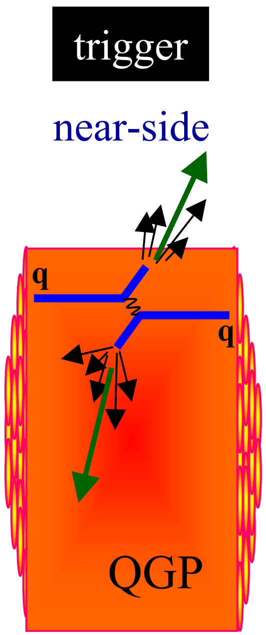
Dynamics of Unstable Quark-Gluon Plasma

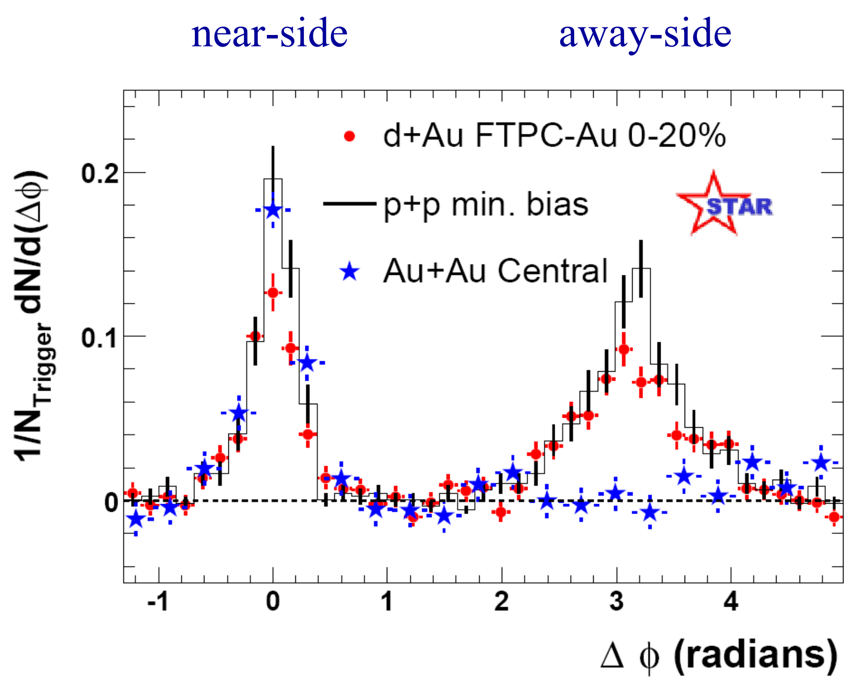
Stanisław Mrówczyński

Jan Kochanowski University, Kielce, Poland
& Institute for Nuclear Studies, Warsaw, Poland

How to compute $\hat{q}$ in unstable QGP ?

Jet quenching @ RHIC





Away-side jet is suppressed in central collisions

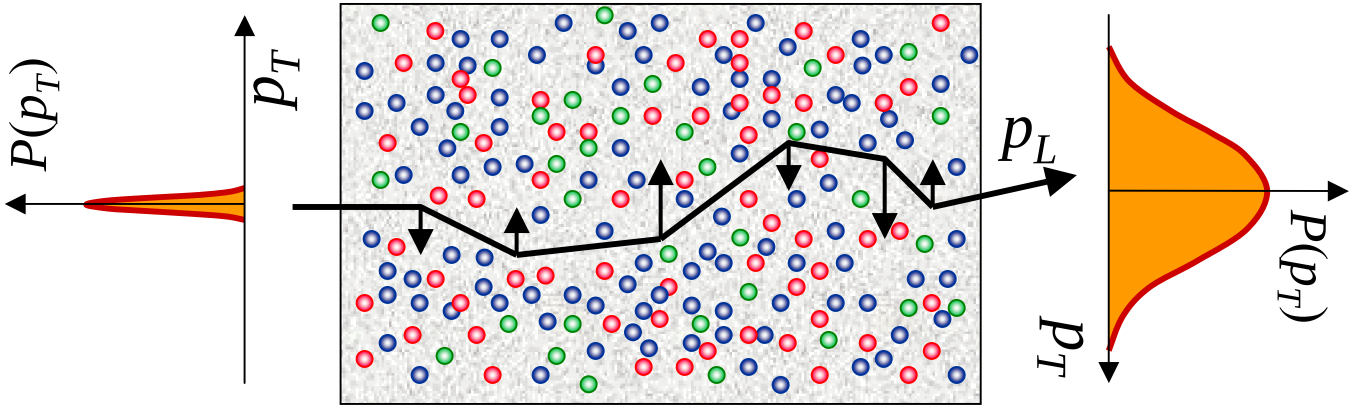
J. Adams et al. [STAR Collaboration], Nucl. Phys. **A757**, 102 (2005)

