QCD Plasma Instabilities and Nonthermal Fixed Points

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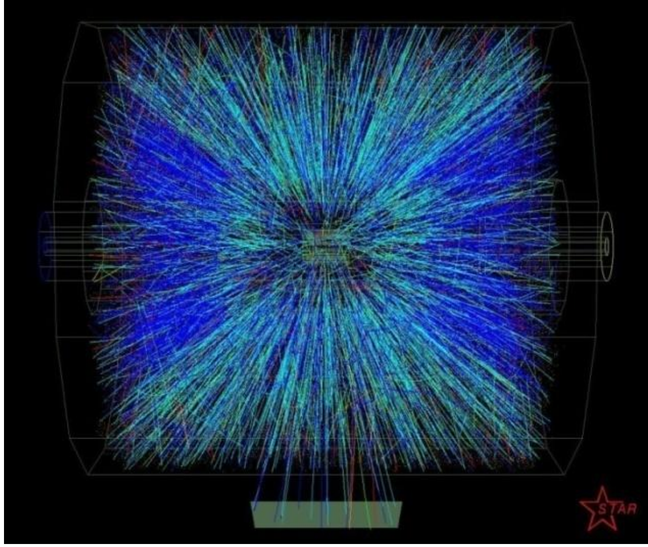
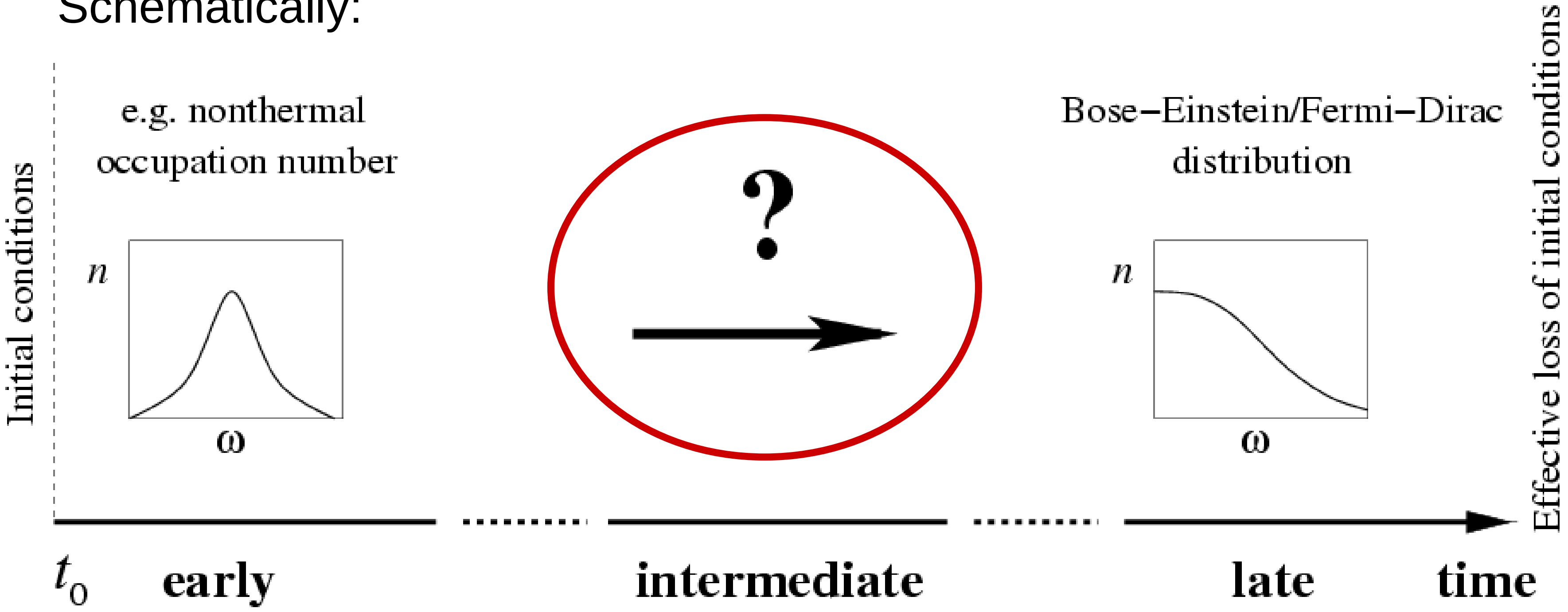
Nonequilibrium dynamics

Relativistic heavy-ion collisions explore strong interaction matter starting from a transient ***nonequilibrium*** state



Thermalization process?

Schematically:



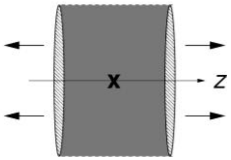
- **Characteristic nonequilibrium time scales? Relaxation? Instabilities?**

Short-time dynamics

- ***Anisotropy*** of the stress tensor T_{ij} in the local fluid rest frame:

  i) prolate anisotropy $\rightarrow T_{zz} \gg T_{xx} \sim T_{yy}$

  ii) oblate anisotropy $\rightarrow T_{xx} \sim T_{yy} \gg T_{zz}$



Isotropization time t_{iso}? In the absence of nonequilibrium instabilities:

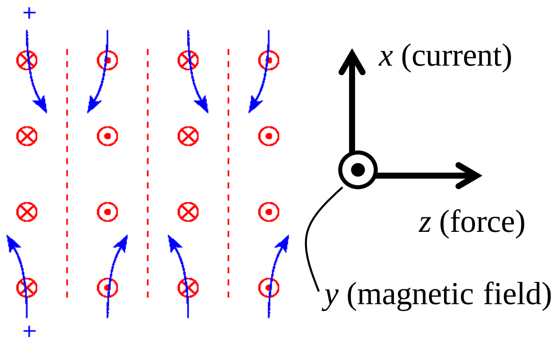
$$t_{iso} \sim \mathcal{O}(1/g^4 T)$$

(T ← characteristic momentum of typical excitation)

- ***Weibel instability:***

Weibel '59; … Mrowczynski '88, '93, '94; Arnold, Lenaghan, Moore '03; Romatschke, Strickland '03; *very many since then*…

$$t_{iso} \sim \mathcal{O}(1/gT)$$

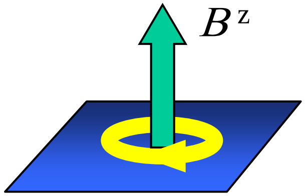


- ***Nielsen-Olesen instability:***

Nielsen, Olesen '78; Chang, Weiss '79; … Iwasaki '08; Fujii, Itakura '08 …

$$t_{iso} \sim \mathcal{O}(1/g^{1/2} B^{1/2})$$

(B ← "homogeneous" background field)



…

Nonperturbative description that contains all possible mechanisms:

Lattice QCD

- ***Quantum simulation*** not possible so far (stochastic quantization?)

