QCD Thermodynamics and the Polyakov Loop

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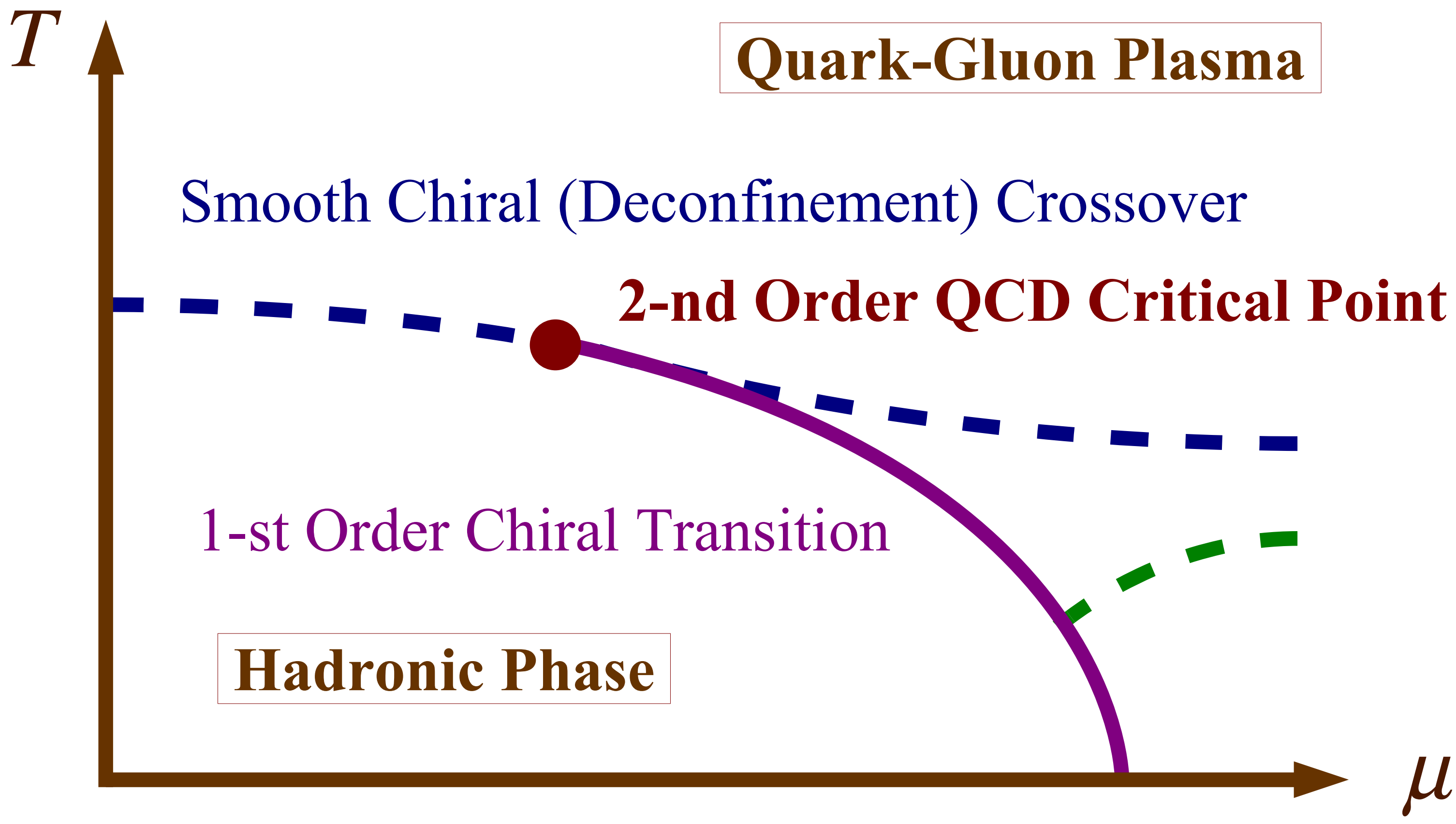
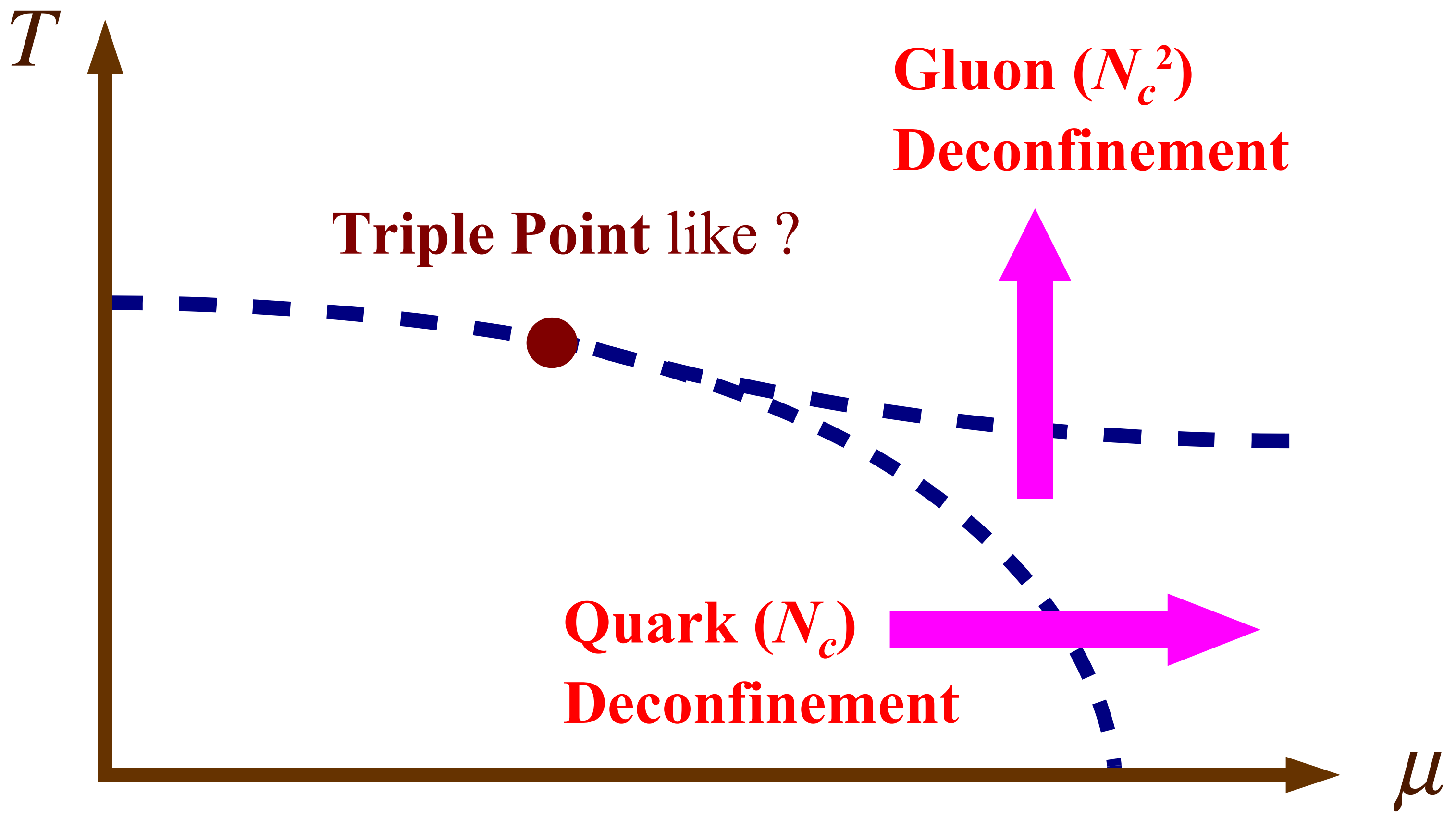
Talk Outline

