PANDA Collaboration Meeting
HIM, Mainz

September 12^{th} 2016

Radiative Corrections on $\bar{p}p \to e^+e^-$ Cross Section and Event Generator

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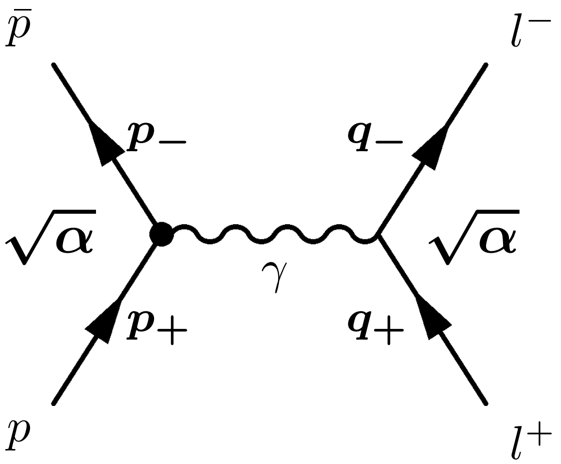
PRISMA

The $\bar{\mathrm{p}}\mathrm{p} \to \mathrm{e}^+\mathrm{e}^-$ cross section : leading order

- **at leading order (BORN approximation), only one diagram contributes to the amplitude:**



$$s = (p_+ + p_-)^2 \qquad t = (p_- - q_-)^2 \qquad u = (p_+ - q_-)^2$$

$$\Rightarrow \mathcal{M}_B \sim \alpha \quad \Rightarrow \quad d\sigma_B \sim |\mathcal{M}_B|^2 \text{ is } \mathcal{O}(\alpha^2)$$

A.Zichichi et al., Nuovo Cimento XXIV, 170 (1962):

$$\frac{d\sigma_B}{d\cos\theta^*} = \frac{\pi\alpha^2}{2s\beta}\left\{|G_M|^2(1+\cos^2\theta^*) + |G_E|^2(1-\beta^2)(1-\cos^2\theta^*)\right\}$$

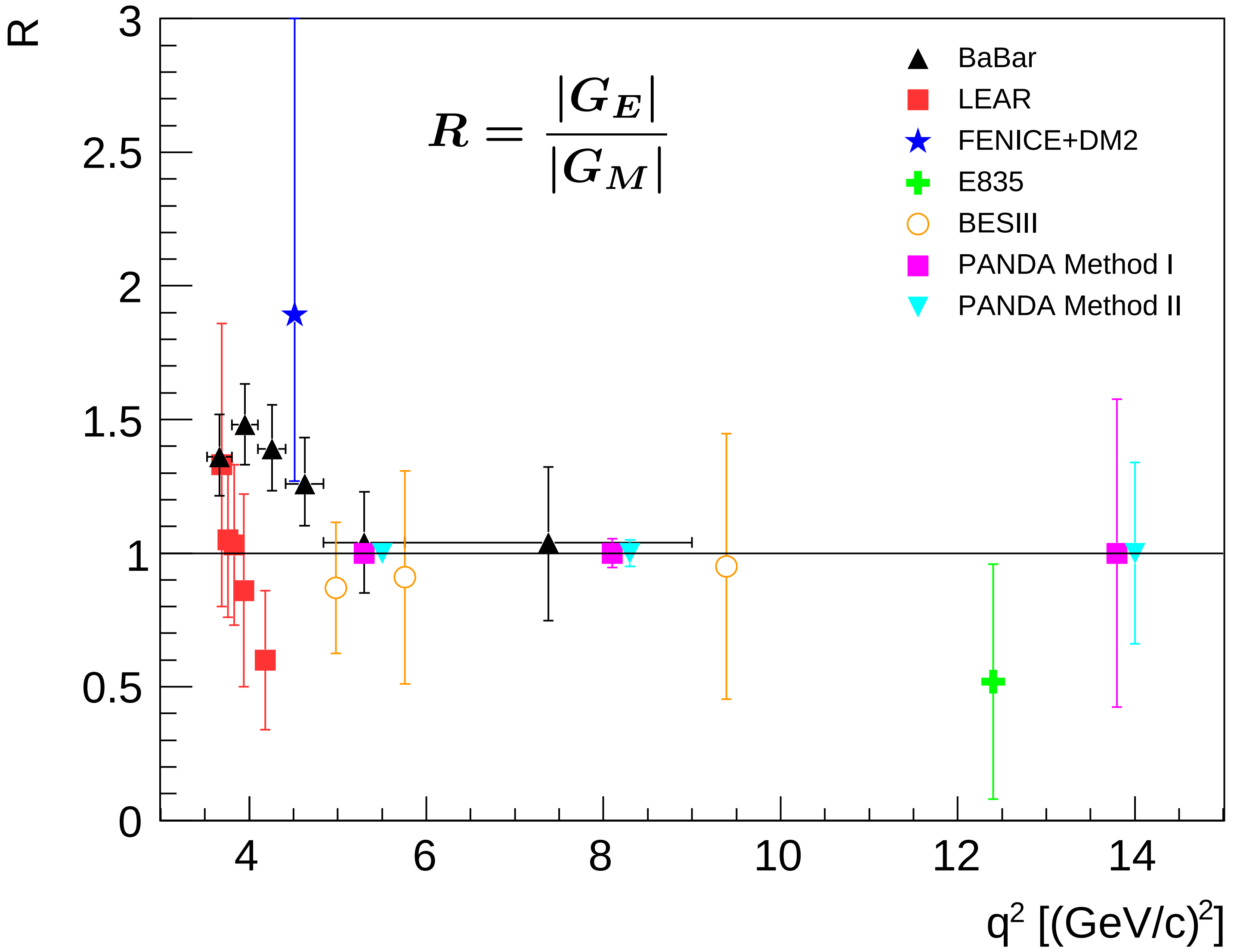
θ^* = angle (electron, antiproton) in $\bar{p}p$ CM frame, $\qquad 1-\beta^2 = 4M_p^2/s$

$\Rightarrow$ **fit function to extract $|G_E|$, $|G_M|$ at leading order**

- **having sufficiently precise data, we would like to attemp a form factor extraction at next to leading order:**

$\Rightarrow$ **we need an $\mathcal{O}(\alpha^3)$ differential cross section**

Future precision measurements of proton FFs with PANDA



at low q^2,

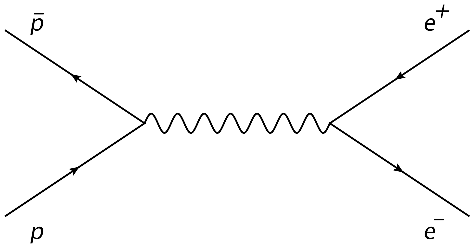
$$\frac{\Delta R}{R} \sim \text{few percent}$$

$\Rightarrow$ radiative corrections will be needed

"Feasibility studies of time-like proton electromagnetic form factors at PANDA at FAIR"
PANDA Collaboration; submitted to European Physical Journal A; [arXiv:1606.01118]

The $\bar{p}p \to e^+e^-$ cross section : next to leading order

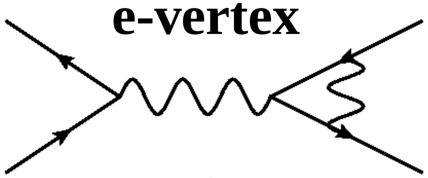
at next to leading order (NLO), many more diagrams contribute to the amplitude:

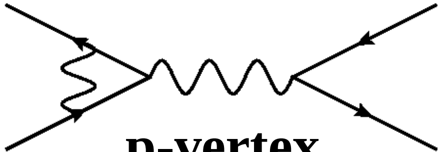


born $\mathcal{M} \sim \alpha$

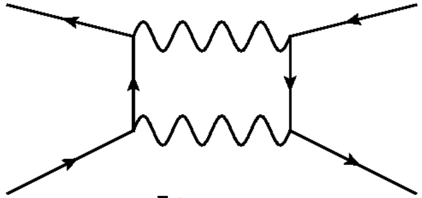
vacuum polarization

e-vertex

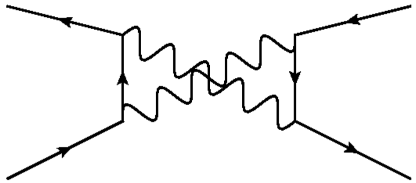




p-vertex

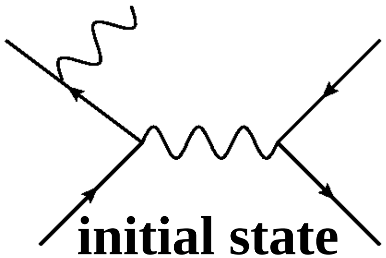


box

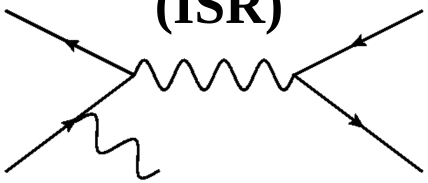


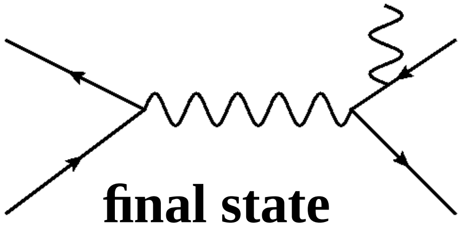
virtual corrections

$\mathcal{M} \sim \alpha^2$

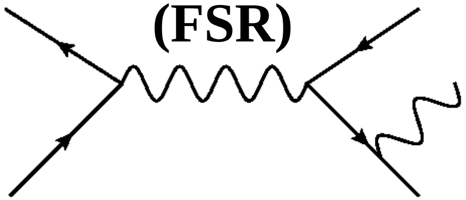


initial state radiation (ISR)





final state radiation (FSR)



real corrections

$\mathcal{M} \sim \alpha^{3/2}$

The $\bar{\mathrm{p}}\mathrm{p} \to \mathrm{e}^+\mathrm{e}^-$ cross section : next to leading order

at NLO, we have two different (i.e. non-interfering) final states:

- $*$ $\underline{e^+e^-}$: **born, virtual corrections (vacuum polarization, ...)**
- $*$ $\underline{e^+e^-\gamma}$: **real corrections (initial state radiation, final state radiation)**

$$\mathcal{O}(\alpha) \qquad \mathcal{O}(\alpha^2) \qquad\qquad \mathcal{O}(\alpha^{3/2})$$

$$\Rightarrow d\sigma \sim |\mathcal{M}_B + \textstyle\sum_i \mathcal{M}_i|^2 + |\mathcal{M}_{ISR} + \mathcal{M}_{FSR}|^2 \qquad \text{truncated at } \mathcal{O}(\alpha^3)$$

$$\sim |\mathcal{M}_B|^2 + \textstyle\sum_i 2\mathrm{Re}(\mathcal{M}_\mathrm{B}\mathcal{M}_\mathrm{i}^*) + |\mathcal{M}_\mathrm{ISR}|^2 + |\mathcal{M}_\mathrm{FSR}|^2 + 2\mathrm{Re}(\mathcal{M}_\mathrm{ISR}\mathcal{M}_\mathrm{FSR}^*)$$

$$\sim |\mathcal{M}_B|^2 \left\{ 1 + \sum_i \frac{2\mathrm{Re}(\mathcal{M}_\mathrm{B}\mathcal{M}_\mathrm{i}^*)}{|\mathcal{M}_B|^2} + \frac{|\mathcal{M}_{ISR}|^2}{|\mathcal{M}_B|^2} + \frac{|\mathcal{M}_{FSR}|^2}{|\mathcal{M}_B|^2} + \frac{2\mathrm{Re}(\mathcal{M}_\mathrm{ISR}\mathcal{M}_\mathrm{FSR}^*)}{|\mathcal{M}_B|^2} \right\}$$

$$\sim |\mathcal{M}_B|^2 \left\{ 1 + \delta_\mathrm{vacuum} + \delta_\mathrm{e\text{-}vertex} + \delta_\mathrm{p\text{-}vertex} + \delta_\mathrm{box} + \underbrace{\delta_\mathrm{ISR} + \delta_\mathrm{FSR} + \delta_\mathrm{interference}}_{\equiv\, \delta_\gamma} \right\}$$

therefore, at $\mathcal{O}(\alpha^3)$, we write

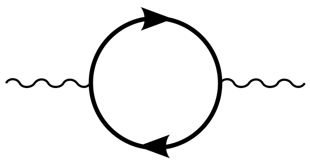
$$\boxed{d\sigma = d\sigma_B \left\{ 1 + \delta_\mathrm{vacuum} + \delta_\mathrm{e\text{-}vertex} + \delta_\mathrm{p\text{-}vertex} + \delta_\mathrm{box} + \delta_\gamma \right\}}$$

Calculation at NLO : virtual corrections

- **virtual corrections relying heavily on**

  A.I. Ahmadov et al. Phys. Rev. D **82**, 094016 (2010)
  G. Bonneu et al. Nucl. Phys. B **27**, 381 (1971)

$*$ **vacuum polarization:** e, μ, τ, **and point-like pion in loop**



$*$ **electron-vertex:** **ultra-relativistic limit** $s \gg m^2$ [G. Bonneu et al. (1971)]

$*$ **proton-vertex:** **point-like proton**

$*$ **box diagram:** **point-like proton**

[J. Van de Wiele and S. Ong, Eur. Phys. J. A **49**, 18 (2013) contains a first order study of radiative corrections to $\bar{p}p \to e^+e^-$, including full virtual and real corrections (soft and hard photon emission) in partial overlapping with A. I. Ahmadov et al. (2010). Our calculation does not make use of this source.]

