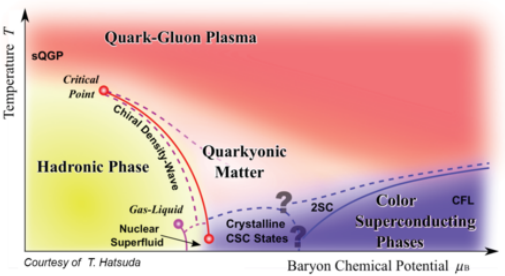
A precision device needs precise simulation: Software description of the CBM Silicon Tracking System

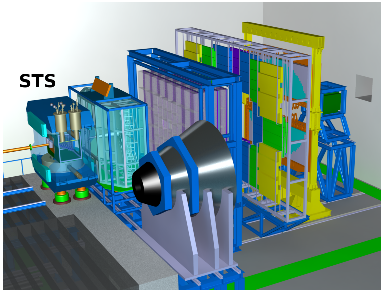
Hanna Malygina[1,2,3], Volker Friese[3]
for the CBM collaboration

[1]Goethe University, Frankfurt, Germany;
[2]KINR, Kyiv, Ukraine;
[3]GSI, Darmstadt, Germany;

CHEP 2016, San-Francisco, 10 - 14 October 2016

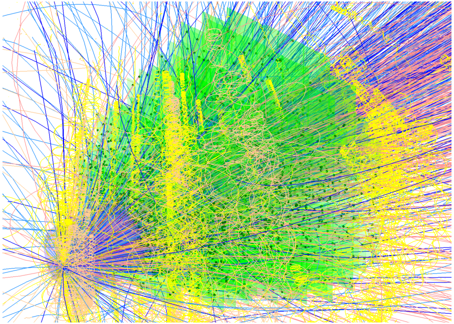
Compressed Baryonic Matter experiment @ FAIR





- QCD-diagram at moderate temperature and high density;
- Au + Au SIS100: $2 - 11\,A\mathrm{GeV}$, $10^5 - 10^7$ interactions/s;
- up to 1000 charged particles per central collision;
- first beam in $\approx$ 2022.

Silicon Tracking System (STS)



Au+Au central collision at $25\,AGeV$

Requirements:

- high efficiency;
- fast: hit rates up to $20\,\mathrm{MHz/cm^2}$;
- radiation hard: $10^{14}\,\mathrm{n_{eq}/cm^2}$;
- low mass: material budget per station $\sim 1\%\,\mathrm{X_0}$.

Design:

- 8 tracking stations in a $1\,\mathrm{T}$ dipole magnet;
- double-sided micro-strip Si sensor: $\sim 300\,\mu\mathrm{m}$ thickness, $58\,\mu\mathrm{m}$ strip pitch, 7.5° stereo-angle;
- self-triggered read-out electronics.

Software chain

- **Digitization:** from particle track in sensor to digital signal in read-out electronics (**`digi`**);
- **Reconstruction:**
  - several (neigbouring) digis from one side of sensor combine to **`cluster`**;
  - combine 2 clusters from opposite sides of sensor to **`hit`**;
  - 4-8 hits from different layers of sensors (stations) combine to **`track`**.

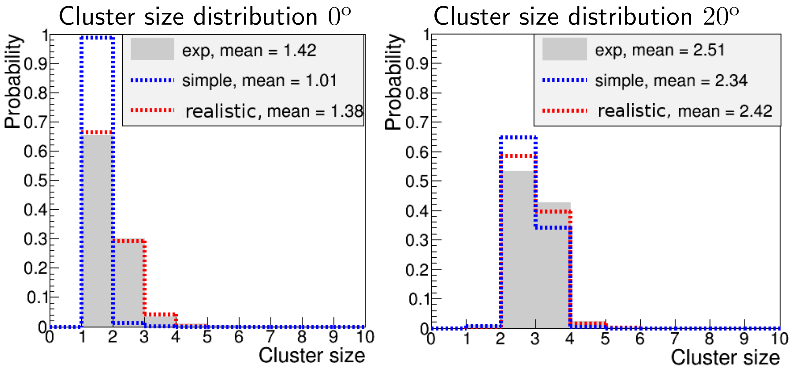
- No hardware trigger $\rightarrow$ no events $\rightarrow$ time-slices (~ 1000 events) $\rightarrow$ time coordinate;
- Do not store all data $\rightarrow$ software trigger: on-line event reconstruction and selection $\rightarrow$ fast algorithms.

