Heavy quark correlators in a thermal medium

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Motivation

- Experimental ("J/Psi suppression")

- Theory. Controversial results.

- No good theory for heavy quark "bound states" in a thermal medium. Ad-hoc approaches/models.

- Our goal : contribute to development of such a theory

WORK IN PROGRESS !

Based on

A. Beraudo, JPB, C. Ratti, NPA 806 (2008) 312 [arXiv: 0712.4394]

A. Beraudo, JPB, P. Faccioli and G. Garberoglio (in preparation)

Outline

- The correlator of a heavy quark-antiquark pair.

- Static approximation (infinite mass limit). Effective interaction potential. Real part and free energy. Imaginary part and damping rate of heavy quark.

- Finite mass effects. One loop spectral function. Exact path integral formulation. Monte Carlo calculation and MEM reconstruction of spectral function.

The heavy quark propagator

$$\begin{aligned} S^{>}(t, r_1; 0, r_1') &= \frac{1}{Z}\mathrm{Tr}\left\{ e^{-\beta H}\psi(t; r_1)\psi^{\dagger}(0; r_1')\right\} \\ &= \frac{1}{Z}\sum_{n,m} e^{-\beta E_n} e^{i(E_n - E_m)t}\langle n|\psi(r_1)|m\rangle\langle|\psi^{\dagger}(r_1')|n\rangle \end{aligned}$$

note analyticity for $-\beta < \mathrm{Im}\, t$

For a free particle

$$S^{>}(t = -i\tau, x) = e^{-M\tau}\left(\frac{M}{2\pi\tau}\right)^{3/2} e^{-\frac{Mx^2}{2\tau}} \qquad (x \equiv r_1 - r_1')$$

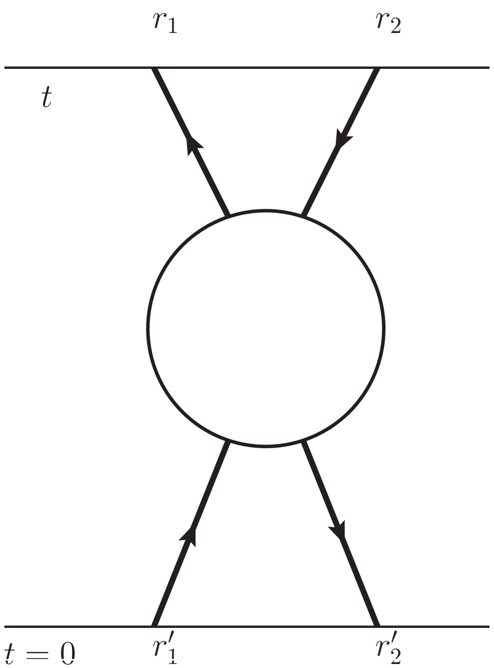
For large M

$$S_0^{>}(t = -i\tau, x) \sim e^{-M\tau}\delta(x)$$

The heavy quark correlator

$$J_Q^\dagger(t;r_1,r_2) \equiv \psi^\dagger(t,r_1)\chi^\dagger(t,r_2)$$

$$\begin{aligned} G^>(t,r_1;t,r_2|0,r_1';0,r_2') &= \frac{1}{Z}\mathrm{Tr}\left\{e^{-\beta H} J_Q(t;r_1,r_2) J_Q^\dagger(0;r_1',r_2')\right\} \\ &= \frac{1}{Z}\sum_{n,m} e^{-\beta E_n} e^{i(E_n-E_m)t}\langle n|J_Q|m\rangle\langle m|J_Q^\dagger|n\rangle \end{aligned}$$



The heavy quarks are treated as a "test particles". They are NOT part of the thermal bath.

