Phases of Strongly-Interacting Matter: the 2+1 PQM model

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EMMI workshop, Wrozlaw, July 9, 2009

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Outline

- primer on symmetries
- the PQM model
- comparison the LQCD data
- finite chemical potential
- outlook

collaborators: M. Wagner, B.-J. Schaefer

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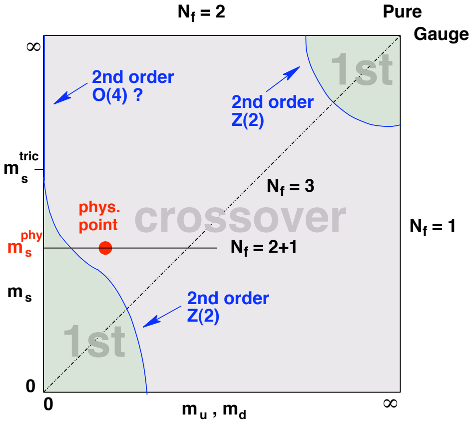
$Z(3)$ center symmetry of $SU(3)_c$ exact for infinitely heavy quarks

$SU(3)_L \times SU(3)_R$ exact for massless m_u, m_d, m_s

Columbia plot'



The 3-flavor QM model

effective chiral theory with quark fields $q(x)$ and meson fields

$\phi_a(x) = (\sigma_a(x) + i\pi_a(x)), \quad a = 0, \ldots 8$

(Gell-Mann Levy 'σ' model)

QM model ($N_f = 3$)

$$\mathcal{L}_{QM} = \bar{q}\left(i\partial\!\!\!/ - GT_a\left(\sigma_a + i\gamma_5\pi_a\right)\right)q + \mathcal{L}_m; \quad T_a = \lambda_a/2$$

$$\begin{aligned}\mathcal{L}_m &= \mathrm{Tr}(\partial_\mu\phi^\dagger\partial^\mu\phi) - m^2\mathrm{Tr}(\phi^\dagger\phi) - \lambda_1[\mathrm{Tr}(\phi^\dagger\phi)]^2 \\ &\quad -\lambda_2\mathrm{Tr}(\phi^\dagger\phi)^2 + c\left(\det(\phi) + \det(\phi^\dagger)\right) \\ &\quad +\mathrm{Tr}[H(\phi+\phi^\dagger)]; \quad \phi = T_a\phi_a;\ H = T_a h_a\end{aligned}$$

only (h_0, h_3, h_8) are non-zero:

$h_0 \neq 0,\ h_3 = h_8 = 0 \rightarrow m_u = m_d = m_s$

$h_0 \neq 0,\ h_3 = 0, h_8 \neq 0 \rightarrow m_u = m_d \neq m_s$

seven parameters fixed to meson sector in the vacuum

in contrast to the NJL model the QM model is renormalizable

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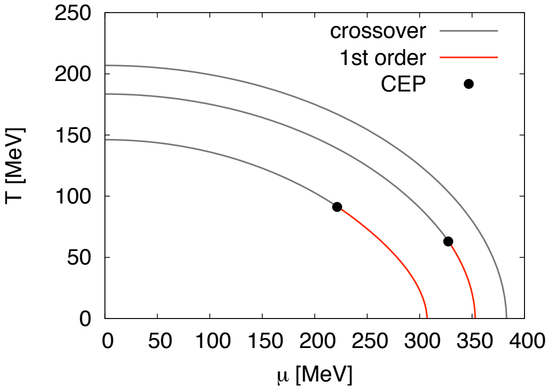
Location of the CEP

location of the (chiral) CEP strongly depends on the mass of the 'sigma' meson

PDG: $f_0(600)$ mass=(400..1200) MeV $\longrightarrow$ broad resonance



$m_\sigma = 600, 800, 900$ MeV

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Including the 'Polyakov loop'

Polyakov loop variable:

$$\Phi(x) \equiv \frac{1}{N_c} \langle Tr_c \mathcal{P}(\vec{x}) \rangle \, ; \qquad \mathcal{P}(\vec{x}) = P \exp\left(i \int_0^{1/T} d\tau A_4(\vec{x}, \tau) \right)$$

