AdS/CFT and Heavy Ion Collisions

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(lecture notes Zakopane '08, arXiv:0812.0500)

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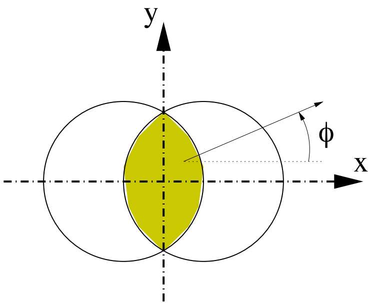
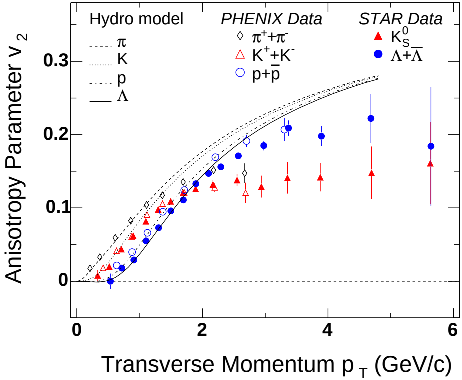
1. Motivations from RHIC
2. Lattice QCD
3. Parton evolution at strong coupling
4. Conclusions

Elliptic flow



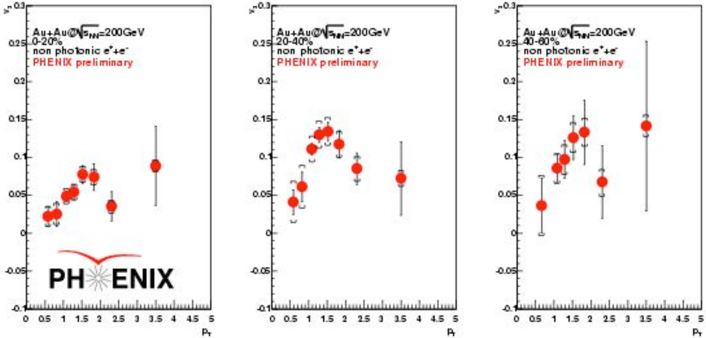


- Non–central *AA* collision: Pressure gradient is larger along x

$$\mathrm{d}N/\mathrm{d}\phi \propto 1 + 2v_2 \cos 2\phi\,, \qquad v_2 = \text{“elliptic flow”}$$

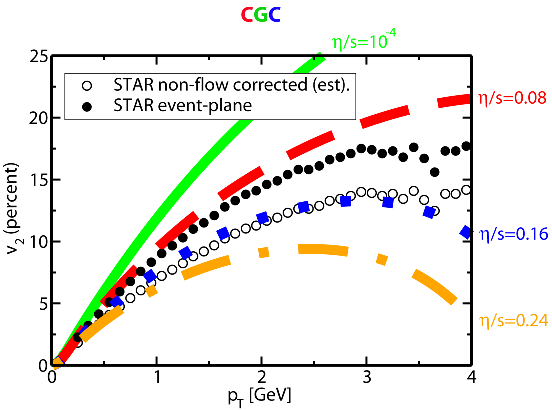
- Large observed flow ! Inconsistent with weak coupling

Elliptic flow



- Even heavy quarks (c, b) seem to flow !

Elliptic flow



- Well described by hydrodynamical calculations with very small viscosity/entropy ratio: "perfect fluid"

Viscosity over entropy density ratio

- A small η/s ratio is a hint towards strong coupling
- Kinetic theory: viscosity is due to collisions among molecules

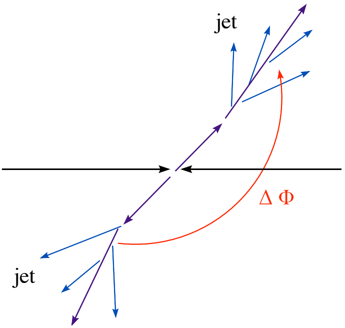
$$\eta \sim \rho\, v\, \ell \;=\; \text{mass density} \;\times\; \text{velocity} \;\times\; \underbrace{\text{mean free path}}_{\sim 1/g^4}$$