A simple model

HQ' s in a plasma of electrons, positrons and photons

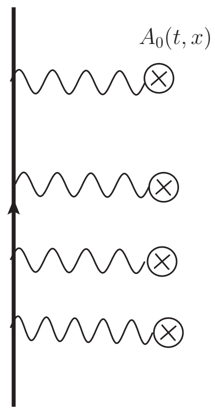
$$(M \to \infty)$$

Hamiltonian of the form

$$H = g \int dx\, A_0(x) \left(-\psi^\dagger(x)\psi(x) + \chi^\dagger(x)\chi(x) \right) + H_{\rm Cb} + H_{\rm f}$$

Propagator in background $A_0(t,x)$

$$S^>_A(t,x) = S^>_0(t,x)\, \exp\left(ig \int_0^t dt' A_0(x,t') \right)$$



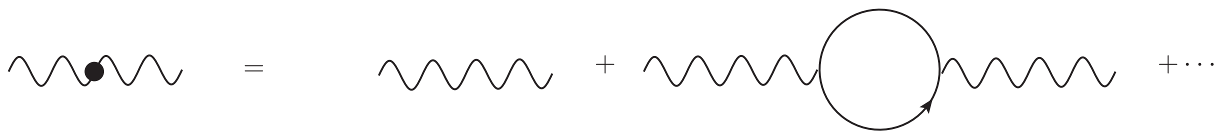
$Q\bar{Q}$ pair in $A_0(t,x)$ background

$$
\begin{aligned}
G_A(t;r_1,r_2;r_1',r_2') &= \delta(r_1-r_1')\delta(r_2-r_2')\times \\
&\times\ \exp\left(ig\int_0^t dt' A_0(r_1,t')\right)\exp\left(-ig\int_0^t dt' A_0(r_2,t')\right)
\end{aligned}
$$

Exact representation of correlator

$$
\begin{aligned}
G^{>}(t;r_1,r_2;r_1',r_2') &= \int [\mathcal{D}A]\, G_A(t;r_1,r_2;r_1',r_2')\, \mathrm{e}^{iS[A]} \\
&\equiv \delta(r_1-r_1')\delta(r_2-r_2')\overline{G}(t,r_1-r_2)
\end{aligned}
$$

$S[A]$ is the hard thermal loop effective action



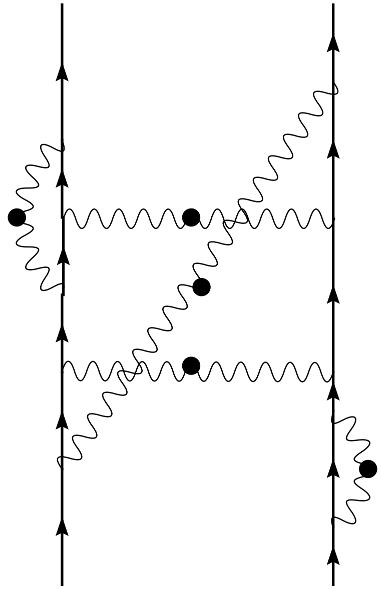
The HTL effective action is quadratic in A

$$S_C^{HTL}[A] = \frac{1}{2}\int_C d^4x \int_C d^4y\, A^\mu(x)\left(D^{-1}\right)^{HTL}_{\mu\nu}(x-y)A^\nu(y).$$

and the path integral can be done exactly

$$\overline{G}(t, r_1 - r_2) = \exp\left[-\frac{i}{2}\int_C d^4x \int_C d^4y\, J^\mu(x) D^{HTL}_{\mu\nu}(x-y) J^\nu(y)\right]$$

$$J^\mu(z) \;=\; \delta^{\mu 0}\theta(z^0)\theta(t-z^0)[-g\delta(z-r_1) + g\delta(z-r_2)]$$



Large time behaviour $(t\, m_D \gg 1)$

$$\overline{G}(t, r_1 - r_2) \underset{t\to\infty}{\sim} \exp[-iV_{\rm eff}(r_1 - r_2)t]$$

V_{eff} has real and imaginary part (*)

$$\begin{aligned} V_{\rm eff}(r_1 - r_2) &\equiv g^2 \int \frac{dq}{(2\pi)^3} \left(1 - e^{iq\cdot(r_1 - r_2)}\right) D_{00}(\omega = 0, q) \\ &= g^2 \int \frac{dq}{(2\pi)^3} \left(1 - e^{iq\cdot(r_1 - r_2)}\right) \left[\frac{1}{q^2 + m_D^2} - i \frac{\pi m_D^2 T}{|q|(q^2 + m_D^2)^2}\right] \\ &= -\frac{g^2}{4\pi}\left[m_D + \frac{e^{-m_D r}}{r}\right] - i \frac{g^2 T}{4\pi} \phi(m_D r) \end{aligned}$$

