Dissipative superfluids from cold atoms to quark matter

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Outline

- ✦ Superfluidity
- ✦ Dissipative processes
- ✦ Bulk viscosity in cold atoms
- ✦ Mutual friction in the CFL phase

Reviews

Cold Atoms

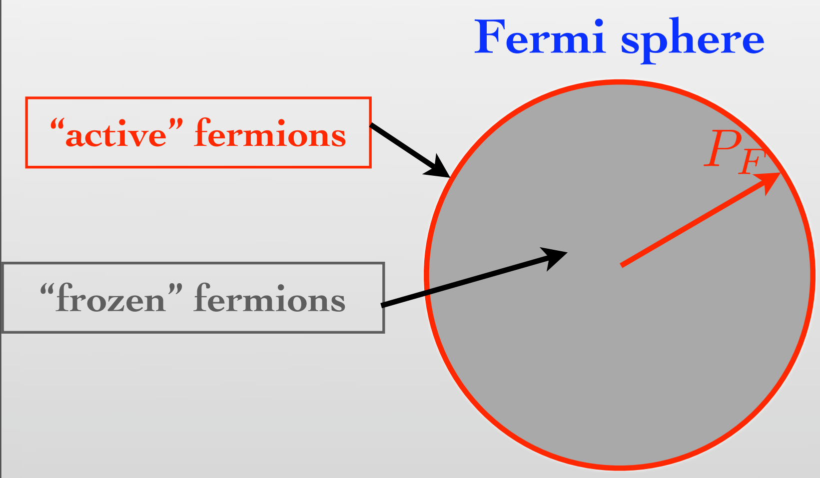
Rev.Mod.Phys. 80, 1215 (2008)
0904.3107

CSC

hep-ph/0011333
0709.4635

Degenerate Fermions

- System of degenerate Fermions at high density and low temperature



- ❖ Fermions fill energy levels up to the Fermi energy
- ❖ Exciting fermions deep in the Fermi sphere has an energy cost
- ❖ Fermions close to the Fermi surface can easily scatter

- Cooper theorem: Any arbitrarily attractive interaction $\longrightarrow$ Cooper pairing

The difermion condensate induces some symmetry breaking

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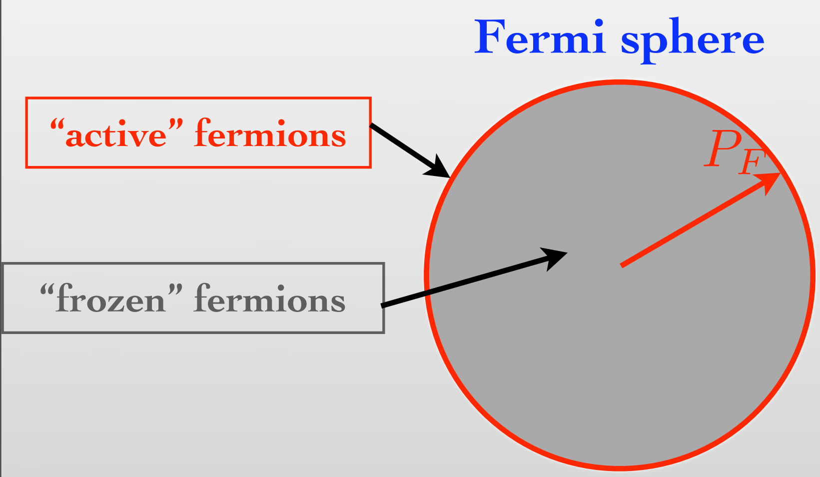

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✱ Breaking of gauge symmetry
(chromo) magnetic field is expelled:
Meissner effect

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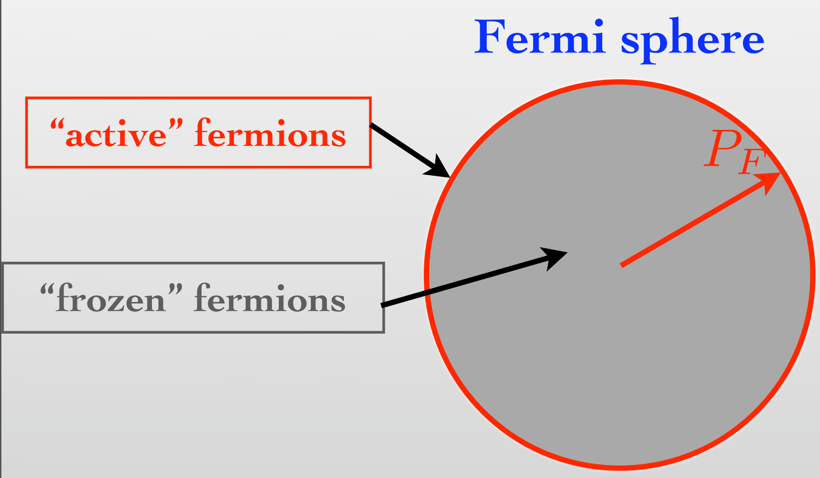

