

# Viscosity of constituent quark matter

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with

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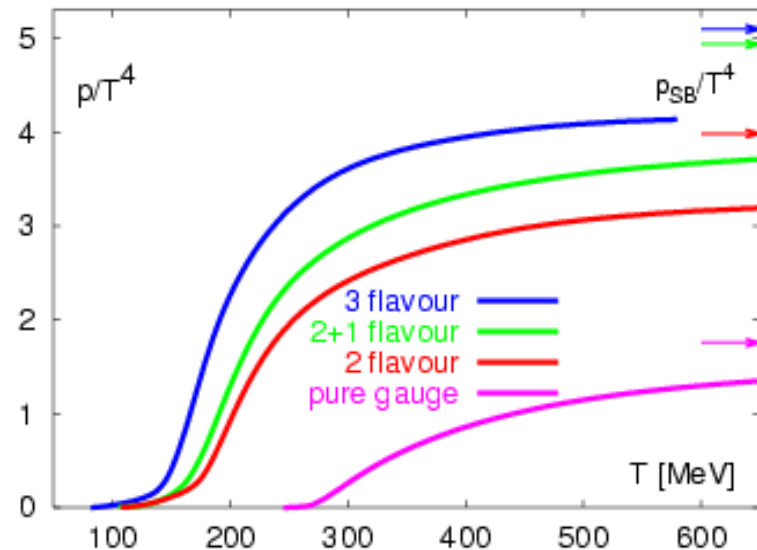
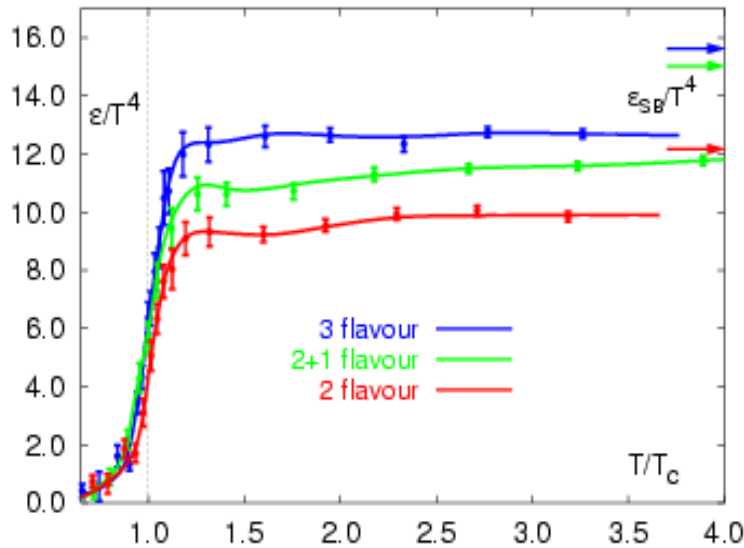
**EMMI Conference,  
Wroclaw, 9 July 2009**

# Fundament-1: EOS for strongly interacting matter from lattice-QCD

zero baryon density (1990-2000)

finite baryon densities (2000 - )

→  $\varepsilon(T, \mu)$ ,  $P(T, \mu)$



LQCD gives realistic EOS for QGP (deconfined matter)

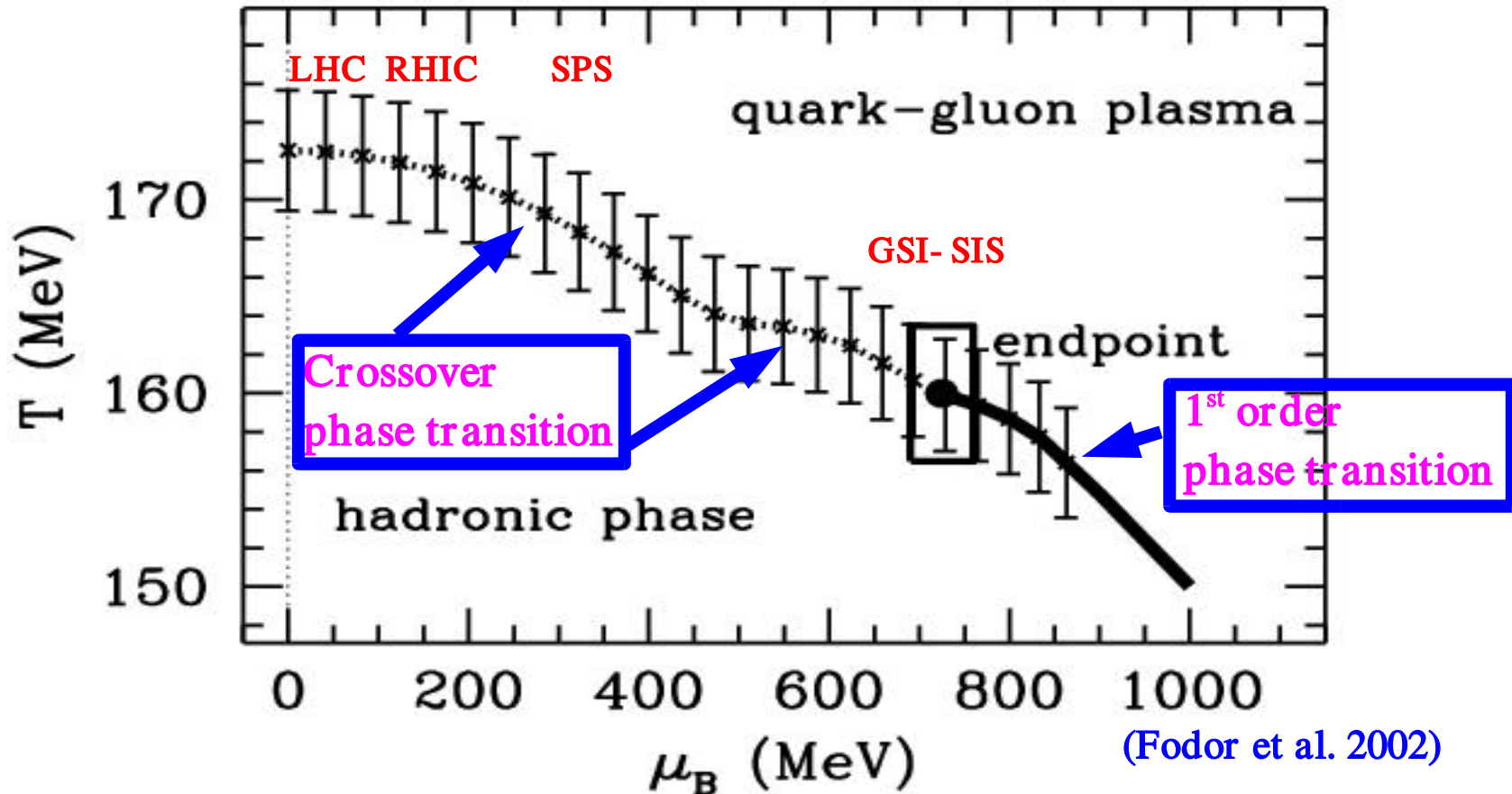
$T_c = 170$  MeV     $T/T_c = 1.1-1.2$     →  $\varepsilon = 2-3$  GeV/fm<sup>3</sup>    (SPS ?)

$T/T_c = 1.5-2.0$     →  $\varepsilon = 6-20$  GeV/fm<sup>3</sup>    (RHIC ?)

$T/T_c = 2.0-3.0$     →  $\varepsilon = 20-100$  GeV/fm<sup>3</sup>    (LHC ?)

## Fundament-2: order of phase transition from lattice-QCD

Lattice-QCD results at finite density, SU(3), Nf=2  $\mu > 0$



Crossover phase transition at small and intermediate baryon densities:



*What is the microscopical mechanism of the hadronization ????*

⇒ **QUARK COALESCENCE is one possibility**

Interacting massive quarks around  $T_c$  !!

## Introducing quasi-particle picture:

**SU(3) Gluon EOS with free quasi-gluons + B(T) bag  
Fix degrees of freedom (d=16)**

$$P(T) = \frac{d}{(2\pi)^3} \int d^3 p \frac{p^2}{3\sqrt{p^2 + M(T)^2}} \left[ \exp\left(\frac{\sqrt{p^2 + M(T)^2}}{T}\right) - 1 \right]^{-1} - B(T)$$

$$\varepsilon(T) = \frac{d}{(2\pi)^3} \int d^3 p \sqrt{p^2 + M(T)^2} \left[ \exp\left(\frac{\sqrt{p^2 + M(T)^2}}{T}\right) - 1 \right]^{-1} + B(T)$$

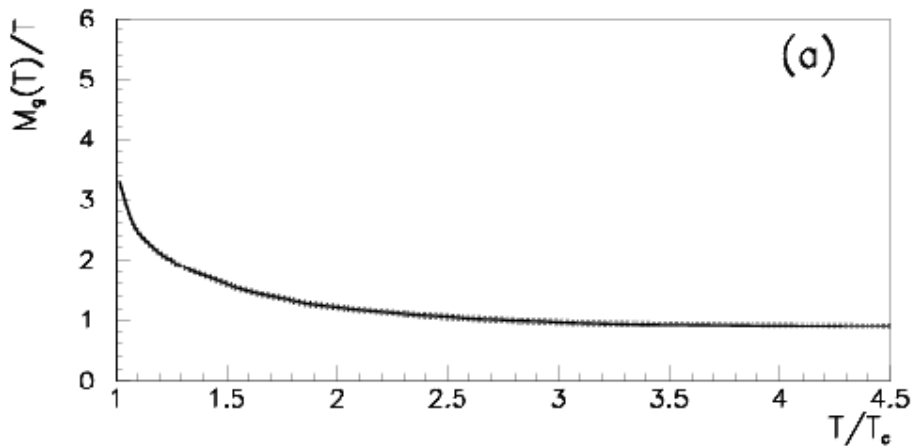
**$\varepsilon(T), P(T) \rightsquigarrow M(T), B(T)$**

**Mass + Interaction**

**P. Lévai, U. Heinz, 1996, PRC51, 3326.**

Extracting  $g(T)$  and  $B(T)$  from lattice-QCD results:

SU(3) Gluon EOS with free quasi-gluons +  $B(T)$  bag  
Fix degrees of freedom ( $d=16$ )

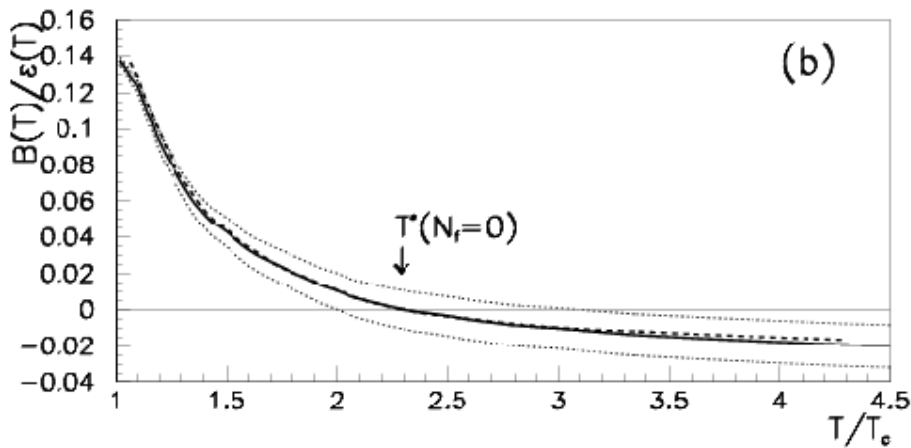


$\epsilon(T), P(T) \rightsquigarrow M(T), B(T)$

$T > 2 T_c$

$M(T)/T = c \cdot g(T) \quad [c=1/\sqrt{2}]$

$T \rightarrow 5 T_c, \quad \alpha_s = g^2/4\pi \rightarrow 0.15 \checkmark$



$B(T)$  is small ( $\approx 0$ )

Free quasi-gluons !