(*first observed by M. Laine et al hep-ph/ 0611300)

Screened potential

$$\begin{aligned} V_{\rm eff}(r) &= -\alpha m_D - \frac{\alpha}{r} e^{-m_D r} \\ &\underset{r\to 0}{\sim} -\alpha m_D - \frac{\alpha}{r} + \alpha m_D = -\frac{\alpha}{r} \\ &\underset{r\to\infty}{\sim} -\alpha m_D \end{aligned}$$

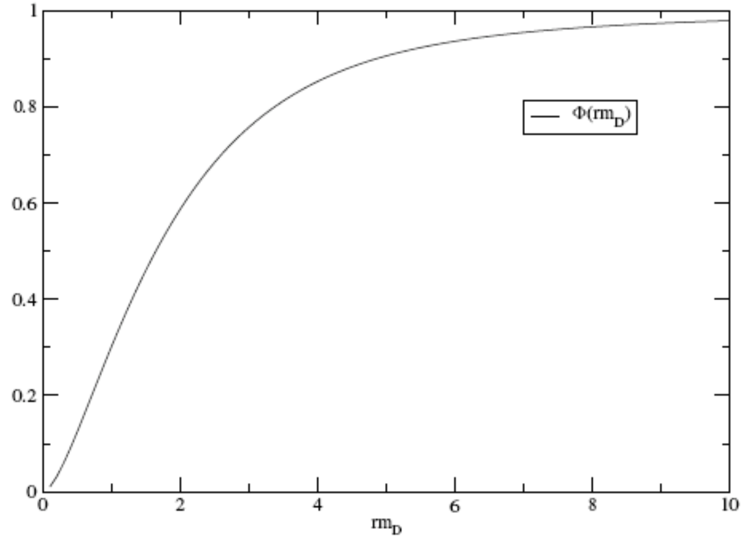
Note: $-\alpha m_D = 2\delta M$

Real part of potential is heavy quark free energy

$$\Delta F_{Q\bar{Q}} = \mathrm{Re} V_{eff}(r)$$

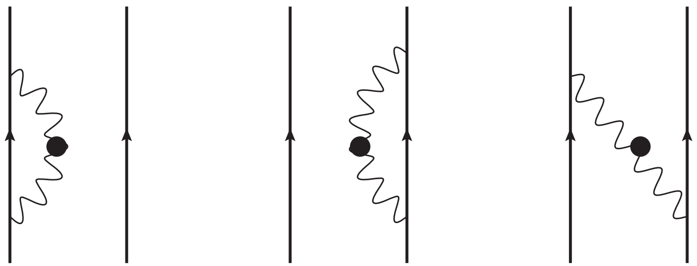
$$S_{Q\bar{Q}} = -\frac{dF}{dT} = \alpha\,[1 - \mathrm{e}^{-m_D r}]\,\frac{dm_D}{dT}$$

The imaginary part of the effective potential



At large distance the imaginary part is twice the damping rate of the heavy quark

At short distance, interference produces cancellation: a small dipole does not "see" the electric field fluctuations.



Finite mass case
Single quark

For a test particle

$$G^{>}(t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \mathrm{e}^{-i\omega t} \sigma(\omega)$$

where $\sigma(\omega)$ is the spectral function

By analytical continuation $(\tau \leq \beta)$

$$G^{>}(t = -i\tau) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \mathrm{e}^{-\omega\tau} \sigma(\omega)$$