Lagrangian:

$$\mathcal{L}_{PQM} \quad = \quad \mathcal{L}_{QM} + \bar{q}\gamma_4 A_4 q - \mathcal{U}(\Phi, \bar{\Phi})$$

effective Polyakov loop potentials:

polynomial (Ratti et al. PRD 2006)

$$\frac{\mathcal{U}_{\text{poly}}}{T^4} = -\frac{b_2(T)}{2}(|\Phi|^2 + |\bar{\Phi}|^2) - \frac{b_3}{6}\left(\Phi^3 + \bar{\Phi}^3\right) + \frac{b_4}{4}(|\Phi|^2 + |\bar{\Phi}|^2)^2$$

with

$$b_2(T) = a_0 + a_1 \left(\frac{T_0}{T}\right) + a_2 \left(\frac{T_0}{T}\right)^2 + a_3 \left(\frac{T_0}{T}\right)^3$$

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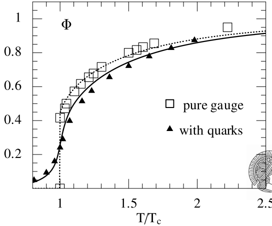
$T_0 = 270$ MeV

a_0	a_1	a_2	a_3	b_3	b_4
6.75	-1.95	2.625	-7.44	0.75	7.5



Including the 'Polyakov loop'

logarithmic (Fukushima PL 2004)

$$\frac{\mathcal{U}_{\log}}{T^4} = -\frac{1}{2}a(T)\bar{\Phi}\Phi + b(T)\ln\left[1 - 6\bar{\Phi}\Phi + 4\left(\Phi^3 + \bar{\Phi}^3\right) - 3\left(\bar{\Phi}\Phi\right)^2\right]$$

with

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 ; \quad b(T) = b_3\left(\frac{T_0}{T}\right)^3$$

$$a_0 = 3.51 \ , \quad a_1 = -2.47 \ , \quad a_2 = 15.2 \ , \quad b_3 = -1.75$$

'strong coupling' (Fukushima PRD 2008)

$$\frac{\mathcal{U}_{\mathrm{Fuku}}}{T^4} = -\frac{b}{T^3}\left[54e^{-a/T}\Phi\bar{\Phi} + \ln\left(1 - 6\Phi\bar{\Phi} - 3(\Phi\bar{\Phi})^2 + 4(\Phi^3 + \bar{\Phi}^3)\right)\right]$$

with

$$a = 664\mathrm{MeV} \ , \quad b = (196.2\mathrm{MeV})^3$$

does not contain contributions from transverse gluons

N_f depencence of T_0 (Schaefer et al. PRD 2007)

N_f	0	1	2	2+1	3
$T_0[\mathrm{MeV}]$	270	240	208	187	178

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Thermodynamic potential

grand canonical potential

$$\Omega(T,\mu;\sigma_x,\sigma_y,\Phi,\bar{\Phi}) = U(\sigma_x,\sigma_y) + \Omega_{\bar{q}q}\left(\sigma_x,\sigma_y,\Phi,\bar{\Phi}\right) + \mathcal{U}\left(\Phi,\bar{\Phi}\right)$$

mesonic potential

$$U(\sigma_x,\sigma_y) = \frac{m^2}{2}\left(\sigma_x^2+\sigma_y^2\right) - h_x\sigma_x - h_y\sigma_y - \frac{c}{2\sqrt{2}}\sigma_x^2\sigma_y + \frac{\lambda_1}{2}\sigma_x^2\sigma_y^2 + \frac{1}{8}\left(2\lambda_1+\lambda_2\right)\sigma_x^4 + \frac{1}{8}\left(2\lambda_1+2\lambda_2\right)\sigma_y^4$$

fermionic part

$$\Omega_{\bar{q}q}(\sigma_x,\sigma_y,\Phi,\bar{\Phi}) = -2T\sum_{f=u,d,s}\int\frac{d^3p}{(2\pi)^3}\left\{\ln\left[1+3(\Phi+\bar{\Phi}e^{-(E_{q,f}-\mu_f)/T})e^{-(E_{q,f}-\mu_f)/T}+e^{-3(E_{q,f}-\mu_f)/T}\right] + \ln\left[1+3(\bar{\Phi}+\Phi e^{-(E_{q,f}+\mu_f)/T})e^{-(E_{q,f}+\mu_f)/T}+e^{-3(E_{q,f}+\mu_f)/T}\right]\right\}$$

mean field approximation

$$\frac{\partial\Omega}{\partial\sigma_x} = \frac{\partial\Omega}{\partial\sigma_y} = \frac{\partial\Omega}{\partial\Phi} = \left.\frac{\partial\Omega}{\partial\bar{\Phi}}\right|_{\min} = 0$$

