Enhancement of quark number susceptibility with an alternative pattern of chiral symmetry breaking in dense matter

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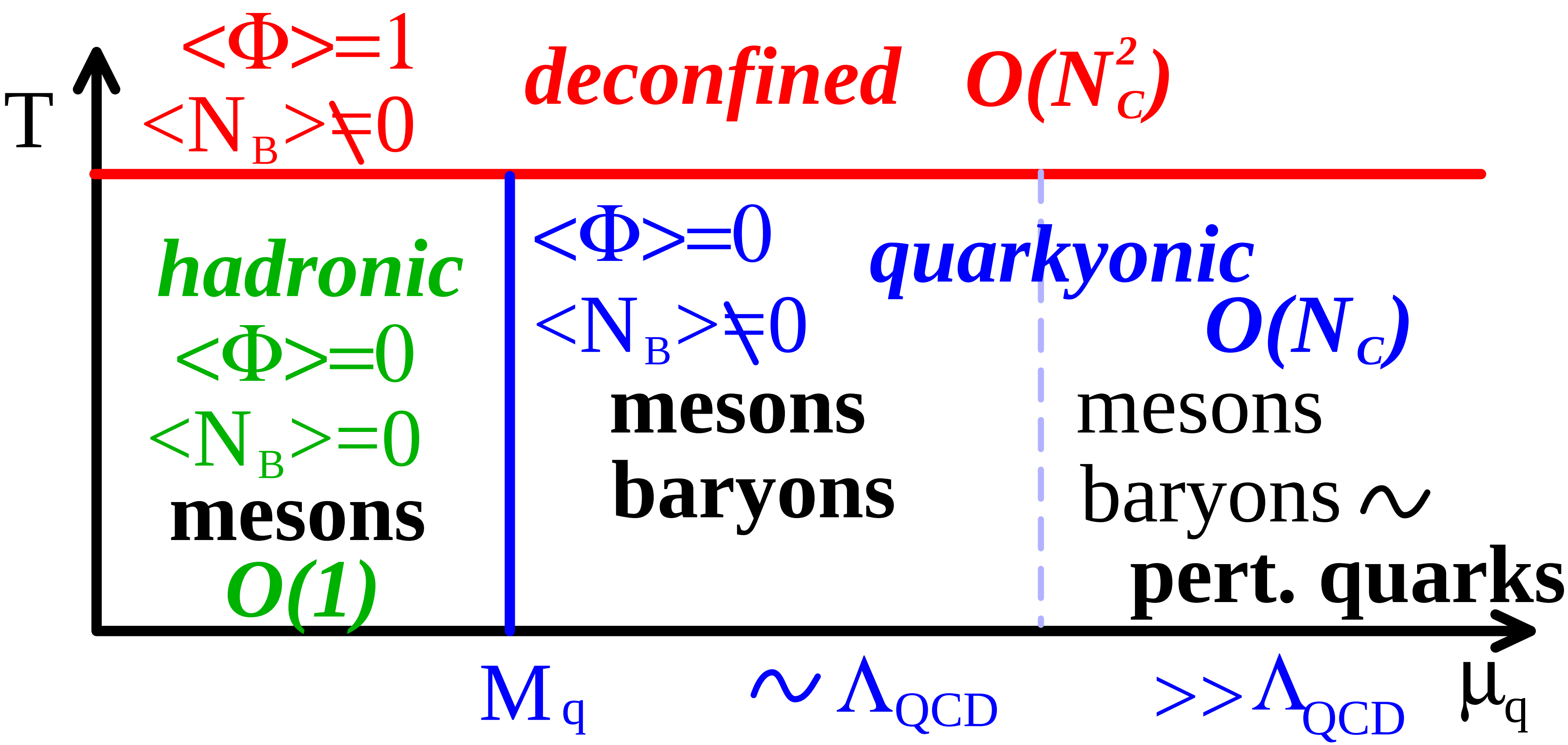
Introduction: what is Quarkyonic Phase for $N_c = 3$?