Calculation at NLO : real corrections, cancellation of divergences

- **new calculation of real corrections (ISR, FSR, interference)**
  [Ahmadov et al. does not separate explicitly ISR/FSR terms;
  [kinematic regime $s, t, u \gg M^2, m^2$ assumed in interference term not valid for PANDA]

  following F. A. Berends et al. Nucl. Phys. B **57** 381 (1973)
  F. A. Berends et al. Nucl. Phys. B **63** 381 (1973) on $e^+e^- \rightarrow \mu^+\mu^-$

  done in the <u>soft photon limit</u>:

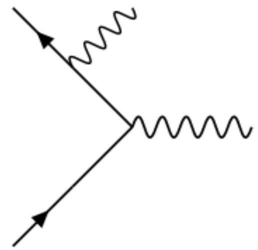
  $* \; E_\gamma < \omega = b\sqrt{s}/2 \qquad b < 10^{-2}$

  $*$ **integration over photon degrees of freedom (no experimental detection)**

- **infrared divergences regularized with a cutoff**

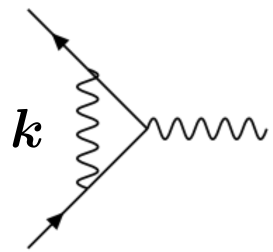
  $*$ **real diagrams:** $E_\gamma > \lambda = a\sqrt{s} \qquad 10^{-6} < a < 10^{-3}$

  $*$ **virtual diagrams:** $\int_0 dk^2 \rightarrow \int_{\lambda^2} dk^2$



- **cancellation of divergences:**

  $$\delta_{\text{e-vertex}} + \delta_{\text{FSR}} = \text{finite}$$
  $$\delta_{\text{p-vertex}} + \delta_{\text{ISR}} = \text{finite}$$
  $$\delta_{\text{box}} + \delta_{\text{interference}} = \text{finite}$$



- **full cross section is λ-independent (more accurately, λ-stable)**

Calculation at NLO : codes

- previous cross section used as a basis for two codes: σ-calculator and event generator, including born amplitude, virtual and real (soft photon only) corrections

- user can freely switch on/off correction terms (as long as cancellation of divergences is not violated), define parameters (center of mass energy $\sqrt{s}$, form factors G_E and G_M, photon cutoffs λ and ω), and kinematic region (angular range)

- "stand alone" version of the codes (i.e. not yet interfaced to PandaRoot) in a three-directory structure: `/inc /src /output`

- `/src`

  → user parameters passed to the program through the infile `mzgen.in`

  → compilation (no need!) by `Makefile` defined rules

  ```
  make clean
  make
  ```

  → run: `./run.sh`

- `/output`

  `output.log` (kinematic information, error messages)
  `output.std` (events in ASCII file, readable format by PandaRoot)

Providing parameters : infile

```
1000000                  ! #events
1                        ! RANLUX seed
1.5                      ! pbar momentum LAB-frame (GeV)
-0.999                   ! cos(theta*) min
0.999                    ! cos(theta*) max
1.279                    ! |G_E|/|G_M|
0.00001                  ! soft photon MIN energy controller
0.01                     ! soft photon MAX energy controller
1                        ! leptons (vacuum polarization)
1                        ! pion    (vacuum polarization)
1                        ! box
1                        ! electron vertex
1                        ! proton vertex
1                        ! final state radiation
1                        ! initial state radiation
1                        ! interference ISR/FSR
../output                ! pathout
output                   ! outfile
```

Providing parameters : infile (information for the user)

```
PARAMETER RANGE
===============

#events:                 > 0 (integer!)
RANLUX seed:             [1,2^31-1]
pbar momentum:           > 0.0 GeV
                         (recommended in PANDA range 1.5 < P < 15.0 GeV)
angular range:           -1 < cos(theta*) min < cos(theta*) max < 1
                         (boundaries excluded!)
|G_E|/|G_M|:             >= 0.0
a-parameter:             [10^{-6},10^{-3}], recommended value is a = 10^{-5}
                         [E_gamma_min = a * sqrt(s)]
b-parameter:             (2.0*a, 0.01]
                         [E_gamma_max = b * sqrt(s) / 2]
leptons:                 0 (OFF), 1 (ON) [vacuum polarization]
pion:                    0 (OFF), 1 (ON) [vacuum polarization]
box:                     0 (OFF), 1 (ON)
electron vertex:         0 (OFF), 1 (ON)
proton vertex:           0 (OFF), 1 (ON)
FSR:                     0 (OFF), 1 (ON)
ISR:                     0 (OFF), 1 (ON)
interference ISR/FSR:    0 (OFF), 1 (ON)


Divergences cancellation forbids the following flags combinations:

(electron vertex, FSR)      = { (0,1), (1,0) }
(proton vertex, ISR)        = { (0,1), (1,0) }
(box, interference ISR/FSR) = { (0,1), (1,0) }
```

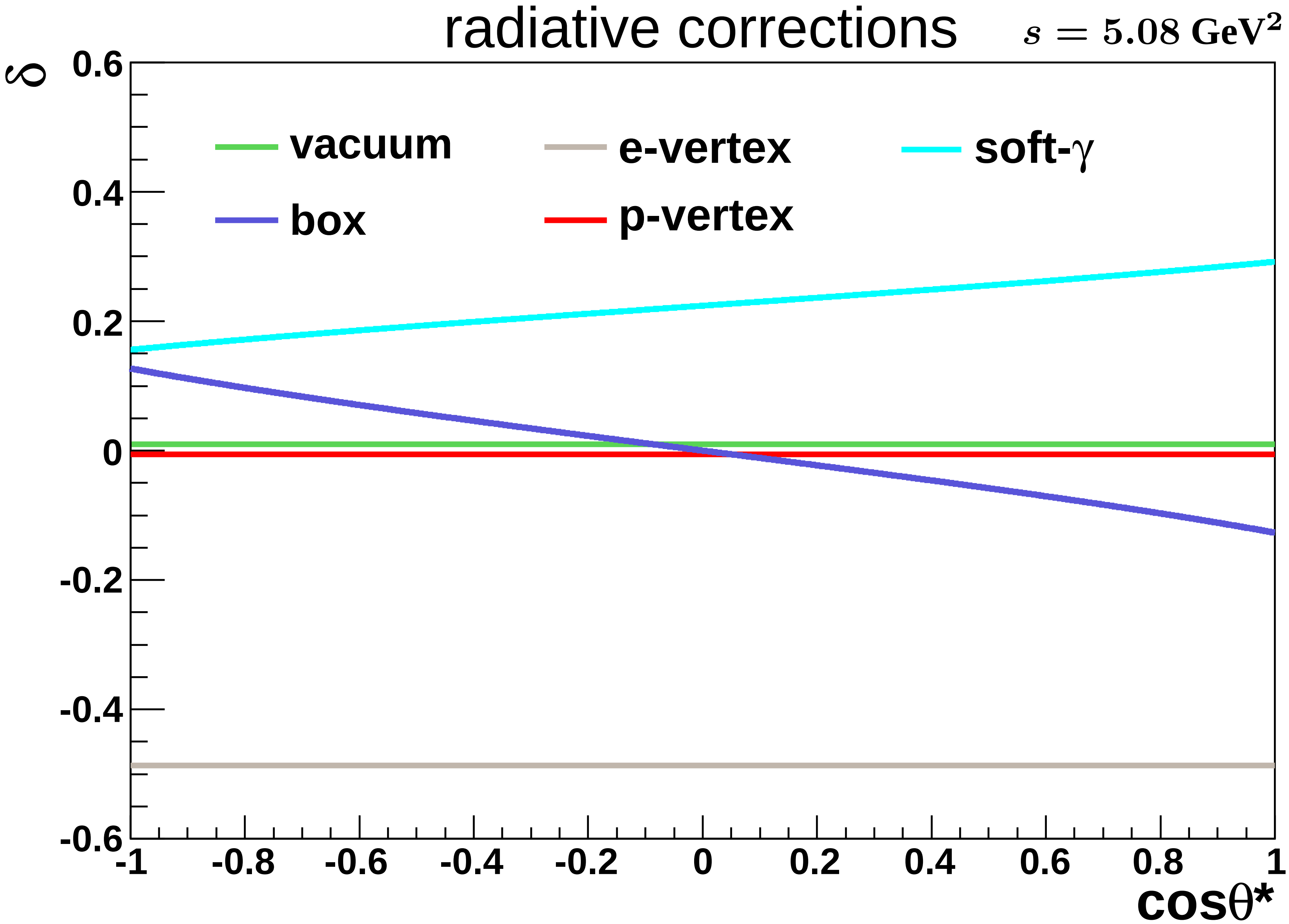
Initialization stage : checking parameters