Momentum transverse broadening

Radiative energy loss of a fast parton is controlled by

$$\hat{q} \equiv \frac{d\left\langle \Delta p_T^2(t) \right\rangle}{dt}$$

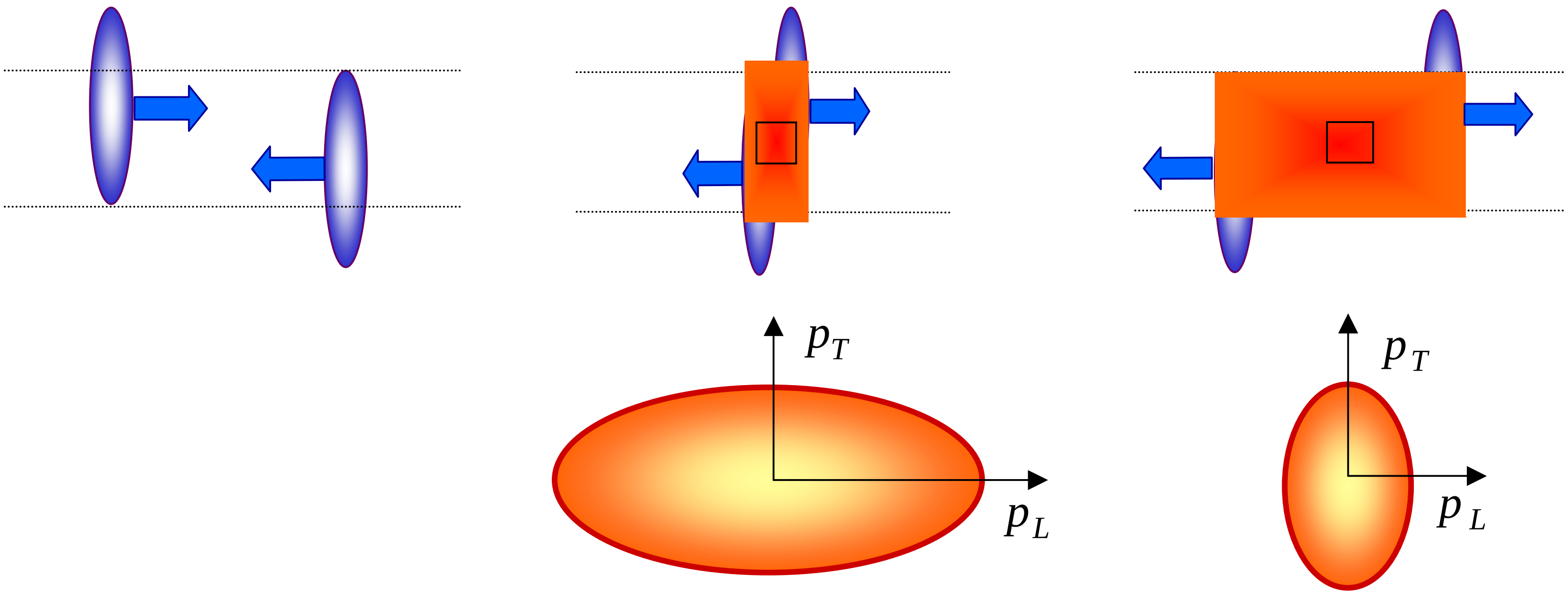
Baier, Dokshitzer, Mueller, Peigne & Schiff 1996



From experiment: $\underbrace{0.5\ \mathrm{GeV}^2/\mathrm{fm}}_{\text{pQGP}} < \hat{q} \leq \underbrace{15\ \mathrm{GeV}^2/\mathrm{fm}}_{?}$

Is QGP strongly coupled ?

Anisotropic QGP @ RHIC



Anisotropic QGP is unstable due to magnetic plasma modes

Momentum broadening in anisotropic QGP

- P. Romatschke, Phys. Rev. **C75**, 014901 (2007)
- R. Baier and Y. Mehtar-Tani, Phys. Rev. **C78**, 064906 (2008)

Anisotropic (unstable) QGP was treated as a static medium

$\hat{q} = \text{const}$

Unstable QGP is generically time dependent

Numerical simulations $\hat{q} \sim e^{2\gamma t}$

- A. Dumitru, Y. Nara, B. Schenke & M. Strickland, Phys. Rev. **C78**, 024909 (2008); B. Schenke, M. Strickland, A. Dumitru, Y. Nara & C. Greiner, Phys. Rev. **C79**, 034903 (2009)

Fast parton in QGP

Wong's equation of motion

$$\begin{cases} \dfrac{dx^\mu(\tau)}{d\tau} = u^\mu(\tau) \\ \dfrac{dp^\mu(\tau)}{d\tau} = gQ_a(\tau)\, F_a^{\mu\nu}(x(\tau))\, u_\nu(\tau) \\ \dfrac{dQ_a(\tau)}{d\tau} = -gf^{abc}\, p_\mu(\tau)\, A_b^\mu(x(\tau))\, Q_c(\tau) \end{cases}$$

Initial value problem

Gauge condition

$$p_\mu(\tau)\, A_b^\mu\big(x(\tau)\big) = 0 \quad \Rightarrow \quad Q_a(\tau) = \text{const}$$

Parton travels with constant velocity: $u^\mu = (\gamma, \gamma\,\mathbf{v}) = \text{const}$

$$p^\mu(\tau) = p^\mu(0) + gQ_a \int_0^\tau d\tau'\, F_a^{\mu\nu}(x(\tau'))\, u_\nu$$

A. Majumder, B. Müller, and St. Mrówczyński, arXiv 0903.3683

Langevin problem

$$\left\langle p^{\mu}(\tau)\, p^{\nu}(\tau)\right\rangle = p^{\mu}(0)\, p^{\nu}(0) + g^2 C \int_0^{\tau} d\tau_1 \int_0^{\tau} d\tau_2 \left\langle F_a^{\mu\nu}(x(\tau_1)) F_a^{\sigma\rho}(x(\tau_2))\right\rangle u_{\sigma} u_{\rho}$$

- $p^{\mu}(0) = \left(E, 0, 0, p_z\right)$
- parton travels with speed of light along axis *z*: $\mathbf{v}(t) = \text{const} = (0,0,1)$

$$C \equiv \begin{cases} \dfrac{1}{2N_c} & -\ \text{fundamenta 1} \\[2mm] \dfrac{N_c}{N_c^2 - 1} & -\ \text{adjoint} \end{cases}$$

$$\left\langle \Delta p_T^2(t)\right\rangle = \left\langle p_x^2(t)\right\rangle + \left\langle p_y^2(t)\right\rangle = g^2 C \int_0^t dt_1 \int_0^t dt_2 \left\langle \left(F_a^{\mu 0}(x(t_1)) - F_a^{\mu 3}(x(t_1))\right)\left(F_a^{\nu 0}(x(t_2)) - F_a^{\nu 3}(x(t_2))\right)\right\rangle$$