- **QCD Thermodynamics from Above**
  - Degrees of freedom = Quarks + Gluons
  - Near $T_c \rightarrow$ Quarks + Polyakov Loop
- **Polyakov Loop = Color Screening Factor**
  - Controls all thermal degrees of freedom
  - Parametrization by the Polyakov loop
- **Suggestion to the "Triple Point"**
  - Phase diagram with a TP and a CP
  - Some thermodynamics and observables

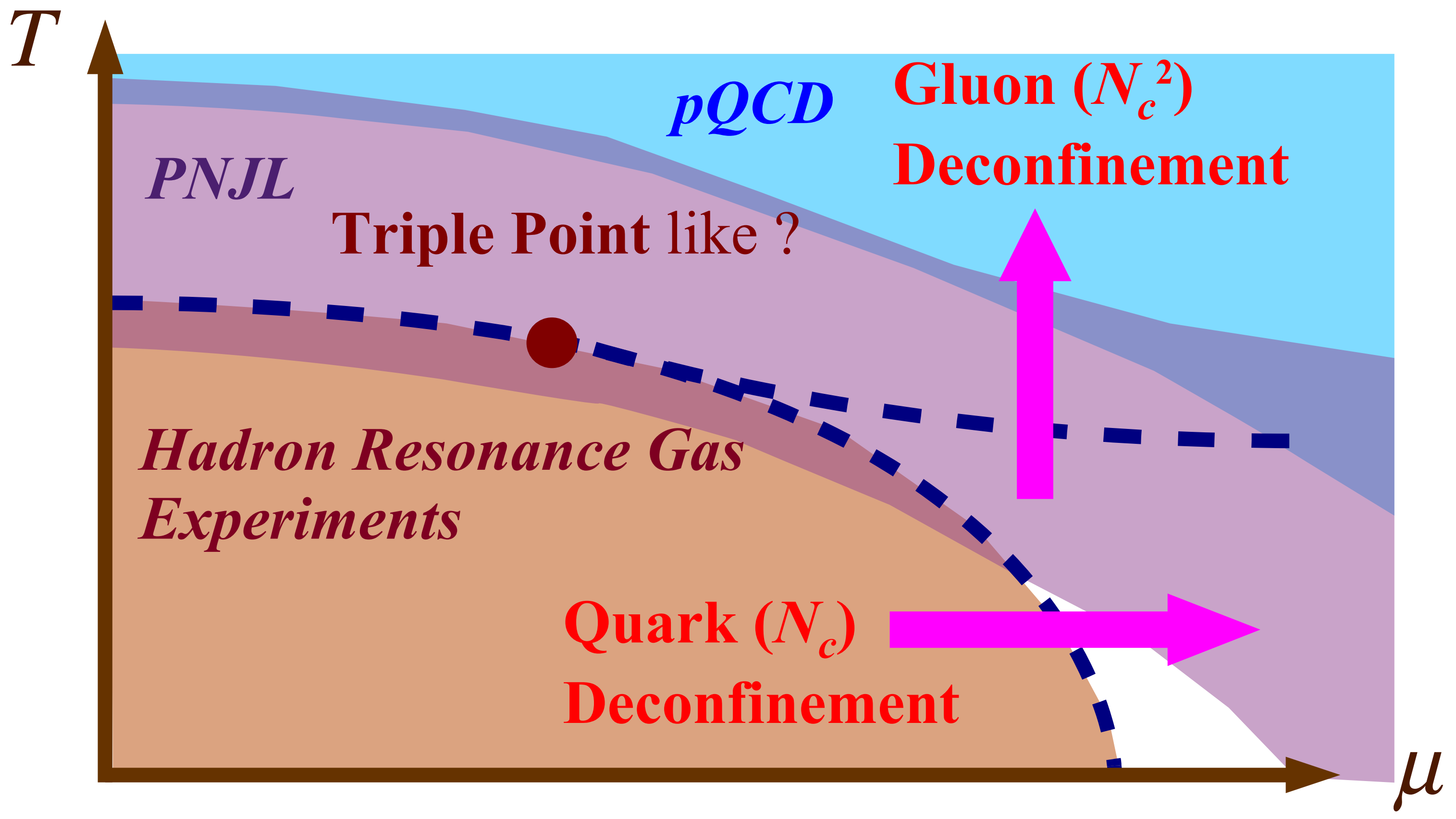
Conventional QCD Phase Diagram



Large N_c Suggestion



Methods and Validity Regions



Degrees of Freedom Above T_c

Quarks and Gluons

- 2 (spin) x 2 (antiparticle) x 2+1 (flavors) x 3 (colors) x (7/8) = **21 ~ 31.5** quarks
- 2 (polarization) x 8 (colors) = **16** gluons

Gluons → Color Phase Factor of Quarks

- Polyakov loop

$$L = P\exp\left[ig\int dx_4 A_4\right]$$
$$\ell = \langle \mathrm{tr}\, L \rangle$$

Only one gluon instead of sixteen... is this enough?