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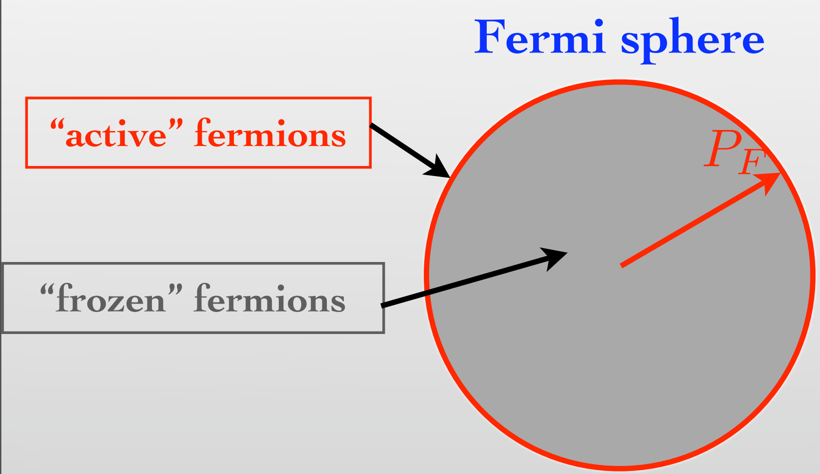

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Goldstone boson with a linear dispersion law $\longrightarrow$ Landau criterion for superfluidity is satisfied

NON-RELATIVISTIC SUPERFLUIDS

Landau two-fluid theory

$$\rho = \rho_n + \rho_s \qquad \mathbf{j} = \rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s$$

Normal component
phonons, rotons...

Superfluid component

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Hydrodynamic equations

$$\begin{aligned}
\frac{\partial j_i}{\partial t} + \partial_j (\Pi_{ij} + \tau_{ij}) &= 0 \\
\frac{\partial \mathbf{v_s}}{\partial t} + \nabla \left(\mu + \frac{\mathbf{v_s}^2}{2} + h \right) &= 0 \\
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Dissipative terms

Dissipative energy flux

$$\mathbf{Q}' = \mathbf{q} + h(\mathbf{j} - \rho \mathbf{v}_n) + \tau \cdot \mathbf{v}_n$$

Entropy production rate

$$R = -h\nabla \cdot (\rho_s(\mathbf{v_n} - \mathbf{v_s})) - \tau_{ik}\partial_k v_{ni} - \frac{1}{T}\mathbf{q} \cdot \nabla T$$

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Non-negative entropy production: $\zeta_1^2 \leq \zeta_2\zeta_3$ and $\kappa, \eta, \zeta_2, \zeta_3$ positive

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$$\begin{aligned}
P &= P_{\text{eq}} - \zeta_1 \text{div}(\rho_s(\mathbf{v_n} - \mathbf{v_s})) - \zeta_2 \text{div}\, \mathbf{v_n} \\
\mu &= \mu_{\text{eq}} - \zeta_3\, \text{div}(\rho_s(\mathbf{v_n} - \mathbf{v_s})) - \zeta_4 \text{div}\, \mathbf{v_n}
\end{aligned}$$

Phonon contribution

phonon dispersion law $\epsilon_p = c_s p + B p^3 + \mathcal{O}(p^5)$

$$\boxed{\partial_t \mathcal{N}_{\rm ph} + \mathrm{div}(\mathcal{N}_{\rm ph}\mathbf{v_n}) = -\frac{\Gamma_{\rm ph}}{T}\mu_{\rm ph}}$$

$$\boxed{\begin{array}{ll} B > 0 & \phi \to \phi\phi \\ B < 0 & \phi\phi \to \phi\phi\phi \end{array}}$$

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Notice that $\zeta_1^2 = \zeta_2\zeta_3$ the system tends toward the state where bulk viscosity does not lead to dissipation

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Son, Wingate, cond-mat/0509786

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then

$$\boxed{\zeta_1 = \zeta_2 = 0 \qquad \zeta_3 \simeq 3695.4 \left(\frac{\xi}{\mu} \right)^{9/2} \frac{(c_1 + \frac{3}{2} c_2)^2}{m^8} T^3 + \mathcal{O}\left(T^5\right)}$$