$$\begin{aligned} \left\langle \Delta p_T^2(t)\right\rangle = g^2 C \int_0^t dt_1 \int_0^t dt_2 \Big\{ & \left\langle E_a^x(x(t_1)) E_a^x(x(t_2))\right\rangle + \left\langle E_a^y(x(t_1)) E_a^y(x(t_2))\right\rangle - \left\langle E_a^x(x(t_1)) B_a^y(x(t_2))\right\rangle \\ & + \left\langle E_a^y(x(t_1)) B_a^x(x(t_2))\right\rangle - \left\langle B_a^y(x(t_1)) E_a^x(x(t_2))\right\rangle - \left\langle B_a^x(x(t_1)) E_a^y(x(t_2))\right\rangle \\ & + \left\langle B_a^x(x(t_1)) B_a^x(x(t_2))\right\rangle + \left\langle B_a^y(x(t_1)) B_a^y(x(t_2))\right\rangle \Big\} \end{aligned}$$

How to compute fluctuations in unstable systems?

- Equilibrium methods are not applicable
- We deal with the initial value problem

The kinetic theory method by Klimontovich & Silin, Rostoker, Tsytovich, see E.M. Lifshitz and L.P. Pitaevskii, *Physical Kinetics*

St. Mrówczynski, Acta Phys. Pol. **B39** (2008) 941 - Electromagnetic Fluctuations
St. Mrówczynski, Phys. Rev. **D77** (2008) 105022 - Chromodynamic Fluctuations

Transport equations

fundamental:

$$p_\mu D^\mu Q - \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu Q\} = C[Q, \overline{Q}, G]$$ quarks

$$p_\mu D^\mu \overline{Q} + \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu \overline{Q}\} = \overline{C}[Q, \overline{Q}, G]$$ antiquarks

adjoint:

$$p_\mu \mathcal{D}^\mu G - \frac{g}{2} p^\mu \{\mathcal{F}_{\mu\nu},(x)\partial_p^\nu G\} = C_g[Q, \overline{Q}, G]$$ gluons

free streaming — mean-field force — collisions

$$D^\mu \equiv \partial^\mu - ig[A^\mu, \ldots.], \quad F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$$

$$D_\mu F^{\mu\nu} = j^\nu[Q, \overline{Q}, G]$$ mean-field generation

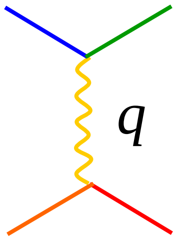
collisionless limit: $C = \overline{C} = C_g = 0$

Time scale of collisional processes

Time scale of processes driven by parton-parton scattering

$$t_{\text{hard}} \sim \frac{1}{g^4 \ln(1/g)\, T}$$

$$t_{\text{soft}} \sim \frac{1}{g^2 \ln(1/g)\, T}$$



hard scattering: $q \sim T$

soft scattering: $q \sim gT$

Time scale of collective phenomena

$$t_{\text{collec}} \sim \frac{1}{g\, T}$$

$$g^2 << 1 \quad \Rightarrow \quad t_{\text{hard}} >> t_{\text{soft}} >> t_{\text{collec}}$$

The instabilities are fast if QGP is weakly coupled

Small fluctuations

The distribution function of quarks

fluctuation

$$Q(t,\mathbf{r},\mathbf{p}) = Q_0(\mathbf{p}) + \delta Q\,(t,\mathbf{r},\mathbf{p})$$

stationary colorless state $Q_0^{ij}(\mathbf{p}) = \delta^{ij} n(\mathbf{p})$

$$|Q_0(\mathbf{p})| >> |\delta Q(t,\mathbf{r},\mathbf{p})|, \quad |\nabla_p Q_0(\mathbf{p})| >> |\nabla_p \delta Q(t,\mathbf{r},\mathbf{p})|$$

$$\mathbf{E}(t,\mathbf{r}), \mathbf{B}(t,\mathbf{r}), A^0(t,\mathbf{r}), \mathbf{A}(t,\mathbf{r}) \sim \delta Q(t,\mathbf{r},\mathbf{p})$$

quarks only, inclusion of antiquarks and gluons: $n(\mathbf{p}) \to n(\mathbf{p}) + \overline{n}(\mathbf{p}) + 2N_c n_g(\mathbf{p})$

Linearized equations

Transport equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\delta Q(t,\mathbf{r},\mathbf{p}) - g\big(\mathbf{E}(t,\mathbf{r}) + \mathbf{v}\times\mathbf{B}(t,\mathbf{r})\big)\nabla_p n(\mathbf{p}) = 0$$

Yang-Mills (Maxwell) equations

$$\nabla\cdot\mathbf{E}(t,\mathbf{r}) = \rho(t,\mathbf{r}), \qquad \nabla\cdot\mathbf{B}(t,\mathbf{r}) = 0,$$

$$\nabla\times\mathbf{E}(t,\mathbf{r}) = -\frac{\partial\mathbf{B}(t,\mathbf{r})}{\partial t}, \quad \nabla\times\mathbf{B}(t,\mathbf{r}) = \mathbf{j}(t,\mathbf{r}) + \frac{\partial\mathbf{E}(t,\mathbf{r})}{\partial t}$$

$$\begin{cases} \rho_a(t,\mathbf{r}) = -g\int\frac{d^3p}{(2\pi)^3}\,\mathrm{Tr}\left[\tau^a\delta Q(t,\mathbf{r},\mathbf{p})\right], \\ \mathbf{j}_a(t,\mathbf{r}) = -g\int\frac{d^3p}{(2\pi)^3}\,\mathbf{v}\,\mathrm{Tr}\left[\tau^a\delta Q(t,\mathbf{r},\mathbf{p})\right], \end{cases}$$