Polyakov Loop

- **Quarks without the Polyakov loop**

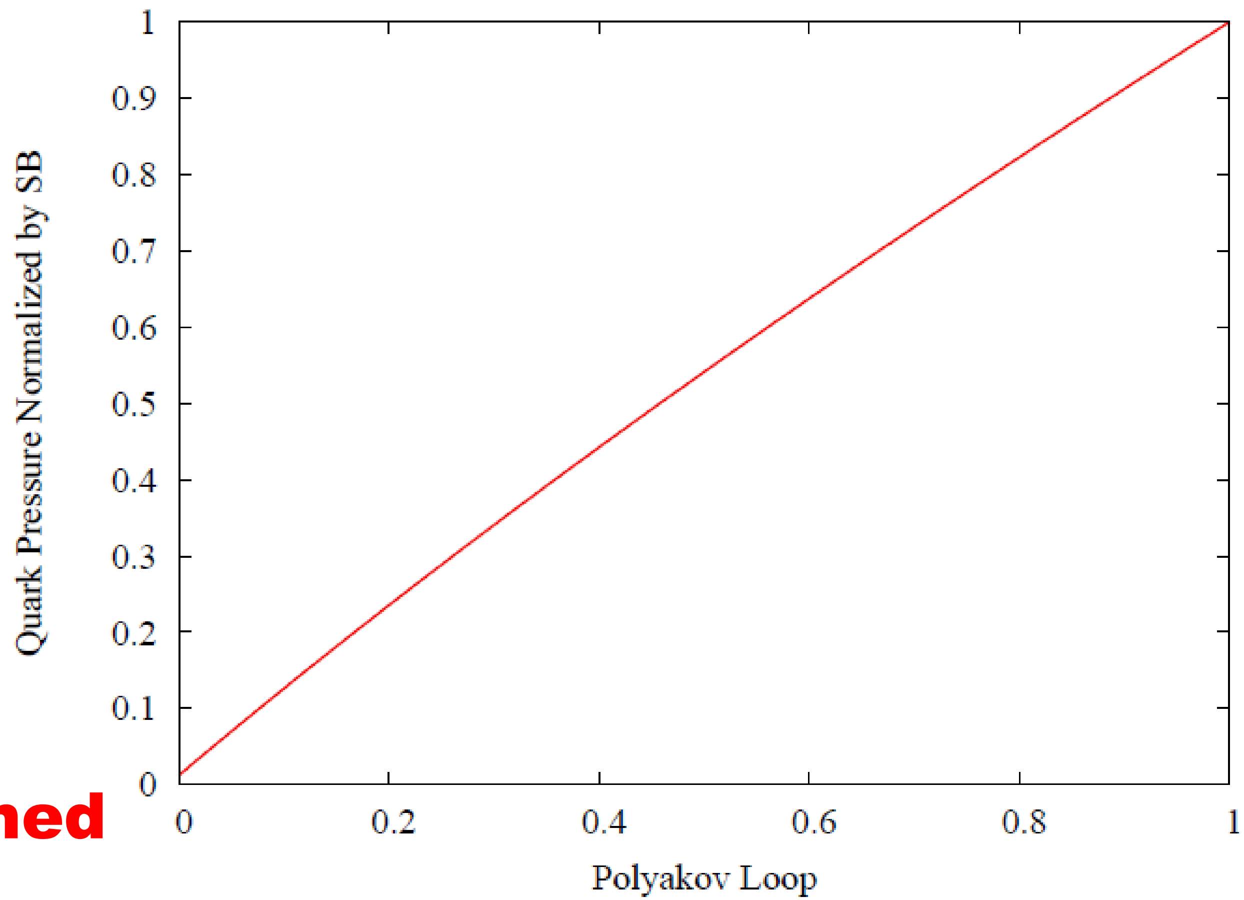
$$2N_f N_c \int \frac{d^3k}{(2\pi)^3} \left[\log\left(1+\mathrm{e}^{-(E-\mu)/T}\right) + \log\left(1+\mathrm{e}^{-(E+\mu)/T}\right) \right]$$

- **Quarks with the Polyakov loop**

$$2N_f \int \frac{d^3k}{(2\pi)^3} \mathrm{tr}\left[\log\left(1+L\,\mathrm{e}^{-(E-\mu)/T}\right) + \log\left(1+L^\dagger\,\mathrm{e}^{-(E+\mu)/T}\right) \right]$$

Massless Quarks

Pressure controled by the Polyakov loop



Confined **Deconfined**

$\sim \ell$

Quarks are not massless $\rightarrow$ Interplay between Chiral and Loop

Polyakov Loop

- **Gluons without the Polyakov loop**

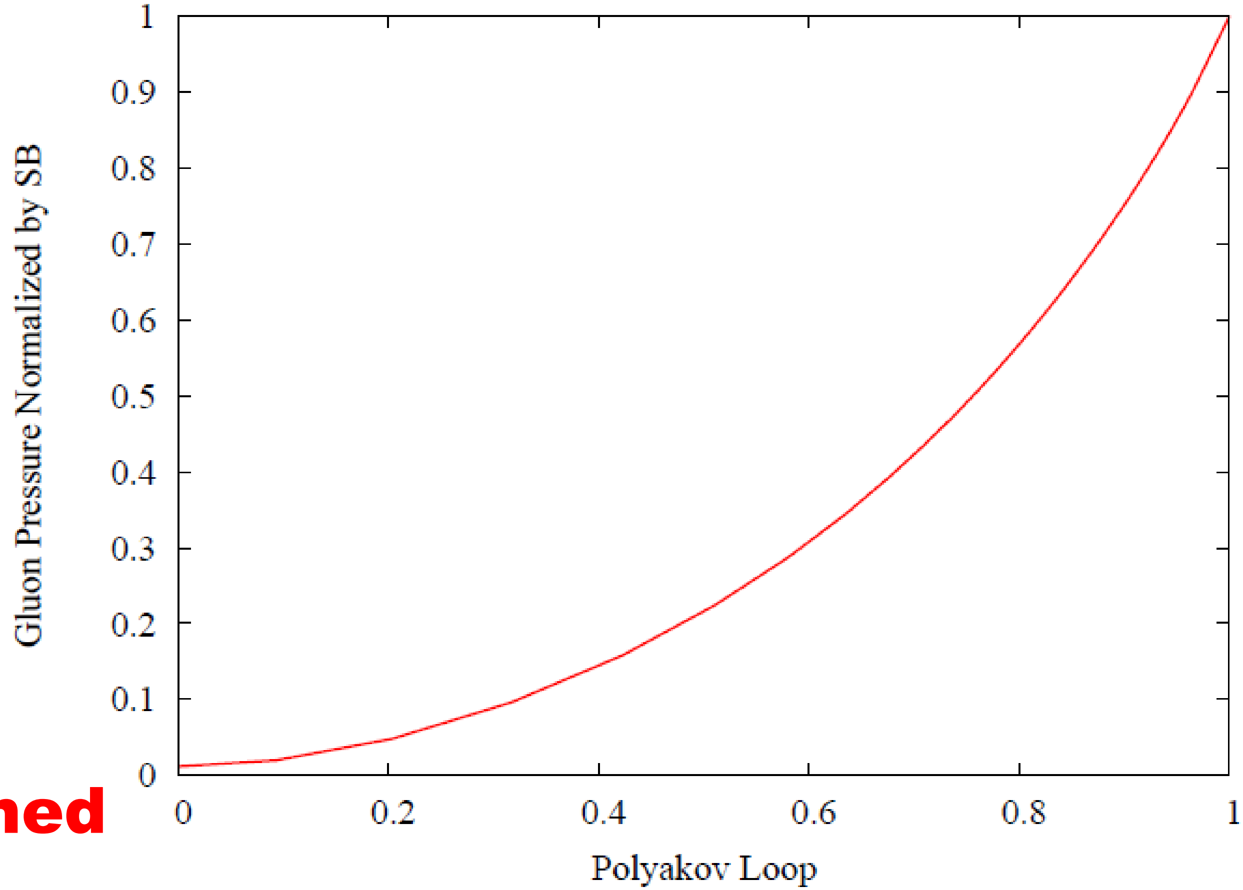
$$-(N_c^2-1)\int\frac{d^3k}{(2\pi)^3}\log\left(1-\mathrm{e}^{-k/T}\right)$$

