Wroclaw, 11. 07. 2009

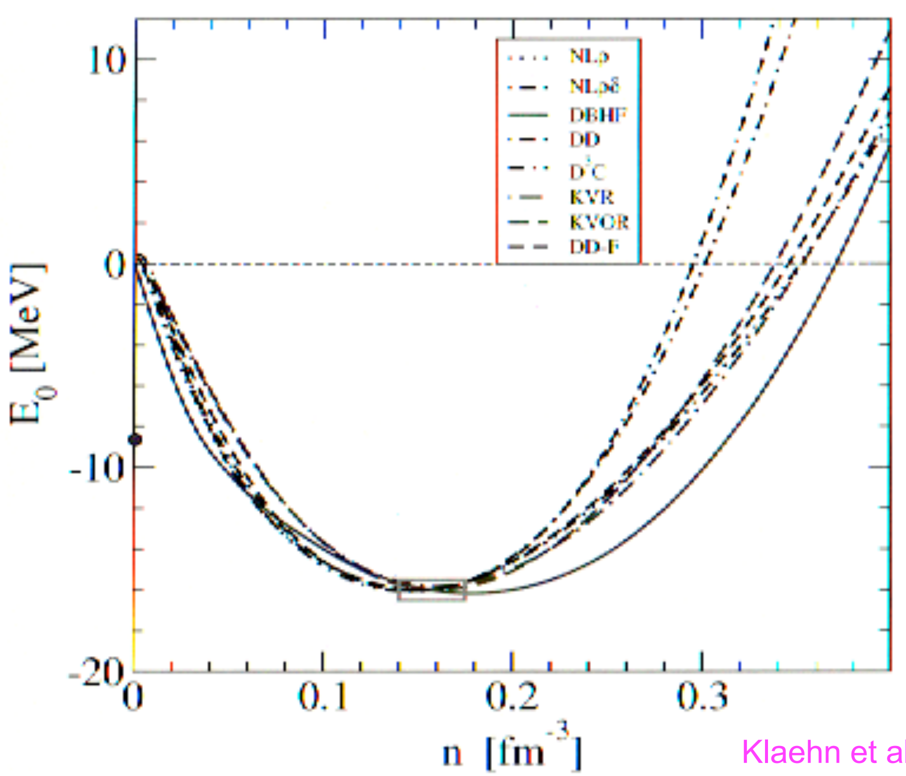
EMMI Workshop and XXVI. Max-Born Symposium

Light Clusters in Nuclear Matter

Gerd Röpke, Rostock

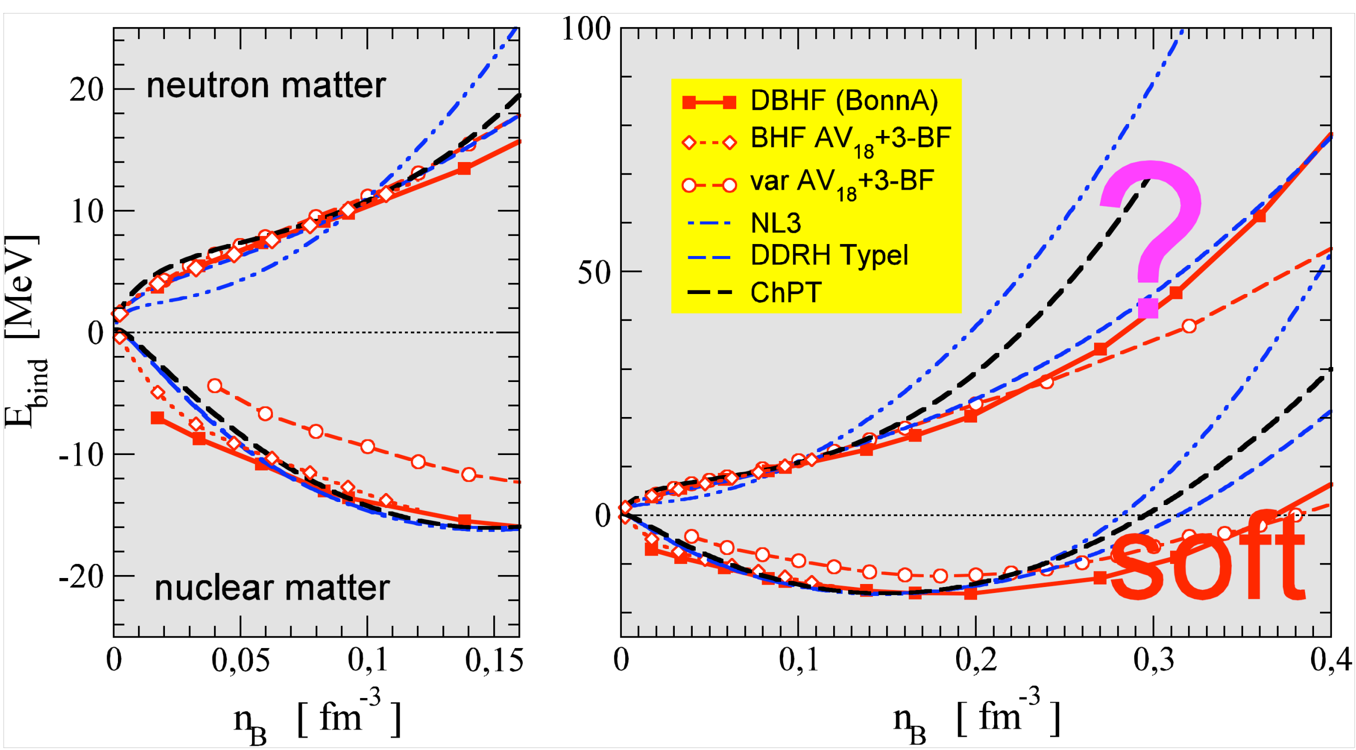
Quasiparticle approximation for nuclear matter

Equation of state for symmetric matter



Klaehn et al., PRC **74**, 035802 (06)

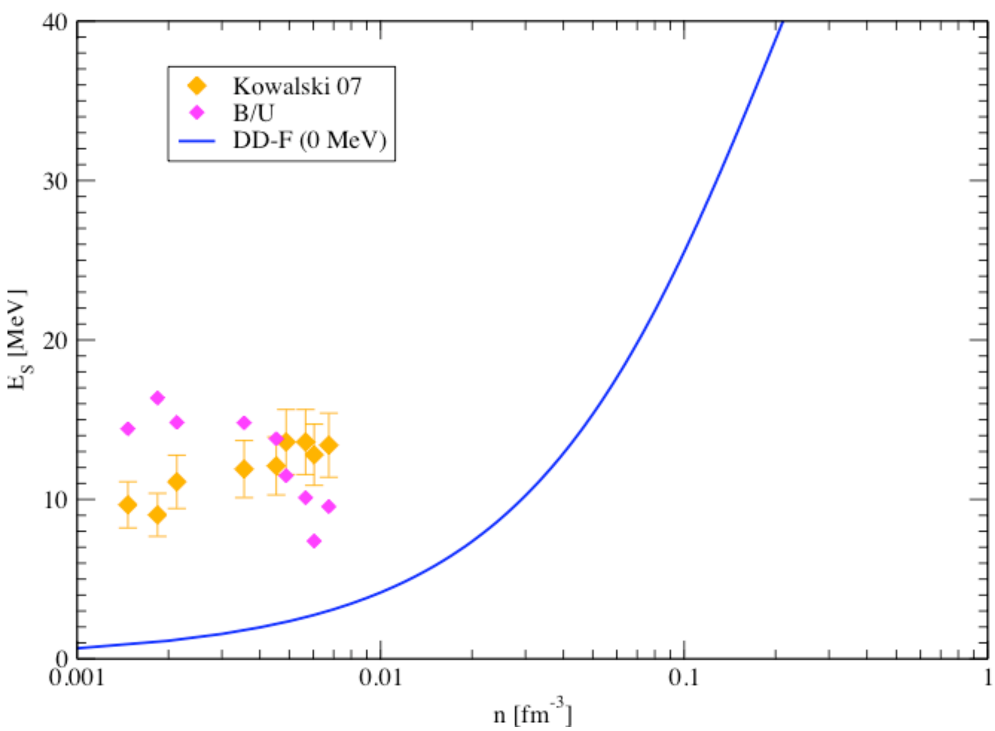
Quasiparticle picture: RMF and DBHF



J.Margueron et al., PRC **76**, 034309 (2007)

Symmetry energy

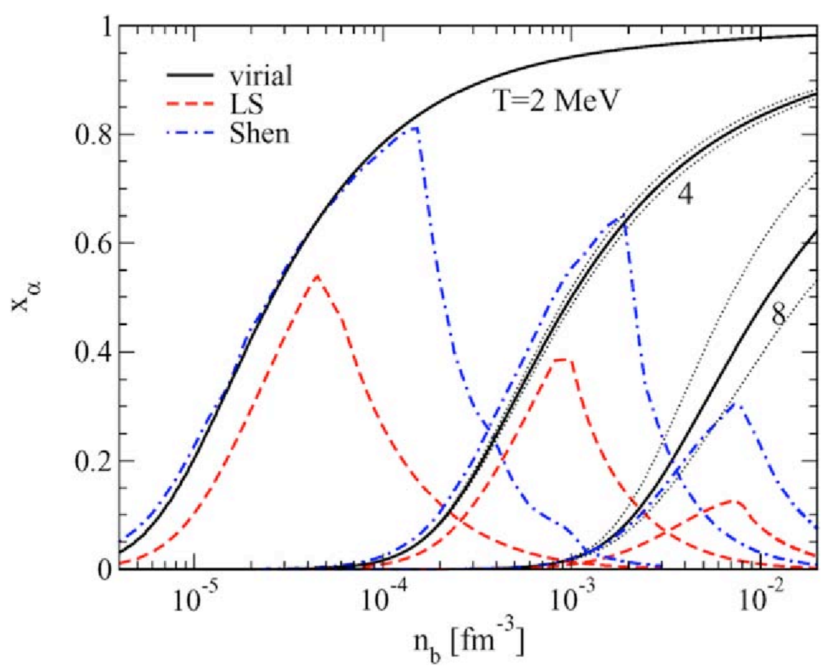
Heavy-ion collisions, spectra of emitted clusters,
temperature (3 - 10 MeV), free energy



