Exact renormalisation group at finite temperature

"Quarks, Hadrons, and the Phase Diagram of QCD"
St Goar
2 Sept. 2009

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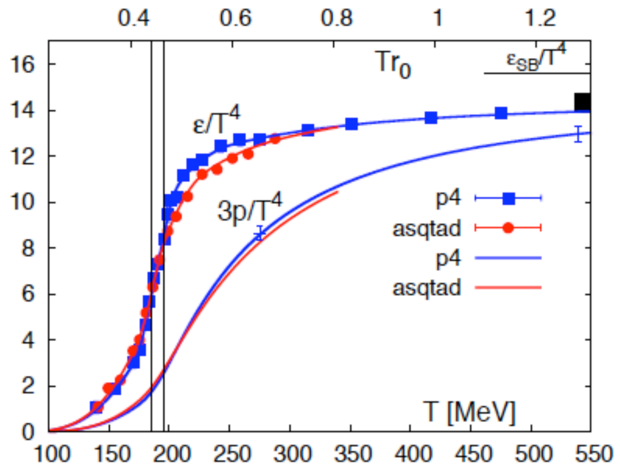
Outline

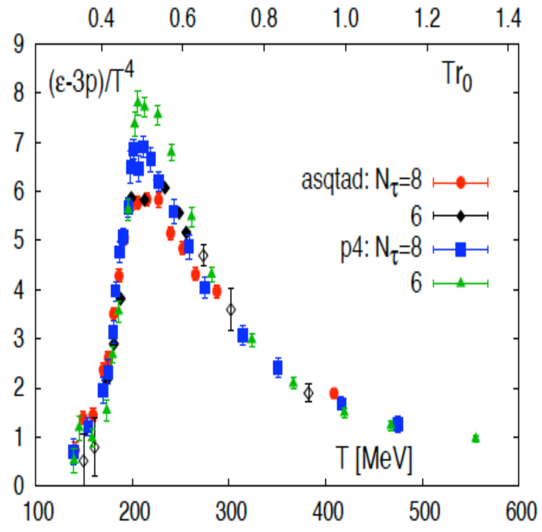
- weak coupling techniques and why they are useful to understand hot QCD

