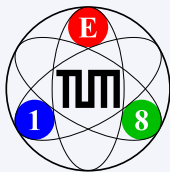


Light-meson Spectroscopy in Multi-Body Final States

Boris Grube

Institute for Hadronic Structure and Fundamental Symmetries
Technische Universität München,
Garching, Germany

EMMI Physics Day 2016
GSI Darmstadt, 15. November 2016



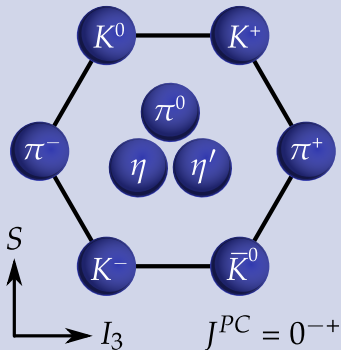
- 1 Introduction
 - Quark model and meson spectrum
 - Beyond the constituent quark model
- 2 How to measure the light-meson spectrum?
 - The COMPASS Experiment
 - Meson production in diffractive dissociation
 - Partial-wave analysis method
- 3 Partial-wave analysis of 3π final state
 - Dominant partial waves
 - Observation of a new meson state $a_1(1420)$
- 4 Conclusions and outlook

Quark-Model Meson Nonets

Light-quark mesons

- u, d , and s quarks $\implies \text{SU}(3)_{\text{flavor}}$ nonets

Ground-state nonets



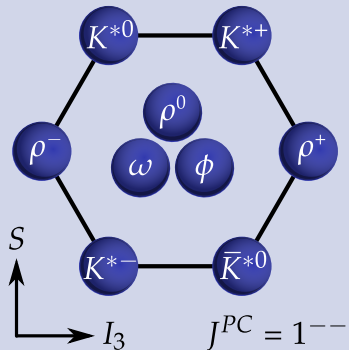
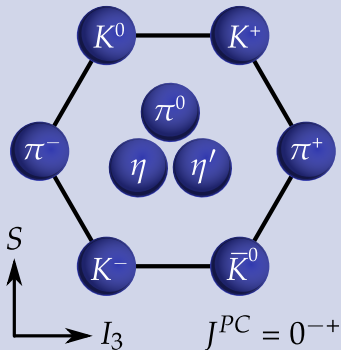
- Many more nonets for orbital and radial excitations

Quark-Model Meson Nonets

Light-quark mesons

- u, d , and s quarks $\implies \text{SU}(3)_{\text{flavor}}$ nonets

Ground-state nonets



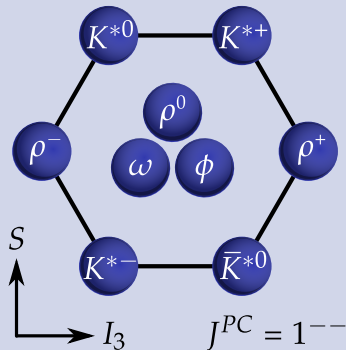
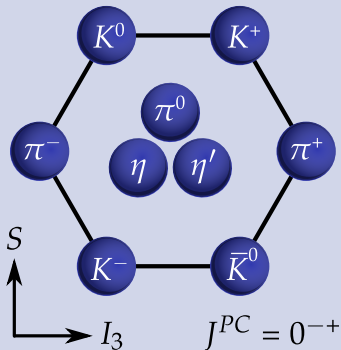
- Many more nonets for orbital and radial excitations

Quark-Model Meson Nonets

Light-quark mesons

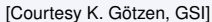
- $u, d,$ and s quarks $\implies \text{SU}(3)_{\text{flavor}}$ nonets

Ground-state nonets



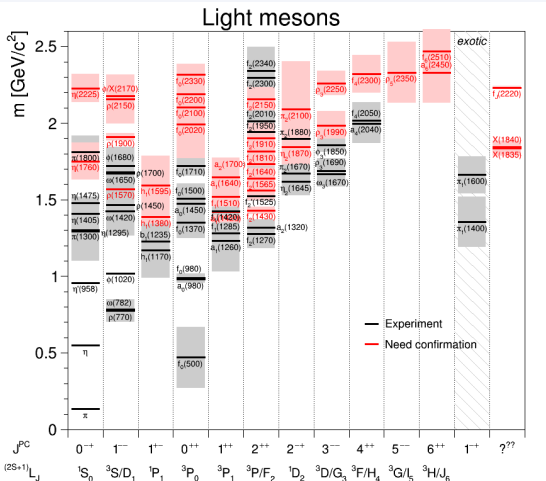
- Many more nonets for orbital and radial excitations

Light mesons



- Many states need **confirmation** in mass region $m \gtrsim 2 \text{ GeV}/c^2$
- Many wide states $\implies$ **overlap and mixing**
- Identification of **higher excitations** becomes exceedingly **difficult**

Light-Quark Meson Spectrum

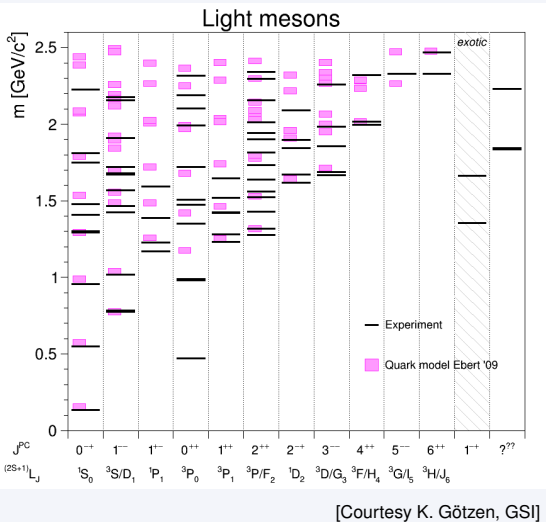


[Courtesy K. Götzen, GSI]

“Light-meson frontier”

- Many states need **confirmation** in mass region $m \gtrsim 2 \text{ GeV}/c^2$
- Many wide states $\implies$ **overlap and mixing**
- Identification of **higher excitations** becomes exceedingly **difficult**

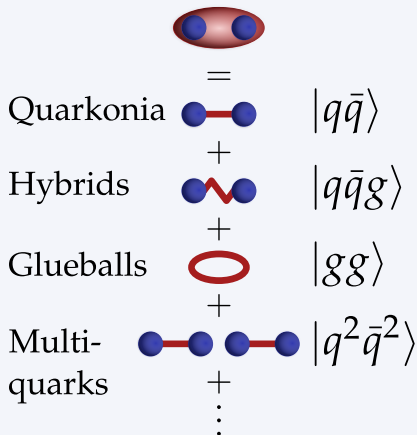
Light-Quark Meson Spectrum



“Light-meson frontier”

- Many states need **confirmation** in mass region $m \gtrsim 2 \text{ GeV}/c^2$
- Many wide states $\Rightarrow$ **overlap and mixing**
- Identification of **higher excitations** becomes exceedingly **difficult**

Beyond the Constituent Quark Model



QCD permits additional color-neutral configurations

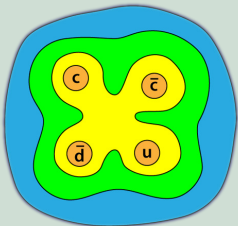
- *Physical mesons*: linear superpositions of *all* allowed basis states
- “Configuration mixing”
- Amplitudes determined by QCD interactions
- Disentanglement of contributions difficult

Beyond the Constituent Quark Model

Best experimental evidence so far from heavy-quark sector

Tetraquark candidates $Z_{c,b}$

- Charged $|c\bar{c}\rangle$ - and $|b\bar{b}\rangle$ -like states
- E.g. $Z_c^\pm(3900) \rightarrow J/\psi + \pi^\pm$



Pentaquark candidates P_c^+

- Heavy baryon
- Decay mode $P_c^+ \rightarrow p + J/\psi$

Light-quark sector

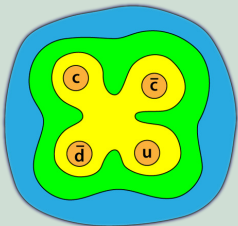
- Model calculations suggest some states to be tetraquarks or hybrids
- No definite experimental evidence

Beyond the Constituent Quark Model

Best experimental evidence so far from heavy-quark sector

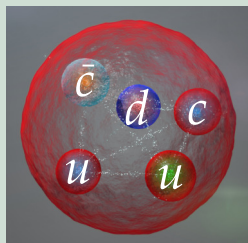
Tetraquark candidates $Z_{c,b}$

- Charged $|c\bar{c}\rangle$ - and $|b\bar{b}\rangle$ -like states
- E.g. $Z_c^\pm(3900) \rightarrow J/\psi + \pi^\pm$



Pentaquark candidates P_c^+

- Heavy baryon
- Decay mode $P_c^+ \rightarrow p + J/\psi$



Light-quark sector

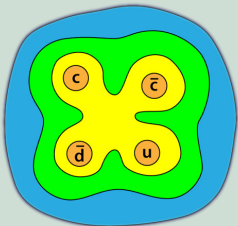
- Model calculations suggest some states to be tetraquarks or hybrids
- No definite experimental evidence

Beyond the Constituent Quark Model

Best experimental evidence so far from heavy-quark sector

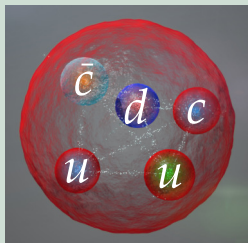
Tetraquark candidates $Z_{c,b}$

- Charged $|c\bar{c}\rangle$ - and $|b\bar{b}\rangle$ -like states
- E.g. $Z_c^\pm(3900) \rightarrow J/\psi + \pi^\pm$



Pentaquark candidates P_c^+

- Heavy baryon
- Decay mode $P_c^+ \rightarrow p + J/\psi$



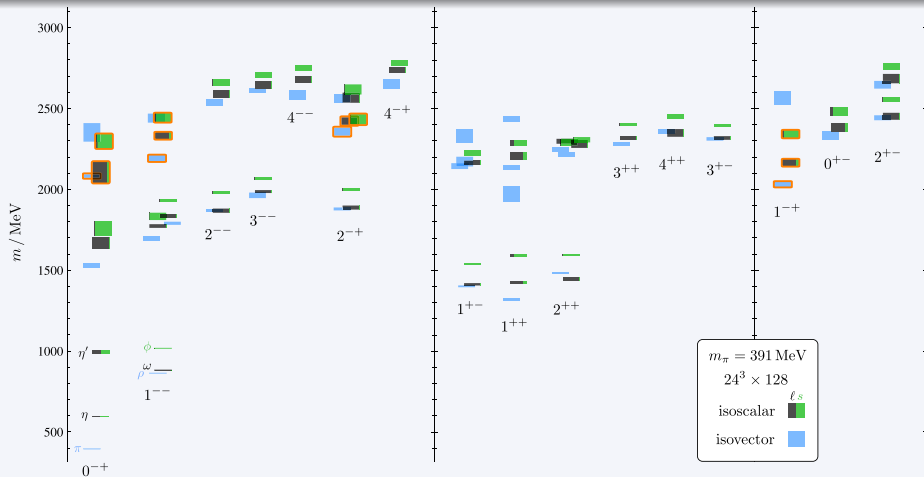
Light-quark sector

- Model calculations suggest some states to be tetraquarks or hybrids
- No definite experimental evidence

Light-Quark Meson Spectrum from Lattice QCD

State-of-the-art calculation with $m_\pi = 391 \text{ MeV}/c^2$

[Dudek *et al.*, PRD **88** (2013) 094505]