- **both programs check that all parameters are correct, otherwise printing an error message and killing the job; examples:**

```
WARNING: anti-proton momentum larger than PANDA upper limit 15.00 GeV
         [P = 20.00 GeV]

ERROR: cos(theta*) range NOT allowed! [-0.80 < cos(theta*) < -0.80]
    => cos(theta*) range [cos(theta*) min, cos(theta*) max]
       should be contained within the interval [-1.0,1.0]
    => job killed!!!

ERROR: b-parameter out of range! [b-parameter = 0.020000]
    => b-parameter should be contained within the interval
       (2.0 * a-parameter, 0.01]
       (with user's choice a-parameter = 0.000010, this interval is
       (0.000020,0.010000] )
    => job killed!!!

ERROR: vacuum polarization flag out of range! [flag = 2]
    => vacuum polarization flag should take the value 0 or 1
    => job killed!!!
```

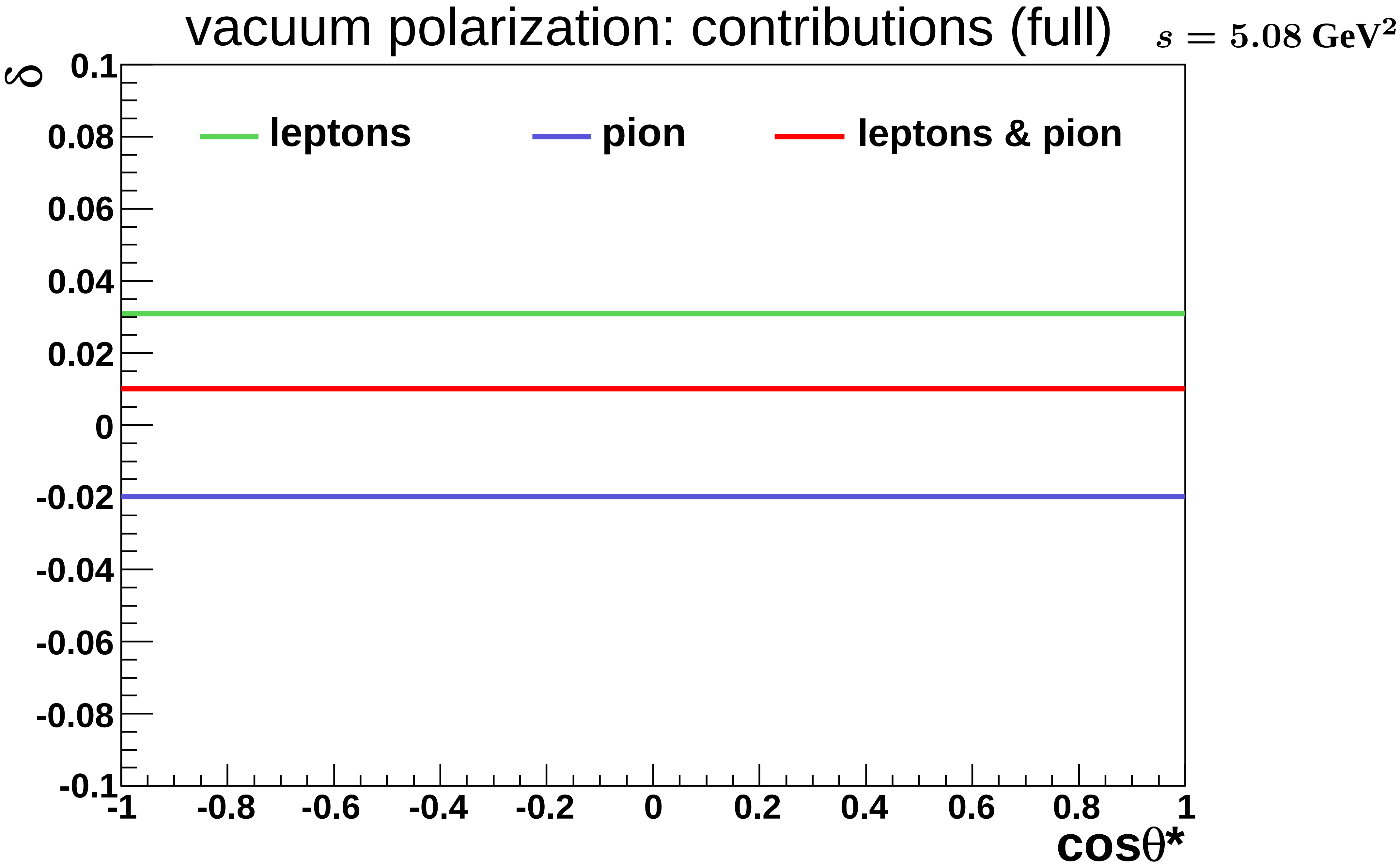
$\Rightarrow$ **no way to run a job with wrong parameters!**