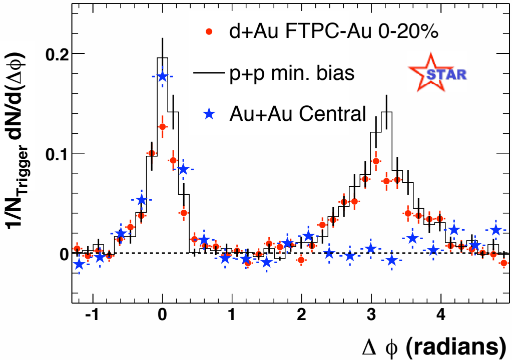
- Weakly interacting systems have $\eta/s \gg \hbar$
- Conjecture from AdS/CFT *(Kovtun, Son, Starinets, 2003)*

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi} \qquad \text{[lower limit = infinite coupling]}$$

- The RHIC value is at most a few times $\hbar/4\pi$!

Jets in proton–proton collisions



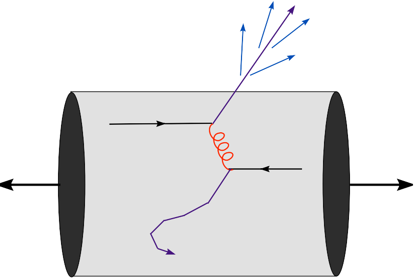


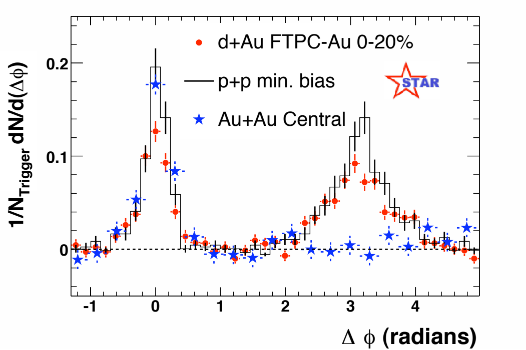
[Nucl.Phys.A783:249-260,2007]

- Azimuthal correlations between the produced jets:

p+p or d+Au : a peak at $\Delta\Phi = 180^\circ$

Nucleus–nucleus collisions at RHIC





- The "away–side" jet has disappeared !

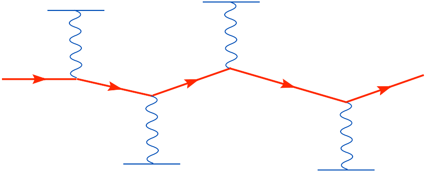
  absorbtion (or energy loss, or "jet quenching") in the medium

- The matter produced in a heavy ion collision is opaque

  high density, strong interactions, ... or both

Jet quenching parameter $\hat{q}$ *(weak coupling)*

- Medium rescattering $\Longrightarrow$ transverse momentum broadening



$$\frac{\mathrm{d}\langle p_\perp^2\rangle}{\mathrm{d}t} \equiv \hat{q} \simeq \alpha_s N_c \, xg(x, Q^2)$$

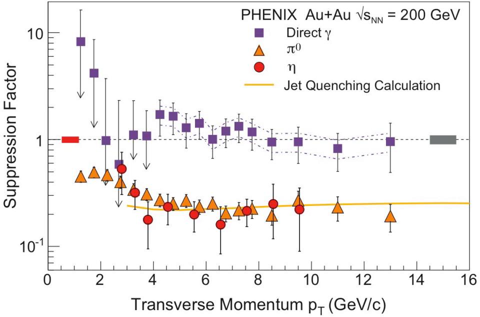
- $xg(x, Q^2)$: gluon distribution per unit volume in the medium

$$xg(x, Q^2) \simeq n_q(T)\, xG_q + n_g(T)\, xG_g \quad \text{with} \quad n_{q,g}(T) \propto T^3$$

- This requires parton evolution from T up to $Q \gg T$
- Lowest order pQCD: $xG_g(x, Q^2) \simeq \alpha_s N_c \ln \frac{Q^2}{T^2}$

Nuclear modification factor

- How to measure $\hat{q}$? Compare *AA* collisions at RHIC to *pp* !