$T < 2 T_c$

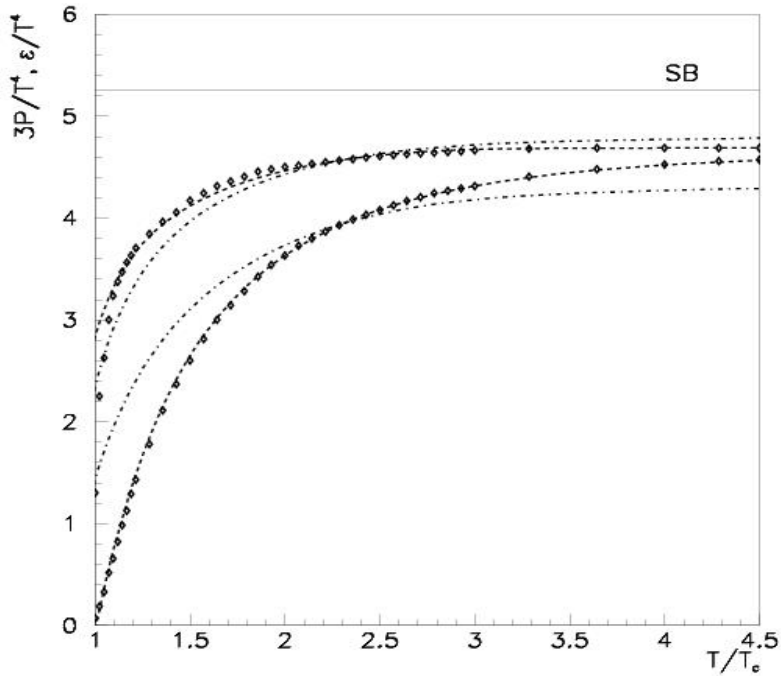
Interacting quasi-gluons  
or new excitations ?

Or both ?

# Quark matter formation in heavy ion collisions

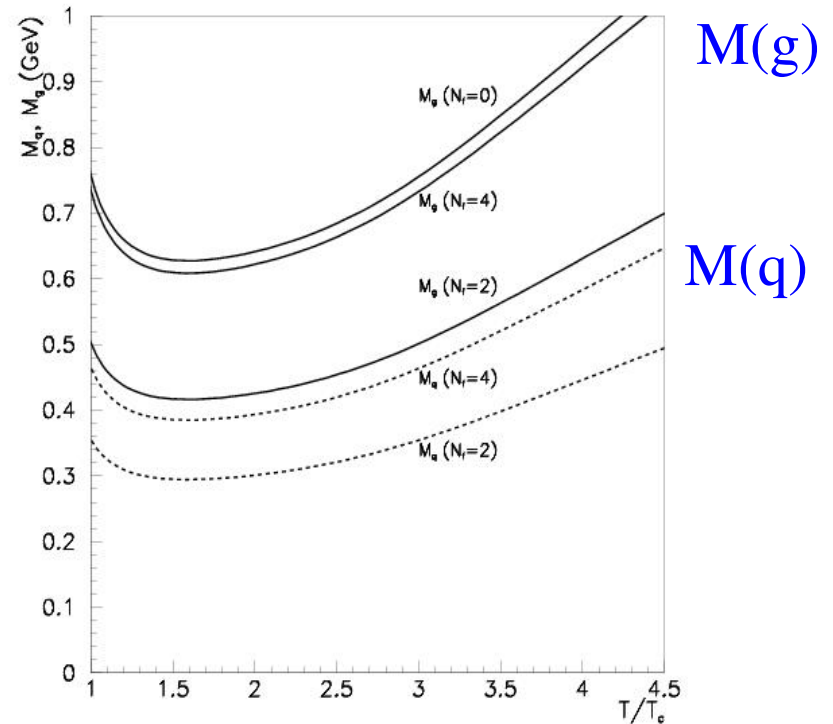
Lattice-QCD results around  $T_c$ , SU(3),  $N_f=0,2,4$   $\mu=0$  (1990 - ...)

Fig.4. SU(3),  $N_f=0$  --- EOS + Lattice QCD data



(Karsch et al. 1992)

Fig.9. SU(3),  $N_f=0,2,4$  ---  $M_g(T)$ ,  $M_q(T)$



Understanding in a quasiparticle picture:  $M(Q) \simeq 300$  MeV,  $M(G) \simeq 500-800$  MeV  
 [L.P, Heinz U., 1996, PRC51,3326]

→ Quark and antiquark dominated matter (QAP)

**HADRONIZATION  $\Leftrightarrow$  QUARK COALESCENCE** (ALCOR '95)

('Cross-over' phase transition) [T.S. Biro, P.L., J. Zimányi]

# Quark matter formation in heavy ion collisions

Lattice-QCD results around  $T_c$ , SU(3),  $N_f=0,2,4$   $\mu=0$

Fig.10. SU(3),  $N_f=0,2,4$  ---  $n_g/n_g^0$ ,  $n_q/n_q^0$

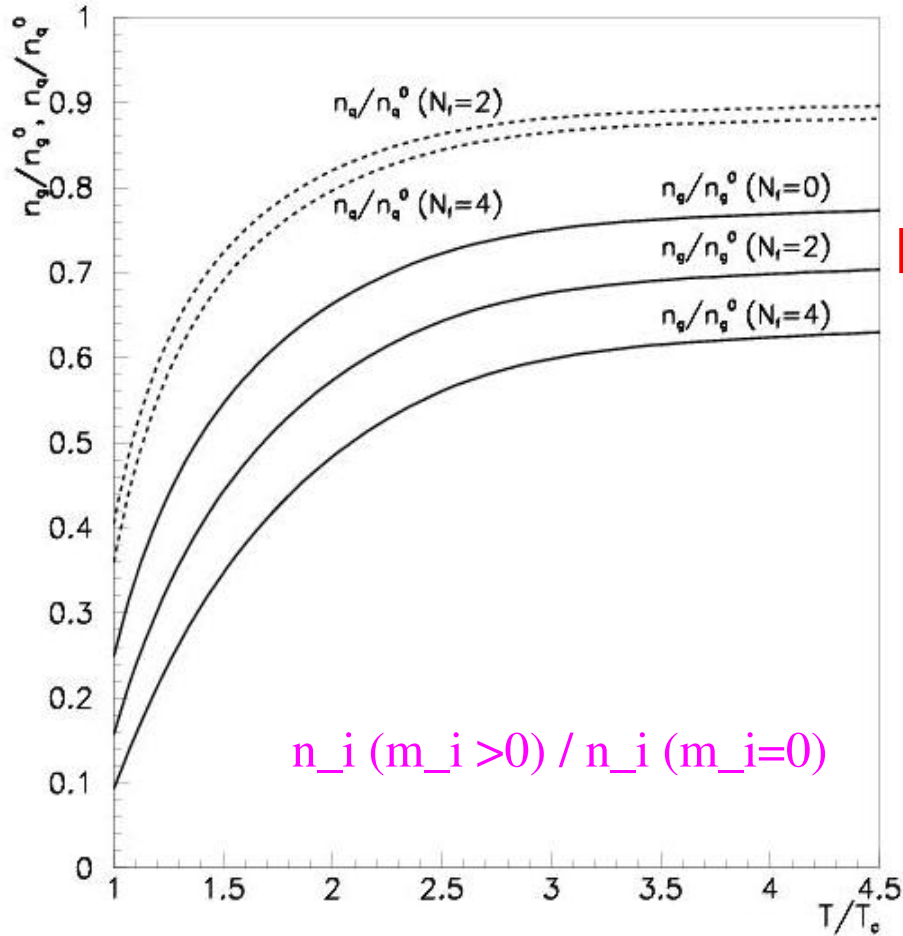
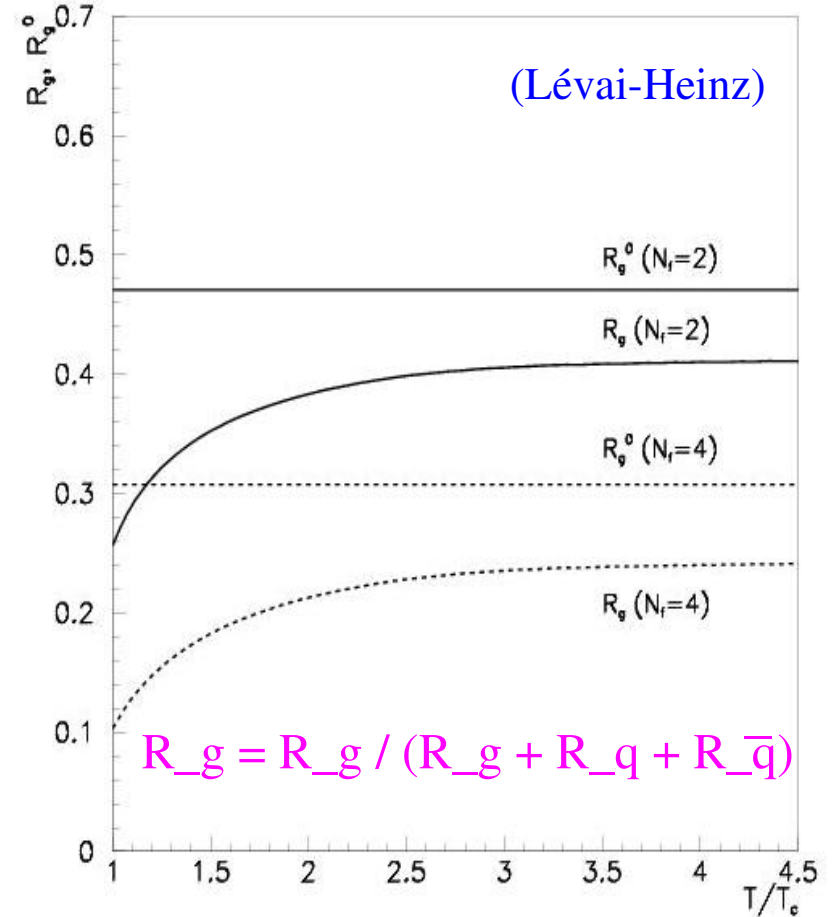


Fig.11. SU(3),  $N_f=2,4$  ---  $R_g(T)$ ,  $R_g^0(T)$



→ GLUON numbers are strongly suppressed at  $T_c$  and they will decay  
**QUARK-ANTIQUARK PLASMA**

## Model for interacting massive gluonic quasi-particles [SU(3), N<sub>f</sub>=0]

**B(T) ↔ interaction between gluons**

$$P_{\text{tot}}(\mathbf{T}) = P_{\text{kin}}(\mathbf{M}(\mathbf{T}), \mathbf{T}) - B(\mathbf{T})$$

$$\varepsilon_{\text{tot}}(\mathbf{T}) = \varepsilon_{\text{kin}}(\mathbf{M}(\mathbf{T}), \mathbf{T}) + B(\mathbf{T})$$

**B(T) → attractive (effective) scalar field**

$$B(T) = \frac{1}{2} m_{\sigma}^2 \sigma^2 = \frac{1}{2} \frac{g^2}{m_{\sigma}^2} n^2$$

**M(T) → effective mass**

$$M(T) = M_0(T) - g\sigma = M_0(T) - \frac{g^2}{m_{\sigma}^2} n$$
$$M_0(T) = \frac{1}{\sqrt{2}} g(T) T$$