- **Gluons with the Polyakov loop**

$$-\int\frac{d^3k}{(2\pi)^3}\,\mathrm{tr}\log\left(1-L_A\,\mathrm{e}^{-k/T}\right)$$

Massless Gluons

Pressure controlled by the Polyakov loop



Confined **Deconfined**

$$\sim \ell^2$$

Tsai-Mueller (2008)

Assumptions

- **Quark Part** – *Quasi-particle description*
  - Quarks = free fermionic particles with mean-field mass
    + Polyakov loop coupling
  - Gluons = parametrized by T dependent potential

- **Gluon Part** – *Polyakov loop potential*
  - Functional form = Vandermonde potential
  - Parameters = fixed by "experimental" data
    (i.e. lattice QCD without quarks)

Polyakov loop model
Matrix model
by Rob Pisarski

Munich Polyakov Loop Potential

Ansatz

$$V(\ell) = -\frac{1}{2}a(T)\ell\bar{\ell} + b(T)\log\left[1-6\ell\bar{\ell}+4(\ell^3+\bar{\ell}^3)-3(\ell\bar{\ell})^2\right]$$

Parameters

$$a(T) = T^4(3.51-2.47t^{-1}+15.2t^{-2})$$
$$b(T) = -1.75t^{-3}\cdot T^4$$
$$t = T/T_c$$

3 parameters

Ratti-Thaler-Weise
Ratti-Roessner-Weise

c.f.

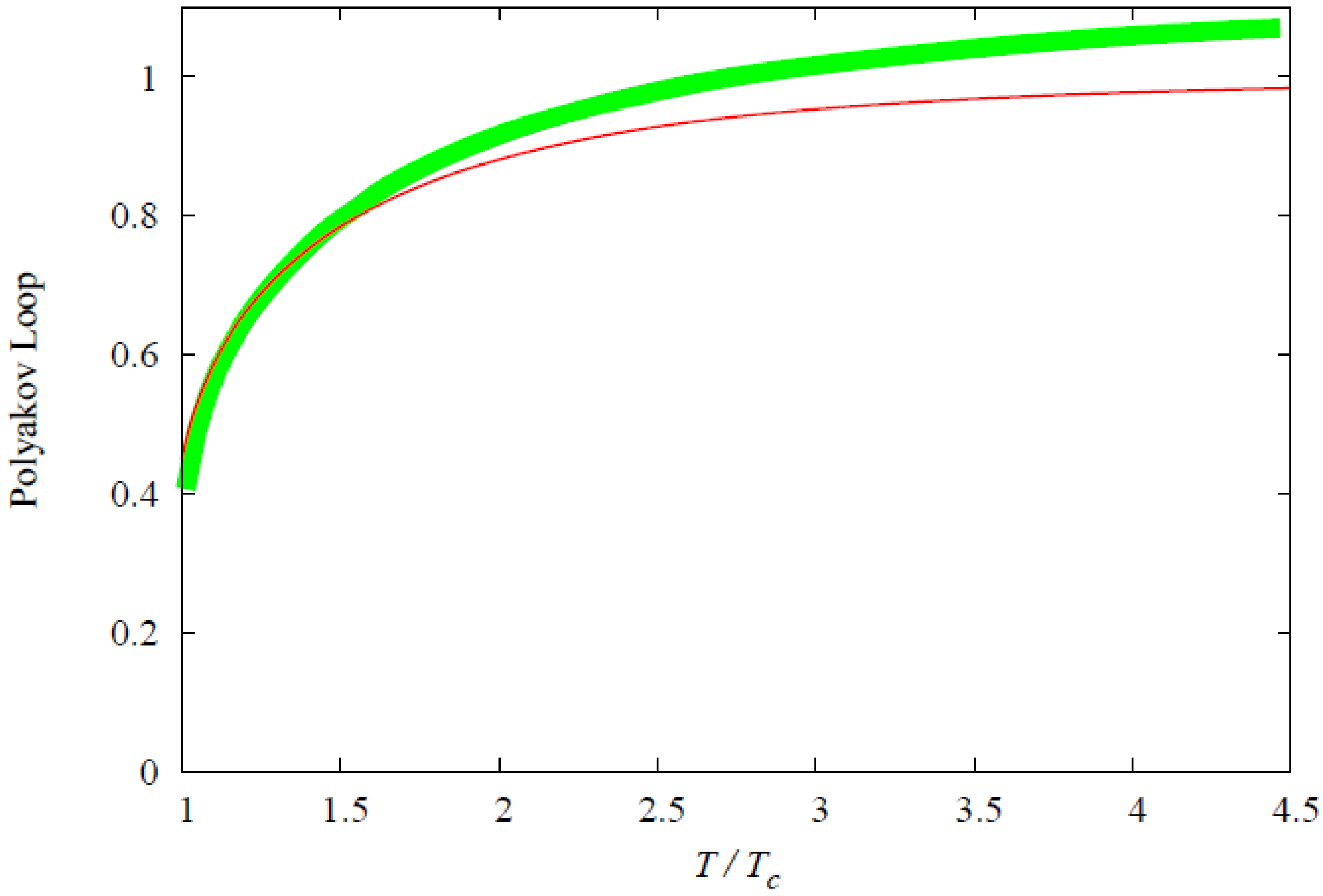
$$a(T) = T\cdot b\cdot 54e^{-a/T}$$
$$b(T) = T\cdot b$$