Detector response model:

- non-uniform energy loss in sensor (Urban model[1]);
- drift of created charge carriers in planar electric field;
- Lorentz shift of e-h pairs in magnetic field;
- diffusion: Gaussian broadening of the charge carrier cloud with time;
- cross-talk due to interstrip capacitance: $Q_{\mathrm{neib\ strip}} = \dfrac{Q_{\mathrm{strip}} C_{\mathrm{i}}}{C_{\mathrm{c}} + C_{\mathrm{i}}}$;
- modeling of the read-out chip including time resolution and dead time.

[1] K. Lassila-Perini and L. Urbán (1995)

Simulation VS experimental data

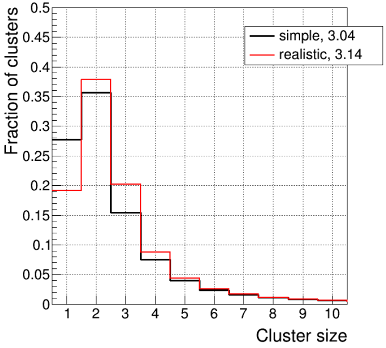


Experiment: 2 GeV protons.

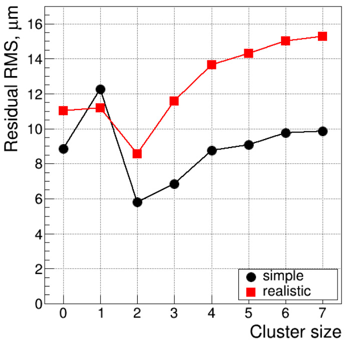
- **realistic** detector response model: described here;
- **simple**: takes into account only noise and threshold.

Detector response model: reconstruction performance

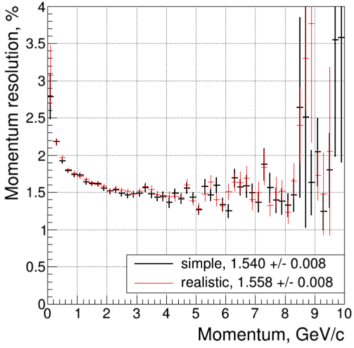
Cluster level



Hit level



Track level



- **realistic** detector response model: described here;
- **simple**: takes into account only average Lorentz shift, noise and threshold.

Cluster position finding algorithm (CPFA)

Center-Of-Gravity:

$$x_{\text{rec}} = \frac{\Sigma x_i q_i}{\Sigma q_i}$$

x_i – the coordinate of ith strip,
q_i – its charge,
$i = 1..n$ – the strip index in the n-strip cluster.

Unbiased CPFA:
2-strip clusters:

$$x_{\text{rec}} = 0.5\,(x_1 + x_2) + \frac{p}{3}\frac{q_2 - q_1}{\max(q_1, q_2)}, \quad p \text{ – strip pitch;}$$

n-strip clusters (head-tail algorithm[2]):

$$x_{\text{rec}} = 0.5\,(x_1 + x_n) + \frac{p}{2}\frac{\min(q_n, q) - \min(q_1, q)}{q}, \quad q = \frac{1}{n-2}\sum_{i=2}^{n-1} q_i$$

[2] R. Turchetta, "Spatial resolution of silicon microstrip detectors", 1993

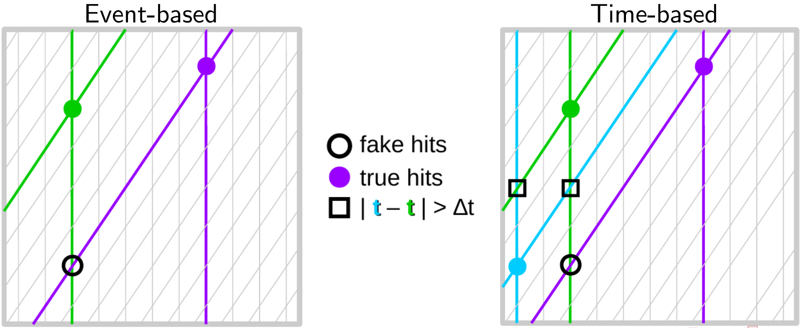
CPFAs comparison

Residuals, μm		
Cluster size	Unbiased	Center-Of-Gravity
1	14.2	14.3
2	10.3	10.2
3	13.3	13.6
all	14.5	15.1

- both CPFAs are implemented: Center-Of-Gravity and the unbiased;
- their performance are comparable;
- unbiased CPFA advantages:
  - faster: less operations;
  - simplifies estimation of position error.