S. Kowalski et al., PRC **75**, 014601 (2007)

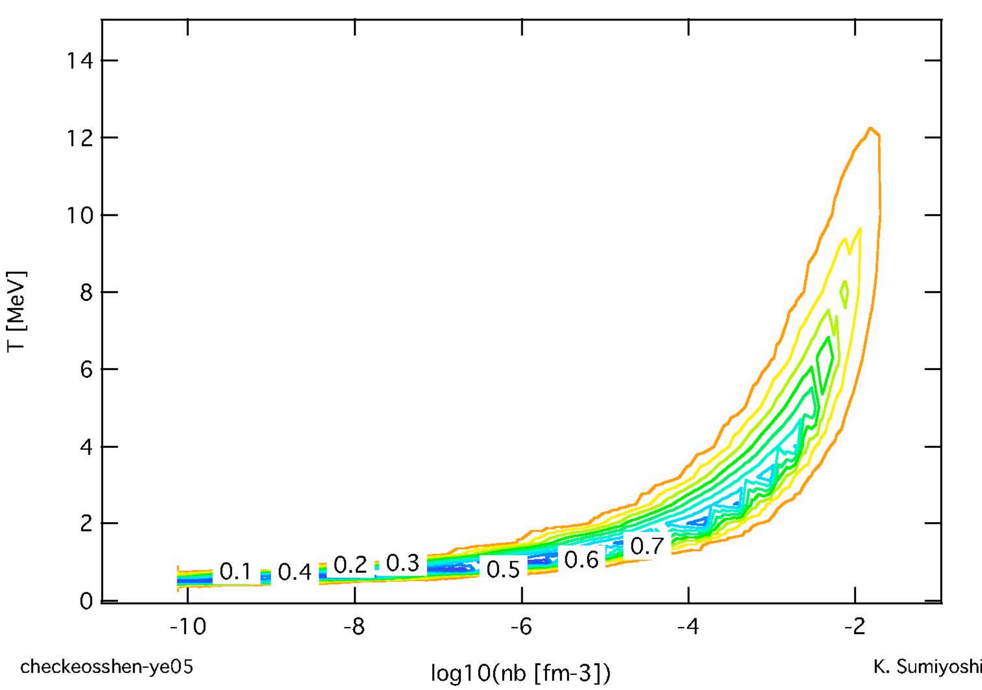
Alpha-particle fraction in the low-density limit

symmetric matter, T=2, 4, 8 MeV



C.J.Horowitz, A.Schwenk, Nucl. Phys. A **776**, 55 (2006)

alpha-fraction in symmetric matter



Problems:

- Warm Dilute Matter: Nuclear matter at subsaturation densities (T, n_p, n_n):

  Temperature $T \leq 16$ MeV = E_s/A, baryon density $n_B \leq 0.17$ fm^{-3} = n_s, asymmetry

- Formation of clusters (nuclei in matter):

  A = 1,2,3,4: free neutrons, free protons, deuterons (^2H), tritons (^3H), helions (^3He), alphas (^4He)

- Low-density, low-temperature limit:

  Virial expansion, non-interacting nuclides, quantum condensates

- Transition to higher densities:

  Medium effects, quasiparticles. Interpolation between Beth-Uhlenbeck and DBHF / RMF

- Cluster formation (correlations) *vs*. mean field:

  Consistent quantum-statistical approach

Outline

- Schrödinger equation with medium corrections:
  Self-energy and Pauli blocking
- Composition of the nuclear gas:
  Generalized Beth-Uhlenbeck equation
- Quantum condensates:
  Pairing and quartetting
- Composition and the EoS of nuclear matter
- Symmetry energy in the low-density region
- Condensates in alpha-matter

Nucleon-nucleon interaction

- general form:

$$V_\alpha(p,p') = \sum_{i,j=1}^{N} w_{\alpha i}(p)\lambda_{\alpha ij} w_{\alpha j}(p') \qquad \text{uncoupled}$$

and

$$V_\alpha^{LL'}(p,p') = \sum_{i,j=1}^{N} w_{\alpha i}^{L}(p)\lambda_{\alpha ij} w_{\alpha j}^{L'}(p') \qquad \text{coupled}$$

p, p'		in- and outgoing relative momentum
α	...	channel
N	...	rank
$\lambda_{\alpha ij}$	.	coupling parameter
L, L'		orbital angular momentum

Many-particle theory

- equilibrium correlation functions

  e.g. equation of state $n(\beta, \mu) = \frac{1}{\Omega_0} \sum_1 \langle a_1^\dagger a_1 \rangle$

  density matrix $\langle a_1^\dagger a_{1'} \rangle = \int \frac{\mathrm{d}\omega}{2\pi}\, \mathrm{e}^{-i\omega t} f_1(\omega) A(1, 1', \omega)$

- Spectral function

$$A(1, 1', \omega) = \mathrm{Im}\left[G(1, 1', \omega + i\eta) - G(1, 1', \omega - i\eta)\right]$$

- Matsubara Green function

$$G(1, 1', iz_\nu), \qquad z_\nu = \frac{\pi\nu}{\beta} + \mu, \quad \nu = \pm 1, \pm 3, \cdots$$

$$1 \equiv \{\mathbf{p}_1, \sigma_1, c_1\}, \quad f_1(\omega) = \frac{1}{\mathrm{e}^{\beta(\omega-\mu)} + 1}, \quad \Omega_0 - \text{volume}$$

Many-particle theory

- Dyson equation and self energy (homogeneous system)

$$G(1, iz_\nu) = \frac{1}{iz_\nu - E(1) - \Sigma(1, iz_\nu)}$$

- Evaluation of $\Sigma(1, iz_\nu)$:
  perturbation expansion, diagram representation

$$A(1, \omega) = \frac{2\mathrm{Im}\,\Sigma(1, \omega + i0)}{[\omega - E(1) - \mathrm{Re}\,\Sigma(1, \omega)]^2 + [\mathrm{Im}\,\Sigma(1, \omega + i0)]^2}$$

approximation for self energy $\longrightarrow$ approximation for equilibrium correlation functions

alternatively: simulations, path integral methods

Different approximations

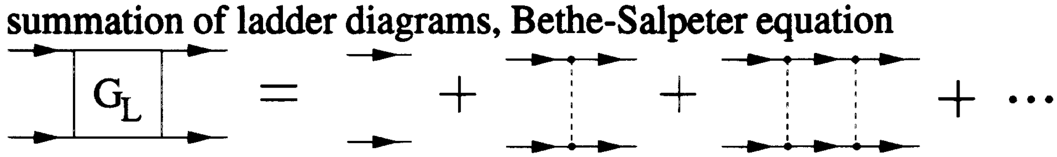
- **Expansion for small Im** $\Sigma(1,\omega+i\eta)$

$$A(1,\omega) \;\approx\; \frac{2\pi\delta(\omega - E^{\text{quasi}}(1))}{1-\frac{\mathrm{d}}{\mathrm{d}z}\text{Re }\Sigma(1,z)|_{z=E^{\text{quasi}}-\mu_1}} \\ -2\text{Im }\Sigma(1,\omega+i\eta)\frac{\mathrm{d}}{\mathrm{d}\omega}\frac{P}{\omega+\mu_1-E^{\text{quasi}}(1)}$$

quasiparticle energy $E^{\text{quasi}}(1) = E(1) + \text{Re }\Sigma(1,\omega)|_{\omega=E^{\text{quasi}}}$

- **chemical picture: bound states** $\hat{=}$ **new species**

summation of ladder diagrams, Bethe-Salpeter equation



Medium effects: Quasiparticle approximation

- Skyrme

- relativistic mean field (RMF)

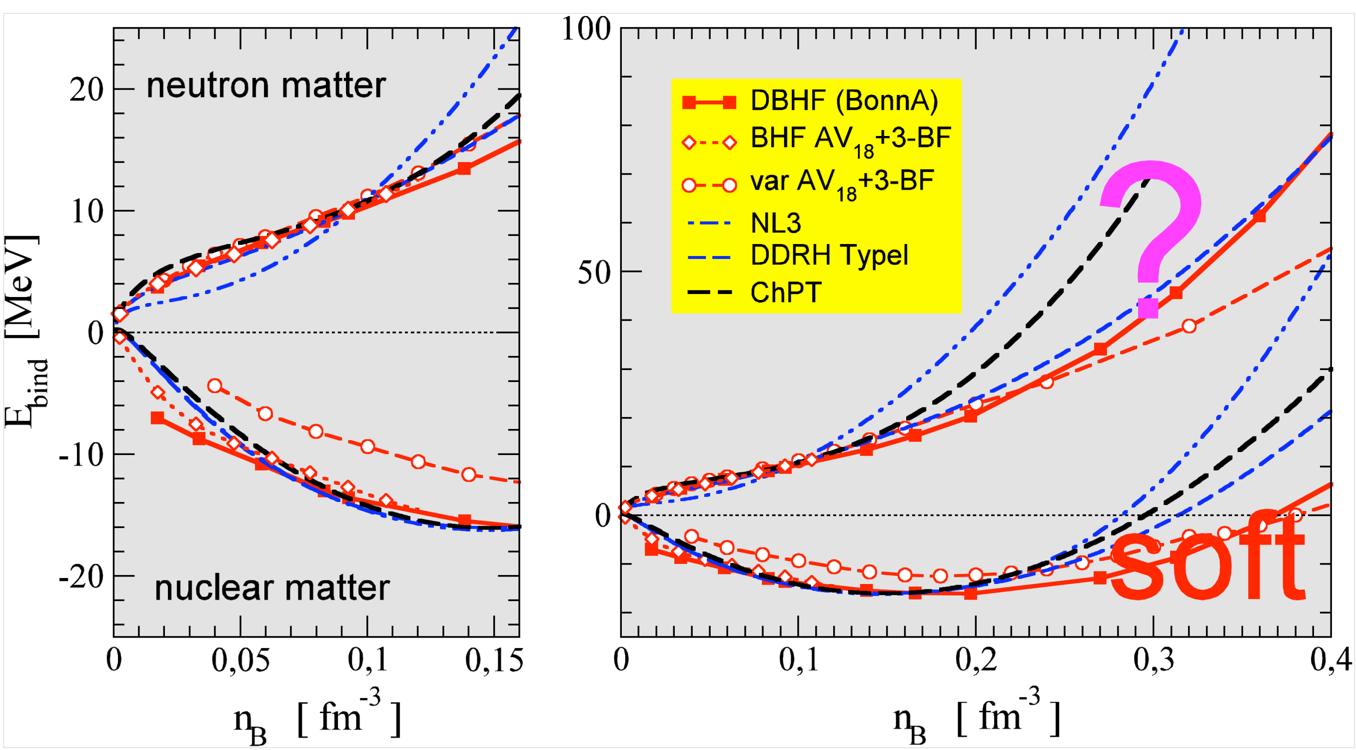
  Lagrangian: non-linear sigma, TM1 parameters,
  single particle modifications, energy shift, effective mass

- DD-RMF [S.Typel, Phys. Rev. C **71**, 064301 (2007)]:

  expansion of the scalar field and the vector fields
  in powers of proton/neutron densities

- Dirac-Brueckner Hartree Fock (DBHF)

Quasiparticle picture: RMF and DBHF

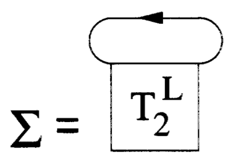


J.Margueron et al., PRC **76**, 034309 (2007)

Different approximations

low density limit:

$$G_2^L(12,1'2',i\lambda) = \sum_{n\mathbf{P}} \Psi_{n\mathbf{P}}(12)\frac{1}{i\omega_\lambda - E_{n\mathbf{P}}}\Psi^*_{n\mathbf{P}}(12)$$

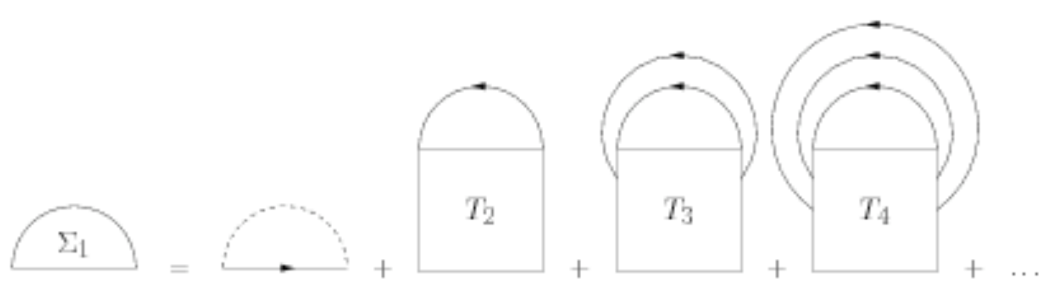


$$n(\beta,\mu) = \sum_1 f_1(E^{\text{quasi}}(1)) + \sum_{2,n\mathbf{P}}^{\text{bound}} g_{12}(E_{n\mathbf{P}})$$

$$+\sum_{2,n\mathbf{P}}\int_0^\infty \mathrm{d}k\; \delta_{\mathbf{k},\mathbf{p}_1-\mathbf{p}_2} g_{12}(E^{\text{quasi}}(1)+E^{\text{quasi}}(2))2\sin^2\delta_n(k)\frac{1}{\pi}\frac{\mathrm{d}}{\mathrm{d}k}\delta_n(k)$$