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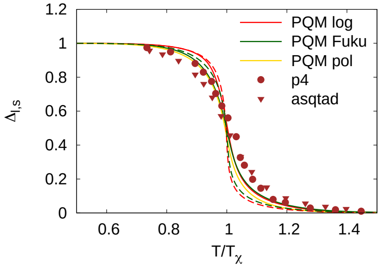
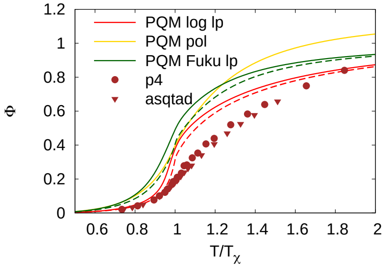
(pseudo) order parameters

lattice data: `Bazavov et al. arXiv:0903.4379 [hep-lat]` $m_\pi \sim 220$ MeV





$$\Delta_{l,s} = \frac{\langle \bar{l}l \rangle(T) - \frac{\hat{m}_l}{\hat{m}_s}\langle \bar{s}s \rangle(T)}{\langle \bar{l}l \rangle(T=0) - \frac{\hat{m}_l}{\hat{m}_s}\langle \bar{s}s \rangle(T=0)} \quad \longrightarrow \quad \Delta_{l,s} = \frac{\sigma_x(T) - \frac{h_x}{h_y}\sigma_y(T)}{\sigma_x(T=0) - \frac{h_x}{h_y}\sigma_y(T=0)}$$

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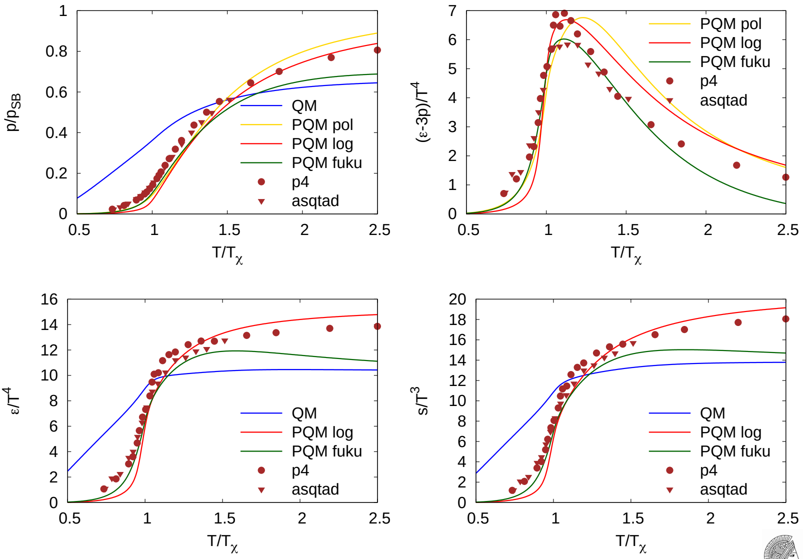
PQM EoS $N_F = 2 + 1 \quad \mu = 0$