Hit finder

- check geometrical overlap and time difference;
- fake hits due to combinatorial strip intersections;
- Δt is chosen based on read-out chip time resolution ($<$ 10 ns).



Time-based finders performance

- cluster finder: sort digis within a time-slice by channel number (**`multimap`**);
- hit finder: additional time difference checking.

	Event-based	Time-based
Efficiency	98 %	97 %
True hits	55 %	53 %
Time/event for cluster finder	22 ms	14 ms

Event-based: mbias events Au+Au @ 25 GeV;
Time-based: time-slices of 10 μs, contains $\approx$ 100 mbias events,
interaction rate 10 MHz — **the highest rate of CBM!**.

Conclusions

- Precise **detector response simulation** for the CBM Silicon Tracking System:
  - important for the detector understanding and obtaining reliable simulation results;
  - includes relevant physical processes in silicon detectors;
  - does not influence tracking performance.
- 2 **cluster position finding algorithms** implemented. The unbiased algorithm has computational advantages.
- **Cluster and hit finders**:
  - algorithms have high-speed performance;
  - the time-based reconstruction has the same quality as the event-based.

Conclusions

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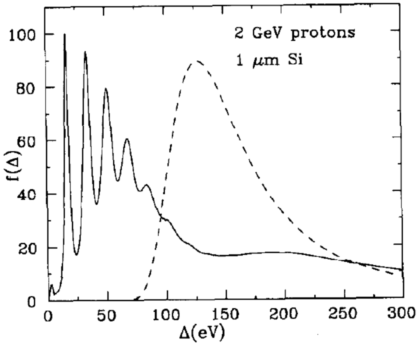
- **Cluster and hit finders**:
  - algorithms have high-speed performance;
  - the time-based reconstruction has the same quality as the event-based.

Thank you for your attention!

Detector response model:

- non-uniform energy loss in sensor:
  divide a track into small steps and simulate energy losses in each of them using Urban model[1] *;*
- drift of created charge carriers in planar electric field
- movement of e-h pairs in magnetic field (Lorentz shift)
- diffusion
- cross-talk due to interstrip capacitance
- modeling of the read-out chip

[1] K. Lassila-Perini and L. Urbán (1995)

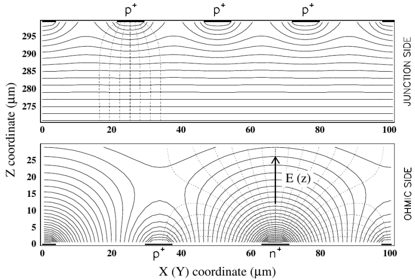


Energy losses of $2\,\mathrm{GeV}$ *protons in* $1\,\mu\mathrm{m}$ *of Si (solid line)*[2]*.*

[2] *H. Bichsel (1990)*

Detector response model:

- non-uniform energy loss in sensor
- drift of created charge carriers in planar electric field:
  non-uniformity of the electric field is negligible in 90% of the volume;
- movement of e-h pairs in magnetic field (Lorentz shift)
- diffusion
- cross-talk due to interstrip capacitance
- modeling of the read-out chip

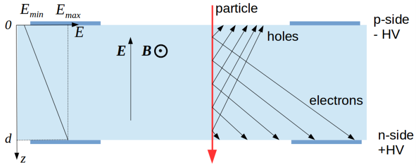


Calculated electric field for sensors with strip pitch $25.5\,\mu\mathrm{m}$ *on the p-side and* $66.5\,\mu\mathrm{m}$ *on the n-side*[1].

[1] *S. Straulino et al. (2006)*

Detector response model:

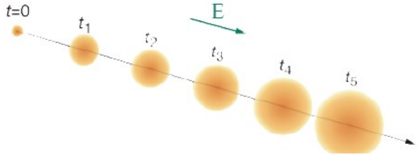
- non-uniform energy loss in sensor
- drift of created charge carriers in planar electric field
- movement of e-h pairs in magnetic field (Lorentz shift):
  taking into account the fact that Lorentz shift depends on the mobility, which depends on the electric field, which depends on the z-coordinate of charge carrier;
- diffusion
- cross-talk due to interstrip capacitance
- modeling of the read-out chip



Lorentz shift for electrons and holes in Si sensor.