gauge dependence discussed *a posteriori*

Initial value problem

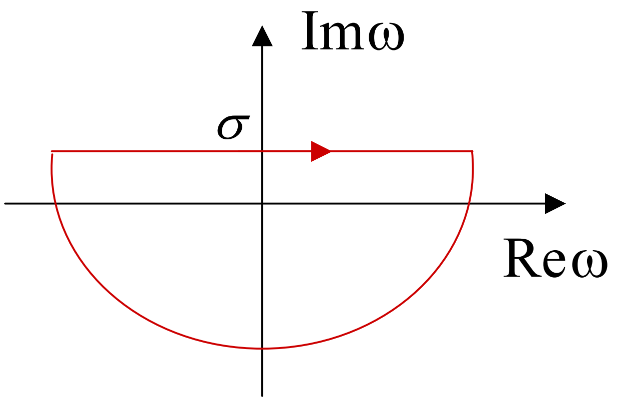
$$\delta Q(t=0,\mathbf{r},\mathbf{p}) = \delta Q_0(\mathbf{r},\mathbf{p}),$$

$$\mathbf{E}(t=0,\mathbf{r},\mathbf{p}) = \mathbf{E}_0(\mathbf{r},\mathbf{p}), \quad \mathbf{B}(t=0,\mathbf{r},\mathbf{p}) = \mathbf{B}_0(\mathbf{r},\mathbf{p})$$

One-sided Fourier transformations

$$\begin{cases} f(\omega,\mathbf{k}) = \int_0^\infty dt \int d^3r\, e^{i(\omega t - \mathbf{kr})} f(t,\mathbf{r}) \\ f(t,\mathbf{r}) = \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3}\, e^{-i(\omega t - \mathbf{kr})} f(\omega,\mathbf{k}) \end{cases}$$

$$0 < \sigma \in R$$



Transformed linear equations

Transport equation

$$-i(\omega - \mathbf{v}\cdot\mathbf{k})\delta Q(\omega,\mathbf{k},\mathbf{p}) - g\big(\mathbf{E}(\omega,\mathbf{k}) + \mathbf{v}\times\mathbf{B}(\omega,\mathbf{k})\big)\nabla_p n(\mathbf{p}) = \delta Q_0(\mathbf{k},\mathbf{p})$$

Yang-Mills (Maxwell) equations

$$i\mathbf{k}\cdot\mathbf{E}(\omega,\mathbf{k}) = \rho(\omega,\mathbf{k}), \qquad i\mathbf{k}\cdot\mathbf{B}(\omega,\mathbf{k}) = 0,$$

$$i\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k}) = i\omega\mathbf{B}(\omega,\mathbf{k}) + \mathbf{B}_0(\mathbf{k}),$$

$$i\mathbf{k}\times\mathbf{B}(\omega,\mathbf{k}) = \mathbf{j}(\omega,\mathbf{k}) - i\omega\mathbf{E}(\omega,\mathbf{k}) - \mathbf{E}_0(\mathbf{k})$$

$$\begin{cases} \rho_a(\omega,\mathbf{k}) = -g\displaystyle\int \frac{d^3p}{(2\pi)^3}\,\mathrm{Tr}\left[\tau^a\,\delta Q(\omega,\mathbf{k},\mathbf{p})\right], \\ \mathbf{j}_a(\omega,\mathbf{k}) = -g\displaystyle\int \frac{d^3p}{(2\pi)^3}\,\mathbf{v}\,\mathrm{Tr}\left[\tau^a\,\delta Q(\omega,\mathbf{k},\mathbf{p})\right], \end{cases}$$

Solution

$$\left[-\mathbf{k}^2\delta^{ij}+k^ik^j+\omega^2\varepsilon^{ij}(\omega,\mathbf{k})\right]E^j(\omega,\mathbf{k})=-g\omega\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega-\mathbf{v}\cdot\mathbf{k}}\delta Q_0(\mathbf{k},\mathbf{p})$$

$$-i\frac{g^2}{2}\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega-\mathbf{v}\cdot\mathbf{k}}\frac{\mathbf{v}\times\mathbf{B}_0(\mathbf{k})}{\omega}\cdot\nabla_p n(\mathbf{p})+i\omega E_0^i(\mathbf{k})-i\left(\mathbf{k}\times\mathbf{B}_0(\mathbf{k})\right)^i$$

$$\Sigma^{ij}(\omega,\mathbf{k})\equiv-\mathbf{k}^2\delta^{ij}+k^ik^j+\omega^2\varepsilon^{ij}(\omega,\mathbf{k})$$

Isotropic system

$$\varepsilon^{ij}(\omega,\mathbf{k})\equiv\varepsilon_L(\omega,\mathbf{k})\frac{k^ik^j}{\mathbf{k}^2}+\varepsilon_L(\omega,\mathbf{k})\left(\delta^{ij}-\frac{k^ik^j}{\mathbf{k}^2}\right)$$

$$\left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k})=\frac{1}{\omega^2\varepsilon_L(\omega,\mathbf{k})}\frac{k^ik^j}{\mathbf{k}^2}+\frac{1}{\omega^2\varepsilon_T(\omega,\mathbf{k})-\mathbf{k}^2}\left(\delta^{ij}-\frac{k^ik^j}{\mathbf{k}^2}\right)$$

Fluctuations of E field

The solution

$$E^{i}(\omega,\mathbf{k}) = \left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k})\big[\ldots\ldots\ \delta Q_0(\mathbf{k},\mathbf{p}) + \ldots\ldots\ \mathbf{E}_0(\mathbf{k}) + \ldots\ldots\ \mathbf{B}_0(\mathbf{k})\big]^{j}$$