Cross section calculator : radiative corrections

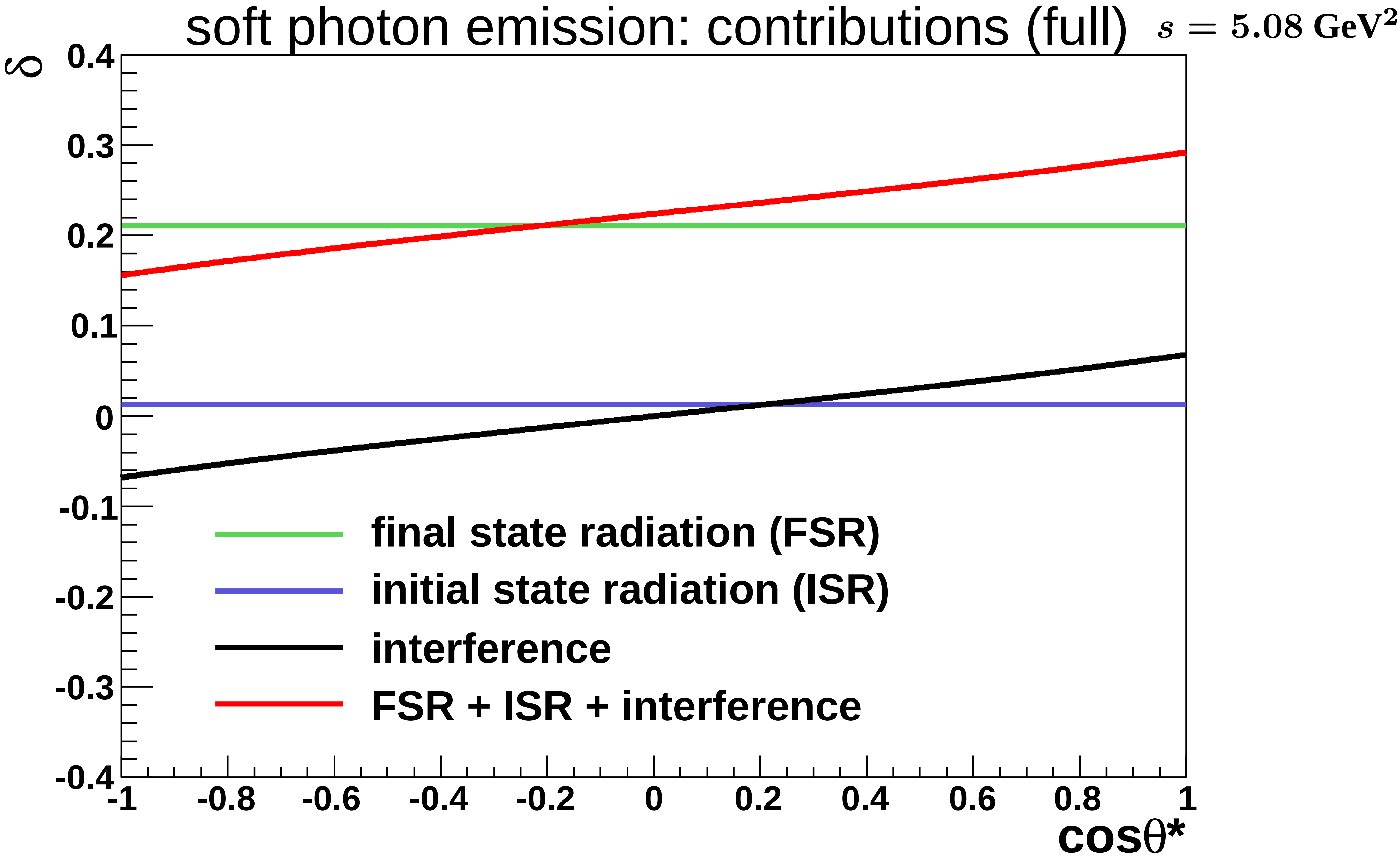


box, proton vertex and ISR/FSR interference are t, u-dependent

Cross section calculator : contributions to vacuum polarization

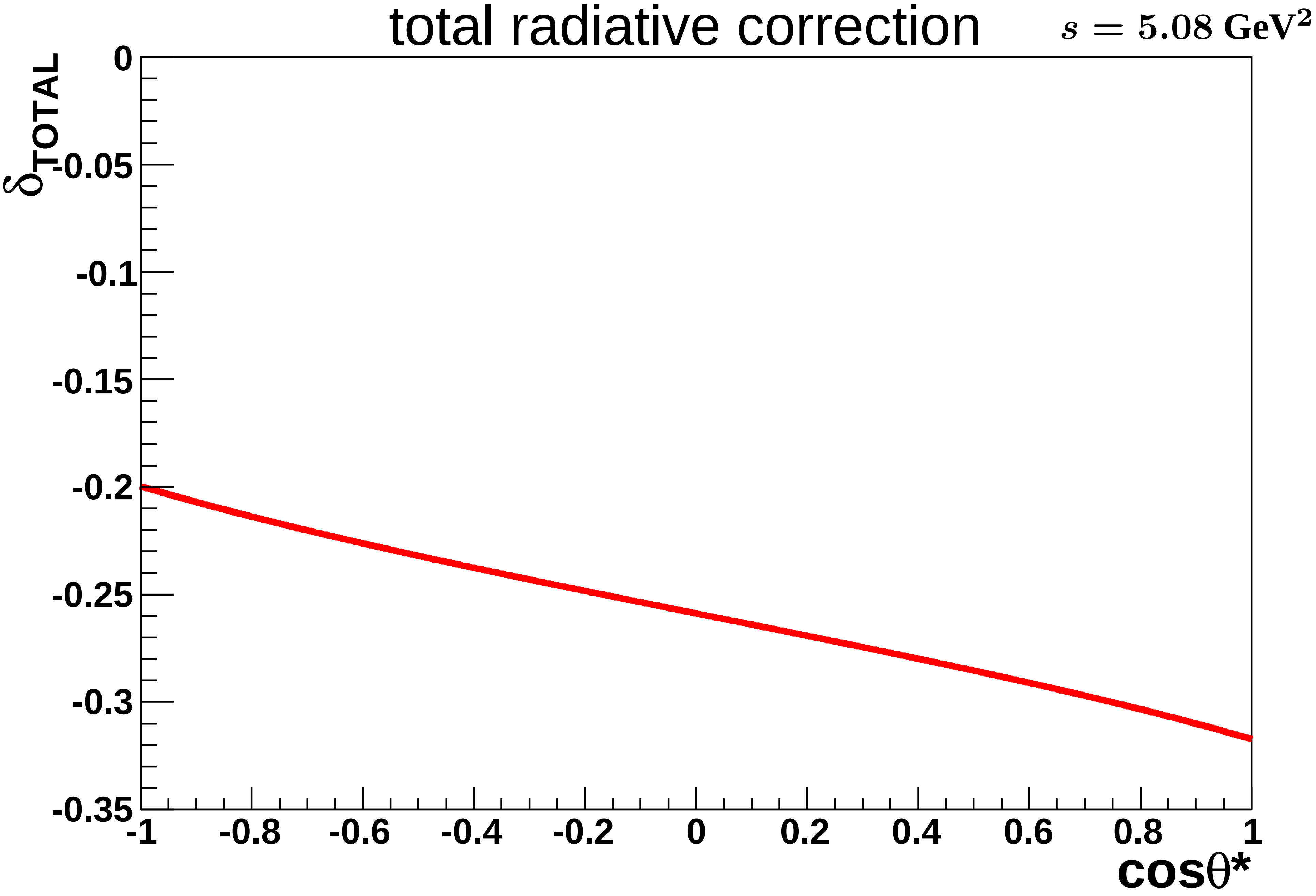


Cross section calculator : contributions to soft photon emission



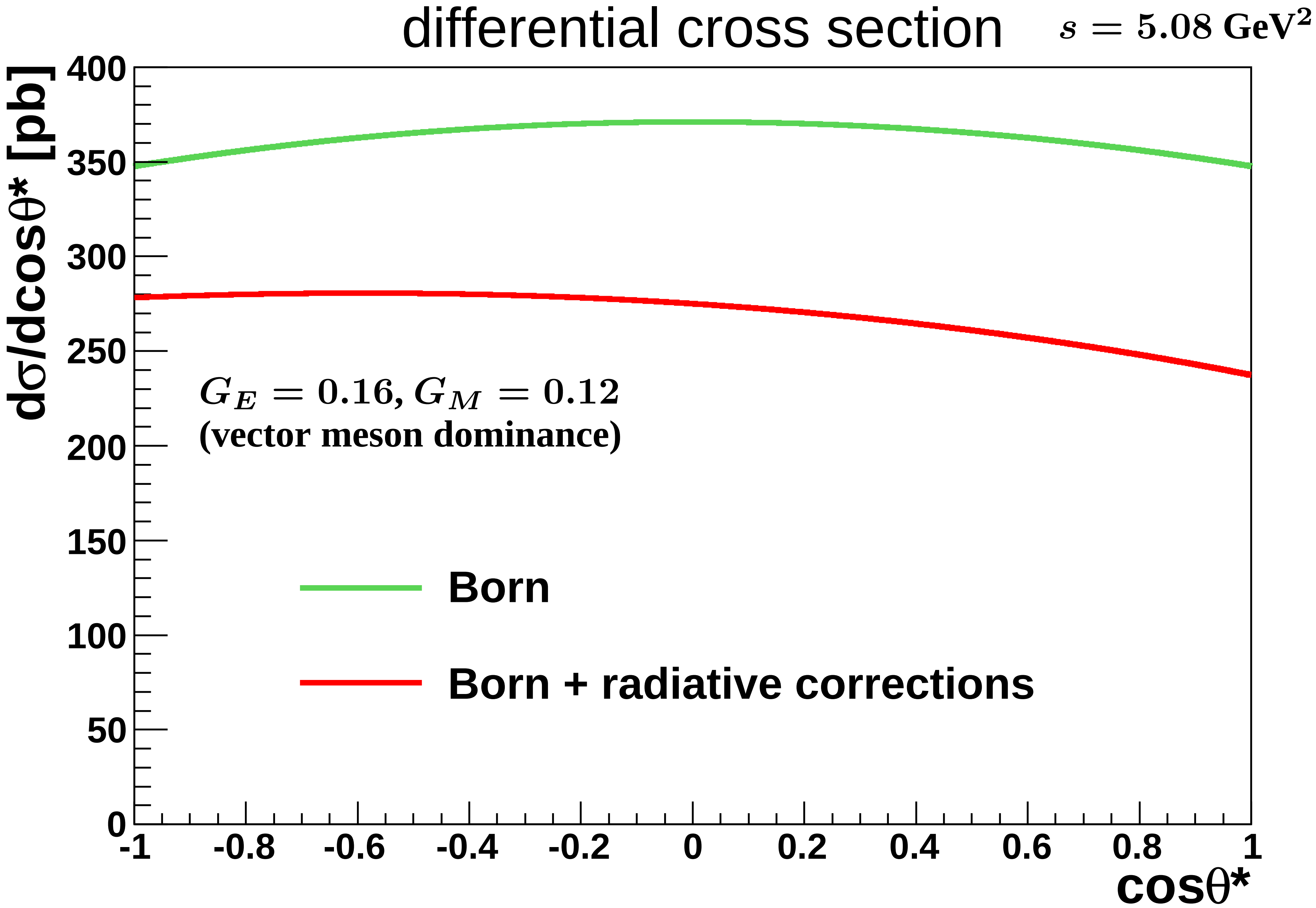
Cross section calculator : total radiative correction