Detector response model:

- non-uniform energy loss in sensor
- drift of created charge carriers in planar electric field
- movement of e-h pairs in magnetic field (Lorentz shift)
- diffusion:
  integration time is bigger than the drift time: estimate the increase of the charge carrier cloud during the whole drift time using Gaussian low;
- cross-talk due to interstrip capacitance
- modeling of the read-out chip



Increasing of charge cloud in time.

Detector response model:

- non-uniform energy loss in sensor
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- movement of e-h pairs in magnetic field (Lorentz shift)
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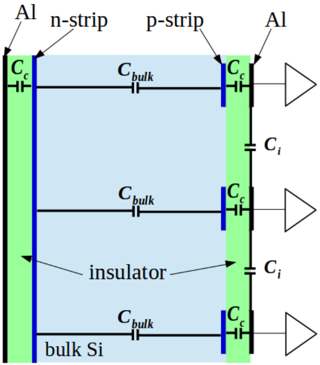
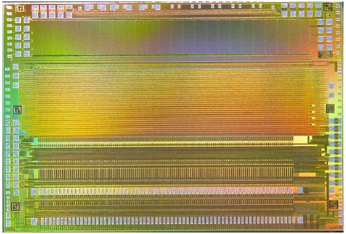
- modeling of the read-out chip



Simplified double-sided silicon microstrip detector layout.

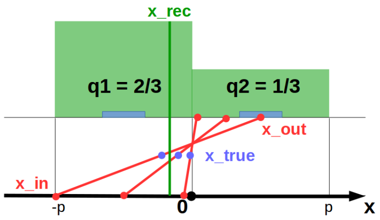
Detector response model:

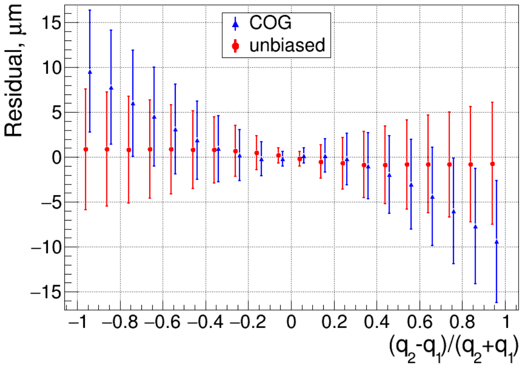
- non-uniform energy loss in sensor
- drift of created charge carriers in planar electric field
- movement of e-h pairs in magnetic field (Lorentz shift)
- diffusion
- cross-talk due to interstrip capacitance
- modeling of the read-out chip:
  - *noise: + Gaussian noise to the signal in fired strip;*
  - *threshold;*
  - *digitization of analog signal;*
  - *time resolution;*
  - *dead time.*



STS-XYTER read-out chip for the CBM Silicon Tracking System.

Residuals comparison for 2 CPFAs: 2-strip clusters





Ideal detector model & uniform energy loss.
Error bars: RMS of the residual distribution.
$q_{1,2}$ – measured charges on the strips.

Unbiased cluster position finding algorithm (CPFA), n-strip clusters

formula for **unifrom** energy loss:

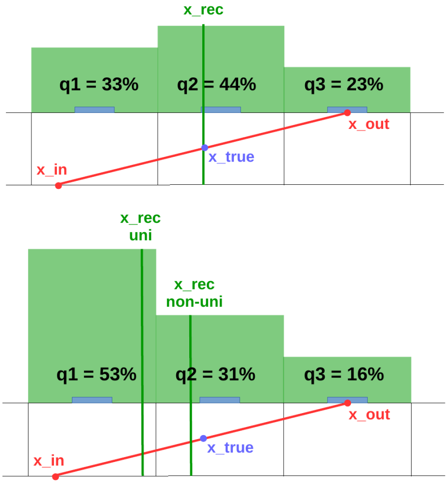
$$x_{\rm rec} = 0.5\,(x_1 + x_n) + \frac{p}{2}\frac{q_n - q_1}{q},$$

$$q = \frac{1}{n-2}\sum_{i=2}^{n-1} q_i;$$

formula for **non-uniform** energy loss (head-tail algorithm[1]):

$$x_{\rm rec} = 0.5\,(x_1 + x_n) + \frac{p}{2}\frac{\min(q_n, q) - \min(q_1, q)}{q},$$

