The Horn and the Unreasonable Success of the Thermal Model

a bit of historical development
and then a recent paper
A. Andronic, P. Braun-Munzinger, J. Stachel,
arXiv:0812.1186, Phys. Lett. B673, 142 (2009)
for some figures see also Acta Phys. Polon. B40, 1005 (2009)

EMMI Workshop and XXVI Max-Born Symposium:
Three Days of Strong Interactions, July 9-11, 2009
Johanna Stachel – Physikalisches Institut – Universitaet Heidelberg

analysis of yields of produced hadronic species in statistical model – grand canonical

partition function: $\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln(1 \pm \exp(-(E_i - \mu_i)/T))$

particle densities: $n_i = N/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2\,\mathrm{d}p}{\exp((E_i - \mu_i)/T) \pm 1}$

for every conserved quantum number there is a chemical potential:

$$\mu_i = \mu_B B_i + \mu_S S_i + \mu_{I_3} I_i^3$$

but can use conservation laws to constrain V, μ_S, μ_{I_3} →

Fit at each energy provides values for T and μ_b

technical details:

van der Waals type interaction via excluded volum correction following Rischke, Gorenstein, Stoecker, Greiner, 1991

finite volume correction a la Balian and Bloch

width of all resonances included by integrating over Breit-Wigner distributions

For a review see: Braun-Munzinger, Redlich, Stachel, QGP3,
R. Hwa ed. (Singapore 2004) 491-599; nucl-th/0304013

successfully applied to AGS Si+Au data in 1994

J.Stachel, P.Braun-Munzinger, workshop in honor of the 75th birthday of R. Hagedorn, Divonne June 1994

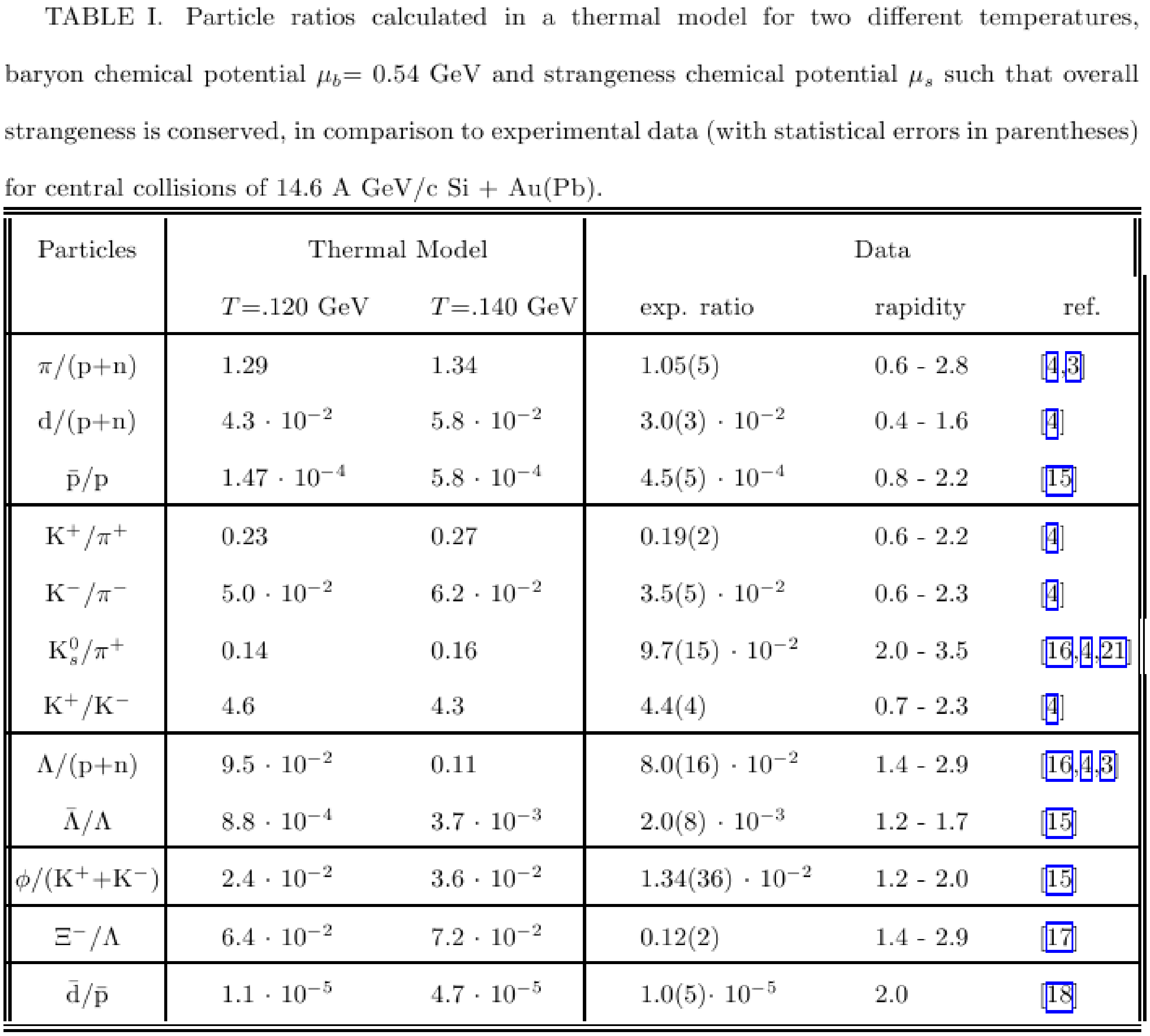


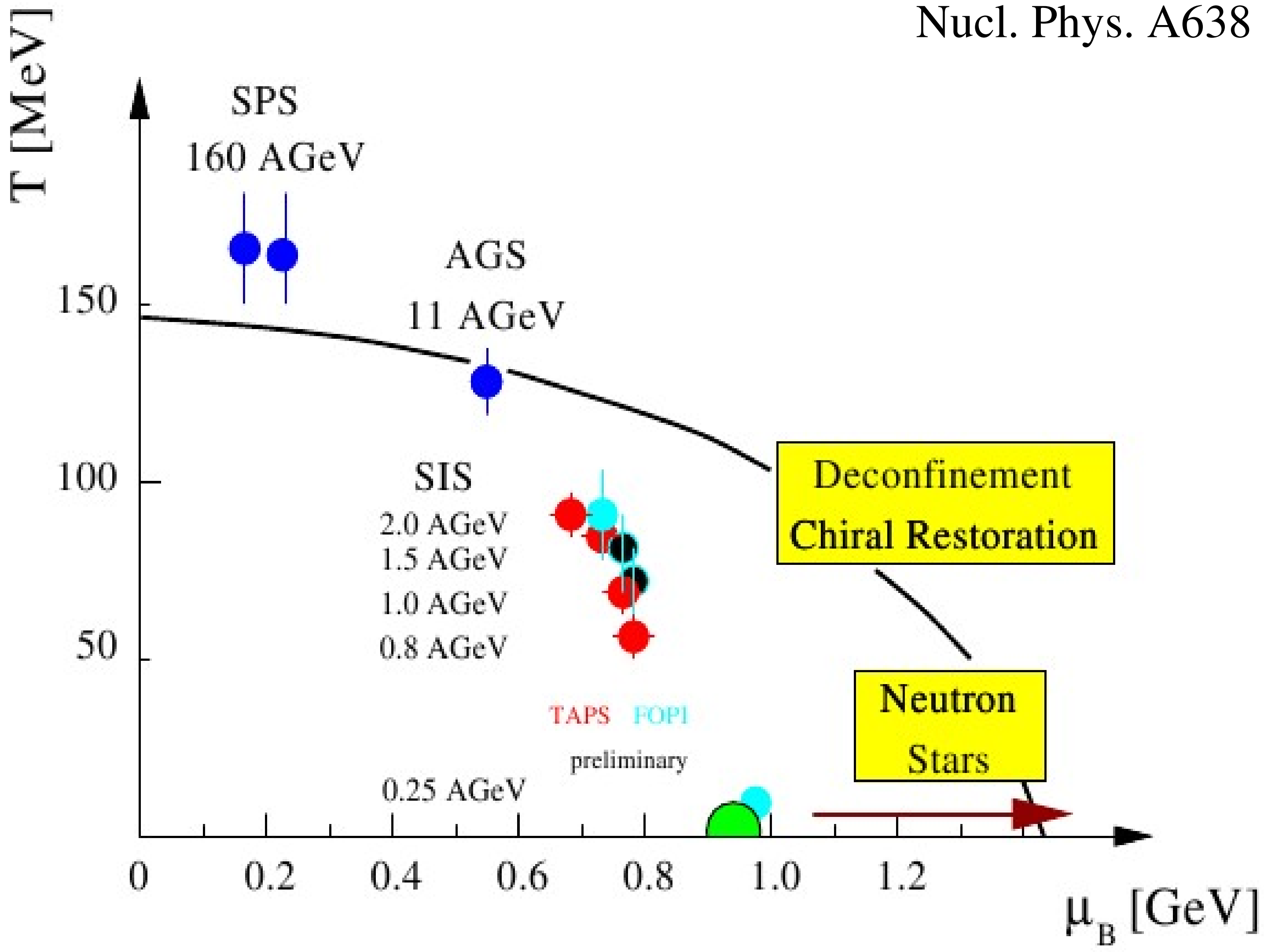
TABLE I. Particle ratios calculated in a thermal model for two different temperatures, baryon chemical potential μ_b= 0.54 GeV and strangeness chemical potential μ_s such that overall strangeness is conserved, in comparison to experimental data (with statistical errors in parentheses) for central collisions of 14.6 A GeV/c Si + Au(Pb).

Particles	Thermal Model		Data		
	T=.120 GeV	T=.140 GeV	exp. ratio	rapidity	ref.
$\pi/(p+n)$	1.29	1.34	1.05(5)	0.6 - 2.8	[4,3]
$d/(p+n)$	$4.3 \cdot 10^{-2}$	$5.8 \cdot 10^{-2}$	$3.0(3) \cdot 10^{-2}$	0.4 - 1.6	[4]
$\bar{p}/p$	$1.47 \cdot 10^{-4}$	$5.8 \cdot 10^{-4}$	$4.5(5) \cdot 10^{-4}$	0.8 - 2.2	[15]
K^+/π^+	0.23	0.27	0.19(2)	0.6 - 2.2	[4]
K^-/π^-	$5.0 \cdot 10^{-2}$	$6.2 \cdot 10^{-2}$	$3.5(5) \cdot 10^{-2}$	0.6 - 2.3	[4]
K^0_s/π^+	0.14	0.16	$9.7(15) \cdot 10^{-2}$	2.0 - 3.5	[16,4,21]
K^+/K^-	4.6	4.3	4.4(4)	0.7 - 2.3	[4]
$\Lambda/(p+n)$	$9.5 \cdot 10^{-2}$	0.11	$8.0(16) \cdot 10^{-2}$	1.4 - 2.9	[16,4,3]
$\bar{\Lambda}/\Lambda$	$8.8 \cdot 10^{-4}$	$3.7 \cdot 10^{-3}$	$2.0(8) \cdot 10^{-3}$	1.2 - 1.7	[15]
$\phi/(K^+ + K^-)$	$2.4 \cdot 10^{-2}$	$3.6 \cdot 10^{-2}$	$1.34(36) \cdot 10^{-2}$	1.2 - 2.0	[15]
Ξ^-/Λ	$6.4 \cdot 10^{-2}$	$7.2 \cdot 10^{-2}$	0.12(2)	1.4 - 2.9	[17]
$\bar{d}/\bar{p}$	$1.1 \cdot 10^{-5}$	$4.7 \cdot 10^{-5}$	$1.0(5) \cdot 10^{-5}$	2.0	[18]

works over 9 oom!