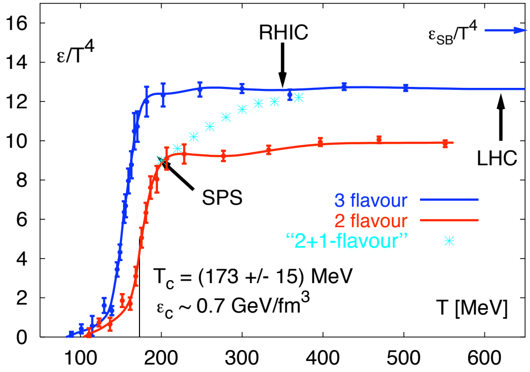
$$R_{AA}(p_\perp) \equiv \frac{Yield(A+A)}{Yield(p+p) \times A^2}$$

- RHIC data seem to prefer a rather large value for $\hat{q}$:

$\hat{q}_{\rm RHIC} \simeq 5 \div 15$ vs. $\hat{q}_{\rm pQCD} \simeq 0.5 \div 1\,{\rm GeV}^2/{\rm fm}$

$\Longrightarrow$ 5 to 10 times larger than the pQCD estimate !

QCD thermodynamics on the lattice *(Bielefeld Coll.)*



$$\mathcal{E}/\mathcal{E}_0 \approx 0.85 \quad \text{for} \quad T = 3T_c$$

- AdS/CFT for $\mathcal{N} = 4$ SYM : $\mathcal{E}/\mathcal{E}_0 \to 0.75$ when $\lambda \to \infty$

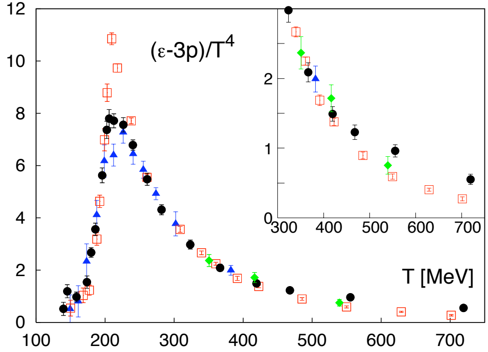
Trace anomaly from lattice QCD

- For $T \gtrsim 2T_c$, the quark–gluon plasma is nearly conformal

$$\beta(g)\,\frac{\mathrm{d}p}{\mathrm{d}g} = \langle T^{\mu}_{\mu} \rangle = \mathcal{E} - 3p$$

- $(\mathcal{E} - 3p)/\mathcal{E}_0 \lesssim 10\%$

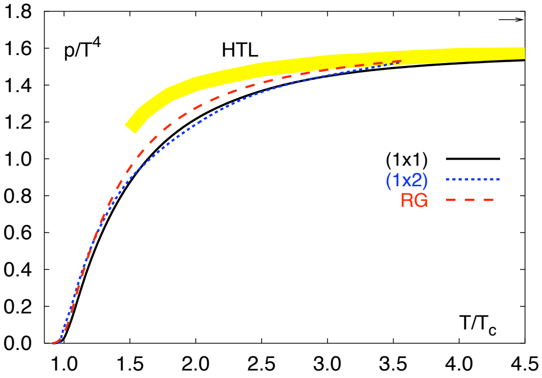
  for any $T \gtrsim 2T_c \simeq 400$ MeV



- AdS/CFT : Better suited for QCD at finite temperature

Resummed perturbation theory

- This ratio $p/p_0 \approx 0.85$ can be also (and better !) explained by resummed perturbation theory ! *(J.-P. Blaizot, A. Rebhan, E. I., 2000)*



- Weakly coupled quasiparticles (quarks and gluons)

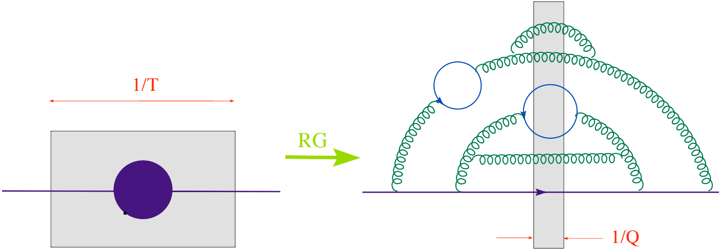
A lattice test of strong coupling *(E.I., A. Mueller 09)*

- Leading–twist, spin n operators (OPE for DIS) :

$$\mathcal{O}_f^{(n)\,\mu_1\cdots\mu_n} \equiv \bar{q}\,\gamma^{\mu_1}(iD^{\mu_2})\cdots(iD^{\mu_n})q \;\sim\; \bar{q}\,P^{n-1}\,q$$

$$\mathcal{O}_g^{(n)\,\mu_1\cdots\mu_n} \equiv -\,F^{\mu_1\nu}(iD^{\mu_2})\cdots(iD^{\mu_{n-1}})F^{\mu_n}{}_{\nu}$$