[1] R. Turchetta, "Spatial resolution of silicon microstrip detectors", 1993



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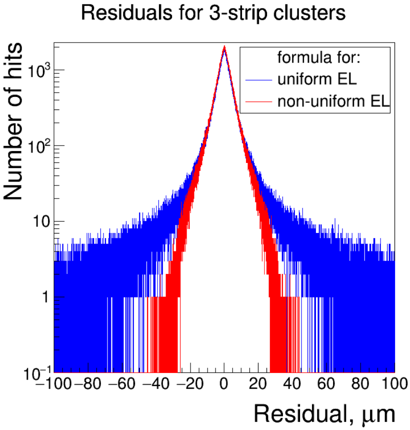

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Motivation

Why care: A reliable estimate of the hit position error $\Rightarrow$
get proper track χ^2 $\Rightarrow$
discard ghost track candidates $\Rightarrow$
improve the signal-to-background ratio and
keep the efficiency high.

Method: Calculations from first principles and independent of:
measured spatial resolution;
simulated residuals.

Verification: Pull distribution: $\text{pull} = \dfrac{\text{residual}}{\text{error}}$:
width must be ≈ 1;
shape must reproduce the shape of the residuals distribution.
χ^2-distribution: mean must be 1.

Needs:
reliable detector response model;
good cluster position finding algorithm (CPFA).

Estimation of hit position error

Hit position error: $\sigma^2 = \sigma^2_{\text{alg}} + \sum_i \left(\frac{\partial x_{\text{rec}}}{\partial q_i}\right)^2 \sum_{\text{sources}} \sigma_j^2,$

σ_{alg} – an error of the unbiased CPFA:

$$\sigma_1 = \frac{p}{\sqrt{24}}, \qquad \sigma_2 = \frac{p}{\sqrt{72}} \frac{|q_2 - q_1|}{\max(q_1, q_2)}, \qquad \sigma_{n>2} = 0.$$

σ_j – errors of the charge registration at one strip, among them already included:

- $\sigma_{\text{noise}} = \text{Equivalent Noise Charge};$
- $\sigma_{\text{discr}} = \dfrac{\text{dynamic range}}{\sqrt{12}\ \text{number of ADC}};$
- $\sigma_{\text{non-uni}}$ is estimated assuming:
  - registered charge corresponds to the most probable value of the energy loss;
  - incident particle is ultrarelativistic ($\beta\gamma \gtrsim 100$).
- σ_{diff} is negligible in comparison with other effects.

1-strip clusters: why not $\sigma_{method} = p/\sqrt{12}$?

In general, for **all** track inclinations:

- $N = \int\limits_{x_{in}} \int\limits_{x_{out}} P_1(x_{in}, x_{out}) dx_{in} dx_{out} = p^2;$
- $\sigma^2 = \frac{1}{N} \int\limits_{x_{in}} \int\limits_{x_{out}} P_1(x_{in}, x_{out}) dx_{in} dx_{out} \Delta x^2 = \frac{p^2}{24}.$

Particullary, for **perpendicular** tracks: $x_{in} = x_{out}$

- $N = \int\limits_{x_{in}} P_1(x_{in}, x_{out}) dx_{in} = p;$
- $\sigma^2 = \frac{1}{N} \int\limits_{x_{in}} P_1(x_{in}, x_{out}) dx_{in} \Delta x^2 = \frac{p^2}{12}$

Error due to non-uniform energy loss

The contribution from the non-uniformity of energy loss is more difficult to take into account because the actual energy deposit along the track is not known. The following approximations allow a straightforward solution:

- the registered charge corresponds to the most probable value (MPV) of energy loss;
- the incident particle is ultrarelativistic ($\beta\gamma \gtrsim 100$).

The second assumption is very strong but it uniquely relates the MPV and the distribution width (Particle Data Group)

$$MPV = \xi[\mathrm{eV}] \times \left(\ln \left(1.057 \times 10^6 \xi[\mathrm{eV}] \right) + 0.2 \right).$$

