{22}\mathrm{C}$ $^{60}\mathrm{Ca}$

^{10}Li or ^{5}He does not bind!



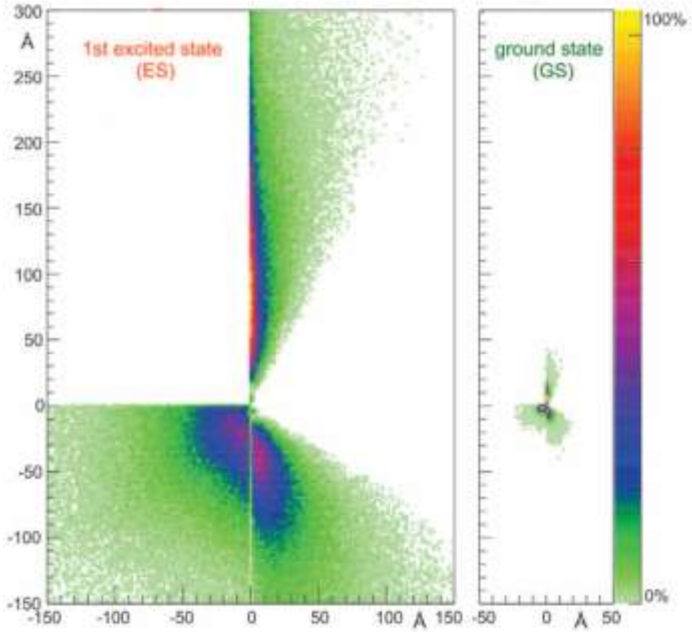


Dobrovolsky *et al.*, Nucl. Phys. A **766**, 1 (2006)

Quantum halos

Efimov molecules

Homonuclear (He_3)



Kunitski *et al.*, Science **348**, 521 (2015)

Heteronuclear (LiCsCs)

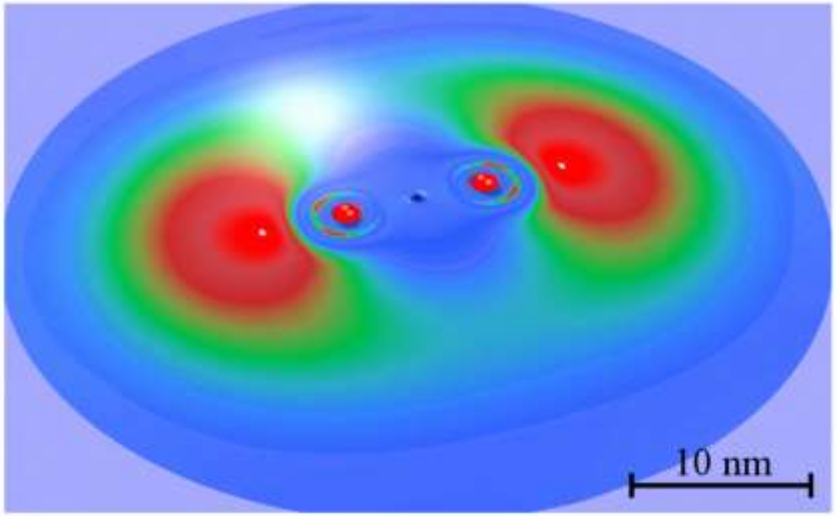
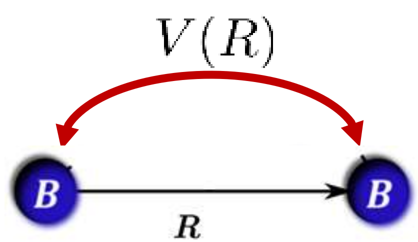


Figure courtesy of Y. Wang

Further universal three-body systems with relevance to solid state physics and biology

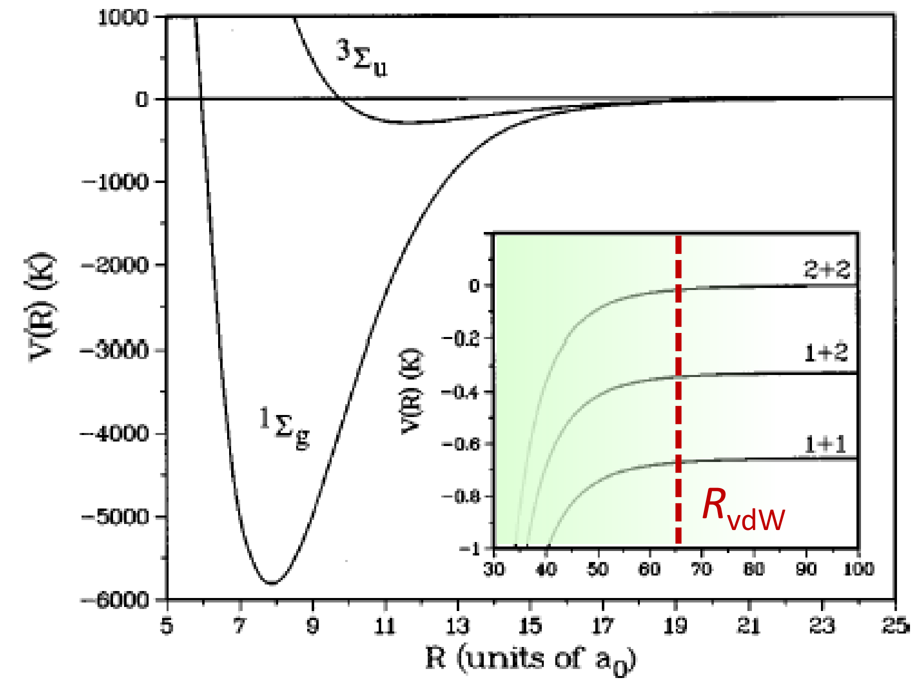
“Normal” two-body system



Long-range potential

(s-wave scattering $\ell = 0$)

$$V_g(R) \simeq -\frac{C_6}{R^6}$$

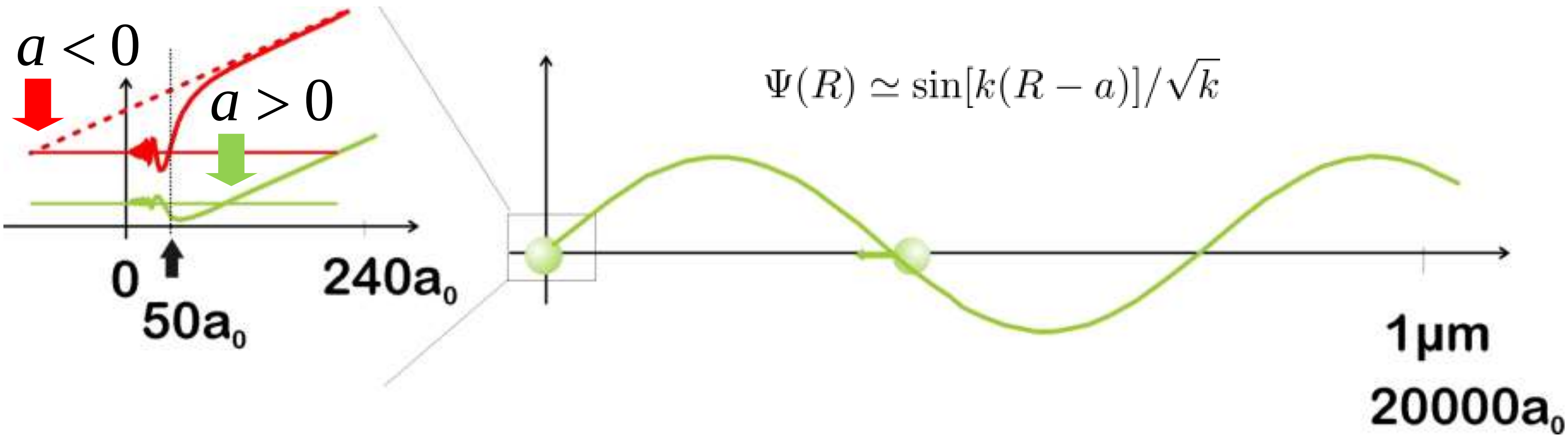


Van-der Waals range

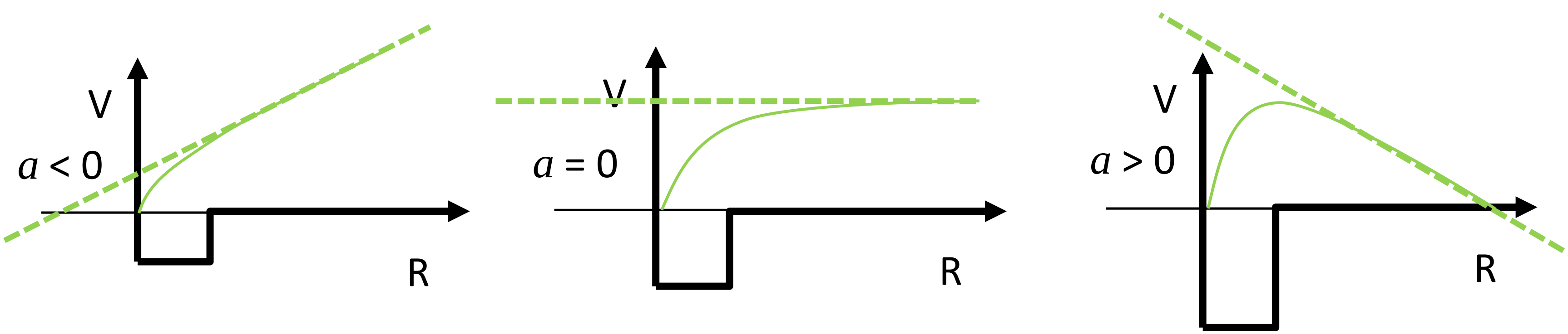
$$V_g(R_{\rm vdW}) \simeq -\frac{\hbar^2}{mR_{\rm vdW}^2} \qquad \Rightarrow R_{\rm vdW} \simeq \left(\frac{mC_6}{\hbar^2}\right)^{1/4}$$

Scattering length

Two-body scattering at low energies

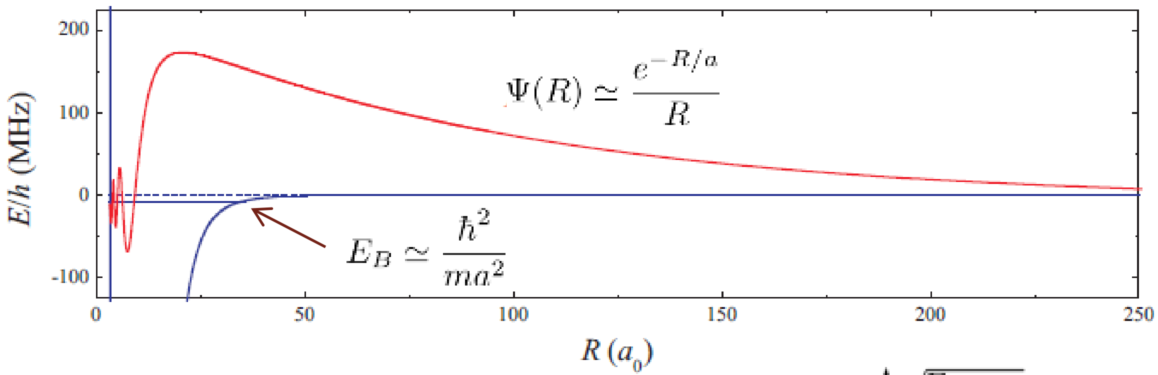


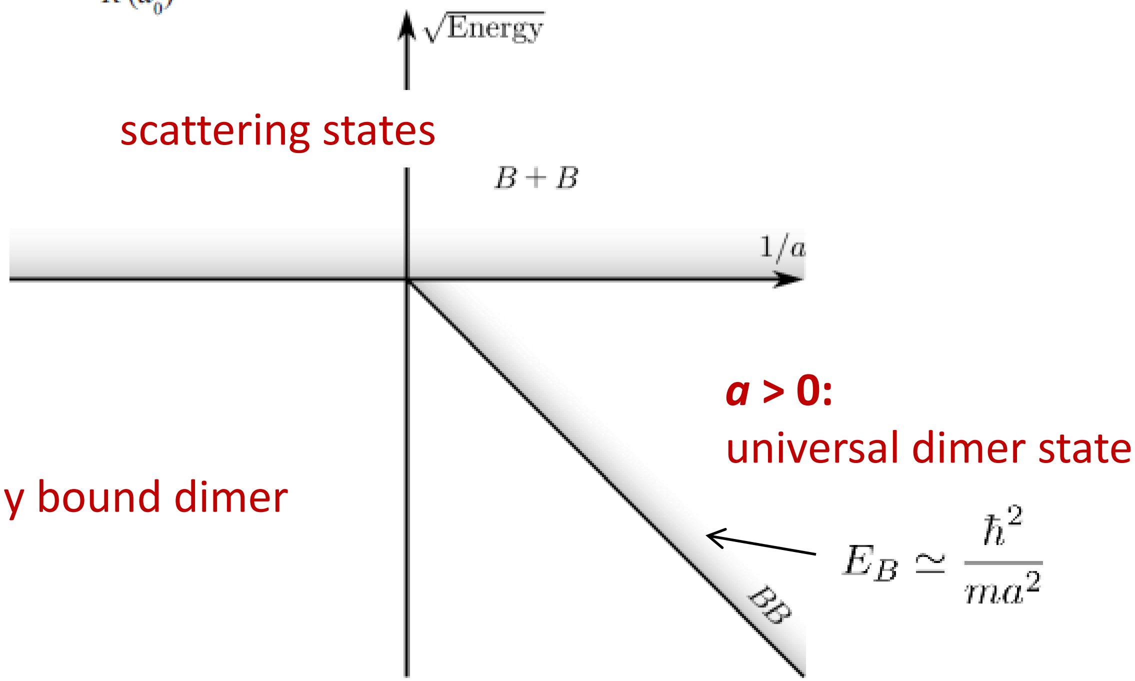
$R \gg R_{\mathrm{vdW}}$: Details of the potential don't matter!