- insights from the exact Renormalization Group

Hot QCD

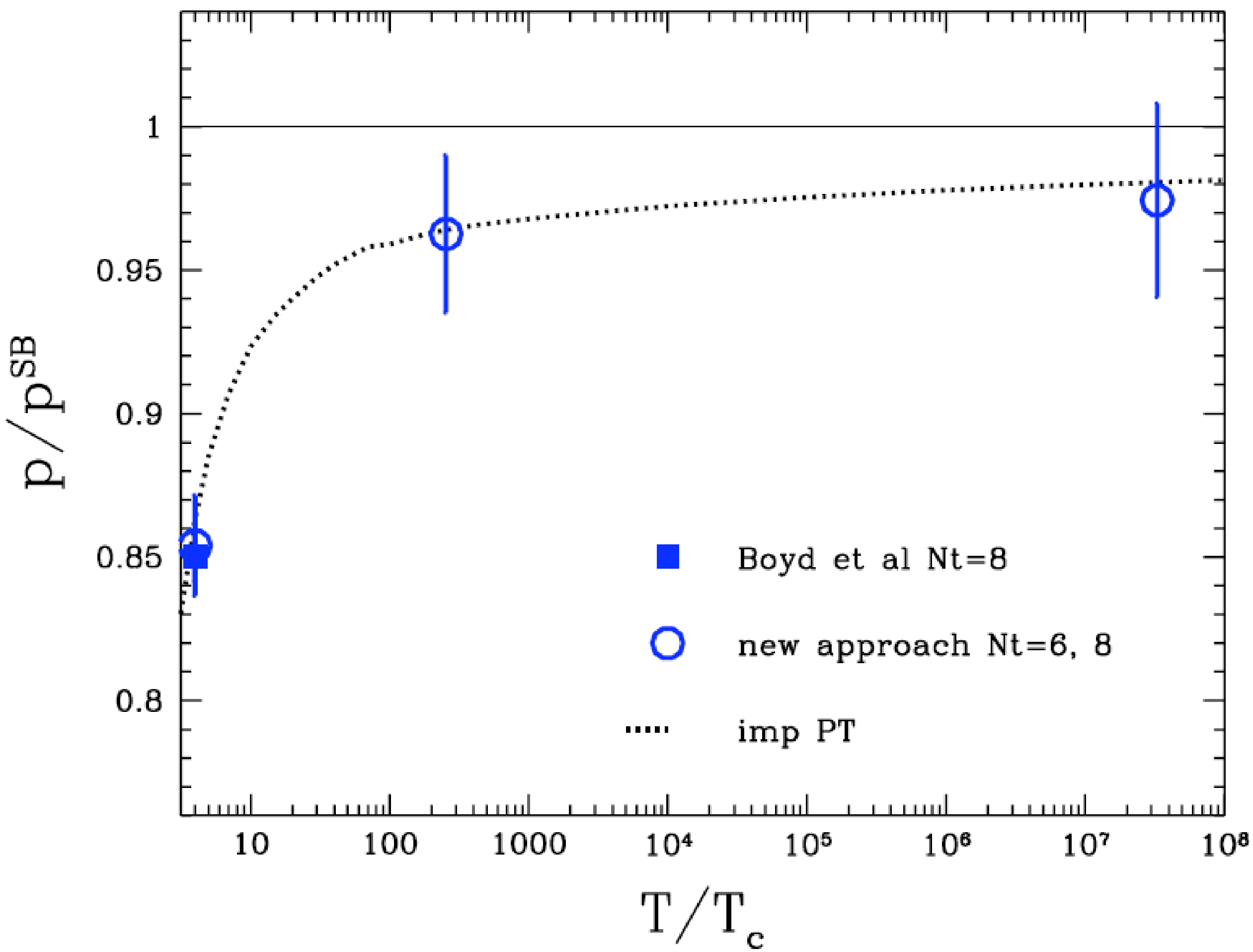
Thermodynamics of hot QCD





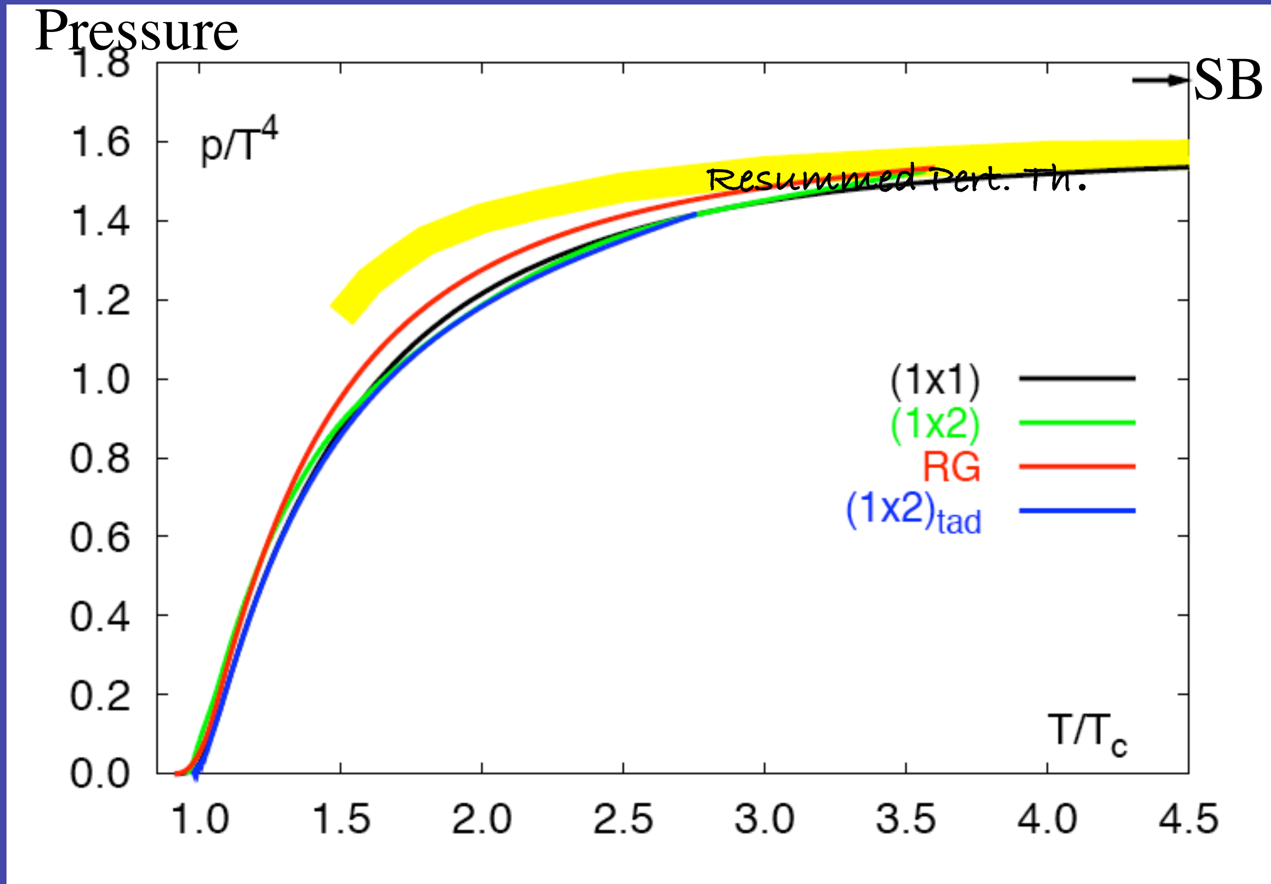
(from M. Bazavov et al, arXiv:0903.4379)

Pressure for SU(3) YM theory at (very) high temperature



(from G. Endrodi et al, arXiv: 0710.4197)

At T>3Tc Resummed Pert. Theory accounts for lattice results



(SU(3) lattice gauge calculation from Karsch et al, hep-lat/0106019)