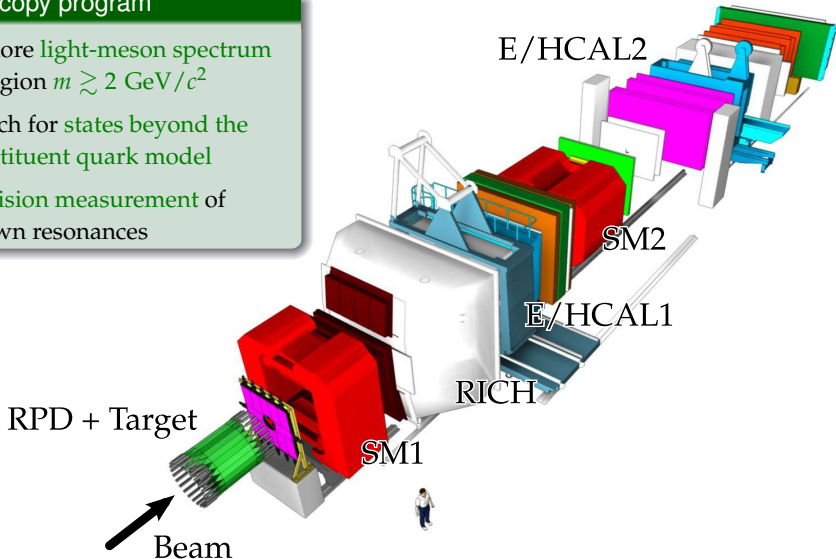


- Essentially recovers **quark-model pattern**
- High towers of **excited states**
- Additional **hybrid-meson super-multiplet**

Experimental Setup

Spectroscopy program

- Explore light-meson spectrum in region $m \gtrsim 2 \text{ GeV}/c^2$
- Search for states beyond the constituent quark model
- Precision measurement of known resonances



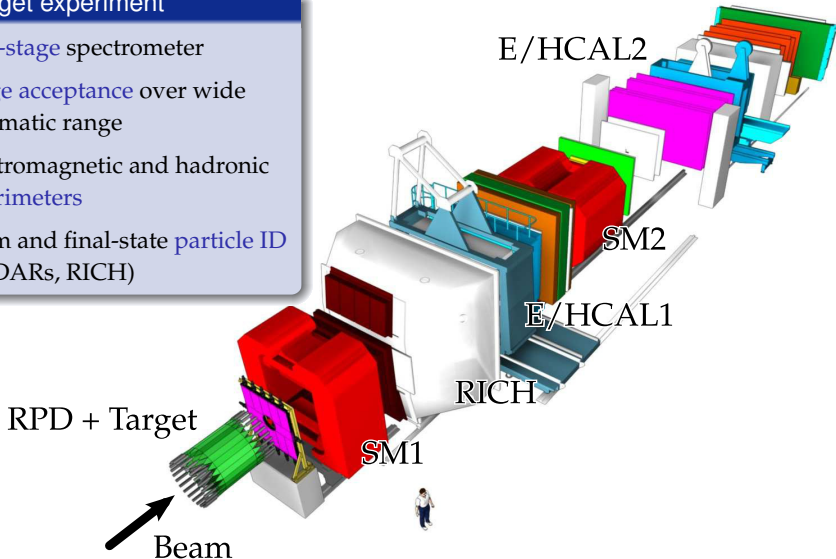
The COMPASS Experiment at the CERN SPS

Experimental Setup

[NIMA 779 (2015) 69]

Fixed-target experiment

- Two-stage spectrometer
- Large acceptance over wide kinematic range
- Electromagnetic and hadronic calorimeters
- Beam and final-state particle ID (CEDARs, RICH)



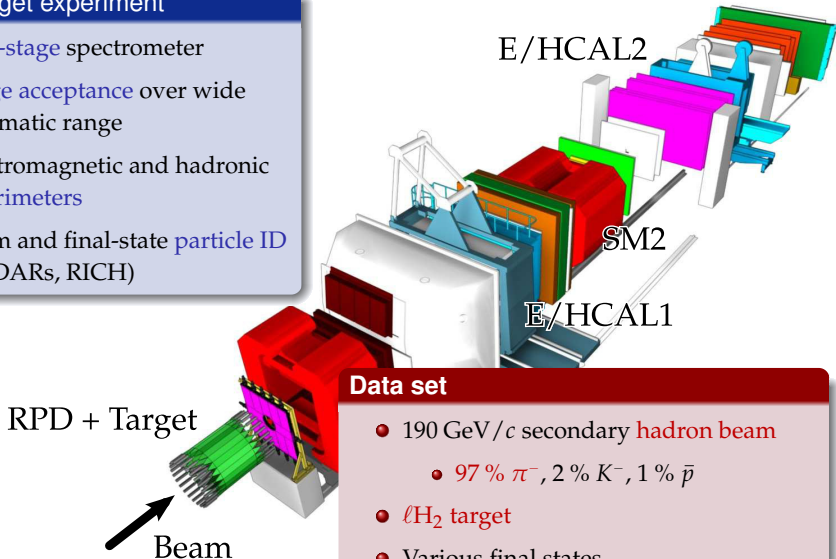
The COMPASS Experiment at the CERN SPS

Experimental Setup

[NIMA 779 (2015) 69]

Fixed-target experiment

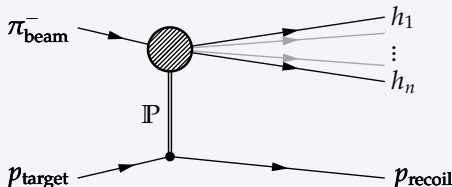
- Two-stage spectrometer
- Large acceptance over wide kinematic range
- Electromagnetic and hadronic calorimeters
- Beam and final-state particle ID (CEDARs, RICH)



Data set

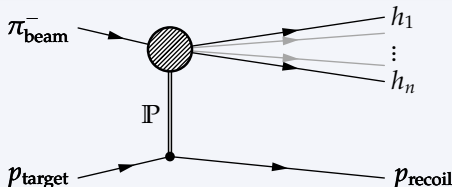
- 190 GeV/c secondary **hadron beam**
 - 97 % π^- , 2 % K^- , 1 % $\bar{p}$
- ℓH_2 target
- Various final states

Meson Production in Diffractive Dissociation



- Soft scattering of beam particle off target
 - Production of n forward-going hadrons
 - Target particle stays intact
- At $190 \text{ GeV}/c$, interaction dominated by space-like pomeron exchange
- All final-state particles are measured

Meson Production in Diffractive Dissociation



- Beam particle gets excited into intermediate resonances X
- X dissociate into n -body hadronic final state
- Rich spectrum of interfering intermediate states X

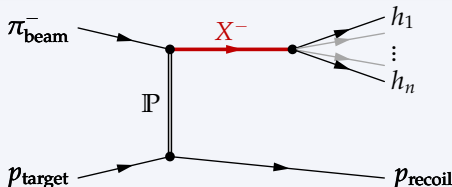
Goal: disentangle all contributing resonances X

- Determine their mass, width, and quantum numbers

Method: **partial-wave analysis (PWA)**

- Uses **full kinematic information** of events
- *Amplitude analysis:* exploits **interference of intermediate states**
 - Additional **phase information** $\implies$ increased **sensitivity**

Meson Production in Diffractive Dissociation



- Beam particle gets excited into intermediate resonances X
- X dissociate into n -body hadronic final state
- Rich spectrum of interfering intermediate states X

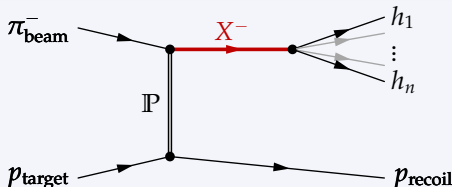
Goal: disentangle all contributing resonances X

- Determine their mass, width, and quantum numbers

Method: partial-wave analysis (PWA)

- Uses full kinematic information of events
- Amplitude analysis: exploits interference of intermediate states
 - Additional phase information $\implies$ increased sensitivity

Meson Production in Diffractive Dissociation



- Beam particle gets excited into intermediate resonances X
- X dissociate into n -body hadronic final state
- Rich spectrum of interfering intermediate states X

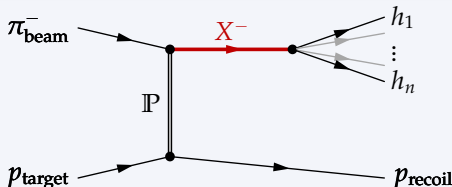
Goal: disentangle all contributing resonances X

- Determine their mass, width, and quantum numbers

Method: partial-wave analysis (PWA)

- Uses full kinematic information of events
- *Amplitude analysis*: exploits interference of intermediate states
 - Additional phase information $\implies$ increased sensitivity

Meson Production in Diffractive Dissociation



- Beam particle gets excited into intermediate resonances X
- X dissociate into n -body hadronic final state
- Rich spectrum of interfering intermediate states X

Goal: disentangle all contributing resonances X

- Determine their mass, width, and quantum numbers

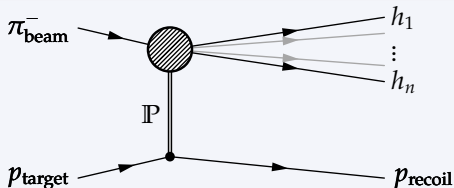
Method: partial-wave analysis (PWA)

- Uses full kinematic information of events
- Amplitude analysis: exploits interference of intermediate states
 - Additional phase information $\implies$ increased sensitivity

Meson Production in Diffractive Dissociation

Example: $\pi^- \pi^+ \pi^-$ final state

[arXiv:1509.00992]

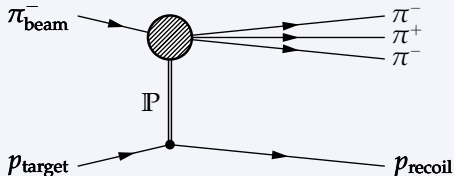


- Exclusive measurement
 - Clean data sample
- Squared four-momentum transfer
 $0.1 < t' < 1.0 (\text{GeV}/c)^2$
- 50 M $\pi^- \pi^+ \pi^-$ events
- Well-known 3π resonances
appear in $m_{3\pi}$ spectrum
- Crosscheck: 3.5 M $\pi^- \pi^0 \pi^0$ events
 - Very different acceptance

Meson Production in Diffractive Dissociation

Example: $\pi^- \pi^+ \pi^-$ final state

[arXiv:1509.00992]

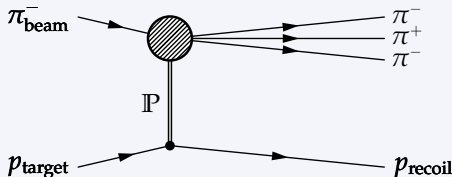


- Exclusive measurement
 - Clean data sample
- Squared four-momentum transfer
 $0.1 < t' < 1.0 (\text{GeV}/c)^2$
- 50 M $\pi^- \pi^+ \pi^-$ events
- Well-known 3π resonances
appear in $m_{3\pi}$ spectrum
- Crosscheck: 3.5 M $\pi^- \pi^0 \pi^0$ events
 - Very different acceptance

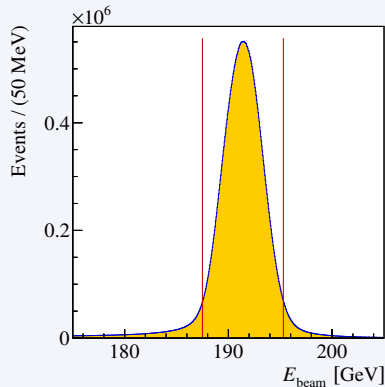
Meson Production in Diffractive Dissociation

Example: $\pi^- \pi^+ \pi^-$ final state

[arXiv:1509.00992]



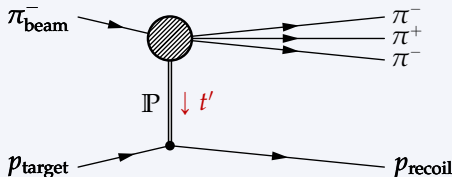
- Exclusive measurement
 - Clean data sample
- Squared four-momentum transfer
 $0.1 < t' < 1.0 (\text{GeV}/c)^2$
- 50 M $\pi^- \pi^+ \pi^-$ events
- Well-known 3π resonances appear in $m_{3\pi}$ spectrum
- Crosscheck: 3.5 M $\pi^- \pi^0 \pi^0$ events
 - Very different acceptance