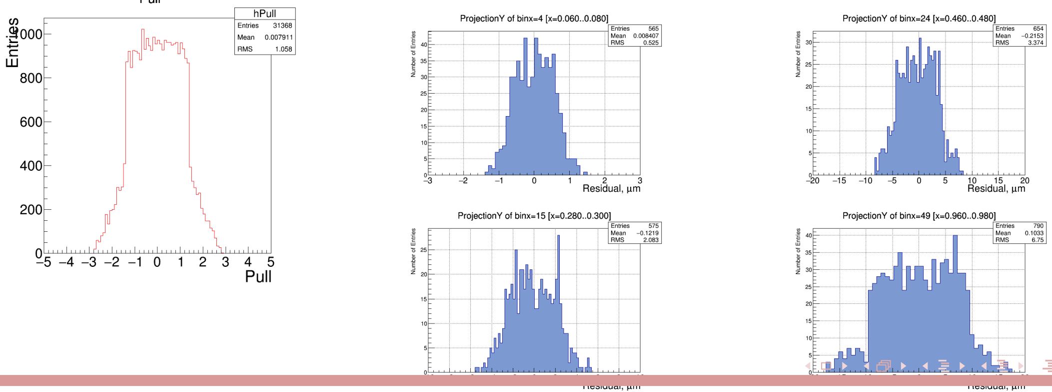
Solving this with respect to ξ gives the estimate for the FWHM (S. Merolli, D. Passeri and L. Servoli, Journal of Instrumentation, Volume 6, 2011)

$$\sigma_{\mathrm{non}} = w/2 = 4.018\xi/2.$$

Error verification

How we can be sure, that we estimate errors correclty?
Shape of pulls distribution must reproduce shape of residuals.

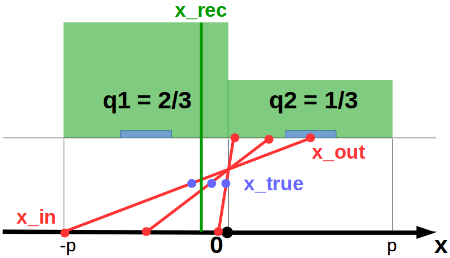
We compare 2-strip cluster pulls and residuals at **fixed** q_i: $\frac{|q_2 - q_1|}{max(q_1, q_2)}$

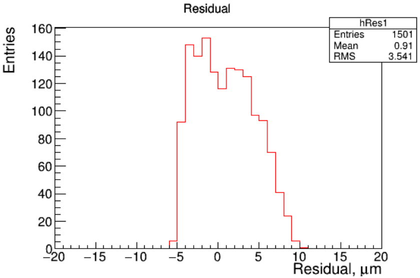


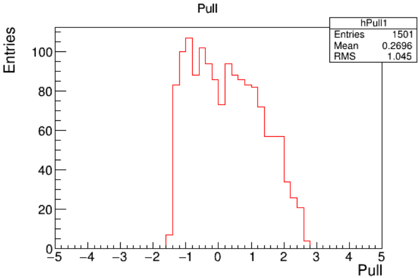
Error verification: Example

Consider:

- 2-strip clusters;
- fixed: $q_1 = 2/3$, $q_2 = 1/3$.