(resummed pert. th. from J.-P. B., E. Iancu, A. Rebhan: Nucl.Phys.A698:404-407,2002)

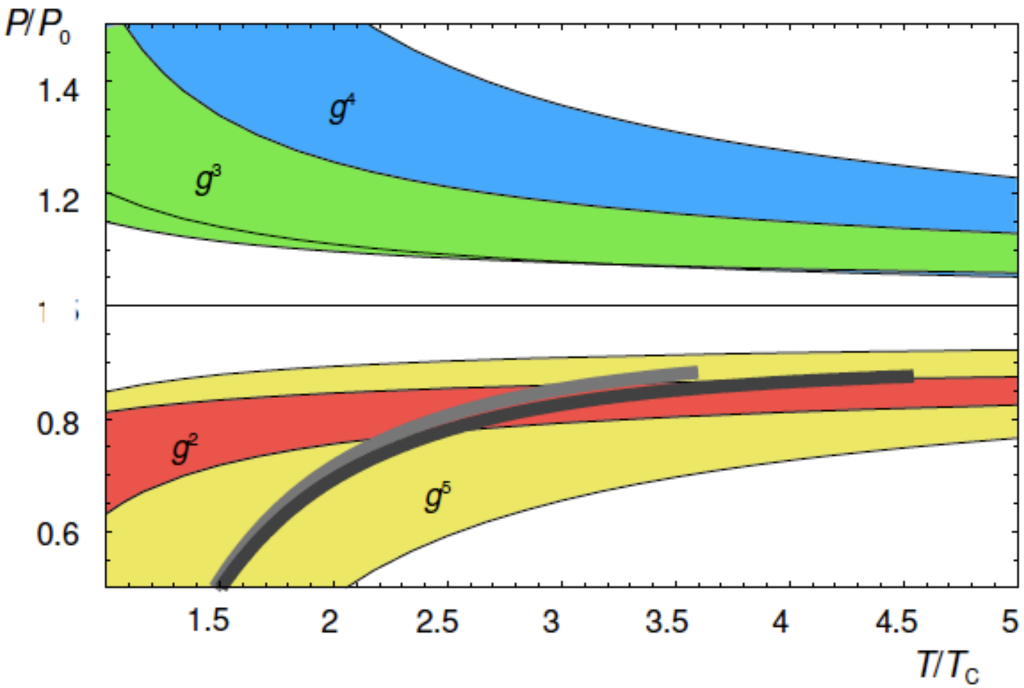
Weak coupling techniques (non perturbative)

-not limited to perturbation theory; in fact weak coupling techniques can be used to study non perturbative phenomena (many degenerate degrees of freedom, strong fields)

Effective theories- Dimensional reduction

Skeleton expansion (2PI formalism)

Perturbation theory is ill behaved



Perturbation theory:

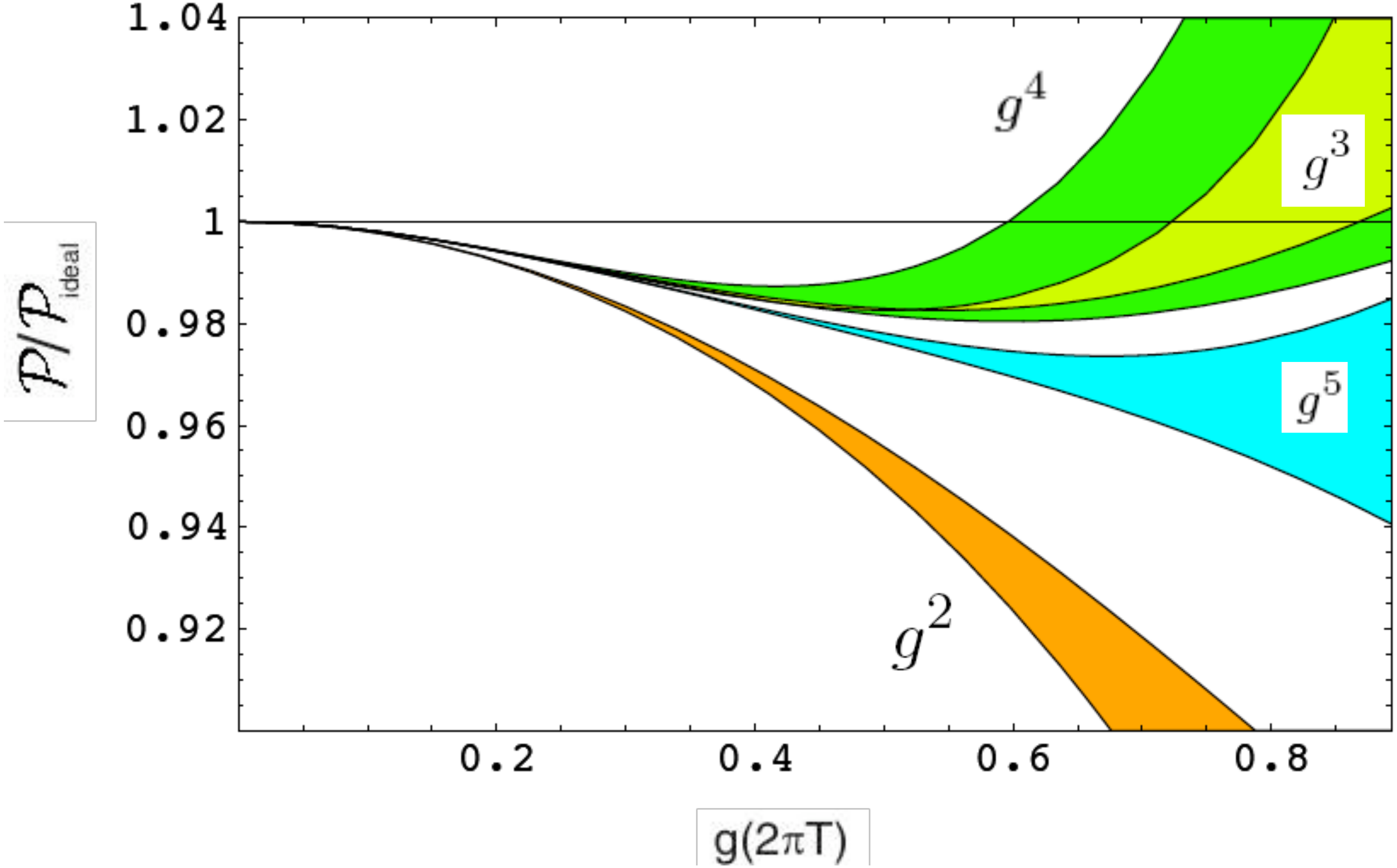
g^2: Shuryak; Chin (1978)
g^3: Kapusta (1979)
g^4 ln g: Toimela (1983)
g^4 : Arnold, Zhai (1994)
g^5 : Zhai, Kastening (1995), Braaten, Nieto (1996)
g^6 ln g: Kajantie, Laine, Rummukainen, Schröder (2002)
g^6 (partly): Di Renzo, Laine, Miccio, Schröder, Torrero (2006)

