Lowest negative parity baryons in the $1/N_c$ expansion

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10th July 2009

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Outline

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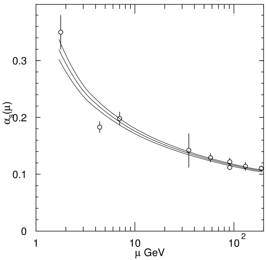
Motivation

- Traditional issue to study **baryons**: **effective theories** and **constituent quark models**
- The $1/N_c$ expansion is a **theoretical method** (1993)
  - Systematic
  - Model independent
  - Predictive
  - Gives support to constituent quark models
- It has been extensively applied to baryon masses, strong and photoproduction

Introduction

- Impossible to solve QCD exactly
- No perturbative expansion of QCD at low energies with respect to g



- 't Hooft suggested to generalize QCD to N_c color (1974)
- $1/N_c$ should be the expansion parameter of QCD
- Witten power counting rules (1979)

Mesons in large N_c QCD

$$|1\rangle_c = \frac{1}{\sqrt{N_c}} \underbrace{(\bar{l}l + \bar{m}m + \ldots + \bar{n}n)}_{N_c \text{ terms}}$$

Large N_c mesons are stable and non-interacting

Baryons in large N_c QCD

$$\varepsilon_{i_1 i_2 i_3 \cdots i_{N_c}} q^{i_1} q^{i_2} q^{i_3} \cdots q^{i_{N_c}}$$

bound states of N_c **valence quarks completely antisymmetric in color** because baryons are colorless

Baryon mass grows with N_c

Baryon weight diagrams

For $N_c = 3$

$n \quad p$

$\Sigma^- \quad \Lambda, \Sigma^0 \quad \Sigma^+$

$\Xi^- \quad \Xi^0$

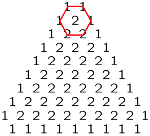
$\Delta^- \quad \Delta^0 \quad \Delta^+ \quad \Delta^{++}$

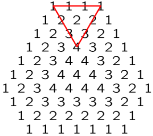
$\Sigma^{*-} \quad \Sigma^{*0} \quad \Sigma^{*+}$

$\Xi^{*-} \quad \Xi^{*0}$

Ω^-

For large N_c





Familiar $N_c = 3$ baryons can be **identified** with **states at the top of the flavor representations**

Exact SU$_f(3)$: all particles in **each weight diagram** have the **same mass** when $m_u = m_d = m_s$

Exact SU(6) : **all the particles** have the **same mass**

ground-state baryons satisfy a **contracted SU$(2N_f)_c$ in $N_c \to \infty$ limit (contracted algebra) [1,2]**

$\Rightarrow$ ground-state baryons form **an infinite tower of degenerate states**

Identification between the SU$(2N_f)_c$ algebra and the SU$(2N_f)$ algebra used in the **quark-shell model** in the $N_c \to \infty$ limit

SU$(2N_f)$ used to classify large N_c baryons *i.e.* **quark-shell model wave functions** (non-relativistic) used in the following

Assume SU(6)$\times$O(3) symmetry at leading order for excited baryons

The total wave function of baryons Ψ

$$\Psi = \psi_{lm}\chi\phi C$$

where ψ_{lm}, χ, ϕ and C are the space, spin, flavor and color part.

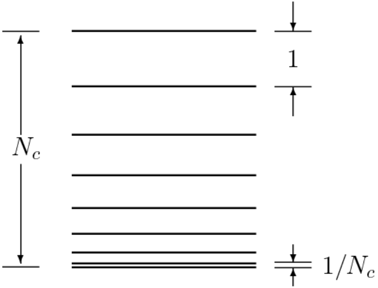
[1] J.-L. Gervais and B. Sakita, Phys. Rev. Lett. **52**, 87 (1984); Phys. Rev. **D30**, 1795 (1984).
[2] R. Dashen and A. V. Manohar, Phys. Lett. **B315**, 425 (1993).

Baryon masses with exact SU(3) symmetry [3]

Baryon mass transforms as a spin-flavor singlet $(1,1)$

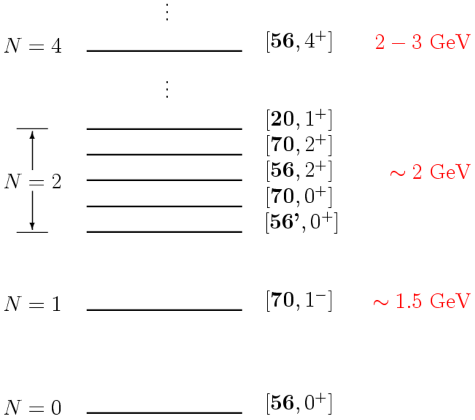
$$M = c_0 N_c \mathbb{1} + c_2 \frac{1}{N_c} S^2 + c_4 \frac{1}{N_c^3} (S^2)^2 + \ldots + c_{N_c-1} \frac{1}{N_c^{N_c-2}} S^{N_c-1}$$

Mass splittings for the tower of large N_c baryon states with $S = \frac{1}{2}, \ldots, \frac{N_c}{2}$