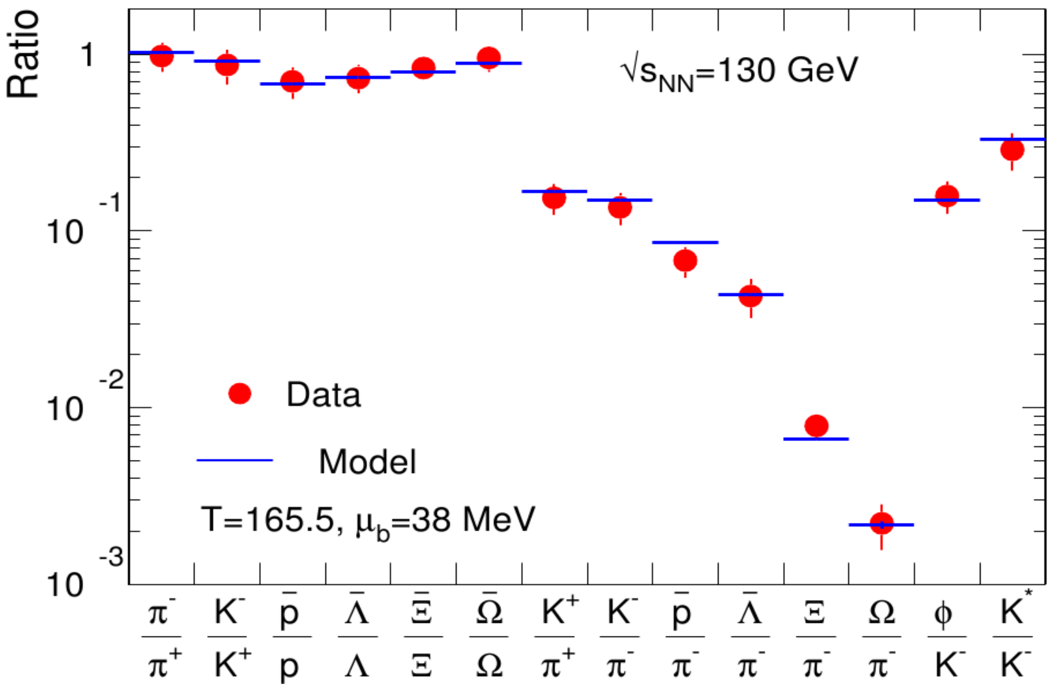
leading to the first phase diagram with experimental points

P.Braun-Munzinger and J. Stachel, nucl-th/9803015,
Nucl. Phys. A638 (1998) 3

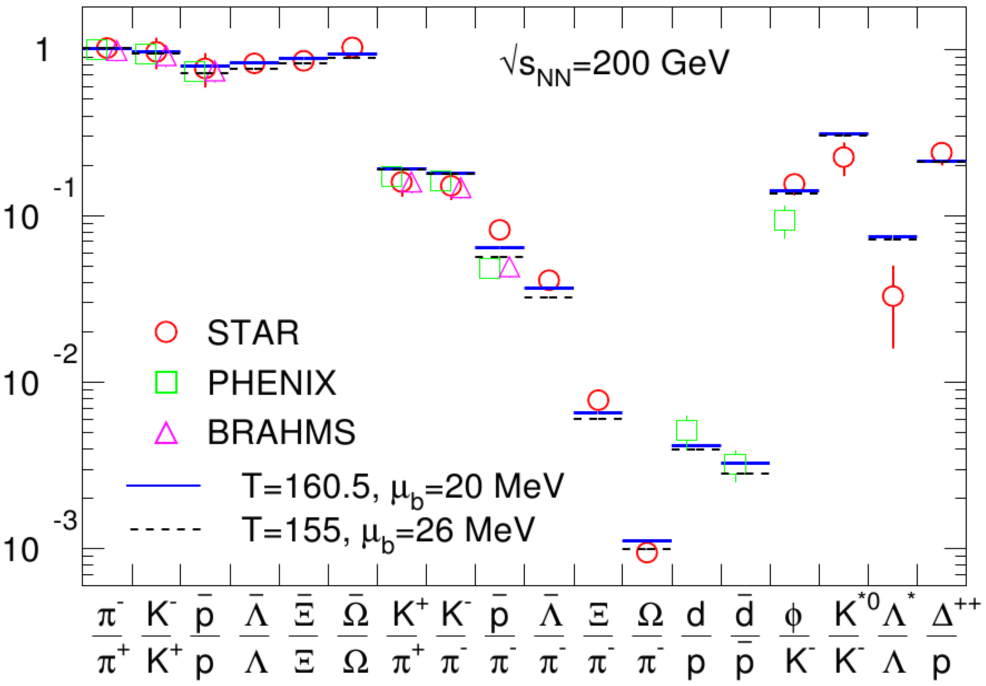


hadron yields at RHIC compared to statistical model (GC)

130 GeV data in excellent agreement
with thermal model predictions



prel. 200 GeV data fully in line
still some experimental discrepancies

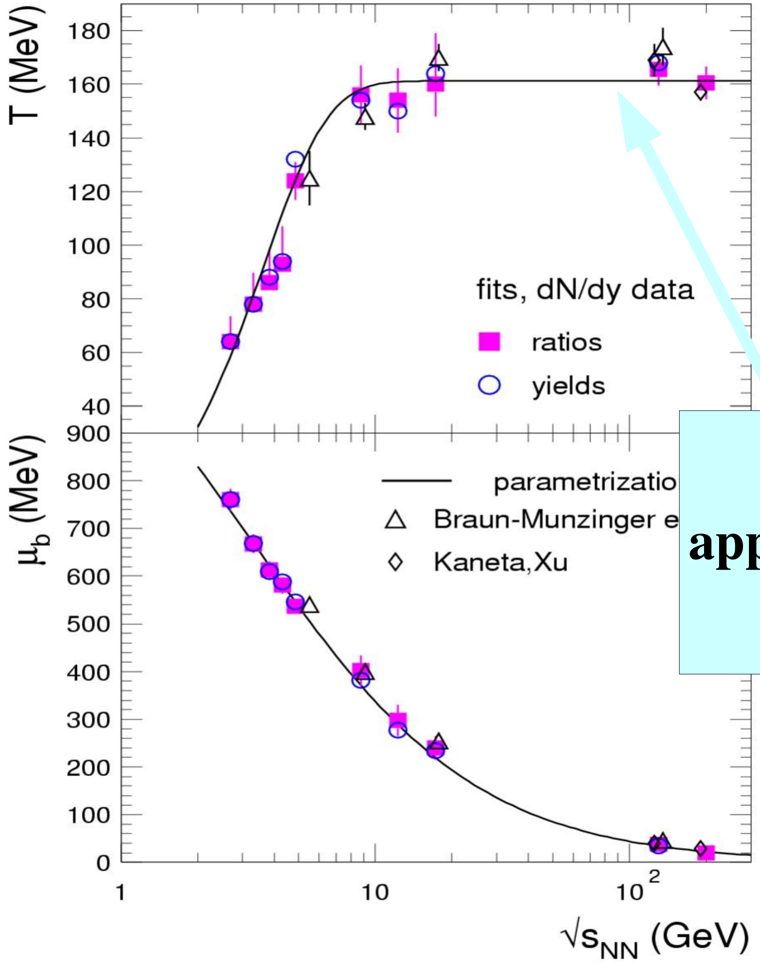


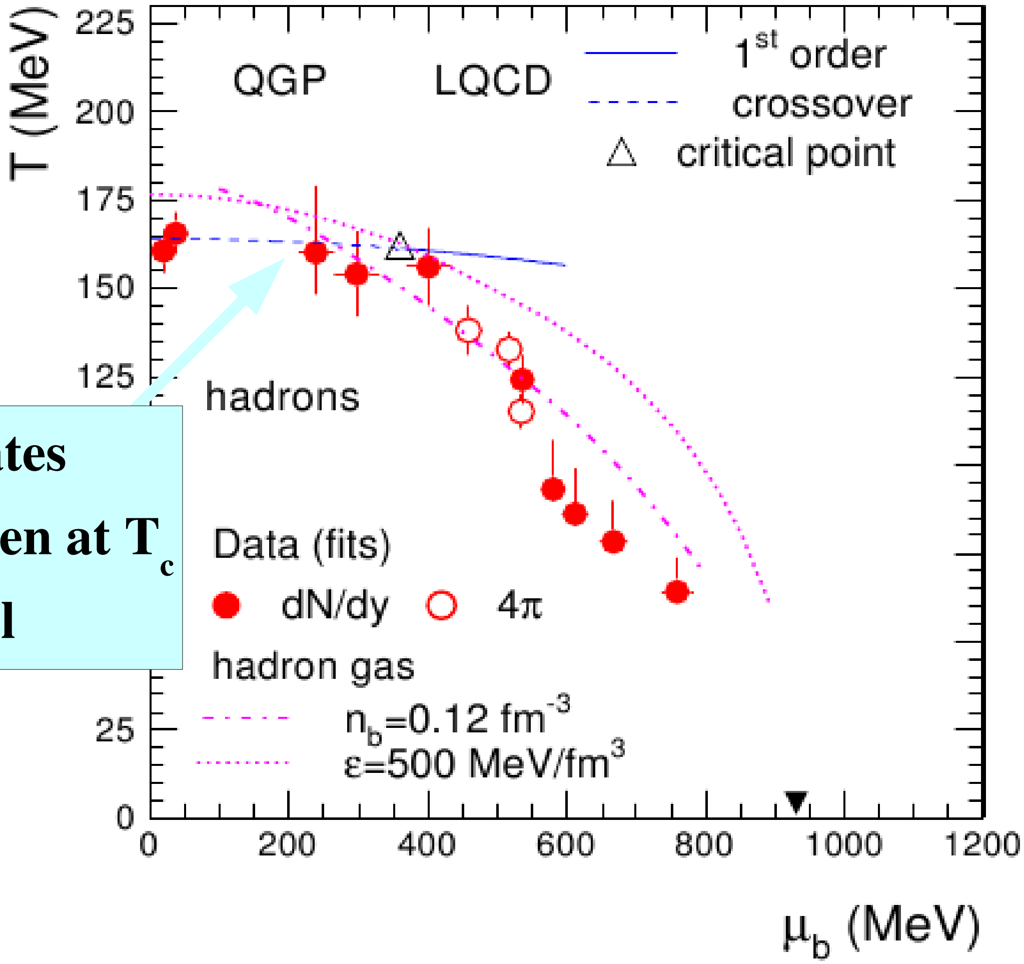
chemical freeze-out at: T = 165 ± 5 MeV

P. Braun-Munzinger, D. Magestro, K. Redlich, J. Stachel, Phys. Lett. B518 (2001) 41
A. Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys. A772 (2006) 167