Lattice data: G. Boyd et al. (1996); M. Okamoto et al. (1999).

A two naive conclusion:

« weak coupling techniques are useless »

Similar difficulty in scalar theory



The bad convergence of Pert. Th. is not related to non abelian features of QCD

Effective theroy
Dimensional reduction

Integration over the hard modes $(gT \leq \Lambda_E \leq T)$

$$\mathcal{L}_E = \frac{1}{2}\mathrm{Tr}F_{ij}^2 + \mathrm{Tr}\left[D_i, A_0\right]^2 + m_E^2 \mathrm{Tr}A_0^2 + \lambda_E \left(\mathrm{Tr}A_0^2\right)^2 + \cdots$$

$$D_i = \partial_i - ig_E A_i$$

In leading order $g_E \approx g\sqrt{T}$ $\quad m_E \approx gT$ $\quad \lambda_E \approx g^4 T$

Integration over the soft modes $(g^2 T \leq \Lambda_M \leq gT)$

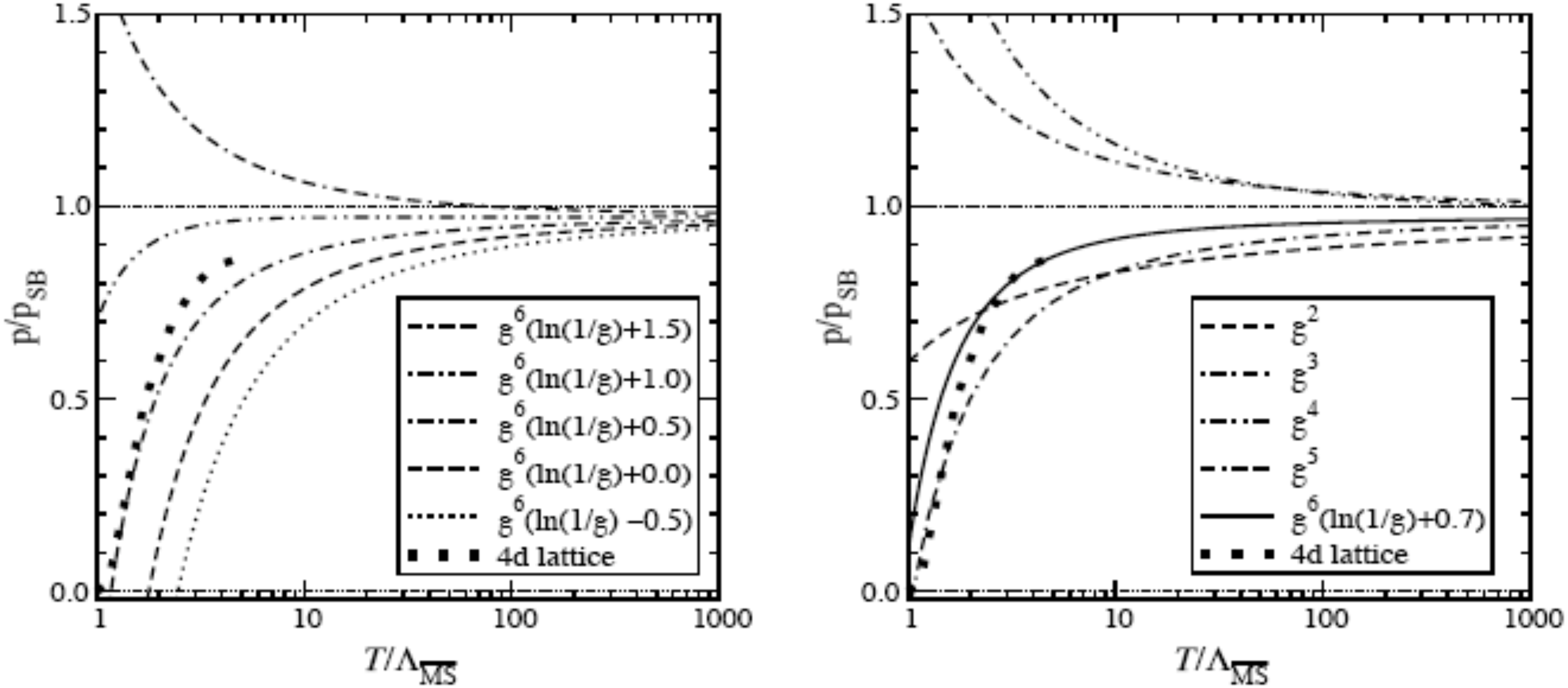
$$\mathcal{L}_M = \frac{1}{2}\mathrm{Tr}F_{ij}^2 + \cdots \qquad D_i = \partial_i - ig_M A_i \qquad g_M^2 \sim g_E^2$$