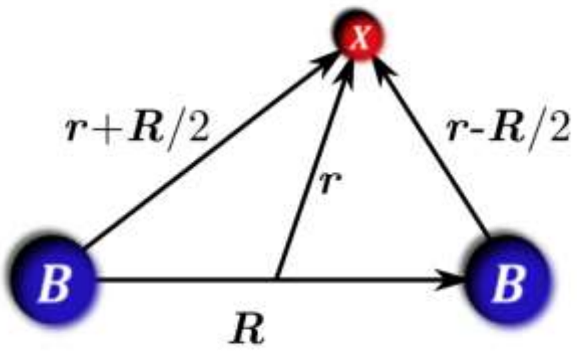
Weakly bound dimer state

Positive scattering length (*a* > 0) and negative energies:





Born-Oppenheimer approach



Wavefunction for the light atom close to a two-body scattering resonance

$$\psi_{\mathbf{R}}(\mathbf{r}) \propto C_1 \frac{e^{-\kappa(R)|\mathbf{r}-\mathbf{R}/2|}}{|\mathbf{r}-\mathbf{R}/2|} + C_2 \frac{e^{-\kappa(R)|\mathbf{r}+\mathbf{R}/2|}}{|\mathbf{r}+\mathbf{R}/2|}$$

Schrödinger equation for the light atom

$$\left[-\frac{1}{2m}\nabla_{\mathbf{r}}^2 + \sum_{\pm} V(|\mathbf{r} \pm \mathbf{R}/2|)\right] \psi_{\mathbf{R}}(\mathbf{r}) = \epsilon(R)\psi_{\mathbf{R}}(\mathbf{r})$$

with energy $\epsilon(R) = -\kappa^2(R)/2m$

Born-Oppenheimer energy for heavy atoms (using contact interaction potential between Li-Cs)

$R \ll |a_{AB}|$: $$\boxed{\varepsilon(R) \simeq -\frac{\beta^2}{2mR^2} \approx -\frac{0.16}{mR^2}}$$

effective $1/R^2$ potential

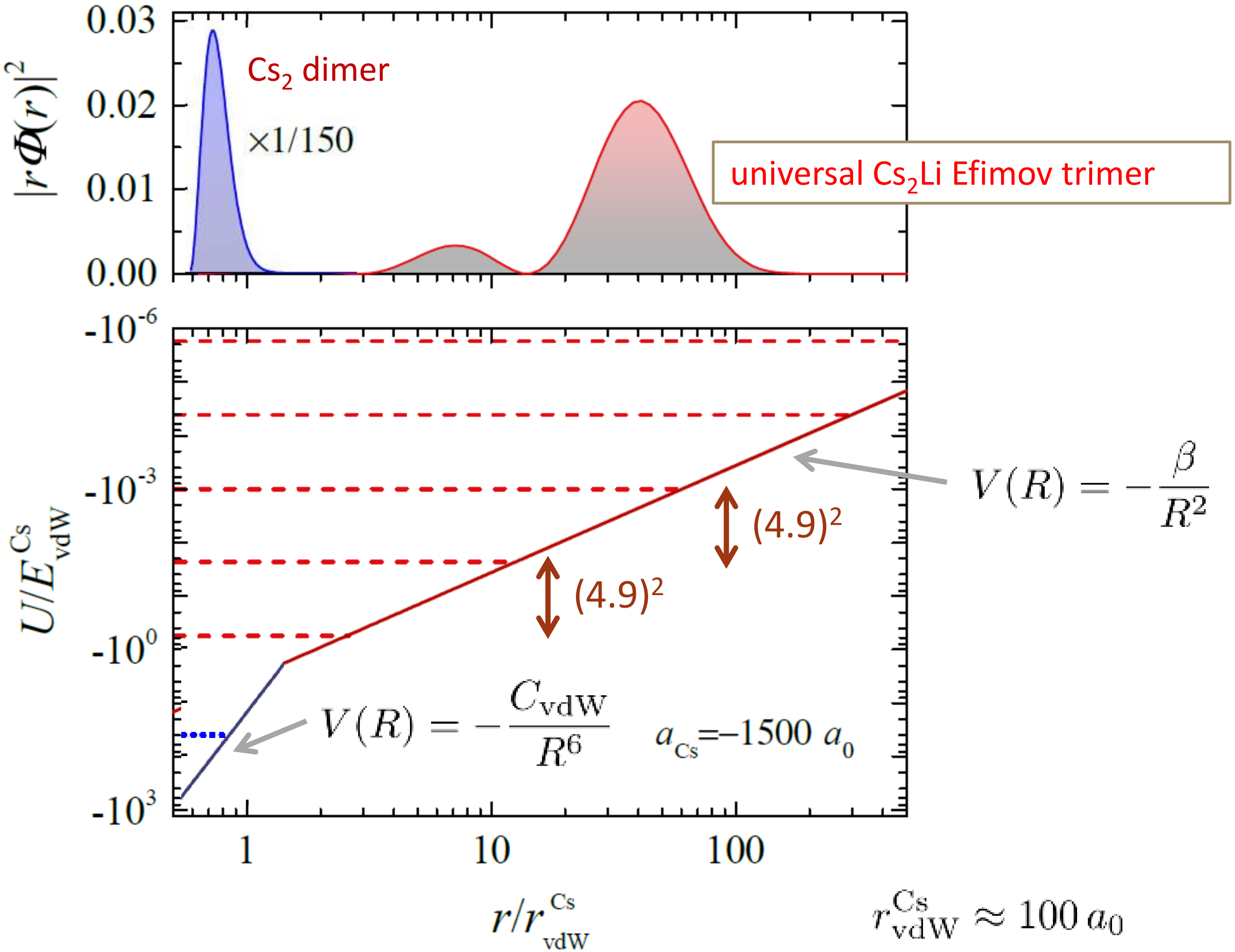
- infinite number of bound states
- discrete scaling symmetry

$$\psi_n(R) = \psi_{n-1}(\lambda R)$$

$$E_n = \lambda^{-2} E_{n-1}$$

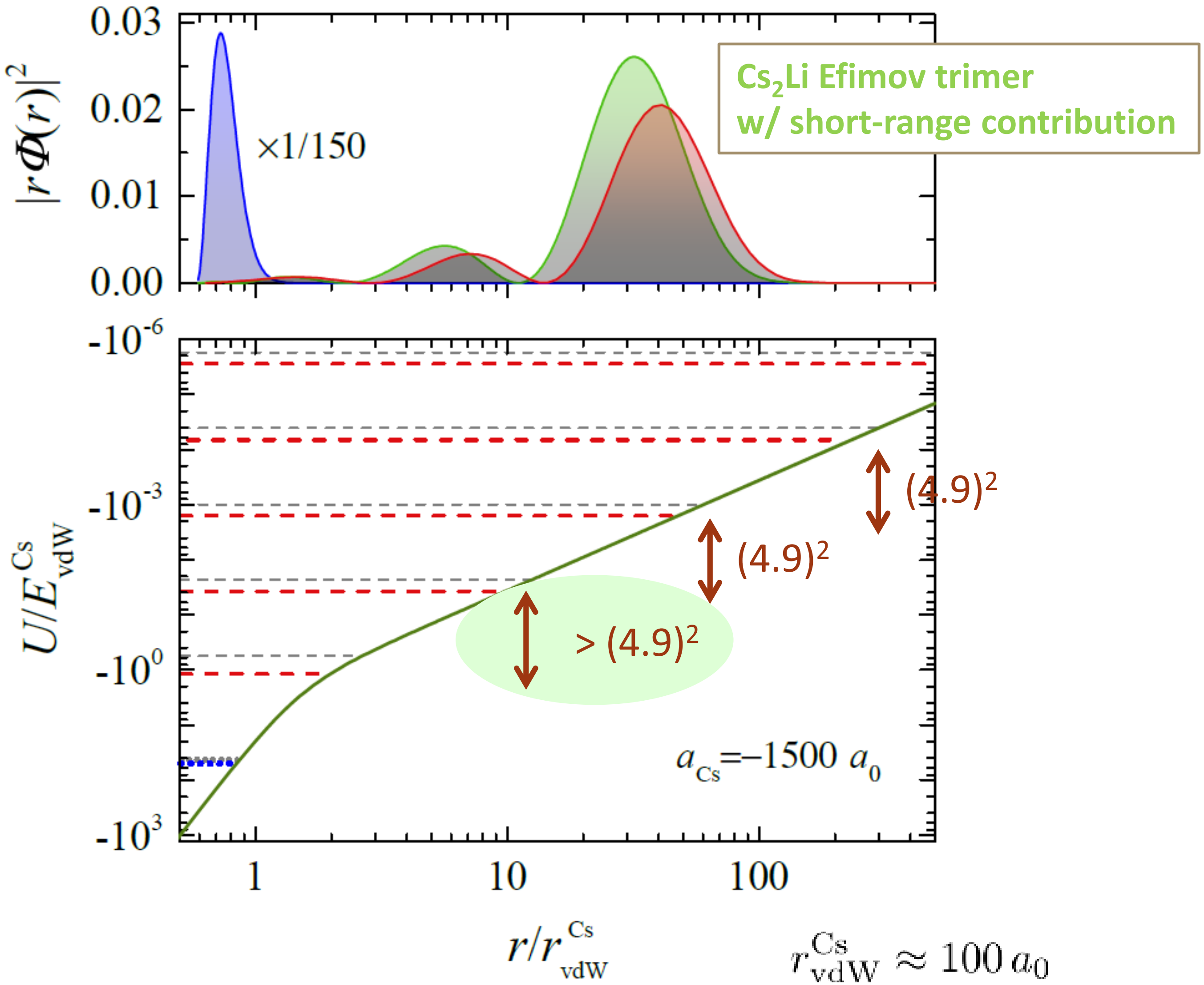
Petrov, D.S.: *The few-body problem*
Lecture Notes from Les Houches School 2010
arXiv:1206.5752

Simplistic Born-Oppenheimer picture



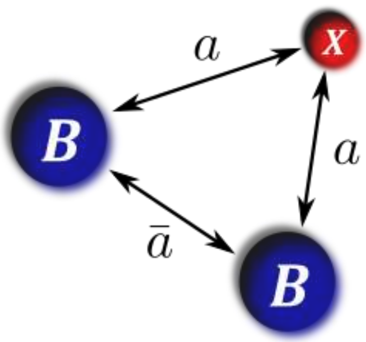
$r_{vdW}^{Cs} \approx 100\, a_0$

Simplistic Born-Oppenheimer picture



$r_{vdW}^{Cs} \approx 100\, a_0$

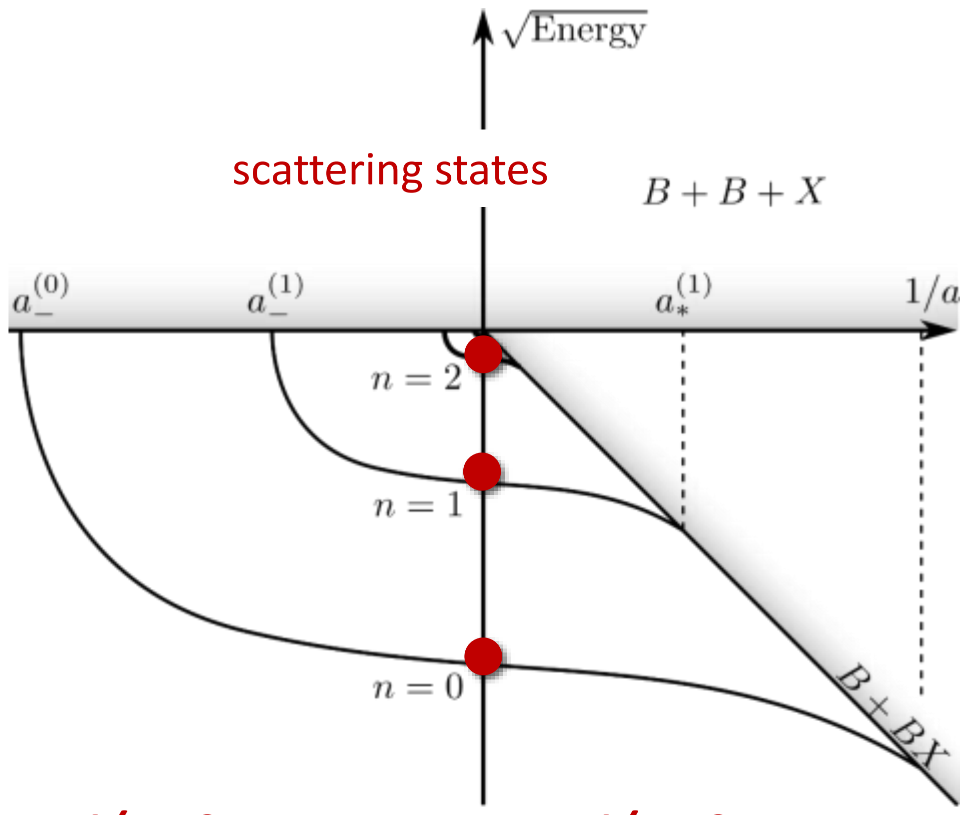
Efimov’s scenario



bound states at infinite *a*

$$\boxed{E^{(n+1)}/E^{(n)} = (22.7)^2}$$

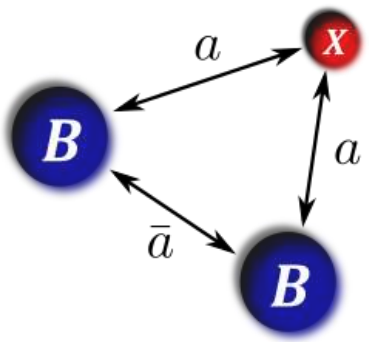
for equal masses!