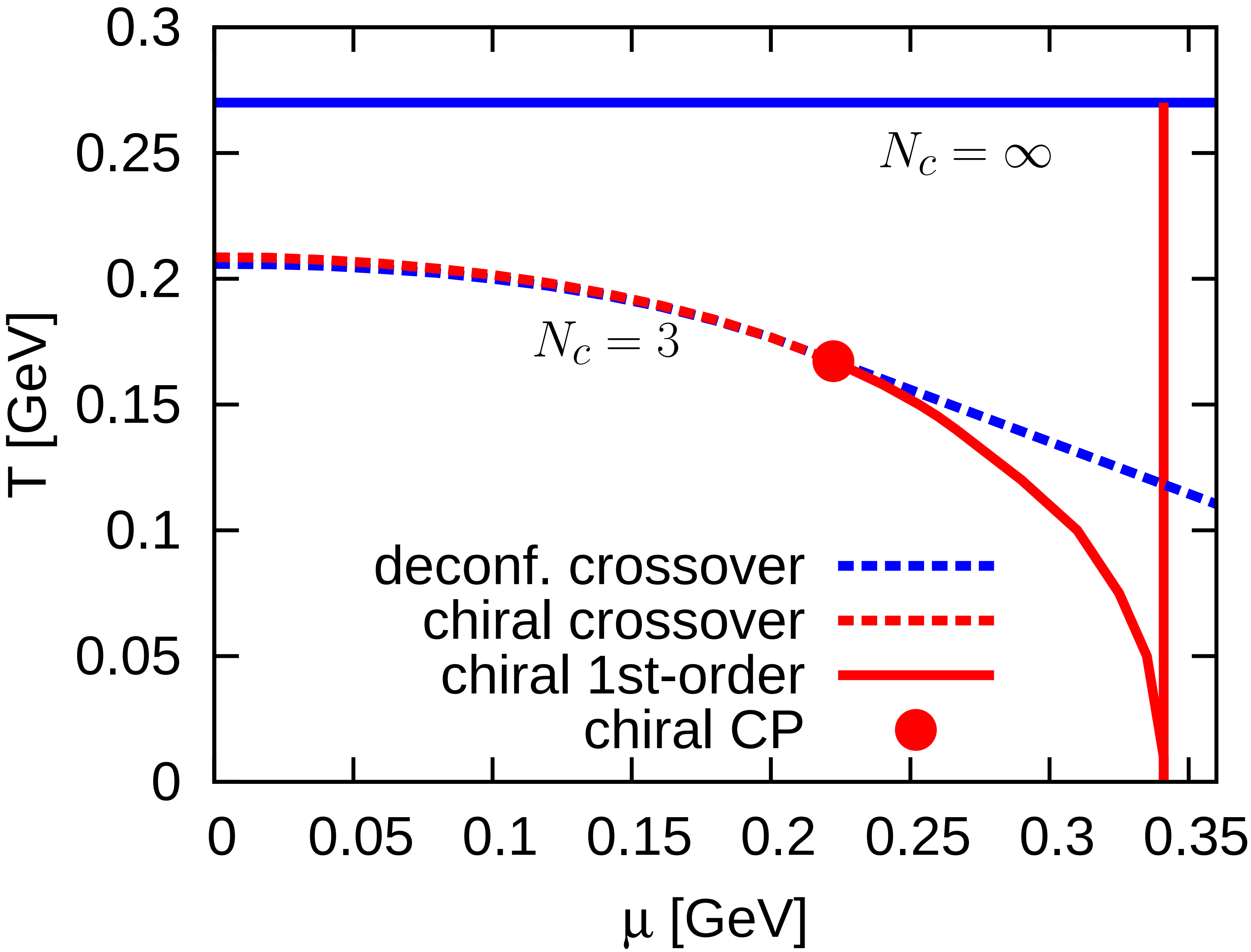
- **3 phases in large N_c** [McLerran-Pisarski (07)]