Non perturbative contribution

E. Braaten and A. Nieto, *Phys. Rev.* D 51 (1995) 6990 , *Phys. Rev.* D 53 (1996) 3421

$$p = T^4 \left[c_0 + c_2\, g^2 + c_3\, g^3 + (c_4' \ln g + c_4)\; g^4 + c_5\, g^5 + c_6\, g^6 \right]$$

$$c_6 = N_c^3 \, \frac{N_c^2 - 1}{(4\pi^4)} \left[\left(\frac{215}{12} - \frac{805}{768}\, \pi^2 \right) \ln \frac{1}{g} + 8\, \delta \right]$$



K. Kajantie, M. Laine, K. Rummukainen, and Y. Schröder, *Phys. Rev. Lett.* 86 (2001) 10 , *Phys. Rev.* D 65 (2002) 045008 , *Phys. Rev.* D 67 (2003) 105008 , *JHEP* 0304 (2003) 036

Skeleton expansion
Phi-derivable approximations
2PI formalism

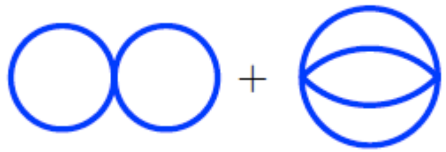
....

- J. M. Luttinger and J. C. Ward, PR 118, 1417 (1960)
- G. Baym, PR127,1391(1962)
- J. M. Cornwall, R. Jackiw, and E. Tomboulis, PR D10, 2428 (1974)

Pressure in terms of dressed propagators $P[G]$

$$-P = \int \frac{d^4k}{(2\pi)^4}\, n(\omega) \left(\mathrm{Im} \log G^{-1} - \mathrm{Im}\, \Pi\, G \right) + T\, \Phi[G]\, /\, V$$

$\Phi[G] =$

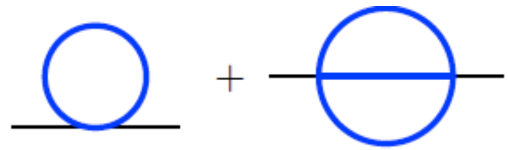


$+ \ldots$

2PI = 2–particle irreducible

$$G^{-1} = G^0 + \Pi[G]$$

$\Pi = 2\frac{\delta\Phi[G]}{\delta G} =$



$+ \ldots$

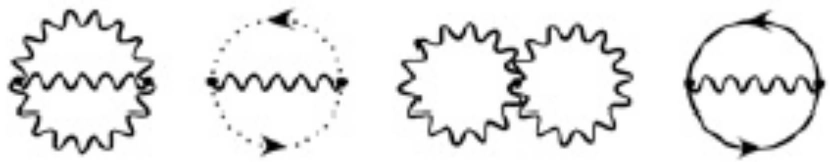
Stationarity property

$$\frac{\delta P}{\delta G} = 0$$

Entropy is simple!

$$S = \frac{dP}{dT} = \left.\frac{dP}{dT}\right|_G$$

THE TWO-LOOP ENTROPY



$$S = -\int \frac{d^4p}{(2\pi)^4} \frac{\partial N}{\partial T} \left\{ \mathrm{Im} \ln D^{-1} - \mathrm{Im}\Pi \, \mathrm{Re} D \right\}$$

