

# Deconfinement and the quark condensate

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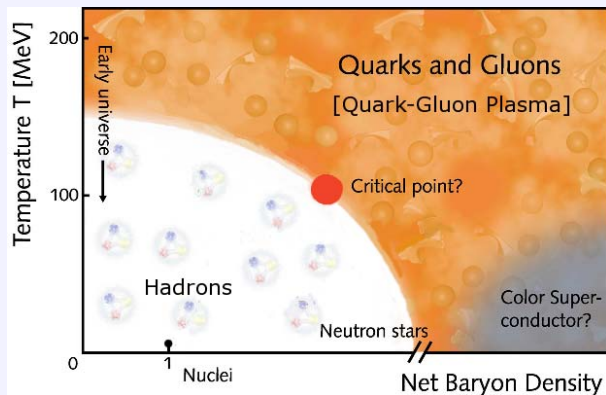
C.F., arXiv:0904.2700 [hep-ph]

C.F. and J. Mueller, in preparation

- 1 Introduction
- 2 Order parameters for chiral symmetry breaking and deconfinement
- 3 Results

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# QCD phase transitions



- Chiral limit ( $M_{weak} = 0$ ): order parameter chiral condensate
- Heavy quarks ( $M_{weak} = \infty$ ): order parameter Polyakov-loop

# Lattice QCD vs. DSE/FRG: Complementary!

- Lattice simulations
  - ▶ Ab initio
  - ▶ Gauge invariant
- Functional approaches:
  - Dyson-Schwinger equations (DSE)
  - Functional renormalisation group (FRG)
  - ▶ Analytic solutions at small momenta
  - ▶ Space-Time-Continuum
  - ▶ Chiral symmetry: light quarks and mesons
  - ▶ Chemical potential: no sign problem

# QCD in covariant gauge

$$\mathcal{Z}_{\text{QCD}} = \int \mathcal{D}[\Psi, A, c] \exp \left\{ - \int_0^{1/T} dt \int d^3x \left( \bar{\Psi} (i\not{D} - m) \Psi \right. \right. \\ \left. \left. - \frac{1}{4} (F_{\mu\nu}^a)^2 + \frac{(\partial A)^2}{2\xi} + \bar{c}(-\partial D)c \right) \right\}$$

Landau gauge ( $\xi = 0$ ) propagators in momentum space,  $q = (\vec{q}, \omega_q)$ :



$$D_{\mu\nu}^{\text{Gluon}}(q) = \frac{Z_T(q)}{q^2} P_{\mu\nu}^T(q) + \frac{Z_L(q)}{q^2} P_{\mu\nu}^L(q)$$



$$S^{\text{Quark}}(q) = \frac{1}{-i \vec{\gamma} \vec{q} A(q) - i \gamma_4 \omega_n C(q) + B(q)}$$

The Goal:

Gauge invariant information from gauge fixed functional approach

- 1 Introduction
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# The ordinary chiral condensate

The diagram shows an equation between three terms. The first term is a horizontal line with a shaded circle in the middle, with a superscript  $-1$  above it. This is followed by an equals sign. The second term is a plain horizontal line with a superscript  $-1$  above it. This is followed by a plus sign. The third term is a loop diagram consisting of a circle with a shaded circle on the left and a white circle on the right. Two horizontal lines enter the circle from the left and right, respectively.

- Consider DSE on torus with  $V = 1/T \times L^3$ 
  - spatial directions: **periodic** boundary conditions
  - temporal direction: **antiperiodic** boundary condition

C.F., arXiv:0904.2700 [hep-ph]

- Consider DSE in infinite volume/continuum

C.F. and J.Mueller, in preparation

- Order parameter for **chiral transition**:

$$\langle \bar{\psi}\psi \rangle = Z_2 N_c \frac{T}{L^3} \text{Tr}_D \sum_{\vec{p}, \omega_p} S(p_{\vec{p}, \omega_p})$$



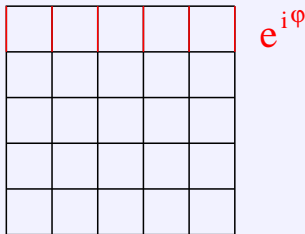
# The dual condensate I

Consider general  $U(1)$ -valued boundary conditions in temporal direction for quark fields  $\psi$ :

$$\psi(\vec{x}, 1/T) = e^{i\varphi} \psi(\vec{x}, 0)$$

Matsubara frequencies:  $\omega_p(n_t) = (2\pi T)(n_t + \varphi/2\pi)$

Lattice:



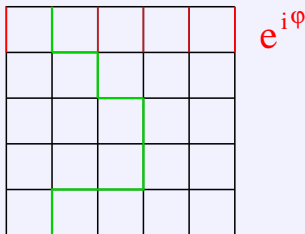
E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007..