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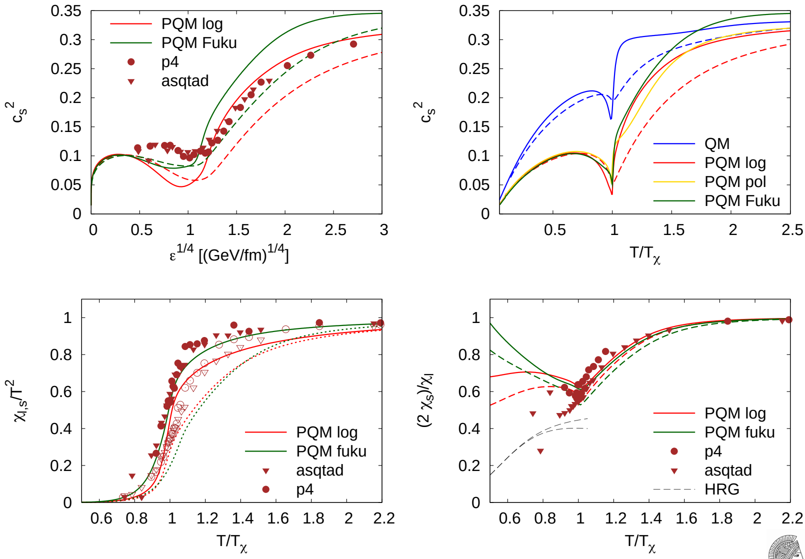
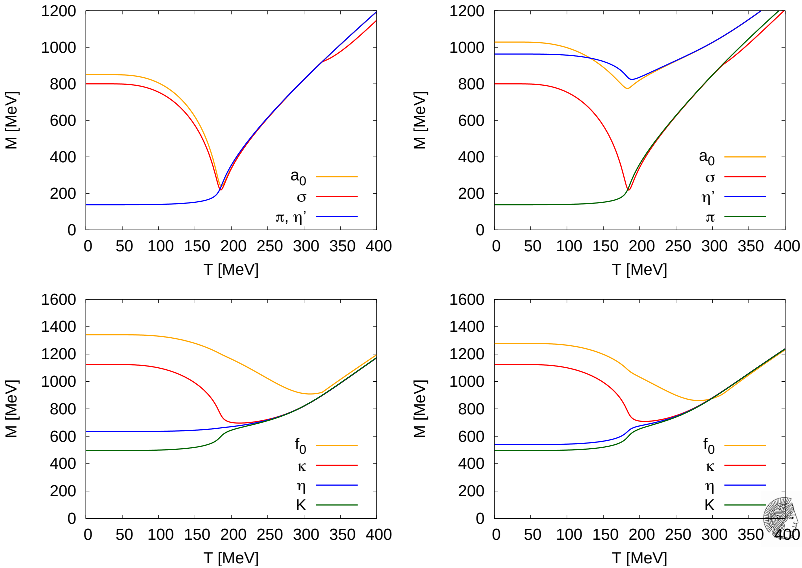


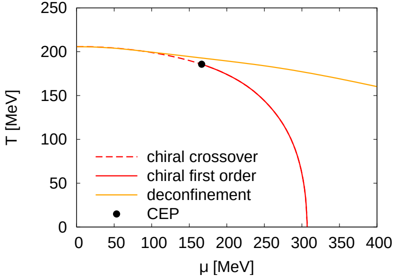
in-medium meson masses $\mu = 0$

without $U_A(1)$ breaking　　　　with $U_A(1)$ breaking

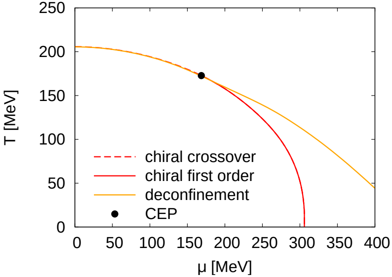


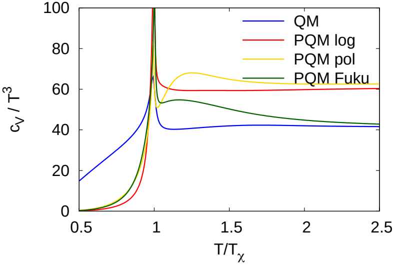
PQM Phase diagram $N_F = 2 + 1$

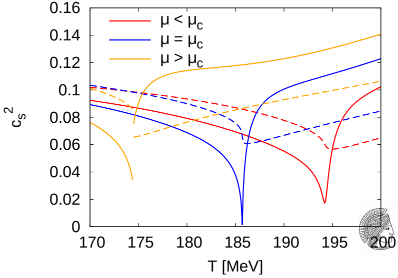
$T_0 = 200$ MeV



$T_0(\mu)$







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EoS at high temperature

at high temperature: p/T^4 is a polynomial in μ/T

high temperature $\leftrightarrow$ ideal relativistic gas

$$\frac{p(T,\mu)}{T^4}_{|\infty} = \frac{N_F}{2\pi T^3}\left(\int_0^\infty dkk^2 \ln(1+z\exp\{-\epsilon(k)/T\}) + \int_0^\infty dkk^2 \ln(1+z^{-1}\exp\{-\epsilon(k)/T\})\right); \qquad z=\exp(\mu/T)$$

QCD at order $O(g^2)$: ($N_F = 2$)
ultra-relativistic limit $\epsilon(k) = k$

$$\frac{p(T,\mu)}{T^4}_{|\infty} = \frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu}{T}\right)^4 - g^2\frac{1}{2\pi^2}\left(\frac{5\pi^2}{36} + \frac{1}{2}\left(\frac{\mu}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu}{T}\right)^4\right)$$

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Taylor expansion

EoS 'only' requires coeff. at $\mu = 0$ (at least in principle)

Taylor expansion:

$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n \quad \text{with} \quad c_n(T) = \frac{1}{n!} \frac{\partial^n (p(T,\mu)/T^4)}{\partial(\mu/T)^n}_{|\mu=0}$$

high-temperature limit:

$$\begin{aligned}
c_0(T \to \infty) &= \frac{7 N_c N_F \pi^2}{180} \\
c_2(T \to \infty) &= \frac{N_c N_F}{6} \\
c_4(T \to \infty) &= \frac{N_c N_F}{12\pi^2} \\
c_n(T \to \infty) &= 0 \quad \text{for} \quad n > 4
\end{aligned}$$

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Thermodynamics for small μ/T

Taylor expansion:

$$\frac{p}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n \simeq c_0 + c_2 \left(\frac{\mu}{T}\right)^2 + c_4 \left(\frac{\mu}{T}\right)^4 + c_6 \left(\frac{\mu}{T}\right)^6 + \cdots$$

number density: $$\frac{n_q}{T^3} = 2c_2 \left(\frac{\mu}{T}\right) + 4c_4 \left(\frac{\mu}{T}\right)^3 + 6c_6 \left(\frac{\mu}{T}\right)^5 + \cdots$$

number susceptibility: $$\frac{\chi_q}{T^2} = 2c_2 + 12c_4 \left(\frac{\mu}{T}\right)^2 + 30c_6 \left(\frac{\mu}{T}\right)^4 + \cdots$$

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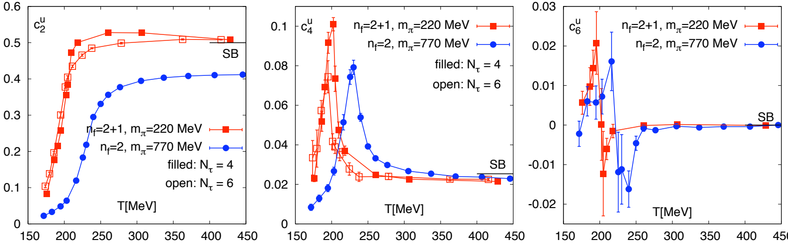
lattice results : `Miao et al. arXiv:0810.0375 [hep-lat]`



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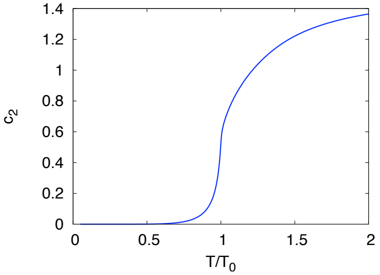
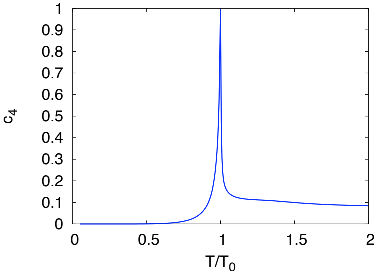
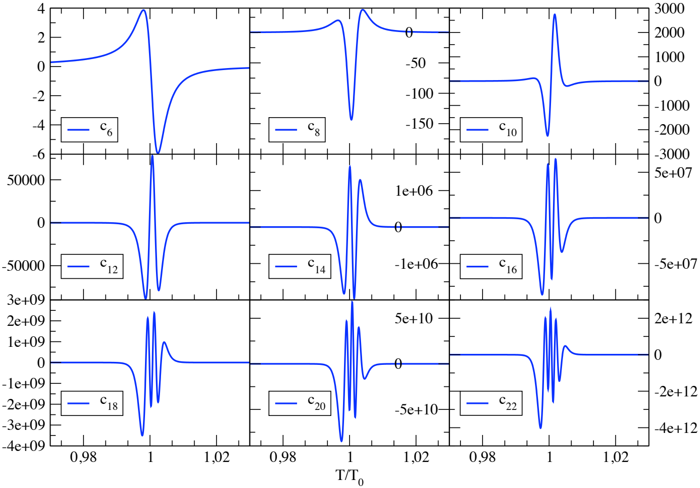
PQM: $(N_F = 2+1)$ M. Wagner et al. '09





Higher-order derivatives

'algorithmic differentiation': M. Wagner et al '09



rapid oscillations for $T \simeq T_c \leftrightarrow$ singularity in the complex plane