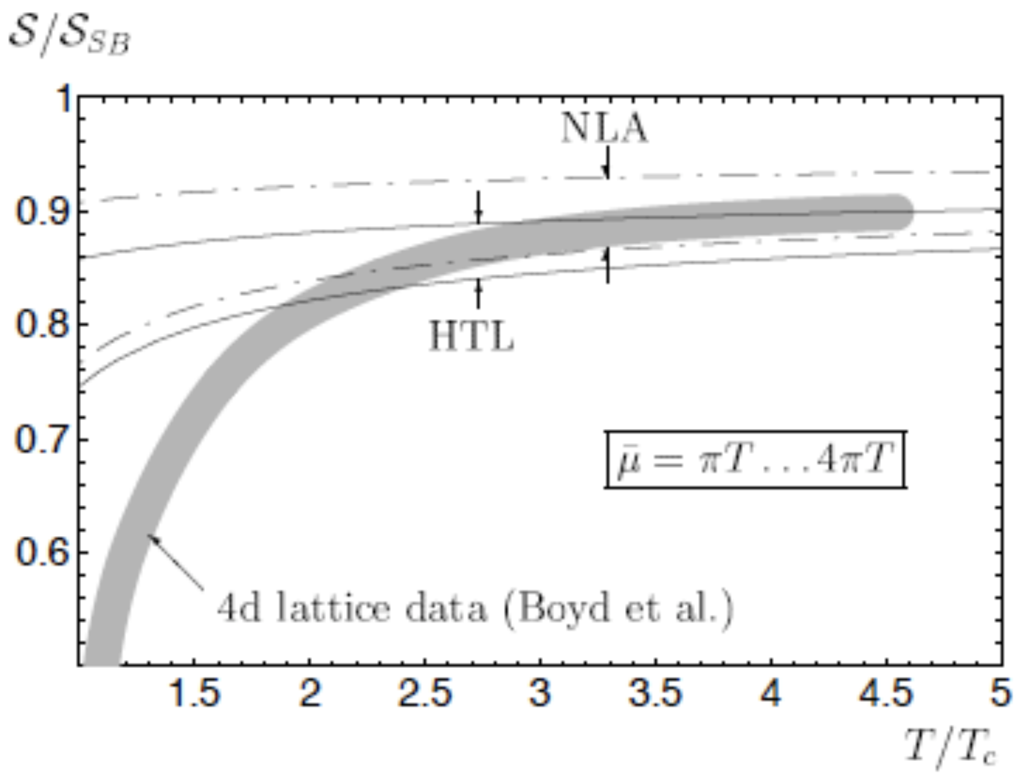
-effectively a one-loop expression; residual interactions start at order three-loop