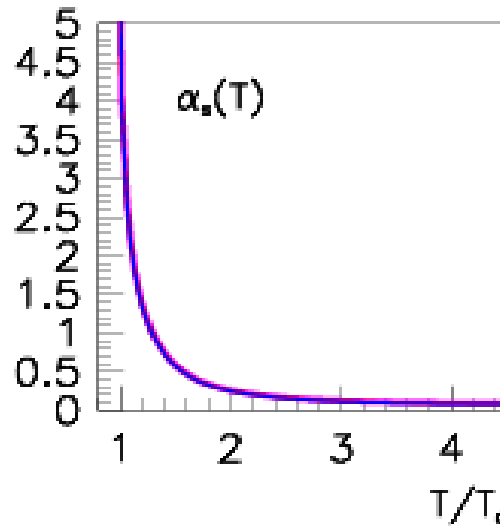
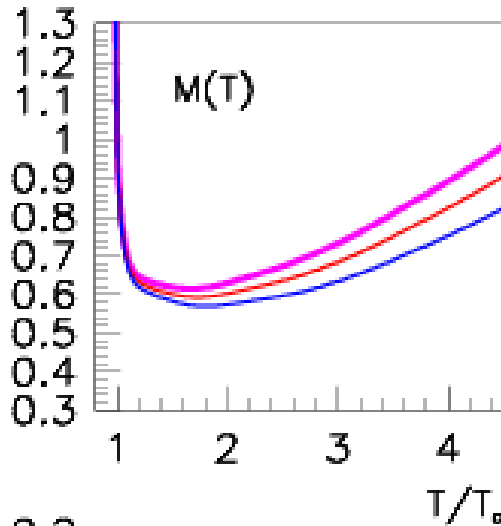
**U(r) → effective potential between octet gluons**

$$U(r) = \langle \lambda_i \lambda_j \rangle \alpha_s \frac{e^{-m_{\sigma} r}}{r} = -\frac{3}{2} \alpha_s \frac{e^{-m_{\sigma} r}}{r}$$



# Numerical results for interacting massive gluonic quasi-particles

$\epsilon(T), P(T)$   $\gggg$   $M(T), B(T)$   $\gggg$   $\alpha_s(T), m_\sigma(T)$

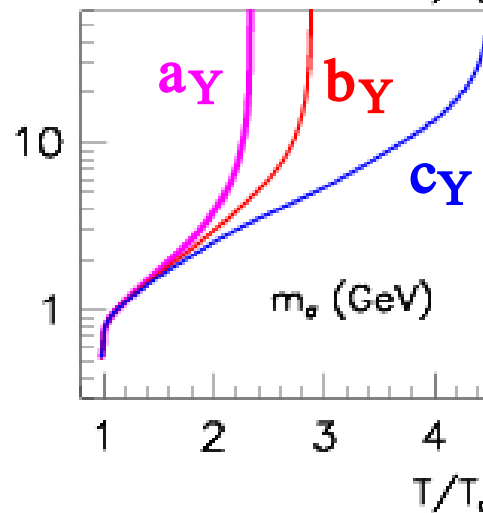
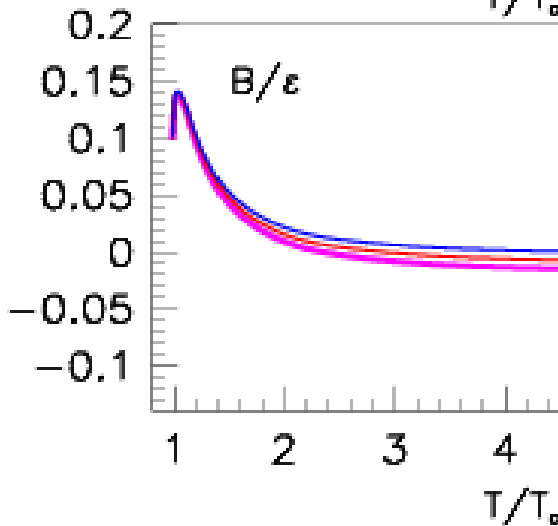


Uncertainties (0-2-4 %) in the lattice results:

$\alpha_s(T)$  is robust

$m_\sigma(T)$  is sensitive

[Yukawa-int.]



$m_\sigma(T)$  is divergent, where  $B$  becomes negative

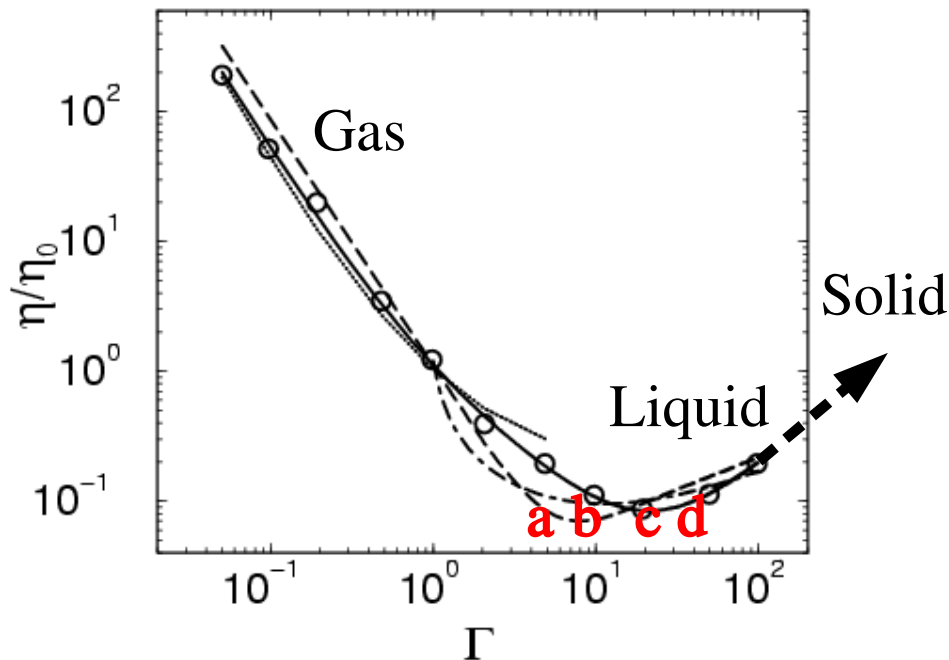
We can compress infinity into a finite  $T/T_c$  region.

**Viscosity of QGP (QAP, ...)**

- Motivations:**
- how large is the viscosity in QGP (around  $T_c$ ) ?
  - how can we determine it ?

## Binary Ionic Mixture (BIM OCP)

S. Bastea, PRE71,2005,056405



$\eta/\eta_0(\Gamma)$  has an U-shape in  $\Gamma$  !!

What about the shape of  $\eta/s(T)$  ???

What about the properties of sQGP ?

## What about sQGP ?

SU(3),  $N_f=2$ ,  $d=16+12+12$

$$\Gamma = \frac{4\pi\alpha_s(T)}{a_{WS}T} = C\alpha_s(T)$$

$$a_{WS}^3 = \frac{3}{4\pi n(T)}$$

$$\eta_0 = \sqrt{\frac{\alpha_s(T)M}{a_{WS}^5} \left(\frac{3}{4\pi}\right)^3}$$

$T=200$  MeV, SU(3),  $N_f=2$

a,  $\alpha_s = 0.17 \Rightarrow \Gamma = 5.1$

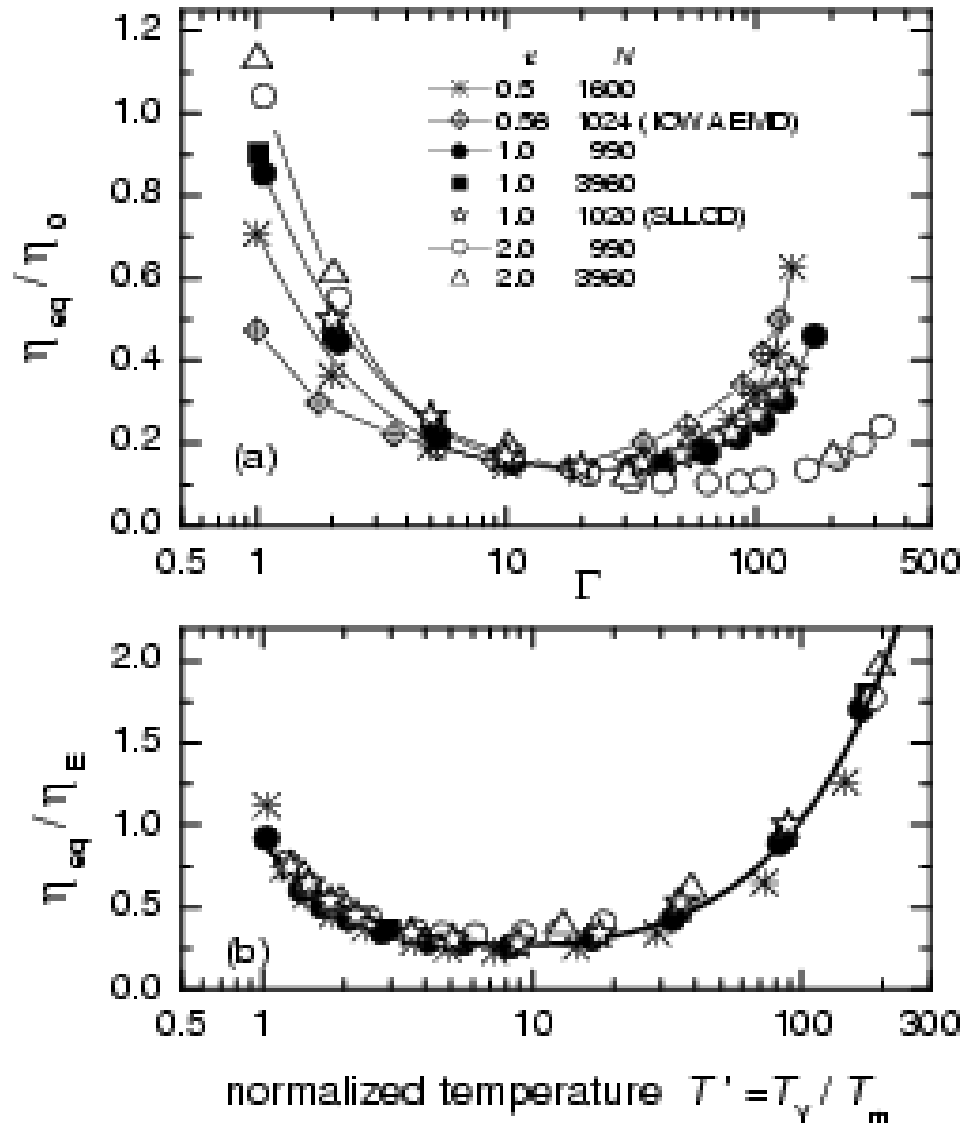
b,  $\alpha_s = 0.25 \Rightarrow \Gamma = 7.3$

c,  $\alpha_s = 1.0 \Rightarrow \Gamma = 24$

d,  $\alpha_s = 2.0 \Rightarrow \Gamma = 40$

Answer in dense plasma physics:

$\eta(T)$  has a U-shape !  $\eta/s$  ?



Z. Donkó, J. Goree,  
P. Hartmann, K. Kutasi:  
*Shear viscosity in 2D  
Yukawa-liquid*  
PRL96(2006)145003

Molecular dynamical  
simulation with a  
finite number of particle.

Can we do it in YM case?  
How large is the viscosity  
in other models?