- The operators depend upon the resolution scale



- A 'quasiparticle' on the scale T may reveal itself as highly composite on the harder scale $Q \gg T$

Renormalization group flow

- RG flow $\Longrightarrow$ negative anomalous dimensions

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2}\, \mathcal{O}^{(n)} = \gamma^{(n)} \mathcal{O}^{(n)} \quad \text{with} \quad \gamma^{(n)} \leq 0$$

- Only exception: energy momentum tensor for which $\gamma_T^{(2)} = 0$

$$T^{\mu\nu} = \mathcal{O}_f^{(2)\,\mu\nu} + \mathcal{O}_g^{(2)\,\mu\nu}$$

- QCD at weak coupling: slow evolution

$$\gamma^{(n)}(\mu^2) = -a^{(n)}\, \frac{\alpha_s(\mu^2)}{4\pi} \quad \Longrightarrow \quad \frac{\mathcal{O}^{(n)}(Q^2)}{\mathcal{O}^{(n)}(\mu_0^2)} = \left[\frac{\ln(\mu_0^2/\Lambda^2)}{\ln(Q^2/\Lambda^2)}\right]^{a^{(n)}/b_0}$$

- Conformal theory, arbitrary coupling: $\frac{\mathcal{O}^{(n)}(Q^2)}{\mathcal{O}^{(n)}(\mu_0^2)} = \left[\frac{\mu_0^2}{Q^2}\right]^{|\gamma^{(n)}|}$

Anomalous dimensions from lattice QCD

- $\mathcal{N} = 4$ SYM at strong 't Hooft coupling: $\lambda \equiv g^2 N_c \gg 1$

$$\gamma^{(n)} \simeq -\sqrt{\frac{n}{2}}\,\lambda^{1/4} \quad \text{for} \quad 1 \ll n \ll \sqrt{\lambda}$$

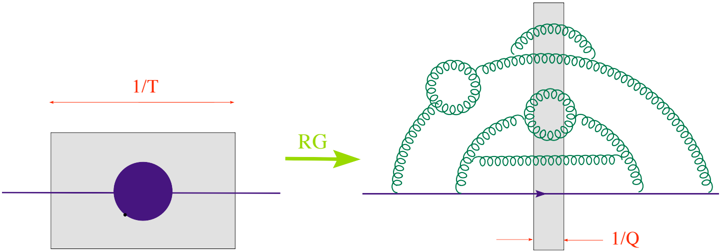
- All the unprotected leading–twist operators are strongly suppressed in the continuum limit $Q \equiv a^{-1} \to \infty$
- Measure unprotected operators in lattice thermal QCD !
- High–spin operators with $n \geq 4$ are difficult to measure ☹
- One $n = 2$ unprotected operator: orthogonal to $T^{\mu\nu}$ ☺

$$\Theta^{\mu\nu}(\mu^2) = \mathcal{O}_f^{(2)\,\mu\nu}(\mu^2) + C(\mu^2)\mathcal{O}_g^{(2)\,\mu\nu}(\mu^2)$$

- ... but we cannot compute $C(\mu^2)$ except at weak coupling ☹

Quenched QCD: not only simpler, but also better

- ... or in quenched QCD (no quark loops), where $C(\mu^2) = 0$ ☺



- Measure the quark energy density in quenched lattice QCD
  ...compare the result with the weak coupling expectation (SB)
  - If the difference is less than 30% $\Longrightarrow$ weak coupling
  - A reduction by a large factor $\gtrsim 5 \Longrightarrow$ strong coupling

The AdS/CFT correspondance *(Maldacena, 1997)*

- Assume a strong coupling scenario :

  How to study parton evolution in a strongly coupled plasma ?