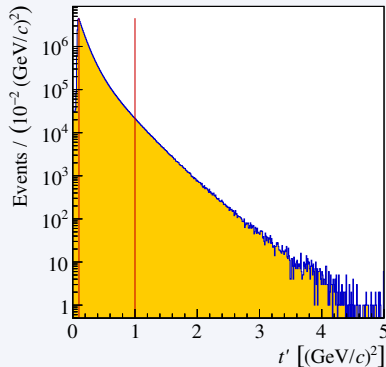
Meson Production in Diffractive Dissociation

Example: $\pi^- \pi^+ \pi^-$ final state

[arXiv:1509.00992]



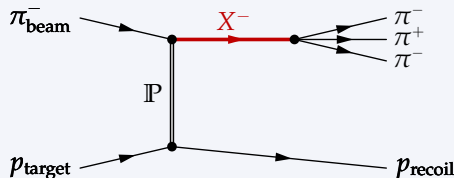
- Exclusive measurement
 - Clean data sample
- Squared four-momentum transfer
 $0.1 < t' < 1.0 \text{ (GeV/c)}^2$
- 50 M $\pi^- \pi^+ \pi^-$ events
- Well-known 3π resonances appear in $m_{3\pi}$ spectrum
- Crosscheck: 3.5 M $\pi^- \pi^0 \pi^0$ events
 - Very different acceptance



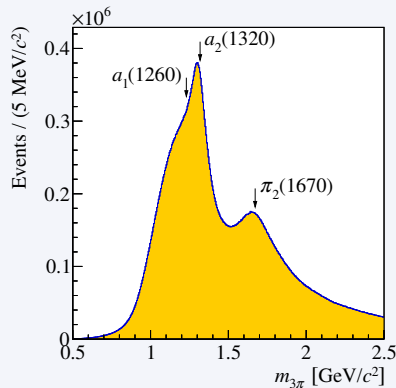
Meson Production in Diffractive Dissociation

Example: $\pi^- \pi^+ \pi^-$ final state

[arXiv:1509.00992]



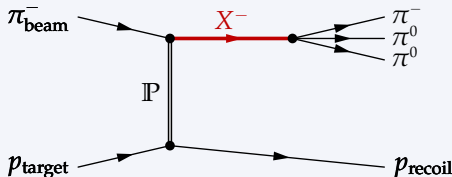
- Exclusive measurement
 - Clean data sample
- Squared four-momentum transfer
 $0.1 < t' < 1.0 \text{ (GeV/c)}^2$
- 50 M $\pi^- \pi^+ \pi^-$ events
- Well-known 3π resonances appear in $m_{3\pi}$ spectrum
- Crosscheck: 3.5 M $\pi^- \pi^0 \pi^0$ events
 - Very different acceptance



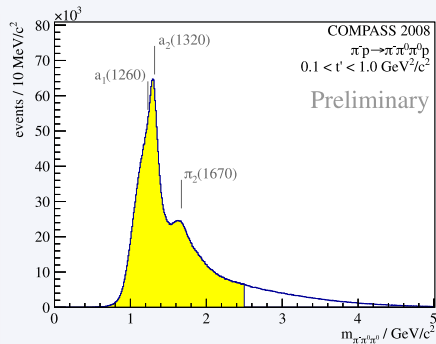
Meson Production in Diffractive Dissociation

Example: $\pi^- \pi^+ \pi^-$ final state

[arXiv:1509.00992]



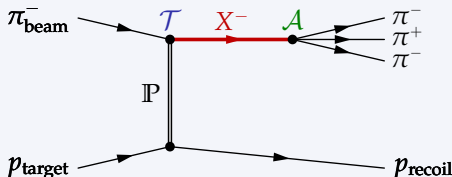
- Exclusive measurement
 - Clean data sample
- Squared four-momentum transfer
 $0.1 < t' < 1.0 \text{ (GeV/c)}^2$
- 50 M $\pi^- \pi^+ \pi^-$ events
- Well-known 3π resonances appear in $m_{3\pi}$ spectrum
- Crosscheck: 3.5 M $\pi^- \pi^0 \pi^0$ events
 - Very different acceptance



Partial-Wave Decomposition

Example: $\pi^- \pi^+ \pi^-$ final state

[arXiv:1509.00992]



Factorization ansatz

- Production of resonance X is independent of its decay
 - Production described by amplitudes $\mathcal{T}_{\text{wave}}(m_X)$
 - Strengths and relative phases of various $X \Rightarrow$ interesting physics
 - Decay of X described by $\mathcal{A}_{\text{wave}}(\tau; m_X)$
 - τ : set of 5 kinematic variables that define 3π final state
 - $\mathcal{A}_{\text{wave}}(\tau; m_X)$ calculable with additional model assumptions

- Model for measured intensity distribution

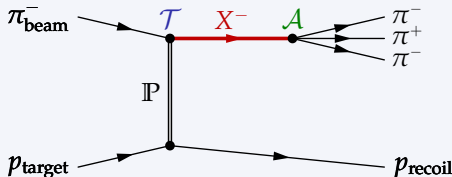
$$\mathcal{I}(\tau; m_X) = \left| \sum_{\text{waves}} \mathcal{T}_{\text{wave}}(m_X) \mathcal{A}_{\text{wave}}(\tau; m_X) \right|^2$$

- Coherent sum of partial-wave amplitudes

Partial-Wave Decomposition

Example: $\pi^- \pi^+ \pi^-$ final state

[arXiv:1509.00992]



Factorization ansatz

- Production of resonance X is independent of its decay
 - Production described by amplitudes $\mathcal{T}_{\text{wave}}(m_X)$
 - Strengths and relative phases of various $X \Rightarrow$ interesting physics
 - Decay of X described by $\mathcal{A}_{\text{wave}}(\tau; m_X)$
 - τ : set of 5 kinematic variables that define 3π final state
 - $\mathcal{A}_{\text{wave}}(\tau; m_X)$ calculable with additional model assumptions

- Model for measured intensity distribution

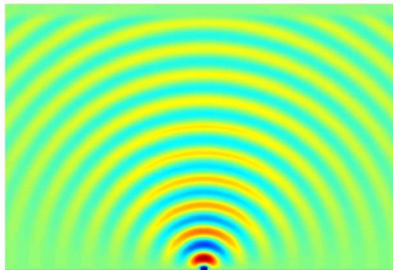
$$\mathcal{I}(\tau; m_X) = \left| \sum_{\text{waves}} \mathcal{T}_{\text{wave}}(m_X) \mathcal{A}_{\text{wave}}(\tau; m_X) \right|^2$$

- Coherent sum of partial-wave amplitudes

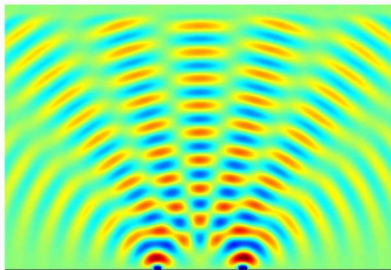
Partial-Wave Analysis: $\pi^- \pi^+ \pi^-$ Final State

Loose Analogy: Interference of Light Waves

Single Slit of Length 0.5λ

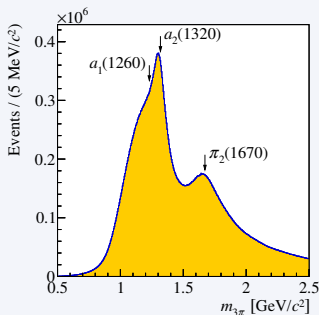
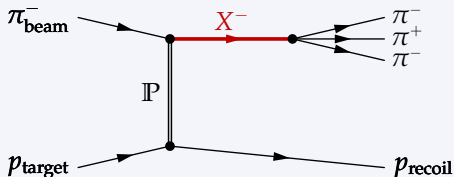


Double Slits of Length 0.5λ



Diffraction on mask with variable number of slits

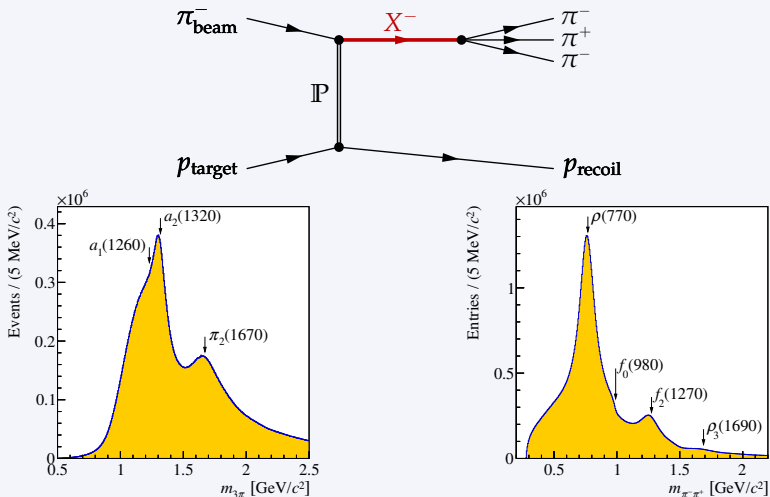
- **Interference pattern** on screen depends on number, position, and width of slits
 - Each slit represents a resonance
 - Width of slit = strength of resonance X
 - Relative position of slits = relative phase difference between X
 - 1D intensity distribution = 5D kinematic distribution of 3π



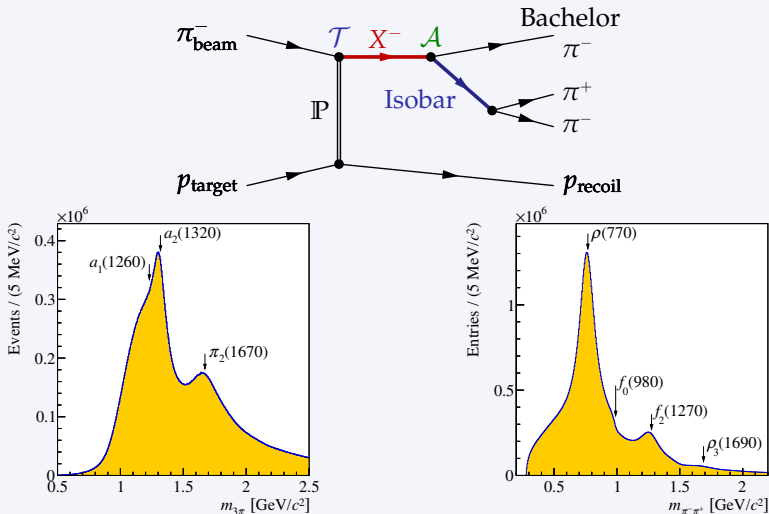
X^- decay via $\pi^+ \pi^-$ resonances = “isobars”

Partial-Wave Analysis: $\pi^- \pi^+ \pi^-$ Final State

[arXiv:1509.00992]



X^- decay via $\pi^+ \pi^-$ resonances = "isobars"

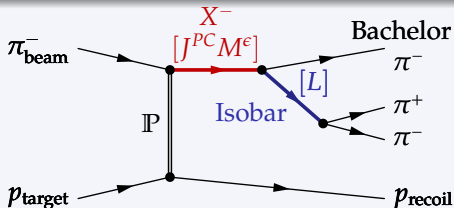


X^- decay via $\pi^+ \pi^-$ resonances = "isobars"

Partial-Wave Analysis: $\pi^- \pi^+ \pi^-$ Final State

Isobar Model

[arXiv:1509.00992]

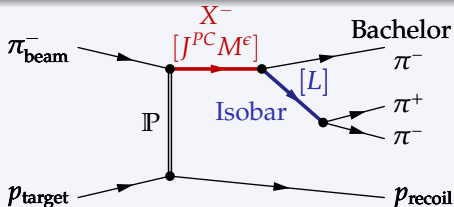


- “Wave” = unique combination of X and isobar quantum numbers
- Notation: $J^{PC} M^E$ isobar πL
- 3-body kinematics fixed by m_X plus 5 phase space variables τ
- Decay amplitude $\mathcal{A}_{\text{wave}}(\tau; m_X)$
 - Describes τ distribution for given wave
 - **Calculable!**