Color superconductor

Cold quark matter at extreme densities

- **Degenerate system of quarks**
- **Attractive interaction between quarks**

$\mu \gg m_s$ CFL phase

Alford, Rajagopal, Wilczek hep-ph/9804403

$$< \psi_{\alpha i} C \gamma_5 \psi_{\beta j} > \sim \Delta\, \epsilon_{I\alpha\beta} \epsilon_{Iij}$$

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- $U(1)_B$ **is spontaneously broken: CFL is a superfluid**

Effective action

Landau/Tisza theory: **Fluid of quasiparticles (phonons) moving on the top of the superfluid background**

Effective Lagrangian (low energy, asymptotic densities) Son hep-ph/0204199

$$\mathcal{L}_{\rm eff} = \frac{3}{4\pi^2}\left[(\partial_0\varphi - \mu_q)^2 - (\partial_i\varphi)^2\right]^2$$

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$$\varphi(x) = \bar{\varphi}(x) + \phi(x)$$

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long-wavelength **"short-wavelength"**

$$S[\varphi] = S[\bar{\varphi}] + \frac{1}{2}\int d^4x \, \frac{\partial \mathcal{L}_{\text{eff}}}{\partial(\partial_\mu\varphi)\partial(\partial_\nu\varphi)}\bigg|_{\bar{\varphi}} \partial_\mu\phi\partial_\nu\phi + \cdots$$

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Acoustic metric

$$g_{\mu\nu} = \eta_{\mu\nu} + (c_s^2 - 1)v_\mu v_\nu$$

Phonons in CFL

Low temperatures $T \lesssim 0.01\ \mathrm{MeV}$

- **Phonon-phonon scattering not effective for producing viscosity**

$\lambda_{\mathrm{ph-ph}} > R_{\mathrm{star}}$

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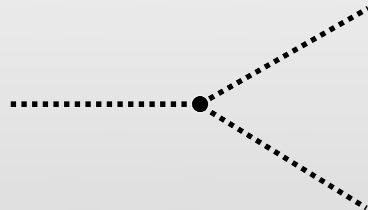

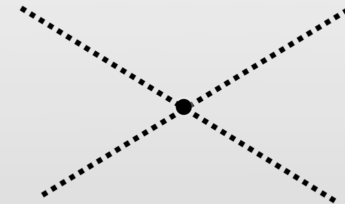

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- **Mutual friction**

Force between the superfluid component and the normal component mediated by phonon-vortex interaction

$$\boxed{\begin{aligned}\rho_s \frac{d\mathbf{v}_s}{dt} &= -\frac{\rho_s}{\rho}\nabla p - \rho_s \nabla\phi - \mathbf{F}^N \\ \rho_n \frac{d\mathbf{v}_n}{dt} &= -\frac{\rho_n}{\rho}\nabla p - \rho_n \nabla\phi + \mathbf{F}^N + \eta\nabla^2\mathbf{v}_n\end{aligned}}$$

Forces acting on a Vortex

- **Magnus force** — Standard hydrodynamic force

$$\mathbf{F}^M = \kappa\rho_s(\mathbf{v}_s - \mathbf{v}_L) \times \hat{\mathbf{z}}$$

- **Friction force** — Scattering of phonons off vortices

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From the equation of motion of the phonon field in the corresponding curved background

$$\frac{d\sigma}{d\theta} = \frac{c_s}{2\pi E}\frac{\cos^2\theta}{\tan^2\frac{\theta}{2}}\sin^2\frac{\pi E}{\Lambda}$$

Where E is the energy of the phonon and $1/\Lambda = \left(1 - c_s^2\right)\frac{\kappa}{2\pi c_s^2}$

CFL star

Suppose strange quark matter is absolutely stable with a lower energy per baryon than nuclear matter Witten PRD 30, 272 (1984), Farhi and Jaffe PRD 30, 2379 (1984)

Stars entirely consisting of strange quark matter should exist

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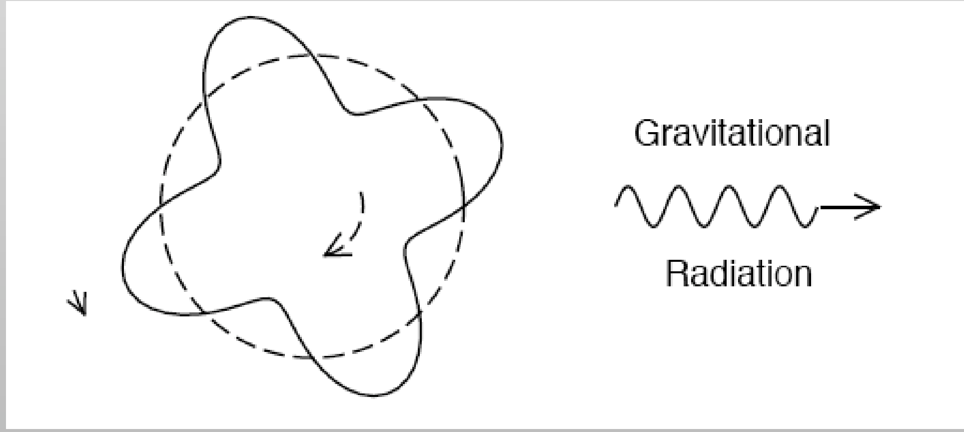
R-mode instability