- 'Duality' : a gauge theory at strong coupling ($\mathcal{N} = 4$ SYM)
  - SU(N_c), conformal invariance, fixed coupling g, no confinement
- ... is equivalent to a string theory at weak coupling
- Strong 't Hooft coupling : $\lambda \equiv g^2 N_c \gg 1 \quad \& \quad g^2 \ll 1$
  - string theory reduces to classical supergravity in AdS_5
- $\mathcal{N} = 4$ SYM plasma at finite temperature: Black Hole in AdS_5
  - a Black Hole has entropy and thermal (Hawking) radiation

DIS off the Black Hole *(Hatta, E.I., Mueller, 07)*

- AdS_5 : Our physical world ($D = 4$) $\times$ a 'radial' dimension χ
- Virtual photon in 4D $\longleftrightarrow$ Maxwell wave A_μ in AdS_5 BH
- DIS cross section $\longleftrightarrow$ absorption of the wave by BH

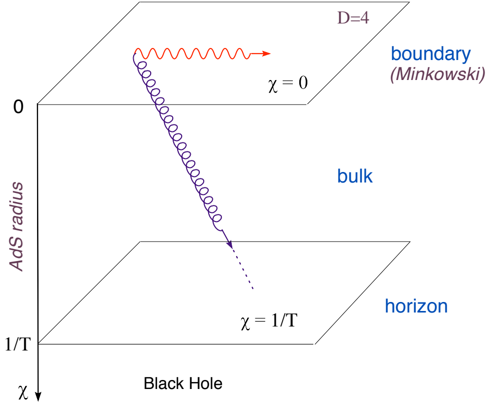
- Physical world: $\chi = 0$

  Black Hole horizon: $\chi = 1/T$

- Maxwell equations in AdS_5 BH

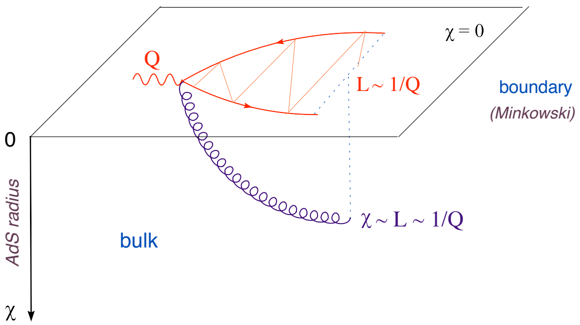
$$\partial_m\left(\sqrt{-g}\, g^{mn} g^{pq} F_{nq}\right) = 0$$

$$F_{mn} = \partial_m A_n - \partial_n A_m$$



The 5th dimension: A reservoir of quantum fluctuations

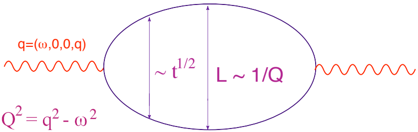
- Radial penetration χ of the wave packet in AdS_5 $\longleftrightarrow$ transverse size L of the partonic fluctuation on the boundary

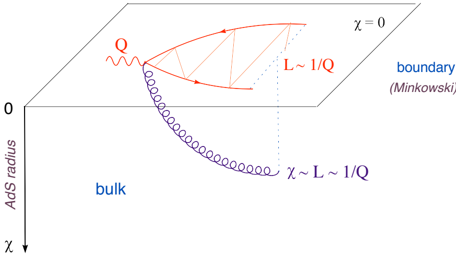


- Space–like photon with virtuality Q : The Maxwell wave penetrates up to a radial distance $\chi \sim 1/Q$

Space–like photon in the vacuum

- A space–like photon cannot decay in the vacuum (energy–momentum conservation)