***1/a* < 0:**
only trimers

***1/a* > 0:**
universal threshold *B*+*BX*

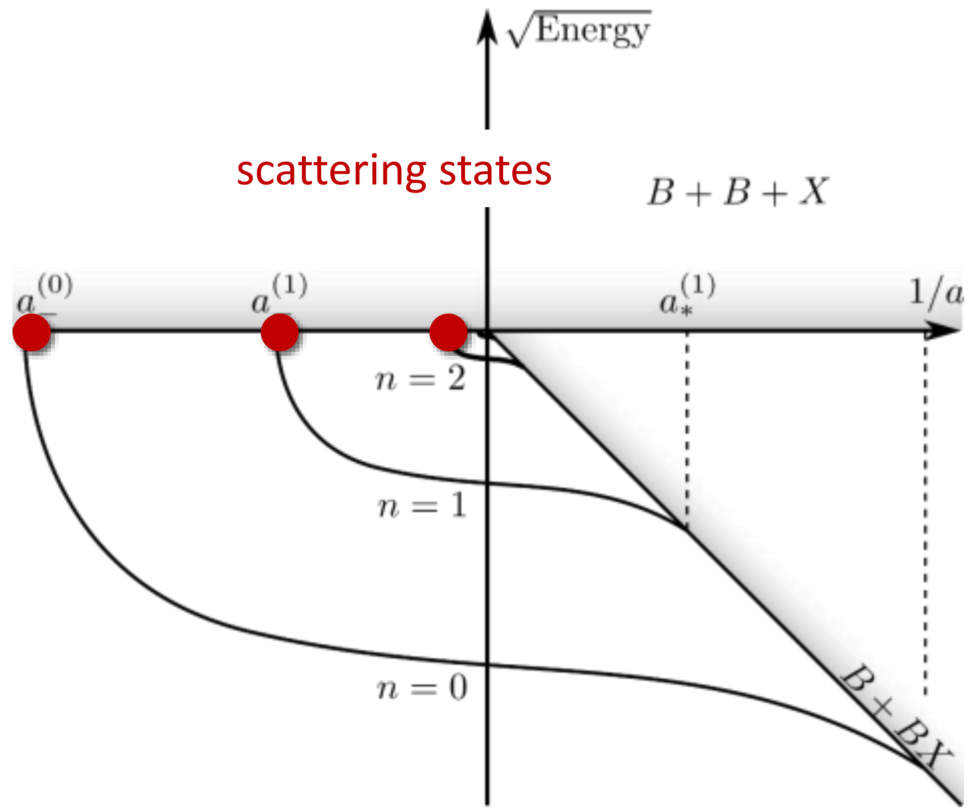
Efimov’s scenario



triatomic resonances

$$\boxed{a_-^{(n+1)}/a_-^{(n)} = 22.7}$$

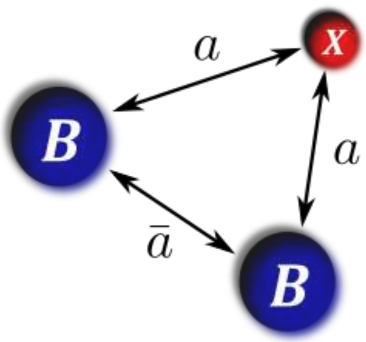
for equal masses!



***1/a* < 0:**
only trimers

***1/a* > 0:**
universal threshold *B+BX*

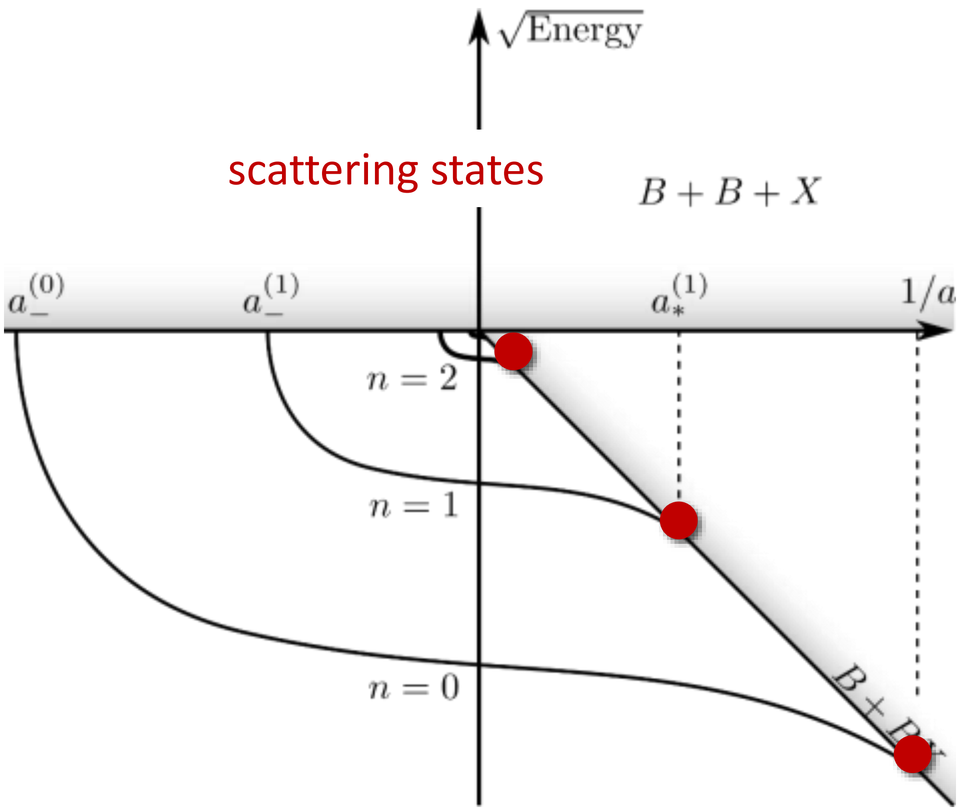
Efimov's scenario



atom-dimer resonances

$$\boxed{a_*^{(n+1)}/a_*^{(n)} = 22.7}$$

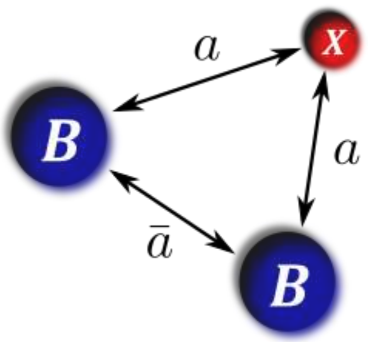
for equal masses!



***1/a* < 0:**
only trimers

***1/a* > 0:**
universal threshold *B+BX*

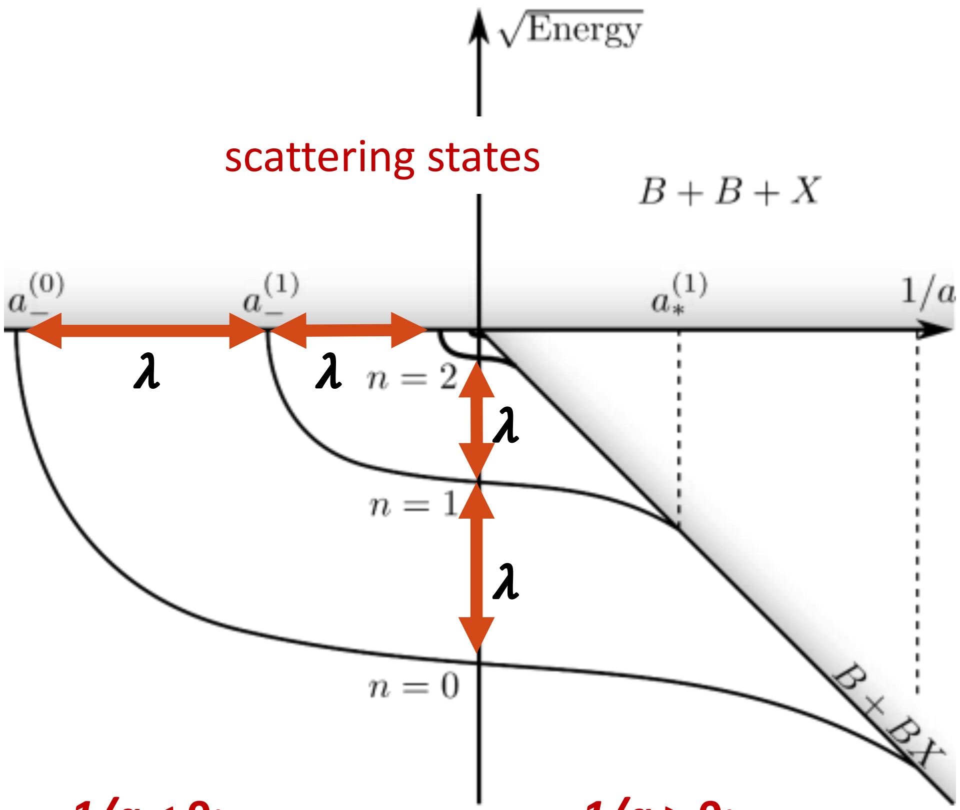
Efimov's scenario



$\boldsymbol{a \to \pm\infty}$ $\boldsymbol{a \gg r_0}$

for mass imabalnce:

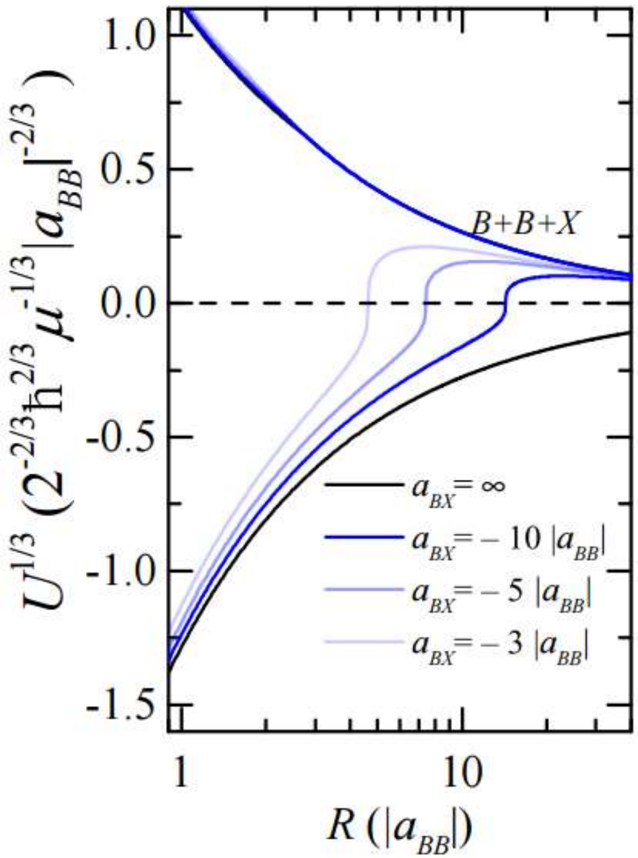
$$\boxed{\boldsymbol{\lambda = \lambda(m_X, m_B)}}$$