Danielewicz-Gyulassy:

$$\eta/s \approx 1 \quad \text{PRD31(1985)}$$

## What about viscosity in QGP?

1. Viscosity in  $\sim$ free massive quark gas [classical physics]  
in weakly interacting QGP [QCD at high-T]
2. Lattice-QCD results  $\Rightarrow \Rightarrow \Rightarrow \Rightarrow$  sQGP
  - viscosity in the quasi-particle picture
  - different models (width, spectral function)
3. Molecular dynamical simulation for  
massive quasi-particles with Yukawa-interaction
  - $\Rightarrow \Rightarrow \Rightarrow$  viscosity at  $1 < T/T_c < 2$

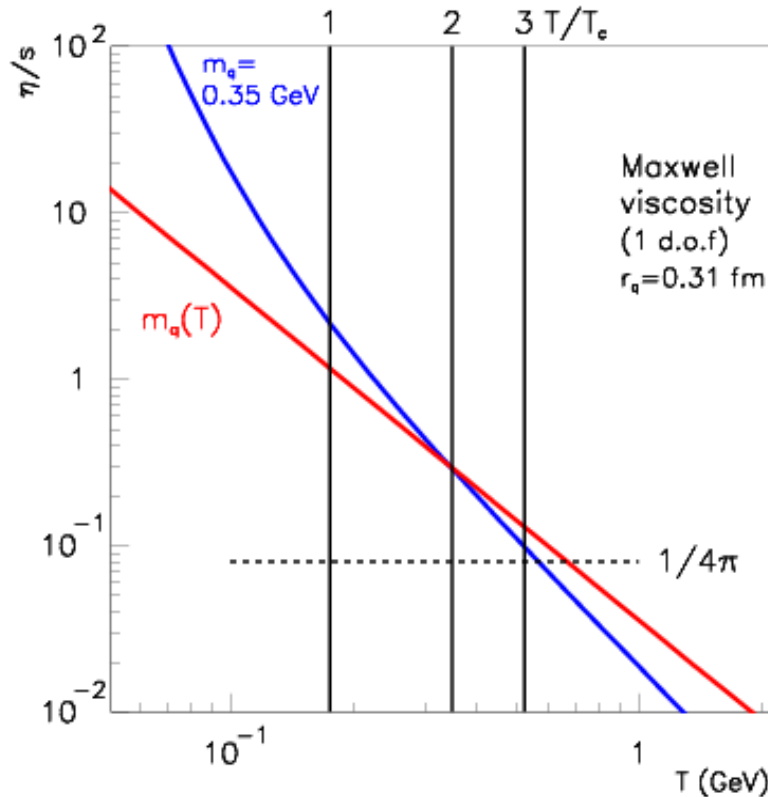
Viscosity of ~free massive fermi (q) gas [1 degree of freedom]

**A, constant mass,  $m_q = 0.35 \text{ GeV}$**

**B, temperature dep. mass,  $m_q(T) = g T/\sqrt{3}$  [ $g=1.77, \alpha_s=0.25$ ]**

$\eta(T)$  : Maxwell viscosity  $\rightarrow \eta = \frac{m_q}{3\sqrt{2}\pi d^2} \sqrt{\frac{8T}{\pi m_q}}$  [ $d=2r_q$ ]  
 [ $r_q=0.31 \text{ fm}, \sigma=3 \text{ mb}$ ]

$s(T) = (\epsilon(T) + P(T))/T$  for free massive fermi gas,  $s(T) \sim T^3$



**NO Minimum (?)**

$\eta/s = 1 / 4\pi$  at  $T \approx 3 T_c$

$\eta/s = 0.2 - 2$  at  $2 > T/T_c > 1$

**Did we find the wanted**

**“perfect fluid”**

**in the classic description of  
a free massive gas ???**

**[Where is the U-shape ???]**

Viscosity of ~free massive fermi (q) gas [1 degree of freedom]

**A, constant mass,  $m_q = 0.35 \text{ GeV}$**

**B, temperature dep. mass,  $m_q(T) = g T/\sqrt{3}$**

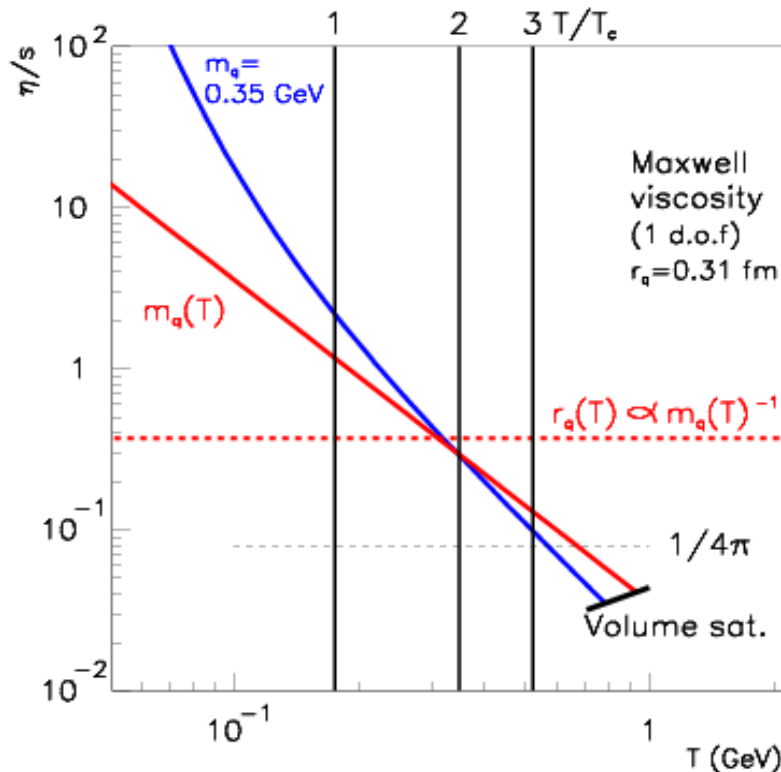
**$[g=1.77, \alpha_s=0.25]$**

$\eta(T)$  : Maxwell viscosity  $\rightarrow \eta = \frac{m_q}{3\sqrt{2}\pi d^2} \sqrt{\frac{8T}{\pi m_q}}$

$[d = 2r_q]$

$[\sigma = \pi r_q^2 \sim (gT)^{-2}]$

**C, -----**  $r_q(T) = \frac{1}{2} \frac{\hbar c}{m_q(T)} \sim \frac{1}{gT}$



**A & B : Constant volume**

$\rightarrow$  Volume saturation

$\eta/s \approx (1/4\pi)/2$  at  $T \approx 5 T_c$

**C, -----**  $\eta \sim T^3, s \sim T^3$

$\frac{\eta}{s} = const.$

**More T-dependence is needed in  $\eta$  for the U-shape**

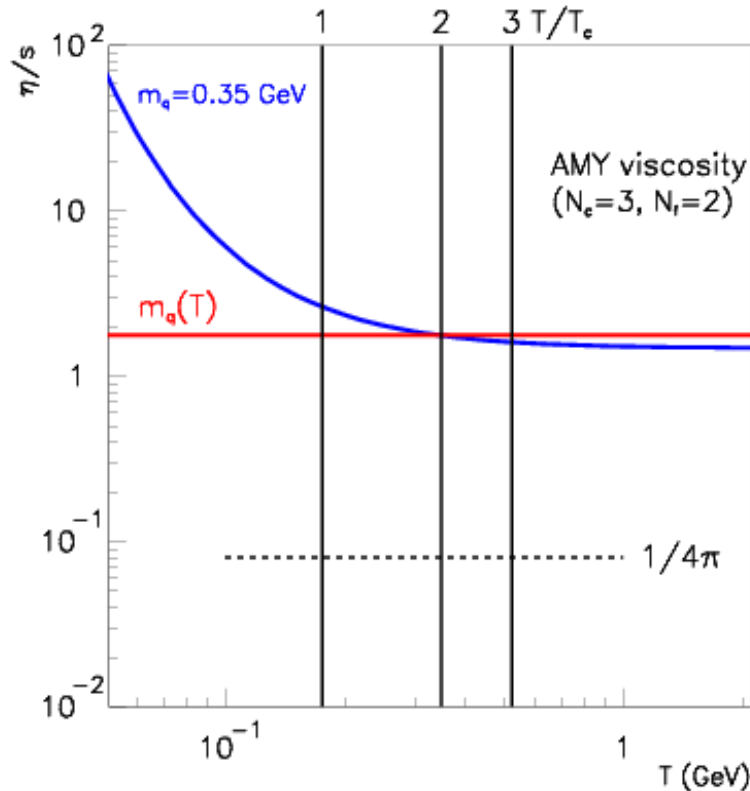
Viscosity in weakly coupled QGP [SU(3), gluon +  $N_f = 2$ ,  $g = \text{const.}$ ]

**A, constant mass,  $m_q = 0.35 \text{ GeV}$ ,  $m_g = \sqrt{2} m_q$**

**B, temperature dep. mass,  $m_q(T) = g T/\sqrt{3}$  [ $g=1.77$ ,  $\alpha_s=0.25$ ]**

$\eta(T)$  : AMY viscosity  $\rightarrow \eta = \kappa \frac{T^3}{g^4 \ln g^{-1}}$  [ $\kappa = 86.47$ ]  
 (Arnold, Moore, Yaffe, JHEP 2003)

$s(T) = (\epsilon(T) + P(T))/T$  for massive fermi and bose gas,  $s(T) \sim T^3$



**NO Minimum**

**If  $T \rightarrow \infty$  then**

$\eta/s \rightarrow 20(1/4\pi)$

**At  $1 < T/T_c < 2$**

$\eta/s = 2 - 3$

**This matter is not a  
“perfect fluid” !**



Viscosity in weakly coupled QGP [SU(3), gluon +  $N_f = 2$ ,  $g(T)$ ]

A, constant mass,  $m_q = 0.35 \text{ GeV}$ ,  $m_g = \sqrt{2} m_q$

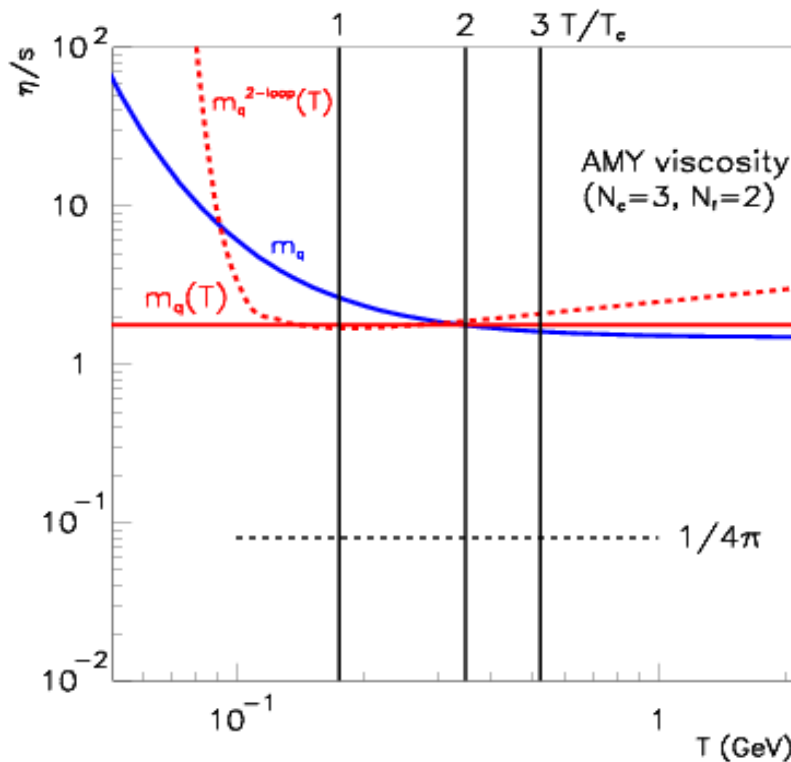
B, temperature dep. mass,  $m_q(T) = g T/\sqrt{3}$  [ $g=1.77$ ,  $\alpha_s=0.25$ ]

$\eta(T)$  : AMY viscosity  $\rightarrow \eta = \kappa \frac{T^3}{g^4 \ln g^{-1}}$  [ $\kappa = 86.47$ ]  
 (Arnold, Moore, Yaffe, JHEP 2003)

C, ---- 2-loop renorm.