Partial-Wave Analysis: $\pi^- \pi^+ \pi^-$ Final State

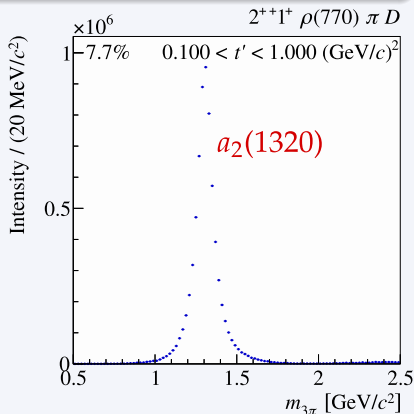
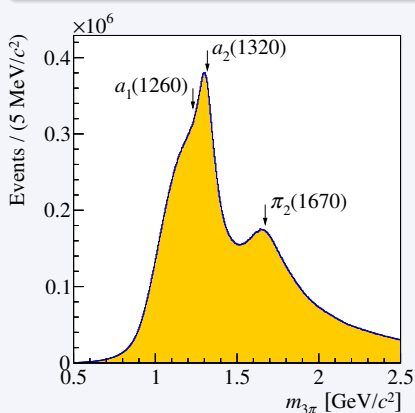
Isobar Model

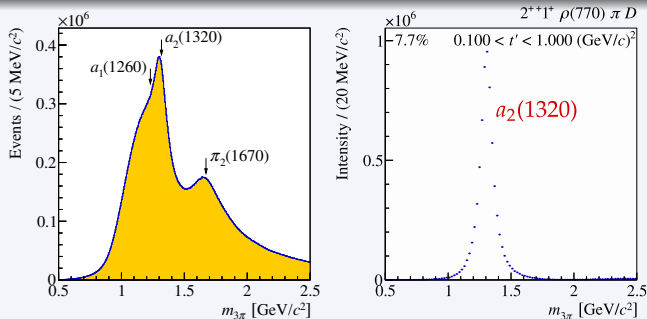
[arXiv:1509.00992]

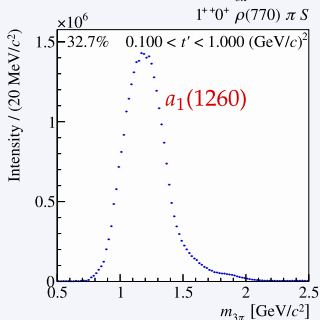
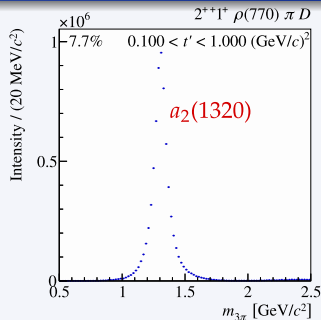
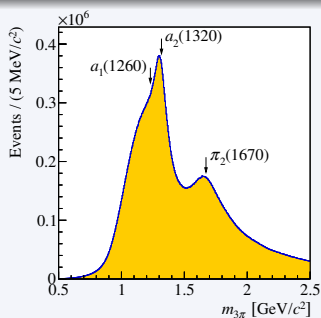


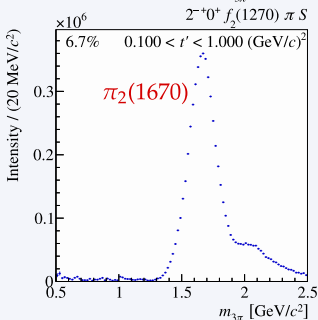
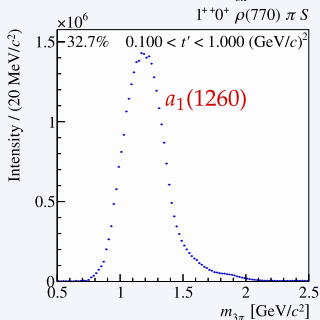
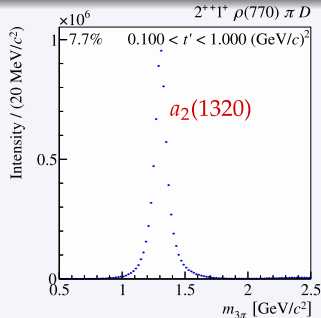
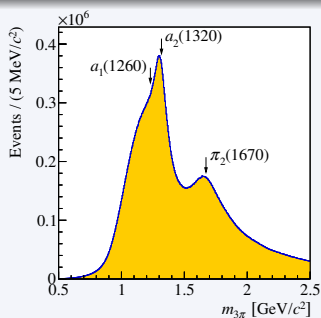
- “Wave” = unique combination of X and isobar quantum numbers
- Notation: $J^{PC} M^E$ isobar πL
- 3-body kinematics fixed by m_X plus 5 phase space variables τ
- Decay amplitude $\mathcal{A}_{\text{wave}}(\tau; m_X)$
 - Describes τ distribution for given wave
 - **Calculable!**

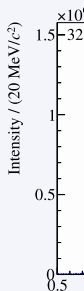
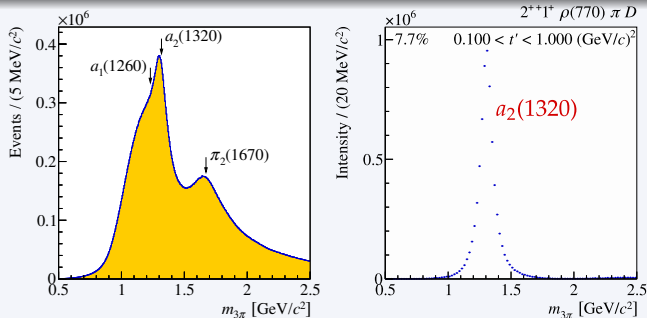
- Partial-wave decomposition performed independently in narrow $m_{3\pi}$ bins
 - In each bin: fit of 5-dimensional intensity distribution
 - Result: production amplitudes $\mathcal{T}_{\text{wave}}(m_{3\pi})$
- PWA makes no assumptions about 3π resonances





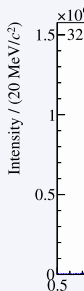
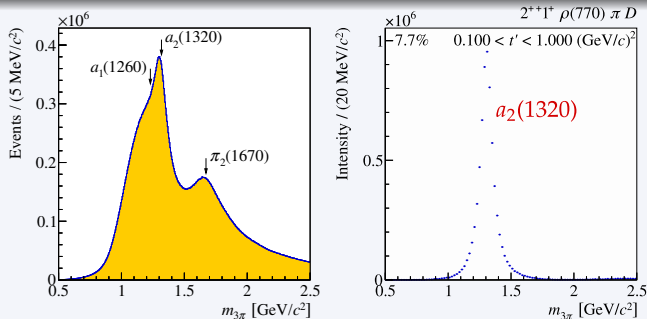






PWA model

- In total 88 partial waves
 - Spin J up to 6
 - Orbital angular momentum L up to 6
- Largest wave set ever used for 3π analysis
- Challenge: model selection
 - Which waves are important, which ones not?
 - Work in progress: automatized procedure using Bayesian extension of maximum-likelihood formalism



PWA model

- In total 88 partial waves
 - Spin J up to 6
 - Orbital angular momentum L up to 6
- Largest wave set ever used for 3π analysis
- Challenge: model selection
 - Which waves are important, which ones not?
 - Work in progress: automatized procedure using Bayesian extension of maximum-likelihood formalism

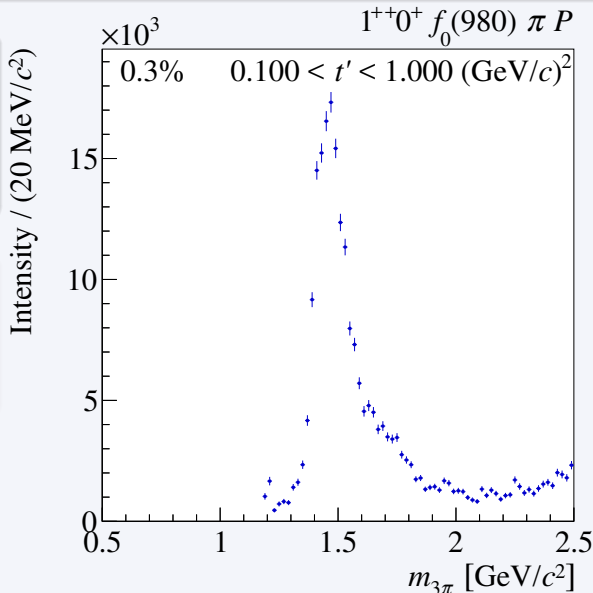
PWA $\pi^- \pi^+ \pi^-$ Final State: A New $a_1(1420)$ Meson?

[arXiv:1509.00992]

- **Unexpected peak around $1.4 \text{ GeV}/c^2$**
- Small intensity: only 0.3% relative contribution

$\pi^- \pi^0 \pi^0$ final state

- Very different acceptance
- **Similar signal**



PWA $\pi^- \pi^+ \pi^-$ Final State: A New $a_1(1420)$ Meson?

Cross Check with $\pi^- \pi^0 \pi^0$ Final State

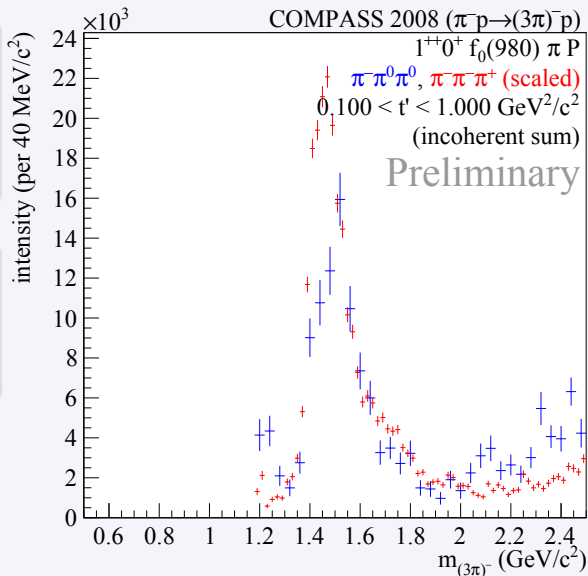
- **Unexpected peak around $1.4 \text{ GeV}/c^2$**
- Small intensity: only 0.3% relative contribution

$\pi^- \pi^0 \pi^0$ final state

- Very different acceptance
- **Similar signal**

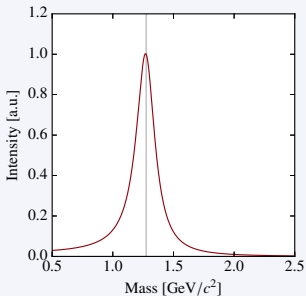
$\pi^- \pi^0 \pi^0$

$\pi^- \pi^+ \pi^-$ scaled



Experimental Signatures of a Resonance

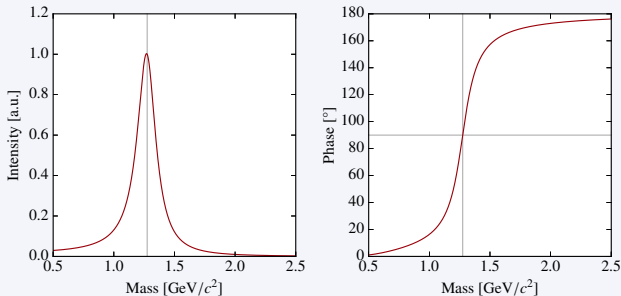
Example: Single Narrow Relativistic Breit-Wigner Resonance



- **Intensity peak** at resonance mass
- **Phase motion:** ϕ rises from 0° to 180° and is 90° at peak position
 - Analogous to mechanical oscillator
- *Argand diagram:* resonance lies on circle