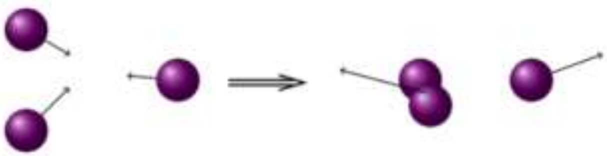
***1/a* < 0:**
only trimers

***1/a* > 0:**
universal threshold *B+BX*

Experimental observation via three-body recombination



Three-body loss coefficient



$$\dot{n} = -L_3 n^3$$

with

$$L_3 = n_l C(a)(\hbar/m)a^4$$

Efimov scaling:

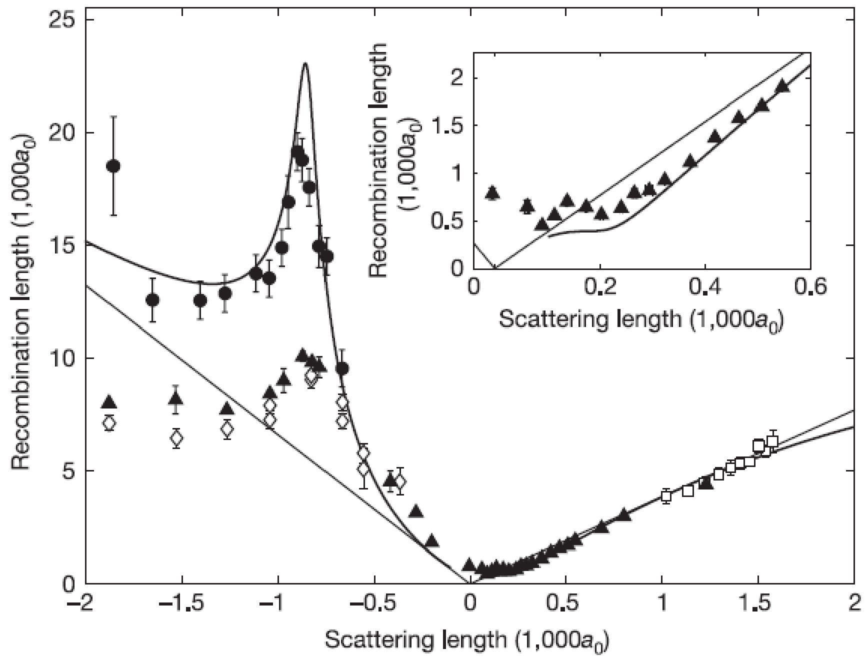
$$\boxed{C(\lambda a) = C(a)}$$

First demonstration: ultracold Cs gas

NATURE|Vol 440|16 March 2006

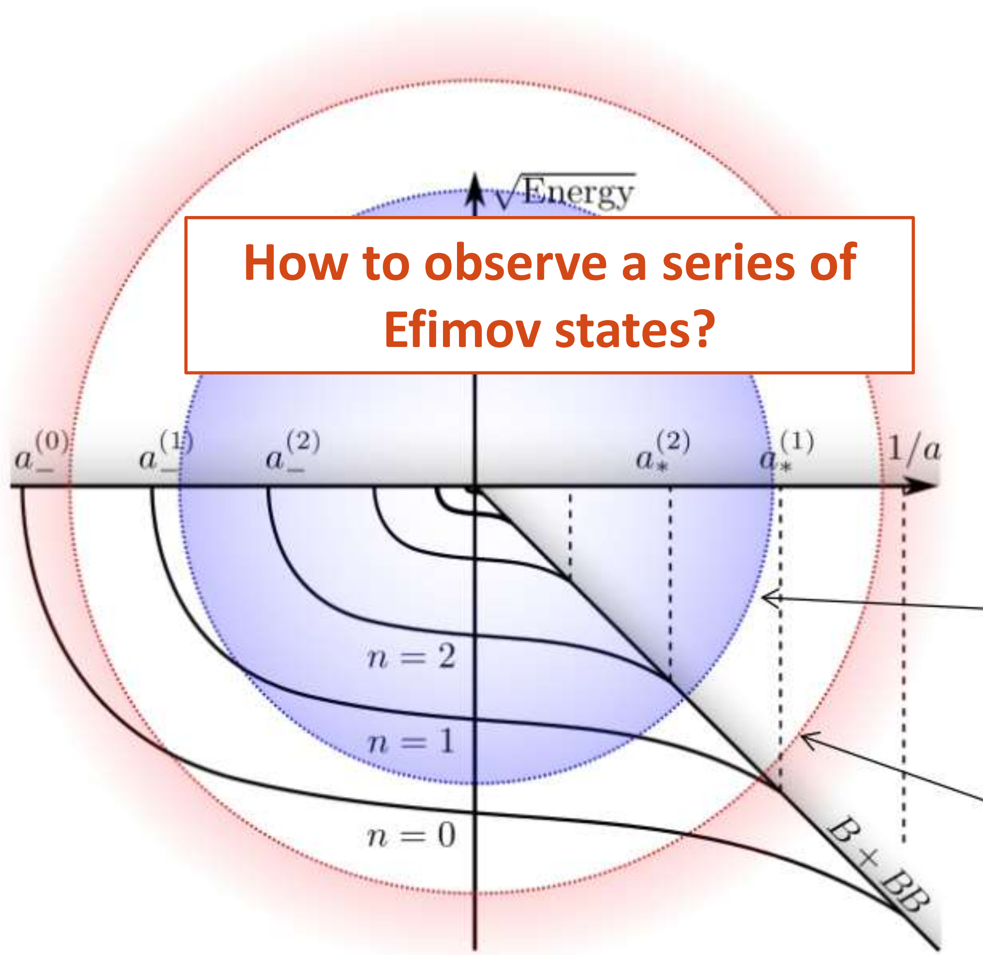
Evidence for Efimov quantum states in an ultracold gas of caesium atoms

T. Kraemer[1], M. Mark[1], P. Waldburger[1], J. G. Danzl[1], C. Chin[1,2], B. Engeser[1], A. D. Lange[1], K. Pilch[1], A. Jaakkola[1], H.-C. Nägerl[1] & R. Grimm[1,3]

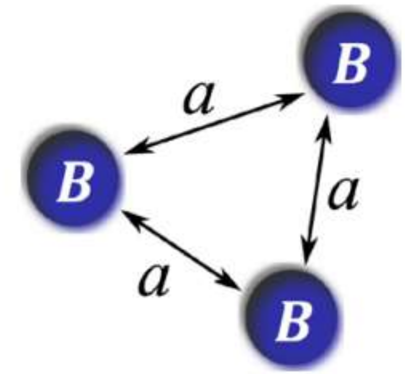


More experiments at (homo- and heteronuclear):
Aarhus, Chicago, ENS, Heidelberg, Houston, JILA, LENS, Pennsylvania, Ramat-Gan, Tokio, Tübingen …

Window of universality



How to observe a series of Efimov states?



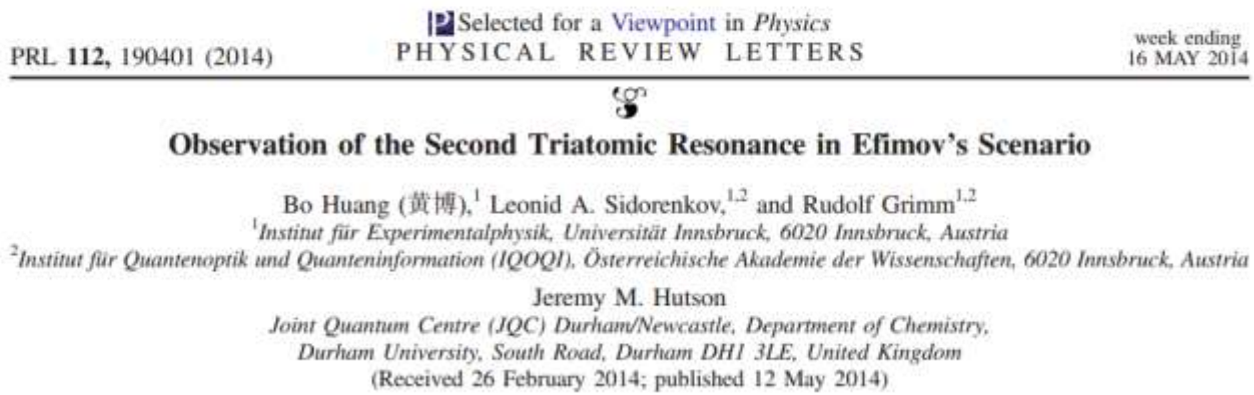
$\lambda = 22.7$

$1/\lambda_{dB}$ **de Broglie wavelength**
~ 5000 a_0 @ 100 nK

$1/\bar{r}_0$ **van der Waals length**
~ 100 a_0

Determines the ground state (three-body parameter)

Lower temperatures…


PRL 112, 190401 (2014)
Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW LETTERS
week ending 16 MAY 2014

Observation of the Second Triatomic Resonance in Efimov's Scenario

Bo Huang (黄博),[1] Leonid A. Sidorenkov,[1,2] and Rudolf Grimm[1,2]
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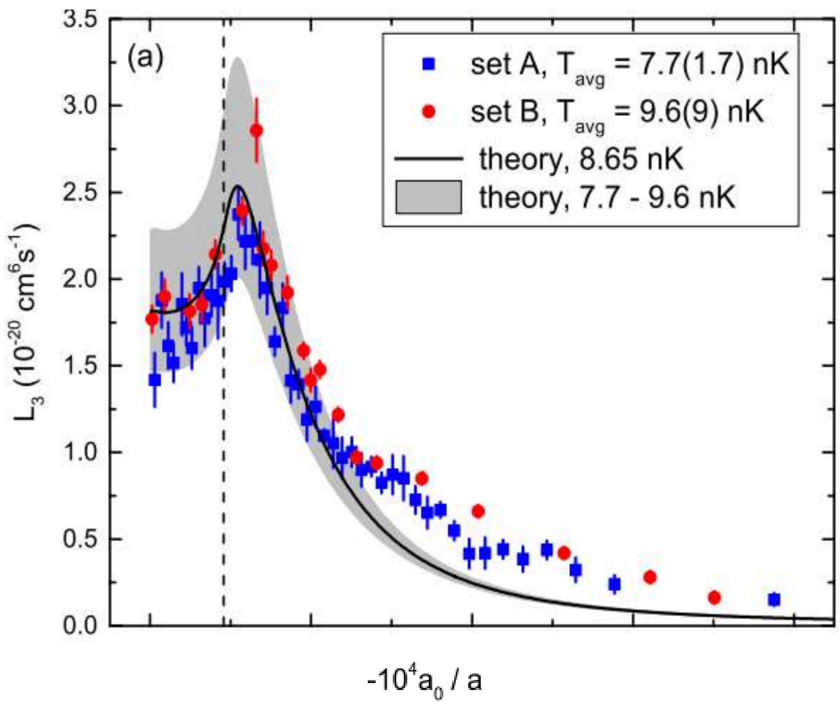
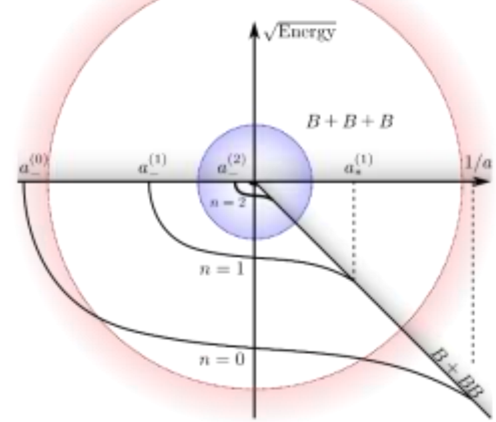
[2]*Institut für Quantenoptik und Quanteninformation (IQOQI), Österreichische Akademie der Wissenschaften, 6020 Innsbruck, Austria*
Jeremy M. Hutson
Joint Quantum Centre (JQC) Durham/Newcastle, Department of Chemistry, Durham University, South Road, Durham DH1 3LE, United Kingdom
(Received 26 February 2014; published 12 May 2014)