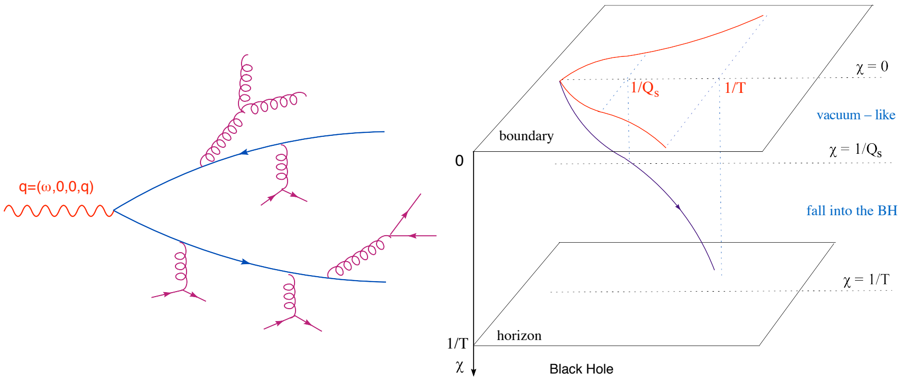




- AdS : The Maxwell wave gets stuck near the boundary $\chi \sim 1/Q$

Space–like photon in the plasma

- … but it can decay in the presence of the plasma

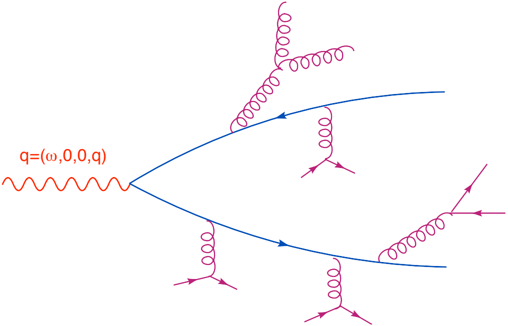


- AdS : The Maxwell wave falls into the Black Hole

  … but what is the physical interpretation ?

Rescattering vs. parton branching

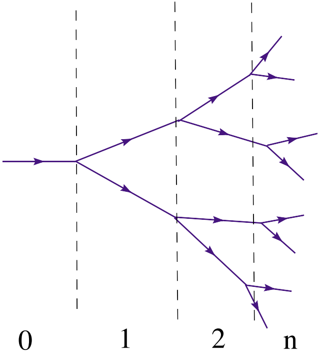
- Two generic mechanisms for decay inside the plasma :



- thermal rescattering
  dominant mechanism at weak coupling
- medium–induced parton branching
  dominant mechanism at strong coupling

Parton branching at strong coupling

- At strong coupling, branching is fast and quasi–democratic



$$\omega_n \sim \frac{\omega_{n-1}}{2} \sim \frac{\omega}{2^n}$$

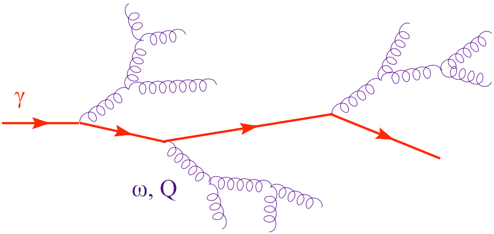
$$Q_n \sim \sim \frac{Q_{n-1}}{2}$$

$$\Delta t_n \sim \frac{\omega_n}{Q_n^2}$$

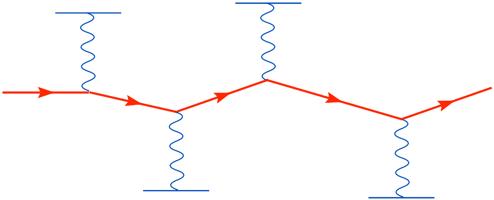
- When $\omega_n \sim Q_n \sim T$, the quanta disappear into the plasma
- Dominant mechanism for energy loss and momentum broadening at strong coupling

Momentum broadening for a heavy quark

- Strong coupling : fluctuations in the emission process

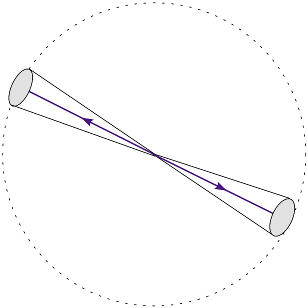


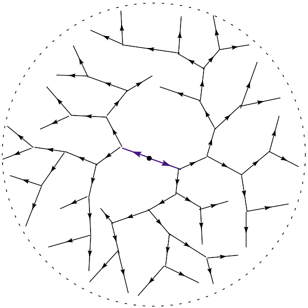
- pQCD : thermal rescattering (different physics !)



No jets at strong coupling !

- A time–like photon can decay already in the vacuum





- No jets in e^+e^- annihilation at strong coupling !