- generalized Beth-Uhlenbeck formula
  correct low density/low temperature limit:
  mixture of free particles and bound clusters

Cluster decomposition of the self-energy



Ideal mixture of reacting nuclides

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$
$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

nuclear statistical equilibrium (statistical multifragmentation)

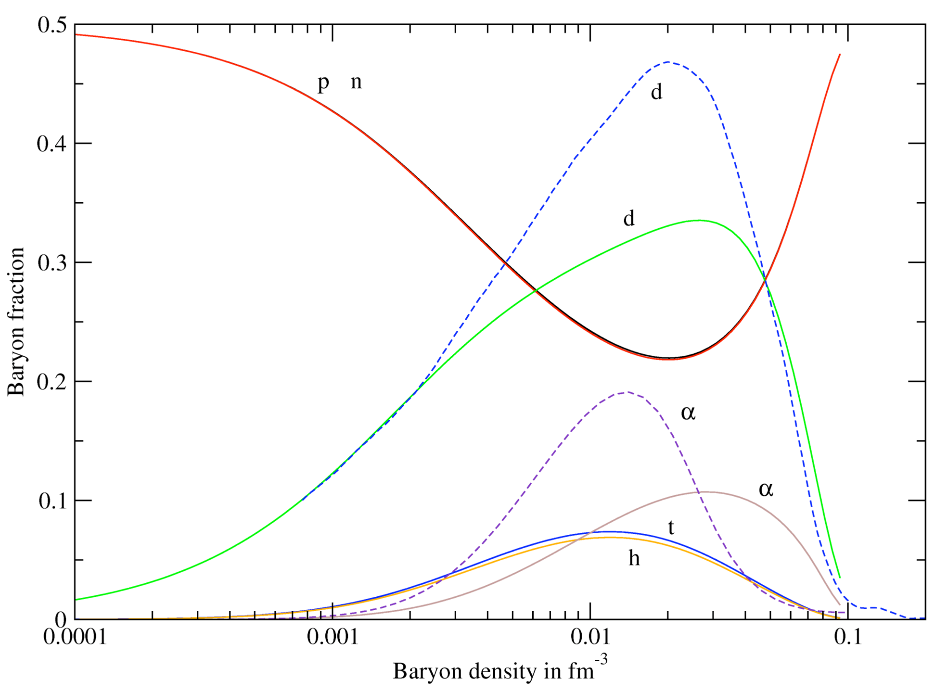
$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

mass number A,
charge Z_A,
energy $E_{A,\nu,K}$,
ν: internal quantum number,
K: center of mass momentum

Composition of symmetric nuclear matter

T=10 MeV

G.Ropke, A.Grigo, K. Sumiyoshi, Hong Shen,
Phys.Part.Nucl.Lett. **2**, 275 (2005)



Effective wave equation for the deuteron in matter

$$\left(\frac{p_1^2}{2m_1}+\frac{p_2^2}{2m_2}\right)\Psi_{n,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{n,P}(p_1',p_2')$$

$$=E_{n,P}\Psi_{n,P}(p_1,p_2)$$

Add self-energy

Pauli-blocking

Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

BEC-BCS crossover:
Alm et al.,1993

Deuteron quasiparticle properties

$$E_d^{\mathrm{qu}}(P) = E_d^{\mathrm{free}} + \Delta E_d + \frac{\hbar^2}{2m_d^*}P^2 + O(P^4)$$

$$\Delta E_d^{\mathrm{Pauli}}(T,n_B,\alpha) = \delta E_d^{(0)}(T,\alpha)n_B + O(n_B^2)$$

$$\frac{m_d^*}{m_d}(T,n_B,\alpha) = 1 + \delta m_d^{(0)}(T,\alpha)n_B + O(n_B^2)$$

T [MeV]	delta E [MeV fm^3]	delta m* [fm^3]
10	364.3	21.3
4	712.9	87.1

$$E_d^{\mathrm{free}} = -2.225\,\mathrm{MeV}$$

G.R.,PRC **79**, 014002 (2009)

Few-particle Schrödinger equation in a dense medium

Four-particle Schrödinger equation with medium effects

$$[E^{mf}(p_1) + E^{mf}(p_2) + E^{mf}(p_3) + E^{mf}(p_4)]\psi_{nP}(p_1,p_2,p_3,p_4)$$

$$+ \sum_{p_1'p_2'p_3'p_4'} \{[1 - f(p_1) - f(p_2)]V(p_1p_2,p_1'p_2')\delta_{p_3p_3'}\delta_{p_4p_4'}$$

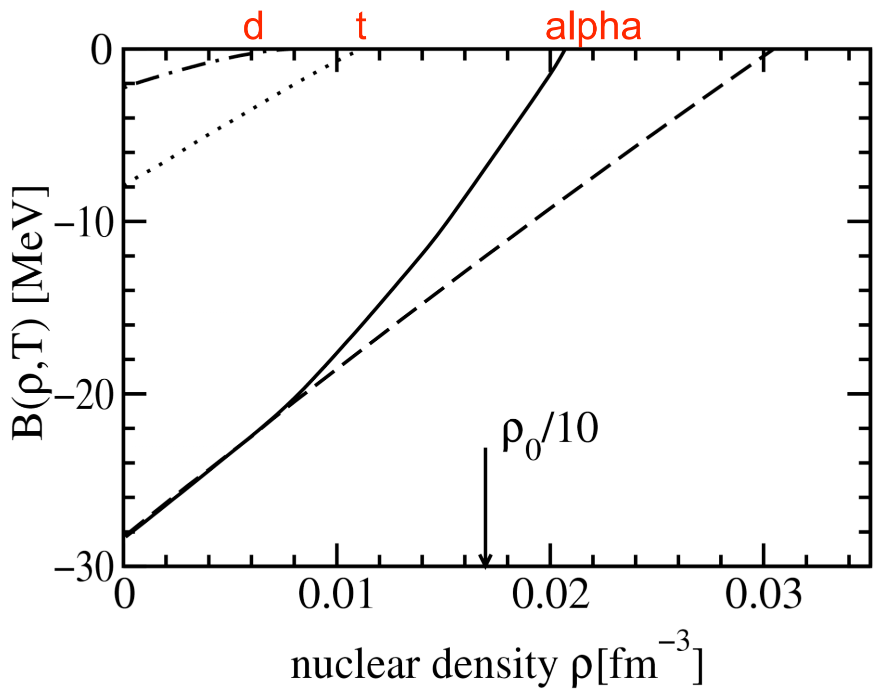
$$+ \text{permutations}\}\psi_{nP}(p_1',p_2',p_3',p_4')$$

$$= E_{nP}^{mf}\psi_{nP}(p_1,p_2,p_3,p_4)$$

In-medium shift of binding energies of clusters

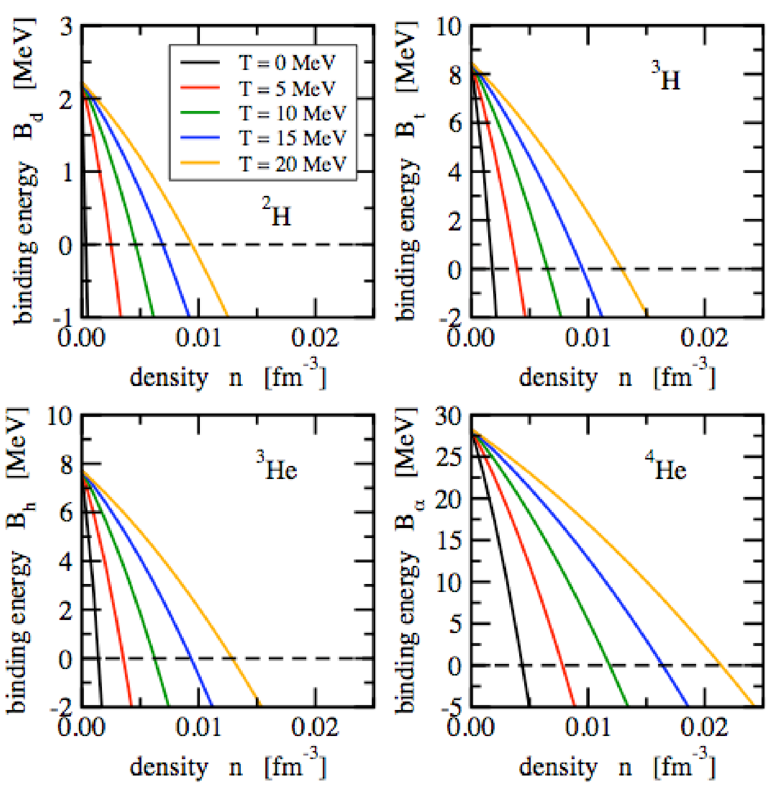
Solution of the Faddeev-Yakubovski equation with Pauli blocking

T=10 MeV



M. Beyer et al., PLB **488**, 247 (00), A. Sedrakian et al., Ann. Phys; PRC **73**, 035803 (06)

Shift of Binding Energies of Light Clusters



Symmetric matter

G.R., PRC **79**, 014002 (2009)
S. Typel et al., in preparation

Composition of dense nuclear matter

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

mass number A,
charge Z_A,
energy $E_{A,\nu,K}$,
ν: internal quantum number,

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

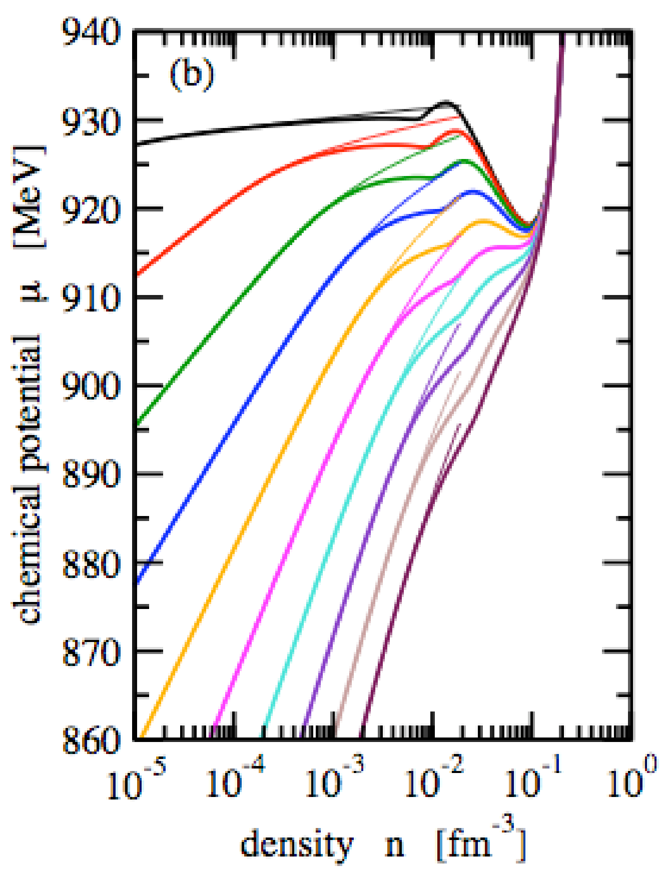
- Inclusion of excited states and continuum correlations