Relation to analytic propagator

$$G(z) = \frac{-1}{z - E_p - \Sigma(z, p)}$$

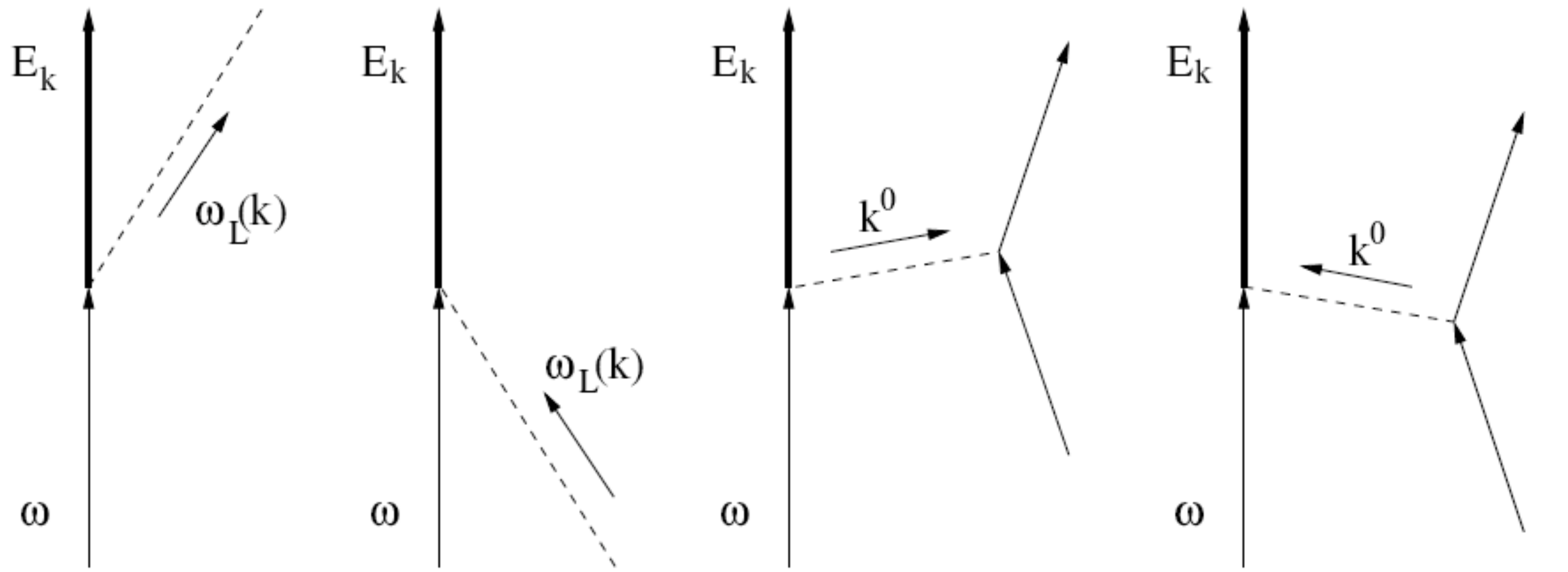
$$z \longrightarrow \omega + i\eta$$

$$\sigma(\omega) \equiv 2\mathrm{Im}\, G^R(\omega) = \frac{\Gamma(\omega)}{[\omega - E_p - \mathrm{Re}\,\Sigma(\omega)]^2 + \Gamma^2(\omega)/4}$$