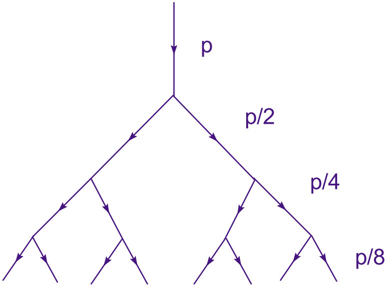
  (similar conclusion by Hofman and Maldacena, 2008)

Parton saturation at strong coupling

- Hadron wavefunction at strong coupling:

  All partons branch down to small values of the longitudinal momentum fraction x and saturate

  (Polchinski and Strassler, 2003; Hatta, E.I., A. Mueller, 2007)

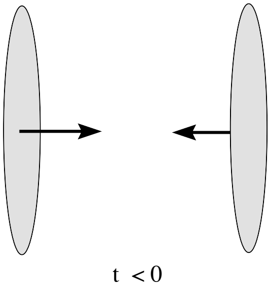


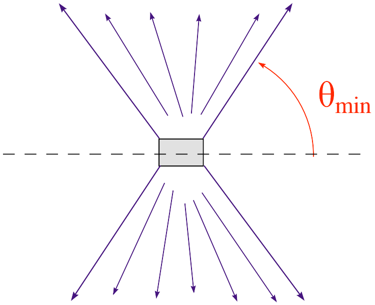
- $x > x_s(Q) \sim \frac{\Lambda^2}{Q^2} \ll 1$: no partons
- $x < x_s(Q)$: occupation numbers ~ 1
- Energy–momentum sum rule

$$\int_0^1 \mathrm{d}x\, F_2(x, Q^2) \sim \mathcal{O}(1)$$

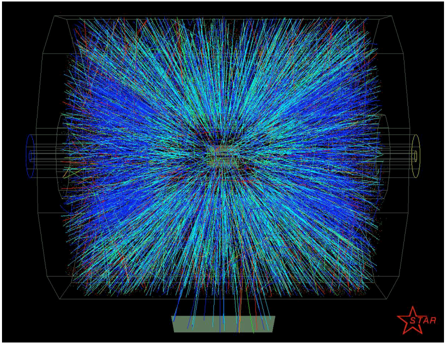
No forward jets !

- No large–x partons $\Longrightarrow$ no forward/backward jets in a hadron–hadron collision at strong coupling





Partons at RHIC



- Partons are actually 'seen' (liberated) in the high energy hadron–hadron collisions
  - central rapidity: small–x partons
  - forward/backward rapidities: large–x partons

Conclusions

- Hard probes & high-energy physics appears to be quite different at strong coupling as compared to pQCD
  - no jets in e^+e^- annihilation
  - no forward/backward particle production in HIC
  - different mechanism for jet quenching
- Are AdS/CFT methods useless for HIC ? Not necessarily so !
  - long–range properties (hydro, thermalization, etc) might be controlled by strong coupling
  - most likely, the coupling is moderately strong, so it useful to approach the problems from both perspectives
- One can test the strong–coupling hypothesis in lattice QCD

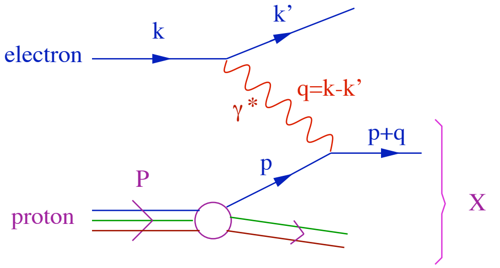
Deep inelastic scattering

- How to probe parton evolution at strong coupling ?

- Space–like photon
- 2 independent variables:

$$Q^2 \equiv -q^\mu q_\mu \geq 0$$

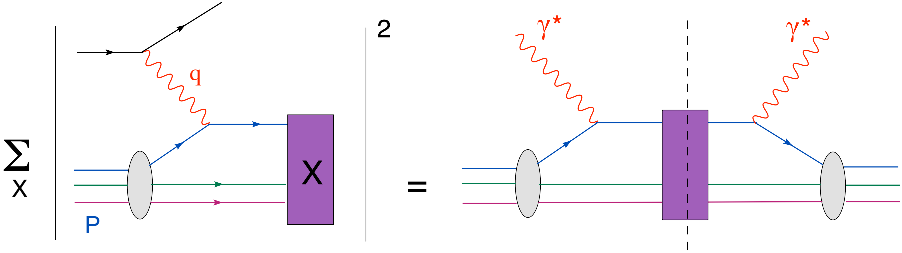
$$x \equiv \frac{Q^2}{2P \cdot q}$$



- Physical picture: γ^* absorbed by a quark excitation with
  - transverse size $\Delta x_\perp \sim 1/Q$
  - and longitudinal momentum $p_z = xP$