$$g(T)^{-2} = \frac{9}{8\pi^2} \ln \frac{T}{\mu_0} + \frac{4}{9\pi^2} \ln \left( 2 \ln \frac{T}{\mu_0} \right)$$

[ $\mu_0 = 0.03 \text{ GeV}$ ]



C: Minimum at  
 $T \approx T_c$ ,  $\eta/s = 20 (1/4\pi)$

$1 < T/T_c < 2$ ,  $\eta/s \approx 2$

$T \rightarrow \infty$ ,  $\eta/s \sim \ln^2 T$

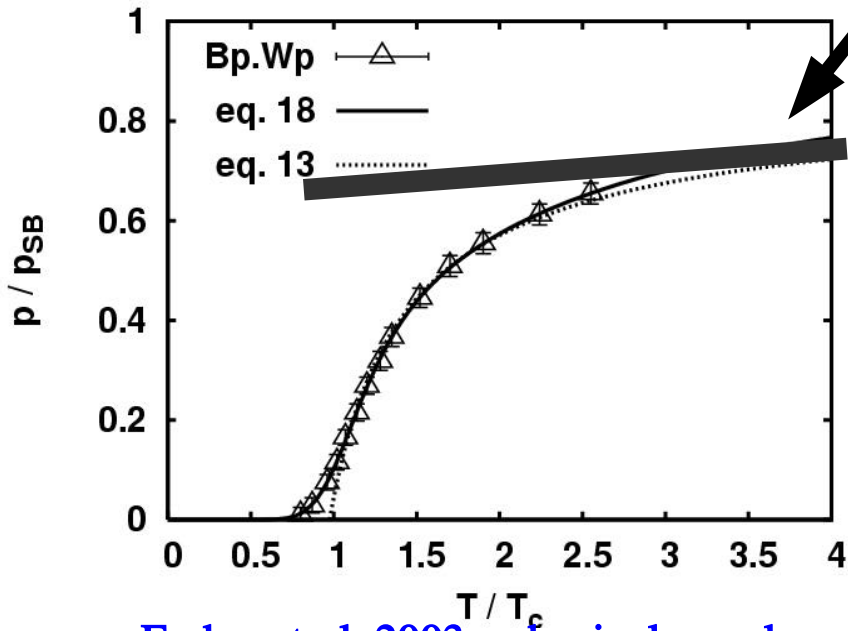
So far this is the best candidate  
 but it is not “perfect fluid”.

“Low-viscosity matter”

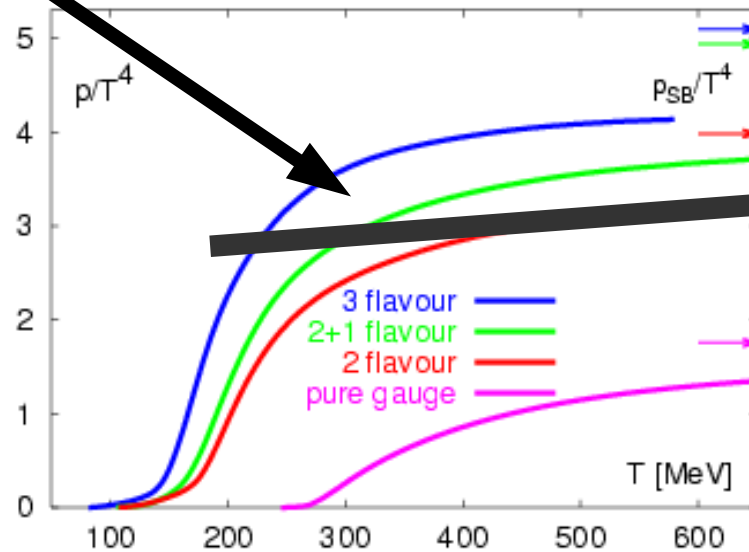
Csernai, Kapusta, McLerran,  
 PRL97, 152303, 2006

# Lattice-QCD results: EOS for strongly interacting QGP

Testing  $g^{2\text{-loop}}(T) \rightarrow \varepsilon(T), P(T)$  in  $SU(3), N_f=2$   
 $m_q(T) = g(T)T/\sqrt{3}; m_g(T) = \sqrt{2} m_q(T)$



Fodor et al. 2003 - physical quark mass



Karsch et al. 1992

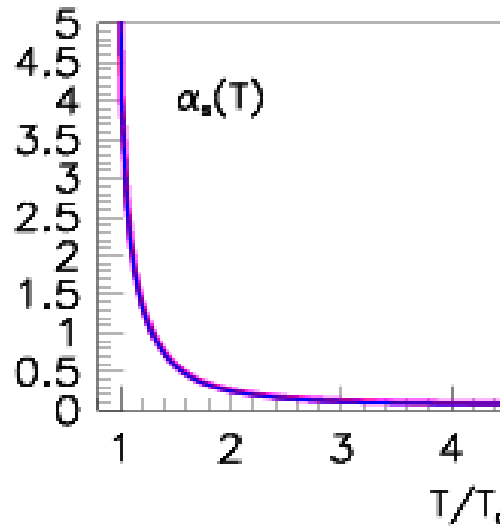
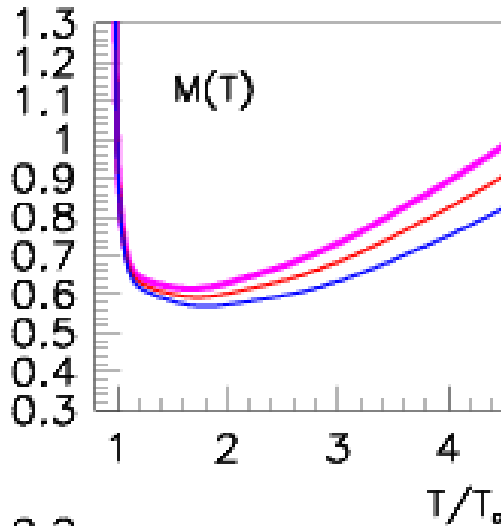
$g^{2\text{-loop}}(T)$  is maybe good asymptotically, but not for  $T < 2 T_c$

Non-ideal EOS  $\rightarrow$  quasi-particle picture of strongly interacting QGP

Can we extract viscosity in the quasi-particle description ???

# Numerical results for interacting massive gluonic quasi-particles

$\epsilon(T), P(T)$   $\gggg$   $M(T), B(T)$   $\gggg$   $\alpha_s(T), m_\sigma(T)$

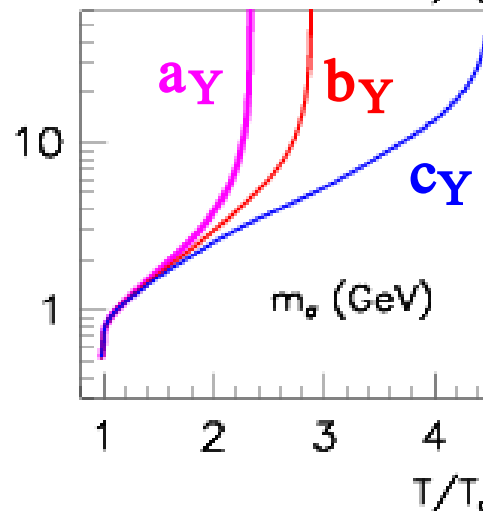
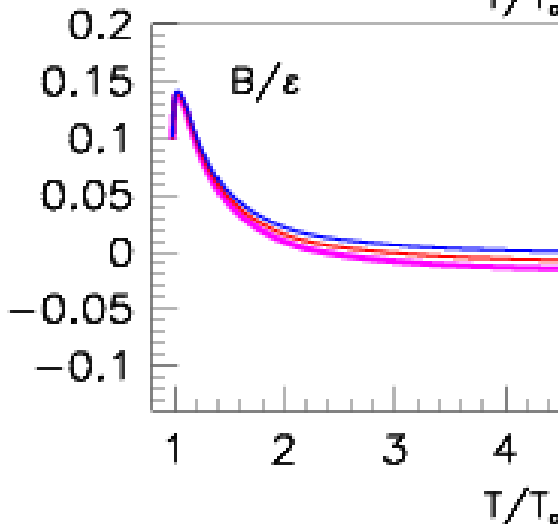


Uncertainties (0-2-4 %) in the lattice results:

$\alpha_s(T)$  is robust

$m_\sigma(T)$  is sensitive

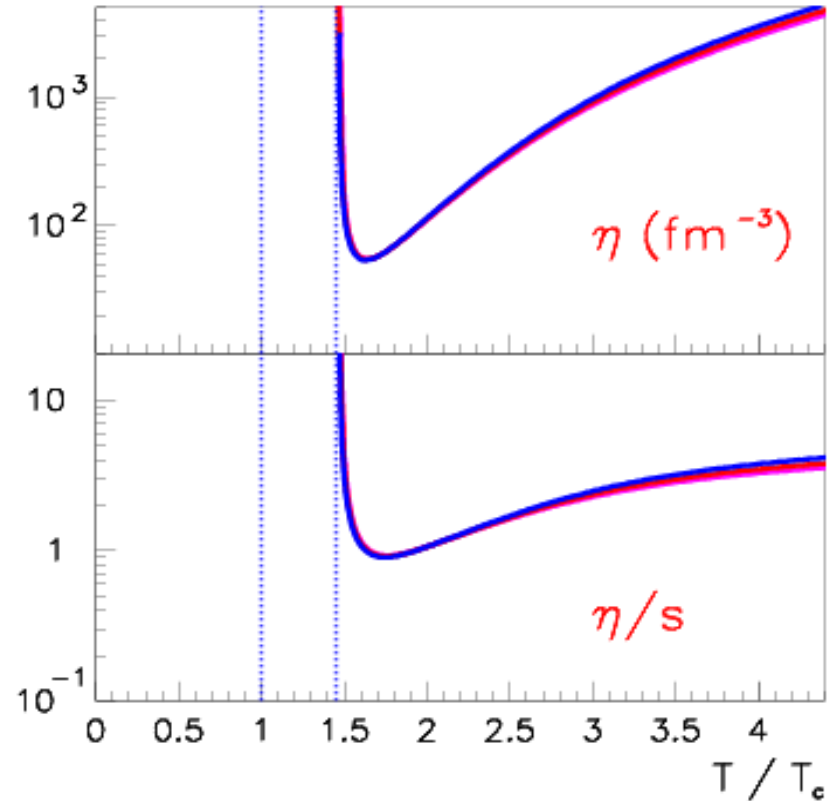
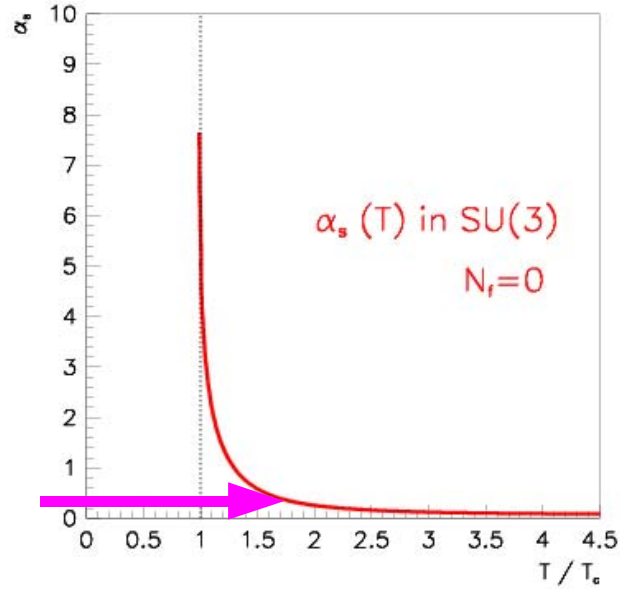
[Yukawa-int.]



$m_\sigma(T)$  is divergent, where  $B$  becomes negative

We can compress infinity into a finite  $T/T_c$  region.