$$d\sigma/d\cos\theta^* = (d\sigma_B/d\cos\theta^*)(1+\delta_T)$$



Cross section calculator : differential cross section

$$d\sigma/d\cos\theta^* = (d\sigma_B/d\cos\theta^*)(1+\delta_T)$$



Event generator : output file

particle 3-momenta in LAB frame

```
2 1 0.0 0.0 0.0
-11   0.245824220500  -0.880338258863  -0.042038863992
11  -0.245824220500   0.880338258863   1.542038863992
2 2 0.0 0.0 0.0
-11   0.060031044323   0.897636407892  -0.065438910784
11  -0.060031044323  -0.897636407892   1.565438910784
2 3 0.0 0.0 0.0
-11  -0.726668658304   0.858063734140   0.657817135674
11   0.726668658304  -0.858063734140   0.842182864326
2 4 0.0 0.0 0.0
-11   0.551317231220  -0.982728435041   0.723022924748
11  -0.551317231220   0.982728435041   0.776977075252
2 5 0.0 0.0 0.0
-11   0.535494626204  -0.916559484986   0.295174287325
11  -0.535494626204   0.916559484986   1.204825712675
2 6 0.0 0.0 0.0
-11   1.032021547949  -0.452798214600   0.737087211981
11  -1.032021547949   0.452798214600   0.762912788019
2 7 0.0 0.0 0.0
-11   0.841641293314   0.739697424717   0.895625342184
11  -0.841641293314  -0.739697424717   0.604374657816
```

$\longrightarrow$ **readable format by PandaRoot**

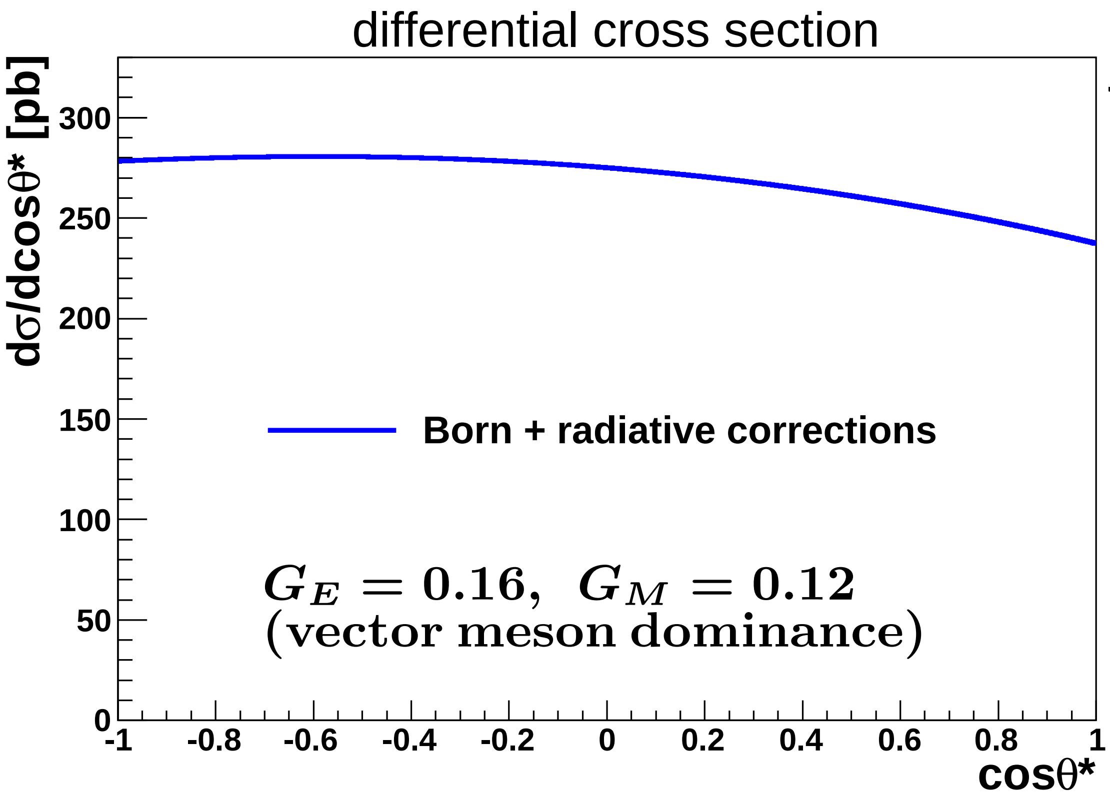
Event generator : $\cos\theta^*$ distribution