Experimental Signatures of a Resonance

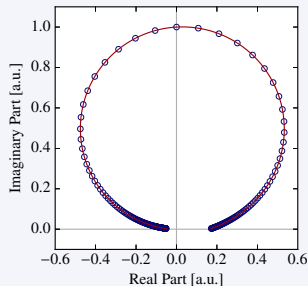
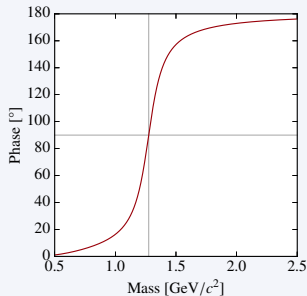
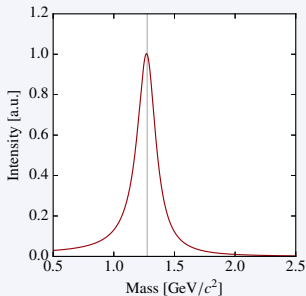
Example: Single Narrow Relativistic Breit-Wigner Resonance



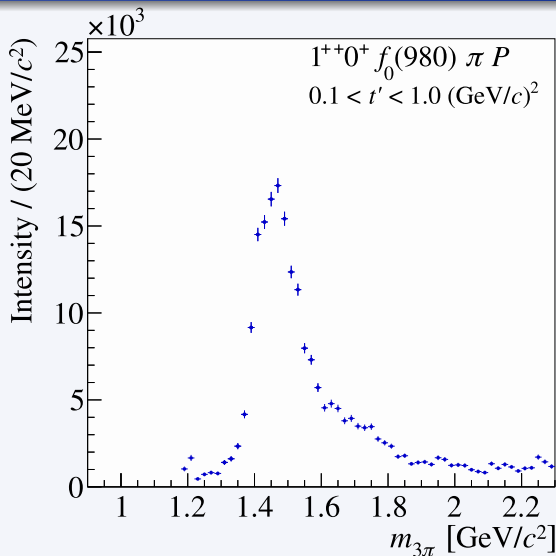
- Intensity peak at resonance mass
- Phase motion: ϕ rises from 0° to 180° and is 90° at peak position
 - Analogous to mechanical oscillator
- Argand diagram: resonance lies on circle

Experimental Signatures of a Resonance

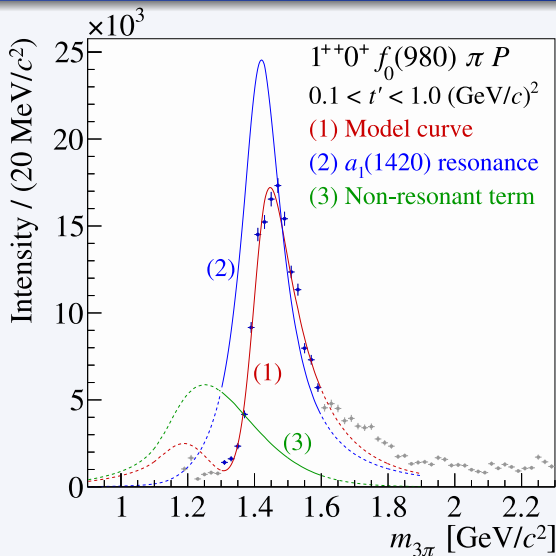
Example: Single Narrow Relativistic Breit-Wigner Resonance



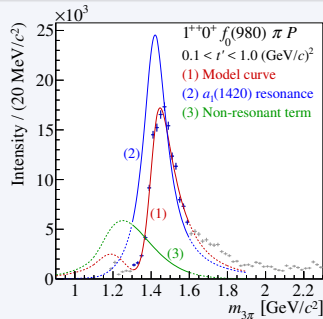
- Intensity peak at resonance mass
- Phase motion: ϕ rises from 0° to 180° and is 90° at peak position
 - Analogous to mechanical oscillator
- Argand diagram: resonance lies on circle



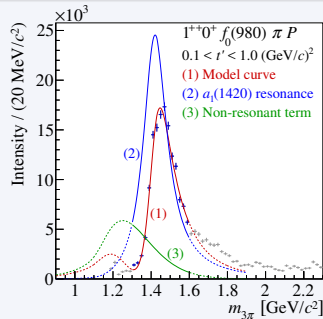
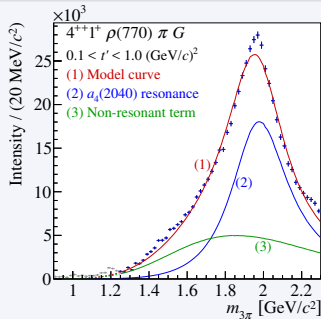
- Coherent sum of resonant (Breit-Wigner) and nonresonant terms



- Coherent sum of resonant (Breit-Wigner) and nonresonant terms



- 1^{++} peak consistent with Breit-Wigner resonance
- $a_1(1420)$ parameters
 Mass: $1414^{+15}_{-13} \text{ MeV}/c^2$
 Width: $153^{+8}_{-23} \text{ MeV}/c^2$
- *Work in progress*: extension to 14 partial waves



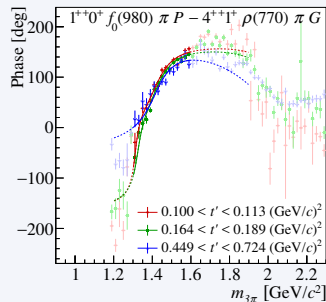
- 1^{++} peak consistent with Breit-Wigner resonance

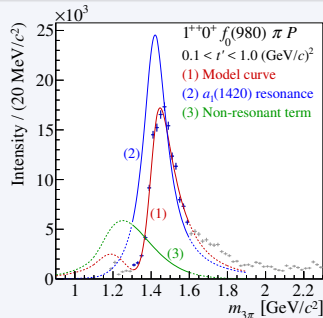
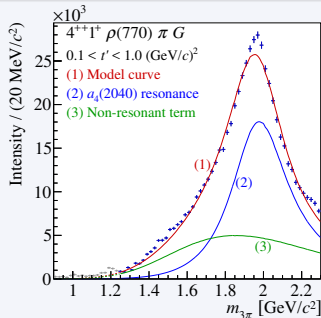
- $a_1(1420)$ parameters

Mass: $1414^{+15}_{-13} \text{ MeV/c}^2$

Width: $153^{+8}_{-23} \text{ MeV/c}^2$

- Work in progress: extension to 14 partial waves





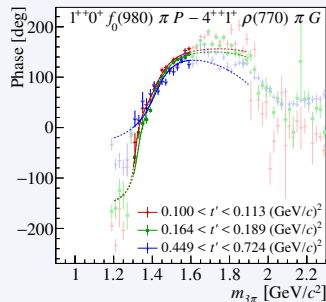
- 1^{++} peak consistent with Breit-Wigner resonance

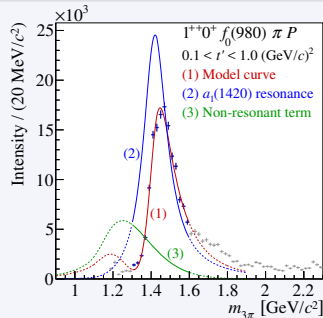
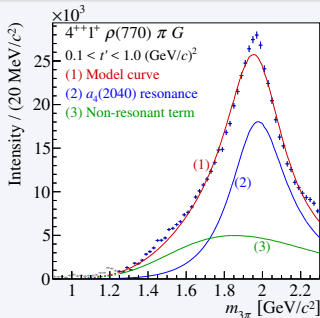
- $a_1(1420)$ parameters

Mass: $1414^{+15}_{-13} \text{ MeV/c}^2$

Width: $153^{+8}_{-23} \text{ MeV/c}^2$

- *Work in progress: extension to 14 partial waves*





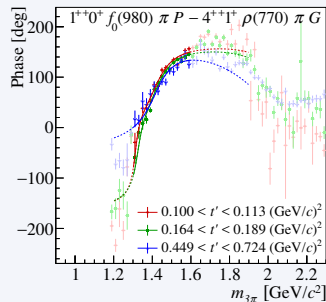
- 1^{++} peak consistent with Breit-Wigner resonance

- $a_1(1420)$ parameters

Mass: $1414^{+15}_{-13} \text{ MeV/c}^2$

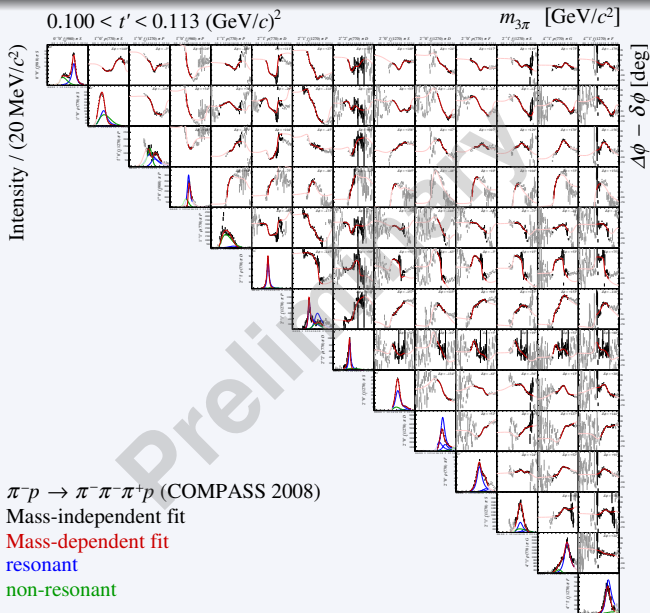
Width: $153^{+8}_{-23} \text{ MeV/c}^2$

- *Work in progress: extension to 14 partial waves*



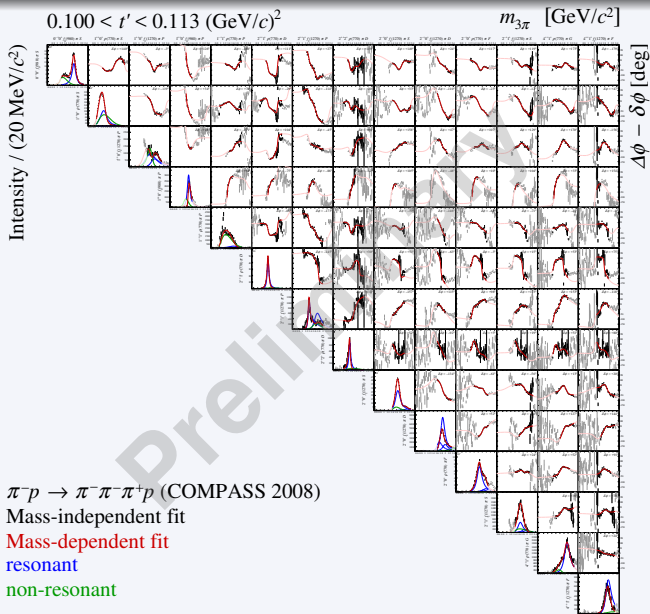
Resonance-Model Fit

14 Waves described by 11 Resonances



Resonance-Model Fit

14 Waves described by 11 Resonances



Virtue of a large Data Set: Analysis in narrow t' Bins

$2^{++} 1^+ \rho(770) \pi D$

- $a_2(1320)$ peak **does not depend on t'**

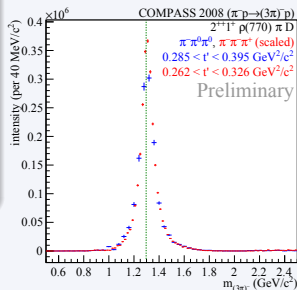
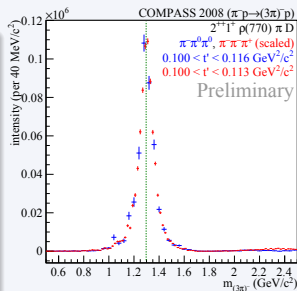
$1^{++} 0^+ \rho(770) \pi S$

- Peak *moves* with t'
- Strong nonresonant contribution

- Better separation of resonant and non-resonant contributions in resonance-model fit