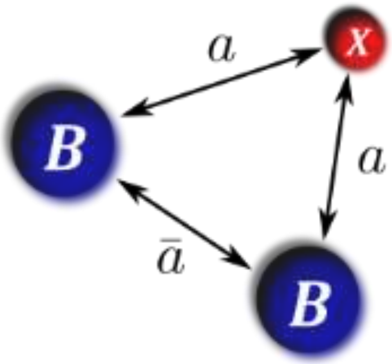


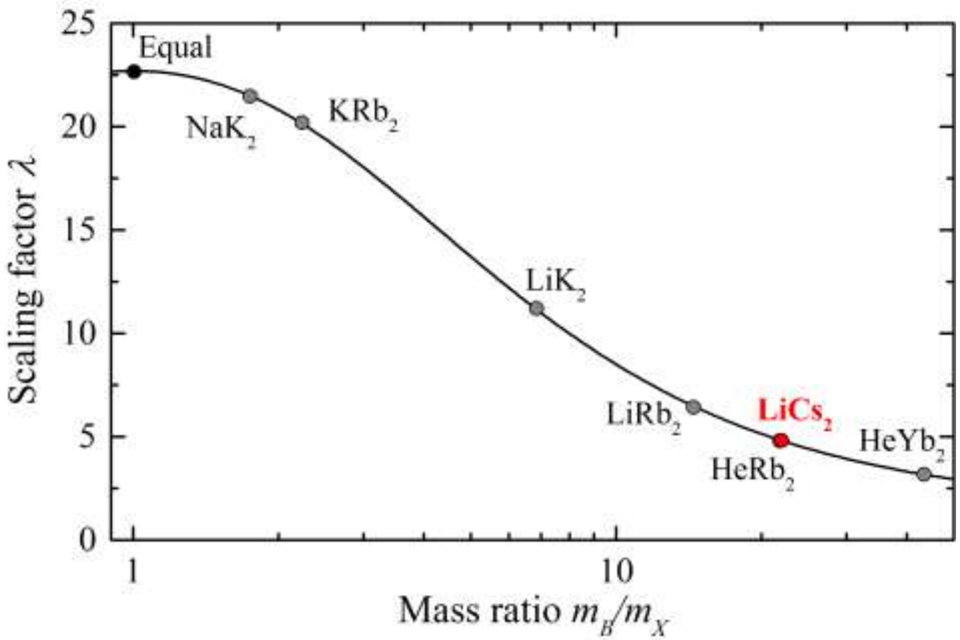


$$\lambda = \frac{a_-^{(1)}}{a_-^{(0)}} = 21.0(1.3)$$

(consistent with universal scaling)

Use two different species





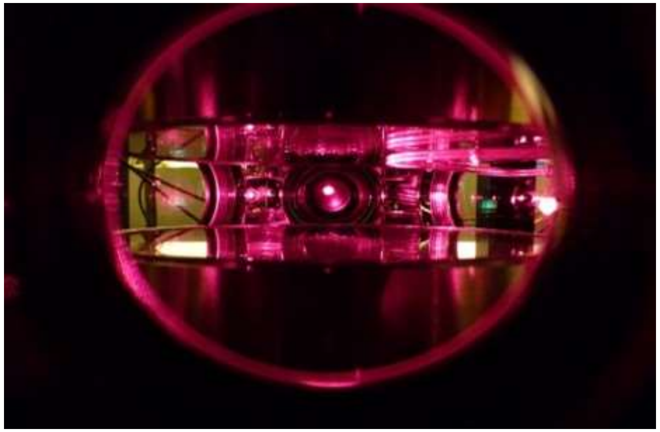
$$\boldsymbol{a} \to \pm\infty \qquad \boldsymbol{a} \gg \boldsymbol{r_0}$$

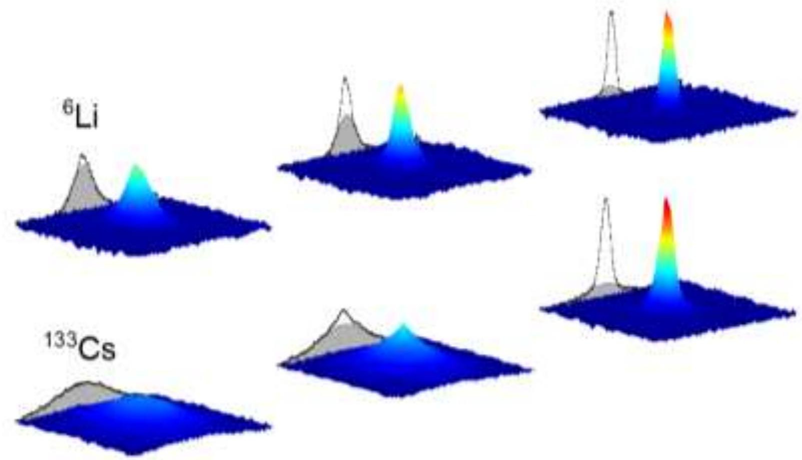
$$\bar{\boldsymbol{a}} \to \pm\infty \qquad \bar{\boldsymbol{a}} \gg \bar{\boldsymbol{r}}_0$$

$$\lambda = \lambda\left(\frac{m_B}{m_X}\right)$$

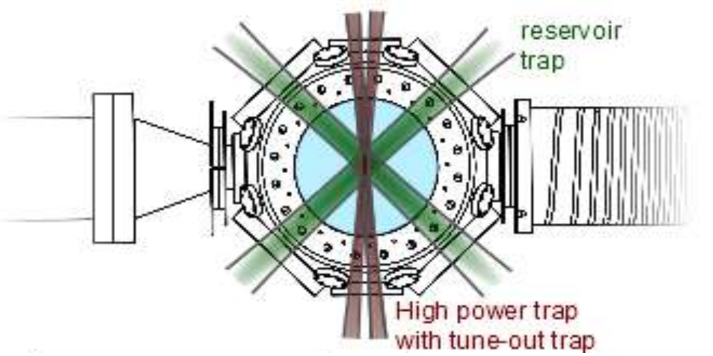
System	λ
^{6}Li - Cs$_2$	**4.877**
^{6}Li - Rb$_2$	6.83
Equal masses	22.7

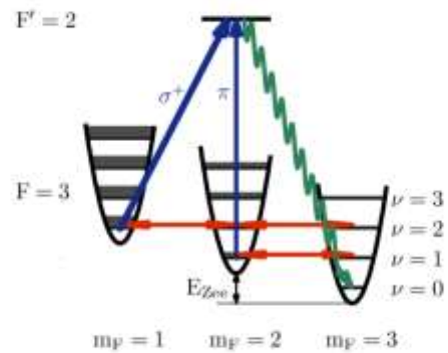
Mixing Li and Cs at nK temperatures

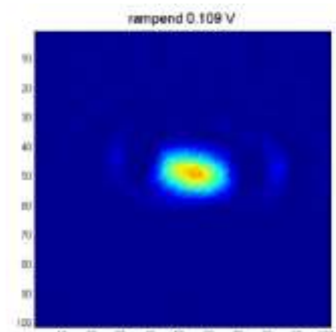




Li MOT → Loading in an optical dipole trap → Forced evaporative cooling → Cs MOT → Degenerate Raman sideband cooling → Mixing of Li and Cs in the ODT



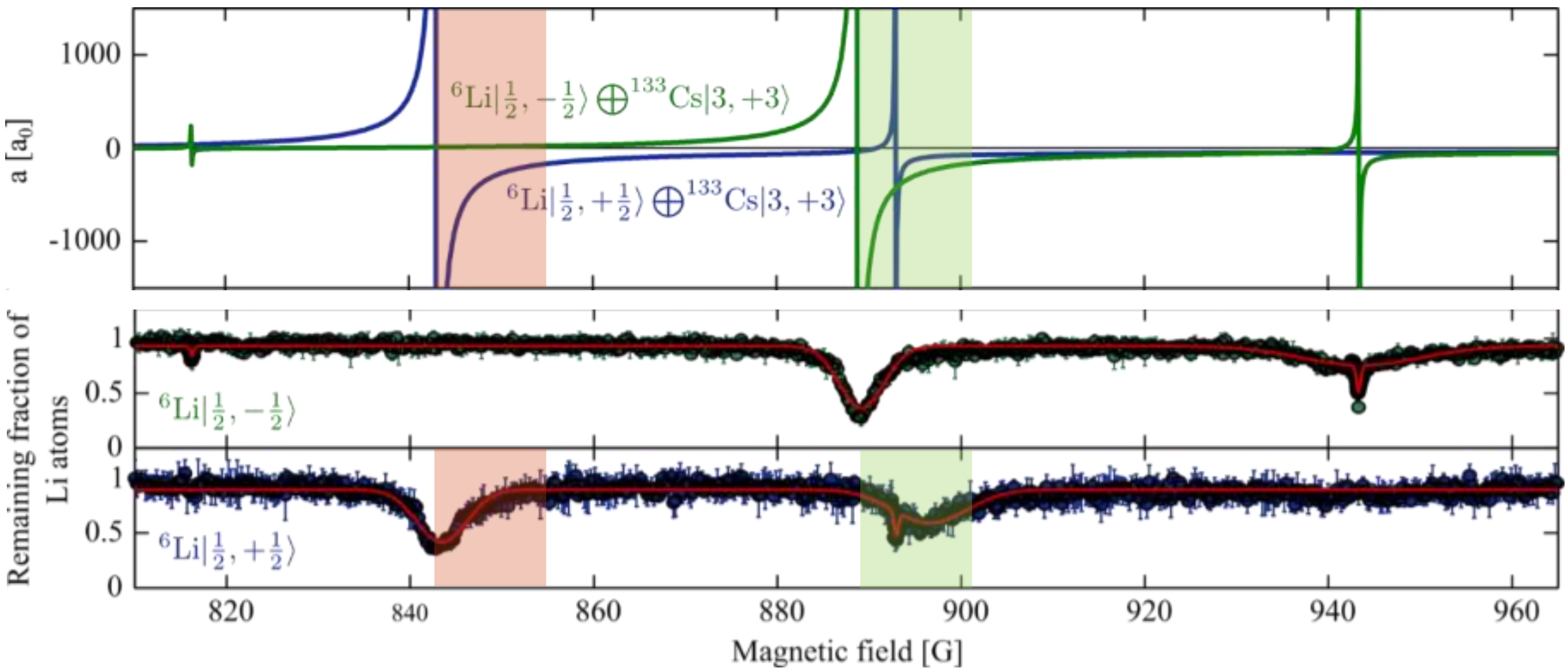




Tuning scattering length in Li-Cs

Feshbach resonances:

coupled-channels calculations by Eberhard Tiemann



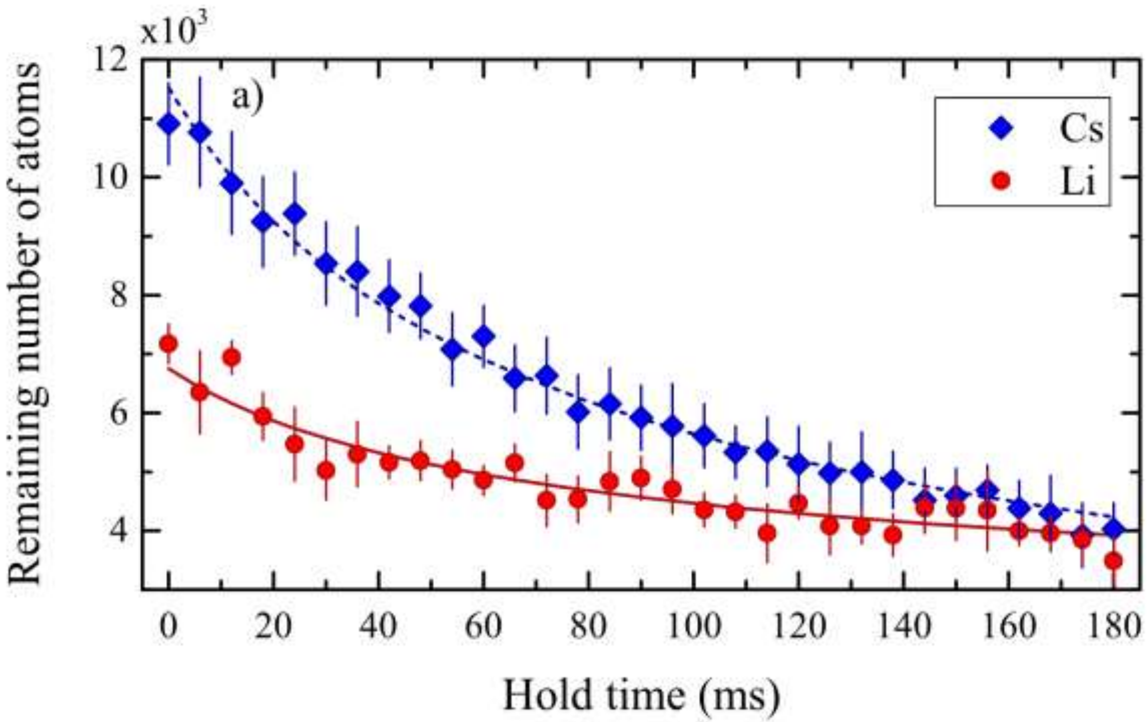
Repp *et al.,* Phys. Rev. A **87**, 010701(R) (2013); see also: Tung *et al., ibid.,* 010702(R)
Pires *et al.,* Phys. Rev. A **90**, 012710 (2014)

Three-body loss rate coefficient

Rate equations:

$$\dot{n}_{Cs} = -L_1^{Cs} n_{Cs} - 2L_3\, n_{Li} n_{Cs}^2 - \cancel{2L_3^{LiLiCs} n_{Li}^2 n_{Cs}} - L_3^{Cs} n_{Cs}^3$$

$$\dot{n}_{Li} = -L_1^{Li} n_{Li} - L_3\, n_{Li} n_{Cs}^2 - \cancel{2L_3^{LiLiCs} n_{Li}^2 n_{Cs} - L_3^{Li} n_{Li}^3}$$