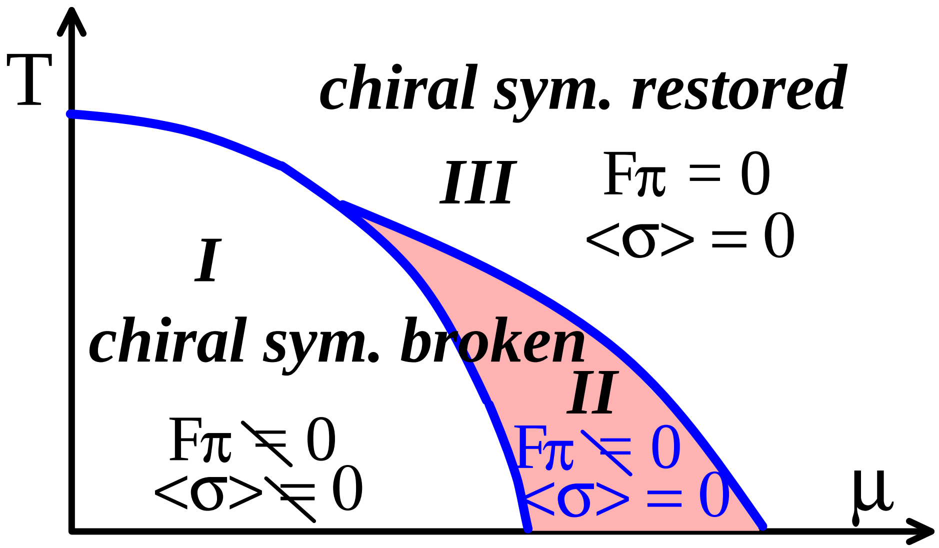
– 3 phases: Polyakov loop $\langle\Phi\rangle$ and baryon number $\langle N_B\rangle \sim e^{(\mu_B - M_B)/T}$.

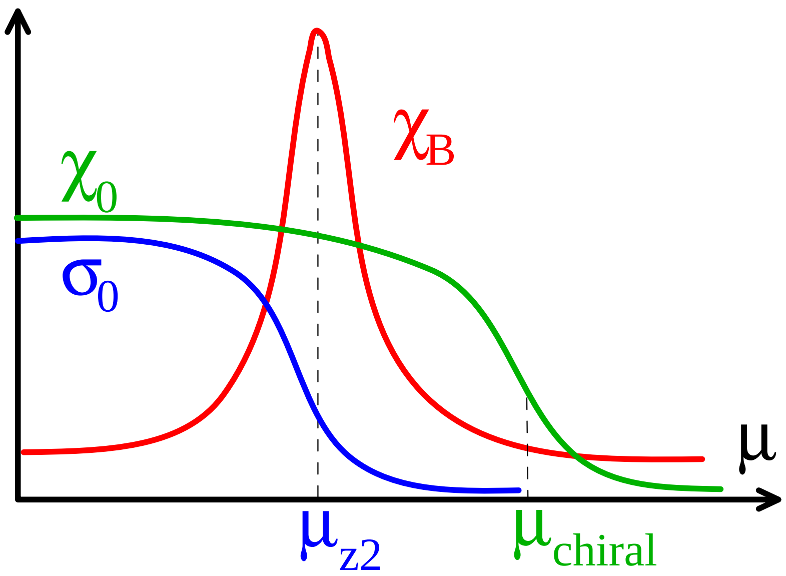
– "quark-yonic": elementary fermionic excitations at very large μ (quarks) and at moderate μ (baryons).

• **chiral & deconf. trs for $N_c = 3$ from PNJL** [McLerran-Redlich-CS (08)]



- **“transition” from hadronic to quarkyonic phase for $N_c = 3$?**
  - large N_c limit: clear distinction of 2 different confined-phases
    baryon number density $\langle N_B \rangle = 0$ (mesonic) and $\neq 0$ (quarkyonic)
    $\Rightarrow$ a rapid increase at $\mu_B = M_B$ in N_B
  - finite N_c: no clear definition
    but it may separate meson dominant from baryon dominant region
    $\Rightarrow$ a rapid change in N_B at $n_{\rm meson}/n_{\rm baryon} \sim \mathcal{O}(1)$ expected
  - enhancement of baryon number susceptibility in chiral models
    Q. $\mu_{\rm quarkyonic} \sim \mu_{\rm chiral}$ or $\mu_{\rm quarkyonic} < \mu_{\rm chiral}$?
    Q. meson & baryon mass spectra and χ-sym. breaking/restoration?





An alternative pattern of chiral symmetry breaking

- **unorthodox pattern of spontaneous χSB** [Knecht-Stern (94,95), Stern (97,98)]

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V \times (Z_{N_f})_A$$

discrete symmetry $(Z_{N_f})_A$: maximal axal subgroup of $SU(N_f)_{L\times R}$

– Z_{N_f} protects a theory from condensate of quark bilinears $\langle \bar{q}q \rangle$

– quartic condensations invariant under $SU(N_f)_V$ and Z_{N_f}

– thus $\langle \bar{q}q \rangle = \mathcal{O}(m_q)$ and $\langle \bar{q}q\bar{q}q \rangle = \mathcal{O}(1)$