hadrochemical freeze-out points and the phase diagram

A. Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys. A772 (2006) 167





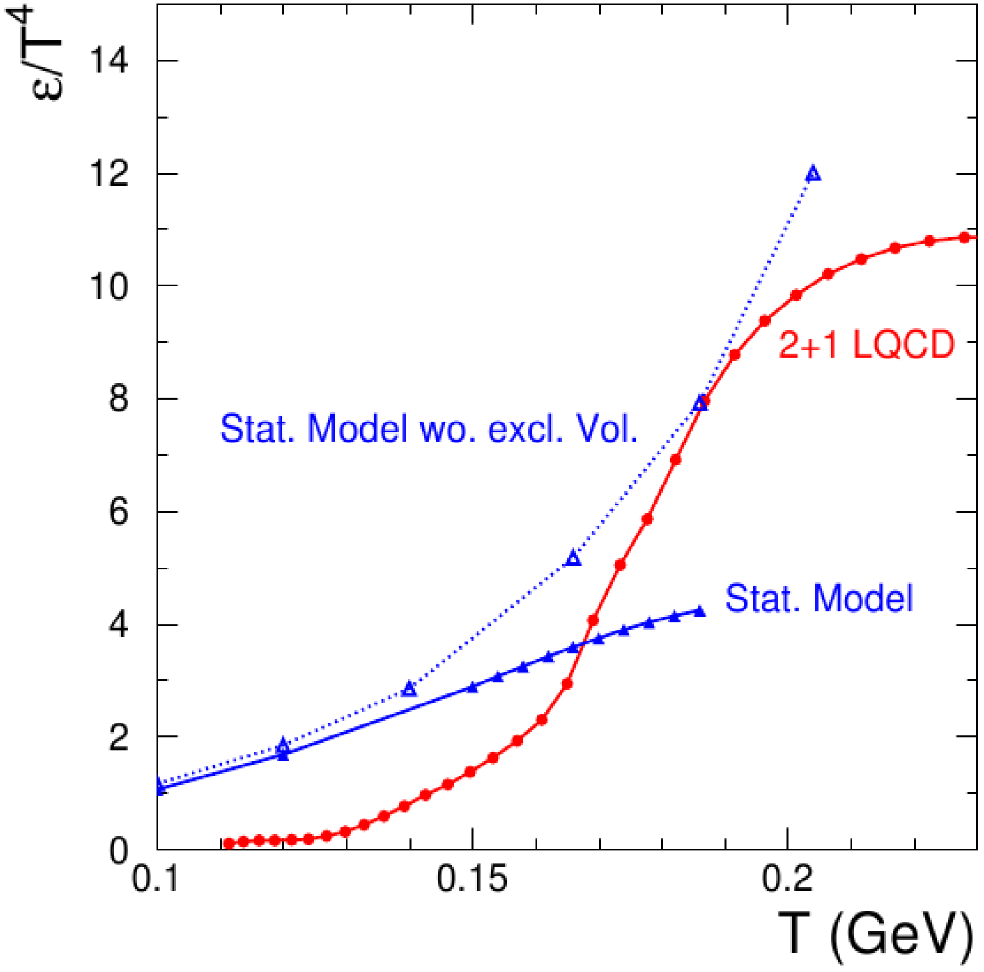
T_{chem} saturates appears to happen at T_c not trivial

in general good fits to all central heavy ion collision data
but K^+/π^+ beam energy dependence shows marked maximum
up to very recently not well reproduced -> focus of this talk

why do all particle yields show one common freeze-out T?

- **The density of particles varies rapidly (factor 2 within 8 MeV) with T near the phase transition due to increase in degrees of freedom.**
- **also: system spends time at Tc -> volume has to triple (entropy cons.)**
- **Multi-particle collisions are strongly enhanced at high density and lead to chem. equilibrium very near to T_c**
- **independently of cross section all particles can freeze out within narrow temperature interval**

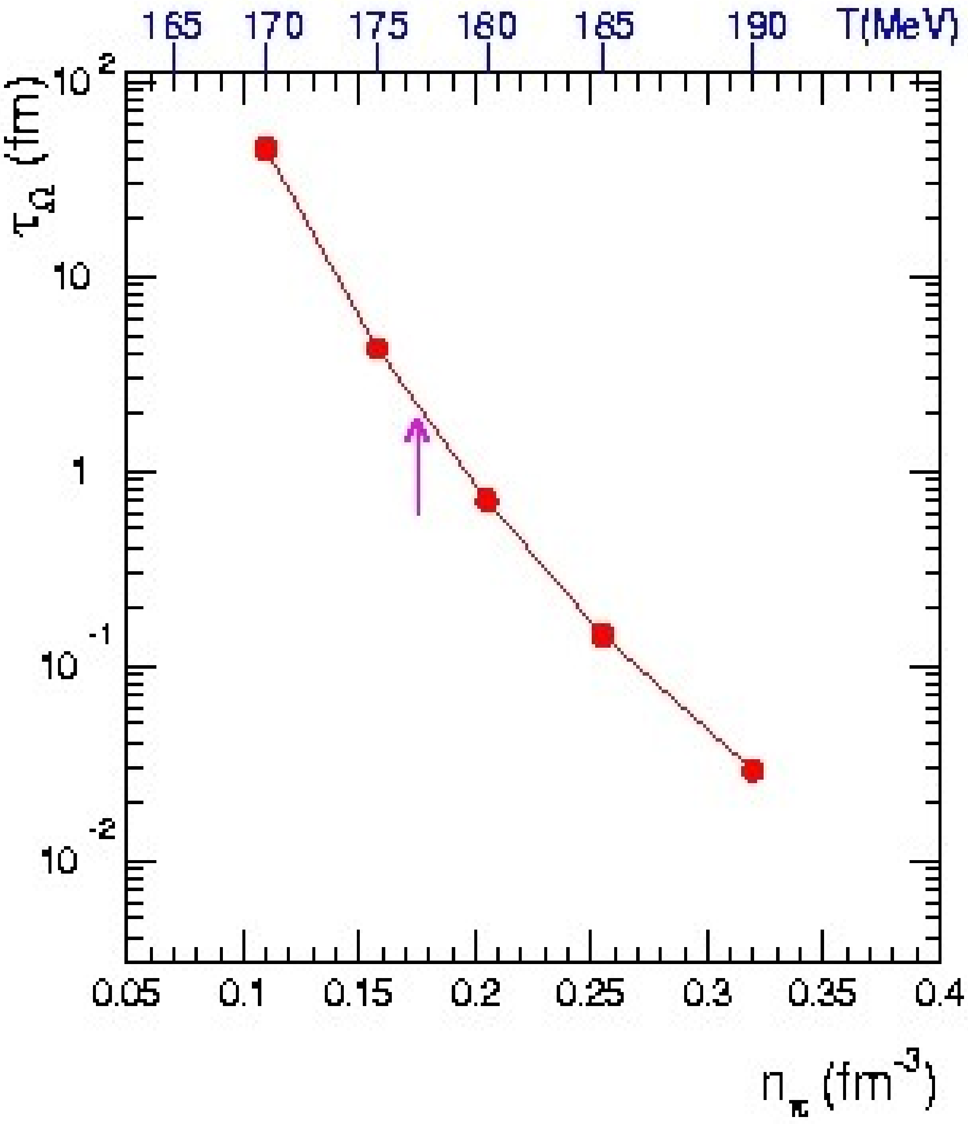
natural consequence that chemical freeze-out takes place at T_c!



Lattice QCD by F. Karsch et al.

P. Braun-Munzinger, J. Stachel, C. Wetterich, Phys. Lett. B596 (2004)61

Density dependence of characteristic time for strange baryon production



- **Near phase transition particle density varies rapidly with T**
- **For small μ_b, reactions such as $2\pi+KKK \rightarrow \Omega$ Nbar bring multi-strange baryons close to equilibrium.**
- **in region around T_c equilibration time $\tau_\Omega \propto T^{-60}$!**
- **increase ρ_π by 1/3 or 8 MeV: $\tau = 0.2$ fm/c**

  decrease ρ_π by 1/3: $\tau = 27$ fm/c
- **All particles freeze out within a very narrow temperature window.**

P. Braun-Munzinger, J. Stachel, C. Wetterich, Phys. Lett. B596 (2004)61

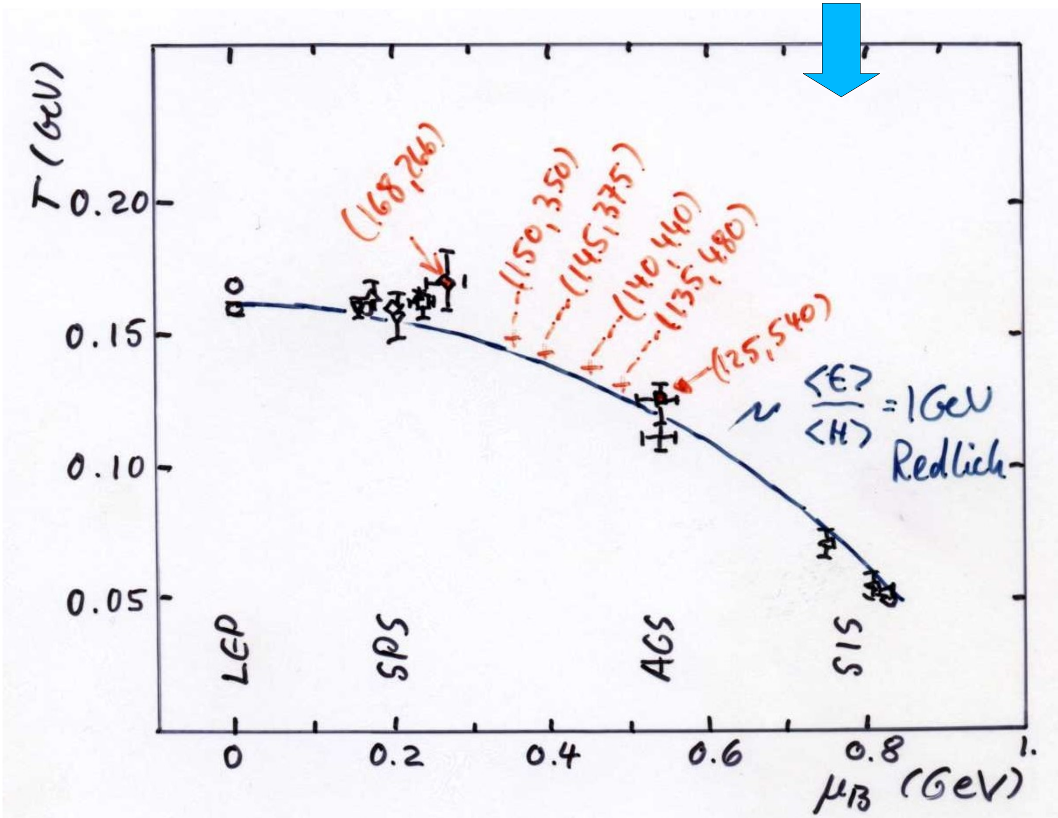
the appearance of the horn

Heavy Ion Forum at CERN October 3, 2000
Marek Gazdzicki reports on first results for 40 GeV/nucleon and energy dependence of pion and strangeness production

K^+/π^+ in PbPb collisions at 40 A GeV larger than at 158 A GeV!