- Medium effects:
  self-energy and Pauli blocking shifts of binding energies,
  Coulomb corrections due to screening (Wigner-Seitz,Debye)

Chemical potential of symmetric matter

Isotherms

T[MeV]
2
4
6
8
10
12
14
16
18
20



thin lines: NSE

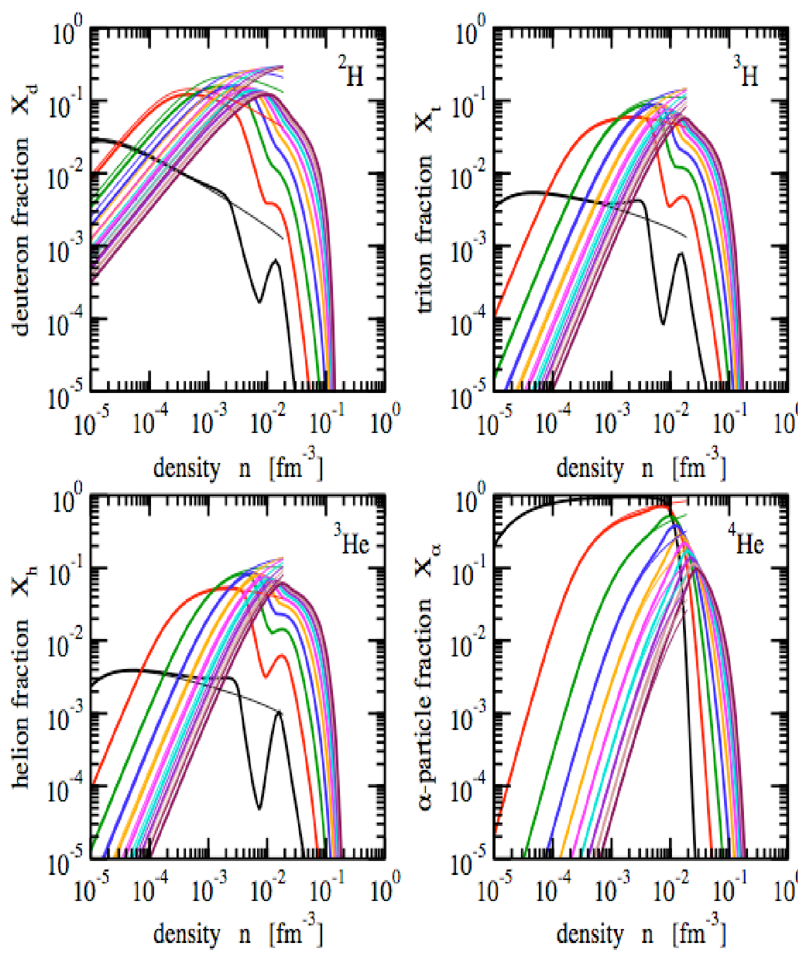
Mass fractions of light clusters

Isotherms

T[MeV]

2
4
6
8
10
12
14
16
18
20

thin lines: NSE

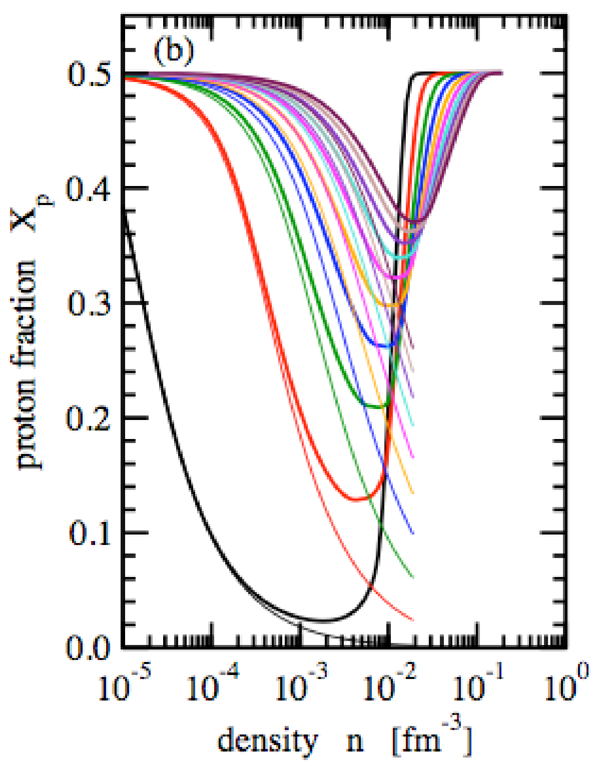


Proton fraction in symmetric matter

Isotherms

T[MeV]
20
18
16
14
12
10
8
6
4
2

thin lines: NSE



Free energy pro nucleon

Isotherms

T[MeV]

2

4

6

8

10

12

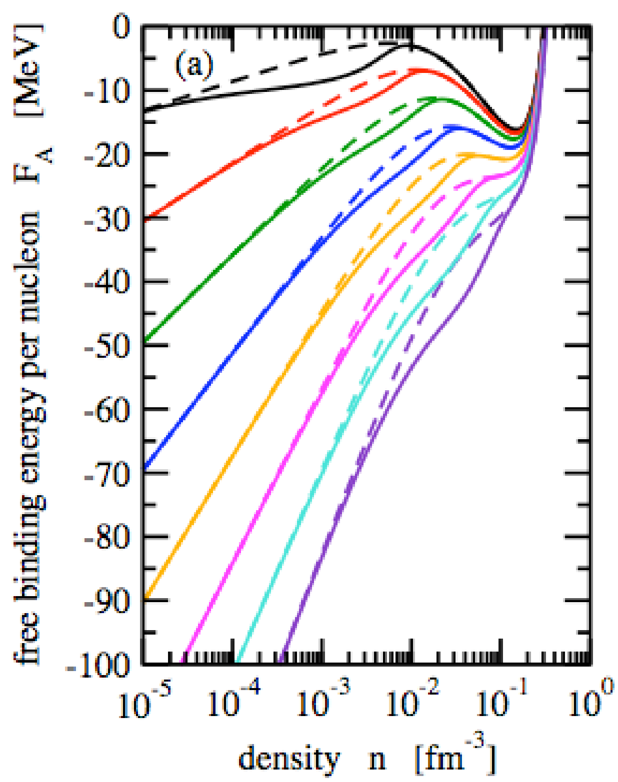
14

16

18

20

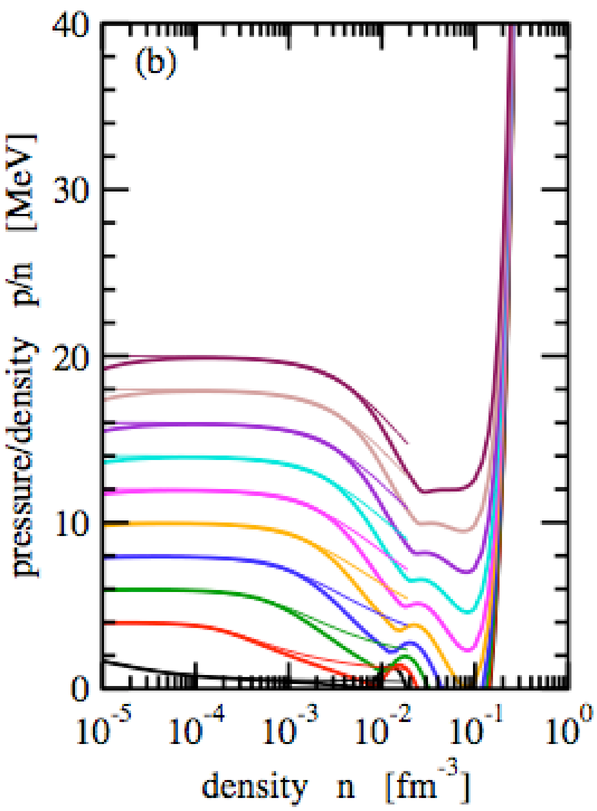
dashed: RMF



Pressure to density ratio

Isotherms

T[MeV]
20
18
16
14
12
10
8
6
4
2

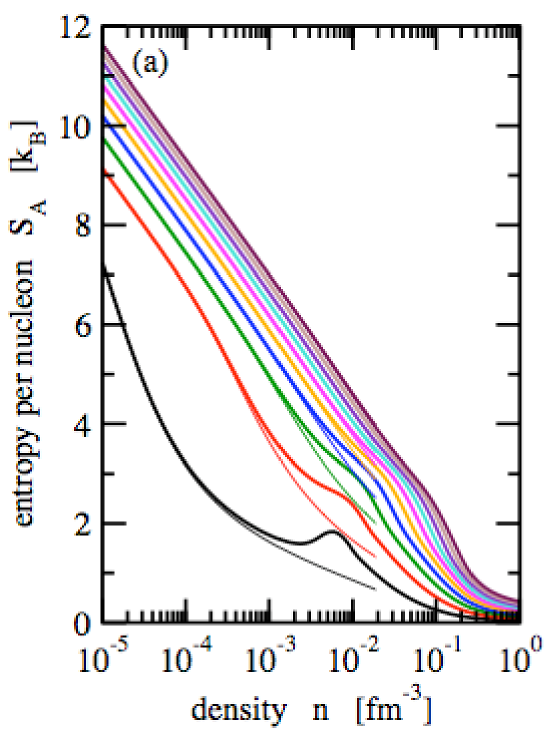


thin lines: NSE

Entropy pro nucleon

Isotherms

T[MeV]
20
18
16
14
12
10
8
6
4
2



thin lines: NSE

Internal energy per nucleon

Isotherms

T[MeV]