- **Nambu-Goldstone theorem**

condition of spontaneous symmetry breaking:

$$\langle 0| \left[iQ, \mathcal{O}(x) \right] |0\rangle = \langle 0|\delta\mathcal{O}(x)|0\rangle \neq 0$$

possible operators $\mathcal{O}$? $\cdots$ not only $\bar{q}q$ but $\bar{q}q\bar{q}q$ etc.
cf. $\bar{q}q\bar{q}q \sim (\bar{q}q)^2$ factorisation *assumption*

- **no-go theorem at $\mu = 0$ ($T = 0$ and $T \neq 0$)** [Kogan-Kovner-Shifman (99)]
  $SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V \times Z_{N_f}$ is ruled out in QCD.
  $\Leftrightarrow$ QCD inequalities $|P(x)P(y)| > |A^\mu(x)A^\nu(y)|$ [Weingarten (83), Witten (83)]

- **phase associated with $\langle \bar{q}q\bar{q}q \rangle \neq 0$ and $\langle \bar{q}q \rangle = 0$ at finite μ**
  operators invariant under Z_{N_f}, e.g.
  - $\mathcal{O}_1 = \bar{q}T_a\gamma_\mu(1-\gamma_5)q \cdot \bar{q}T_a\gamma^\mu(1+\gamma_5)q \sim V_\mu V^\mu - A_\mu A^\mu$
  - $\mathcal{O}_2 = \bar{q}T_a(1-\gamma_5)q \cdot \bar{q}T_a(1+\gamma_5)q \sim \sigma^2 - \pi^2$
  - scalar diquark & scalar anti-diquark $\Rightarrow$ chiral singlet
  - axial-vector diquark & axial-vector anti-diquark $\Rightarrow$ chiral non-singlet

- **evidence from dynamical model calculations?**
  - O(2) model (scalar $\phi = \phi_1 + i\phi_2$) in CJT: [Watanabe-Fukushima-Hatsuda (03)]
    meta-stable phase where $\langle \phi^2 \rangle \neq 0$ while $\langle \phi \rangle = 0$
  - Skyrme model [Park et al. (02), Lee et al. (03)]
    similar intermediate phase where $\langle \sigma \rangle \neq 0$ but $F_\pi \neq 0$

A chiral model for 2- and 4-quark states

- assume any suitable 4-quark operators invariant under Z_{N_f}.
- symmetry breaking pattern:

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V \times (Z_{N_f})_A \to SU(N_f)_V$$

- ingredients
  - 2-quark state $M \sim \bar{q}q$ and 4-quark state $\Sigma \sim (\bar{q}q)^2$

$$M_{ij} = \frac{1}{\sqrt{2}}\left(\sigma\delta_{ij} + i\phi^a\tau^a_{ij}\right) \,, \quad \Sigma_{ab} = \frac{1}{\sqrt{3}}\chi\delta_{ab} + \frac{1}{\sqrt{2}}\epsilon_{abc}\psi_c$$

    non-singlet under chiral trnsf, and $M \to -M \, , \Sigma \to +\Sigma$ under Z_2
  - Lagrangian $\mathcal{L}_{\rm kin} - V$ up to 4th order in the fields:

$$V(M,\Sigma) = -\frac{m^2}{2}\mathrm{Tr}\left[MM^\dagger\right] - \frac{\bar{m}^2}{2}\Sigma_{ab}\Sigma^T_{ba}$$
$$+ \frac{\lambda^2}{4}\mathrm{Tr}\left[\left(MM^\dagger\right)^2\right] + \frac{\bar{\lambda}_1^2}{4}\Sigma_{ab}\Sigma^T_{bc}\Sigma_{cd}\Sigma^T_{da} + \frac{\bar{\lambda}_2^2}{4}\left(\Sigma_{ab}\Sigma^T_{ba}\right)^2$$
$$+ g_1\left(\Sigma_{ab}\mathrm{Tr}\left[T_aMT_bM^\dagger\right] + \mathrm{h.c.}\right) + g_2\Sigma_{ab}\Sigma^T_{ba}\mathrm{Tr}\left[MM^\dagger\right] + g_3\mathrm{Det}\Sigma$$