# The dual condensate II

Relation of condensate to loops of link variables  $U_\mu(x)$ :

$$\langle \bar{\psi}\psi \rangle_\varphi = \text{Tr} [m + D_\varphi]^{-1} = \frac{1}{Vm} \sum_{l \in \mathcal{L}} \frac{e^{i\varphi n(l)}}{(2am)^{|l|}} \text{Tr}_c \prod_{(x,\mu) \in l} s(l) U_\mu(x).$$

- geometric series of inverse staggered Dirac operator
- winding number  $n(l)$  of loop  $l$  around temporal direction



# The dual condensate III

Then define dual condensate  $\Sigma_n$ :

$$\Sigma_n = - \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi n} \langle \bar{\psi}\psi \rangle_\varphi$$

- $n = 1$  projects out loops with  $n(l) = 1$ : dressed Polyakov loop
- transforms under center transformation exactly like ordinary Polyakov loop
- $\Sigma_1$  is order parameter for center symmetry/deconfinement

E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.

- $\Sigma_1$  is accessible with functional methods

C.F., arXiv:0904.2700 [hep-ph]

# The (dual) scalar quark dressing

Inverse quark propagator:

$$S^{-1}(q) = -i \vec{\gamma} \vec{q} A(q) - i \gamma_4 \omega_n C(q) + B(q)$$

- Another order parameter for **chiral symmetry breaking**:  
**Scalar quark dressing function  $B(0, \pi T)$**
  
- Another order parameter for **confinement/deconfinement**:  
**Dual scalar quark dressing:**

$$\Sigma_B = - \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi} B(0, \varphi T),$$



$$\begin{array}{c} -1 \\ \text{---} \bullet \text{---} \\ \text{---} \end{array} = \begin{array}{c} -1 \\ \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \bullet \text{---} \\ \text{---} \bullet \text{---} \\ \text{---} \bullet \text{---} \end{array}$$

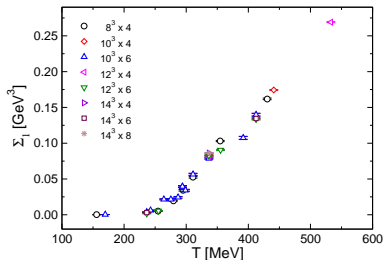
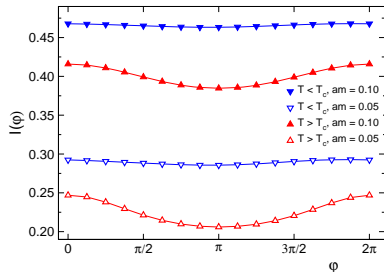
The diagram shows the Dyson-Schwinger equation for the quark propagator. On the left is a quark propagator with a shaded blob representing the self-energy. This is equal to the sum of two terms: a bare quark propagator (a simple line) and a loop diagram. The loop diagram consists of a quark line forming a circle with a gluon loop (represented by a wavy line) attached to it. The vertices in the loop are shaded, indicating a ghost loop contribution.

- $T$ -dependent ansatz for quark-gluon vertex

$$\Gamma_\nu(q, k, p) = \tilde{Z}_3 \left( \delta_{4\nu} \gamma_4 \frac{C(k) + C(p)}{2} + \delta_{j\nu} \gamma_j \frac{A(k) + A(p)}{2} \right) \times \left( \frac{d_1}{d_2 + q^2} + \frac{q^2}{\Lambda^2 + q^2} \left( \frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2 + 1]}{4\pi} \right)^{2\delta} \right).$$

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# Lattice results

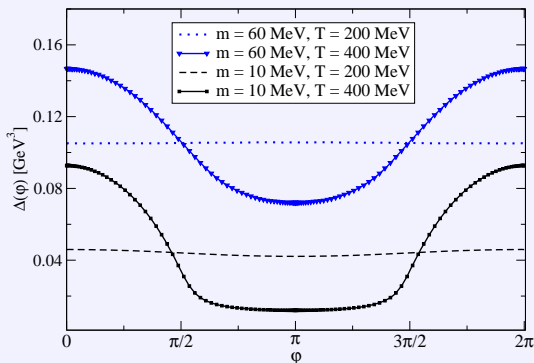


$$\Sigma_1 = - \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi} \langle \bar{\psi}\psi \rangle_\varphi$$

E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.