The correlation function

$$\begin{aligned}\left\langle E^{i}(\omega,\mathbf{k})E^{j}(\omega',\mathbf{k}')\right\rangle = \left(\Sigma^{-1}\right)^{ik}(\omega,\mathbf{k})\left(\Sigma^{-1}\right)^{jl}(\omega',\mathbf{k}')\Big[\ldots\ldots\ &\left\langle \delta Q_0(\mathbf{k},\mathbf{p})\delta Q_0(\mathbf{k}',\mathbf{p}')\right\rangle \\ &+ \ldots\ldots\ \left\langle \delta Q_0(\mathbf{k},\mathbf{p})E_0^{m}(\mathbf{k}')\right\rangle + \ldots\ldots\ \left\langle \delta Q_0(\mathbf{k},\mathbf{p})B_0^{m}(\mathbf{k}')\right\rangle \\ &+ \ldots\ldots\ \left\langle E_0^{m}(\mathbf{k})E_0^{n}(\mathbf{k}')\right\rangle + \ldots\ldots\ \left\langle E_0^{m}(\mathbf{k})B_0^{n}(\mathbf{k}')\right\rangle \\ &+ \ldots\ldots\ \left\langle B_0^{m}(\mathbf{k})B_0^{n}(\mathbf{k}')\right\rangle\Big]^{kl}\end{aligned}$$

$\langle \cdots \rangle$ - statistical ensemble average

B, ρ, j are given by E

From Maxwell equations

$$\begin{cases} \mathbf{B}(\omega,\mathbf{k}) = \dfrac{\mathbf{k}}{\omega}\times\mathbf{E}(\omega,\mathbf{k}) + \dfrac{i}{\omega}\mathbf{B}_0(\mathbf{k}) \\ \rho(\omega,\mathbf{k}) = i\mathbf{k}\cdot\mathbf{E}(\omega,\mathbf{k}) \\ \mathbf{j}(\omega,\mathbf{k}) = i\omega\mathbf{E}(\omega,\mathbf{k}) - i\mathbf{k}\times\mathbf{B}(\omega,\mathbf{k}) + \mathbf{E}_0(\mathbf{k}) \end{cases}$$

Initial values

Using Maxwell equations

$\mathbf{E}_0(\mathbf{k})$, $\mathbf{B}_0(\mathbf{k})$, $\rho_0(\mathbf{k})$, $\mathbf{j}_0(\mathbf{k})$ can be expressed through $\delta Q_0(\mathbf{k},\mathbf{p})$

Initial fluctuations

color indices $i, j, k, l = 1, 2, \ldots, N_c$

$$\left\langle \delta Q_0^{ij}(\mathbf{r},\mathbf{p})\, \delta Q_0^{kl}(\mathbf{r}',\mathbf{p}') \right\rangle = ?$$

Assumption

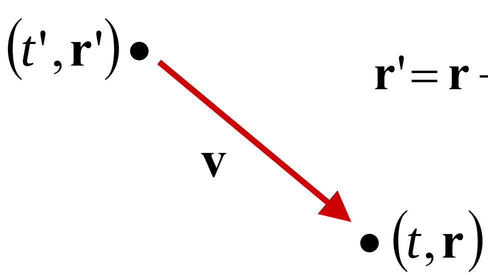
The initial fluctuations are given by $\left\langle \delta Q^{ij}(t=0,\mathbf{r},\mathbf{p})\, \delta Q^{kl}(t'=0,\mathbf{r}',\mathbf{p}') \right\rangle_{\text{free}}$

colorless state

$$\delta Q^{ij}(t,\mathbf{r},\mathbf{p}) \equiv Q^{ij}(t,\mathbf{r},\mathbf{p}) - \left\langle Q^{ij}(t,\mathbf{r},\mathbf{p}) \right\rangle = Q^{ij}(t,\mathbf{r},\mathbf{p}) - \delta^{ij} n(\mathbf{p})$$

Classical limit

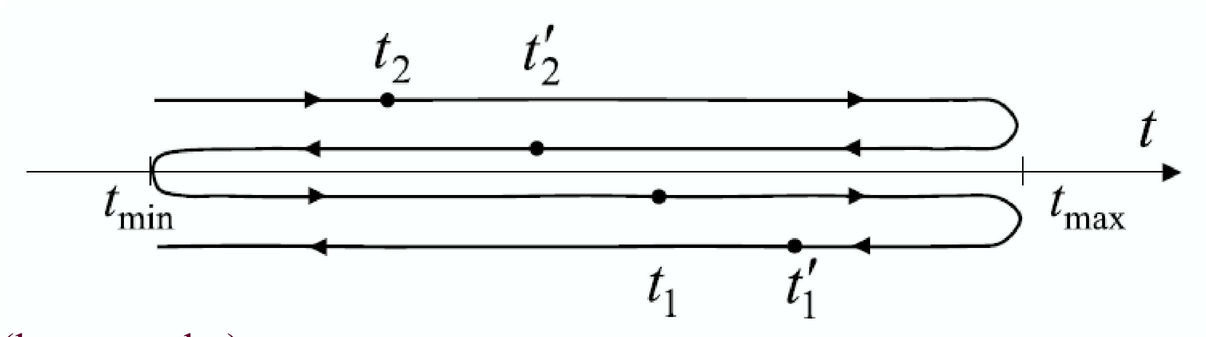
$$\left\langle \delta Q^{ij}(t,\mathbf{r},\mathbf{p})\, \delta Q^{kl}(t',\mathbf{r}',\mathbf{p}') \right\rangle_{\text{free}} = \delta^{il}\delta^{jk}(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}')(2\pi)^3\delta^{(3)}(\mathbf{r}'-\mathbf{r}-\mathbf{v}(t'-t))\,n(\mathbf{p})$$



$$\mathbf{r}' = \mathbf{r} + \mathbf{v}(t'-t)$$

Fluctuations of free distribution functions cont.

$$\left\langle \varphi_j^*(x'_1)\varphi_i(x_1)\,\varphi_l^*(x'_2)\varphi_k(x_2)\right\rangle = \left\langle T_c\left(\varphi_j^*(x'_1)\varphi_i(x_1)\,\varphi_l^*(x'_2)\varphi_k(x_2)\right)\right\rangle$$