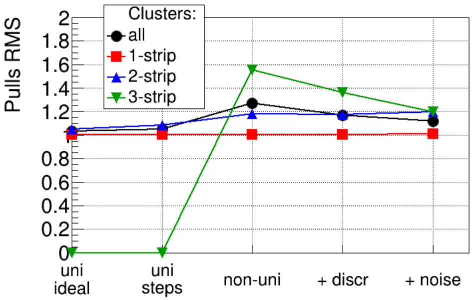




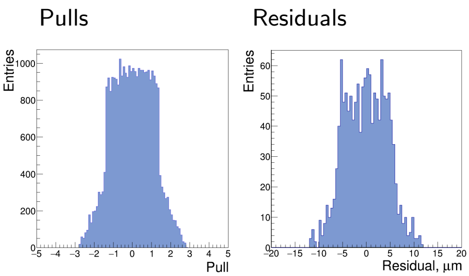


Verification: pull distribution

Width

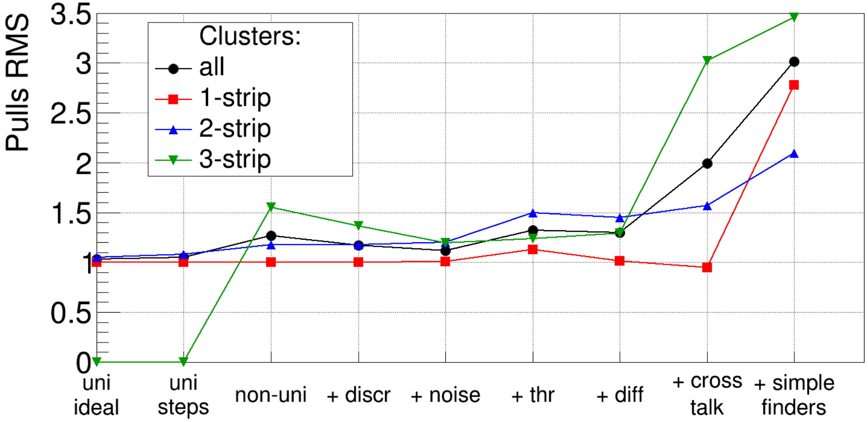


Shape



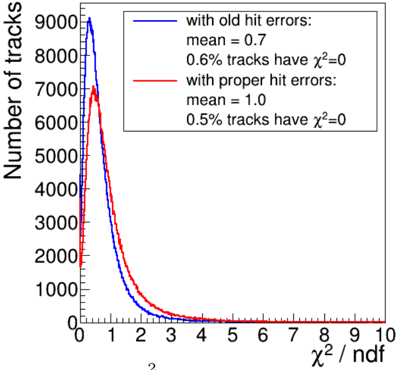
- ideal detector;
- 2-strip clusters;
- residuals at fixed: $\frac{|q_2 - q_1|}{\max(q_1, q_2)}$.

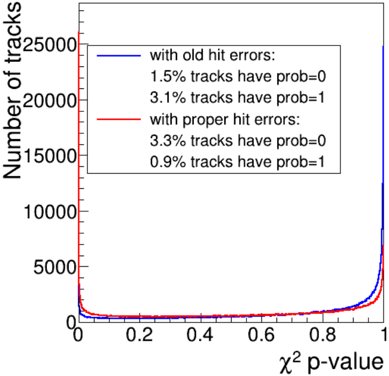
Pull width with all the effects included



Into errors, only effects till noise is included.

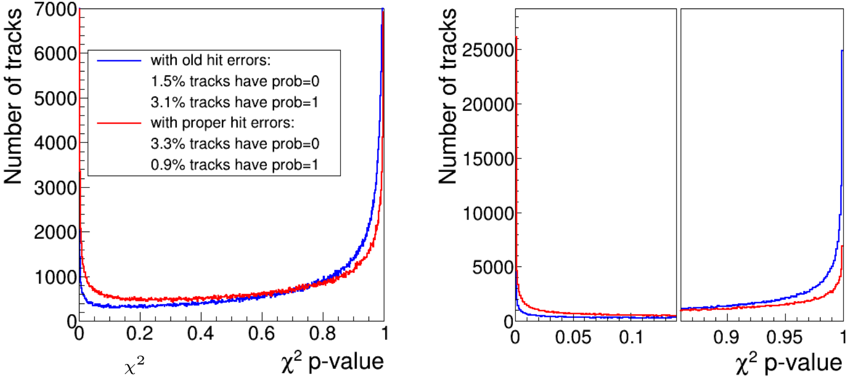
Verificati





- p-value $= 1 - \int_{-\infty}^{\chi^2} f_{\chi^2}(t)dt = 1 - \mathrm{CDF}$ (cumulative distribution function);
- with new error estimate p-value distribution is flat.

Verification: track fit performance



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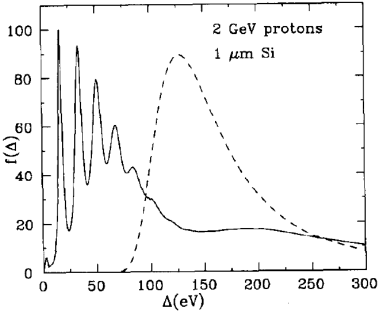
Energy loss process



The energy loss spectrum for 2 GeV protons transversing a Si absorber of thickness $1\mu m$ as calculated by [Bichsel,1990] (solid line): only inelastic interactions with plasmons and electrons in the detector material are included (no bremsstrahlung, no nuclear interactions).
The function $f(\Delta)$ extends to a maximum value $\Delta_M = 9\ MeV$. The separate peaks at 17, 34, 51 ... eV correspond to 1, 2, 3 ... plasmon excitations. The Landau function is shown as dashed line.

Energy loss models

A particle losses its energy non-uniformly along the path. Divide the track in a sensor into thin layers and calculate the deposited energy for each layer independently.