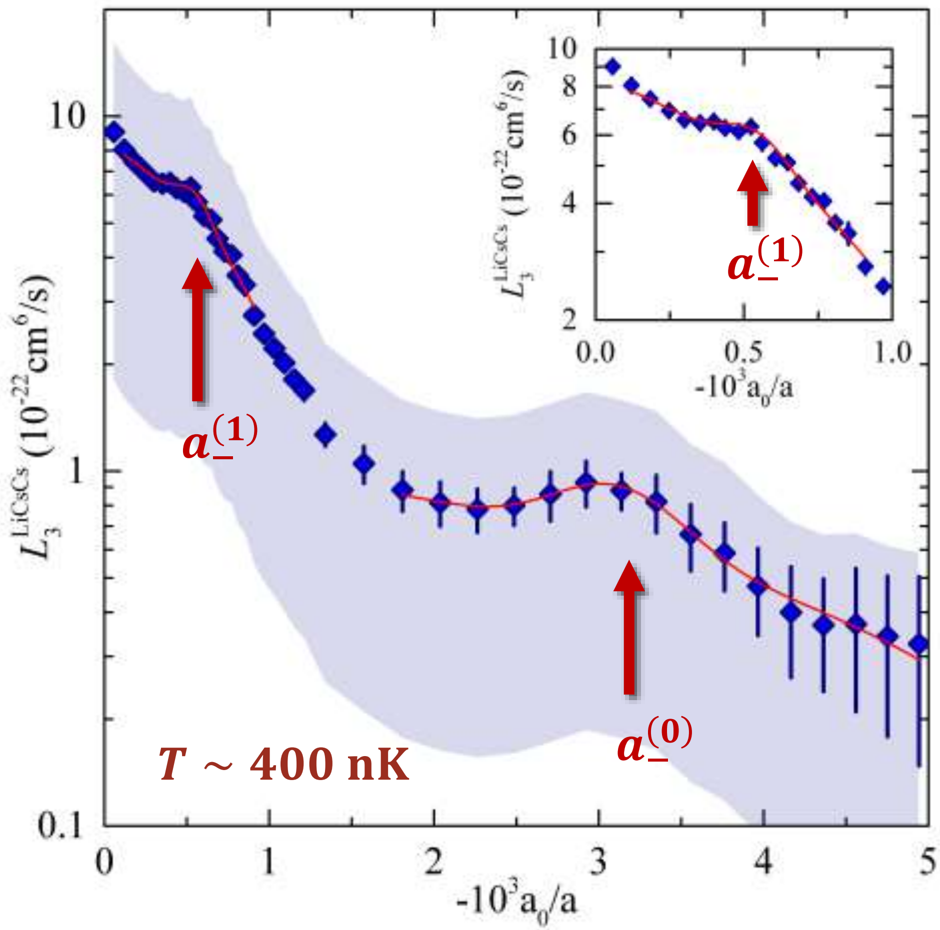


Assumptions:

- Fermionic Li
  → suppression of L_3^{LiLiCs} and L_3^{Li}
- constant temperature
- L_3^{Cs} → constant

$$\mathrm{Li+Cs+Cs} \rightarrow \begin{cases} \mathrm{Li+Cs_2} \\ \mathrm{LiCs+Cs} \end{cases}$$

Successive Efimov resonances



$a_-^{(0)} \approx -310\ a_0$

$$\frac{a_-^{(1)}}{a_-^{(0)}} = 5.48(7)_{\mathrm{stat}}(16)_{\mathrm{sys}}(10)_{\mathrm{rf}}$$

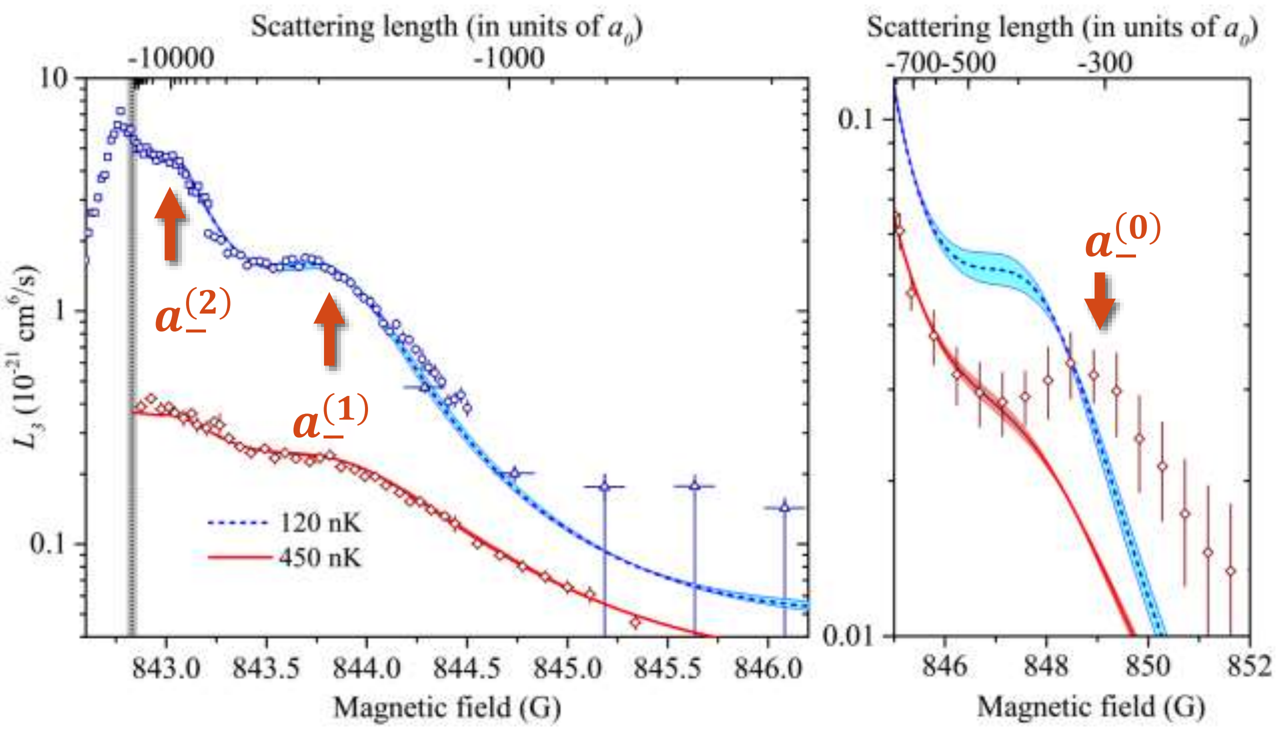
≈ 4.88 ?

R. Pires *et al.*, PRL **112**, 250404 (2014);

See also: S. Tung *et al.,* PRL **113**, 240402 (2014); R.A.W. Mayer *et al.*, PRL **115**, 043201 (2015)

Observation of three Efimov resonances

Use delta potentials to model the two-body potential (zero-range theory)

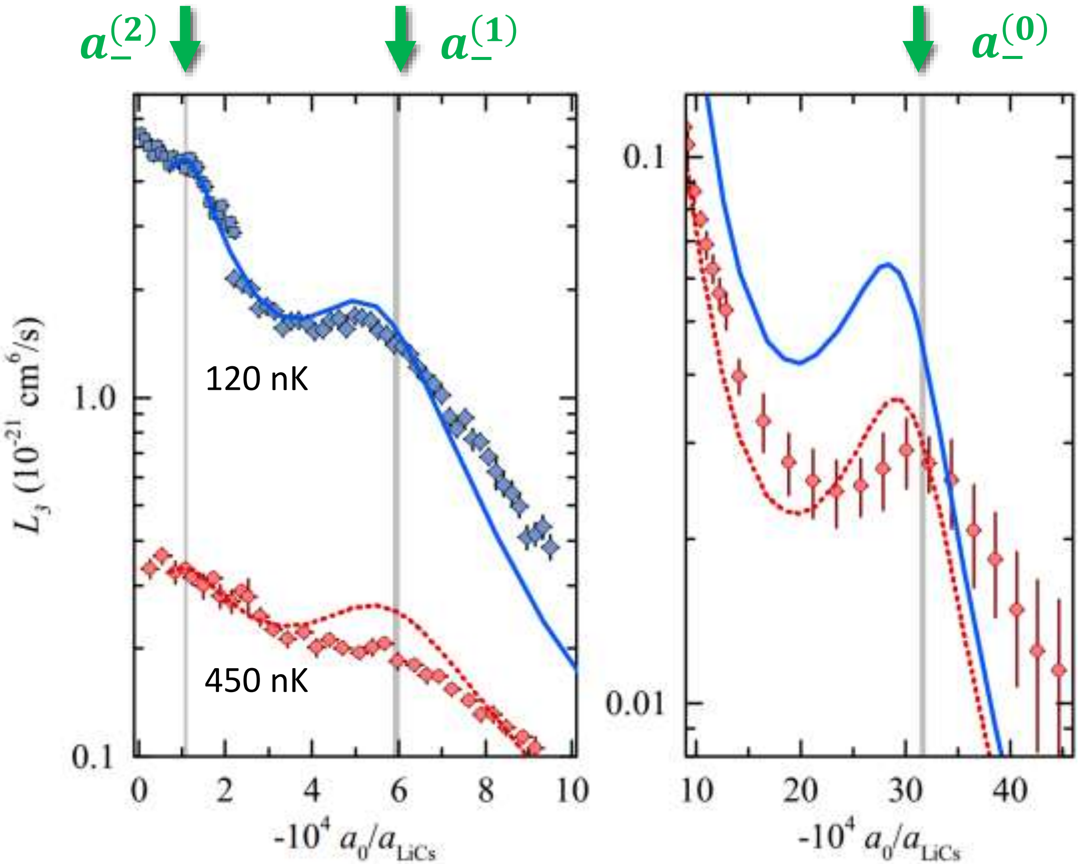


- Consistent description of excited state recombination resonances
- Universal scaling factor ≈ 5
- Non-universal ground state resonance $\boldsymbol{a_-^{(0)}}$

J. Ulmanis *et al.*, PRA **93**, 022707 (2016)

Comparison with van der Waals model

Use Lennard-Jones potentials to model the two-body potential (vdW theory)



vdW theory:

$$\lambda_1 = 5.3 \pm 0.1$$

$$\lambda_2 = 5.1 \pm 0.2$$

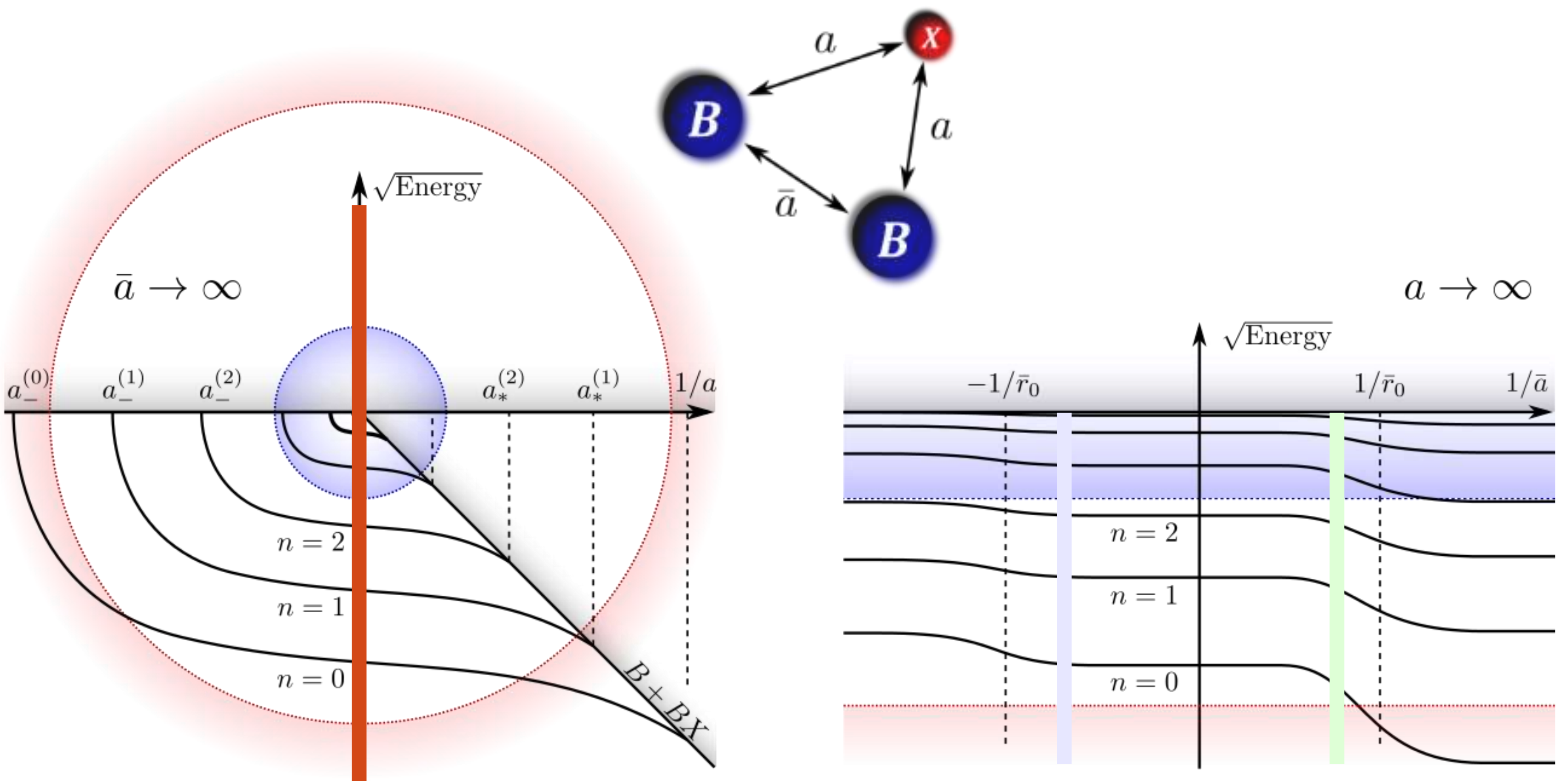
Universal zero-range theory:

$$\lambda_1 = 5.08$$

$$\lambda_2 = 5.18$$

Small short-range effects

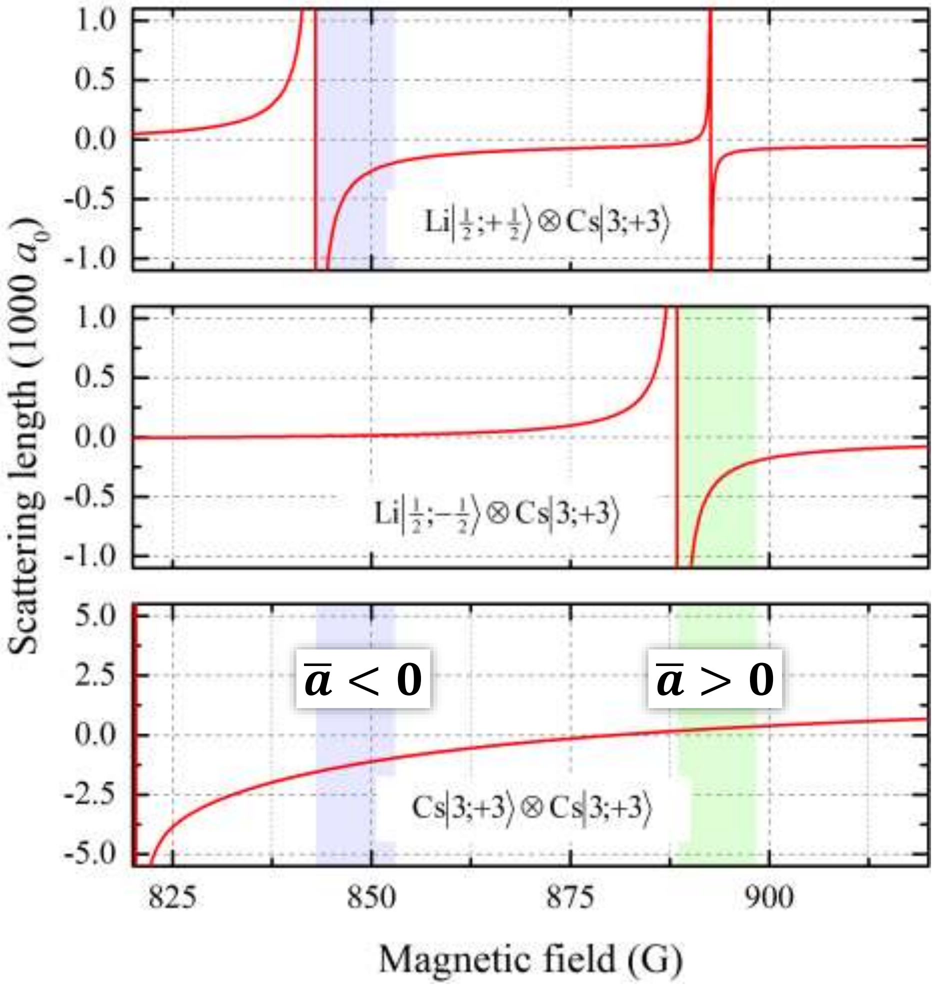
Heteronuclear Efimov scenario



For a review see: Ulmanis *et al.*, Nat. Sci. Rev. 3, 174 (2016)

Tuning scattering length in Li-Cs

Feshbach resonances:



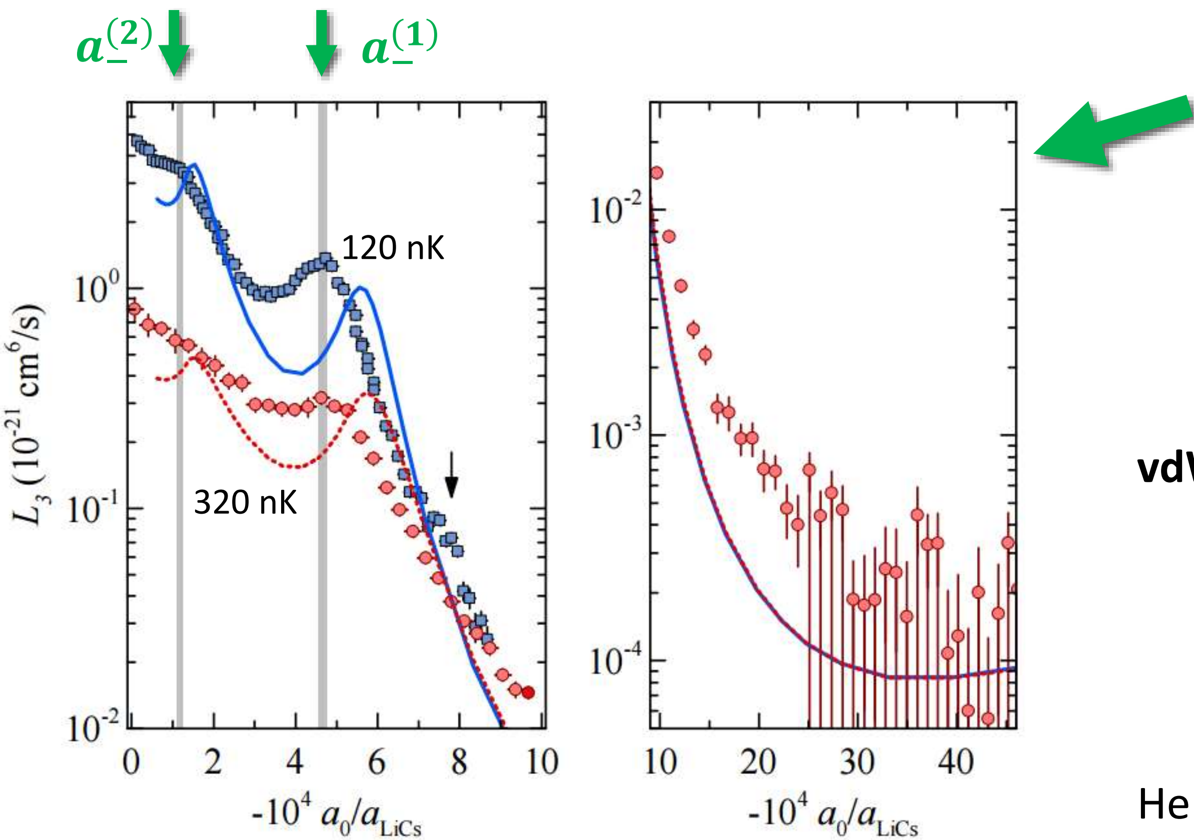
Mapping $a_{\mathrm{LiCs}}(B)$ and $a_{\mathrm{Cs}}(B)$

Cs-Cs interactions (Grimm group):
Berninger et al., Phys. Rev. A 87, 032517 (2013)

Li-Cs interactions (our work):
Repp et al., Phys. Rev. A 87, 010701(R) (2013)
Pires et al., Phys Rev. A 90, 012710 (2014)
Ulmanis et al., New J. Phys. 17, 055009 (2015)

Li-Cs interactions (Chicago):
Tung et al., Phys. Rev. A 87, 010702(R) (2013)

Three-body recombination: $\bar{a} > 0$



Ground state resonance missing???

$\boldsymbol{a_-^{(1)} \approx -2100\ a_0}$

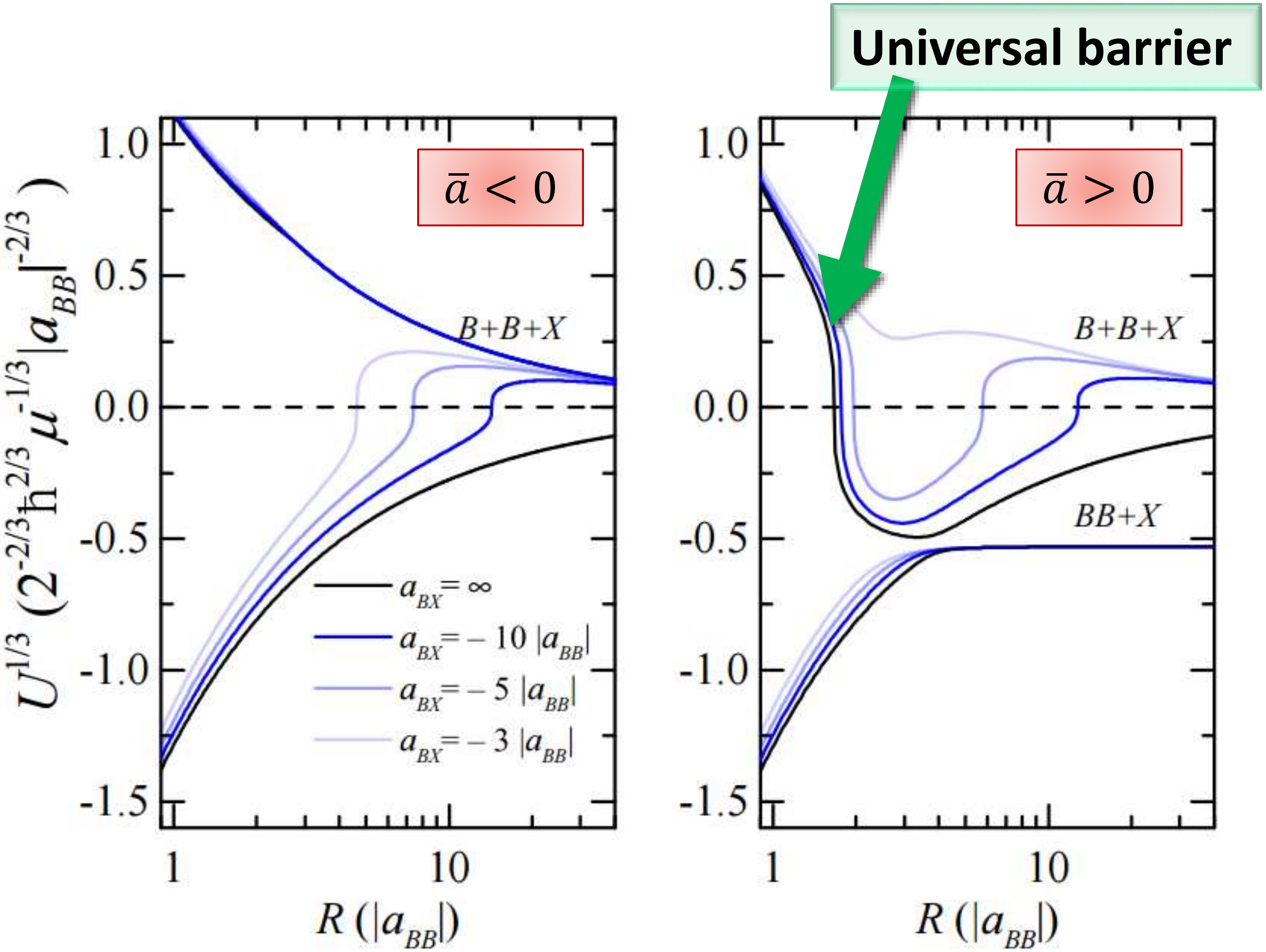
vdW scaling factor:
$\lambda_2 = 4.3^{+0.5}_{-0.4}$

Here: $\boldsymbol{\bar{a} \approx +190\ a_0}$

Ulmanis *et al.*, PRL **117**, 153201 (2016)