Lindblom, astro-ph/0101136

Modes retrograde in the corotating frame (but prograde in the inertial frame) are unstable.

See also Andersson, Kokkotas gr-qc/0010102

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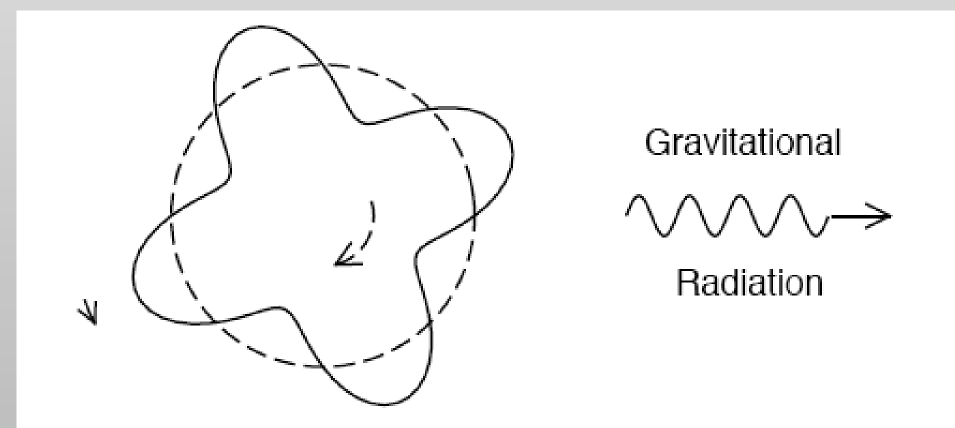
R-mode oscillation difficult to damp in CFL stars

Madsen, Phys. Rev. Lett. 85, 10 (2000)

Emitting gravitational radiation the star quickly spins down

R-mode instability



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Stars entirely consisting of strange quark matter should exist

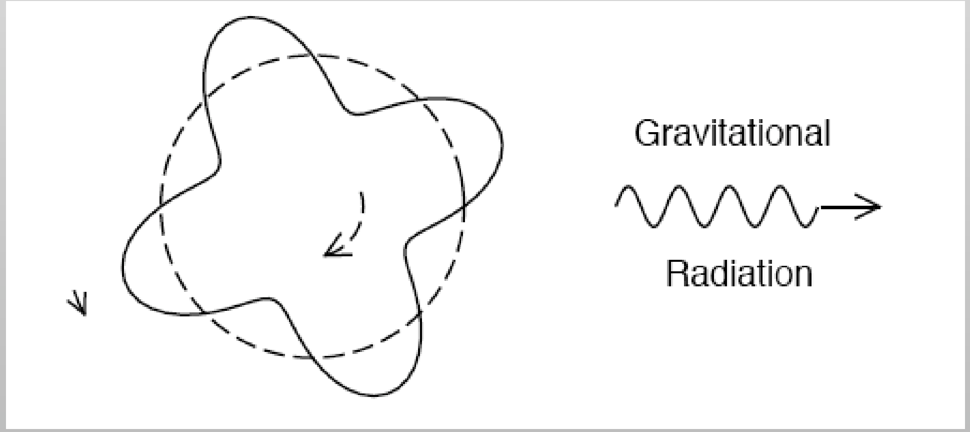
R-mode oscillation difficult to damp in CFL stars

Madsen, Phys. Rev. Lett. 85, 10 (2000)

Emitting gravitational radiation the star quickly spins down

Madsen neglected mutual friction effects

R-mode instability



Lindblom, astro-ph/0101136

Modes retrograde in the corotating frame (but prograde in the inertial frame) are unstable.

See also Andersson, Kokkotas gr-qc/0010102

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Neutron Stars rotating at higher frequencies are observed $\longrightarrow$ **CFL stars are ruled out?**

Summary

- Superfluidity appears in a variety of places
- Cold atoms experiments are an interesting playground for our understanding of superfluids
- CSC matter is a place where we can apply our understanding
- Gravity analogs are helpful tools in describing the low energy properties of (quantum) fluids