# AMY formula for the quasi-gluon gas [SU(3) $N_f=0$ ]



$$\eta_{NLL} = \frac{27.12 T^3}{g^4 \ln(2.765/g)}$$

$\eta/s = 0.9$  at  $T = 1.75 T_c$

$T \rightarrow T_c \gg \gg \gg \gg \alpha_s(T)$  becomes extremely large

If  $\alpha_s(T) > 1$ , then the simple viscosity formula does not work.

AMY: formula is good for  $\alpha_s(T) < 0.3$  !!!  $T/T_c \geq 1.8, \eta/s \geq 0.9$

How to improve quasi particle description,  
especially in the region  $1 < T/T_c < 1.5$

1. Multi-gluon and multi-quark states around  $T_c$

E.V. Shuryak, I. Zahed, PRC70 (2004) 021901.

P. Lévai, A. Németh, in preparation [10-15 %]

2. Width of the quasi particles .

A. Peshier, W. Cassing, PRL94(2005)172301. [Diverg.]

3. Mass distribution of the quasi particles (spectral function)

T.S. Bíró, P. Lévai, P. Ván, J. Zimányi

JPG31,2005,711

hep-ph/0606076

[Interaction in  
the  $F(m_q)$ .]

OR: Molecular dynamical simulations for SU(3) quark matter!

P. Hartmann, Z. Donkó, G. Kalman, P. Lévai

Nucl. Phys. A774, 2006, 881

Talks on the QM05 and QM06.

**INTERACTION  $\rightarrow$  Multi-gluon states: 2g, 3g, 4g, ...**

$$M_0(T) = \frac{1}{\sqrt{2}} g(T) T$$

**Singlet (1) is attractive**

**Octet (8) is attractive**

**Decuplet (10) is neutral**

**Higher multiplets are repulsive**

$$M_i(T) = i M_0(T) - g \sigma = i M_0(T) - \frac{g^2}{m_\sigma} \sum n_i$$

Multi-gluon states	1	8	10	27	28	35	64	80 ...
$8 \otimes 8$	1	2	2	1	0	0	0	0
$8 \otimes 8 \otimes 8$	2	8	8	6	0	4	1	0
$8 \otimes 8 \otimes 8 \otimes 8$	8	32	40	33	4	30	12	0
$8 \otimes 8 \otimes 8 \otimes 8 \otimes 8$	32	145	200	180	40	200	94	10
$8 \otimes 8 \otimes 8 \otimes 8 \otimes 8 \otimes 8$	145	702	1050	999	322	1260	660	140
$8 \otimes 8 \otimes 8 \otimes 8 \otimes 8 \otimes 8 \otimes 8$	702	3598	5712	5670	2352	7840	4424	1400

Suppression of the multi-gluon states:  $\Gamma * (2g, 3g, 4g, \dots)$

$$P(T) = P_1(M_1, T) + C(T) * \sum_2^{\infty} P_i(M_i, T) - B(T)$$

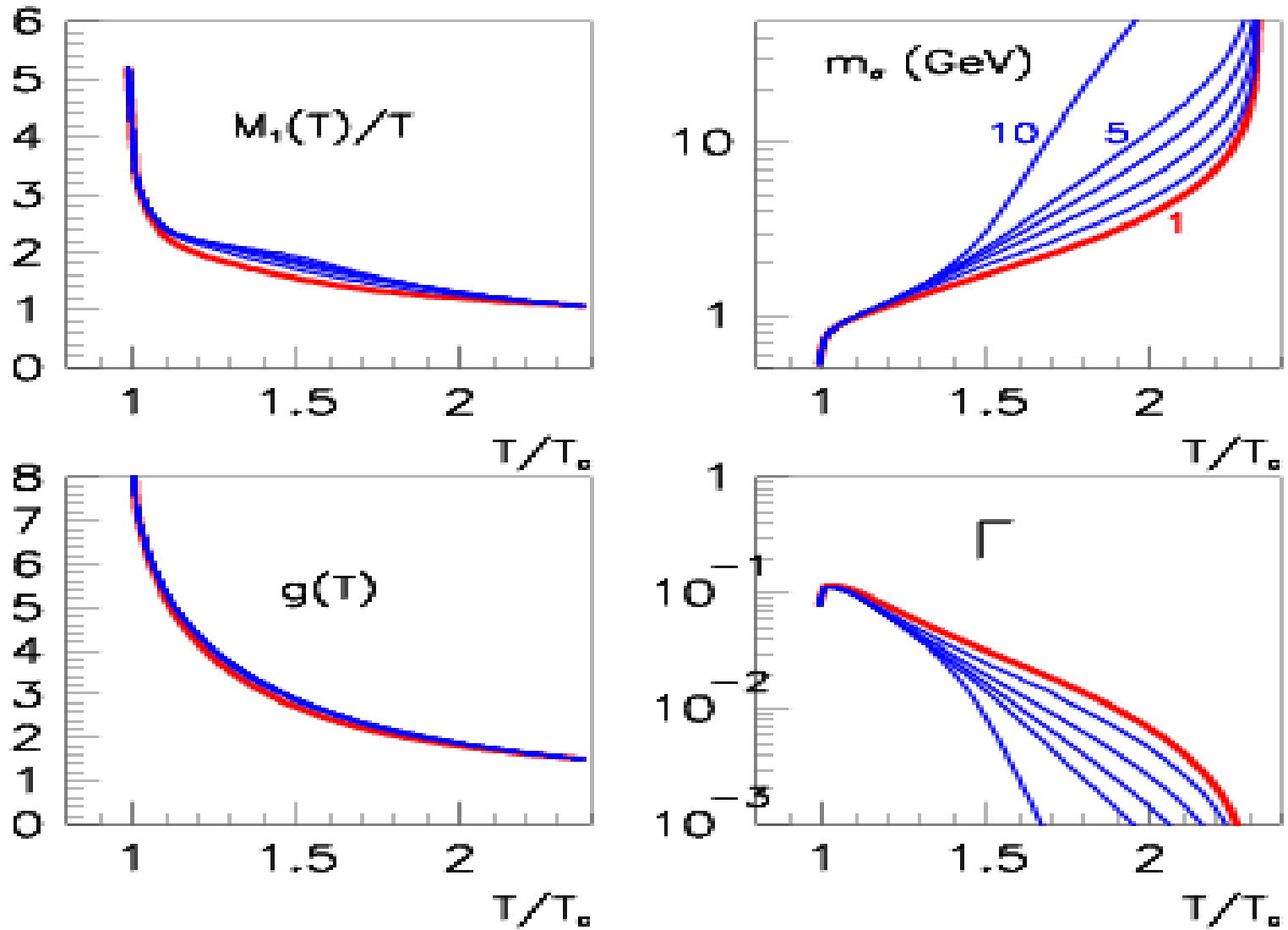
$$\varepsilon(T) = \varepsilon_1(M_1, T) + C(T) * \sum_2^{\infty} \varepsilon_i(M_i, T) + B(T)$$

$$n(T) = n_1(M_1, T) + C(T) * \sum_2^{\infty} n_i(M_i, T)$$

$$C(T) = \kappa * \Gamma(T)$$

$$\Gamma = E_{\text{pot}} / E_{\text{kin}} = \frac{3}{m_{\sigma}^2} \frac{n^2}{\varepsilon_{\text{tot}} - B}$$

Interacting multi-gluon states (2,3,4,5,...,10) and  $\Gamma = E_{\text{pot}} / E_{\text{kin}}$







## Interpretation:

Multi-gluon states has very small contribution at  $T_c$

Multi-gluon states has contribution at large  $T$ ,

but  $\Gamma(T)$  -factor cuts this out,

cut-off moving closer to  $T_c$  as  $N_g$  is increasing

Finally there is a small window ( $1.05 < T/T_c < 1.15$ )

where multi-gluons can exist – depending on  $\Gamma$

One-gluon states are good degrees of freedom

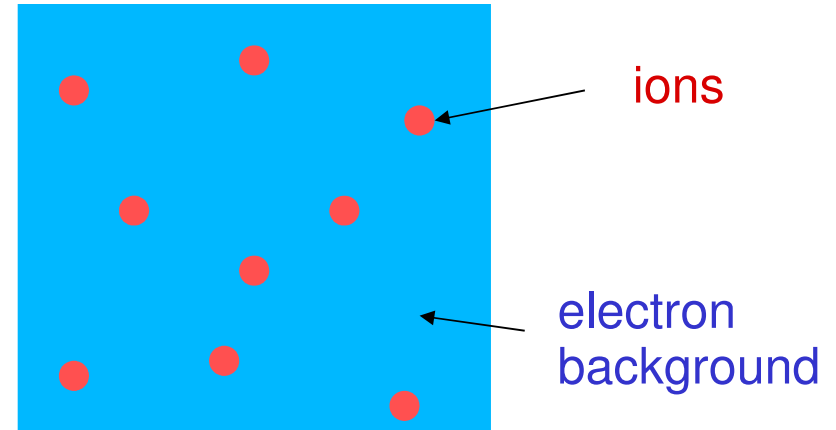
at  $T_c$  and at high temperature

# Classical strongly coupled plasmas

The simplest system: classical one-component plasma (OCP).

OCP: charged heavy particles immersed into a homogeneous neutralizing background.

interaction (Coulomb) potential:  $V(r) = \frac{Q^2}{r}$



particle density $n$	}	plasma coupling parameter	$\Gamma = Q^2 l (a_{WS} k_B T)$
particle mass $m$		ion sphere radius	$a_{WS} = \sqrt[3]{3 / (4 \pi n)}$
electric charge $Q$		plasma frequency	$\omega_p = \sqrt{4 \pi Q^2 n / m}$
Temperature $T$			

investigated properties:

- structure (pair correlation function, static structure function)
- thermodynamics (internal energy, compressibility, equation of state)
- transport phenomena (thermal conductivity, shear viscosity, diffusion)
- collective dynamics (density and current fluctuations, dispersion relations)

Our sQGP model is rooted on the classical OCP model. The links are:

<u>classical OCP</u>		<u>QGP model</u>
<i>ions</i>	→	<i>quarks (massive)</i>
<i>electron background</i> (neutralizing)	→	<i>gluon background</i> <b>interacting !!!</b>
$V(r_{ij}) = \frac{Q^2}{r_{ij}}$	→	$V(r_{ij}) = \frac{\langle \lambda_i \lambda_j \rangle \alpha_s}{r_{ij}}$
$\Gamma = \frac{Q^2}{a_{WS} k_B T}$	→	$\Gamma = \frac{C_s}{a_{WS} k_B T}$

The numerical simulation is based on the classical molecular dynamics scheme:

- calculating the forces acting on each particle due to all other particles
- integrating the equation of motion for all particles in each time-step
- using periodic boundary conditions to handle long range forces
- implementing color rotation due to random gluonic interaction

# Potential model for QCD interaction

color dependent interaction potential between quark  $i$  and  $j$ :  $V = \langle \lambda_i \lambda_j \rangle \frac{\alpha}{r_{ij}}$

possible two-quark states ( $R$ ,  $G$  and  $B$  are the single-quark color states):