$p = 1.5$ GeV $\Rightarrow$ $s = 5.08$ GeV^2
$\sqrt{s} = 2.25$ GeV

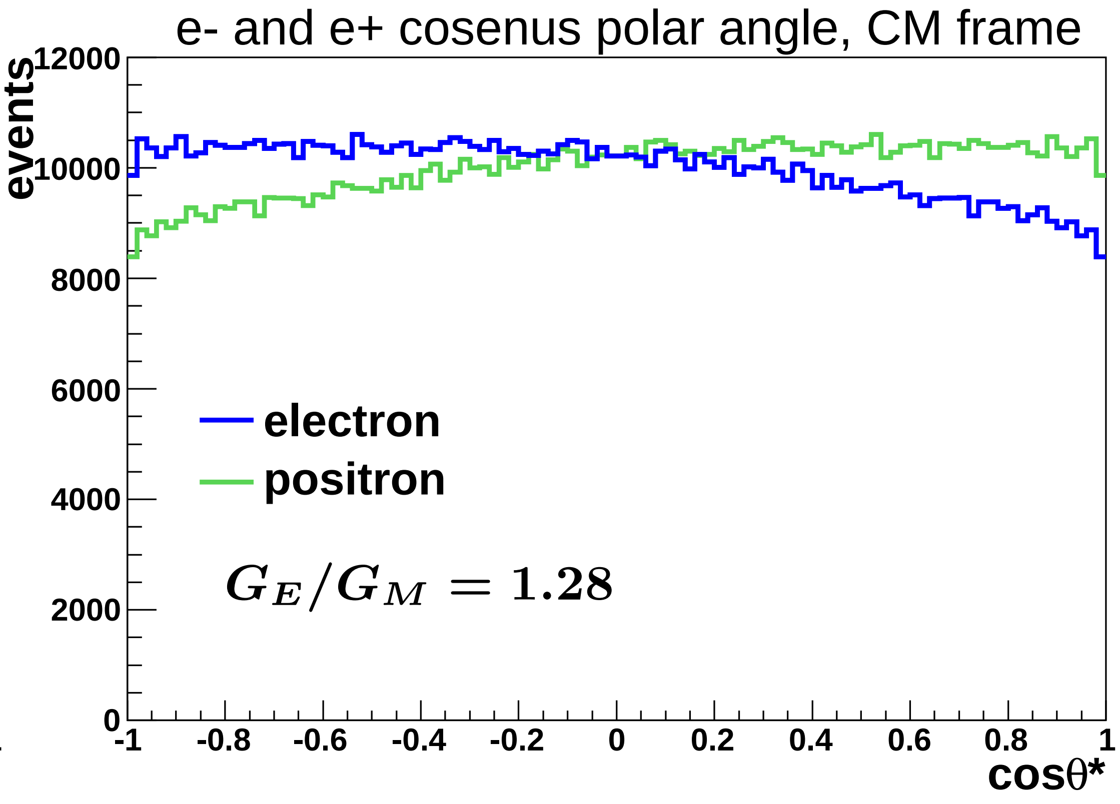
$N = 10^6$ generated events

$\theta^* =$ angle(electron, antiproton) CM frame

radiative corrected cross section

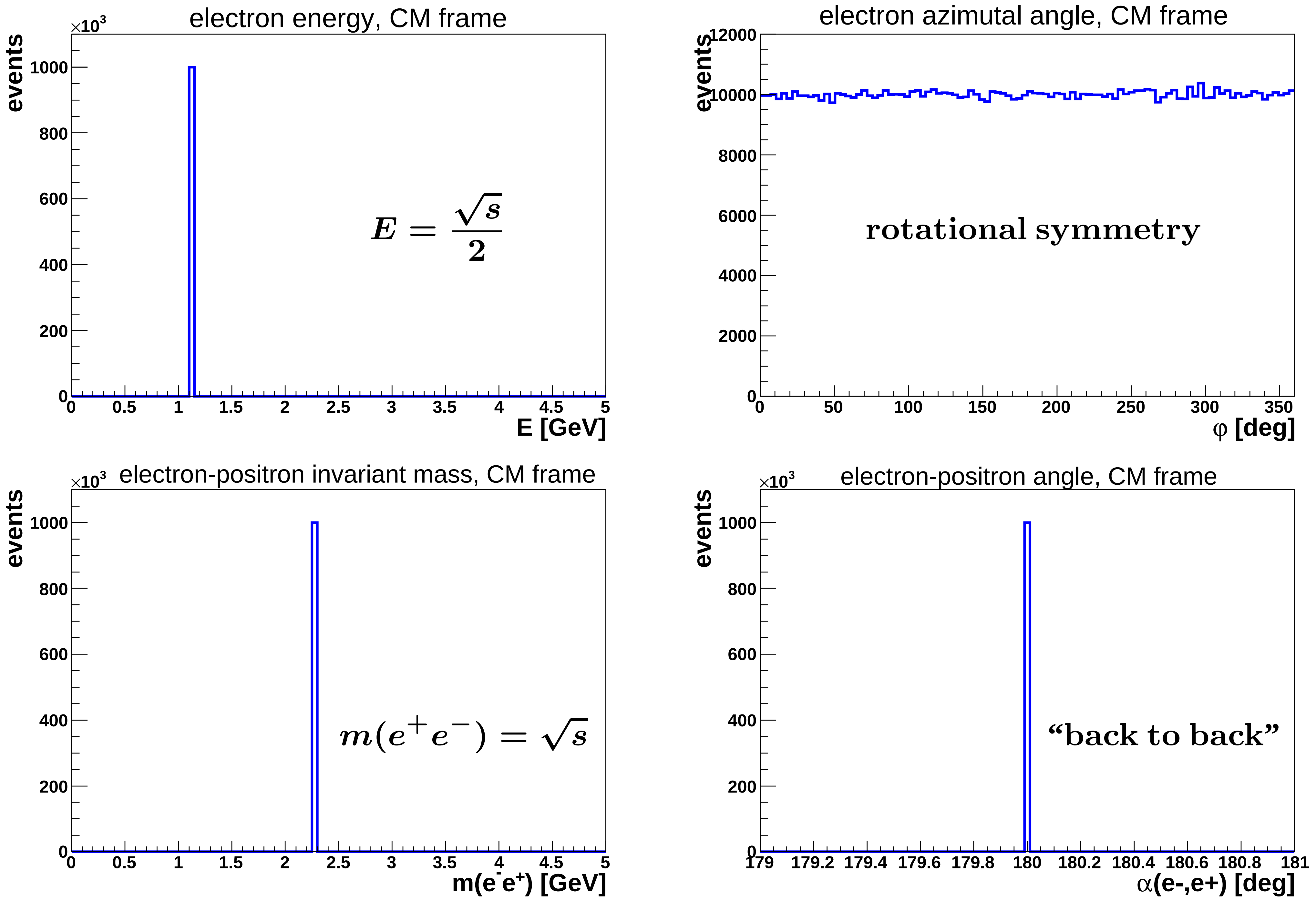


generated events



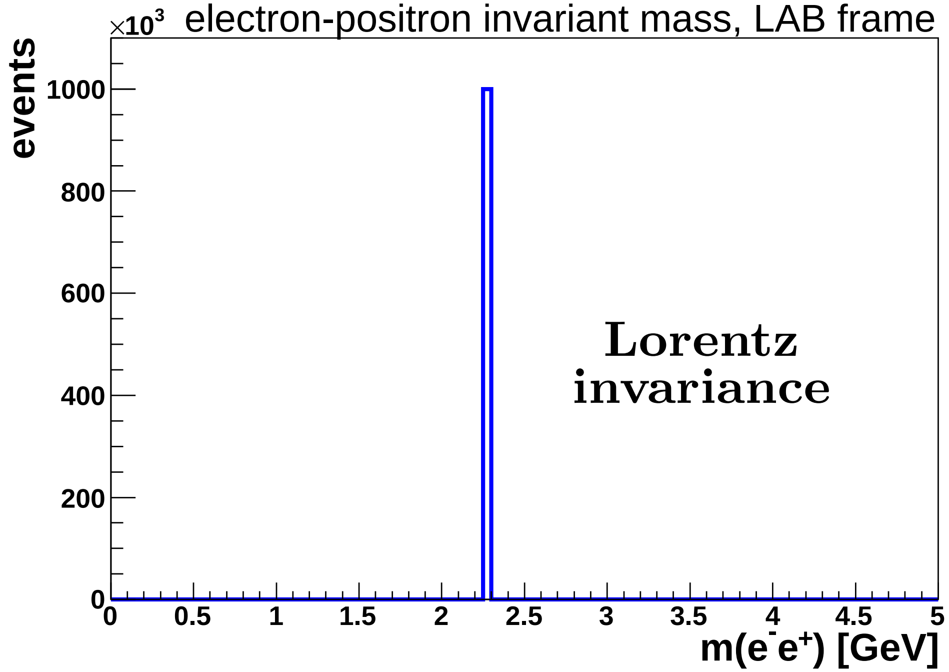
Event generator : example distributions (CM frame)