$\pi^- \pi^0 \pi^0$

$\pi^- \pi^+ \pi^-$ scaled for each plot



Virtue of a large Data Set: Analysis in narrow t' Bins

$2^{++} 1^+ \rho(770) \pi D$

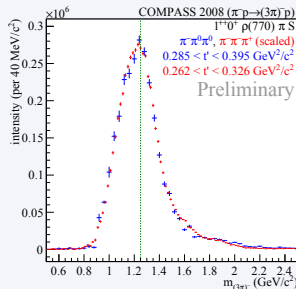
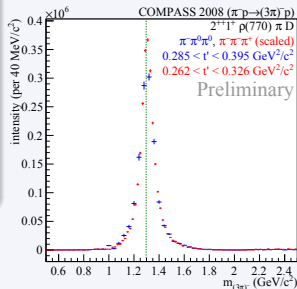
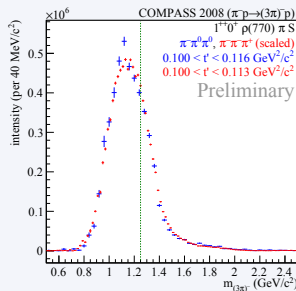
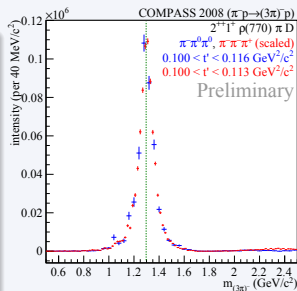
- $a_2(1320)$ peak does not depend on t'

$1^{++} 0^+ \rho(770) \pi S$

- Peak *moves* with t'
- Strong nonresonant contribution
- Better separation of resonant and non-resonant contributions in resonance-model fit

$\pi^- \pi^0 \pi^0$

$\pi^- \pi^+ \pi^-$ scaled for each plot



Virtue of a large Data Set: Analysis in narrow t' Bins

$2^{++} 1^+ \rho(770) \pi D$

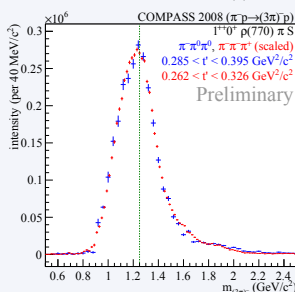
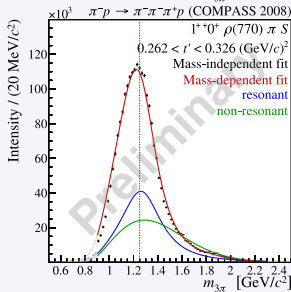
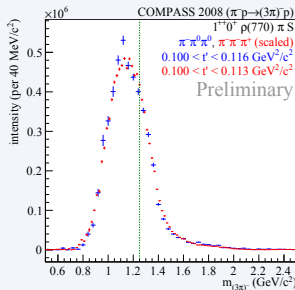
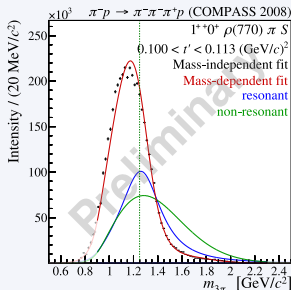
- $a_2(1320)$ peak **does not depend on t'**

$1^{++} 0^+ \rho(770) \pi S$

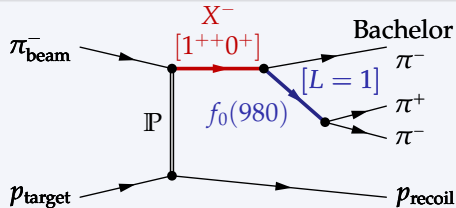
- **Peak moves with t'**
- **Strong nonresonant contribution**
- **Better separation of resonant and non-resonant contributions in resonance-model fit**

$\pi^- \pi^0 \pi^0$

$\pi^- \pi^+ \pi^-$ scaled for each plot

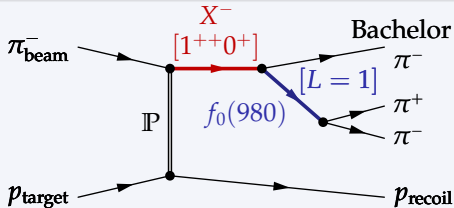


Is the $a_1(1420)$ a Model Artifact?

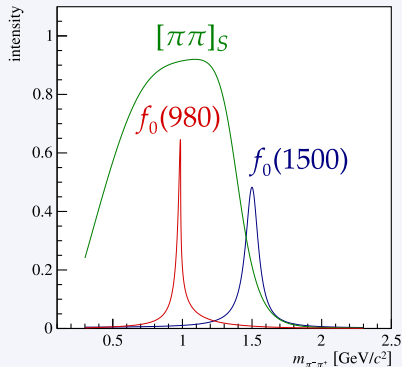


- Calculation of decay amplitudes $\mathcal{A}_{\text{wave}}(\tau)$ needs precise knowledge of isobar $\rightarrow \pi^+ \pi^-$ amplitude
- At least 3 isobars with $J^{PC} = 0^{++}$
 - $[\pi\pi]_{S\text{-wave}}$
 - $f_0(980)$
 - $f_0(1500)$
- Parametrization of $m_{\pi^+ \pi^-}$ dependence difficult

Is the $a_1(1420)$ a Model Artifact?



- Calculation of decay amplitudes $\mathcal{A}_{\text{wave}}(\tau)$ needs precise knowledge of isobar $\rightarrow \pi^+ \pi^-$ amplitude
- At least 3 isobars with $J^{PC} = 0^{++}$
 - $[\pi\pi]_S$ -wave
 - $f_0(980)$
 - $f_0(1500)$
- Parametrization of $m_{\pi^+ \pi^-}$ dependence difficult

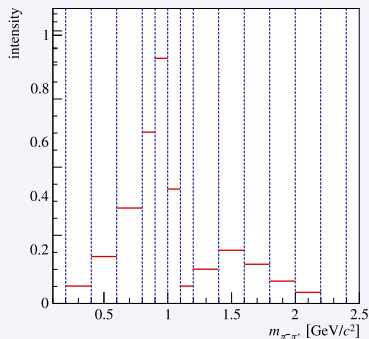
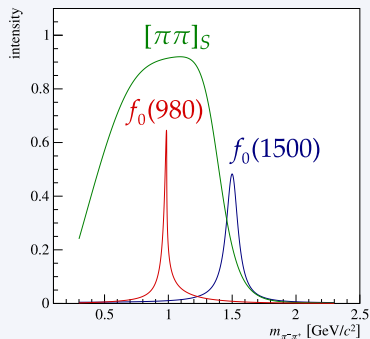


Is the $a_1(1420)$ a Model Artifact?

Novel analysis method

inspired by E791 analysis [PRD **73** (2006) 032204]

- Replace $J^{PC} = 0^{++}$ isobar parametrizations by **piece-wise constant amplitudes** in $m_{\pi^+\pi^-}$ bins
- Extract $m_{3\pi}$ dependence of $J^{PC} = 0^{++}$ isobar amplitude from data
 - *Advantage:* drastic reduction of model bias
 - *Caveat:* significant increase in number of fit parameters

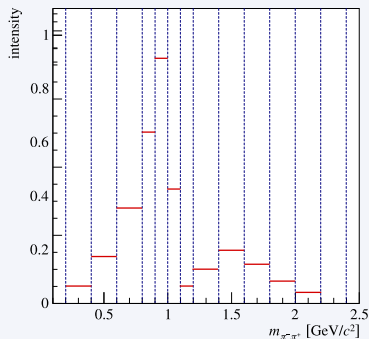
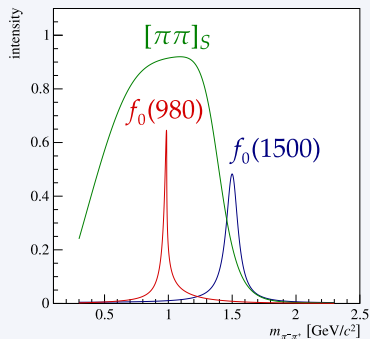


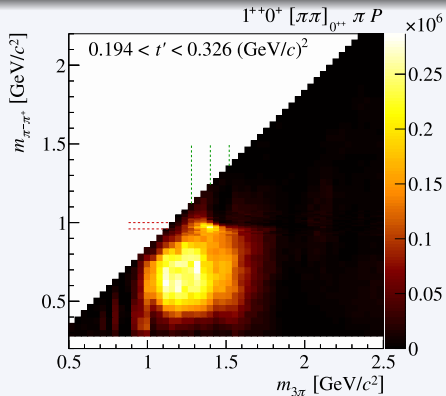
Is the $a_1(1420)$ a Model Artifact?

Novel analysis method

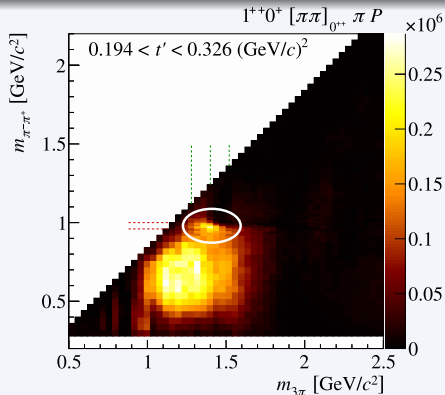
inspired by E791 analysis [PRD **73** (2006) 032204]

- Replace $J^{PC} = 0^{++}$ isobar parametrizations by **piece-wise constant amplitudes** in $m_{\pi^+\pi^-}$ bins
- Extract $m_{3\pi}$ dependence of $J^{PC} = 0^{++}$ isobar amplitude from data
 - *Advantage:* drastic **reduction of model bias**
 - *Caveat:* significant **increase in number of fit parameters**



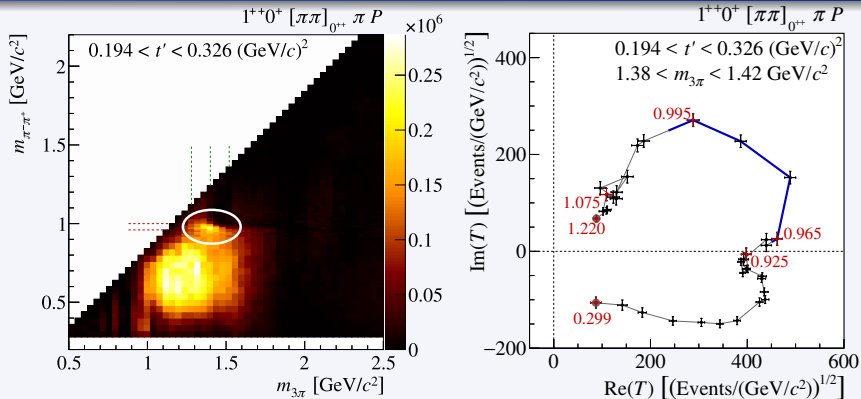


- Correlation of 3π intensity around $1.4 \text{ GeV}/c^2$ with $f_0(980)$
- $f_0(980)$ semicircle in Argand diagram
- Confirms that $f_0(980)\pi$ signal is *not* an artifact of isobar parametrization



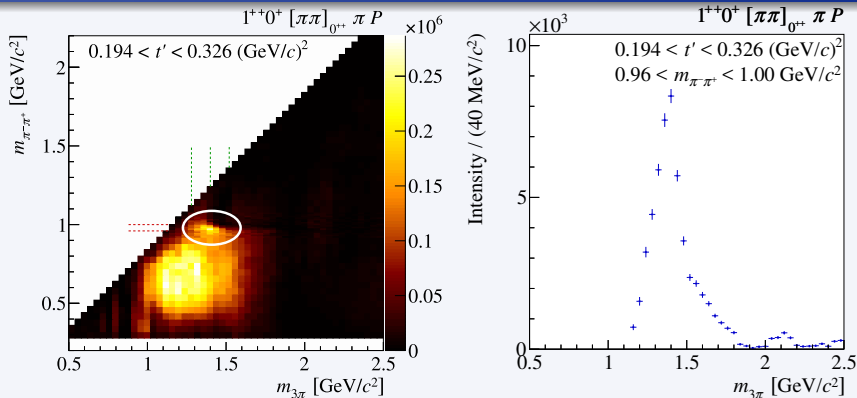
- **Correlation** of 3π intensity around 1.4 GeV/c² with $f_0(980)$
- $f_0(980)$ semicircle in Argand diagram
- Confirms that $f_0(980)\pi$ signal is *not* an artifact of isobar parametrization