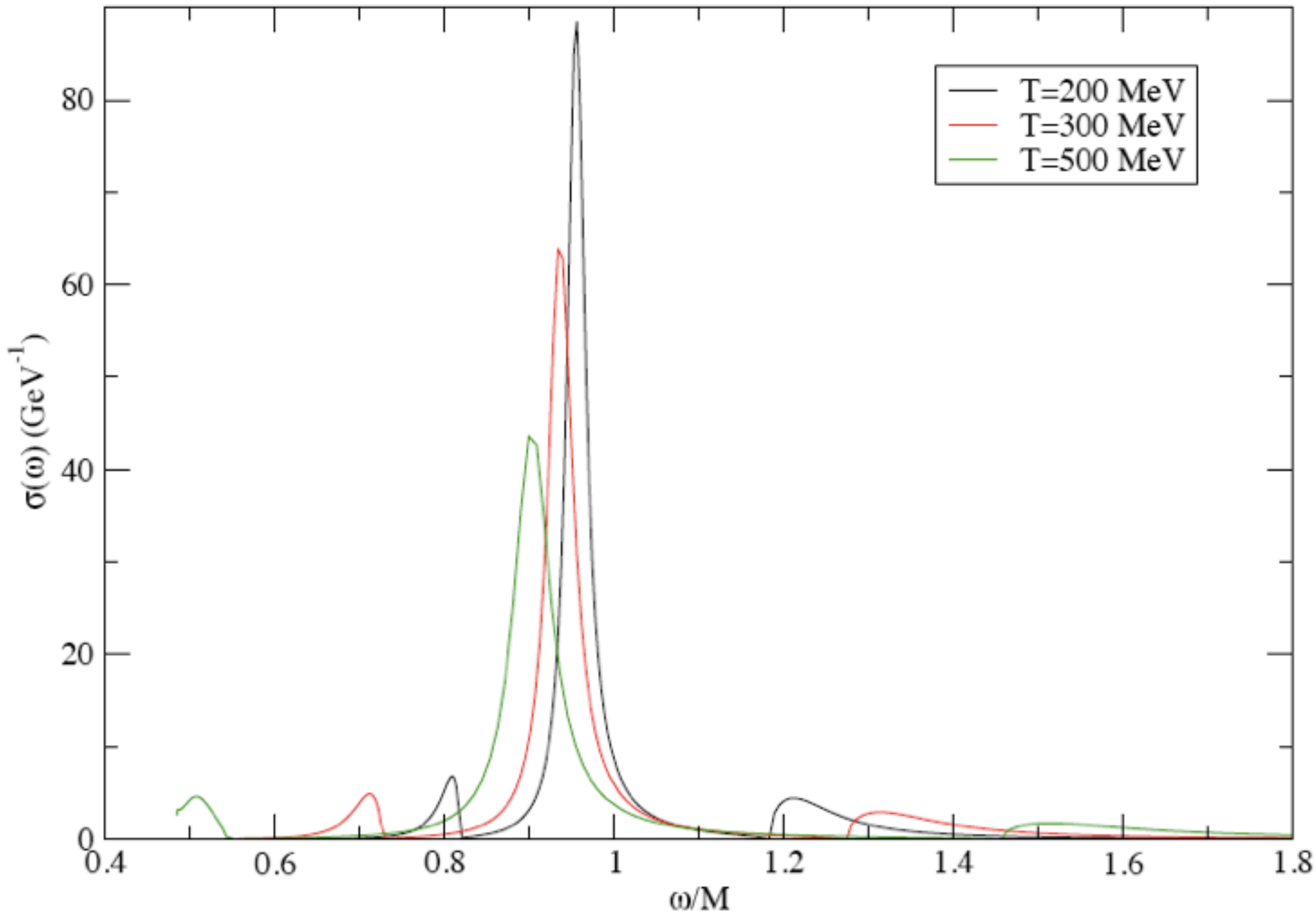
$$\Gamma(\omega) \equiv -2\mathrm{Im}\,\Sigma^R(\omega)$$