2 parameters

Fukushima

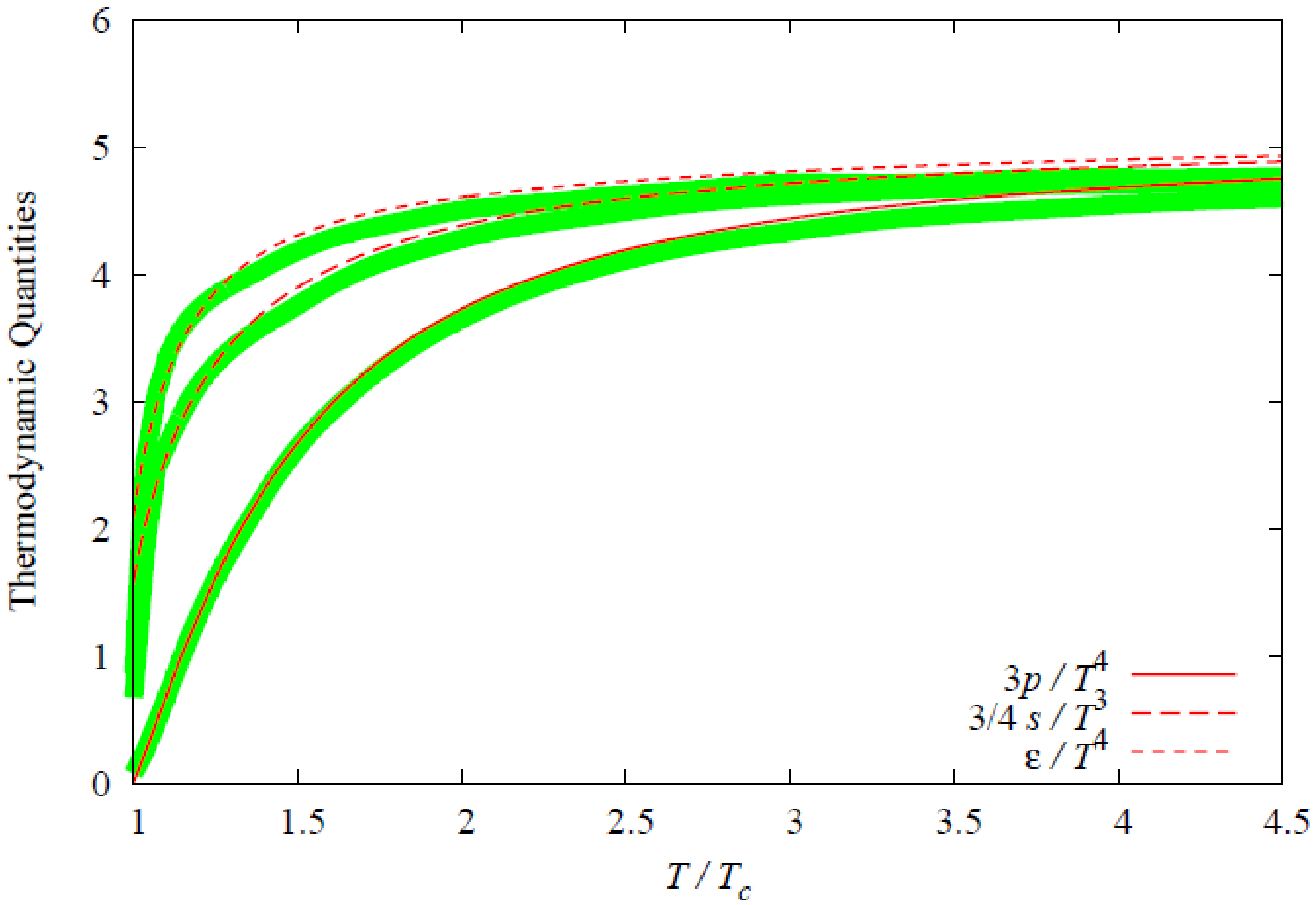
Polyakov Loop

Reproducing the “experimental” data



Thermodynamics

Reproducing the "experimental" data



Mean-Field Mass of Quarks

Nambu--Jona-Lasinio model

$$L = \bar{\psi}(i\gamma\cdot\partial - m) + \frac{g_S}{2}\left[(\bar{\psi}\lambda\psi)^2 + (\bar{\psi}i\gamma_5\lambda\psi)^2\right]$$
$$+ g_D\left[\det\bar{\psi}(1-\gamma_5)\psi + \text{h.c.}\right]$$