[3] E. Jenkins and R.F. Lebed, Phys. Rev. **D53**, 2625 (1996)

Baryon spectrum (SU(6) notation: $[\mathbf{X}, l^P]$)



Excited baryons: an approximate approach [4]

- Witten's suggestion: **Hartree approximation exact in $N_c \to \infty$ limit**
  - Each quark moves in an **average potential** generated by the other $N_c - 1$ quarks
  - Total potential experienced by **each quark** is of order $\mathcal{O}(1)$
  - Interaction between **any given pair of quarks** of order $\mathcal{O}(1/N_c)$
- $[\mathbf{70}, 1^-]$ multiplet (first excited band): baryons composed of $N_c - 1$ **ground-state quarks** (the core) and **1 excited quark**
- The core quarks are described by **symmetric wave functions in spin-flavor parts** as ground-state baryons
- **Decouple the wave function: excited quark** coupled to **a symmetric core**

[4] C.E. Carlson, C.D. Carone, J.L. Goity and R.F. Lebed, Phys. Rev. **D59**, 114008 (1999)

Has been applied to $[\mathbf{70}, 1^-]$, $[\mathbf{70}, \ell^+]$ $(\ell = 0, 2)$

Mass operator

$$M_{[\mathbf{70},\ell^+]} = \sum_{i=1}^{6} c_i O_i + \sum_{j=1}^{4} d_j B_j$$

Operator	Fitted coef. (MeV)			
$O_1 = N_c \mathbb{1}$	$c_1 =$	556	$\pm$	11
$O_2 = \ell_q^i s^i$	$c_2 =$	-43	$\pm$	47
$O_3 = \frac{3}{N_c} \ell_q^{(2)ij} g^{ia} G_c^{ja}$	$c_3 =$	-85	$\pm$	72
$O_4 = \frac{4}{N_c+1} \ell^i t^a G_c^{ia}$				
$O_5 = \frac{1}{N_c}(S_c^i S_c^i + s^i S_c^i)$	$c_5 =$	253	$\pm$	57
$O_6 = \frac{1}{N_c} t^a T_c^a$	$c_6 =$	-25	$\pm$	86
$B_1 = t^8 - \frac{1}{2\sqrt{3}N_c} O_1$	$d_1 =$	365	$\pm$	169
$B_2 = T_c^8 - \frac{N_c-1}{2\sqrt{3}N_c} O_1$	$d_2 =$	-293	$\pm$	54
$B_4 = 3\ell_q^i g^{i8} - \frac{\sqrt{3}}{2} O_2$				

$$\chi^2_{\text{dof}} = 1.0$$

[5] N. Matagne and Fl. Stancu, Phys. Rev. **D74**, 034014 (2006)

A totally symmetric orbital-spin-flavor state is given by

$$\Phi_S = \frac{1}{\sqrt{N_c-1}} \sum_Y |[N_c-1,1]Y\rangle_O |[N_c-1,1]Y\rangle_{SF}$$

Illustration for $N_c = 5$ ($[\mathbf{70}, 1^-]$) (center of mass motion removed)

$$\begin{array}{|c|c|c|c|c|}\hline 1&2&3&4&5\\\hline\end{array} = \frac{1}{\sqrt{4}}\left(\begin{array}{|c|c|c|c|}\hline 1&2&3&4\\\hline 5\\\cline{1-1}\end{array}\;\begin{array}{|c|c|c|c|}\hline 1&2&3&4\\\hline 5\\\cline{1-1}\end{array} + \begin{array}{|c|c|c|c|}\hline 1&2&3&5\\\hline 4\\\cline{1-1}\end{array}\;\begin{array}{|c|c|c|c|}\hline 1&2&3&5\\\hline 4\\\cline{1-1}\end{array} + \begin{array}{|c|c|c|c|}\hline 1&2&4&5\\\hline 3\\\cline{1-1}\end{array}\;\begin{array}{|c|c|c|c|}\hline 1&2&4&5\\\hline 3\\\cline{1-1}\end{array} + \begin{array}{|c|c|c|c|}\hline 1&3&4&5\\\hline 2\\\cline{1-1}\end{array}\;\begin{array}{|c|c|c|c|}\hline 1&3&4&5\\\hline 2\\\cline{1-1}\end{array} \right)$$

Young tableau	Young-Yamanouchi basis vectors of [41]
$\begin{array}{\|c\|c\|c\|c\|}\hline 1&2&3&4\\\hline 5\\\cline{1-1}\end{array}$	$\frac{1}{\sqrt{20}}(4ssssp - sssps - sspss - spsss - pssss)$
$\begin{array}{\|c\|c\|c\|c\|}\hline 1&2&3&5\\\hline 4\\\cline{1-1}\end{array}$	$\frac{1}{\sqrt{12}}(3sssps - sspss - spsss - pssss)$
$\begin{array}{\|c\|c\|c\|c\|}\hline 1&2&4&5\\\hline 3\\\cline{1-1}\end{array}$	$\frac{1}{\sqrt{6}}(2sspss - spsss - pssss)$
$\begin{array}{\|c\|c\|c\|c\|}\hline 1&3&4&5\\\hline 2\\\cline{1-1}\end{array}$	$\frac{1}{\sqrt{2}}(spsss - pssss)$