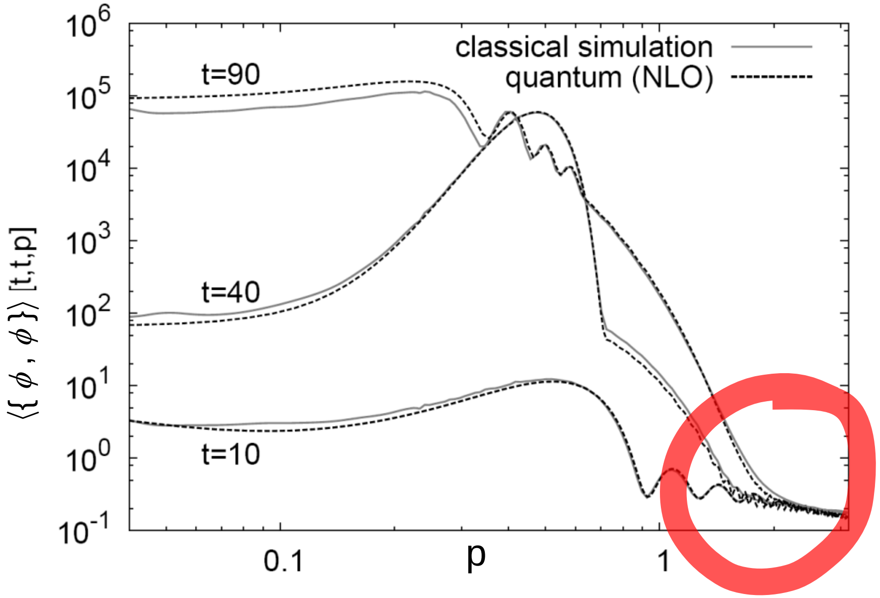
- ***Classical-statistical simulation:*** includes all fluctuations with

$$\langle\{ A(x) , A(y) \}\rangle \gg \langle[A(x) , A(y)]\rangle$$

i.e. “occupation numbers“ $\gg 1$ (includes, in particular, HTL / Vlasov)

physics of nonequilibrium instabilities $\rightarrow$ high occupation numbers

- Classical simulations well tested for quantum evolutions in scalar theories:



(N=4)-component $\lambda\phi^4$

parametric/spinodal instability

Berges, Rothkopf, Schmidt, *PRL* 101 (2008) 041603

Classical-statistical lattice gauge field simulations

Romatschke, Venugopalan (CGC); Berges, Gelfand, Scheffler, Sexty

Wilson action:

$$S[U] = -\beta_0 \sum_x \sum_i \left\{ \frac{1}{2\mathrm{Tr}\,\mathbf{1}} \left(\mathrm{Tr}\, U_{x,0i} + \mathrm{Tr}\, U_{x,0i}^{-1} \right) - 1 \right\}$$
$$+\beta_s \sum_x \sum_{\substack{i,j \\ i<j}} \left\{ \frac{1}{2\mathrm{Tr}\,\mathbf{1}} \left(\mathrm{Tr}\, U_{x,ij} + \mathrm{Tr}\, U_{x,ij}^{-1} \right) - 1 \right\}$$

Here: $\beta = \beta_0 / \gamma = \beta_s \gamma = 4$, axial-temporal/Coulomb gauge

Initial conditions: Normalized Gaussian probability functional

$P[A(t{=}0),\partial_t A(t{=}0)]$: $\langle\, A(t)\, A(t')\, \rangle = \int DA(0)\, D\partial_t A(0)\, P[A(0),\partial_t A(0)]\, A(t)\, A(t')$

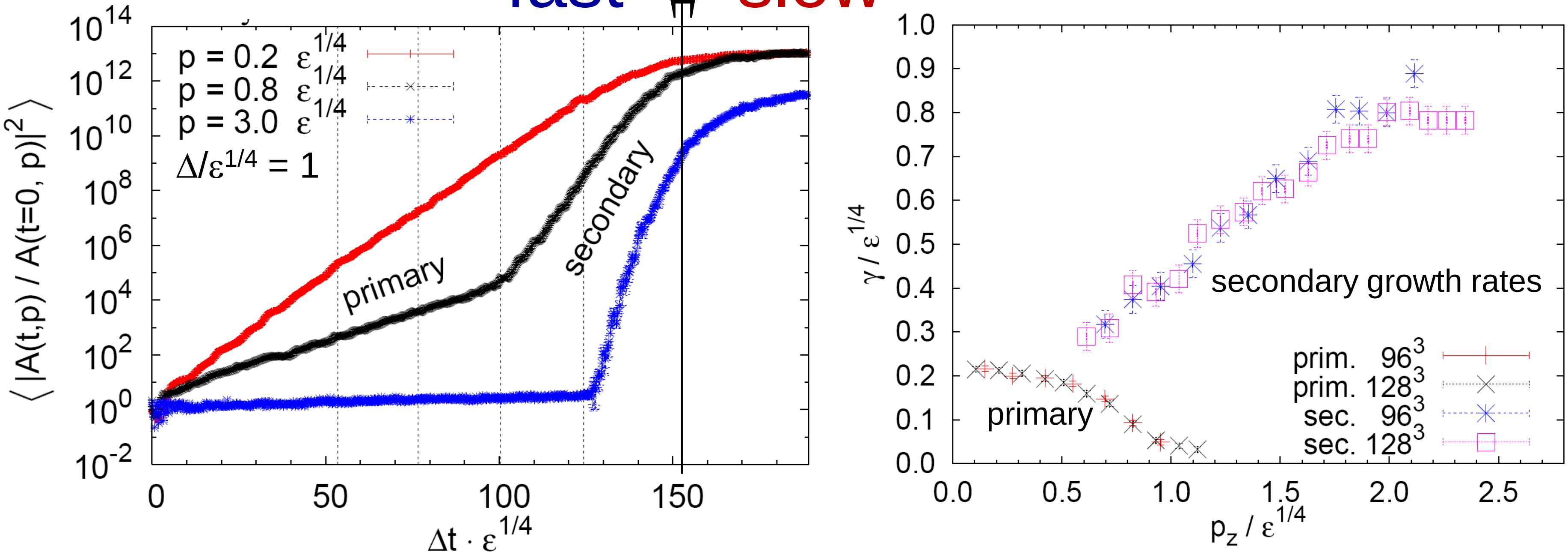
$$\langle\, |A_j^a(t=0,\vec{k})|^2\, \rangle \sim C \exp\left\{ -\frac{k_x^2 + k_y^2}{2\Delta^2} - \frac{k_z^2}{2\Delta_z^2} \right\}$$

with $\Delta \gg \Delta_z$ (extreme anisotropy), $\Delta \sim Q_s$ (CGC type)