Wick theorem (lowest order)

$$\left\langle T_c\left(\varphi_j^*(x'_1)\varphi_i(x_1)\,\varphi_l^*(x'_2)\varphi_k(x_2)\right)\right\rangle = \left\langle T_c\left(\varphi_j^*(x'_1)\varphi_i(x_1)\right)\right\rangle\left\langle T_c\left(\varphi_l^*(x'_2)\varphi_k(x_2)\right)\right\rangle$$
$$+\left\langle T_c\left(\varphi_j^*(x'_1)\varphi_k(x_2)\right)\right\rangle\left\langle T_c\left(\varphi_l^*(x'_2)\varphi_i(x_1)\right)\right\rangle$$

$$\left\langle \varphi_j^*(x'_1)\varphi_i(x_1)\,\varphi_l^*(x'_2)\varphi_k(x_2)\right\rangle = \left\langle \varphi_j^*(x'_1)\varphi_i(x_1)\right\rangle\left\langle \varphi_l^*(x'_2)\varphi_k(x_2)\right\rangle$$
$$+\left\langle \varphi_j^*(x'_1)\varphi_k(x_2)\right\rangle\left\langle \varphi_i(x_1)\varphi_l^*(x'_2)\right\rangle$$

Fluctuations in isotropic (stable) system

$$\left\langle E_a^i(\omega,\mathbf{k})E_b^j(\omega',\mathbf{k}')\right\rangle = \frac{g^2}{2}\delta^{ab}\left(2\pi\right)^3\delta^{(3)}\left(\mathbf{k}+\mathbf{k}'\right)\int\frac{d^3p}{(2\pi)^3}n(\mathbf{p})F(\omega,\mathbf{k},\omega',\mathbf{k}',\mathbf{p})$$

colorless background

translational invariance

$F(\omega,\mathbf{k},\omega',\mathbf{k}',\mathbf{p})$ has poles at:

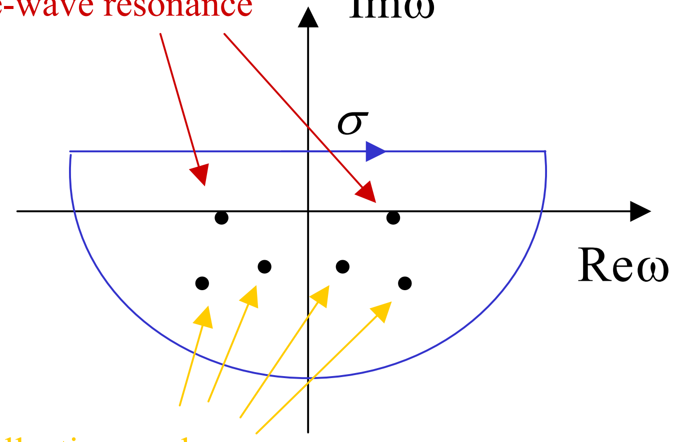
particle-wave resonance
$$\begin{cases}\omega-\mathbf{v}\cdot\mathbf{k}=0\\ \omega'-\mathbf{v}'\cdot\mathbf{k}'=0\end{cases}$$

collective longitudinal modes
$$\begin{cases}\varepsilon_L(\omega,\mathbf{k})=0\\ \varepsilon_L(\omega',\mathbf{k}')=0\end{cases}$$

collective transverse modes
$$\begin{cases}\omega^2\varepsilon_T(\omega,\mathbf{k})-\mathbf{k}^2=0\\ \omega'^2\,\varepsilon_T(\omega',\mathbf{k}')-\mathbf{k}'^2=0\end{cases}$$

Fluctuations in isotropic (stable) system

$$\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle = \int_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega}{2\pi}\int_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega'}{2\pi}\int\frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\,e^{-i(\omega t+\omega' t'-\mathbf{kr}-\mathbf{k}'\mathbf{r}')}\times\left\langle E_a^i(\omega,\mathbf{k})E_b^j(\omega',\mathbf{k}')\right\rangle$$



$$\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle \sim f(\mathbf{r}-\mathbf{r}')$$

$$\left\langle E_a^i(\omega,\mathbf{k})E_b^j(\omega',\mathbf{k}')\right\rangle \sim \delta^{(3)}(\mathbf{k}+\mathbf{k}')$$

$$\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle = \begin{pmatrix}\text{collective}\\ \text{modes}\end{pmatrix}\left(e^{-\gamma t}\ \text{ or }\ e^{-\gamma t'}\right) + \begin{pmatrix}\text{particle-wave}\\ \text{resonance}\end{pmatrix} f(t-t')$$

$$\gamma \equiv \mathrm{Im}\,\omega > 0$$

Fluctuations in equilibrium system

Long time limit $\quad t, t' \to \infty \quad \left\langle E_a^i(t,\mathbf{r}) E_b^j(t',\mathbf{r}') \right\rangle_\infty = f(t'-t, \mathbf{r}'-\mathbf{r})$

$$\left\langle E_a^i(t,\mathbf{r}) E_b^j(t',\mathbf{r}') \right\rangle_\infty = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega(t-t')-\mathbf{k}(\mathbf{r}-\mathbf{r}'))} \left\langle E_a^i E_b^j \right\rangle_{\omega,\mathbf{k}}$$

fluctuation spectrum

Fluctuation dissipation relation

$$\left\langle E_a^i E_b^j \right\rangle_{\omega,\mathbf{k}} = 2\delta^{ab} T \omega^3 \left[\frac{k^i k^j}{\mathbf{k}^2} \frac{\operatorname{Im} \varepsilon_L(\omega,\mathbf{k})}{|\omega^2 \varepsilon_L(\omega,\mathbf{k})|^2} + \left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2} \right) \frac{\operatorname{Im} \varepsilon_T(\omega,\mathbf{k})}{|\omega^2 \varepsilon_T(\omega,\mathbf{k}) - \mathbf{k}^2|^2} \right]$$