- **phase structure in mean field approximation**
  - replace fields with their VEV:

$$M_{ij} \sim \sigma\delta_{ij}\,, \quad \Sigma_{ab} \sim \chi\delta_{ab}$$

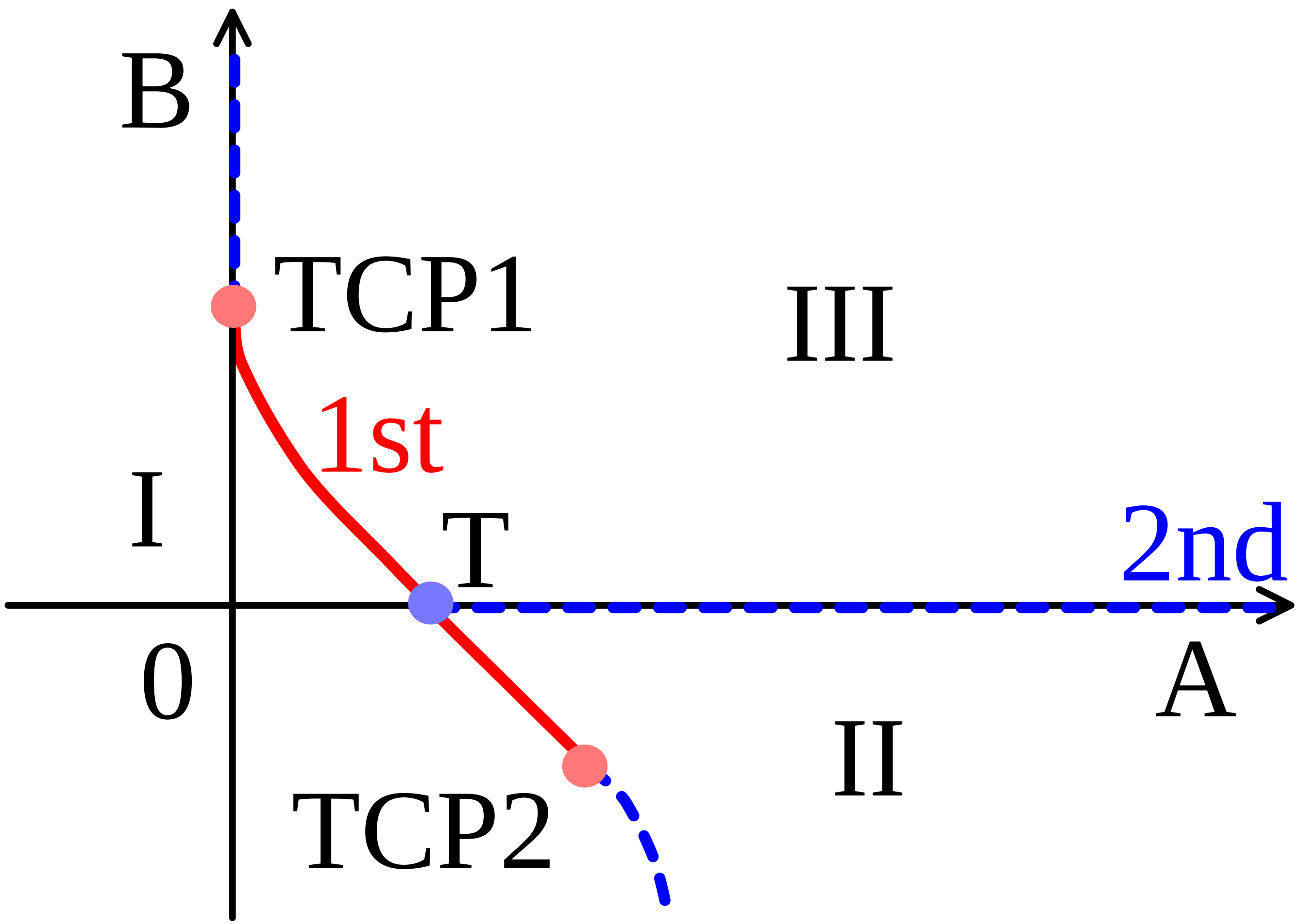
  - Ginzburg-Landau effective potential

$$V(\sigma,\chi) = \lambda_1\sigma^4 + \lambda_2\chi^4 + A\sigma^2 + B\chi^2 + C\sigma^2\chi + D\chi^3 + F\sigma^2\chi^2$$

    take $\lambda_1 = \lambda_2 = -C = 1$ without loss of generality

  - 3 phases expected
    * phase I: $\langle\sigma\rangle \neq 0$ & $\langle\chi\rangle \neq 0 \;\Rightarrow\; SU(2)_V$
    * phase II: $\langle\sigma\rangle = 0$ & $\langle\chi\rangle \neq 0 \;\Rightarrow\; SU(2)_V \times Z_2$
    * phase III: $\langle\sigma\rangle = 0$ & $\langle\chi\rangle = 0 \;\Rightarrow\; SU(2)_L \times SU(2)_R \times Z_2$