C is adjusted to obtain a given energy density ε ,e.g. $\varepsilon_{\mathrm{RHIC}} \sim 5\text{–}25$ GeV/fm^3

Characteristic time scales

fast ⟷ slow



Here: $\Delta / \varepsilon^{1/4} = 1$

Berges, Scheffler, Sexty, *PRD* 77 (2008) 034504

Inverse primary growth rates: e.g. $\varepsilon_{\text{RHIC}} \sim 5\text{–}25$ GeV/fm³, $\varepsilon_{\text{LHC}} \sim 2 \times \varepsilon_{\text{RHIC}}$

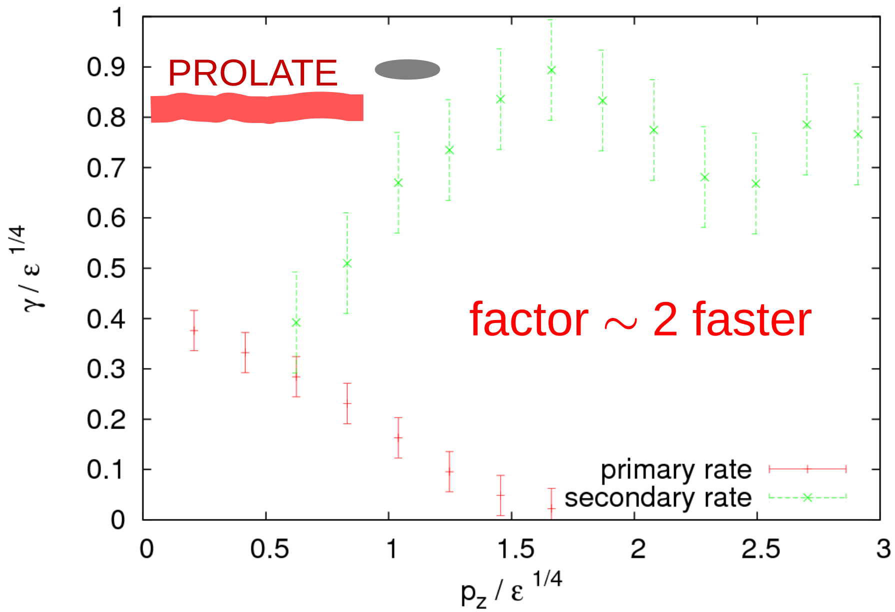
fast:

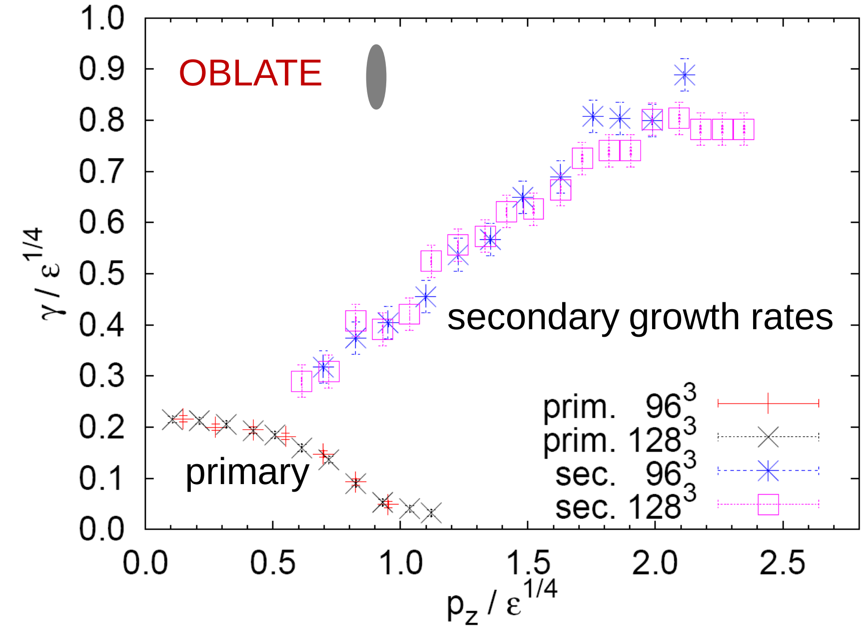
$$\gamma_{\max}^{-1} \simeq 1.2 - 1.8\,\text{fm}/c \ \ (\text{RHIC})$$

$$\gamma_{\max}^{-1} \simeq 1.0 - 1.5\,\text{fm}/c \ \ (\text{LHC})$$

SU(2)

Characteristic time scales





Berges, Scheffler, Sexty, *PRD* 77 (2008) 034504

Inverse primary growth rates: e.g. $\varepsilon_{\rm RHIC} \sim 5$–$25$ GeV/fm^3, $\varepsilon_{\rm LHC} \sim 2 \times \varepsilon_{\rm RHIC}$
(oblate)

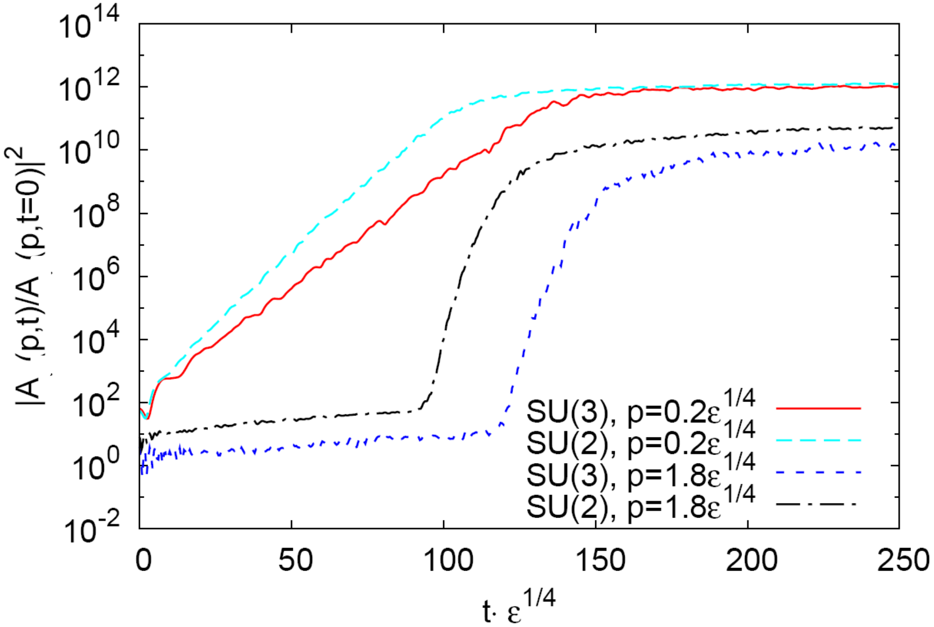
fast:

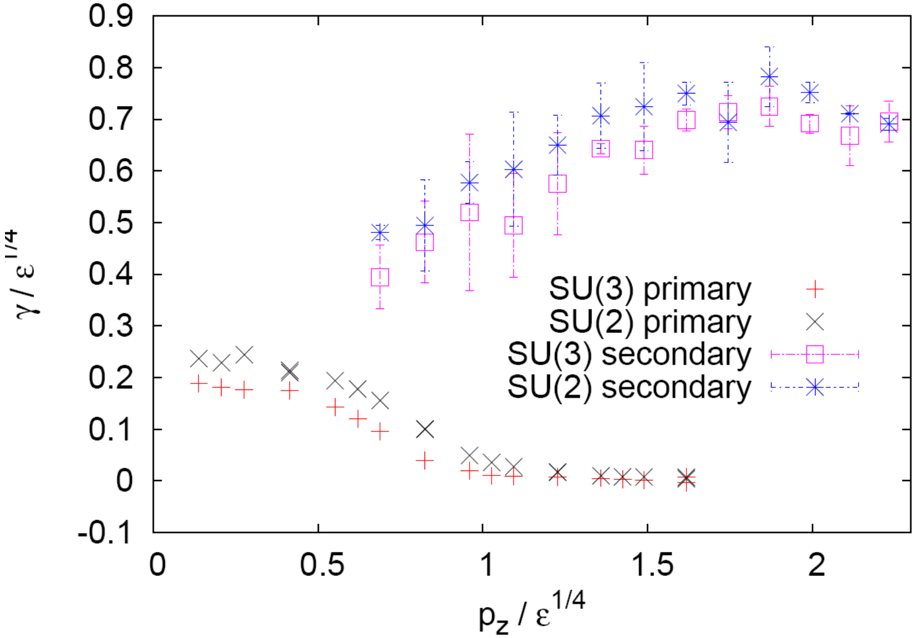
$$\gamma_{\max}^{-1} \simeq 1.2 - 1.8\,{\rm fm}/c \;\; ({\rm RHIC})$$

$$\gamma_{\max}^{-1} \simeq 1.0 - 1.5\,{\rm fm}/c \;\; ({\rm LHC})$$

SU(2)

Comparison SU(2) vs. SU(3)





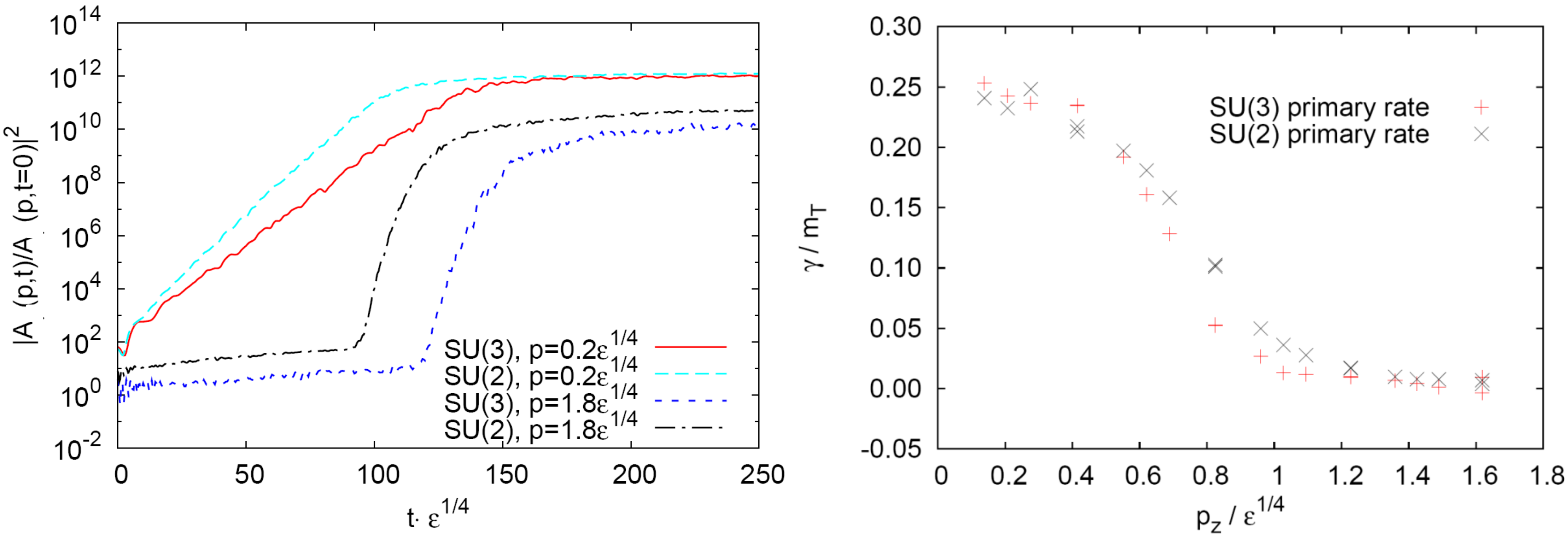
Berges, Gelfand, Scheffler, Sexty, *PLB 677* (2009) 210

$$\gamma^{-1}_{\text{max. pr.}} \simeq 1.6 - 2.4\,\text{fm/c} \qquad (\text{RHIC}),$$
$$\gamma^{-1}_{\text{max. pr.}} \simeq 1.3 - 2.0\text{fm/c} \qquad (\text{LHC}).$$

SU(3)

$\Rightarrow$ SU(3) shows *reduced* primary growth rates by about 25% for given ε