Excited baryons: the exact wave function

Two approaches:

1. **Split the wave function** into a core of $N_c - 1$ quarks + 1 quark [6]

   Two main differences:
   core not always in the ground state and **symmetric (S-F part)**, **last quark not always excited**

2. Treat the multiplet **without splitting** the wave function [7]

$$|\ell S I I_3; J J_3\rangle = \sum_{m_\ell, S_3} \left(\begin{array}{cc|c} \ell & S & J \\ m_\ell & S_3 & J_3 \end{array} \right) |[N_c - 1, 1] S S_3 I I_3\rangle |[N_c - 1, 1] \ell m_\ell\rangle$$

[6] N. Matagne, Fl. Stancu, Phys. Rev. **D77**, 054026 (2008)
[7] N. Matagne, Fl. Stancu, Nucl. Phys. **A811**, 291 (2008)

Results of the fits (nonstrange baryons) $[70, 1^-]$

Only with the **7 resonances**: ${}^2N_{1/2}(1538 \pm 18)$, ${}^4N_{1/2}(1660 \pm 20)$, ${}^2N_{3/2}(1523 \pm 8)$, ${}^4N_{3/2}(1700 \pm 50)$, ${}^4N_{5/2}(1678 \pm 8)$, ${}^2\Delta_{1/2}(1645 \pm 30)$ and ${}^2\Delta_{3/2}(1720 \pm 50)$

Operator		Approx. w.f. (MeV)	Exact w.f. (MeV)
$O_1 = N_c \mathbb{1}$	$c_1 =$	211 ± 23	299 ± 20
$O_2 = \ell^i s^i$	$c_2 =$	3 ± 15	3 ± 15
$O_3 = \frac{1}{N_c} s^i S_c^i$	$c_3 =$	-1486 ± 141	-1096 ± 125
$O_4 = \frac{1}{N_c} S_c^i S_c^i$	$c_4 =$	1182 ± 74	1545 ± 122
$O_5 = \frac{1}{N_c} t^a T_c^a$	$c_5 =$	-1508 ± 149	417 ± 79
$\chi^2_{\rm dof}$		1.56	1.56

Operator		Fit (MeV)
$O_1 = N_c \mathbb{1}$	$c_1 =$	481 ± 5
$O_2 = \ell^i s^i$	$c_2 =$	-31 ± 26
$O_3 = \frac{1}{N_c} S^i S^i$	$c_3 =$	161 ± 16
$O_4 = \frac{1}{N_c} T^a T^a$	$c_4 =$	169 ± 36
$O_5 = \frac{15}{N_c} \ell^{(2)ij} G^{ia} G^{ja}$	$c_5 =$	-29 ± 31
$O_6 = \frac{3}{N_c} \ell^i T^a G^{ia}$	$c_6 =$	32 ± 26
χ^2_{dof}		0.43

	Part. contrib. (MeV)						Total (MeV)	Exp. (MeV)	Name, status
	c_1O_1	c_2O_2	c_3O_3	c_4O_4	c_5O_5	c_6O_6			
$^2N_{\frac{1}{2}}$	1444	10	40	42	0	-8	1529 ± 11	1538 ± 18	$S_{11}(1535)$****
$^4N_{\frac{1}{2}}$	1444	26	201	42	-31	-20	1663 ± 20	1660 ± 20	$S_{11}(1650)$****
$^2N_{\frac{3}{2}}$	1444	-5	40	42	0	4	1525 ± 8	1523 ± 8	$D_{13}(1520)$****
$^4N_{\frac{3}{2}}$	1444	10	201	42	25	-8	1714 ± 45	1700 ± 50	$D_{13}(1700)$***
$^4N_{\frac{5}{2}}$	1444	-16	201	42	-6	12	1677 ± 8	1678 ± 8	$D_{15}(1675)$****
$^2\Delta_{\frac{1}{2}}$	1444	-10	40	211	0	-40	1645 ± 30	1645 ± 30	$S_{31}(1620)$****
$^2\Delta_{\frac{3}{2}}$	1444	5	40	211	0	20	1720 ± 50	1720 ± 50	$D_{33}(1700)$****

Conclusions

- **Two approaches** for the study of excited baryons in the $1/N_c$ expansion
  - One based on the Hartree approximation, technicaly easier, but approximate (old approach)
  - One considering the wave function in one block and without splitting the operators into core + excited quark operators
- Contribution of I^2 **as important as** S^2
- Results **at variance** with that of Pirjol and Schat [8] (**claim that the inclusion of core and excited quark operators is necessary**)
- New approach should be extended to SU(6). More involved, isoscalar factors of SU(6) generators of a mixed symmetric wave function needed [9]

[8] D. Pirjol and C. Schat, Phys. Rev. **D78**, 034026 (2008)

[9] N. Matagne and Fl. Stancu, Nucl. Phys. **A826**, 161 (2009)