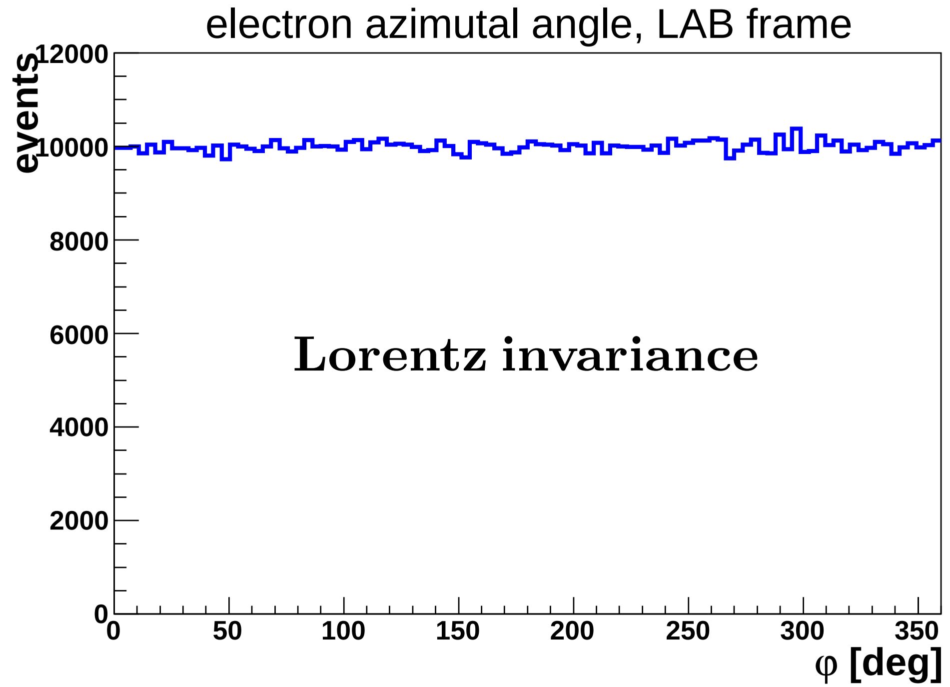
$\sqrt{s} = 2.25$ GeV

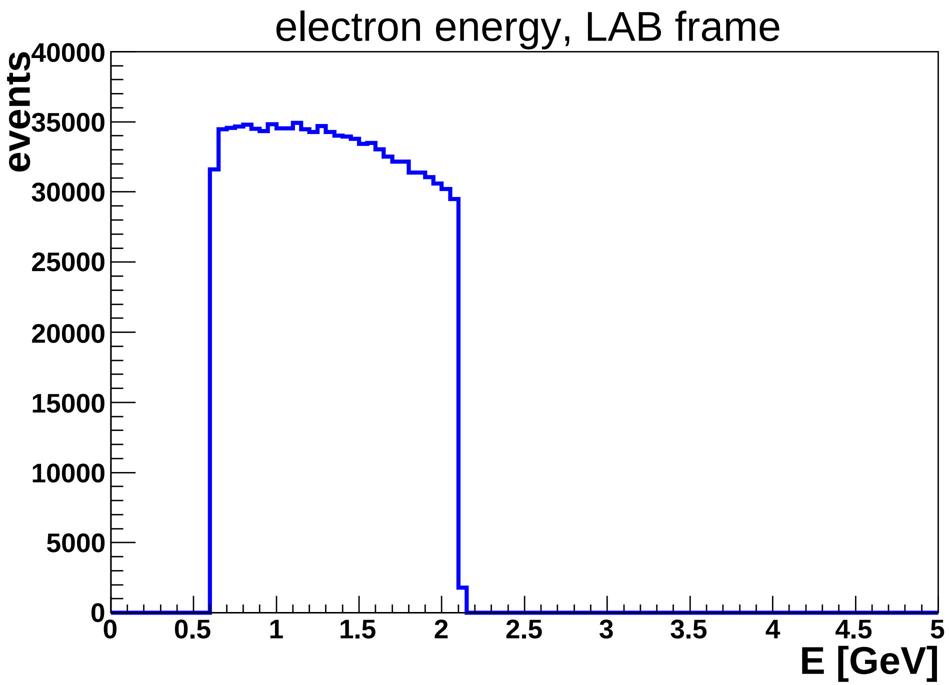


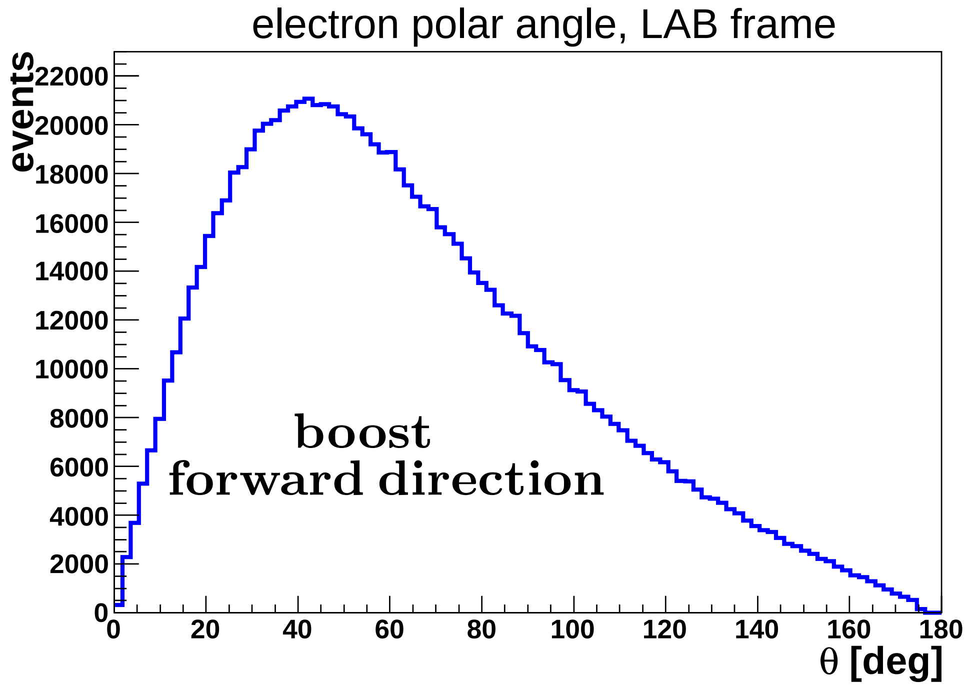
Event generator : example distributions (LAB frame)

$\sqrt{s} = 2.25$ GeV









Summary, conclusions, and perspectives

- **cross section calculator and ("stand alone") event generator for the channel $\bar{p}p \to e^+e^-$, including Born amplitude, virtual corrections, and real corrections (soft photon only), finished and ready to use for physics analyses**

next steps

- **interfacing the generator to PandaRoot**
- **including hard photons (both programs)**
- **full documentation (both programs)**