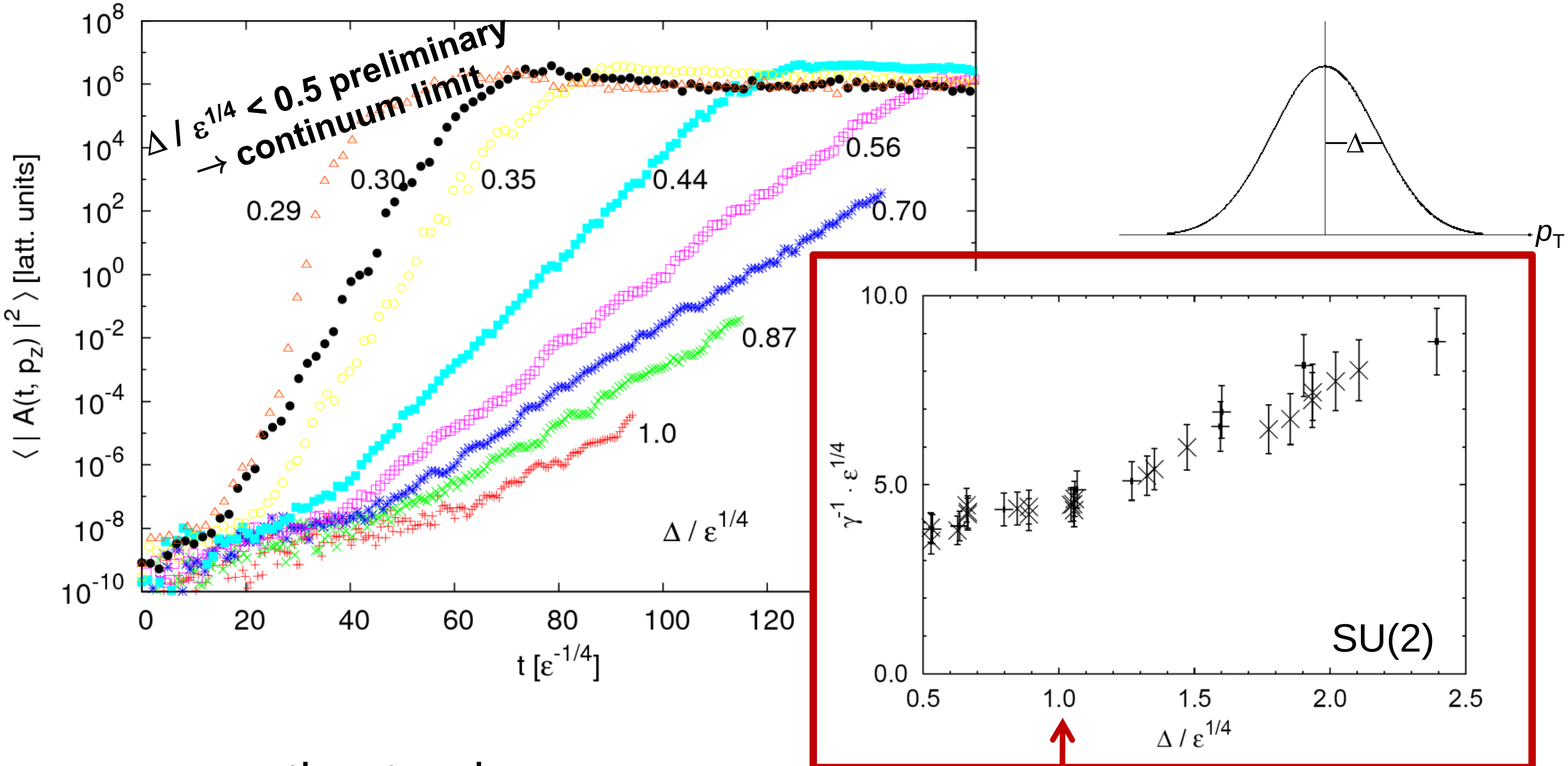
Comparison SU(2) vs. SU(3)



Berges, Gelfand, Scheffler, Sexty, *PLB 677* (2009) 210

⇒ Measured in units of the characteristic screening masses $m_{T,SU(2)}$ and $m_{T,SU(3)}$, respectively, the primary growth rate is independent of N_c

- Initial conditions with **faster isotropization/thermalization**?



⇒ growth rate γ inceases for smaller $\Delta / \varepsilon^{1/4}$

$\Delta / \varepsilon^{1/4} \ll 1$ corresponds to rather “homogeneous“ field configurations ($\Delta_z / \varepsilon^{1/4} \ll 1$)

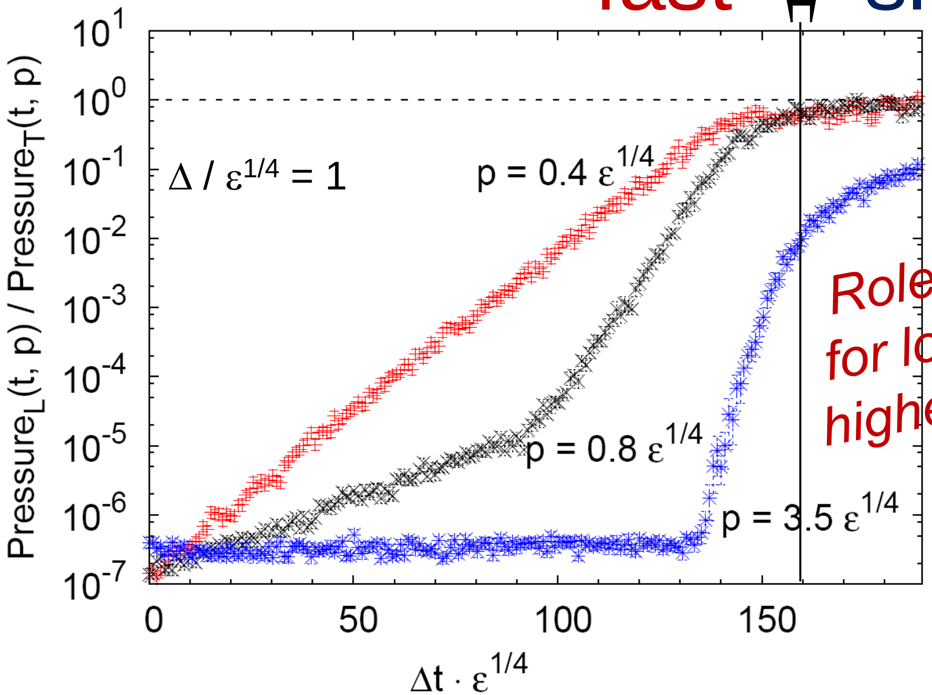
$\rightarrow$ Nielsen-Olesen?