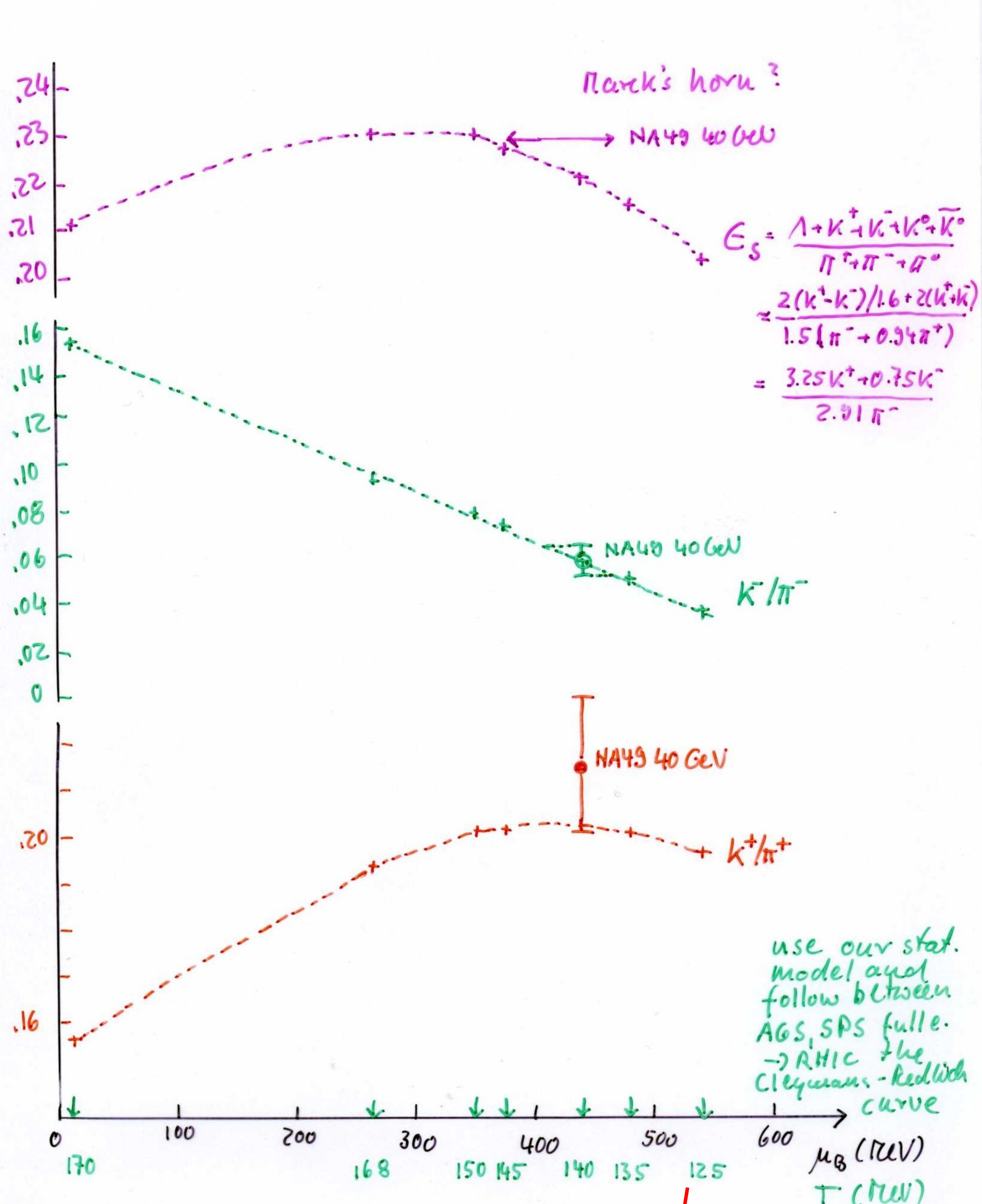
Discussion of CERN heavy ion forum talk: 2 figs. shown by J.S.

statistical model fits to SIS, AGS, top SPS energy data yield systematic evolution of T and μ_B



interpolate (red crosses) and compute K/π:

maximum in K^+/π^+ because with increasing T more kaons and at same time with decreasing μ_B reduced associated production



← √s

a first beam energy dependence calculation:

P. Braun-Munzinger, J. Cleymans, H. Oeschler, K. Redlich, NPA 697 (2002) 902   hep-ph/0106066

K^+/π^+ looks allright   and   an even more dramatic horn predicted in Λ/π

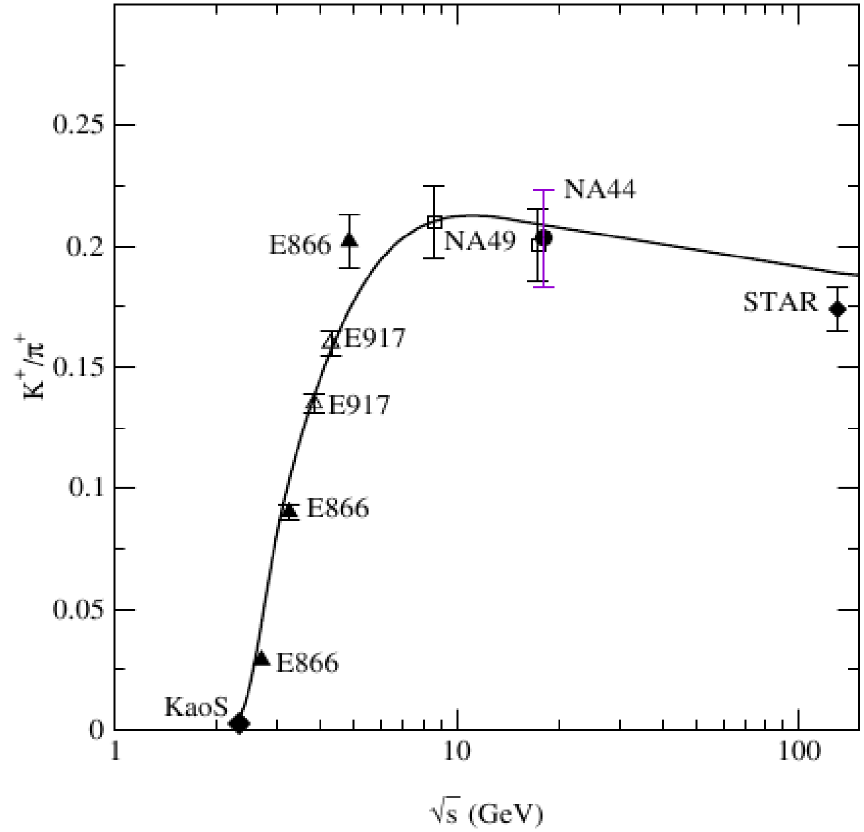


Figure 5: K^+/π^+ ratio obtained around midrapidity as a function of $\sqrt{s}$ from the various experiments. For the references for all data points see [3, 6] . The full line shows the results of the statistical model in complete equilibrium. The value at RHIC was estimated using results from the STAR collaboration on the K^-/π^- and K^+/K^- ratios, assuming $\pi^-/\pi^+ = 1.007$.

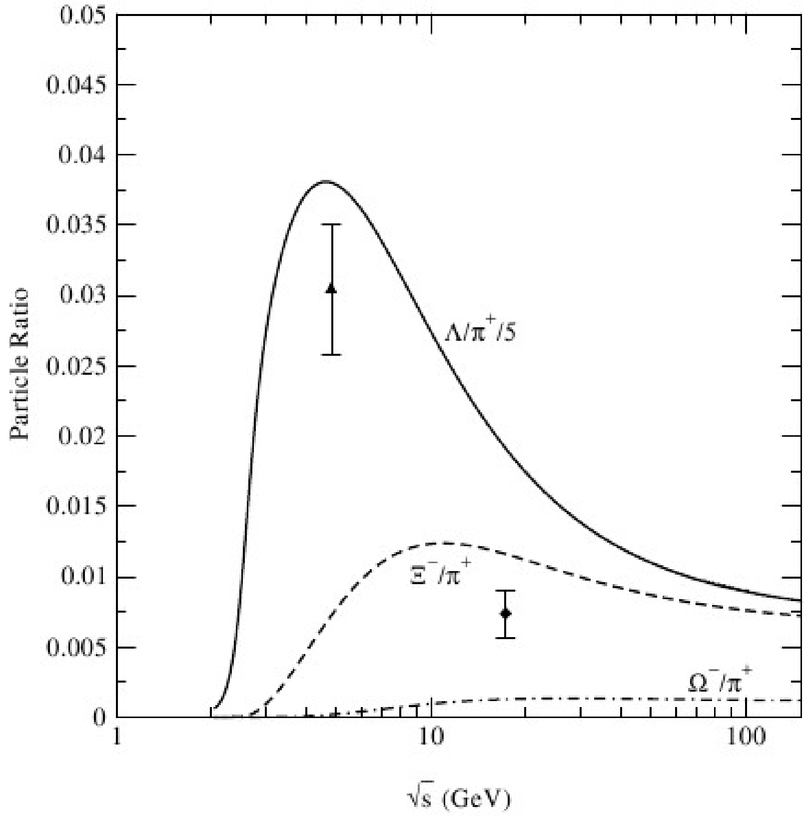
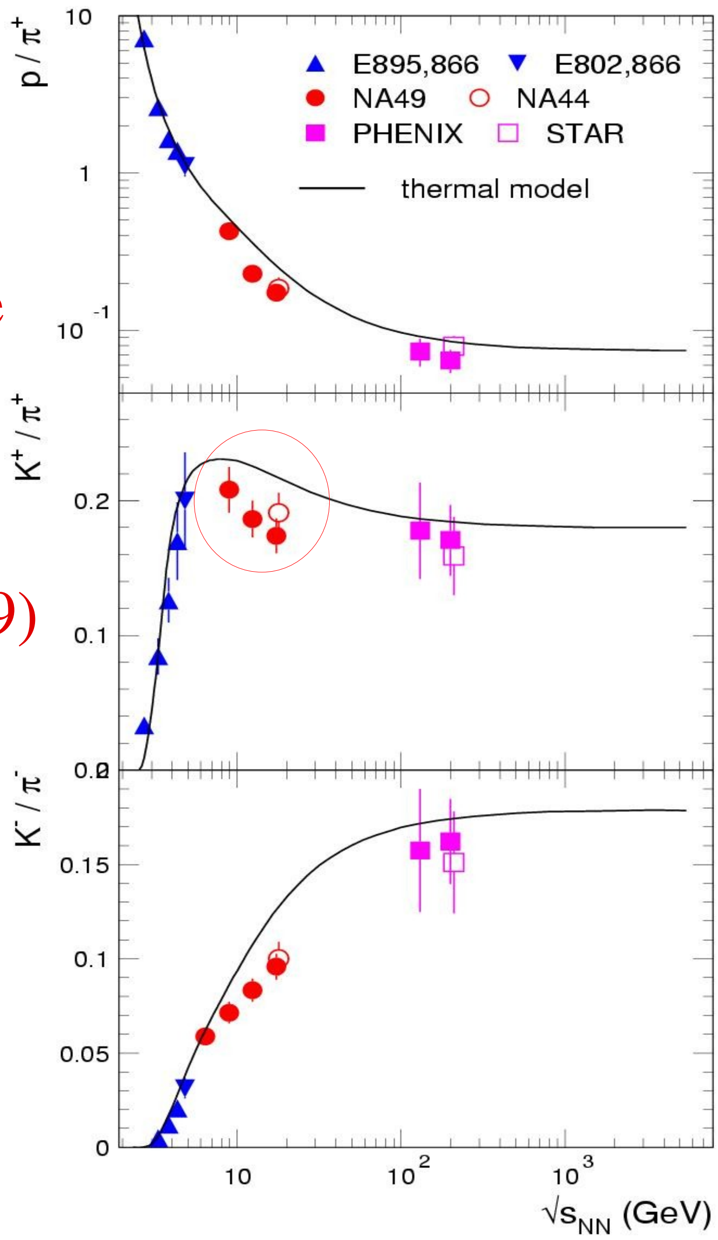