$\pi\pi$ S-Wave Amplitude in $J^{PC} = 1^{++}$ 3π Wave [arXiv:1509.00992]



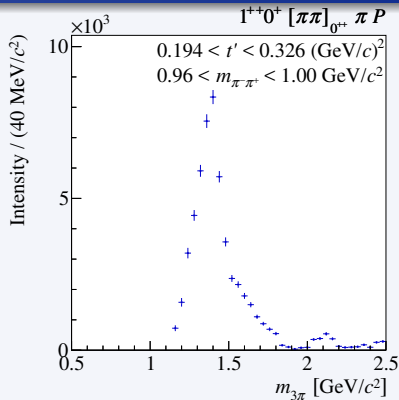
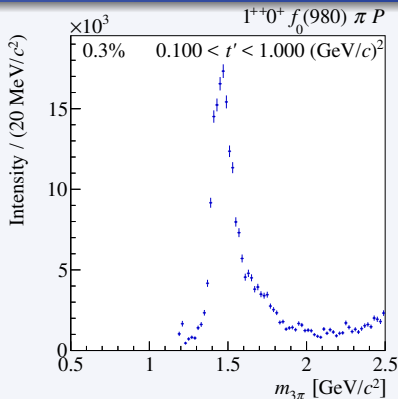
- Correlation of 3π intensity around $1.4 \text{ GeV}/c^2$ with $f_0(980)$
- $f_0(980)$ semicircle in Argand diagram
- Confirms that $f_0(980)\pi$ signal is *not* an artifact of isobar parametrization

$\pi\pi$ S-Wave Amplitude in $J^{PC} = 1^{++}$ 3π Wave [arXiv:1509.00992]



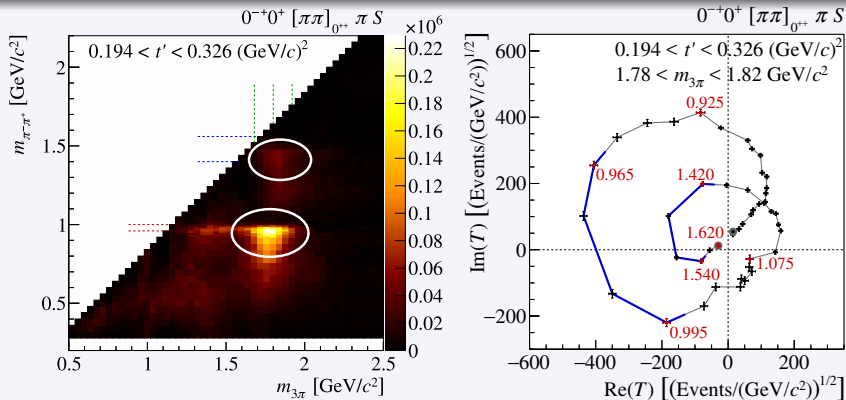
- Correlation of 3π intensity around $1.4 \text{ GeV}/c^2$ with $f_0(980)$
- $f_0(980)$ semicircle in Argand diagram
- Confirms that $f_0(980)\pi$ signal is *not* an artifact of isobar parametrization

$\pi\pi$ S-Wave Amplitude in $J^{PC} = 1^{++}$ 3π Wave [arXiv:1509.00992]

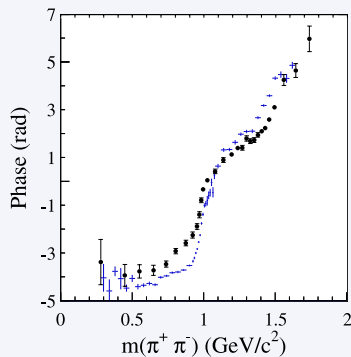
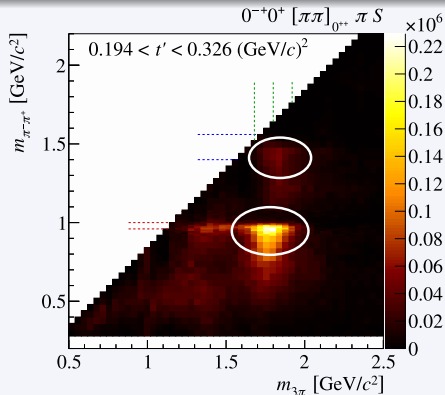


- Correlation of 3π intensity around 1.4 GeV/c^2 with $f_0(980)$
- $f_0(980)$ semicircle in Argand diagram
- Confirms that $f_0(980)\pi$ signal is *not* an artifact of isobar parametrization

$\pi\pi$ S-Wave Amplitude in $J^{PC} = 0^{-+} 3\pi$ Wave [arXiv:1509.00992]



- Coupling of $\pi(1800)$ to $f_0(980)\pi$ and $f_0(1500)\pi$ decay modes
- $\pi\pi$ S-wave phase similar to the one extracted from $D_s^+ \rightarrow \pi^+\pi^-\pi^+$ (black) BABAR, PRD 79 (2009) 032003
- Input/constraint for CP violation analyses in multi-body heavy-meson decays?



- Coupling of $\pi(1800)$ to $f_0(980)\pi$ and $f_0(1500)\pi$ decay modes

- $\pi\pi$ S-wave phase similar to the one extracted from

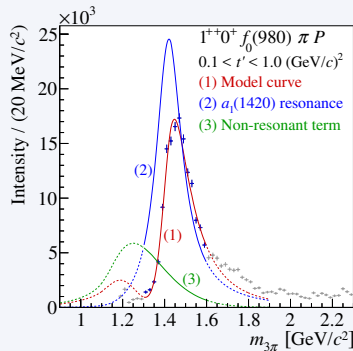
$D_s^+ \rightarrow \pi^+\pi^-\pi^+$ (black)

BABAR, PRD **79** (2009) 032003

- Input/constraint for CP violation analyses in multi-body heavy-meson decays?

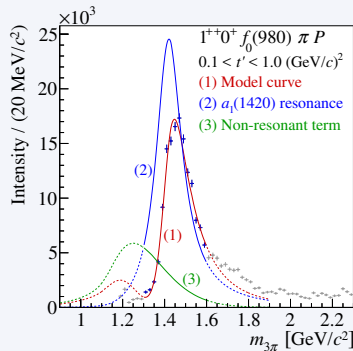
Still unclear

- $J^{PC} = 1^{++}$ ground state is $a_1(1260)$
 - Mass: $1230 \pm 40 \text{ MeV}/c^2$
 - Width: 250 to 400 MeV/c^2
- No quark-model states expected at $1.4 \text{ GeV}/c^2$
 - First excited 1^{++} state expected to be heavier and wider
- Isospin partner of narrow $f_1(1420)$?
- $a_1(1420)$ has peculiar decay mode
 - Only seen in $f_0(980)\pi$ decay
 - $f_0(980)$ has large $s\bar{s}$ content
 - Some models explain $f_0(980)$ as tetra-quark state
- $a_1(1420)$ lies suspiciously close to $K\bar{K}^*$ threshold



Still unclear

- $J^{PC} = 1^{++}$ ground state is $a_1(1260)$
 - Mass: $1230 \pm 40 \text{ MeV}/c^2$
 - Width: 250 to 400 MeV/c^2
- No quark-model states expected at $1.4 \text{ GeV}/c^2$
 - First excited 1^{++} state expected to be heavier and wider
- Isospin partner of narrow $f_1(1420)$?
- $a_1(1420)$ has peculiar decay mode
 - Only seen in $f_0(980)\pi$ decay
 - $f_0(980)$ has large $s\bar{s}$ content
 - Some models explain $f_0(980)$ as tetra-quark state
- $a_1(1420)$ lies suspiciously close to $K\bar{K}^*$ threshold



What is the Nature of the $a_1(1420)$?

Several proposed explanations

A genuine resonance

- Two-quark-tetraquark mixed state
- Tetraquark with mixed flavor symmetry

[Wang, arXiv:1401.1134]

[Chen *et al.*, PRD **91** (2015) 094022]

Effect in production of $a_1(1260)$

- Two-channel unitarized Deck amplitude + direct $a_1(1260)$ production

[Basdevant and Berger, PRL **114** (2015) 192001 and arXiv:1501.04643]

What is the Nature of the $a_1(1420)$?

Several proposed explanations

A genuine resonance

- Two-quark-tetraquark mixed state
- Tetraquark with mixed flavor symmetry

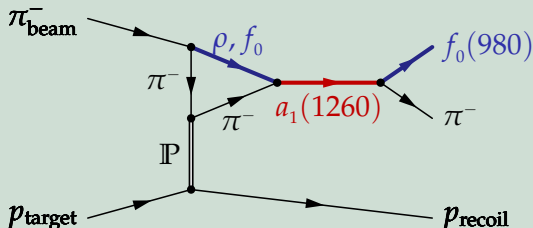
[Wang, arXiv:1401.1134]

[Chen *et al.*, PRD **91** (2015) 094022]

Effect in production of $a_1(1260)$

- Two-channel **unitarized Deck** amplitude + direct $a_1(1260)$ production

[Basdevant and Berger, PRL **114** (2015) 192001 and arXiv:1501.04643]



What is the Nature of the $a_1(1420)$?

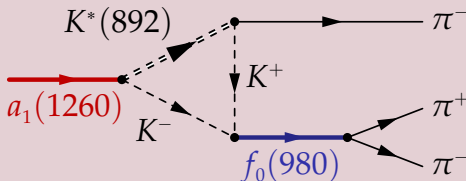
Several proposed explanations

Effect in decay of $a_1(1260)$

- Singularity in **triangle diagram**

[Mikhasenko *et al.*, PRD **91** (2015) 094015]

[Aceti *et al.*, arXiv:1606.06893]



- Similar diagrams proposed to explain some **X, Y, Z states** and **pentaquark candidate P_c** in heavy-meson sector

Multi-body final states

- Ideal laboratory to study **hadronic resonances** and **hadron dynamics**
- For **large data sets**, PWA extracts even **very small signals** reliably
 - E.g. new axial-vector signal $a_1(1420)$

Novel analysis techniques

- *Freed-isobar method*: new insights into **dynamics of $\pi\pi$ subsystem** in the presence of third π
 - *Work in progress*: **extension to $\pi\pi$ subsystems with $J^{PC} = 1^{--}$ and 2^{++}**
 - *Challenge*: resolution of **mathematical ambiguities** caused by Bose symmetrization of final state
- *t' -resolved analysis*: **better separation of resonant and nonresonant contributions**
- **Tight collaboration with theorists** to improve analysis model

Multi-body final states

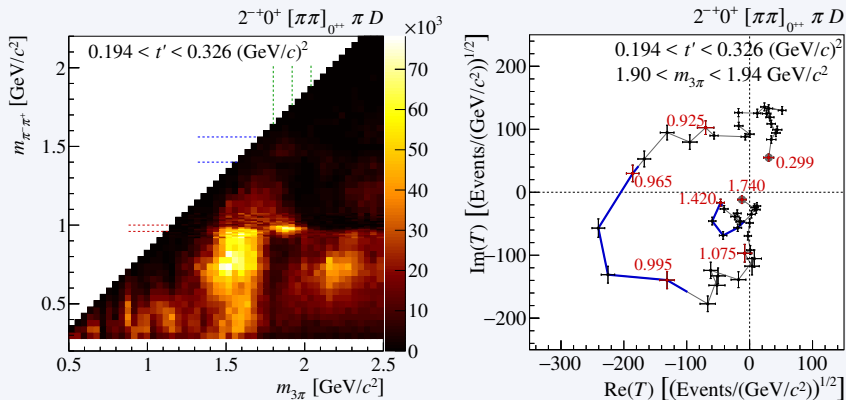
- Ideal laboratory to study **hadronic resonances** and **hadron dynamics**
- For **large data sets**, PWA extracts even **very small signals** reliably
 - E.g. new axial-vector signal $a_1(1420)$