Parameters

$\Lambda = 631.4\,\text{MeV}$

$m_{ud} = 5.5\,\text{MeV}$

$m_s = 135.7\,\text{MeV}$

$g_S\Lambda^2 = 3.67$

$g_D\Lambda^5 = -9.29$

⇐

m_π

f_π

m_K

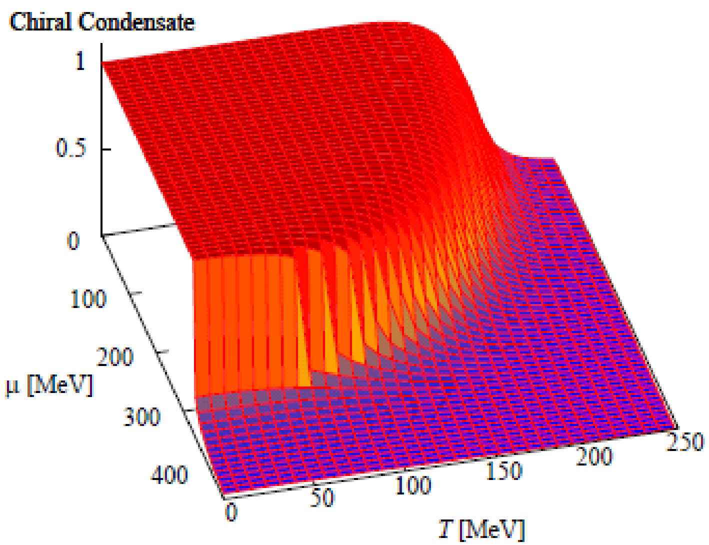
$m_{\eta'}$

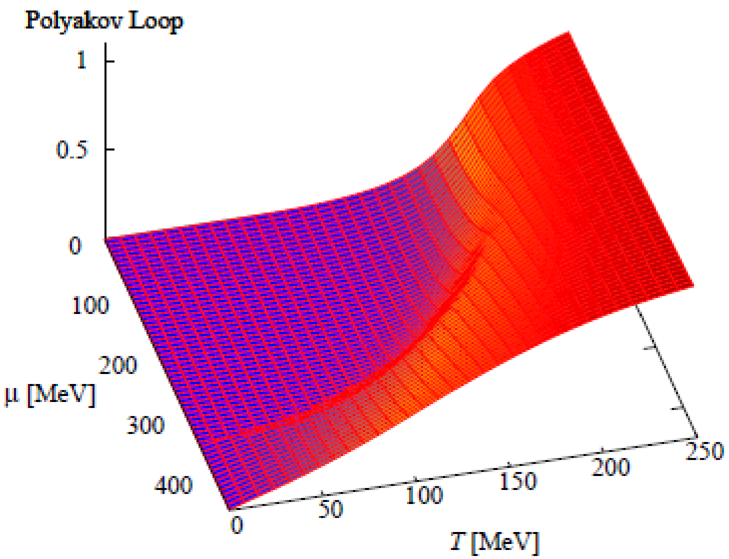
one more? M_{ud}

Dangerous at high μ_B

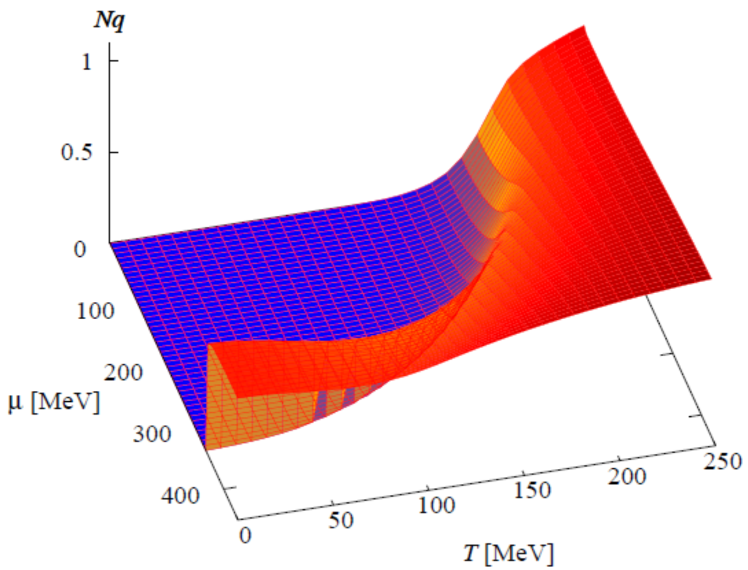
Phase Structure

3D Plots – Chiral Condensate and Polyakov Loop





Quark Density



KF (2008)

Remarks

Chiral Transition

- Chiral condensate melts at high T and high μ
- First order phase transition at high μ

Deconfinement Transition

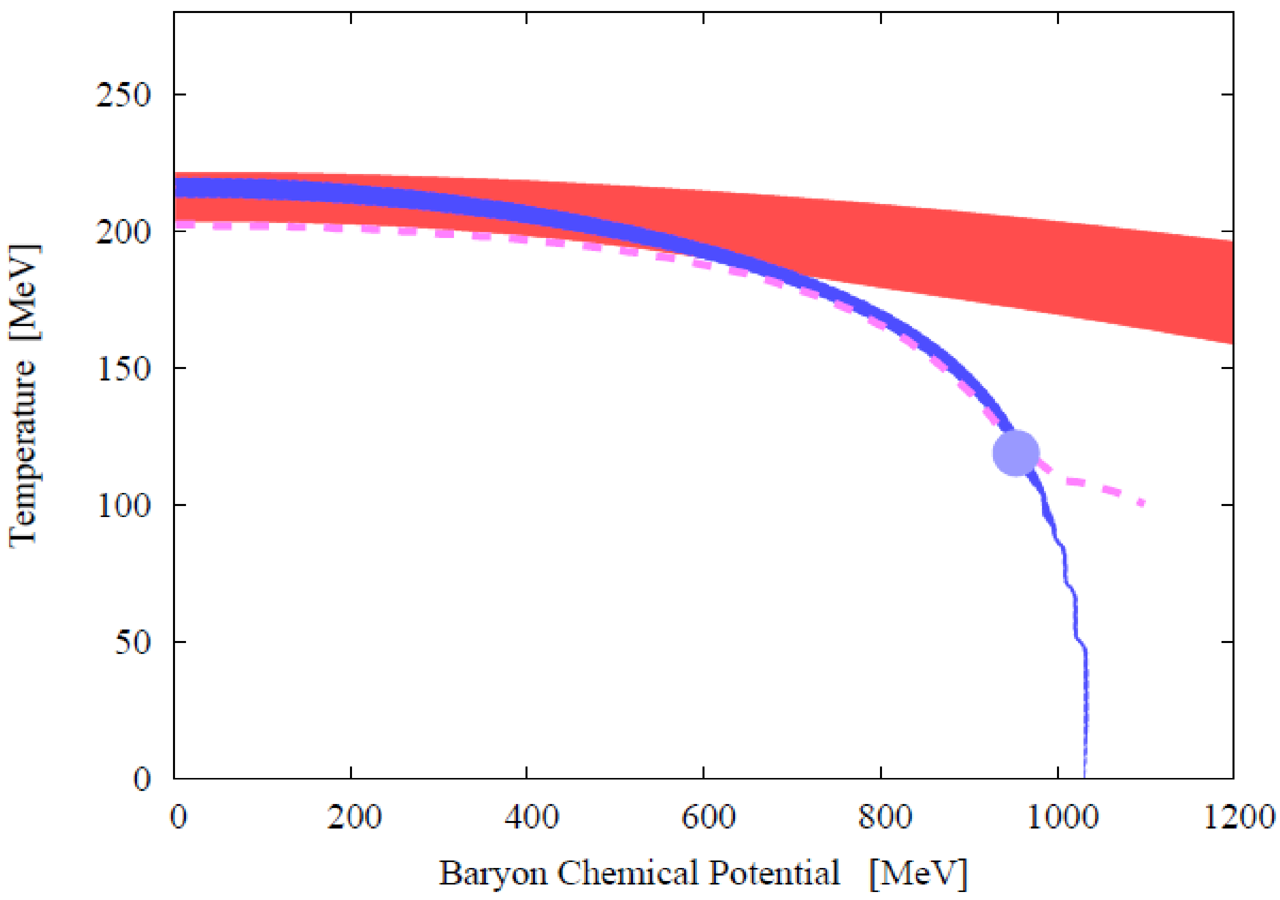
- Polyakov loop grows at high T and high μ
- Only smoothly change at high μ

Quarkyonic Transition

- Quark (baryon) density jumps at high T and high μ
- Almost associated with chiral transition (?)