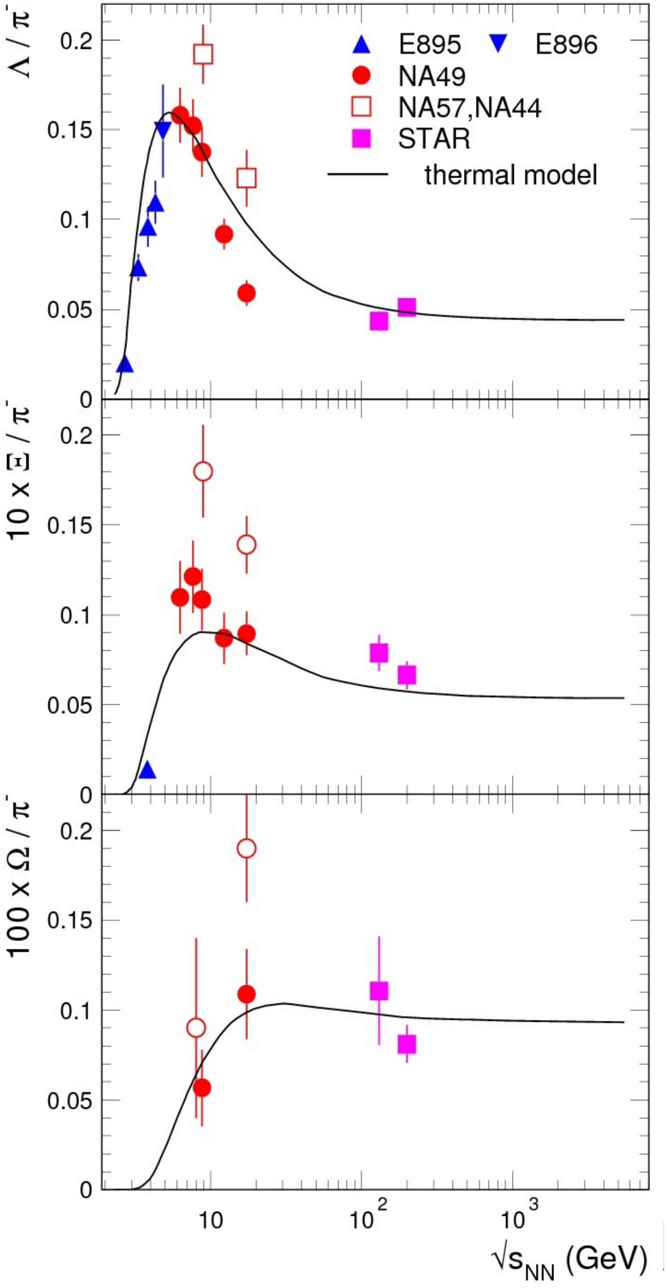
Figure 6: Prediction for the Λ/π^+ (note the factor 5), Ξ^-/π^+ and the Ω^-/π^+ ratios as a function of $\sqrt{s}$. For compilation of data see [15].

but the data evolve and among the different √s dependences K^+/π^+ cannot be well reproduced

A. Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys. A772 (2006) 167

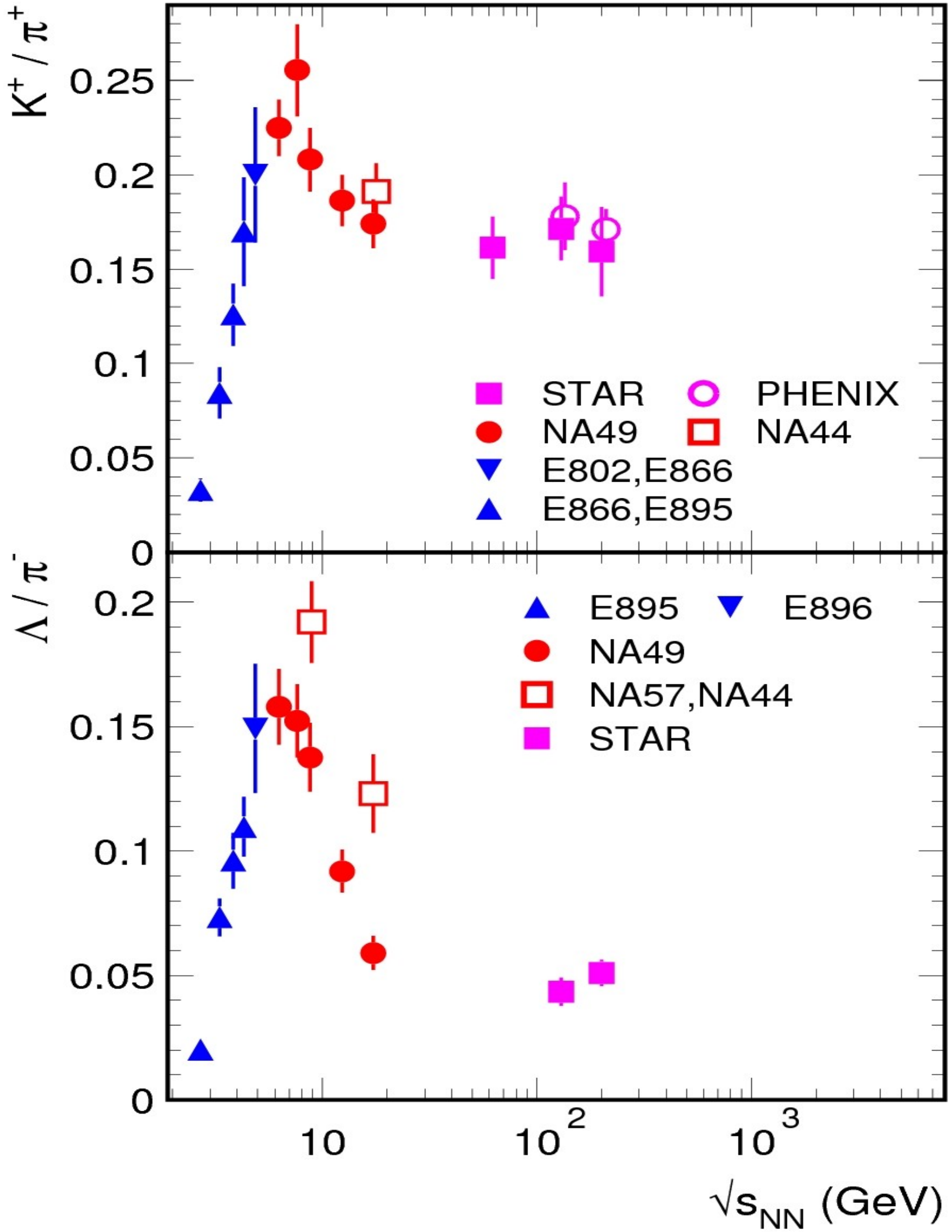
instead: suggestion of evidence for the onset of deconfinement (Gazdzicki, Gorenstein 1999)





why address the issue again now?

- extended and finalized data by NA49 and low energy point from STAR



why address the issue again now?

- study effect of consequences of extended hadronic mass spectrum using all information as of PDG2008

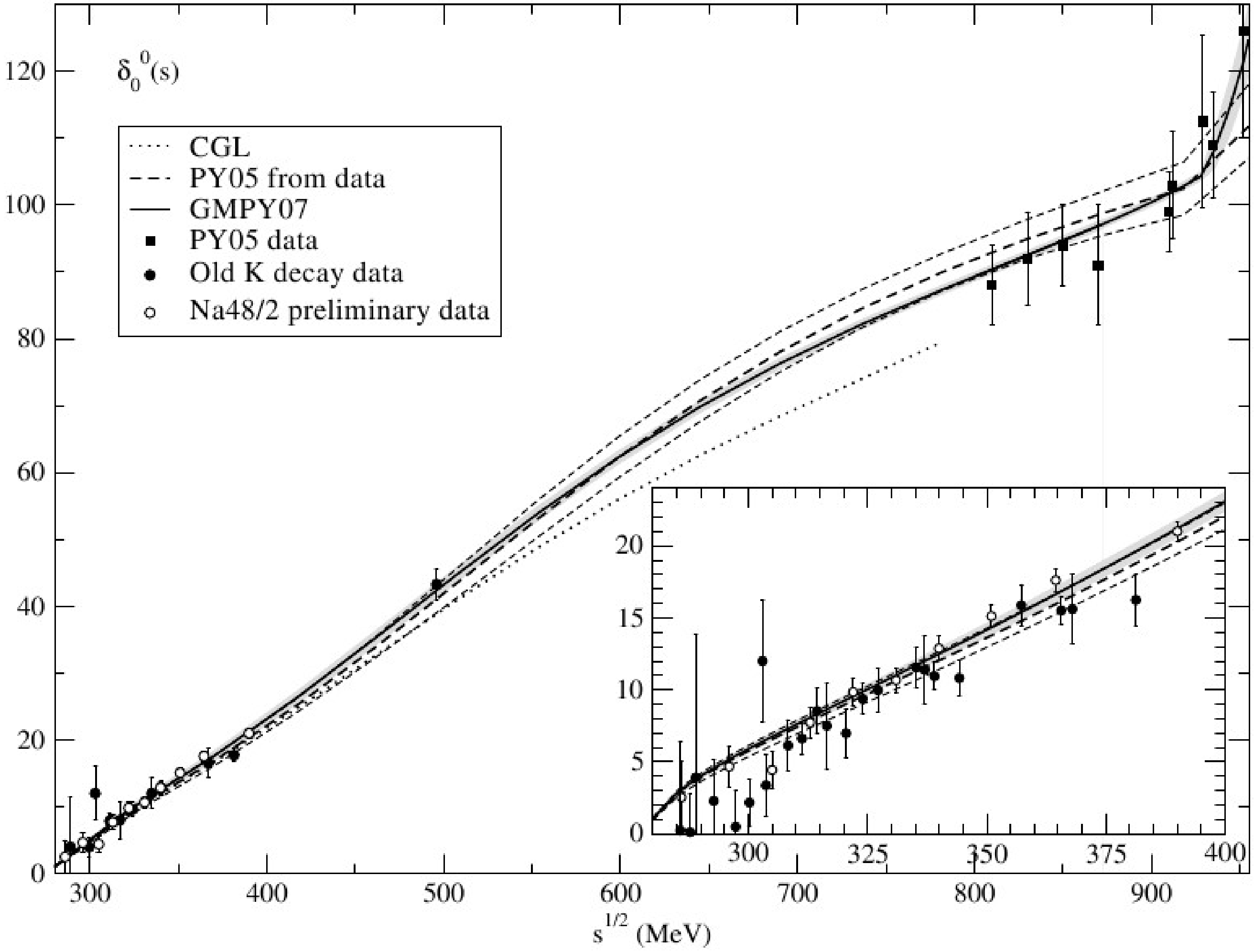
		2005 paper	now		
mesons	nonstrange	37	+ 86	=	123
	strange	28	+ 4		32
	charmed	15	+ 25		40
	beauty	16	+ 12		28
baryons	nonstrange	30	+ 36		66
	strange	33	+ 30		63
	charmed	10	+ 22		32
	beauty	0	+ 14		14
composites		28			28
total		197	+ 229		426

why address the issue again now?