20

18

16

14

12

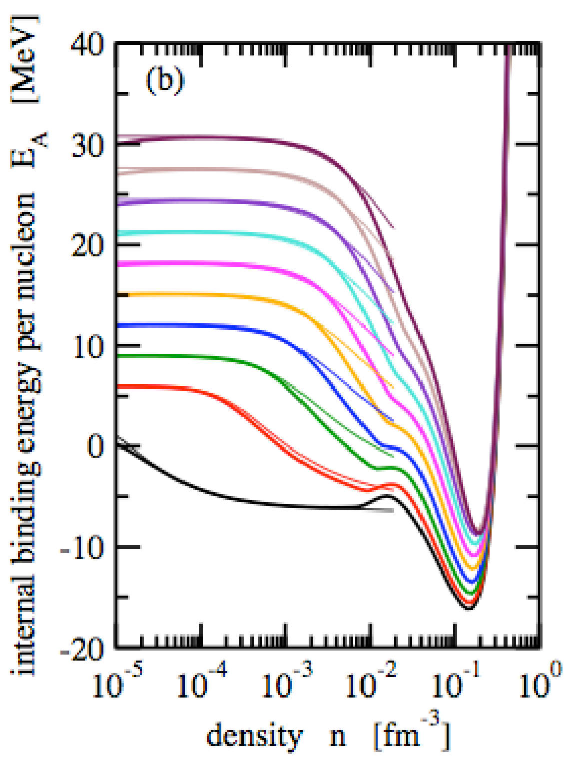
10

8

6

4

2



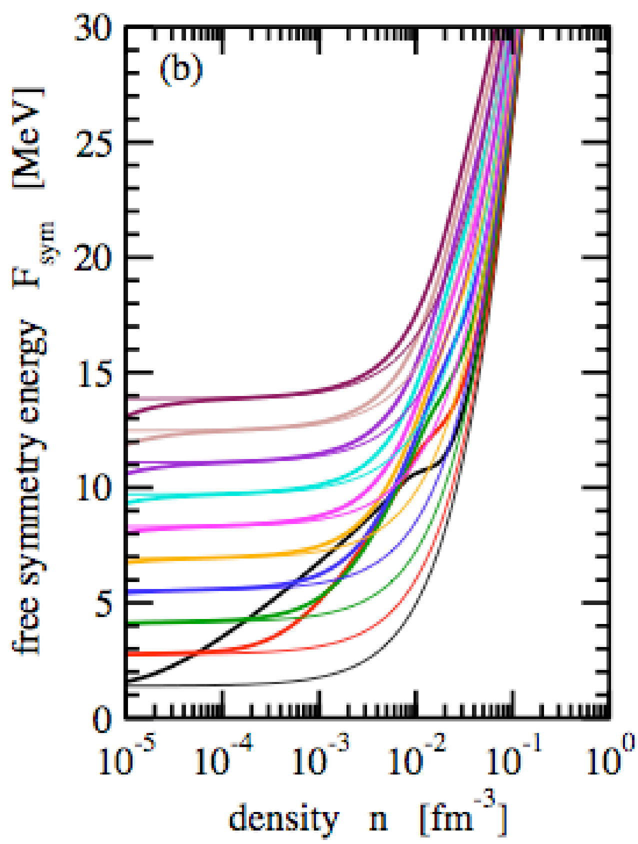
thin lines: NSE

Free symmetry energy per nucleon

Isotherms

T[MeV]
20
18
16
14
12
10
8
6
4
2

thin lines: RMF

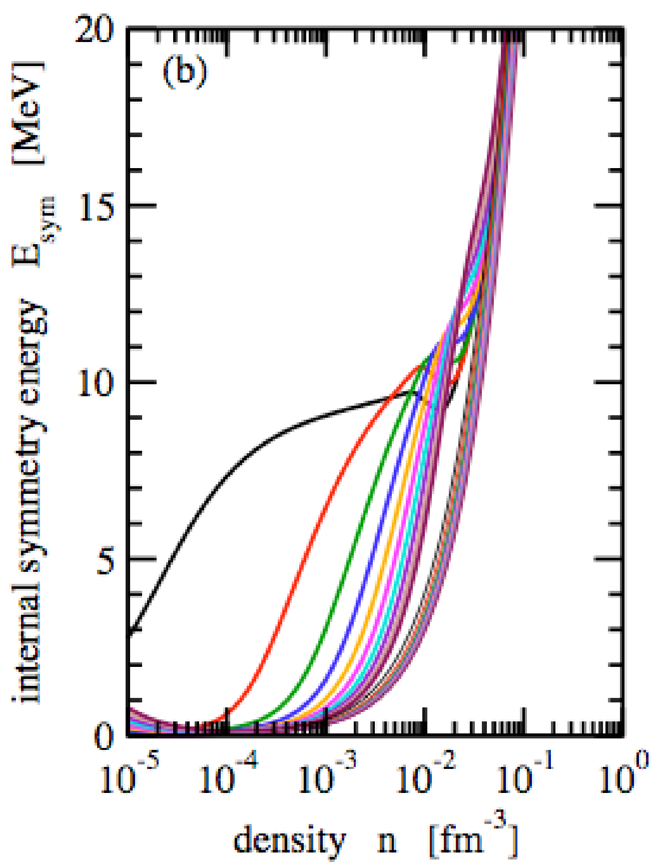


Internal symmetry energy pro nucleon

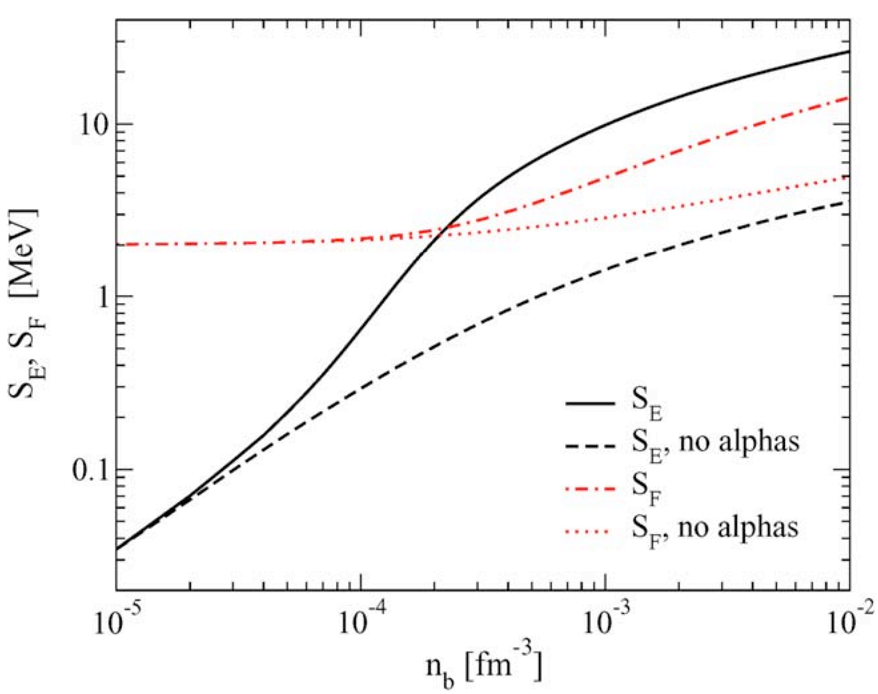
Isotherms

T[MeV]
2
4
6
8
10
12
14
16
18
20

dashed: RMF



Symmetry energy and symmetry free energy

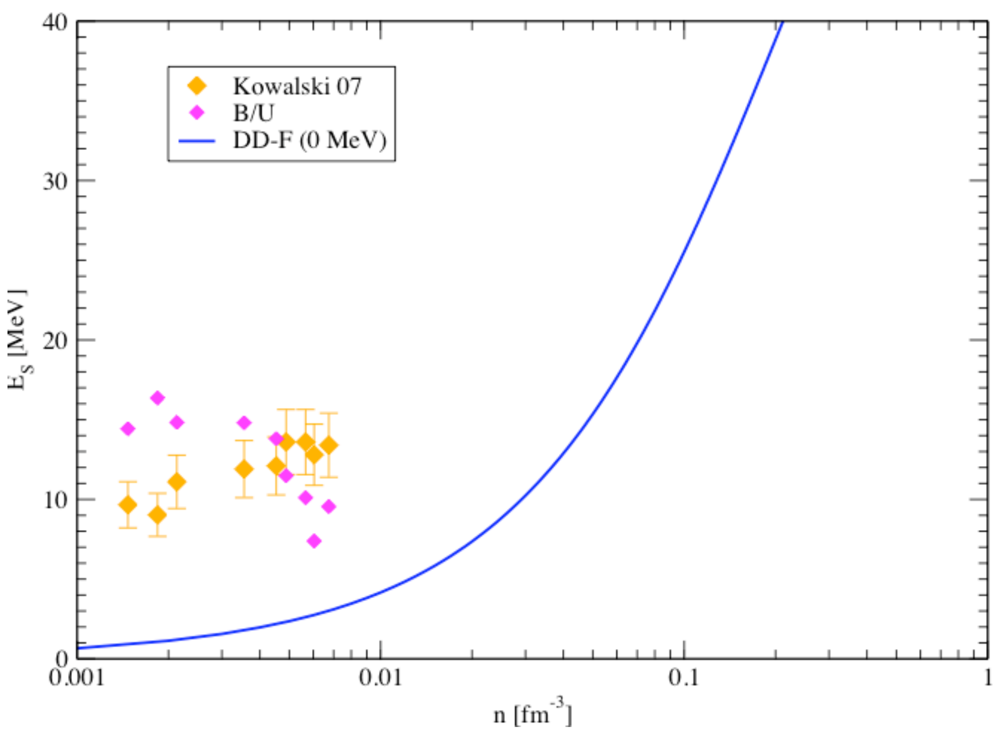


T=4MeV

Horowitz & Schwenk,
NPA (2006)

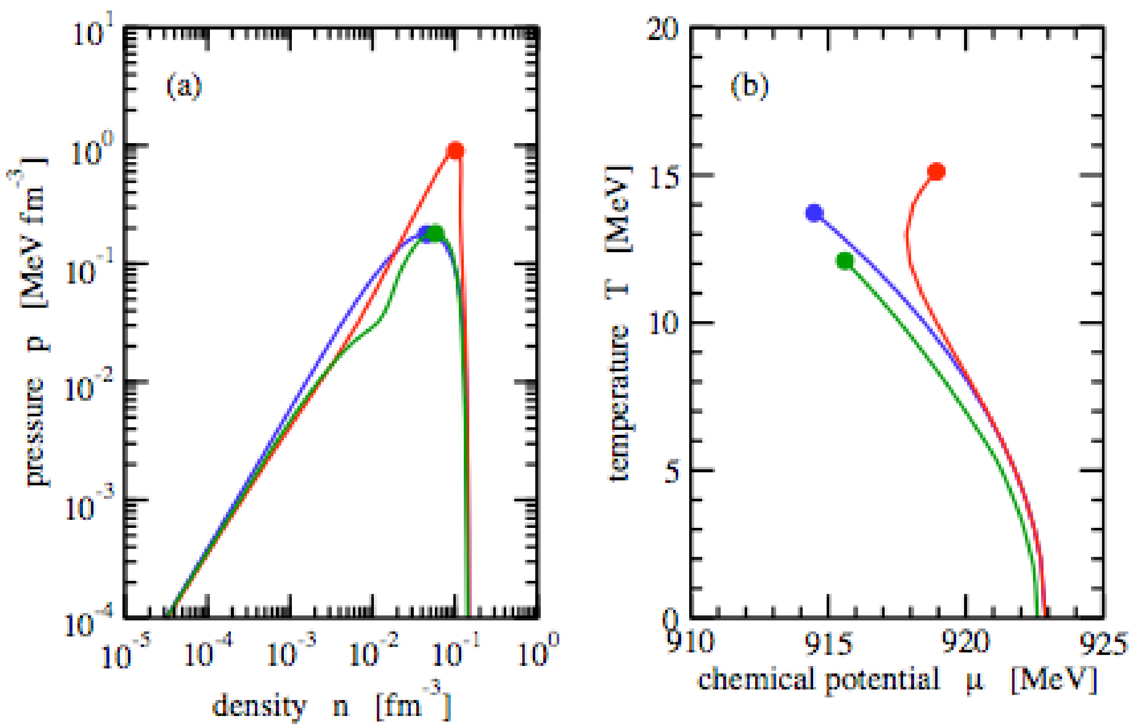
Symmetry energy

Heavy-ion collisions, spectra of emitted clusters,
temperature (3 - 10 MeV), free energy



S. Kowalski et al., PRC **75**, 014601 (2007)

Liquid-vapor phase transition



blue: no light cluster, green: with light clusters

Cluster yields in Heavy Ion Collisions

- Thermodynamic parameters (temperature and nucleon density) at freeze out from cluster yields