-manifestly ultraviolet finite

-perturbatively correct up to order g^3

State of the art

pure-glue SU(3) Yang-Mills theory

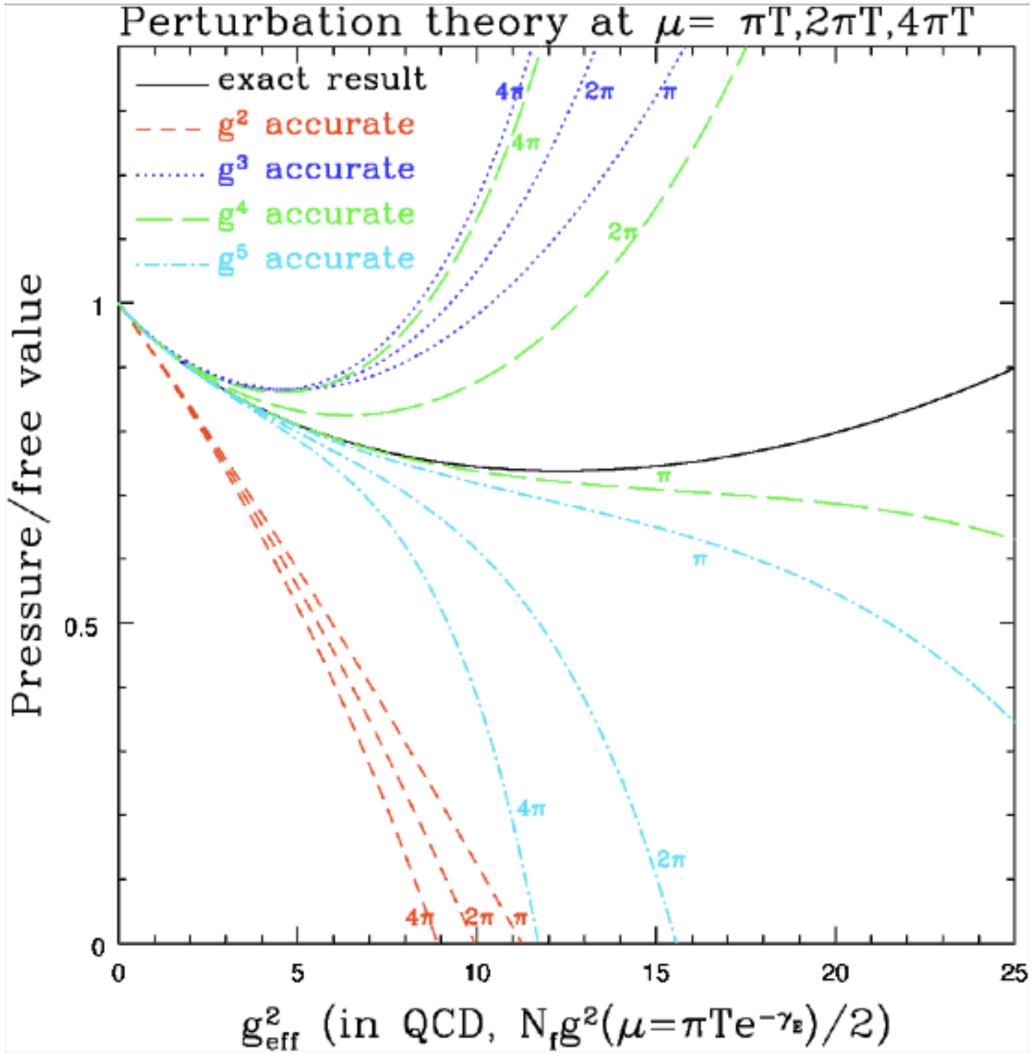


from J.-P. B., E. Iancu, A. Rebhan: Nucl.Phys.A698:404-407,2002

- J.-P. B., E. Iancu, A. Rebhan: Phys.Rev.D63:065003,2001
- G. Boyd *et al.*, Nucl. Phys. B469, 419 (1996).