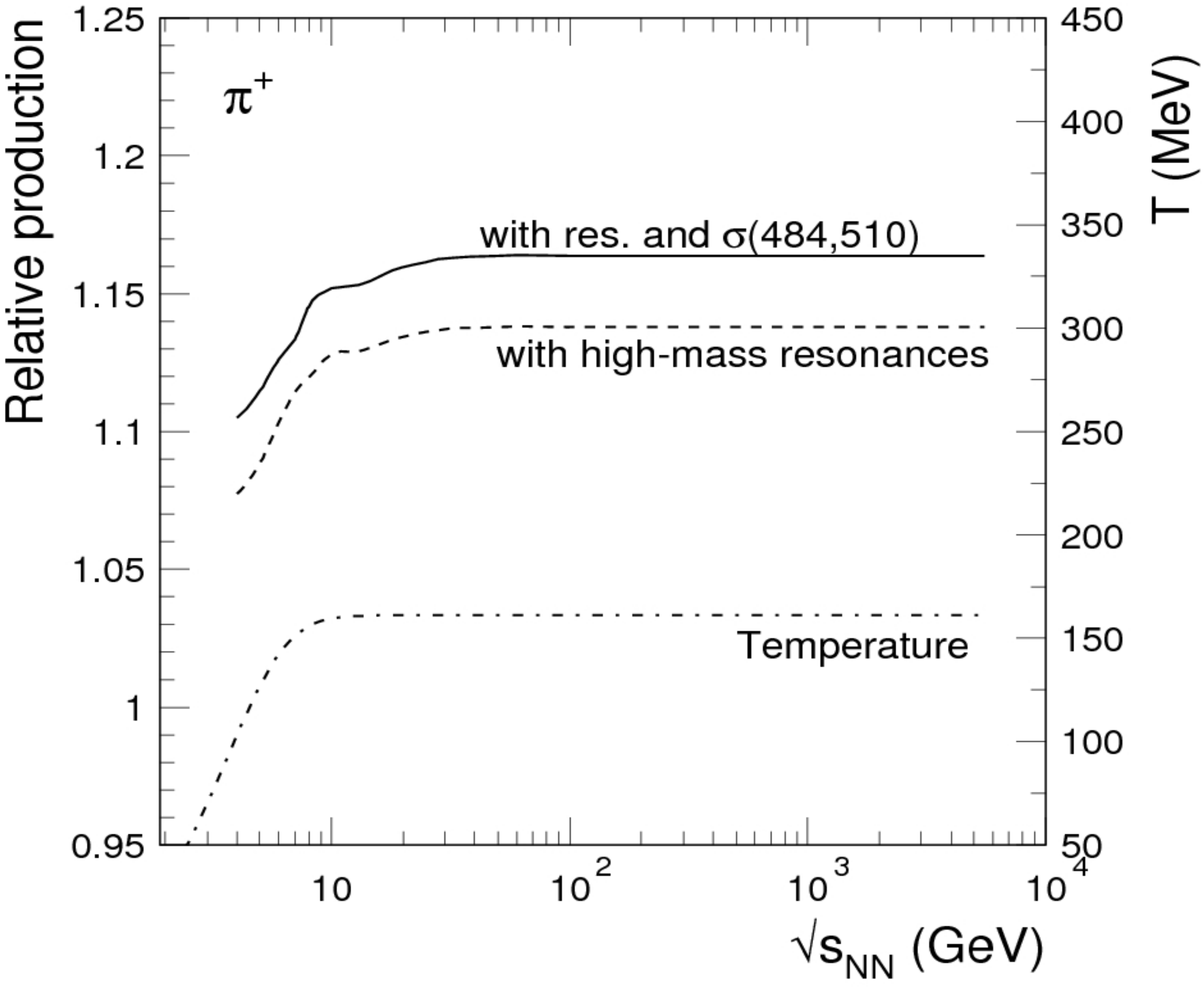
- solidified evidence for the σ meson

analysis by Garcia-Martin, Pelaez, Yndurain Phys. Rev. D76 (2007) 074034, hep-ph/0701025

$$M_\sigma = 484 \pm 17 \text{ MeV}, \quad \Gamma_\sigma/2 = 255 \pm 10 \text{ MeV}$$

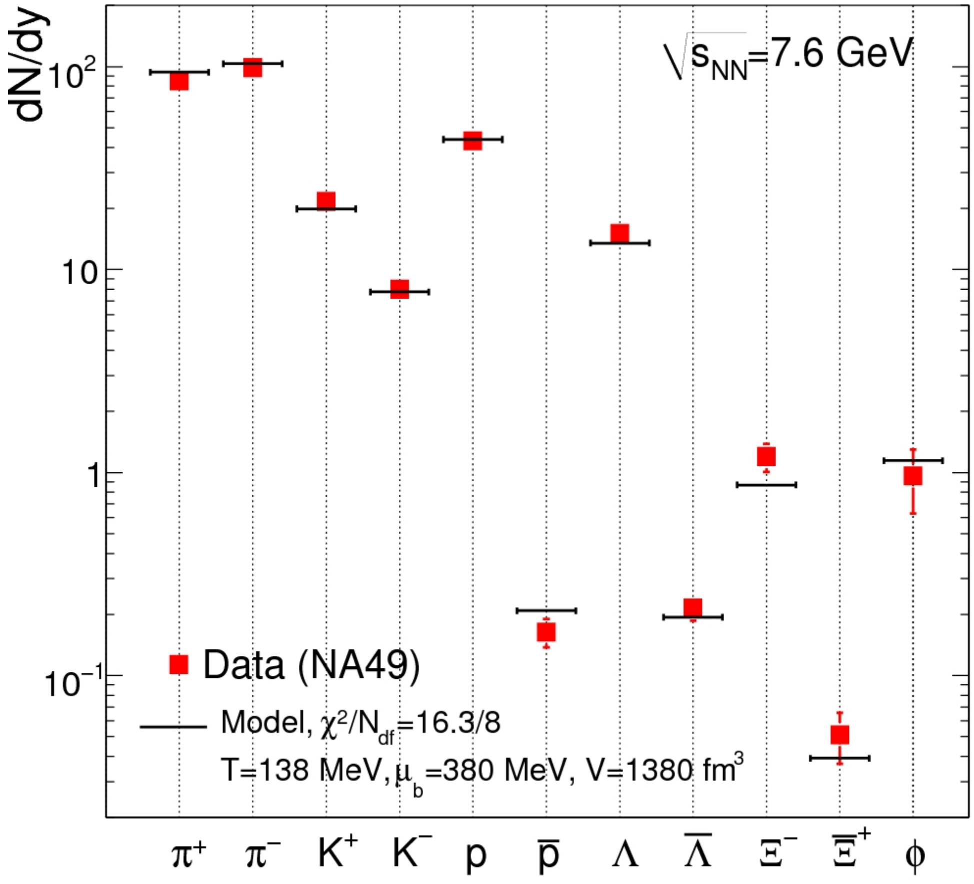


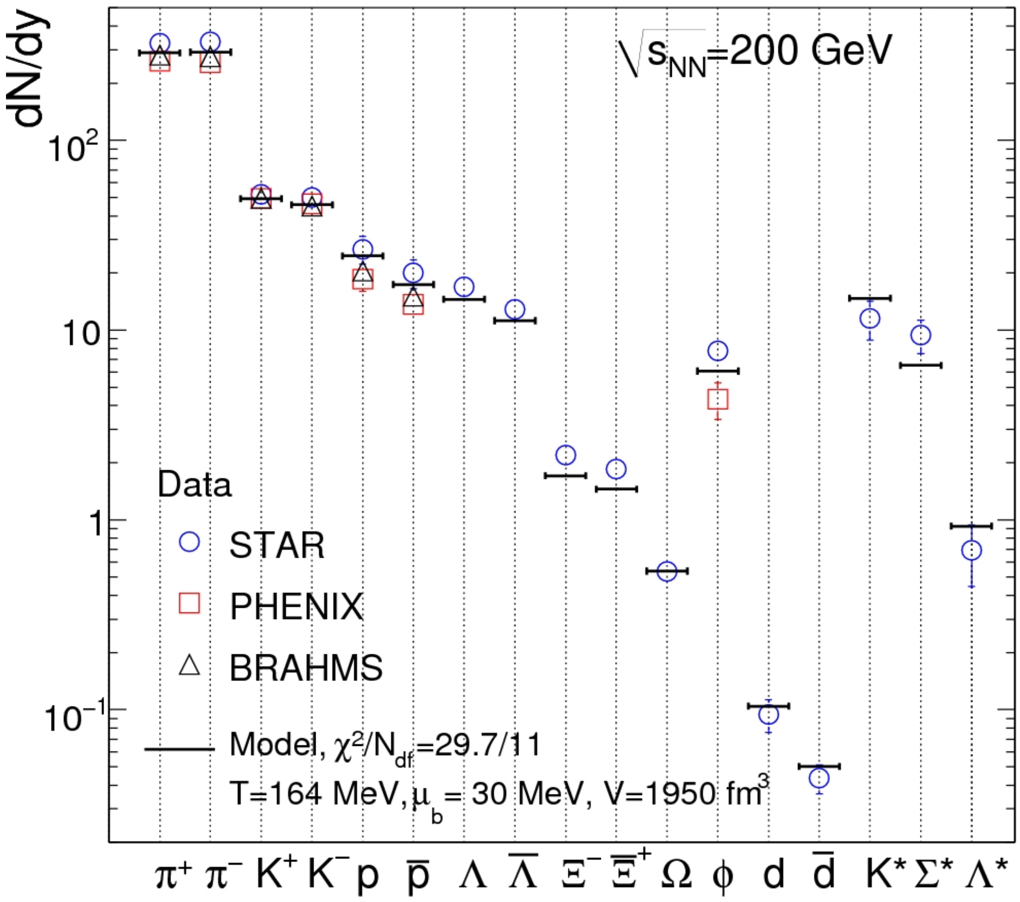
relative change in pion yield with more high mass resonances and the σ



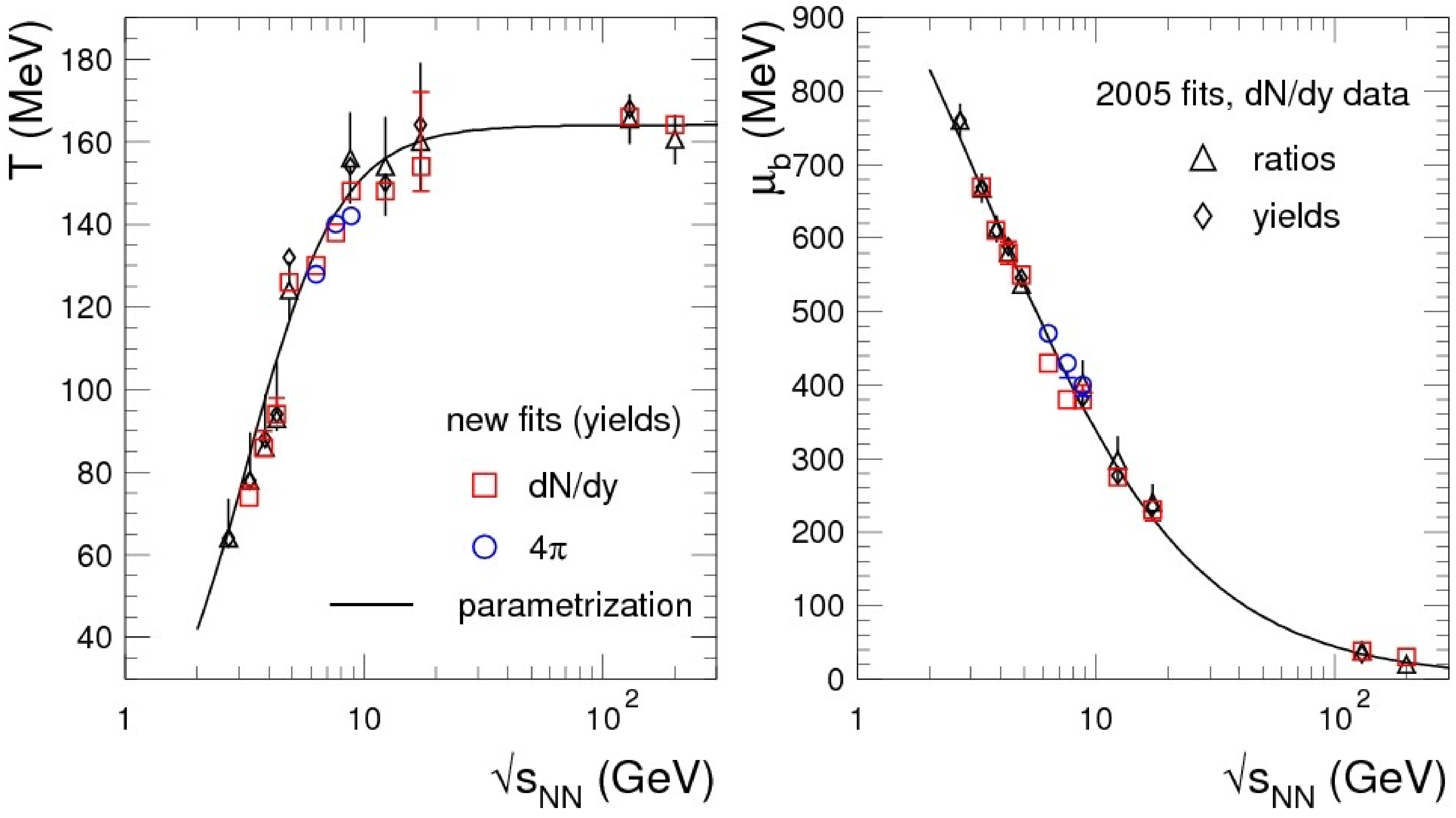
as T levels off,
so does the increase in
pion yield