Pressure

Spatial Fourier transform of the stress tensor $T^{\mu\nu}(x)$: $P_L(t,p)$ for $\mu=\nu=3$, $P_T(t,p)$

gauge invariant!

fast ⇔ slow



Role of quantum corrections for lower occupied modes at higher momenta?

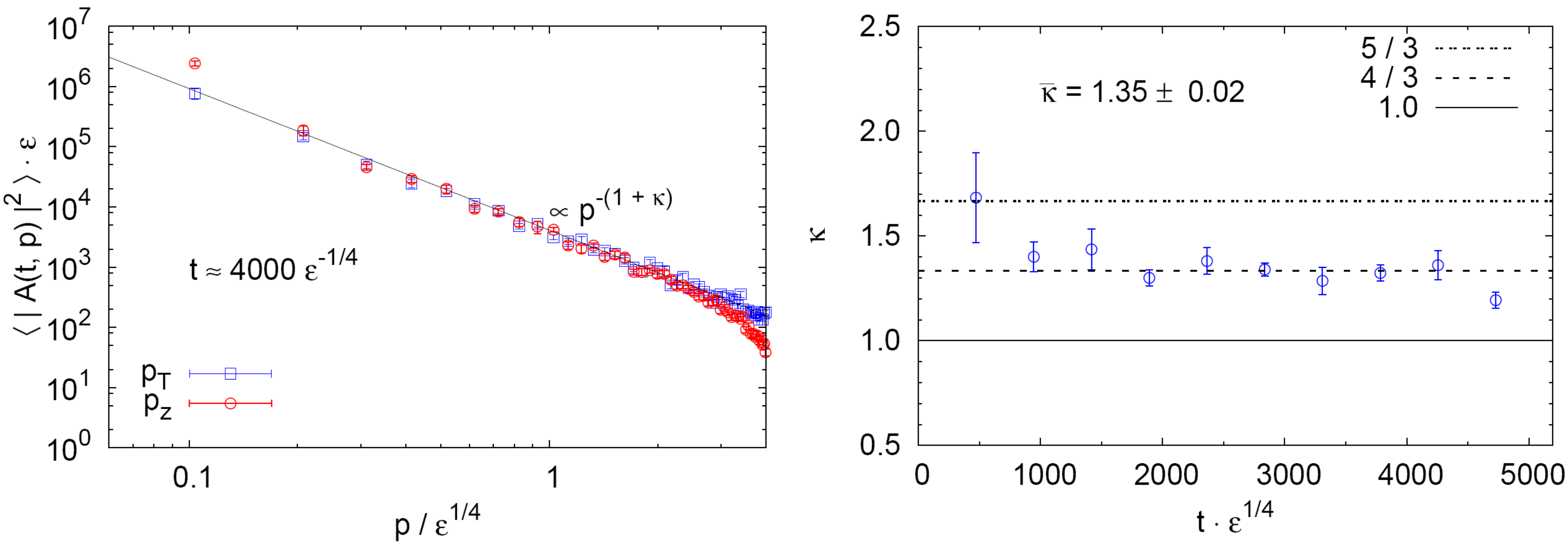
Fast bottom-up isotropization for momenta:

$p_z \lesssim 1$ GeV

'enough' for hydro?

Still far from equilibrium at this stage!

Slow: Kolmogorov wave turbulence



Berges, Scheffler, Sexty, arXiv:0811.4293 [hep-ph]

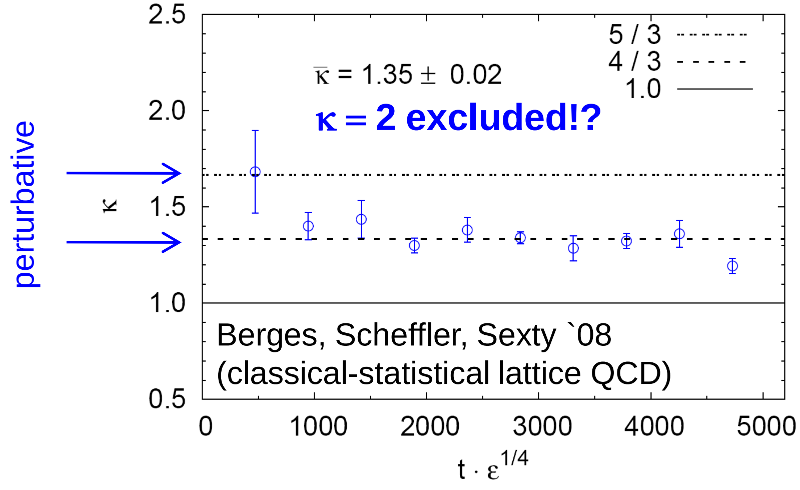
- Scaling exponent κ close to the perturbative value $\kappa = 4/3$

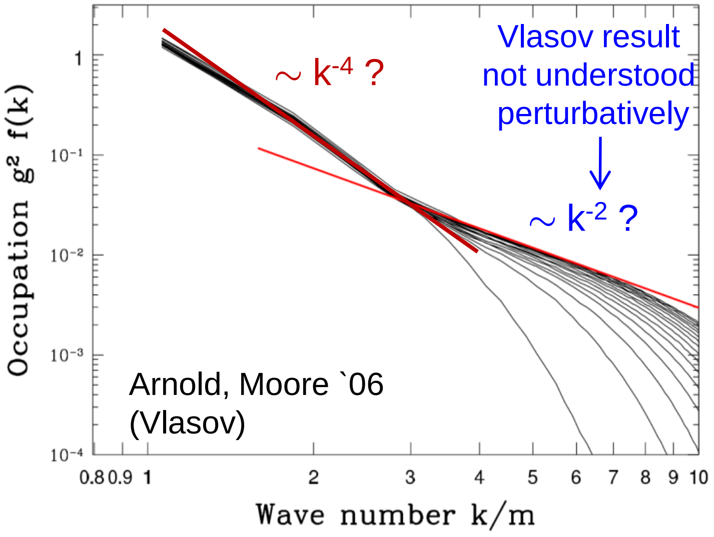
See however: Arnold, Moore *PRD* 73 (2006) 025006; Mueller, Shoshi, Wong, *NPB* 760 (2007) 145

- Different infrared behavior? Nonthermal IR fixed point?