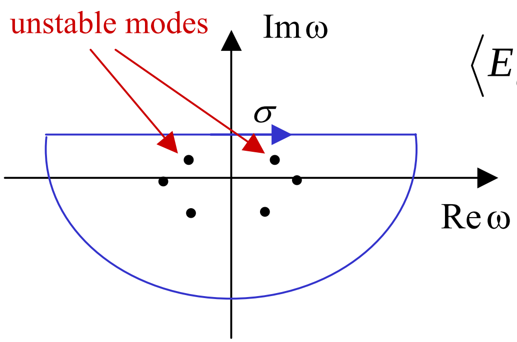
$$\left\langle B_a^i B_b^j \right\rangle_{\omega,\mathbf{k}} = 2\delta^{ab} T \omega \left(\mathbf{k}^2 \delta^{ij} - k^i k^j \right) \frac{\operatorname{Im} \varepsilon_T(\omega,\mathbf{k})}{|\omega^2 \varepsilon_T(\omega,\mathbf{k}) - \mathbf{k}^2|^2}$$

Fluctuations in unstable systems

Two-stream system

$$n(\mathbf{p}) = (2\pi)^3 n\left[\delta^{(3)}(\mathbf{p}-\mathbf{q}) + \delta^{(3)}(\mathbf{p}+\mathbf{q})\right]$$

Longitudinal electric field: $\omega_+(\mathbf{k})$ - stable mode, $\omega_-(\mathbf{k})$ - unstable mode



$$\left\langle E_a^i(\omega,\mathbf{k}) E_b^i(\omega',\mathbf{k}')\right\rangle = \frac{g^2}{2}\delta^{ab}(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')\frac{\mathbf{k}\cdot\mathbf{k}'}{\mathbf{k}^2\mathbf{k}'^2}$$

$$\times \frac{1}{\varepsilon_L(\omega,\mathbf{k})}\frac{1}{\varepsilon_L(\omega',\mathbf{k}')}\int\frac{d^3p}{(2\pi)^3}\frac{n(\mathbf{p})}{(\omega-\mathbf{v}\cdot\mathbf{k})(\omega'-\mathbf{v}'\cdot\mathbf{k}')}$$

broken time translational invariance

$$\left\langle E_a^i(t,\mathbf{r}) E_b^i(t',\mathbf{r}')\right\rangle_{\text{unstable}} = \frac{g^2}{2}\delta^{ab} n\int\frac{d^3k}{(2\pi)^3}\frac{e^{-i\mathbf{k}(\mathbf{r}-\mathbf{r}')}}{\mathbf{k}^2}\frac{1}{\left(\omega_+^2-\omega_-^2\right)^2}\frac{\left(\gamma_\mathbf{k}^2+(\mathbf{k}\mathbf{u})^2\right)^2}{\gamma_\mathbf{k}^2}$$

$$\times\left[\left(\gamma_\mathbf{k}^2+(\mathbf{k}\mathbf{u})^2\right)\cosh\left(\gamma_\mathbf{k}(t+t')\right)+\left(\gamma_\mathbf{k}^2-(\mathbf{k}\mathbf{u})^2\right)\cosh\left(\gamma_\mathbf{k}(t-t')\right)\right]$$

$\mathbf{u} \equiv \frac{\mathbf{q}}{E_\mathbf{q}}$, $\gamma_\mathbf{k} \equiv \mathrm{Im}\,\omega_-(\mathbf{k})$

Gauge dependence

Generic correlation function: $L_{ab}(x,x') \equiv \langle H_a(x) K_b(x') \rangle$

Infinitesimal gauge transformation

$$H_a(x) \to H_a(x) + f_{abc}\lambda_b(x) H_c(x)$$

$$L_{ab}(x,x') \to L_{ab}(x,x') + f_{acd}\lambda_c(x) L_{db}(x,x') + f_{bcd}\lambda_c(x') L_{ad}(x,x')$$

colorless background

Actual correlation function: $L_{ab}(x,x') \equiv \delta^{ab} L(x,x')$

$$L_{ab}(x,x') \to \left(\delta^{ab} + f_{acb}\lambda_c(x) + f_{bca}\lambda_c(x')\right) L(x,x')$$

$L_{aa}(x,x') = \left(N_c^2 - 1\right) L(x,x')$ **- gauge invariant!**

Momentum broadening in equilibrium QGP

$$\hat{q} \equiv \frac{d\left\langle \Delta p_T^2(t) \right\rangle}{dt}$$

$$\left\langle \Delta p_T^2(t) \right\rangle = g^2 C \int_0^t dt_1 \int_0^t dt_2 \Big\{ \left\langle E_a^x(x(t_1)) E_a^x(x(t_2)) \right\rangle + \left\langle E_a^y(x(t_1)) E_a^y(x(t_2)) \right\rangle - \left\langle E_a^x(x(t_1)) B_a^y(x(t_2)) \right\rangle$$
$$+ \left\langle E_a^y(x(t_1)) B_a^x(x(t_2)) \right\rangle - \left\langle B_a^y(x(t_1)) E_a^x(x(t_2)) \right\rangle - \left\langle B_a^x(x(t_1)) E_a^y(x(t_2)) \right\rangle$$
$$+ \left\langle B_a^x(x(t_1)) B_a^x(x(t_2)) \right\rangle + \left\langle B_a^y(x(t_1)) B_a^y(x(t_2)) \right\rangle \Big\}$$

translational invariance

$$\left\langle E_a^i(t, \mathbf{r}) E_b^j(t', \mathbf{r}') \right\rangle_\infty = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} \, e^{-i(\omega(t-t') - \mathbf{k}(\mathbf{r}-\mathbf{r}'))} \left\langle E_a^i E_b^j \right\rangle_{\omega,\mathbf{k}}$$

fluctuation spectrum

$\mathbf{r}(t) = (0,0,t)$ - parton's trajectory

$$\int_0^t dt_1 \int_0^t dt_2 e^{-i(\omega - k_z)(t_1 - t_2)} = \frac{4}{(\omega - k_z)^2} \sin\left(\frac{(\omega - k_z)t}{2} \right) \underset{t\to\infty}{\longrightarrow} 2\pi t \, \delta(\omega - k_z)$$