overall, good fit of hadron yields



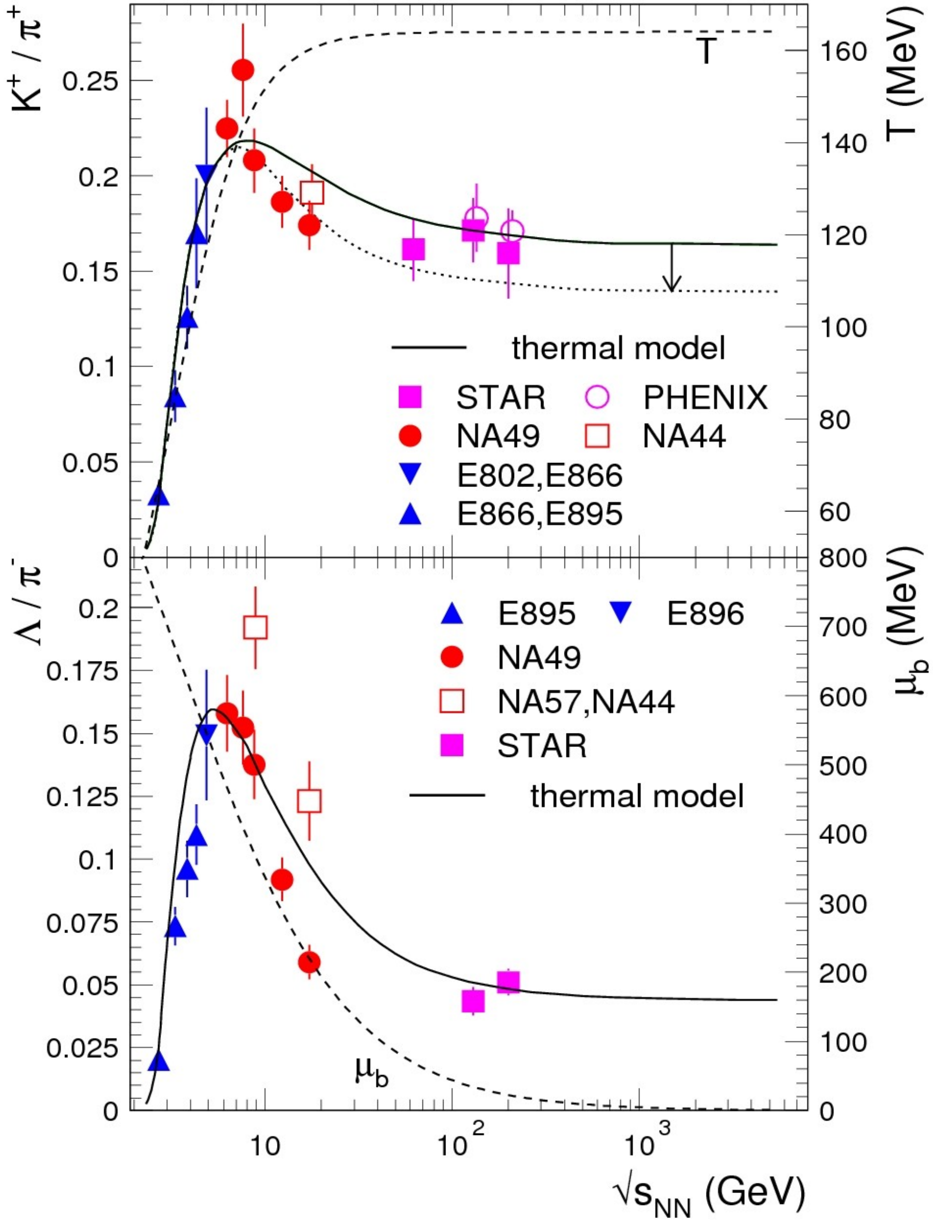


little change in systematic beam energy dependence of T and μ

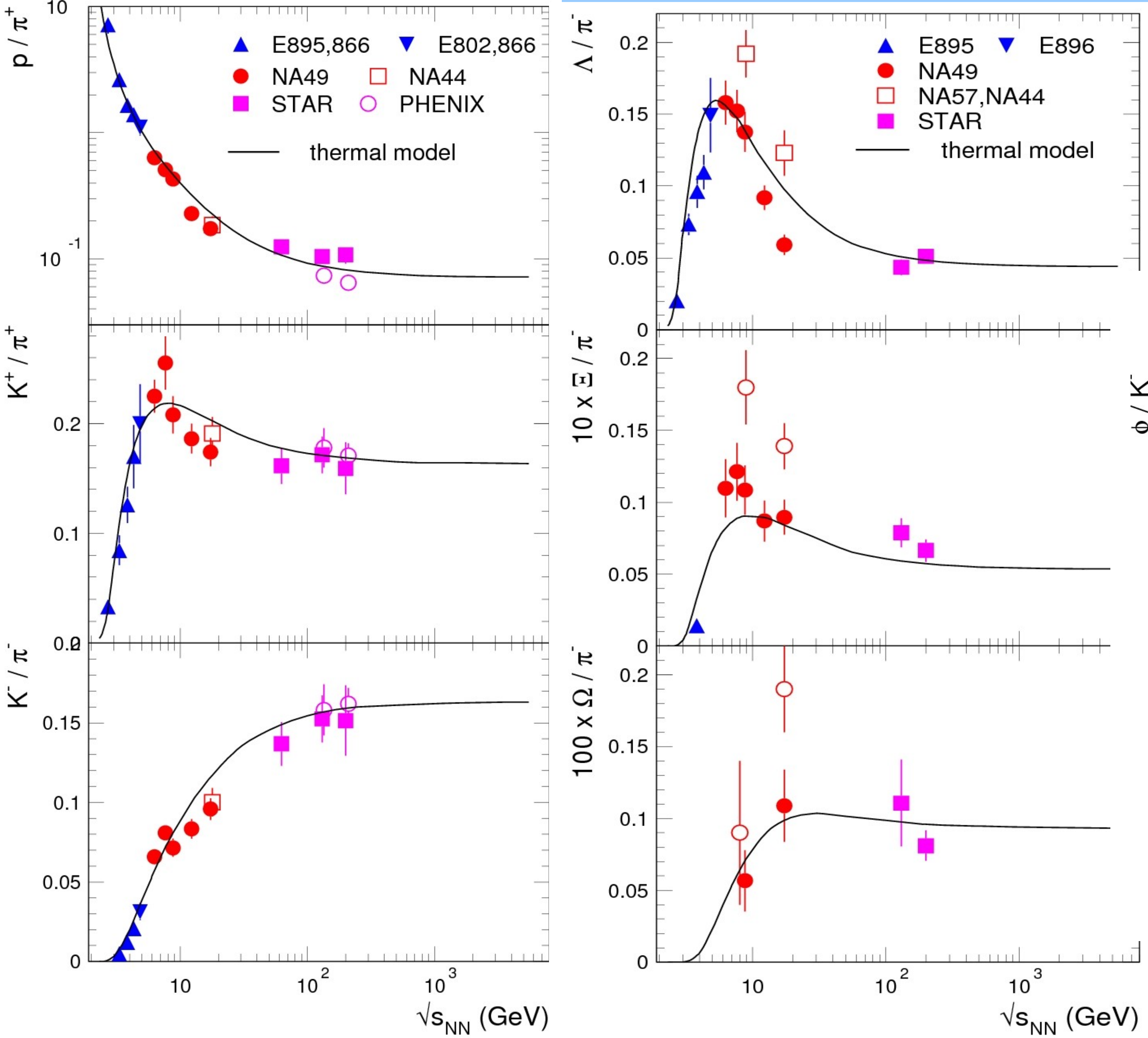


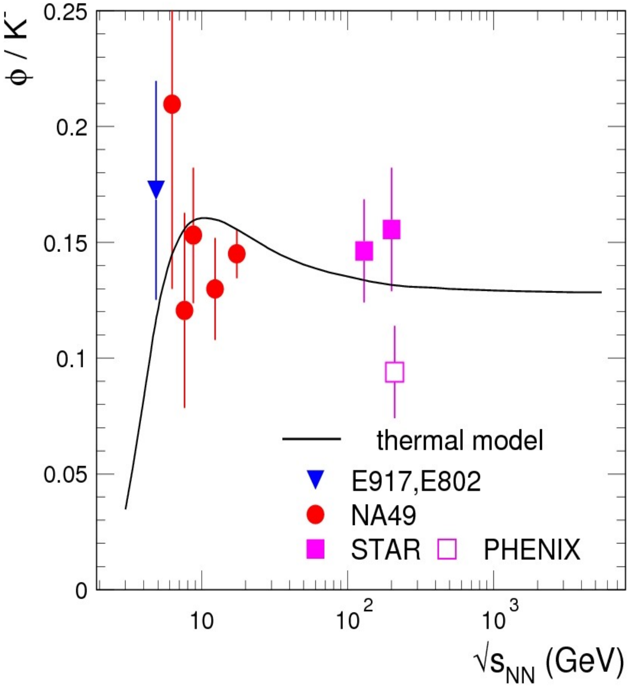
the horn becomes more pronounced in statistical model

levelling off of T and concomittant levelling off of increase in pion yield sharpens the maximum



beam energy dependence of hadron production well reproduced





what is consequence of potentially still incomplete hadron spectrum?

R. Hagedorn's statistical bootstrap model Nuovo Cimento 3 (1965) 147; also CERN-TH.4100/85

strongly interacting particles form resonances (3,4,5,...n) and those may combine to form new resonances

only low lying ones experimentally known

all particles form mass spectrum ρ(m) of which lower part is known

infer asymptotic behaviour from some new principle - ρ(m) should generate itself from a self-consistency condition

density of states
$$\sigma(E,V) = \sum_{n=1}^{\infty} \frac{V^n}{n!} \int \delta\left(E - \sum_{i=1}^{n} E_i\right) \prod_{i=1}^{n} \rho(m_i)\, dm_i\, d^3p_i$$

both ρ(m) and σ(E,V) should be the same if E=m and V is resonance volume V_0

single out one 'elementary' input state m_0 and identity of ρ and σ yields the bootstrap equation

$$\rho(m) = \delta(m - m_0) + \sum_{n=2}^{\infty} \frac{V_0^n}{n!} \int \delta\left(m - \sum_{i=1}^{n} E_i\right) \prod_{i=1}^{n} \rho(m_i)\, dm_i\, d^3p_i$$

'clusters consist of clusters'

physical solution
$$\rho(m) = f(m)\, e^{m/T_0}$$

partition function has singularity at T_0 and possibly exhibits a phase transition

Hagedorn Spectrum

Note: an exponential mass spectrum is expected also in QCD

at least in the large Nc limit (T.L Cohen, arXiv:0901.0494)

what do we know today about Hagedorn spectrum?