The spectral function is determined by the imaginary part of the self-energy

One loop processes



One loop spectral function



Finite mass case
Exact path integral for the propagator

Heavy quark in a background field A

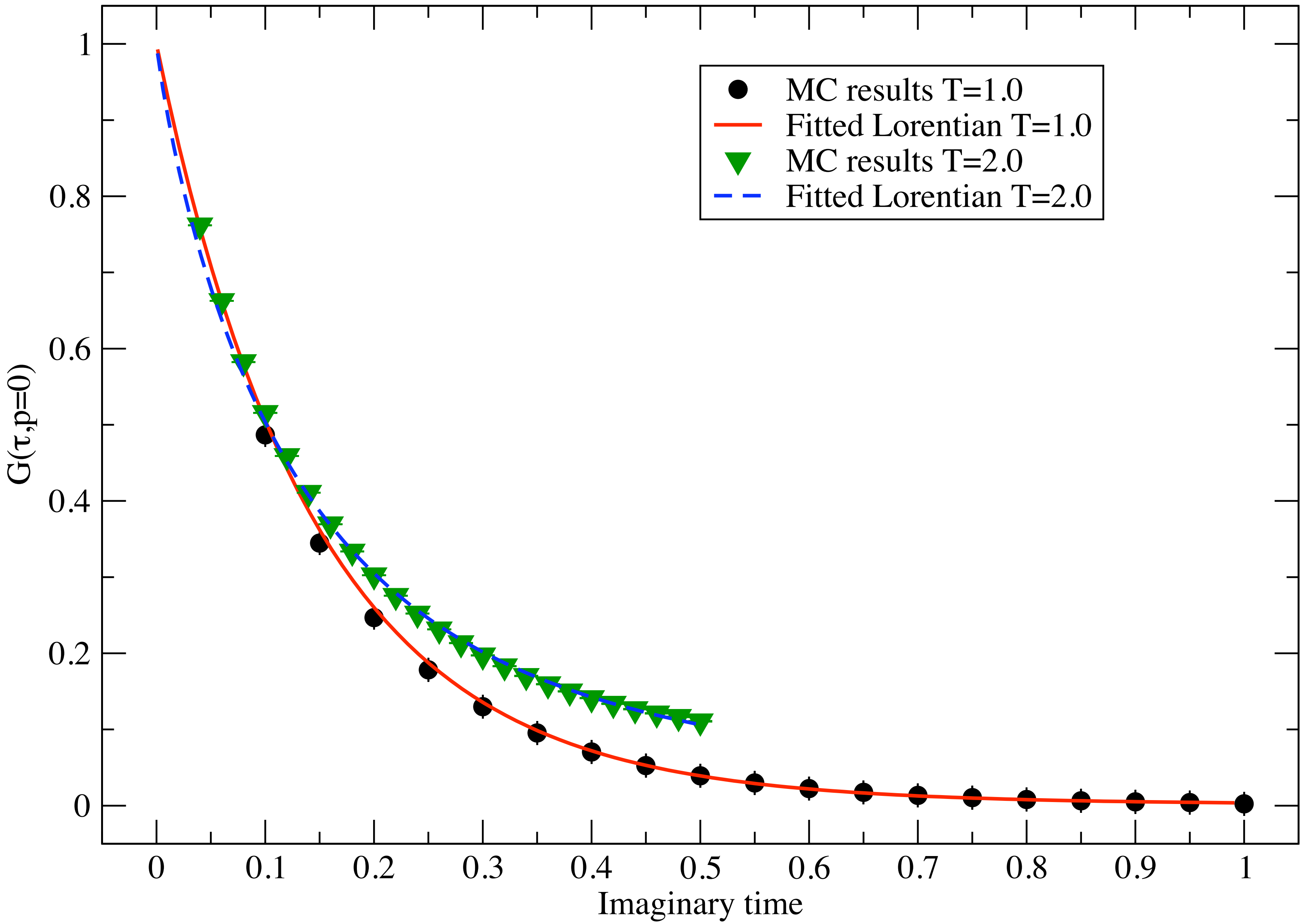
$$\langle r_1 \tau | r_1' 0 \rangle = \int_{x(0)=r_1'}^{x(\tau)=r_1} [\mathcal{D}x(\tau')] \exp\left[-\int_0^\tau d\tau' \left(\frac{1}{2} M \dot{x}^2 + g A_0(\tau', x) \right) \right]$$

Averaging over A yields

$$G^>(-i\tau, r_1 | 0, r_1') = \int_{z(0)=r_1'}^{z(\tau)=r_1} [\mathcal{D}z] \exp\left[-\int_0^\tau d\tau' \frac{1}{2} M \dot{z}^2 \right] \times$$

$$\times \exp\left[\frac{g^2}{2} \int_0^\tau d\tau' \int_0^\tau d\tau'' \Delta(\tau' - \tau'', z(\tau') - z(\tau'')) \right]$$