Momentum broadening in equilibrium QGP

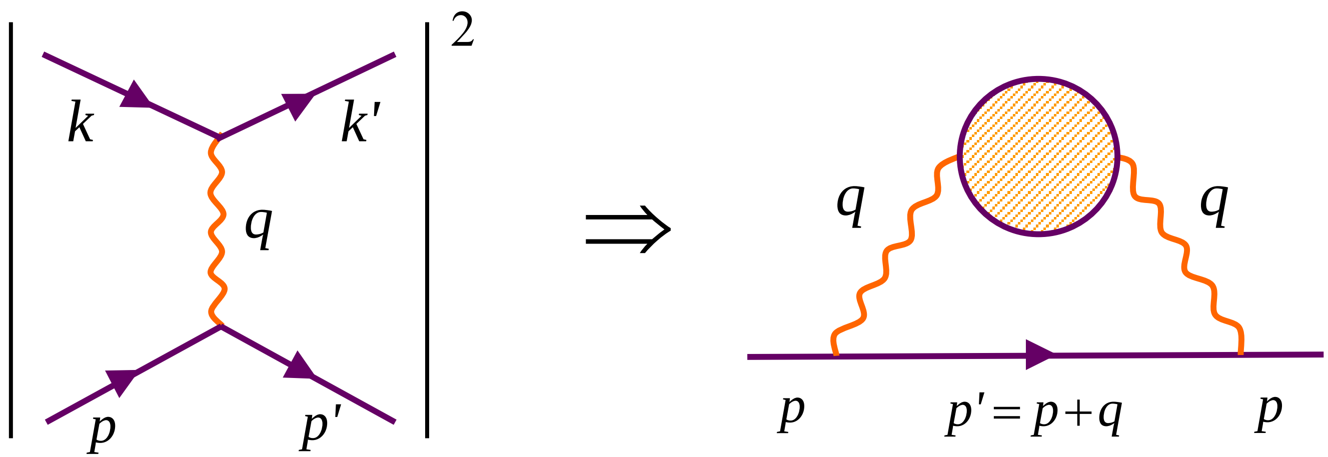
$$\hat{q} = g^2 C \int \frac{d^3k}{(2\pi)^3} \Big\{ \langle E_a^x E_a^x \rangle_{k_z,\mathbf{k}} + \langle E_a^y E_a^y \rangle_{k_z,\mathbf{k}} - \langle E_a^x B_a^y \rangle_{k_z,\mathbf{k}} + \langle E_a^y B_a^x \rangle_{k_z,\mathbf{k}} - \langle B_a^y E_a^x \rangle_{k_z,\mathbf{k}} - \langle B_a^x E_a^y \rangle_{k_z,\mathbf{k}} + \langle B_a^x B_a^x \rangle_{k_z,\mathbf{k}} + \langle B_a^y B_a^y \rangle_{k_z,\mathbf{k}} \Big\}$$

$$C_R \equiv C\,(N_c^2 - 1) = \begin{cases} \frac{1}{2} & -\text{ quark} \\ N_c & -\text{ gluon} \end{cases}$$

$$\hat{q} = 2 g^2 C_R T \int \frac{d^3k}{(2\pi)^3} \frac{k_T^2}{k_z \mathbf{k}^2} \left[\frac{\operatorname{Im} \varepsilon_L(k_z,\mathbf{k})}{|\varepsilon_L(k_z,\mathbf{k})|^2} + \frac{k_z^2 k_T^2 \operatorname{Im} \varepsilon_T(k_z,\mathbf{k})}{|k_z^2 \varepsilon_T(k_z,\mathbf{k}) - \mathbf{k}^2|^2} \right]$$

$$\hat{q} \approx \frac{g^2}{2\pi} C_R\, m_D^2\, T \ln(1/\mathrm{g})$$

Momentum broadening in thermal field approach



$$\hat{q} = \frac{1}{4E_p}\mathrm{Tr}[\not{p}\Sigma^{>}_{w}(p)]$$

$q \sim gT, \quad k \sim T$

Hard-Loop propagator

$$\Sigma^{>}_{w}(p) = g^2 C_F \int \frac{d^3 p'}{(2\pi)^3 2E'} q_T \gamma^{\mu} \not{p}' \gamma^{\nu} D^{<}_{\mu\nu}(q)$$

$$\hat{q} \approx \frac{g^2}{2\pi} C_R\, m_D^2\, T \ln(1/\mathrm{g})$$

Moore & Teaney 2005; Baier & Mehtar-Tani 2008

Momentum broadening in unstable QGP

Two-stream system

longitudinal electric fields only

$$\left\langle \Delta p_T^2(t) \right\rangle = g^2 C \int_0^t dt_1 \int_0^t dt_2 \left\{ \left\langle E_a^x(x(t_1)) E_a^x(x(t_2)) \right\rangle + \left\langle E_a^y(x(t_1)) E_a^y(x(t_2)) \right\rangle \right\}$$

$$\left\langle \Delta p_T^2(t) \right\rangle \approx \frac{g^4}{4} C_R\, n \int \frac{d^3k}{(2\pi)^3} e^{2\gamma_{\mathbf{k}} t} \frac{k_T^2 \left(\gamma_{\mathbf{k}}^2 + (\mathbf{ku})^2\right)^3}{\mathbf{k}^4 \left(\omega_+^2 - \omega_-^2\right)^2 \gamma_{\mathbf{k}}^2 \left(k_z^2 + \gamma_{\mathbf{k}}^2\right)}$$

$$\hat{q} \sim e^{2\gamma t}$$

Conclusions

- Unstable QGP is generically time dependent
- $\hat{q} \sim e^{2\gamma t}$