Maciej Sobczak – analysis of states listed in PDG2008 compilation

$$f_{FIT}(m) = log_{10}\left(\int_0^m \frac{c}{(x^2+m_0^2)^{5/4}} exp(x/T_H)\right)$$

$$\rho(m) = \frac{c}{(m^2+m_0^2)^{5/4}} exp(m/T_H)$$

$$N_{exp}(m) = \sum_i g_i \Theta\left(m - m_i\right)$$

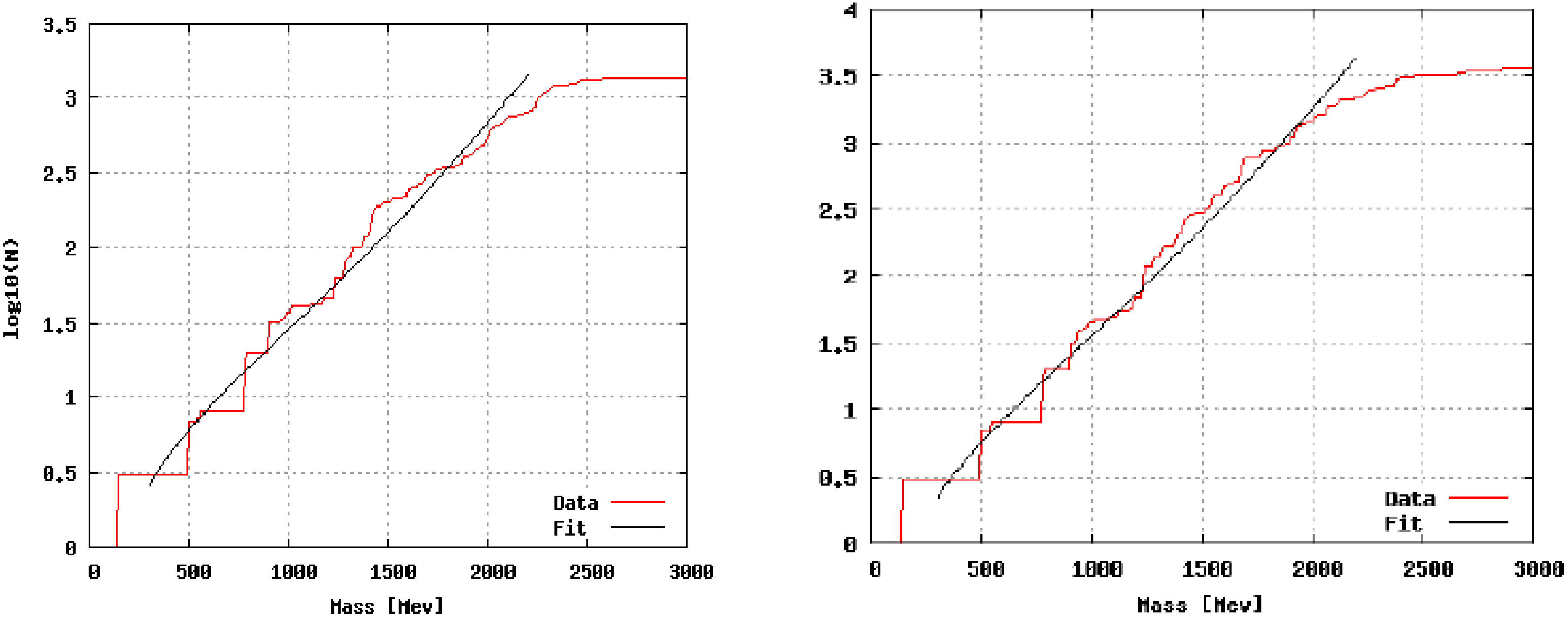


Figure 2: All mesons $T_H = 203.315$, $c = 25132.674$, range: $300 - 2200\ MeV$ All hadrons $T_H = 177.086$, $c = 18726.494$, range: $300 - 2200\ MeV$

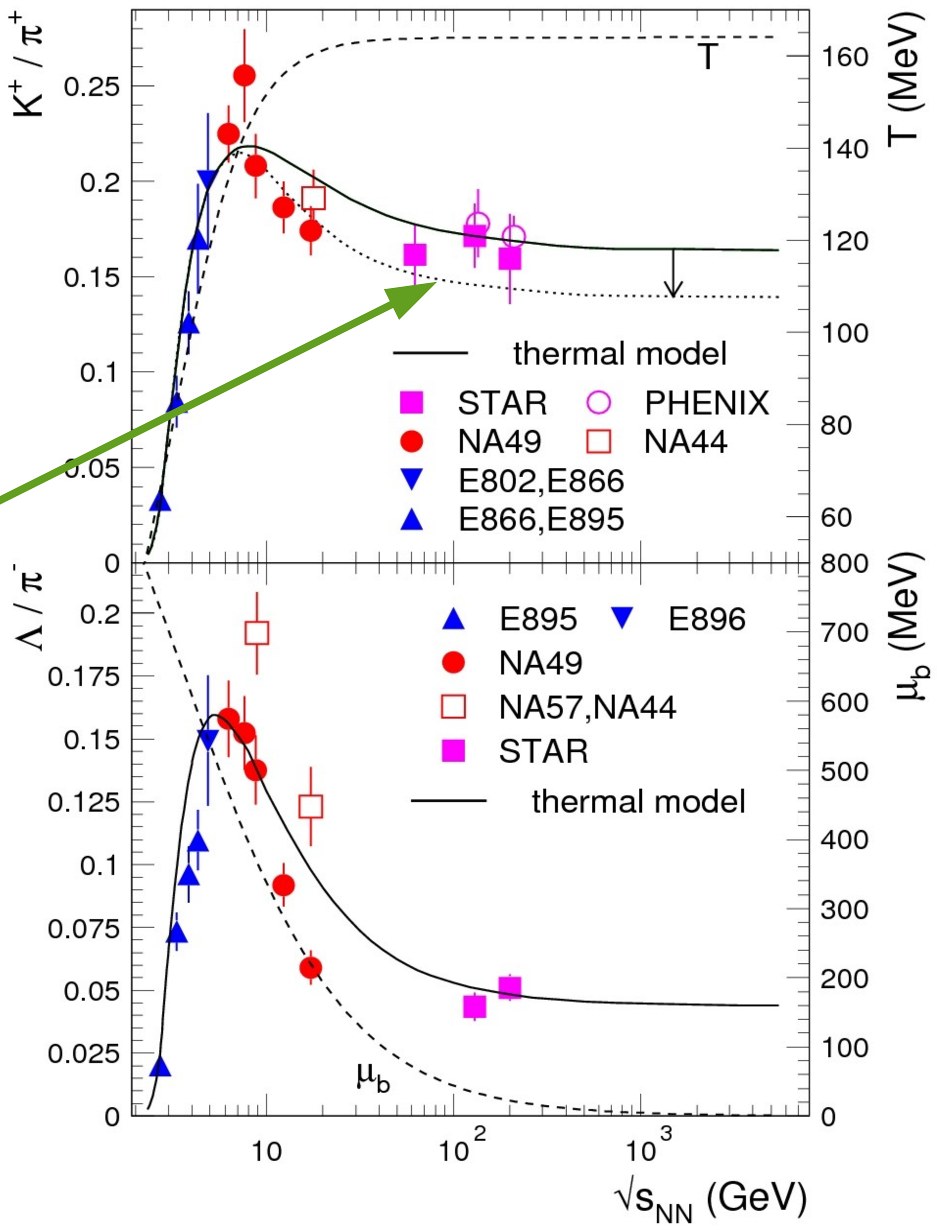
How would this affect K^+/π^+?

estimate effect by extending mass spectrum beyond 3 GeV based on T_H = 200 MeV and assumption how states decay

strongest contribution to kaon from K* producing one K

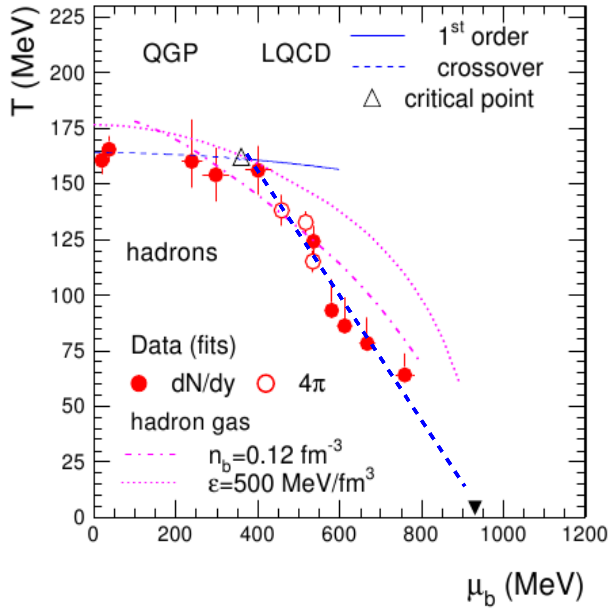
all high mass resonances produce multiple pions

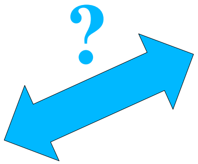
-> further reduction of K^+/π^+

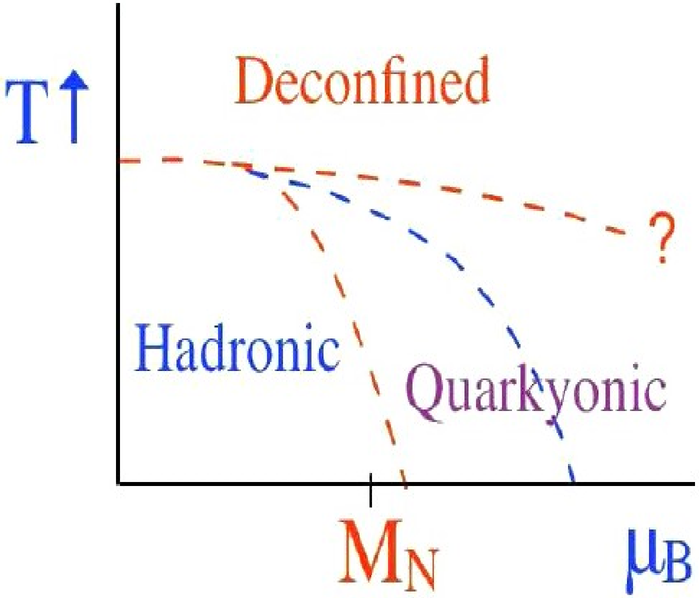


Conclusions

- the horn is produced by the statistical hadronization and the specific variation of the thermal parameters with beam energy
- in nature, rapid equilibration and freeze-out at a common T is achieved at the phase boundary due to steep density dependence on T and μ
  (shown for top SPS energy and above; P. Braun-Munzinger, J. Stachel, C. Wetterich, Phys. Lett. B596 (2004) 61)
- the thermal parameters delineate the phase boundary and thus the horn is due to the location of the phase boundary in the T-μ landscape







From Small $N_c = 2$ to
Large N_c : Proposed Phase Diagram
McLerran, Pisarski, 2007
and Sasaki, Redlich, McLerran 2008

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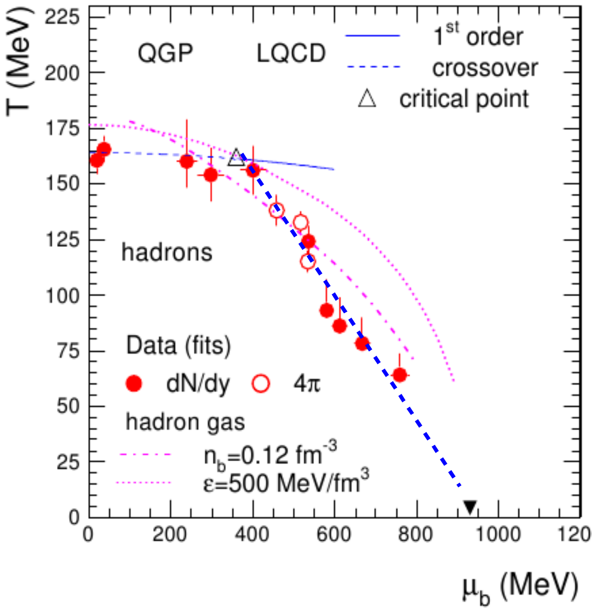

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subject of a publication in preparation by D. Blaschke, P. Braun-Munzinger, J. Cleymans, K. Fukushima, H. Oeschler, R.D. Pisarski, L. McLerran, K. Redlich, C. Sasaki and J.S.