- **Landau theory**. However, it is not valid for a layer as thin as 1μm or even 10μm.
- **Urban method**. It is a Monte Carlo method that is applicable to thin layers. It is used in GEANT to compute the energy loss, when the Landau theory is not valid.
- **The photoabsorption ionization (PAI) model**.
- **"Bichsel's"-model**.

We use Urban model, because it is employed in Geant for thin layers by default and it doesn't require huge tables of cross-sections.

Urban method

Landau theory has 2 limits of validity:

- the number of collisions in which a particle loses an amount of energy close to the maximum value of the transferable energy in one collision should be small compared to the total number of collisions;
- the number of collisions in which a particle loses a small amount of energy should be large in the path length under consideration

For very thin layers second condition is violated.

Urban method assumes that the atoms have only two energy levels with binding energy E_1 and E_2. The particle-atom interaction will then be an excitation with energy loss E_1 or E_2, or an ionization with an energy loss distributed according to a function $g(E) \sim 1/E^2$

Charge carriers motion in silicon sensor

The charge carriers produced in the silicon sensor move in the electric (and magnetic) field and undergo thermal diffusion:

- in zero approximation e-h pairs move in **planar** electric field;
- the charge carriers **deviate** due to the Lorentz force by the angle:

$$tan\theta_{L,i} = \frac{\Delta x_i}{d} = \mu_i B \tag{1}$$

where μ is the Hall mobility, Δx is the Lorentz shift, i denotes the carrier type;

- the spatial distribution of the charge carriers becomes **broader** after time t according to the Gaussian law with the standard deviation:

$$\sigma = \sqrt{2Dt}, \quad D = \frac{kT}{e}\mu \tag{2}$$

D is the diffusion coefficient, T is the temperature, e is an elementary charge and μ is a mobility of the charge carrier.

Thermal diffusion

After time t the distribution of the charge carriers becomes broader according to the Gaussian law with standard deviation:

$$\sigma = \sqrt{2Dt}, \quad D = \frac{kT}{e}\mu \tag{3}$$

D – diffusion coefficient, T – temperature, e – elementary charge and μ – mobility of the charge carrier. Since $D \sim \mu$ and the collection time $t \sim 1/\mu$, the resulting σ doesn't depend on the type of the charge carrier.

Possibilities for the electric field modelling

- To approximate the electric field with analytic expression for a field in a **planar** abrupt p-n junction:

$$E(z) = -\left(\frac{V_{bias} + V_{dep}}{d} - \frac{2z}{d^2} V_{dep}\right), \tag{4}$$

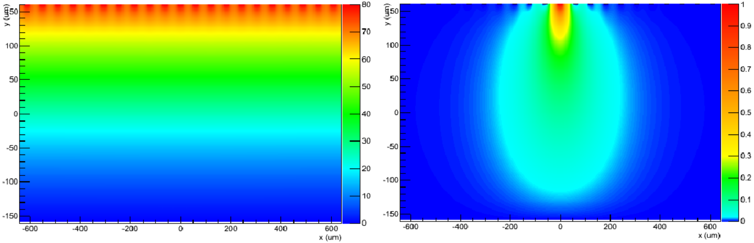
  where d – thickness of the sensor, V_{dep} – depletion voltage, V_{bias} – bias voltage. And then project produced charge onto read-out plane;
- To calculate the **detailed map** of the electric potential and to solve the equation of motion for each hole and electron: $\vec{v} = \mu\vec{E}$, where μ — mobility of the charge carrier.

Finally, to evaluate the current, induced at time t on the kth electrode by a moving carrier from the Shockley-Ramo theorem:

$$i_k(t) = -q\vec{v} \cdot \vec{E}_{wk} \tag{5}$$

where q – the charge of the carrier, $\vec{v}$ – its velocity and $\vec{E}_{wk}$ - the weighting field associated to the kth electrode which is determined by setting the electrode k to unit potential and all others to zero potential.

Weighting field



(a) Drift potential (b) Weighting potential

- When pre-amplifier integration time $>$ collection time of all charge carriers, the measured electrode gets only the current, induced by those charge carriers, which moving terminates on this electrode, while other electrodes get zero net current.
- STS-XYTER integration time in slow channel is $\sim 80\ ns$.
- At bias voltage of $100\ V$, depletion voltage of $60\ V$, detector thickness of $300\ \mu m$: electrons collection time $\leqslant 8\ ns$, holes collection time $\leqslant 22\ ns$.
- That is why the effect of the weighting field will be integrated out.