- Saha equation in plasma physics and Albergo thermometer (mass action law)

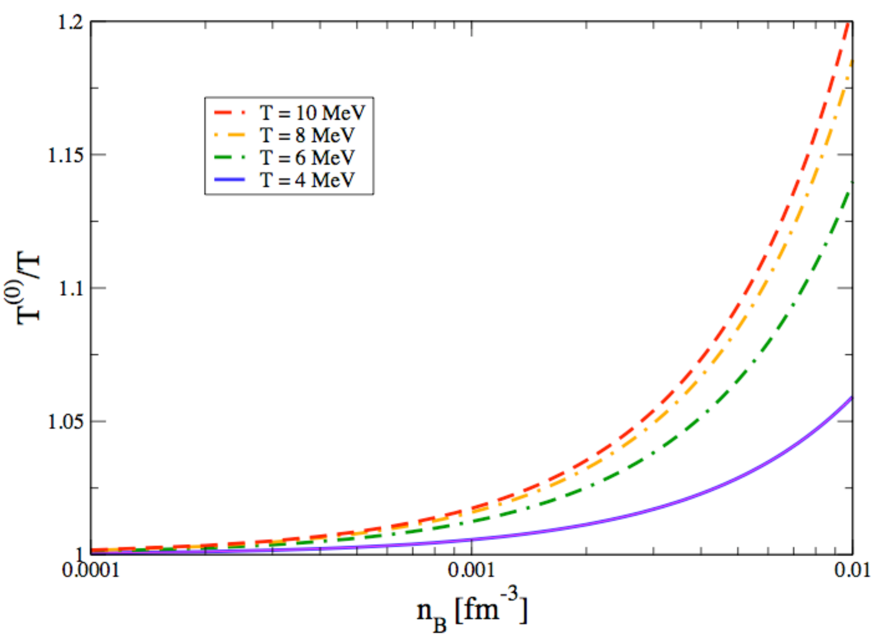
- Albergo temperature: double ratios of yields to eliminate the chemical potentials (e.g. 2H, 4He vs. 3H, 3He)

- Is nuclear statistical equilibrium (NSE) or statistical multifragmentation justified?

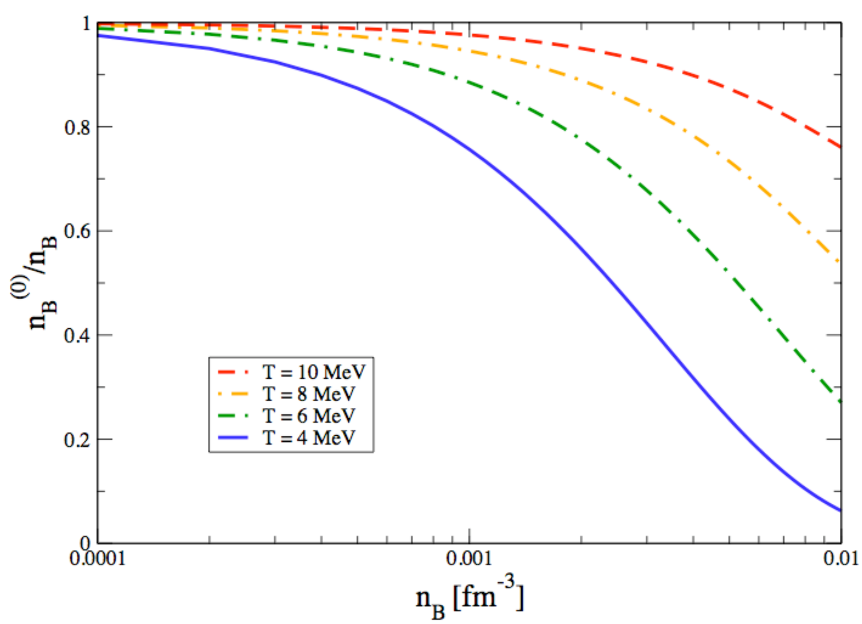
- Account of medium effects changes the inferred values for temperature and density.

S. Shlomo, G. R., J.B. Natowitz,
PRC **79**, 034604 (2009)

Albergo Temperature Misfit

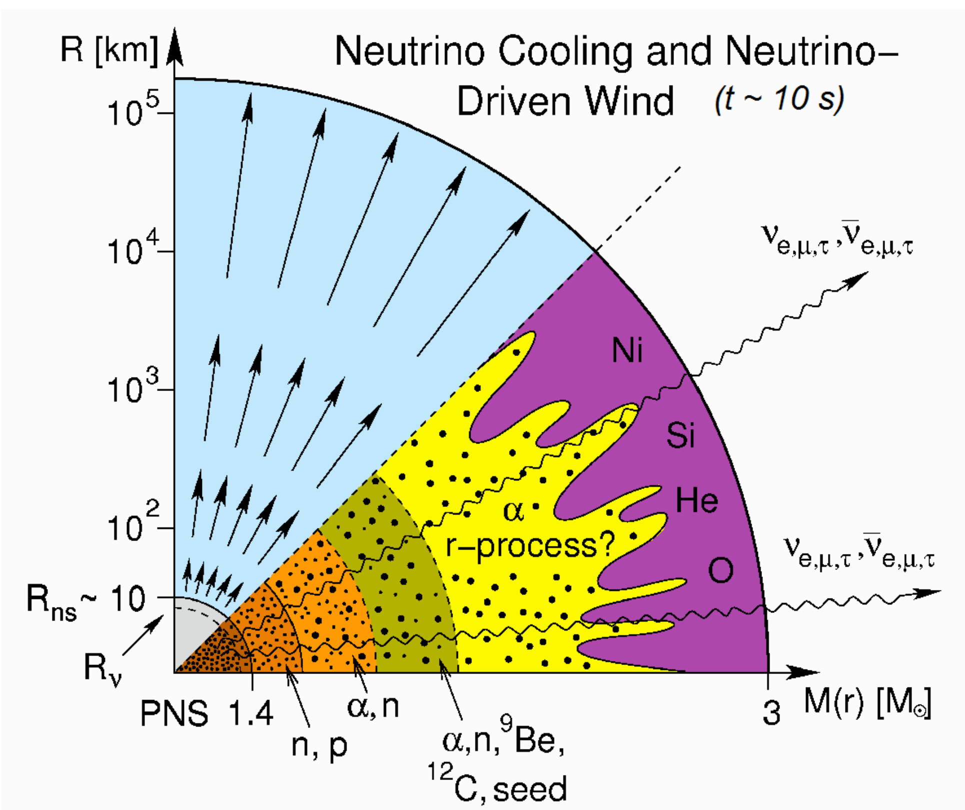


Albergo Density Misfit



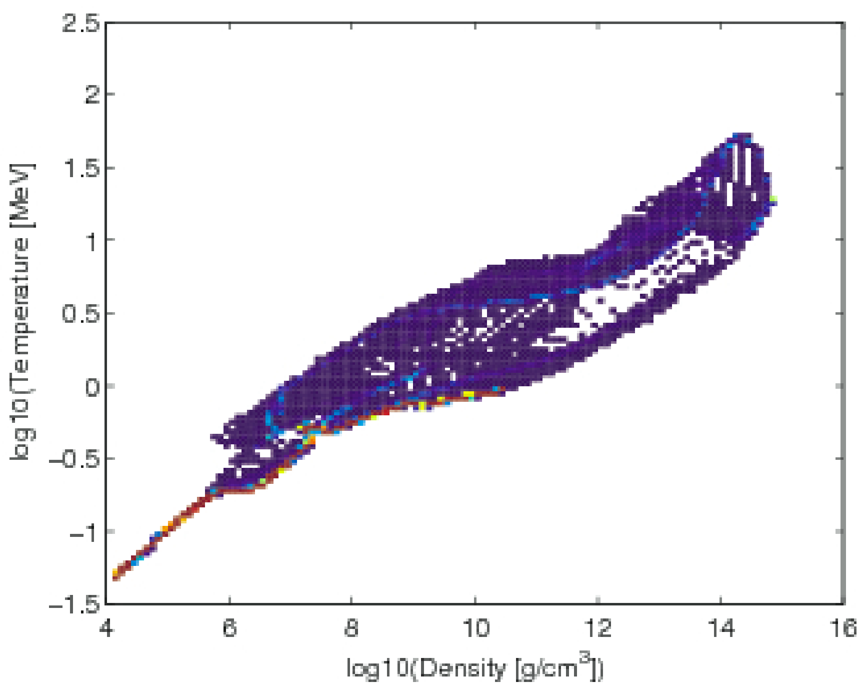
S. Shlomo, G. R., J.B. Natowitz,
PRC **79**, 034604 (2009)