Phase Diagram

Phase Boundaries by Order Parameters

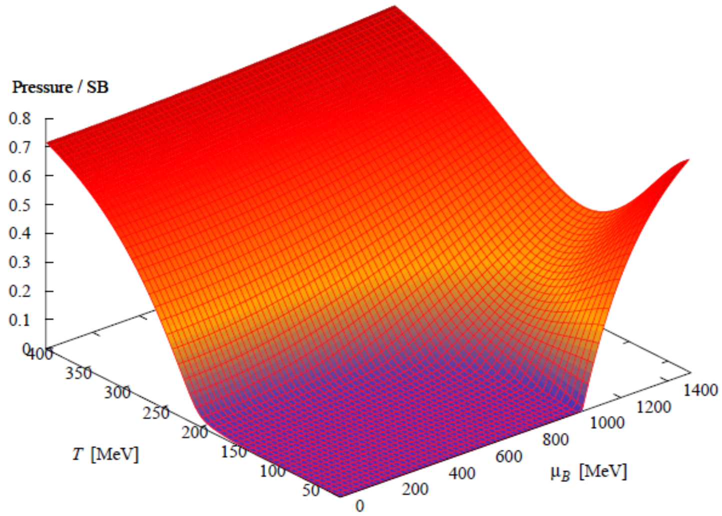


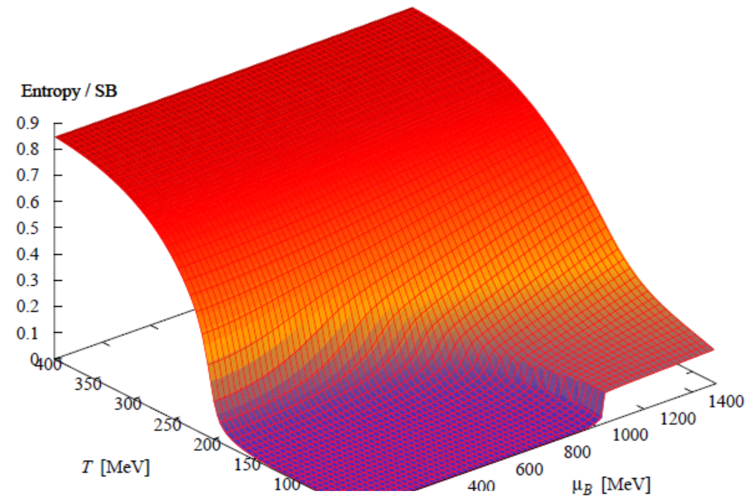
(Normalized) order parameters ranging between 0.3 ~ 0.5

c.f If susceptibility (or slope) is used, two lines meet up to some density...

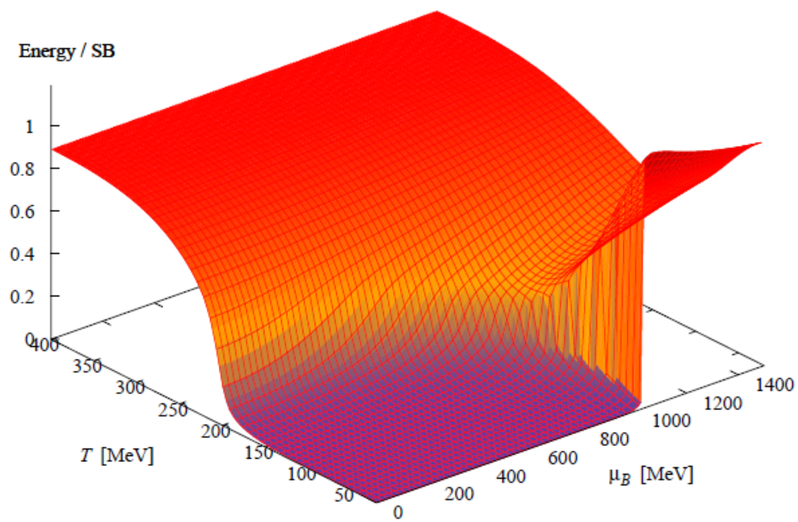
Thermodynamics

3D Plots – Pressure and Entropy (density)





Energy (density)



Thermodynamic Relations

$$P = -\left(\frac{\partial \Omega}{\partial V}\right)_{T,\mu}, \quad S = -\left(\frac{\partial \Omega}{\partial T}\right)_{V,\mu}, \quad N = -\left(\frac{\partial \Omega}{\partial \mu}\right)_{T,V}$$

$$E = -PV + TS + \mu N$$

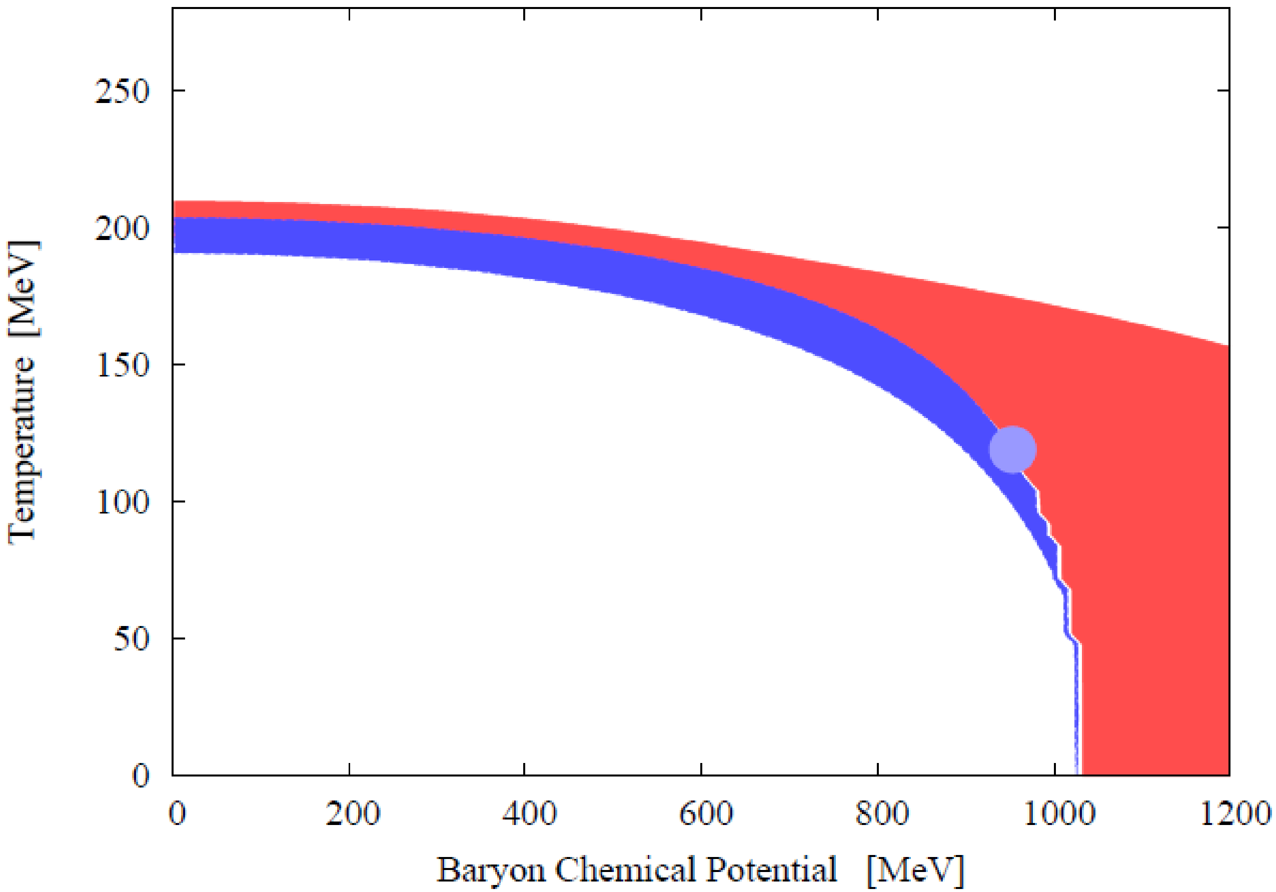
It would be appropriate to use S (dual to T) for deconfinement and N (dual to μ) for quarkyonic transitions.