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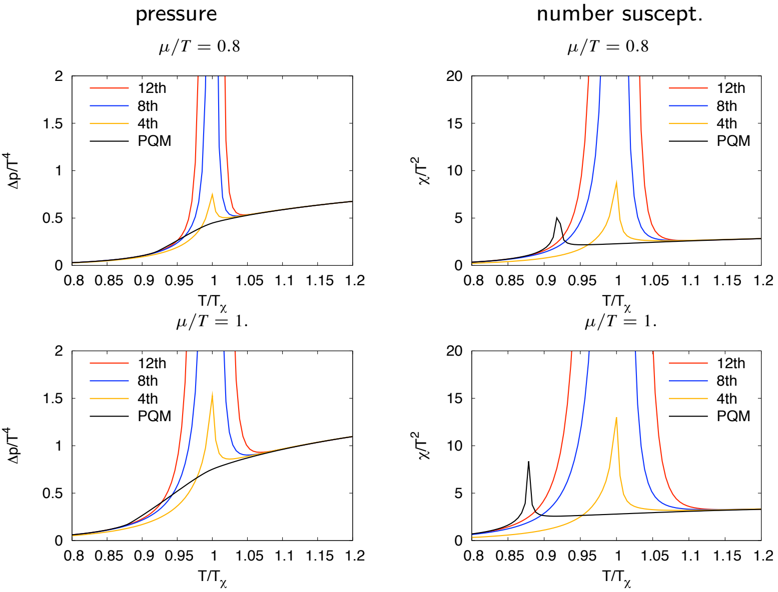
Finite μ extrapolations $N_F = 2 + 1$



Radius of convergence

Taylor expansion:

$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n \quad \text{with} \quad c_n(T) = \frac{1}{n!} \frac{\partial^n (p(T,\mu)/T^4)}{\partial(\mu/T)^n}\Big|_{\mu=0}$$

radius of convergence:

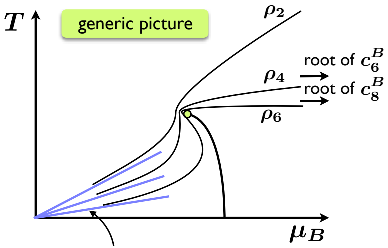
$$r_n = \left|\frac{c_{2n}}{c_{2n+2}}\right|^{1/2}; \quad \text{or} \quad \rho_n = \left|\frac{c_2}{c_{2n}}\right|^{1/(2n-2)}$$

estimate CEP from Taylor coefficients?

$$\left(\frac{\mu}{T}\right)_c = \lim_{n\to\infty} \sqrt{\left|\frac{c_{2n}}{c_{2n+2}}\right|}$$

$$= \lim_{n\to\infty} r_n = \lim_{n\to\infty} \rho_n$$

need $c_n(T) > 0$ for singularity on the real axis

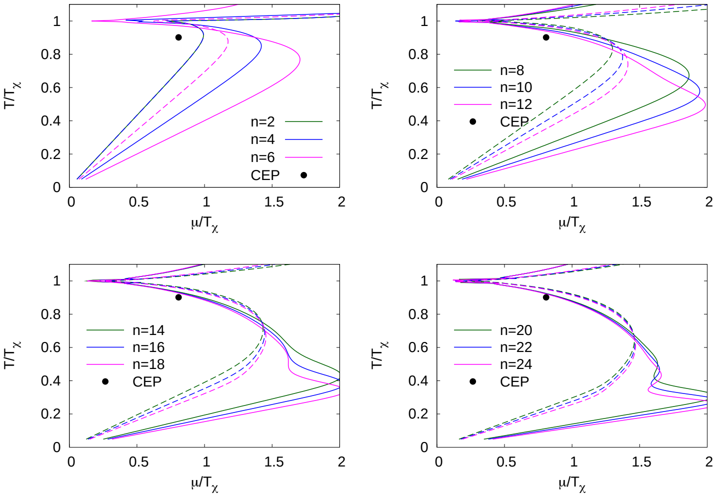


[C. Schmidt '08]

Radius of convergence

M. Wagner et al '09

r_n: solid lines $\quad$ ρ_n : dashed lines



Summary and Conclusions

findings:

- based on relevant symmetries of QCD the PQM model incorporates chiral symmetry breaking and (stastistical) confinement
- parameters determined in the vaccum
- location of the CEP depends on σ-mass
- lattice EoS and fluctuations at $\mu = 0$ well reproduced by mean-field approx.
- assess properties not easily accessible on the lattice (pole masses, chiral limit,..)
- test latttice procedures for the CEP (convergence of Taylor expansion)

outlook:

- singularity in the complex μ-plane
- fluctuations beyond mean-field via RG
- transport properties
- .....