Supernova explosion



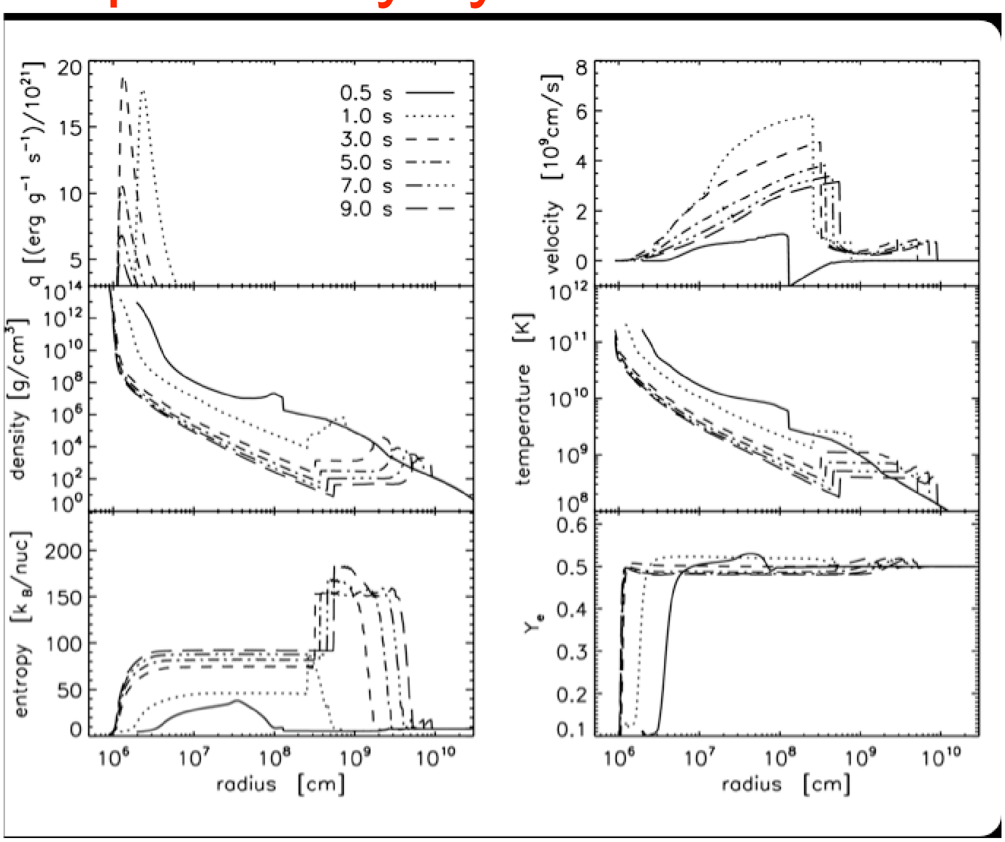
T.Janka

Parameter range: Explosion



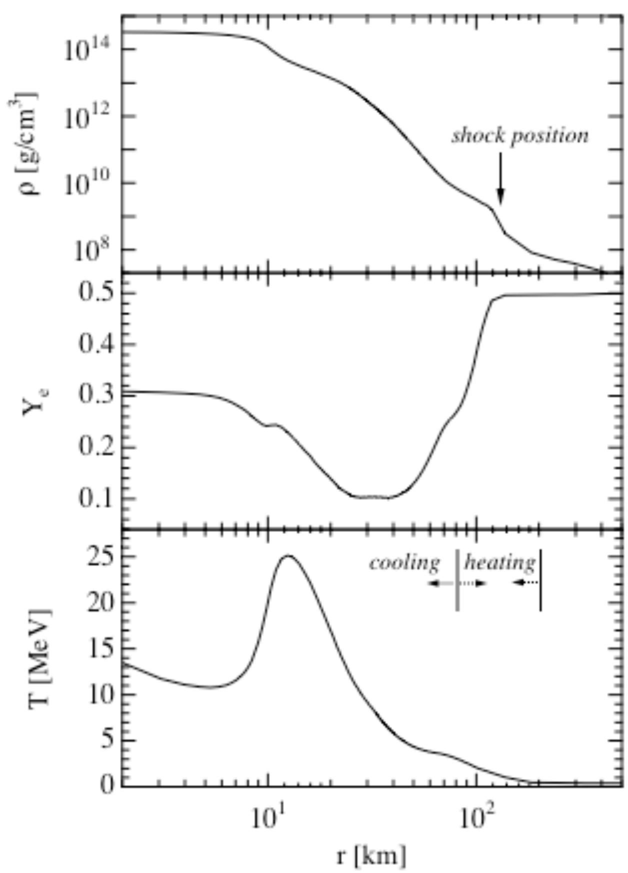
T. Fischer, On the possible fate of massive progenitor stars, Talk 25.2.08 Ladek

Supernova collapse: spherically symmetric simulations



A. Arcones et al.
Neutrino driven winds,
Talk 25. 2. 08 Ladek;
PRC **78**, 015806 (08)

Core-collapse supernovae



Density.

electron fraction, and

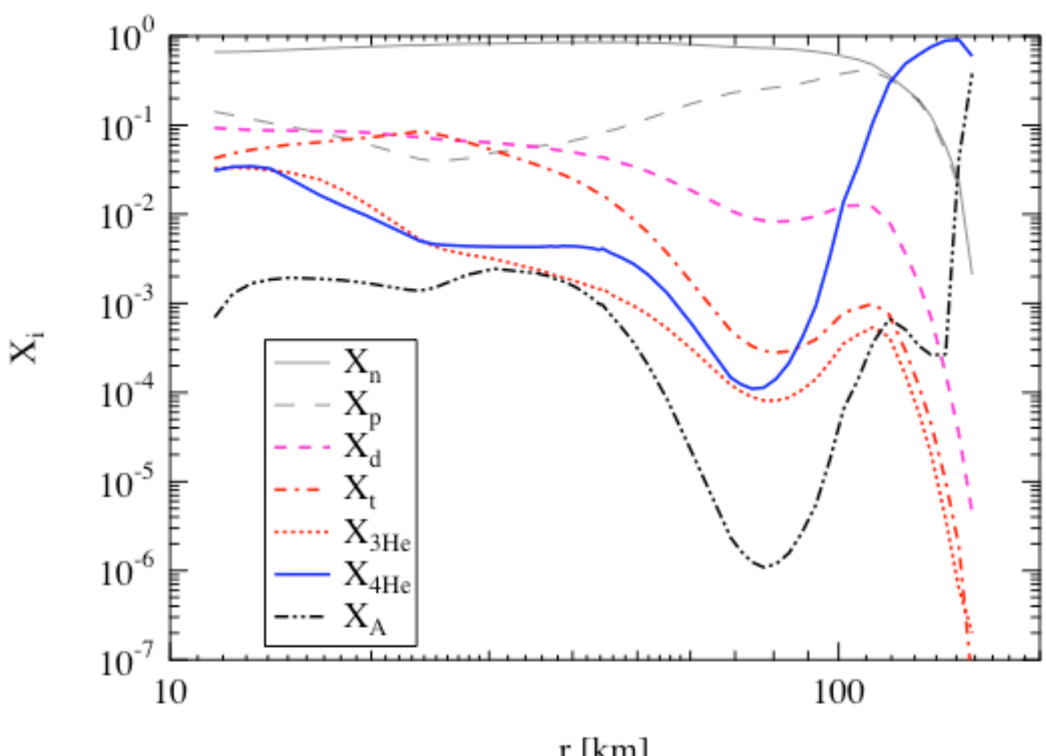
temperature profile

of a 15 solar mass supernova
at 150 ms after core bounce
as function of the radius.

Influence of cluster formation
on neutrino emission
in the cooling region and
on neutrino absorption
in the heating region ?

K.Sumiyoshi et al.,
Astrophys.J. **629**, 922 (2005)

Composition of supernova core

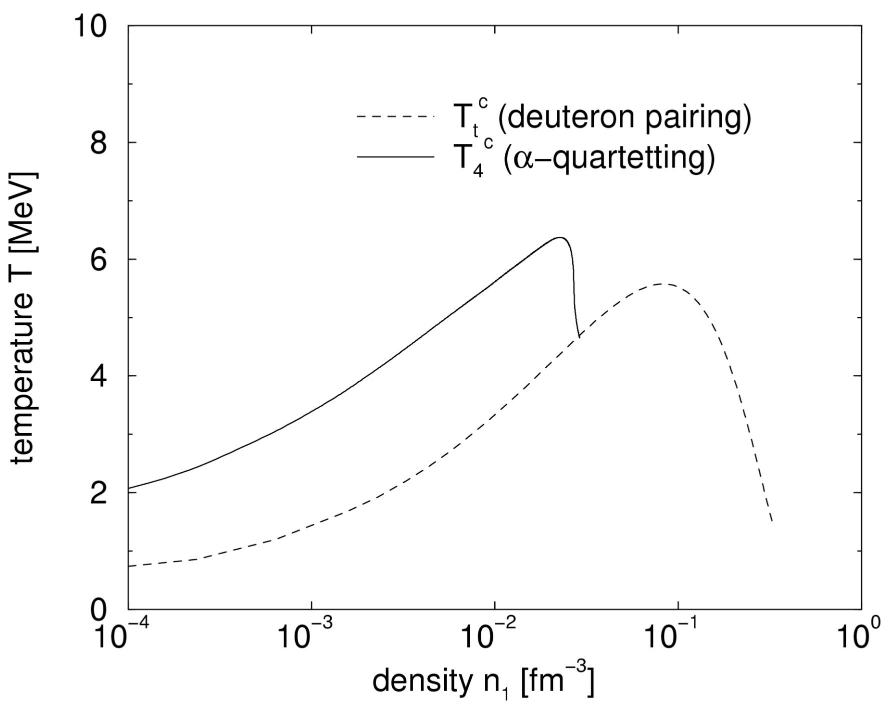


Mass fraction X of light clusters for a post-bounce supernova core

K. Sumiyoshi, G. R., PRC **77**, 055804 (08)

α-cluster-condensation

(quartetting)



G. Röpke, A. Schnell, P. Schuck, and P. Nozieres, PRL **80**, 3177 (98)

Self-conjugate 4n nuclei

$n\alpha$ nuclei: ^{8}Be, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ...

Single-particle shell model, or

Cluster type structures

ground state, excited states

$n\alpha$ break up at the threshold energy $E_{n\alpha}^{\mathrm{thr}} = nE_{\alpha}$

Hoyle state:
dilute gas like state of alpha clusters in 12C near threshold.
Wave function similar to Bose condensate
(identical orbital for the center of mass motion of alpha particles)

Variational ansatz

$$|\Phi_{n\alpha}\rangle = \left(C_\alpha^\dagger\right)^n |\mathrm{vac}\rangle$$

α- particle creation operator

$$C_\alpha^\dagger = \int d^3R e^{-\vec{R}^2/R_0^2}$$

$$\times \int d^3r_1 \ldots d^3r_4 \phi_{0s}(\vec{r}_1 - \vec{R}) a^\dagger_{\sigma_1\tau_1}(\vec{r}_1) \ldots \phi_{0s}(\vec{r}_4 - \vec{R}) a^\dagger_{\sigma_4\tau_4}(\vec{r}_4)$$

with

$$\phi_{0s}(\vec{r}) = \frac{1}{(\pi b^2)^{3/4}} e^{-\vec{r}^2/(2b^2)}$$

A. Tohsaki, H. Horiuchi, P. Schuck, G. Röpke, PRL **87**, 192501 (2001)

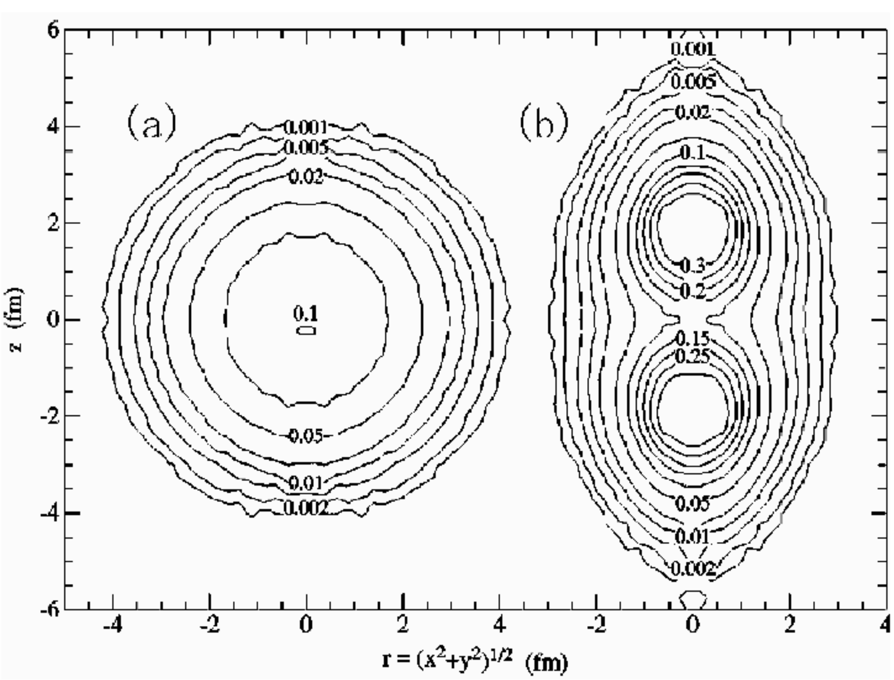
Results

		E_k (MeV)	E_{exp} (MeV)	$E_k - E_{n\alpha}^{\text{thr}}$ (MeV)	$(E - E_{n\alpha}^{\text{thr}})_{\text{exp}}$ (MeV)	$\sqrt{\langle r^2 \rangle}$ (fm)	$\sqrt{\langle r^2 \rangle}_{\text{exp}}$ (fm)
^{12}C	$k=1$	-85.9	-92.16 (0_1^+)	-3.4	-7.27	2.97	2.65
	$k=2$	-82.0	-84.51 (0_2^+)	+0.5	0.38	4.29	
	$E_{3\alpha}^{\text{thr}}$	-82.5	-84.89				
^{16}O	$k=1$	-124.8 (-128.0)*	-127.62 (0_1^+)	-14.8 (-18.0)*	-14.44	2.59	2.73
	$k=2$	-116.0	-116.36 (0_3^+)	-6.0	-3.18	3.16	
	$k=3$	-110.7	-113.62 (0_5^+)	-0.7	-0.44	3.97	
	$E_{4\alpha}^{\text{thr}}$	-110.0	-113.18				
^{8}Be				- 0.17	+ 0.1		

Tabelle 1: Comparison of the generator coordinate method calculations with experimental values. $E_{n\alpha}^{\text{thr}} = nE_\alpha$ denotes the threshold energy for the decay into α-clusters, the values marked by * correspond to a refined mesh.