– phase diagram of $V(\sigma, \chi; D = F = 0)$

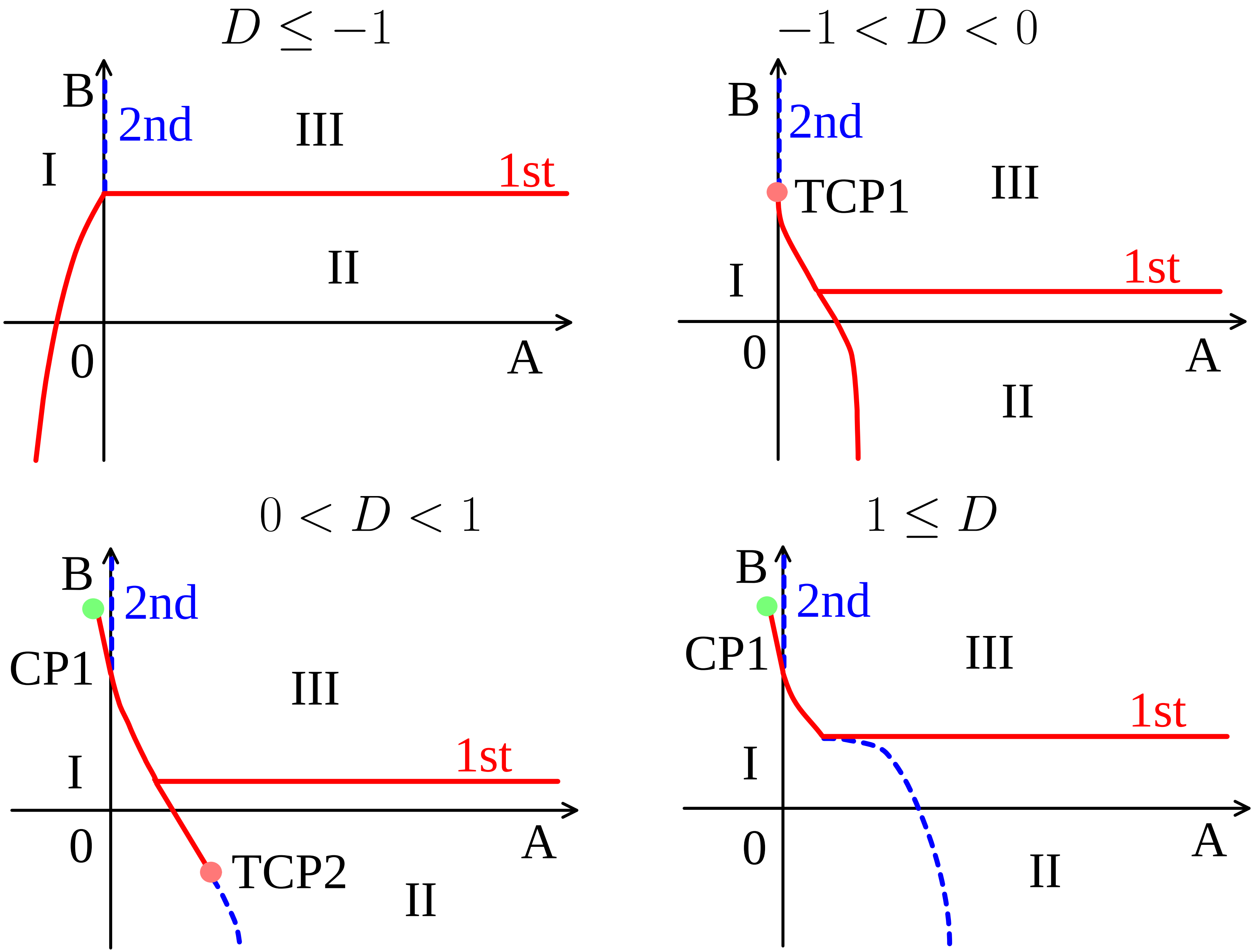


– phase I: $\langle\sigma\rangle \neq 0$ & $\langle\chi\rangle \neq 0 \Rightarrow SU(2)_V$