Why should we rely on the order parameters which are NOT direct observables?

Phase diagram by S and N would be more physical??

Phase Diagram

Phase Boundaries by Thermodynamics



(Normalized) s and n_B ranging between 0.1 ~ 0.3 (Onset)

Similar to the one by order parameters...

K^+/π^+ Description

Non-trivial structure in a quark description

$$u \quad d \quad \to \quad u\bar{s}\,(K^+) \quad d\bar{s}\,(K^0)$$
$$\to \quad uds\,(\Lambda)$$

More u and d at high μ_B but eventually deconfined...
Correlation among u d and s is possible by the Polyakov loop
L and $L^\dagger$ describe how much strange quarks are screened by u d

$$\int \frac{d^3k}{(2\pi)^3} \mathrm{tr}\left[\log\left(1+L\,\mathrm{e}^{-(E-\mu_s)/T}\right)+\log\left(1+L^\dagger\,\mathrm{e}^{-(E+\mu_s)/T}\right)\right]$$

Mixed Flavor Correlation

PNJL description

- If no constraint at all...

$$u \quad d \quad \rightarrow \quad u\,\bar{s} \quad d\,\bar{s}$$

induces anti-s-quarks $\quad n_s - n_{\bar{s}} < 0$

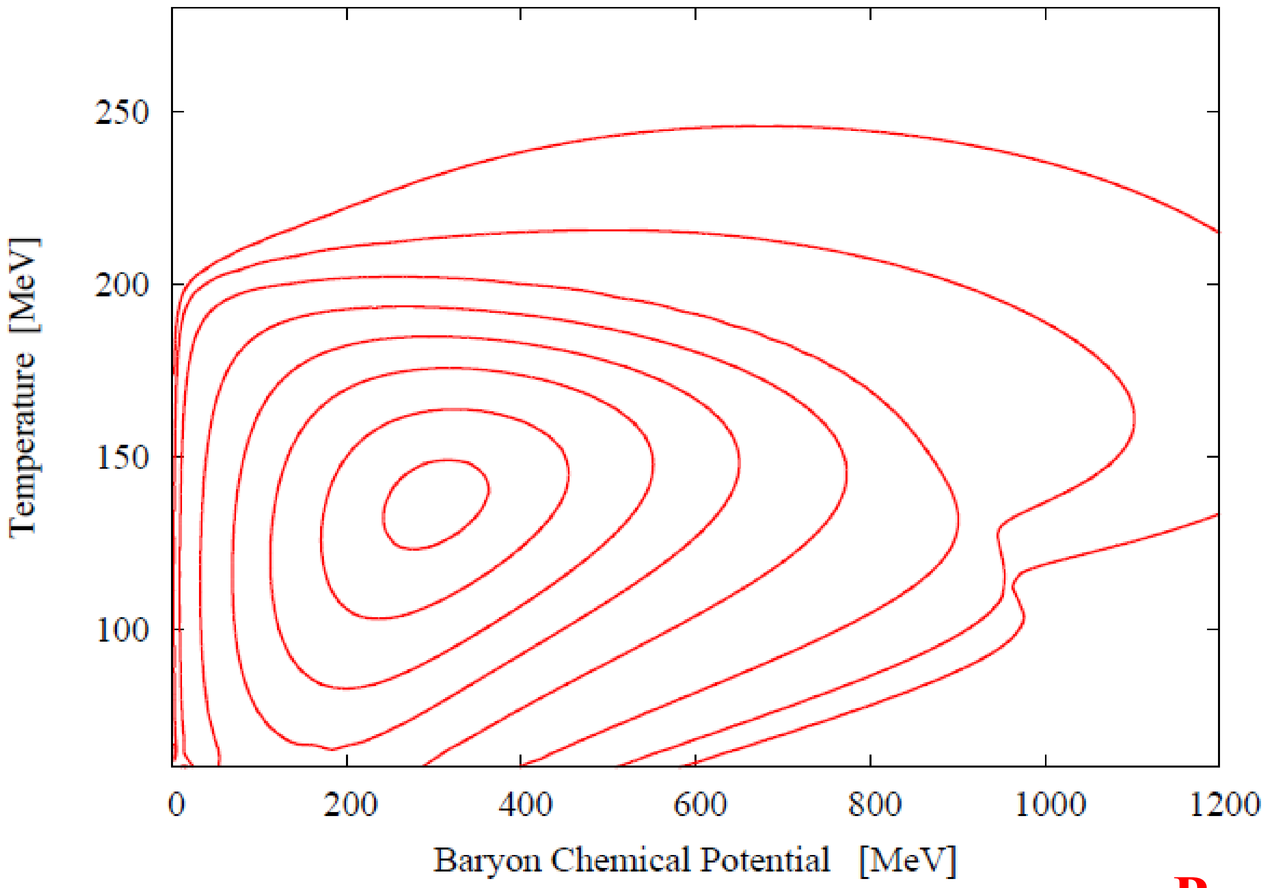
- If net strangeness is constrained to be zero...

$$u \quad d \quad \rightarrow \quad u\,\bar{s} \quad d\,\bar{s} \quad + \quad s$$

realizing $\quad n_s' - n_{\bar{s}}' = 0 \qquad \sim \delta n_s = n_s' - n_{\bar{s}}$

Suggestive???

Ratio of δn_s and $n_u + n_{\bar{u}}$



Peak structure?

Summary

- PNJL = Natural setup leading to
  the Stefan-Boltzmann limit
  containing correct degrees of freedom
- Phase diagram with a TP and a CP
  - Phase diagram by order parameters
  - Phase diagram by thermodynamic quantities
- Tendency toward K^{+}/π^{+} singular structure...
- Questions
  - Why is chiral restoration close to quarkyonic transition?
  - Right description both above and below T_c ?