which can be evaluated by Monte Carlo



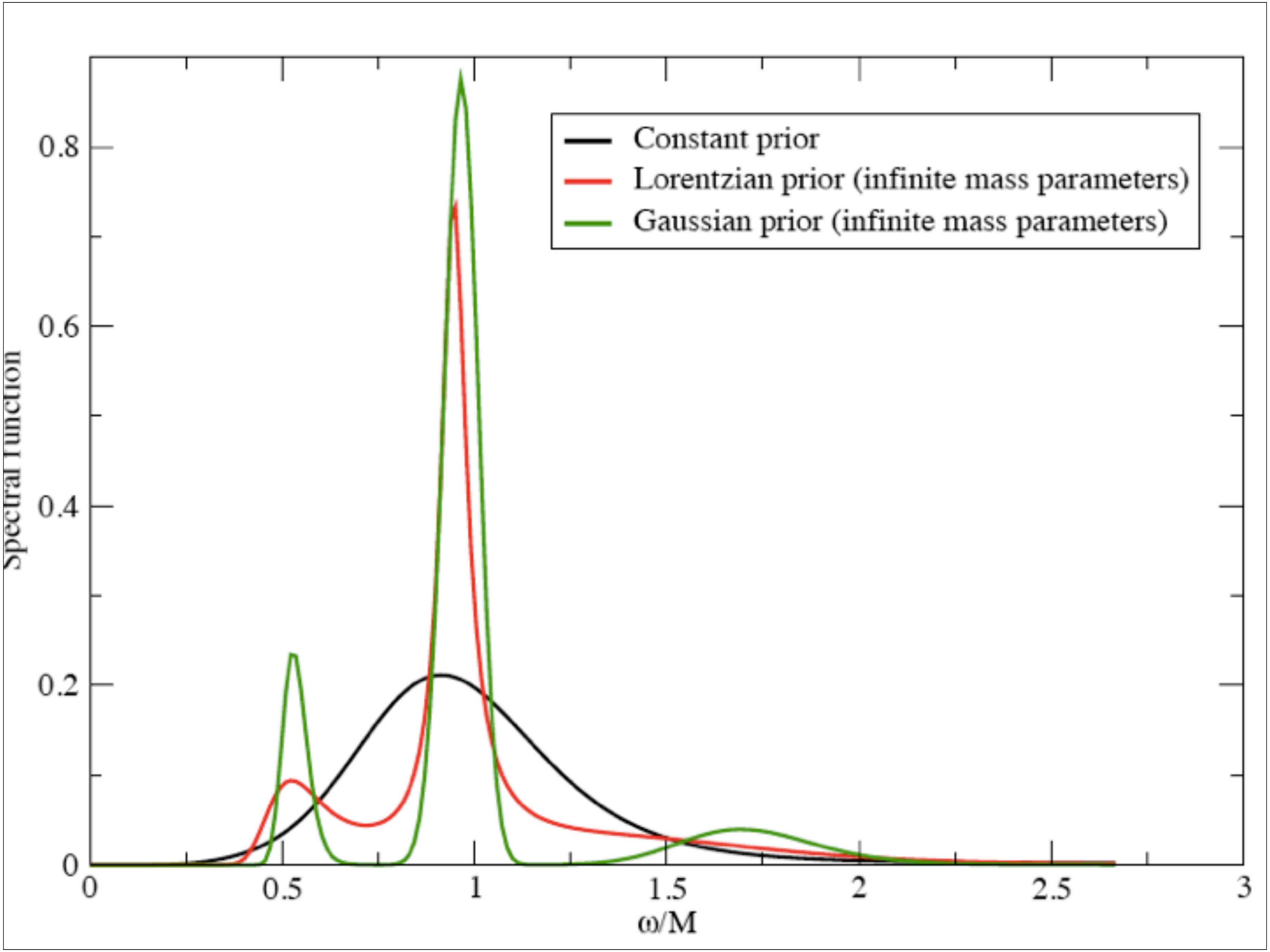
MC results T=1.0
Fitted Lorentian T=1.0
MC results T=2.0
Fitted Lorentian T=2.0
G(τ,p=0)
Imaginary time
0
0.2
0.4
0.6
0.8
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Recovering the spectral function from the (numerical) correlator is a difficult problem

$$G^{>}(t=-i\tau)=\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\mathrm{e}^{-\omega\tau}\sigma(\omega)$$

We are using the Maximum Entropy Method.

Preliminary results (still sensitive to the choice of the prior - see plot)



Constant prior
Lorentzian prior (infinite mass parameters)
Gaussian prior (infinite mass parameters)
Spectral function
ω/M
0
0.2
0.4
0.6
0.8
0.5
1
1.5
2
2.5
3

Conclusions

- A simple model was constructed, which takes into account exactly all essential many-body effects in the interaction of a heavy quark pair with a thermal bath

- The model leads to a simple path integral that can be calculated using Monte Carlo techniques (in the Euclidean)

- To reconstruct (numerically) the real time information one has to face similar difficulties as in usual lattice gauge theory, but in a much simpler context.