Novel analysis techniques

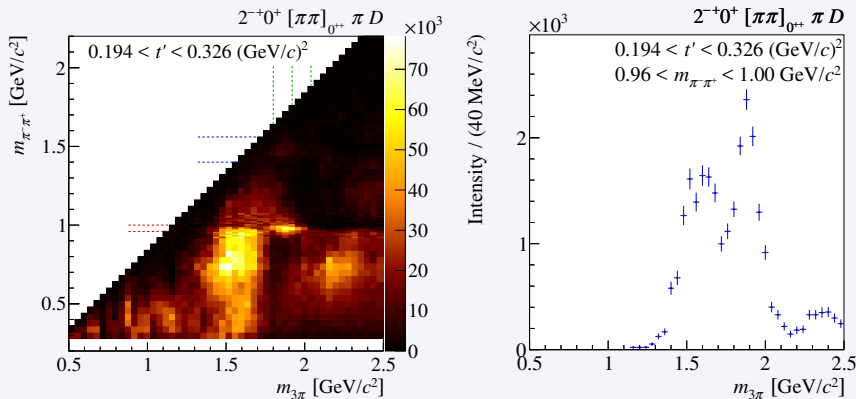
- *Freed-isobar method*: new insights into **dynamics of $\pi\pi$ subsystem** in the presence of third π
 - *Work in progress*: **extension to $\pi\pi$ subsystems with $J^{PC} = 1^{--}$ and 2^{++}**
 - *Challenge*: resolution of **mathematical ambiguities** caused by Bose symmetrization of final state
- *t' -resolved analysis*: better **separation of resonant and nonresonant contributions**
- Tight **collaboration with theorists** to improve analysis model

5 Backup slides

- $\pi\pi$ S -wave amplitude in $J^{PC} = 2^{-+}$ 3π wave
- $J^{PC} = 1^{-+}$ spin-exotic partial wave
- Resonance-model fit



- Correlation of intensity around $m_{3\pi} = 1.9$ GeV/c² with $f_0(980)$
- $f_0(980)$ semicircle in Argand diagram
- Coupling of $\pi_2(1880)$ to $f_0(980)\pi$ decay mode



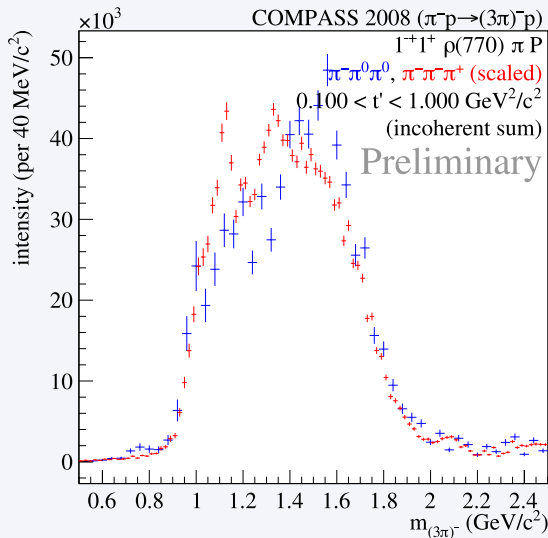
- Correlation of intensity around $m_{3\pi} = 1.9 \text{ GeV}/c^2$ with $f_0(980)$
- $f_0(980)$ semicircle in Argand diagram
- Coupling of $\pi_2(1880)$ to $f_0(980)\pi$ decay mode

Spin-Exotic $J^{PC} = 1^{-+}$ Signal in $(3\pi)^{-}$ PWA

- Broad intensity bump
- Similar in both 3π channels

● Similar to $\pi\eta'$ channel

[PLB 740 (2015) 303]



$\pi^{-}\pi^{0}\pi^{0}$

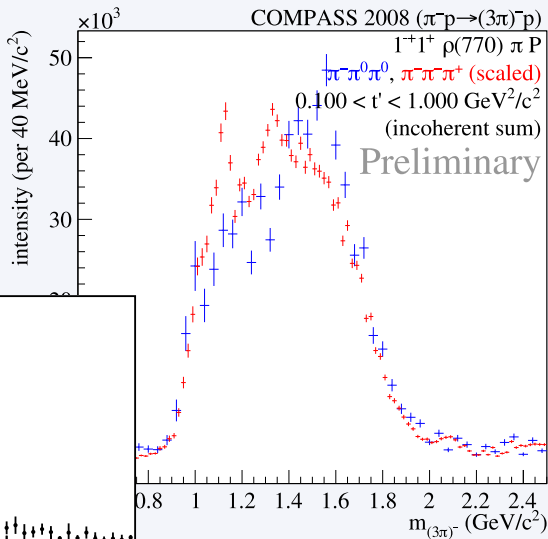
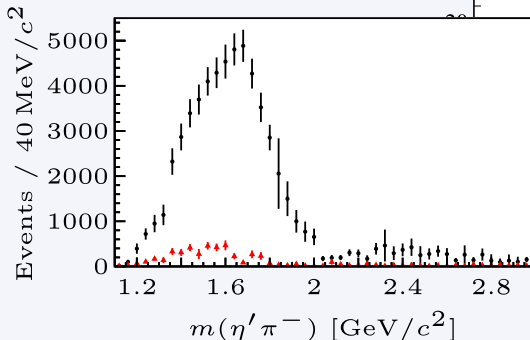
$\pi^{-}\pi^{+}\pi^{-}$ scaled

Spin-Exotic $J^{PC} = 1^{-+}$ Signal in $(3\pi)^-$ PWA

- Broad intensity bump
- Similar in both 3π channels

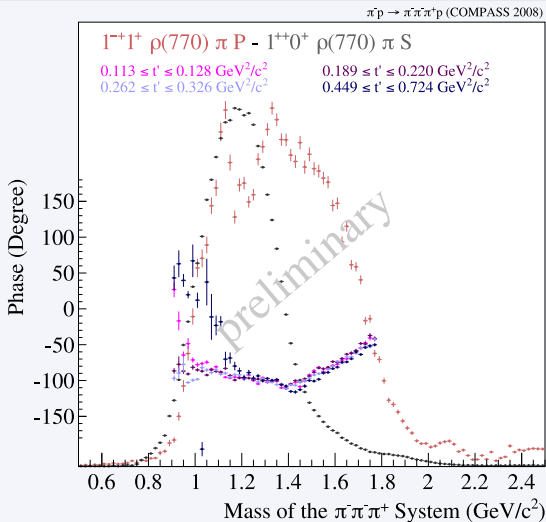
- Similar to $\pi\eta'$ channel

[PLB 740 (2015) 303]



Spin-Exotic $J^{PC} = 1^{-+}$ Signal in $\pi^{-}\pi^{+}\pi^{-}$ PWA

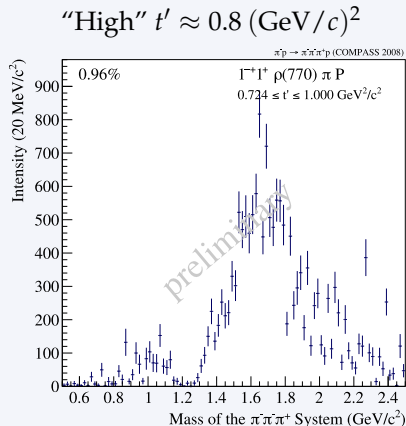
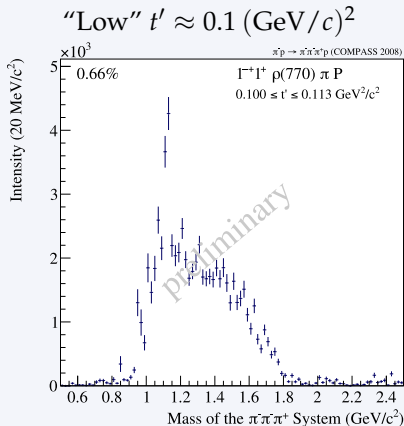
Relative Phase w.r.t. $1^{++}0^{+}\rho(770)\pi$ S Wave



- Slow phase 60° motion in $1.6 \text{ GeV}/c^2$ region independent of t'

Spin-Exotic $J^{PC} = 1^{-+}$ Signal in $\pi^{-}\pi^{+}\pi^{-}$ PWA

Drastic Change of Mass Spectrum with t'

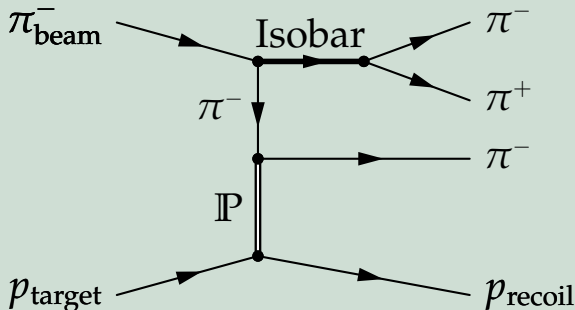


- Dominant nonresonant contribution
 - Needs to be better understood in order to extract resonance content

Spin-Exotic $J^{PC} = 1^{-+}$ Signal in $\pi^{-}\pi^{+}\pi^{-}$ PWA

Model for Nonresonant Component

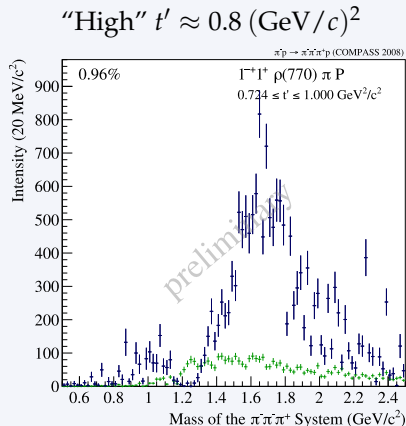
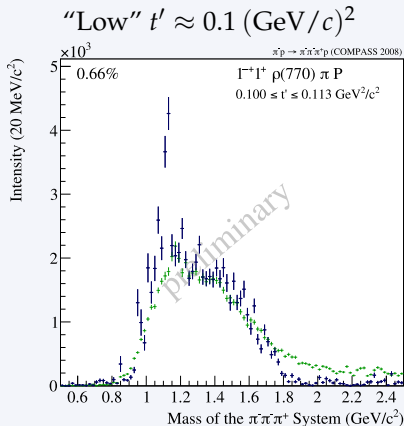
Deck effect



- MC pseudodata generated according to model of Deck amplitude
based on [ACCMOR, NPB **182** (1981) 269]
- Analyzed like real data

Spin-Exotic $J^{PC} = 1^{-+}$ Signal in $\pi^{-}\pi^{+}\pi^{-}$ PWA

Deck-Model for Nonresonant Component

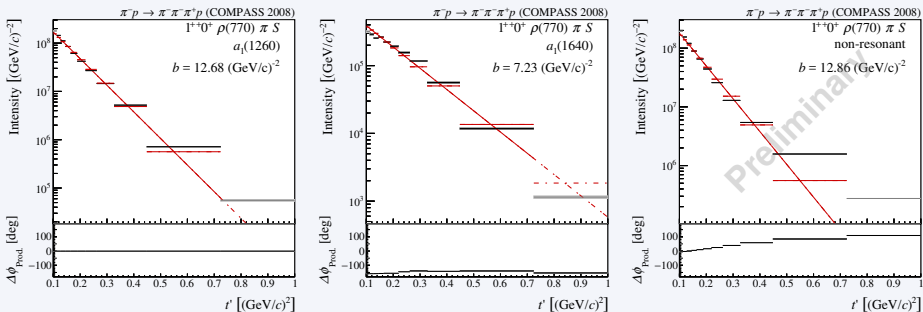


- Deck MC scaled to t' -summed intensity
 - Similar mass spectrum at low t'
 - Different shape at high t'

$\pi^- \pi^+ \pi^-$ Resonance-Model Fit

t' Dependence of Model Components

Example: model components in $1^{++} 0^+ \rho(770) \pi S$ wave



Can t' dependences of amplitudes be modelled?