$ RR\rangle$	$\Psi = \Psi_1(R)\Psi_2(R)$
$ GG\rangle$	$\Psi = \Psi_1(G)\Psi_2(G)$
$ BB\rangle$	$\Psi = \Psi_1(B)\Psi_2(B)$
$ RG\rangle$	$\Psi = \frac{1}{\sqrt{2}}[\Psi_1(R)\Psi_2(G) + \Psi_1(G)\Psi_2(R)]$
$ RB\rangle$	$\Psi = \frac{1}{\sqrt{2}}[\Psi_1(R)\Psi_2(B) + \Psi_1(B)\Psi_2(R)]$
$ GB\rangle$	$\Psi = \frac{1}{\sqrt{2}}[\Psi_1(G)\Psi_2(B) + \Psi_1(B)\Psi_2(G)]$
$ RG\rangle_{Anti}$	$\Psi = \frac{1}{\sqrt{2}}[\Psi_1(R)\Psi_2(G) - \Psi_1(G)\Psi_2(R)]$
$ RB\rangle_{Anti}$	$\Psi = \frac{1}{\sqrt{2}}[\Psi_1(R)\Psi_2(B) - \Psi_1(B)\Psi_2(R)]$
$ GB\rangle_{Anti}$	$\Psi = \frac{1}{\sqrt{2}}[\Psi_1(G)\Psi_2(B) - \Psi_1(B)\Psi_2(G)]$

color factor:

$$\langle \lambda_i \lambda_j \rangle = \frac{1}{2} \left[ \langle (\lambda_i + \lambda_j)^2 \rangle - \langle \lambda_i^2 \rangle - \langle \lambda_j^2 \rangle \right] \begin{cases} +1/3 & \text{symmetric (6)} \\ -2/3 & \text{antisymmetric (}\bar{3}\text{)} \end{cases}$$



# MD results for pair correlation

In the following we present molecular dynamics results for quark plasma with physical parameters:

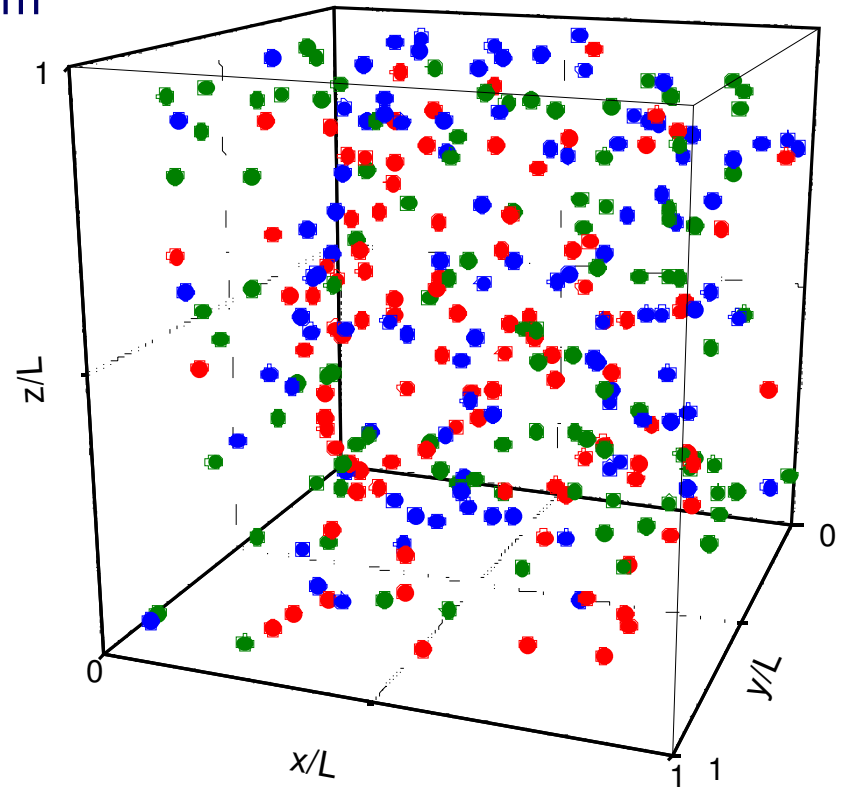
- kinetic temperature,  $T_0 = 180 - 350$  MeV
- particle density,  $n = 5-100$  quarks / fm<sup>3</sup>
- interaction strength,  $\alpha_s = 1-2$
- quark mass,  $m = 300-400$  MeV

and technical parameters:

- number of particles,  $N = 300$
- starting positions = random
- initialization time,  $t_i = 10^{6+1} dt$
- measure time,  $t_m = 2 \times 10^5 dt$
- time-step,  $dt = 5 \times 10^{-5}$  fm

measured parameters are:

- kinetic temperature,  $T(t)$
- pair correlation function,  $g(r)$
- viscosity,  $\eta$



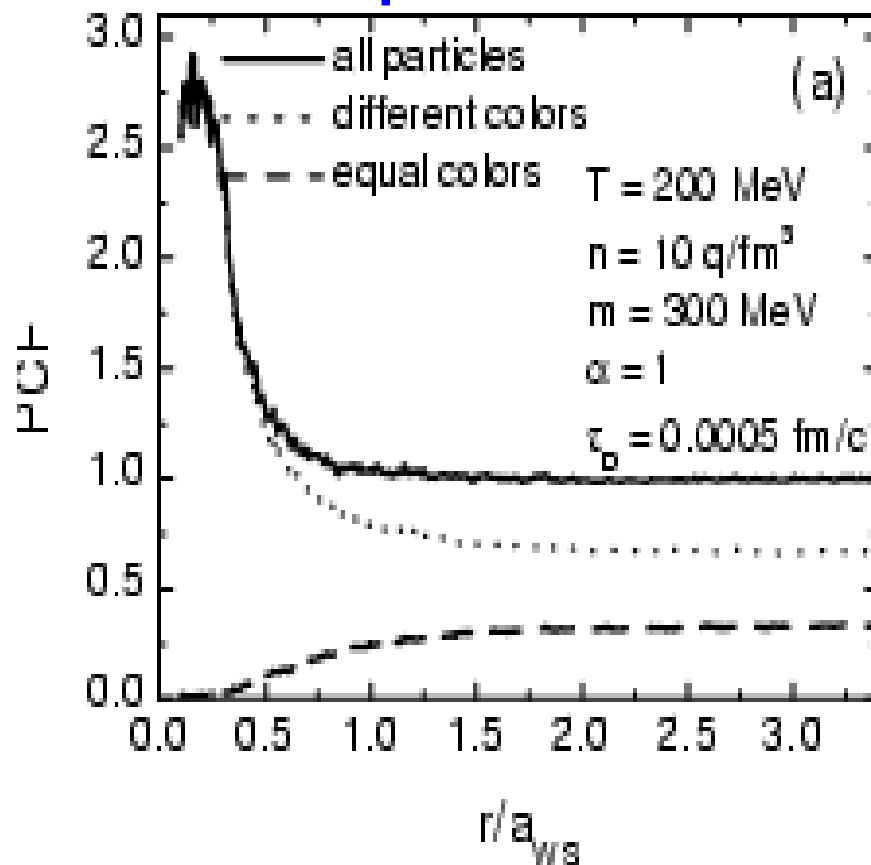
# Correlations

$g(r)$  pair-correlation function: liquid or gas?  $\Rightarrow\Rightarrow\Rightarrow$  "gas" !

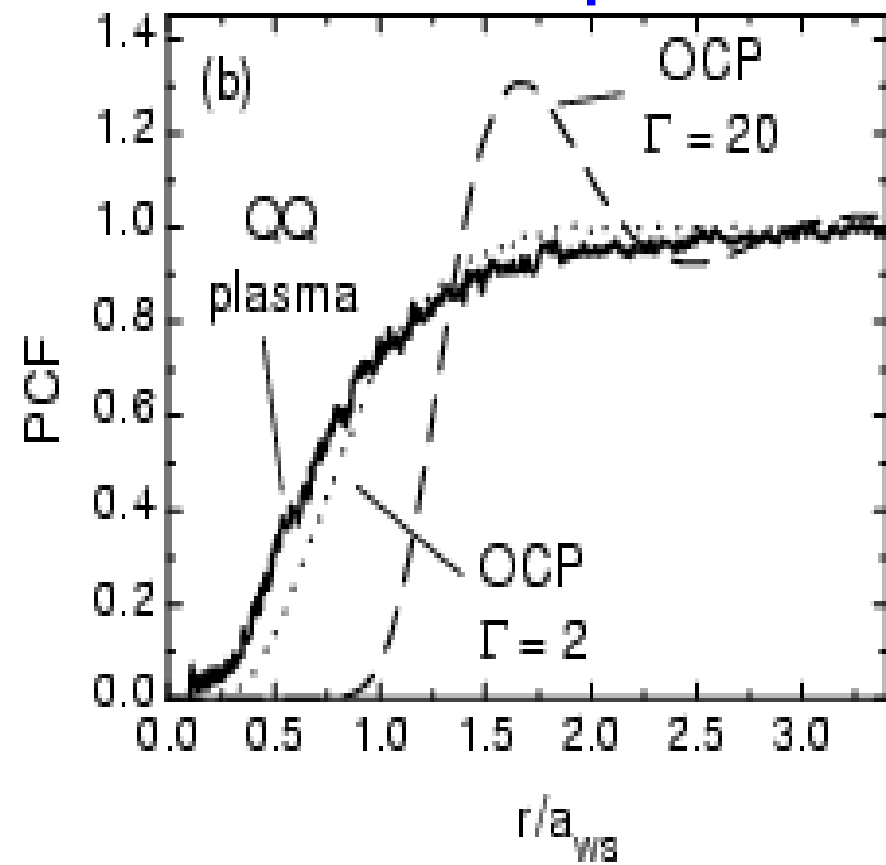
repulsive species: gas-like behaviour

attractive species: strong correlation (clusterization)