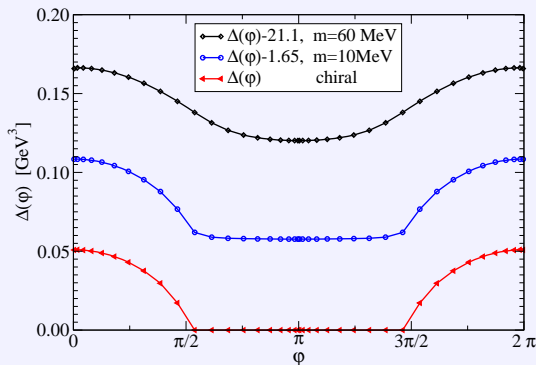
# Angular dependence of condensate



$$\Delta(\varphi) \equiv \langle \bar{\psi}\psi \rangle_{\varphi} = \text{Tr} [m + D_{\varphi}]^{-1} = \frac{1}{Vm} \sum_{l \in \mathcal{L}} \frac{e^{i\varphi n(l)}}{(2am)^{|l|}} \text{Tr}_c \prod_{(x,\mu) \in l} s(l) U_{\mu}(x).$$

- Smaller mass: more contributions from loop with larger  $n(l)$ !

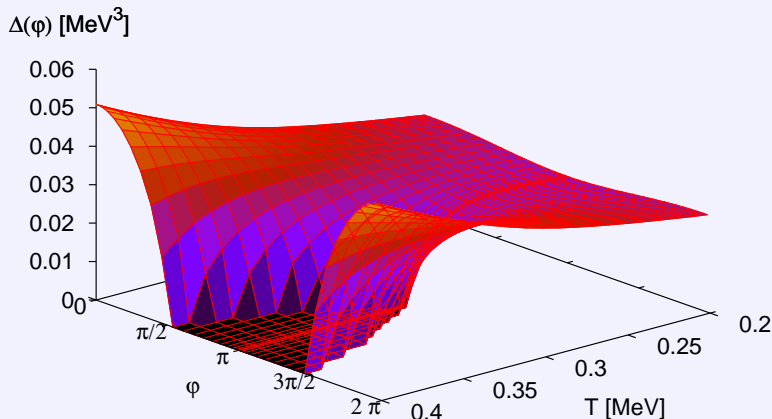
# Angular dependence in chiral limit



- Chiral limit: need continuum DSEs
- Chiral limit: loop expansion breaks down
- Width of plateau is  $T$ -dependent!