M. Chernykh et al., PRL **98**, 032501 (07); Y. Funaki et al., PRL **101**, 082502 (08)

Alpha cluster structure of Be 8



R.B. Wiringa et al., PRC **63**, 034605 (01)

Contours of constant density, plotted in cylindrical coordinates, for 8Be(0+) .
The left side is in the laboratory frame while the right side is in the intrinsic frame.

Estimation of condensate fraction in zero temperature α-matter

$$n_0 = \frac{\langle \Psi | a_0^\dagger a_0 | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$

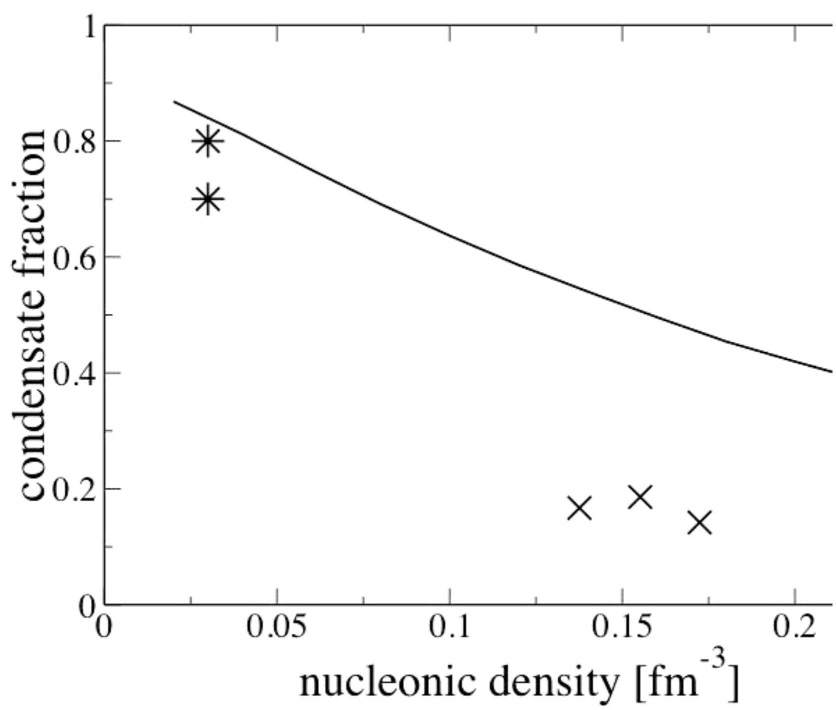
destruction of the BEC of the ideal Bose gas:
thermal excitation, but also correlations

"excluded" volume for α-particles ≈ 20 fm^3
at nucleon density $\rho = 0.048$ fm^{-3} filling factor ≈ 28 %
(liquid ^{4}He: 8 % condensate),
destruction of the condensate at $\approx \rho_0/3$

Suppresion of condensate fraction

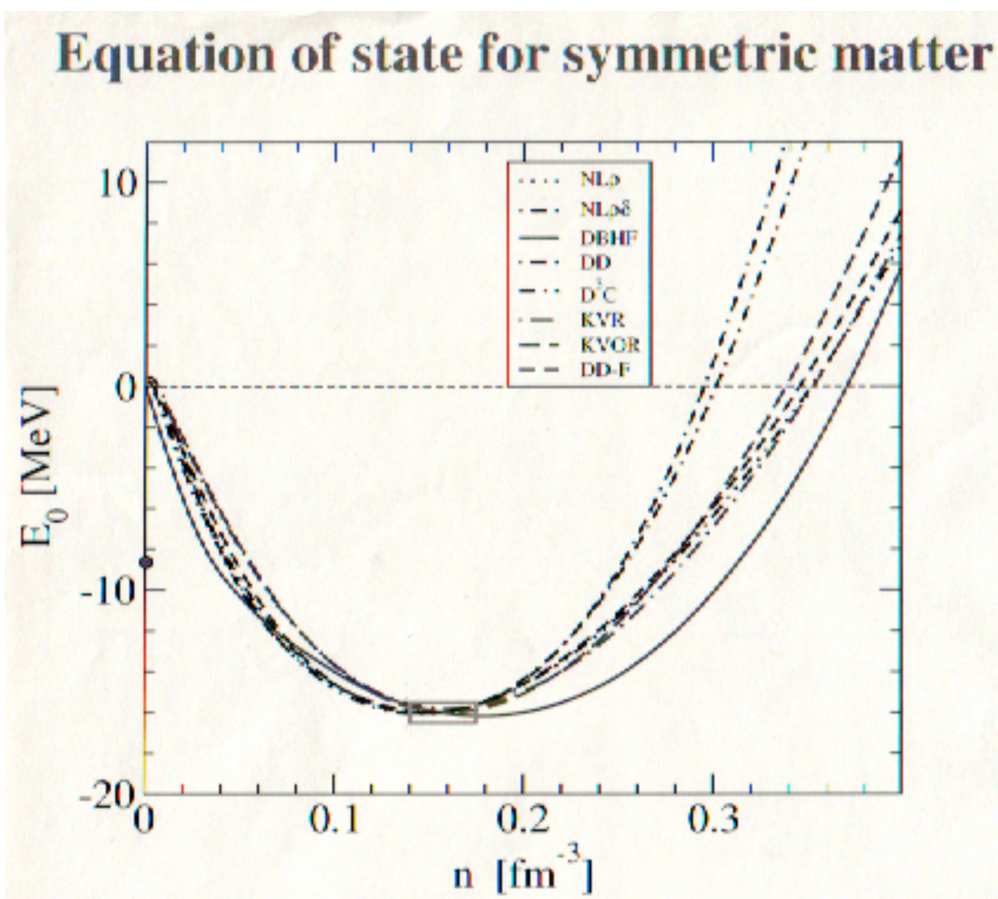
- Alpha-alpha interaction (Ali/Bodmer),
  no Pauli blocking:
- Variational calculation (Clark/Jastrow approach to the alpha-particle condensate amplitude) (crosses)
- First order approximation (full line)

- Yamada/Schuck's result for condensate in C12 - O2+ (stars)

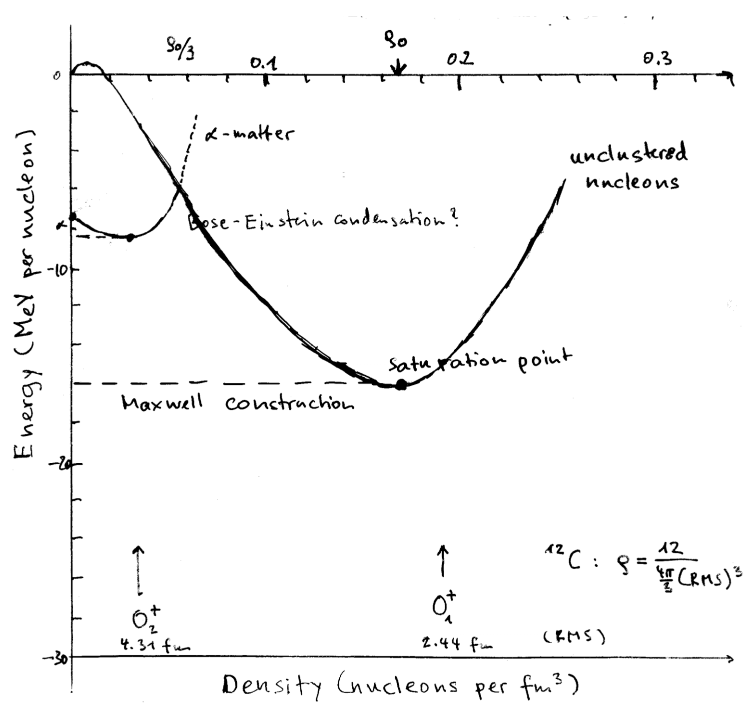


Y. Funaki et al., PRC **77**, 064312 (08)

Quasiparticle approximation for nuclear matter



Low-density limit: alpha matter?



Internal energy per nucleon

Isotherms

T[MeV]

20

18

16

14

12

10

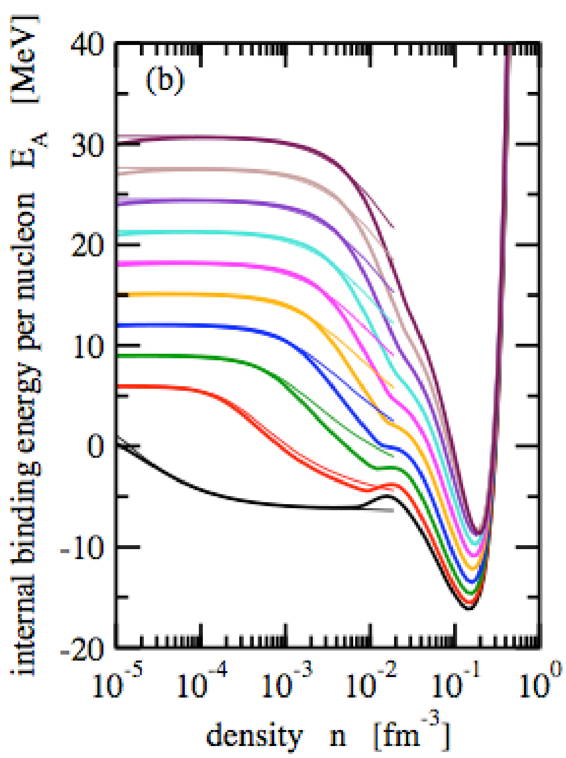
8

6

4

2

thin lines: NSE



Exploring Hot Dense Matter

- EMMI: No nuclear matter in laboratory experiments
- Nuclei: inhomogeneous, finite systems. Restricted region of asymmetry, no neutron matter.
  Large variety: Exotic nuclei, halo nuclei,…
  Local density approximation. Signatures of correlations in nuclear matter are also found in finite nuclei, e.g. pairing.
- Heavy ion collisions: excited finite systems, expanding.
  Fireball, non-equilibrium, transport equations.
  Local thermal equilibrium, freeze out, statistical multifragmentation.
- Probing hot and dense matter in laboratory experiments, i.e. finite systems in non-equilibrium, gives signatures of correlations that are also present in nuclear matter.

Summary

- The low-density limit of the nuclear matter EoS can be rigorously treated. The Beth-Uhlenbeck virial expansion is a benchmark.

- An extended quasiparticle approach can be given for single nucleon states and nuclei. In a first approximation, self- energy and Pauli blocking is included. An interpolation between low and high densities is possible.

- Compared with the standard quasiparticle approach, significant changes arise in the low-density limit due to clustering. Examples are Bose-Einstein condensation (quartetting), and the behavior of the symmetry energy.

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