QQ plasma

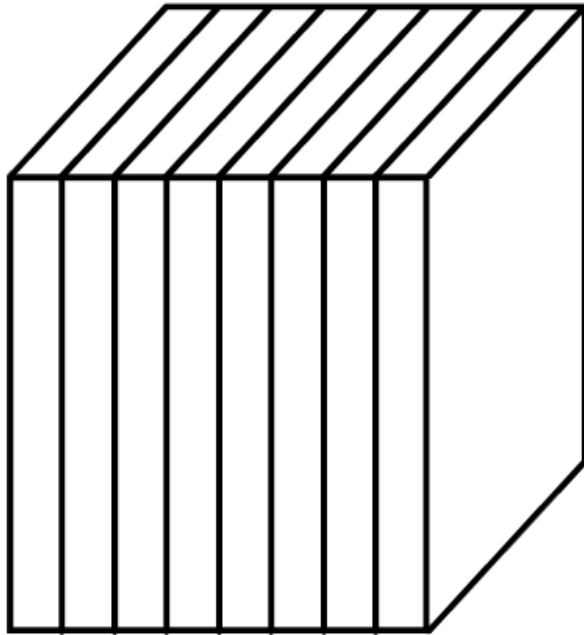


QQ and ee plasma



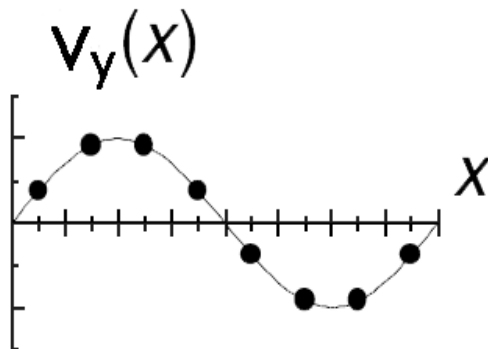


## MD measurement for viscosity – 1



Z. Donkó, B. Nyíri, L. Szalai, S. Holló,  
Phys. Rev. Lett. 81 (1998) 1622.

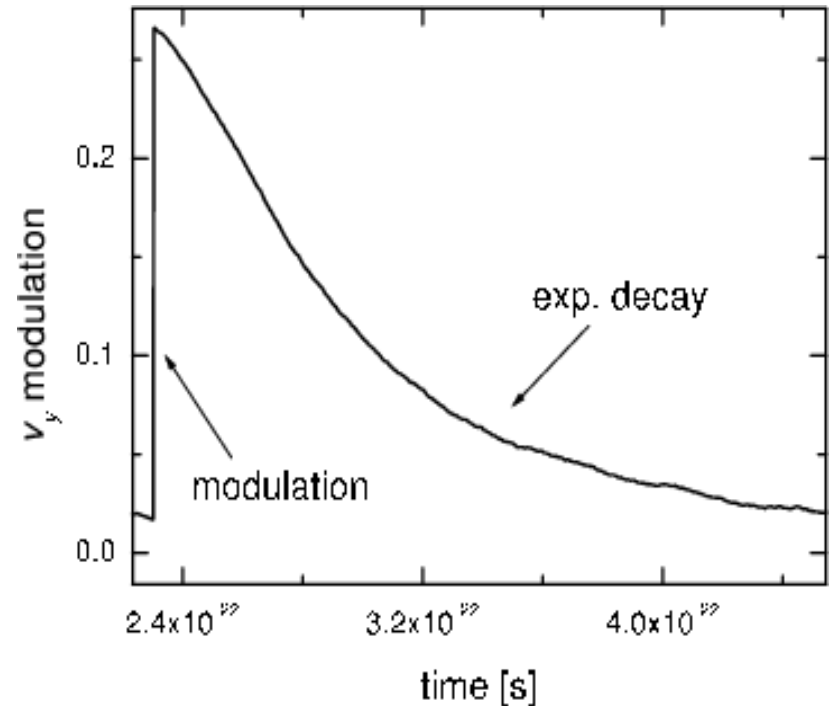
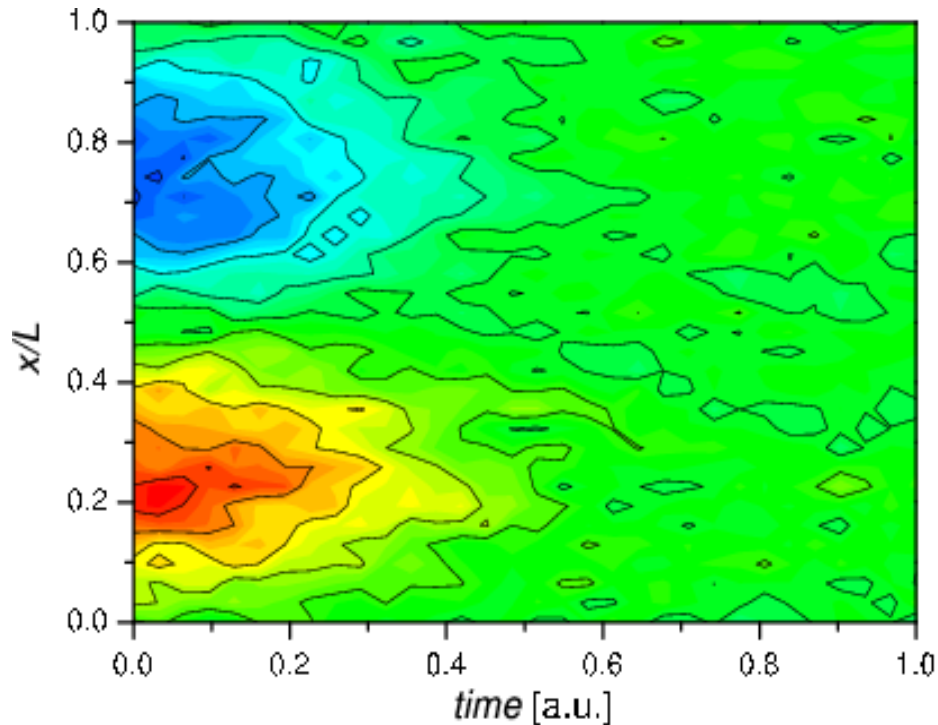
**Periodic boundary condition.  
(Separation into 8 bin.)**



**Sinusoid velocity perturbation  
(artificial shear in the system):**

$$\Delta v_y(x, t=t_0) = v_{m0} \sin(2\pi x/L)$$

## MD measurement for viscosity – 2



**The relaxation of this shear is exponential (see Navier-Stokes):**

$$v_m(t) = v_{m0} \exp(-(t - t_0)/\tau)$$

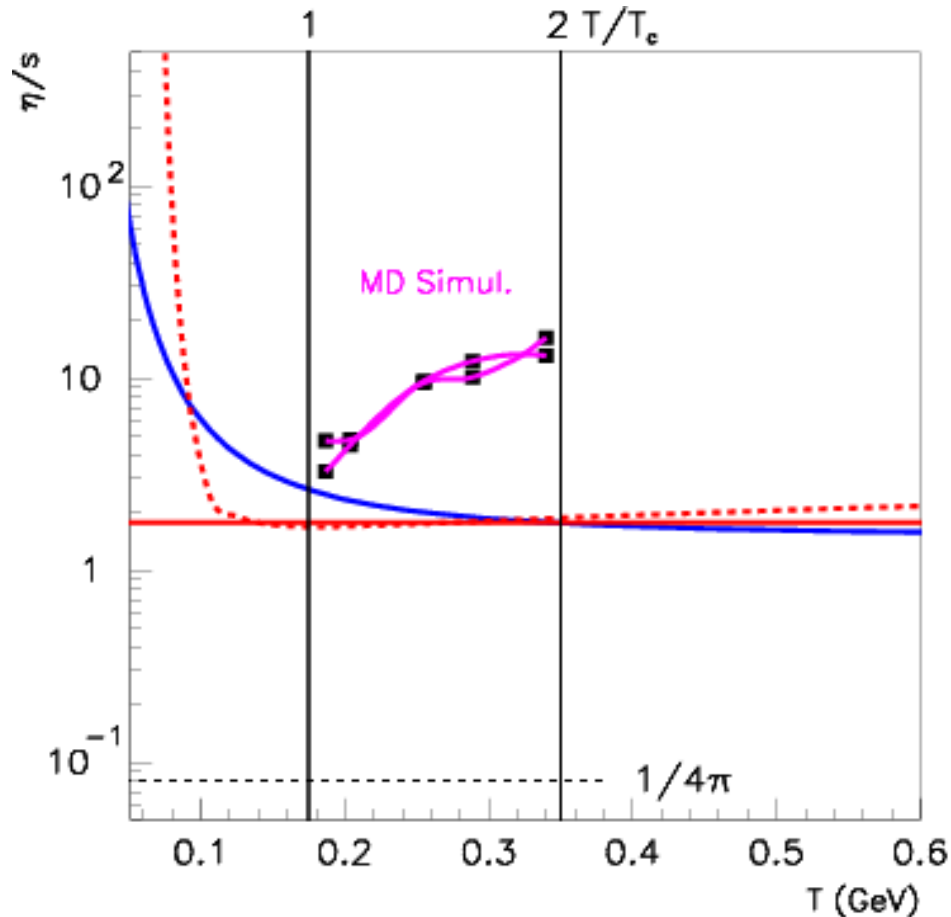
**Here  $\tau$  is related to the dynamical viscosity  $\eta_D$  :**

$$\eta_D = \frac{\rho m_q}{\tau} (L/2\pi)^2$$

$\rho$  : density [ $1/\text{fm}^3$ ]

$L$  : simulation box size [fm]

# RESULTS from MD simulation for viscosity (100 000 part.):



MD simulation

for quark matter:

Minimum

$T = T_c \Rightarrow \eta/s \approx 3$

Result is larger than AMY

$1 < T/T_c < 2 \Rightarrow \eta/s = 3 - 15$

Introduction

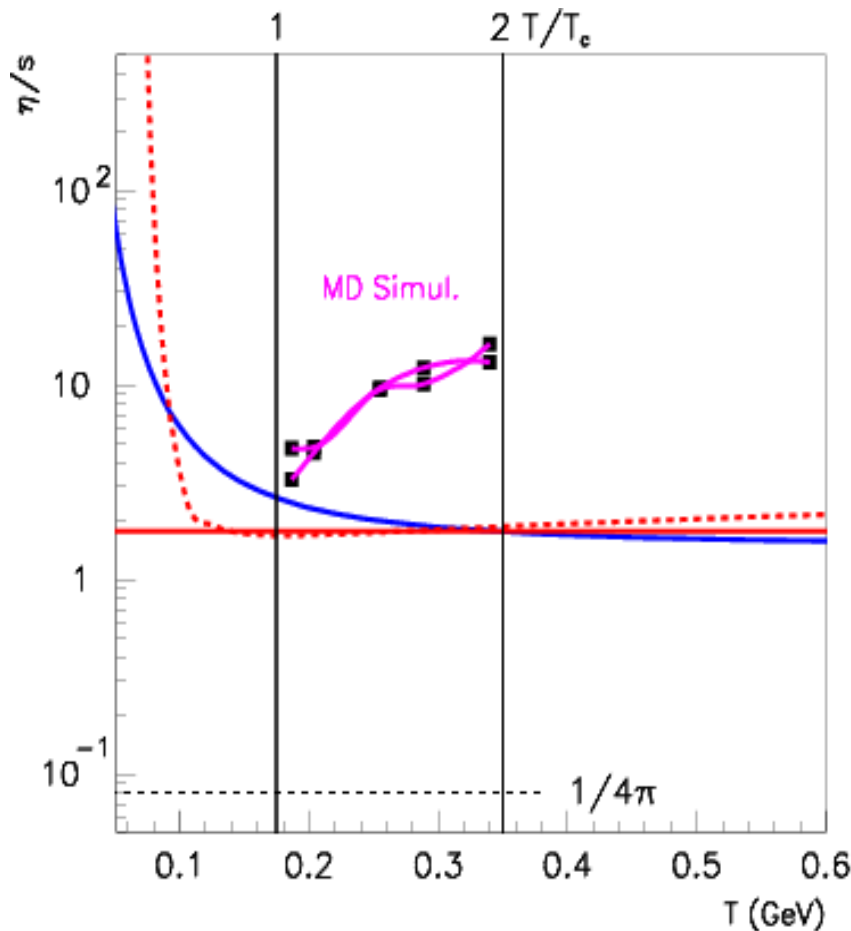
of antiquarks and gluons will

increase the entropy

decreases  $\eta/s$

$\eta/s \approx 1-5$  : even this is large

# RESULTS from MD simulation for viscosity:



If Yukawa  $\rightarrow 0$  at  $T/T_c \rightarrow \infty$   
 then  $\eta/s \rightarrow \approx 20$

Massive quark plasma  
 will not approach  
 dense gluon field  
 [ $\eta/s = 2$ ]

Dense gluon field:  
 Anomalous viscosity ( $\eta_A$ )  
 in turbulent gluon fields,  
 $\eta_A \ll \eta$

What happen at  $T/T_c \rightarrow 1$  ?

Cross-over phase transition in MD?

$$\eta_A/s \approx C (1/4\pi) \quad C=?? \quad (2-3)$$

[Asakawa, Bass, Müller, PRL96,2006]

## COMMENT after the MD simulation:

we obtain too large viscosity,  
because we are using “particles” and  
a reduced interaction between them

small viscosity: interaction energy is large  
kinetic energy is small

'non-particle' picture of strongly interacting QGP ?  
we loose hadronization successful models  
we loose (particle) EOS and particle transport

Laminar -> turbulent flow is a good solution.

But what can we calculate in a turbulent state?

## Summary:

### 1. Viscosity can be calculated in many ways:

Classical description can give very small value for  $\eta/s$   
seems to generate “perfect fluid” (?)

QM and QCD give  $\eta/s \approx 2$  at  $T/T_c \geq 1$

Quasi-particle picture gives  $\eta/s \geq 0.9$  at  $T/T_c \geq 1.8$

MD for quark matter gives  $\eta/s \geq 3$  at  $T/T_c \geq 1.1$

Further studies are needed, especially including antiquarks, g.

### 2. How large is the anomalous viscosity, $\eta_A/s$ ?

### 3. Viscosity and jets, direct experimental measurement ?

Especially, if the viscosity is large:  $\eta/s \geq 1$  at  $T \approx 200$  MeV

$\eta/s \geq 2$  at  $T > 250$  MeV!

### 4. 'Non-particle' based descriptions?