Universal van der Waals model: Yujun Wang and Chris Greene

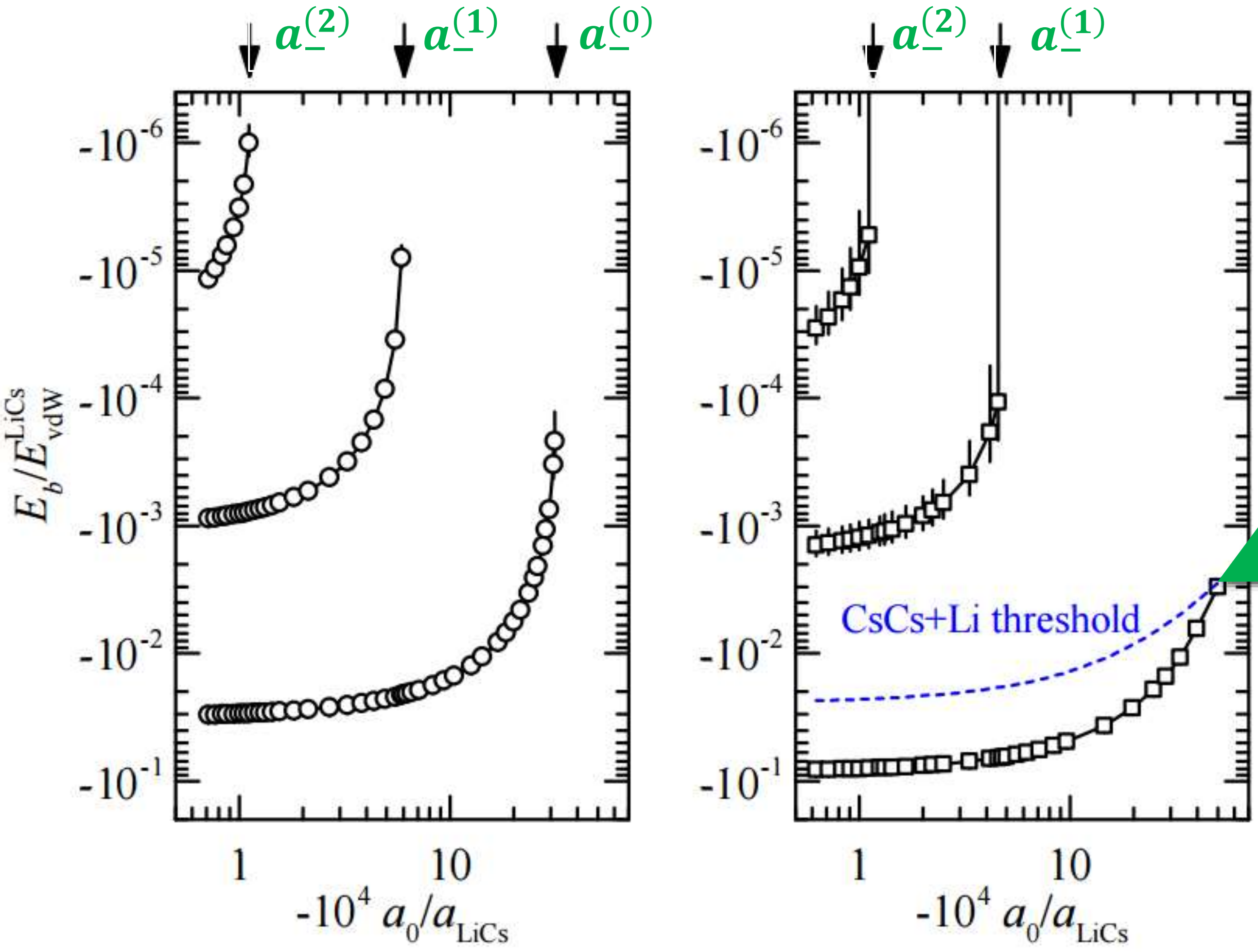
Universal three-body states



Ulmanis *et al.*, PRL **117**, 153201 (2016)

Efimov states in the upper branch independent of two-body phsysics!

Efimov spectrum for Li-Cs-Cs



Ground state trimer merges with the $\mathbf{CsCs + Li}$ threshold!

$\bar{a} \approx -1500\, a_0$

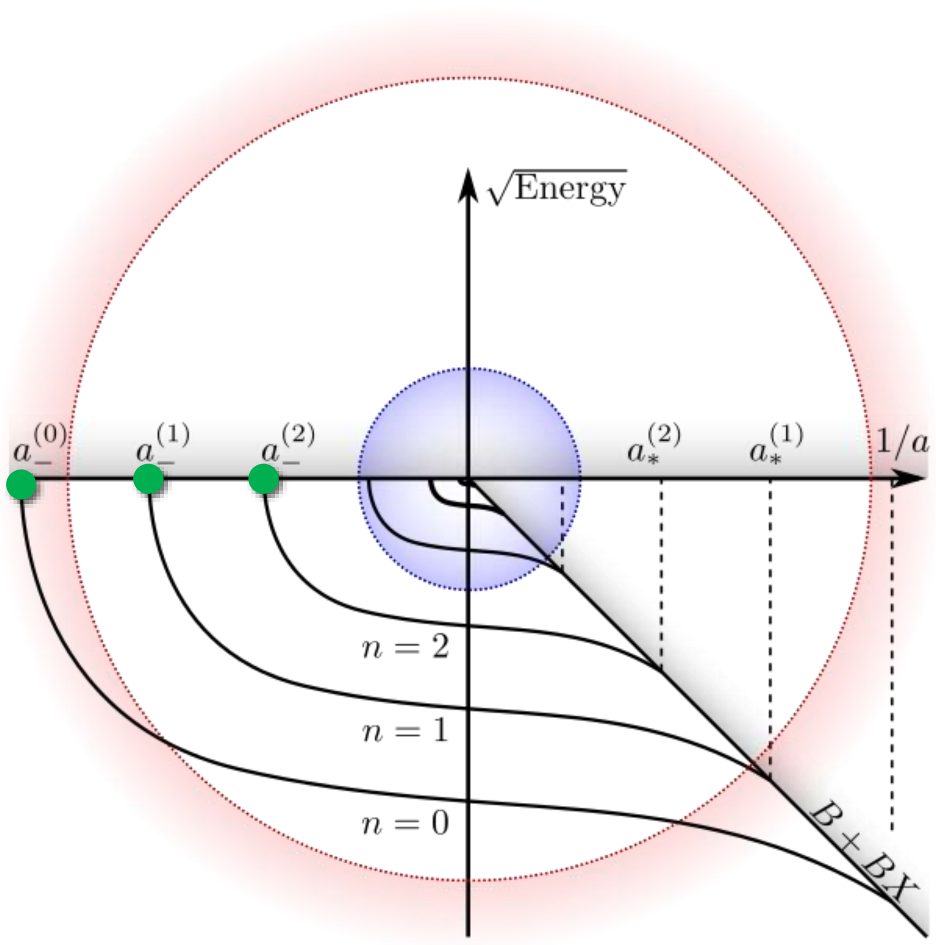
$\bar{a} \approx +190\, a_0$

Ulmanis *et al.*, PRL **117**, 153201 (2016)

Dissappearance of the resonance

$\boldsymbol{a} \to \pm\infty$

$\bar{\boldsymbol{a}} < \boldsymbol{0}$



Dissappearance of the resonance

$\boldsymbol{a} \to \pm\infty$

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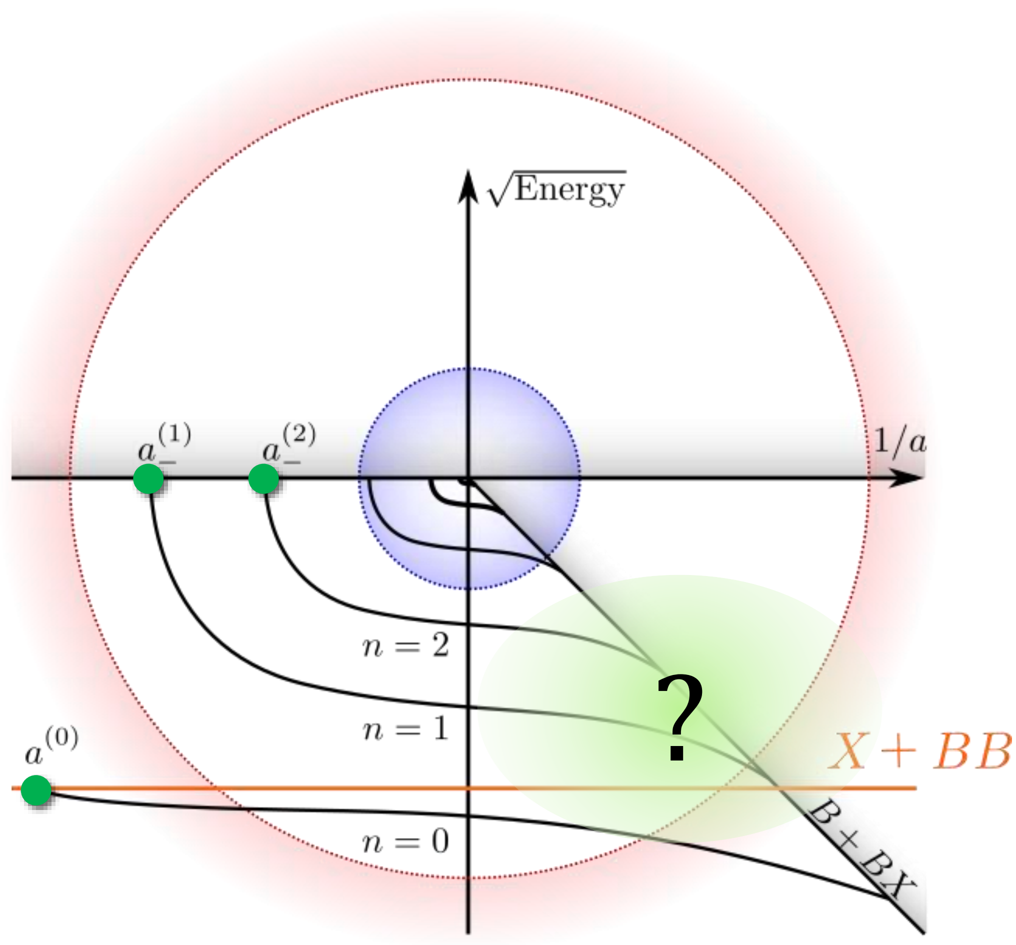
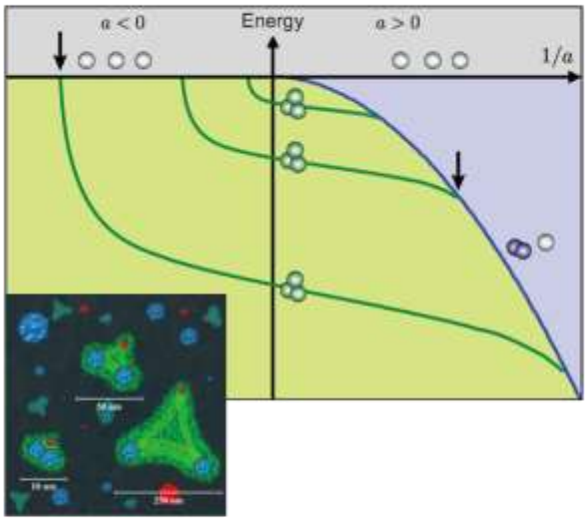
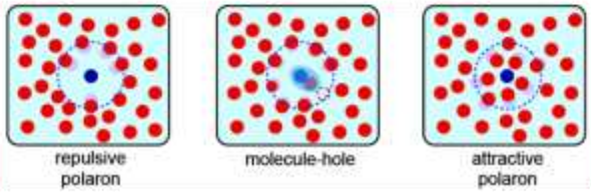


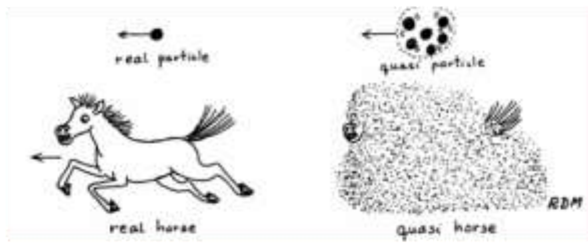
Outlook

Few-to many bound states: Efimov effect, N-body states …

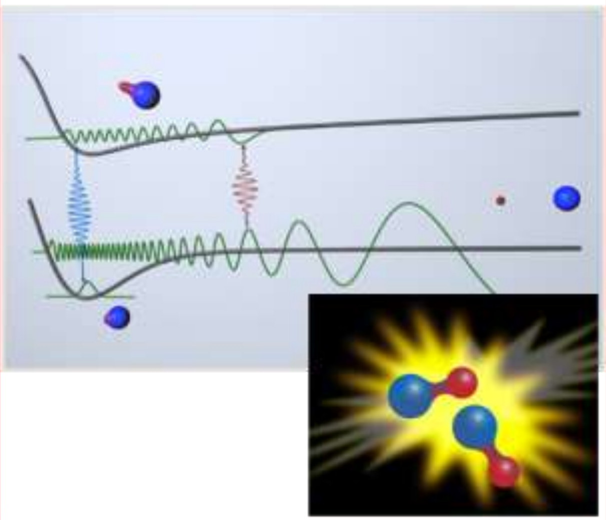


Impurity in a bath: Polarons, simulation of solid-state systems …





Polar ground state molecules: Dipolar interactions, ultracold chemistry …



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