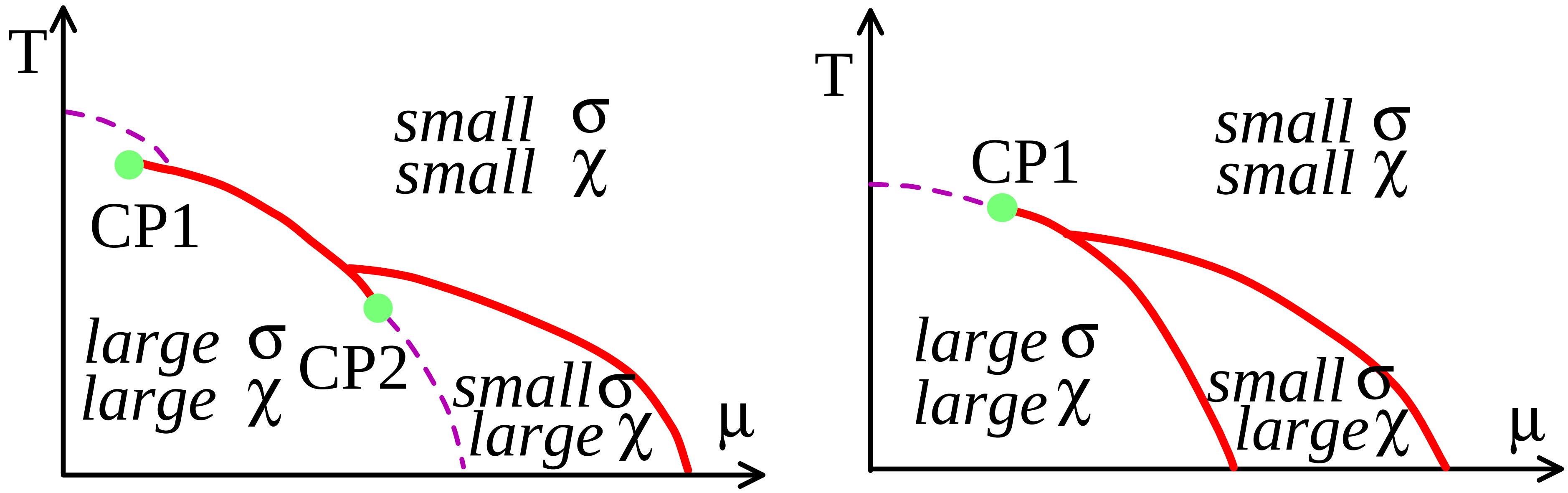
– phase II: $\langle\sigma\rangle = 0$ & $\langle\chi\rangle \neq 0 \Rightarrow SU(2)_V \times Z(2)$

– phase III: $\langle\sigma\rangle = 0$ & $\langle\chi\rangle = 0 \Rightarrow SU(2)_L \times SU(2)_R \times Z(2)$