Current–current correlator

- Total cross–section ("structure functions"): optical theorem



$$F_{1,2}(x, Q^2) \;\sim\; \mathrm{Im} \int \mathrm{d}^4x \, \mathrm{e}^{-iq\cdot x} \, i \, \langle P \,|\, \mathrm{T}\,\{J_\mu(x) J_\nu(0)\}\,|\, P\rangle$$

$$J^\mu \;=\; \sum_f e_f \, \bar{q}_f \, \gamma^\mu \, q_f$$: quark electromagnetic current

- Valid to leading order in $\alpha_{\rm em}$ but all orders in α_s

DIS off the strongly coupled plasma

- Thermal expectation value $(Q^2 \equiv |q^2| \gg T^2)$

$$\Pi_{\mu\nu}(q) \equiv \int \mathrm{d}^4x \, \mathrm{e}^{-iq\cdot x} \, i\theta(x_0) \, \langle \, [J_\mu(x), J_\nu(0)] \, \rangle_T$$

- $\mathcal{N} = 4$ SYM at finite temperature & $\lambda \equiv g^2 N_c \to \infty$:
  classical gravity in the $AdS_5 \times S^5$ Black Hole geometry

$$\mathrm{d}s^2 = \frac{R^2}{\chi^2}\big(-f(\chi)\mathrm{d}t^2 + \mathrm{d}\boldsymbol{x}^2\big) + \frac{R^2}{\chi^2 f(\chi)}\mathrm{d}\chi^2 + R^2 \mathrm{d}\Omega_5^2$$

where $f(\chi) = 1 - (\chi/\chi_0)^4$ and $\chi_0 = 1/T =$ BH horizon

- A Black Hole has entropy and thermal (Hawking) radiation

Space–like photon in the plasma

- Gravitational interactions are proportional to the energy density in the wave (ω) and in the plasma (T)

- High Q^2/large Bjorken x

  The wave gets stuck near the boundary

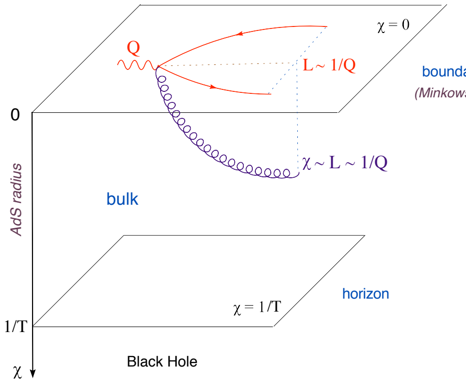
  $$\chi \lesssim 1/Q \ll 1/T$$

  $\Longrightarrow$ No interaction with the BH

- Low Q^2/small x

  $$x \equiv \frac{Q^2}{2\omega T} \lesssim x_s(Q) \simeq \frac{T}{Q}$$

  $\Longrightarrow$ The wave falls into the BH



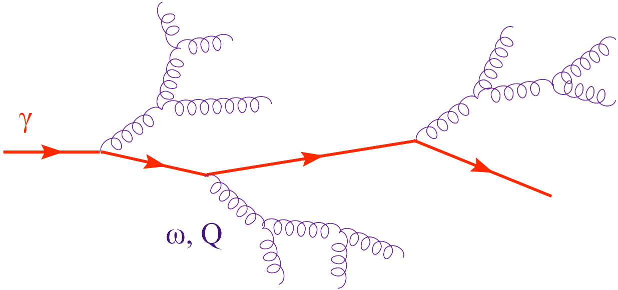
The energy–momentum sum rule

$$\int_0^1 \mathrm{d}x\, F_2(x, Q^2) = \mathit{const.} \qquad \text{as} \quad Q^2 \to \infty$$

- ... is still dominated by the few partons remaining at $x \sim \mathcal{O}(1)$
- As $x \to 0$, F_2 rises 'only' like $F_2(x, Q^2) \sim x^{-\lambda}$ with $\lambda \lesssim 0.3$
- The small–x gluons are numerous, but carry very little energy
- Pointlike valence quarks

... to be contrasted with the situation at strong coupling !

Heavy Quark in a strongly–coupled plasma



- Medium–induced radiation
  - virtual quanta with $Q \lesssim Q_s$ are liberated into the plasma
  - energy loss, momentum broadening
  - Langevin equation from AdS/CFT

  Casalderrey-Solana, Teaney, 2006; Gubser, 2006; Dominguez et al, 2008