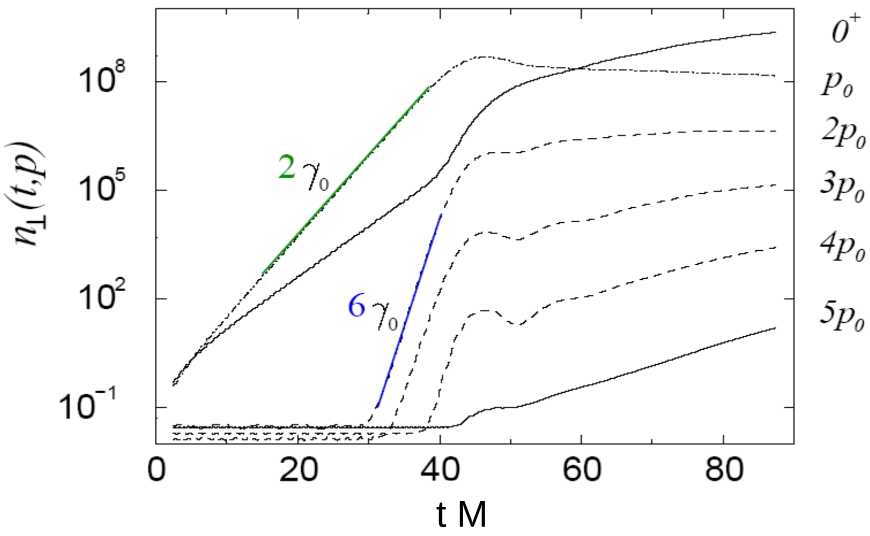
(Infrared occupation number $\sim 1/g^2$ $\Rightarrow$ strongly correlated)

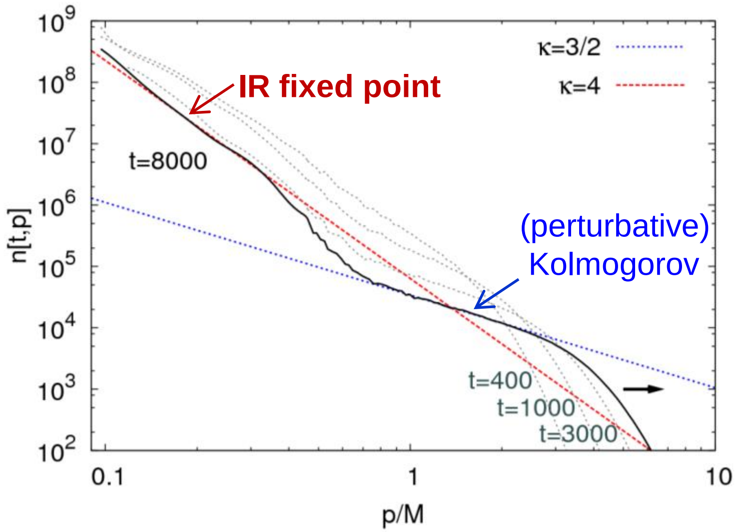
• Apparent discrepancy:





• Compare scalar (inflaton) instability dynamics

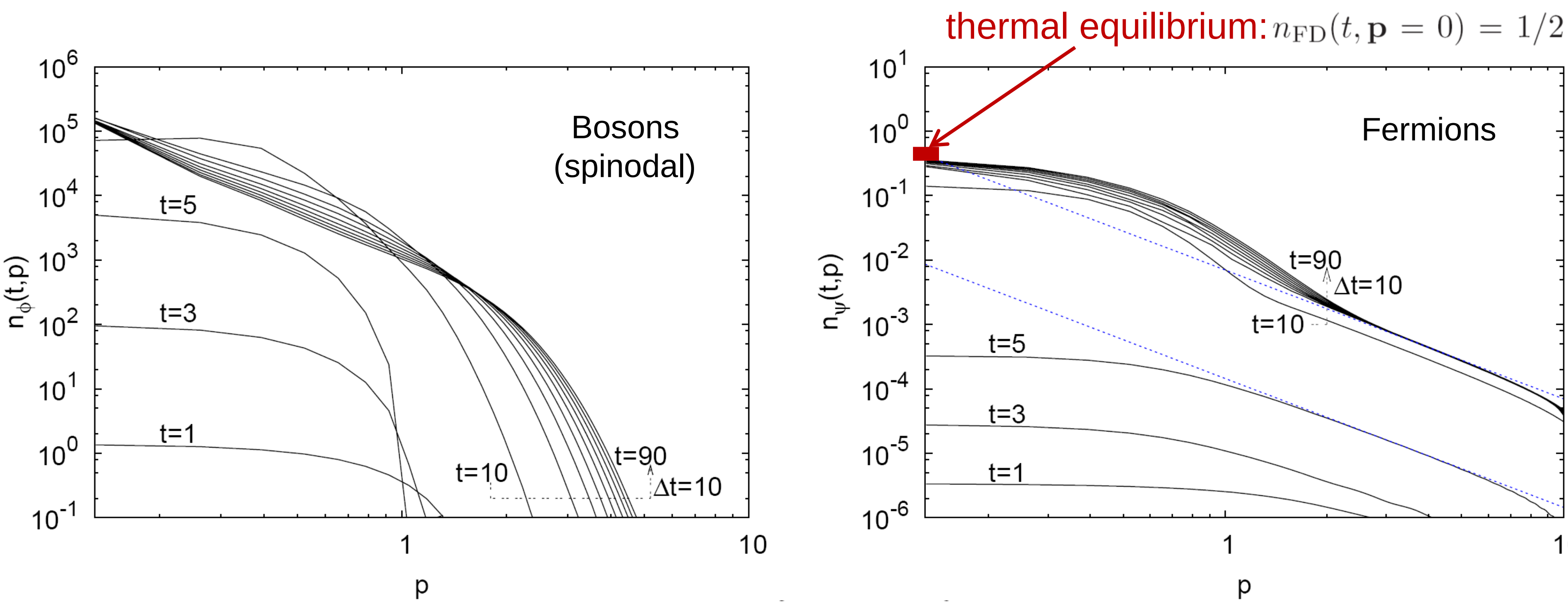




Berges, Rothkopf, Schmidt, *PRL* 101 (2008) 041603

Instability-induced fermion production

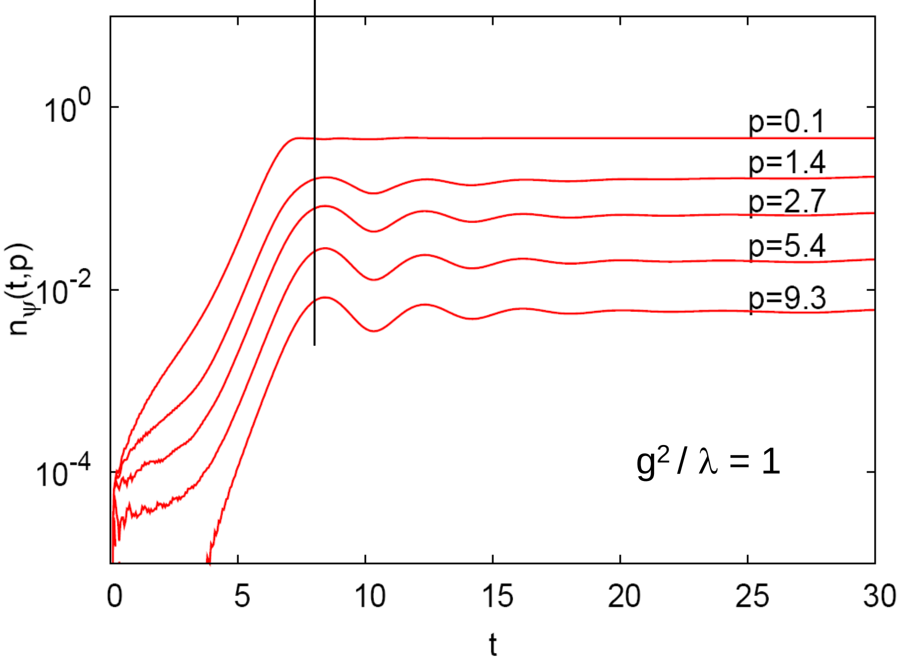
Quantum evolution of $SU(2)_L \times SU(2)_R$ linear sigma model (2PI *1/N* to NLO):

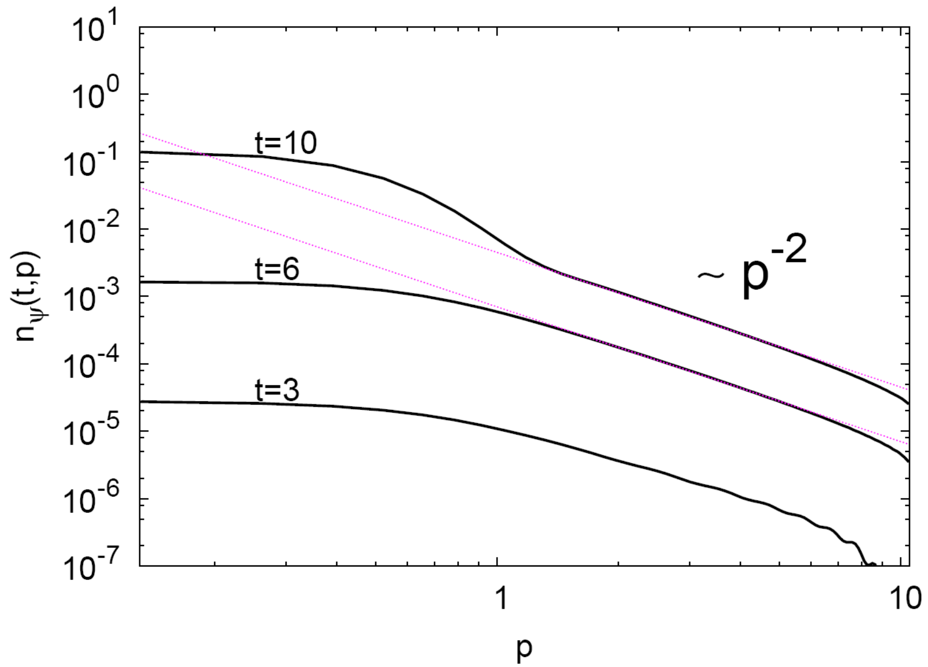


Berges, Pruschke, Rothkopf, PRD to appear, arXiv:0904.3073 [hep-ph]

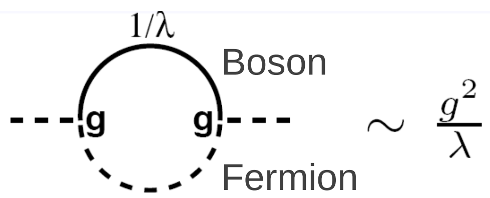
- Fermion production proceeds with the maximum primary boson growth rate!
- Fast approach to Fermi-Dirac distribution in the infrared!

fast ↔ slow



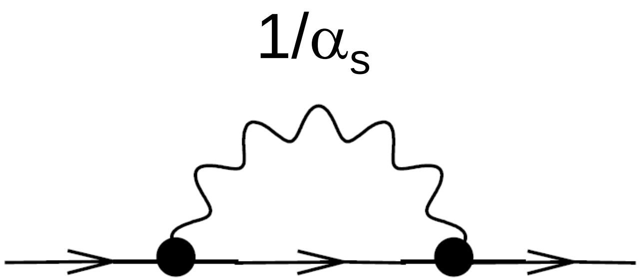


- Fast fermion growth induced via boson-fermion loop:



- Scaling behavior at higher momenta (no IR scaling $\rightarrow$ Pauli principle)
- Bosons practically unaffected for Yukawa couplings $\lesssim O(1)$

→ **Instability-induced fermion production** can lead to substantial deviations from standard production processes



$\sim\ O(1)$

Quantitatively: **Classical-statistical lattice QCD with** (quantum) **fermions** can be simulated with well established techniques!

Conclusions

- **Plasma instabilities** for CGC type initial conditions (Δ / $\varepsilon^{1/4}$ = 1)

$$\gamma^{-1}_{\text{max. pr.}} \simeq 1.6 - 2.4\,\text{fm/c} \qquad (\text{RHIC})\,,$$
$$\gamma^{-1}_{\text{max. pr.}} \simeq 1.3 - 2.0\text{fm/c} \qquad (\text{LHC})\,.$$

SU(3)

- **'Bottom-up' isotropization** of stress tensor for **$p \lesssim$ 1 GeV**
  i.e. (optimistically) about the range where hydro 'works'

- (Perturbative) **Kolmogorov wave turbulence** with $\kappa \simeq 4/3$
  Nonthermal **IR fixed point** in QCD?

- Initial conditions with **faster isotropization/thermalization**?
  $\rightarrow \quad \Delta \sim O(\Lambda_{\text{QCD}})$?

- **Instability-induced fermion production** can lead to substantial deviations from standard production processes

  $\rightarrow$ **fast thermalization of low-momentum fermions** on time scale of maximum boson growth rate seen in linear sigma model