– phase diagram of $V(\sigma, \chi; D \neq 0, F \neq 0)$

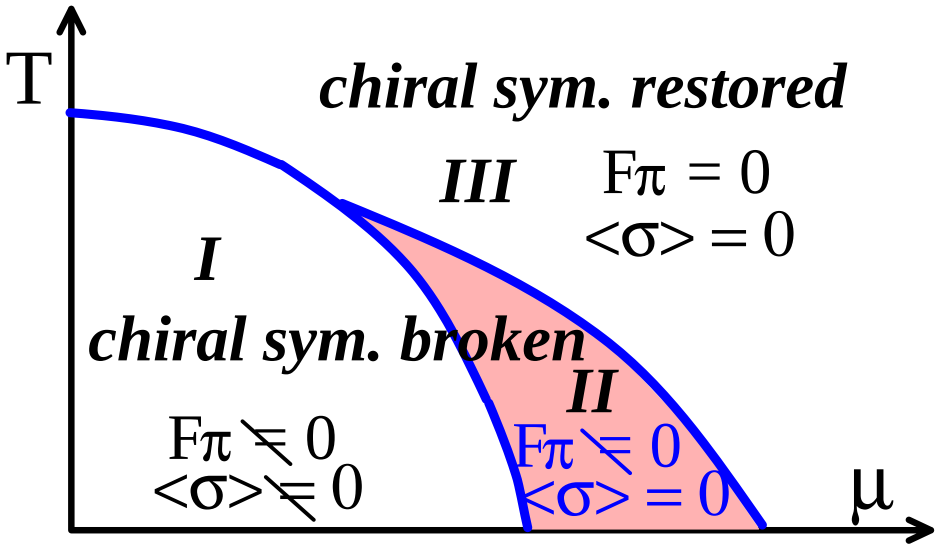


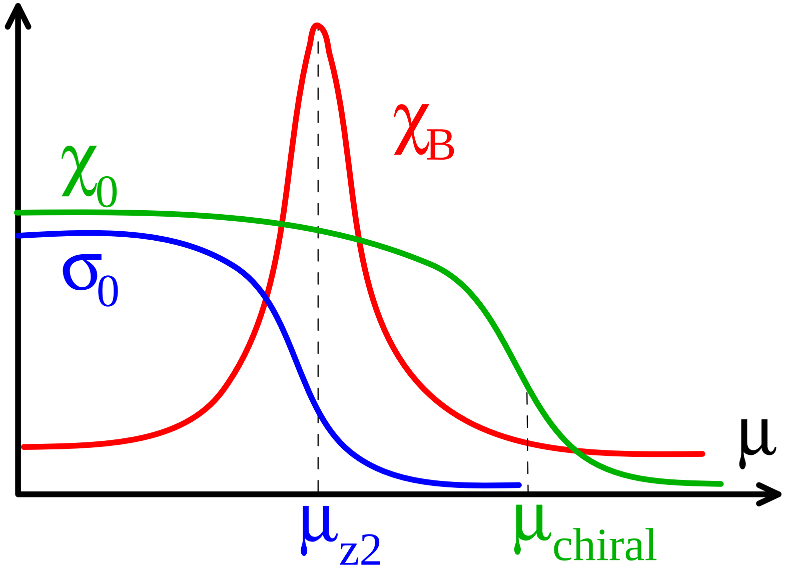
- hypothetical phase diagram in T-μ plane (w/ explicit breaking)



– 2 order parameters: σ (2-quark) and χ (4-quark)
  $\longrightarrow$ 2 phase transitions: restoration of Z_2 and chiral symmetries
  $\longrightarrow$ max. of corresponding susceptibilites
– multiple critical points: χ_B diverges with exponent $\gamma = 2/3$.

- **baryon number susceptibility**





– I-II: χ_B finite but max. (a jump if 1st order) $\cdots$ $\sigma \to 0$

– II-III: χ_B no much change $\cdots$ no Yukawa term $\bar{N}N\chi$ in phase II (Z_2)
true both in standard and mirror assignment of chirality

$*$ if standard, the lowest baryon state massless

$*$ if mirror and χ-inv. mass $m_0 \neq 0$, massive parity partners:
N^+: nucleon N^-: N′(1535)

– χ_B max. along Z_2 restoration line
can be interpreted as "quarkyonic transition" for $N_c = 3$

• hadron mass spectra and pion decay constant $(N_f = 2, 3)$

(I) $\sigma_0 \neq 0\,, \chi_0 \neq 0$	(II) $\sigma_0 = 0\,, \chi_0 \neq 0$	(III) $\sigma_0 = \chi_0 = 0$
$SU(N_f)_V$	$SU(N_f)_V \times (Z_{N_f})_A$	$SU(N_f)_L \times SU(N_f)_R$
$m_S \neq 0\,, m_P = 0$ $m_V \neq m_A$	$m_S = m_P = 0$ $m_V \neq m_A$	$m_S = m_P$ $m_V = m_A$
$F_\pi = \sqrt{\sigma_0^2 + (8/3)\chi_0^2}$	$F_\pi = \sqrt{8/3}\,\chi_0$	no residue
$m_N \neq 0$	(i) standard $(\psi_{R,L} \to g_{R,L}\psi_{R,L})$ small m_N (ii) mirror $(\psi_{2R,L} \to g_{L,R}\psi_{2R,L})$ $m_{N^+} \overset{?}{\neq} m_{N^-} \neq 0$	(i) standard $m_N = 0$ (ii) mirror $m_{N^+} = m_{N^-} \neq 0$

Summary and prospects

- **a chiral model with** $SU(N_f)_L \times SU(N_f)_R \times Z_{N_f}$
  - a model for 2- and 4-quark states
  - topology of phase structure
  - enhancement of χ_B associated with Z_{N_f} symmetry restoration
  - hadron masses and pion decay constant
- **to be done using effective Lagrangians**
  - e.g. linear sigma model plus χ plus nucleons
  - chiral restoration of baryons (mirror vs. standard)
  - anomaly matching