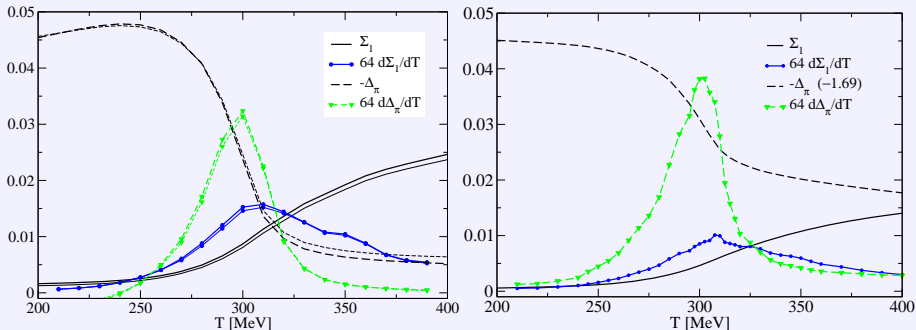
C.F. and Jens Mueller, in preparation

# Angular and temperature dependence in chiral limit



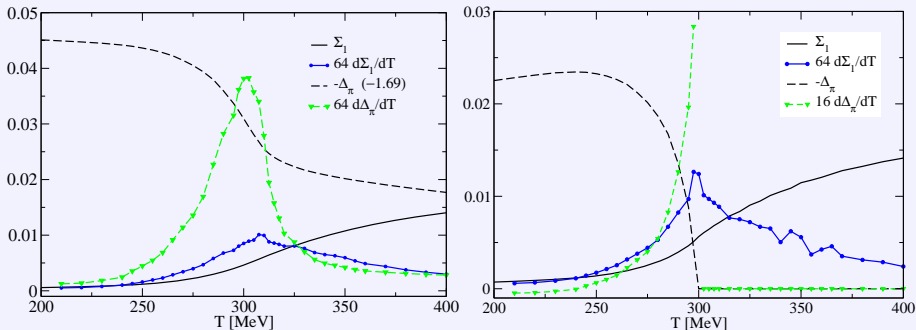
- Width of plateau grows with  $T$  but saturates at  $T \approx 2 T_c$
- $\Delta(\varphi = 0)$  grows with  $T^2/\ln(T)$  for  $T > 3/2 T_c$

# Dressed Polyakov loop $\Sigma_1$ for finite quark masses



- $T_{deconf} > T_{chiral}$
- Almost similar  $T_{chiral}$  from chiral susceptibility
- Systematic study of volume and discretisation effects possible.

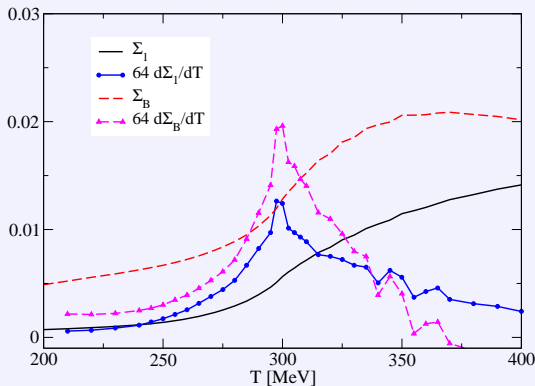
# Dressed Polyakov loop $\Sigma_1$ in chiral limit



Chiral limit:

- Second order chiral phase transition
- Transition temperatures coincide

# Dual scalar quark dressing $\Sigma_B$



- Transition temperatures of **dual condensate** and **dual scalar quark dressing** agree

$$\Sigma_B = - \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi} B(0, \varphi T),$$

## Techniques:

- **Dual condensate** → deconfinement order parameter
- **Dual scalar quark dressing** → deconfinement order parameter
- **Calculable with functional methods!**

C.F., arXiv:0904.2700 [hep-ph]; C.F. and J. Mueller, in preparation

J. Braun, L. Haas, F. Marhauser, J. M. Pawłowski, in preparation

J. Braun, H. Gies and J. M. Pawłowski, arXiv:0708.2413 [hep-th].

## Results for $D\chi$ SB and Deconfinement:

- **Different** transition temperatures **at finite quark mass**
- **Same** transition temperatures **in chiral limit**

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 **LOEWE** – Landes-Offensive zur Entwicklung  
Wissenschaftlich-ökonomischer Exzellenz



Helmholtz-Alliance: Extremes of density and  
temperature; cosmic matter in the laboratory



# Gluon and Quark-Gluon-Vertex

Fit function for gluon:

$$Z_{T,L}(\vec{q}, \omega_q, T) = \frac{q^2 \Lambda^2}{(q^2 + \Lambda^2)^2} \left\{ \left( \frac{c}{q^2 + \Lambda^2 a_{T,L}(T)} \right)^2 + \frac{q^2}{\Lambda^2} \left( \frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2 + 1]}{4\pi} \right)^\gamma \right\}$$

Ansatz for Quark-Gluon-Vertex:

$$\Gamma_\nu(q, k, p) = \tilde{Z}_3 \left( \delta_{4\nu} \gamma_4 \frac{C(k) + C(p)}{2} + \delta_{j\nu} \gamma_j \frac{A(k) + A(p)}{2} \right) \times \left( \frac{d_1}{d_2 + q^2} + \frac{q^2}{\Lambda^2 + q^2} \left( \frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2 + 1]}{4\pi} \right)^{2\delta} \right).$$