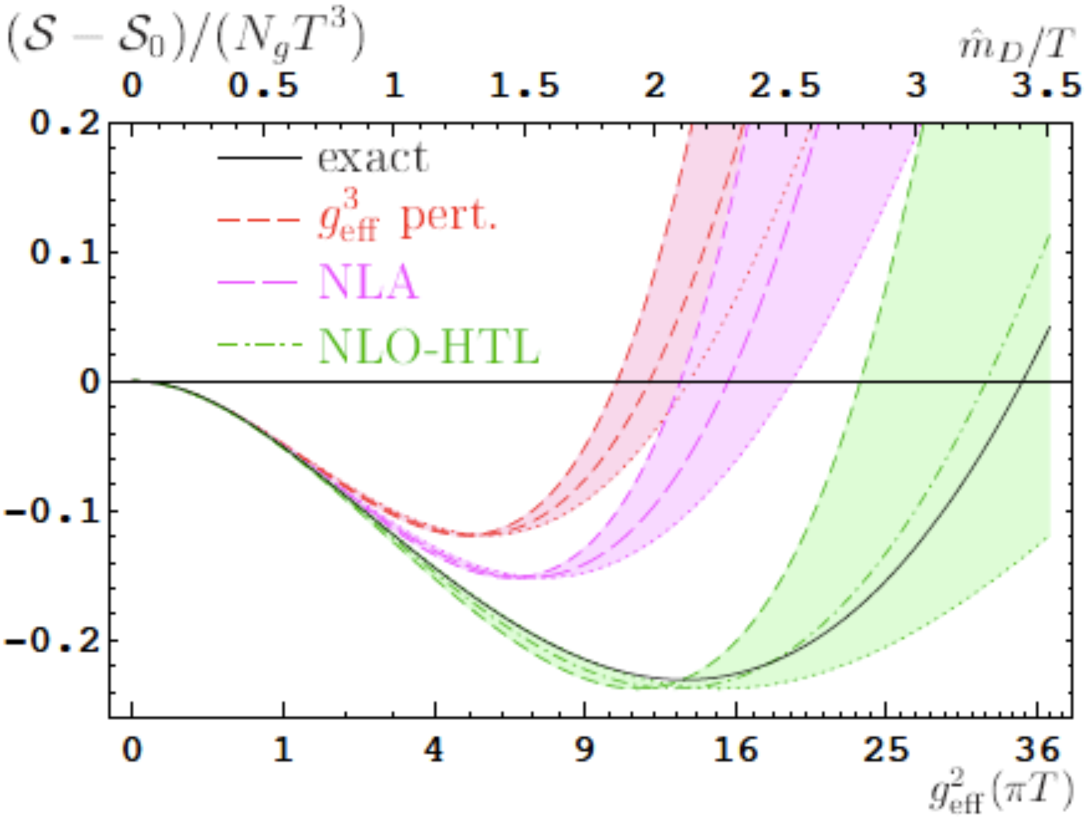
Large N_f

Pressure at Large N_f



Moore, JHEP 0210
A.Ipp, Moore, Rebhan, JHEP 0301

Entropy in large N_f



Good agreement, even at large coupling

J.-P. B., A.Ipp, A. Rebhan, U. Reinosa,
hep-ph/0509052

Strong coupling
Comparison with SSYM

Hard-thermal-loop entropy of supersymmetric Yang-Mills theories

J.-P. B, E. Iancu, U. Kraemmer and A. Rebhan, hep-ph/0611393

Entropy formula

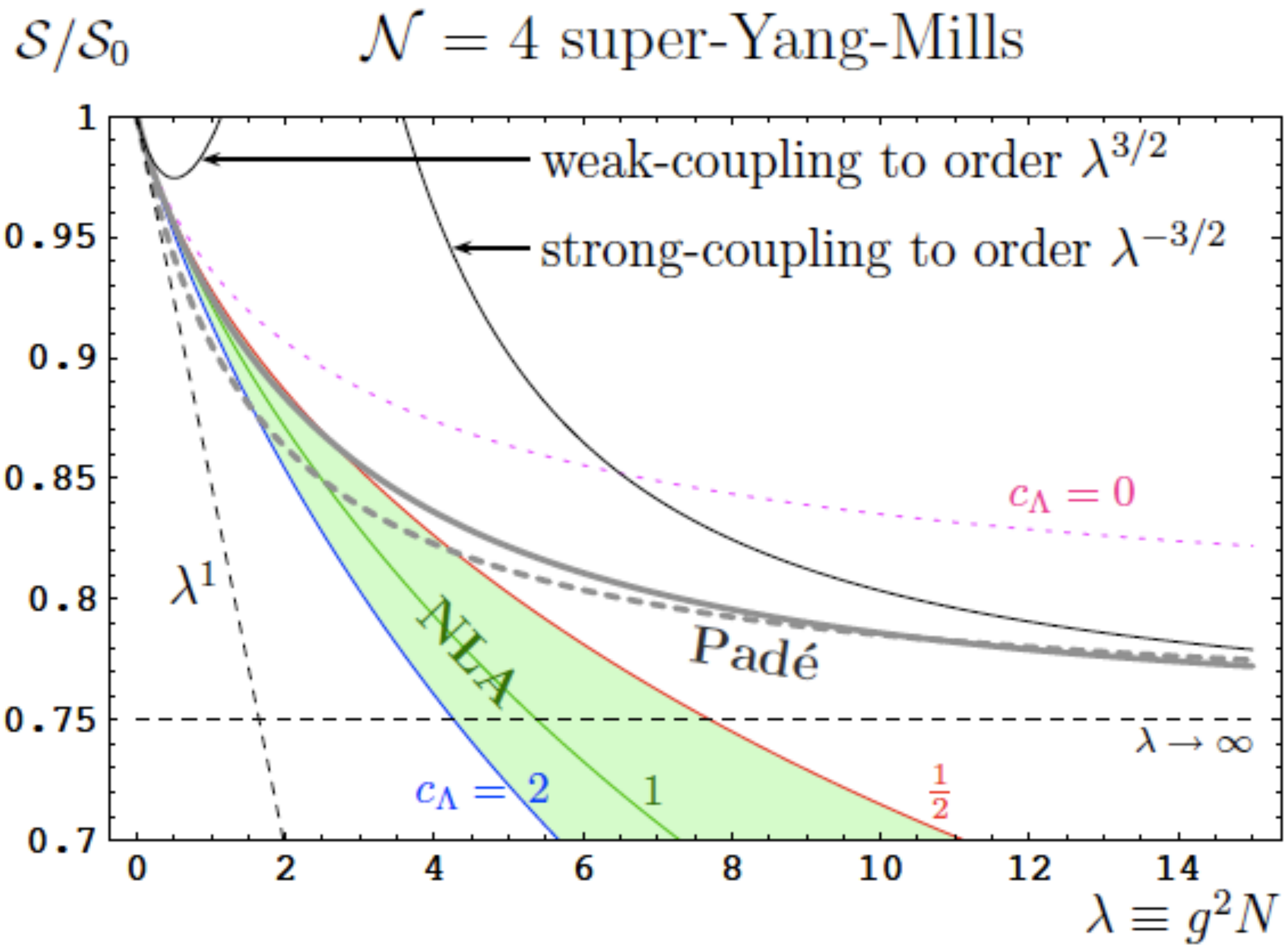
$$\mathcal{S} = -\operatorname{tr}\int \frac{d^4k}{(2\pi)^4}\frac{\partial n(\omega)}{\partial T}\Big[\operatorname{Im}\log D^{-1} - \operatorname{Im}\Pi\operatorname{Re}D\Big]$$
$$-2\operatorname{tr}\int \frac{d^4k}{(2\pi)^4}\frac{\partial f(\omega)}{\partial T}\Big[\operatorname{Im}\log S^{-1} - \operatorname{Im}\Sigma\operatorname{Re}S\Big] + \mathcal{S}'$$

Leading order correction

$$\mathcal{S}_2 = -T\left\{\sum_b \frac{m^2_{\infty(b)}}{6} + \sum_f \frac{m^2_{\infty(f)}}{12}\right\}$$

At strong coupling (AdS/CFT)

$$\frac{S}{S_0} = \frac{3}{4} + \frac{45}{32}\varsigma(3)\frac{1}{\lambda^{3/2}} \qquad \left(\lambda \equiv g^2 N_c\right)$$



$\mathcal{S}/\mathcal{S}_0$
$\mathcal{N} = 4$ super-Yang-Mills
weak-coupling to order $\lambda^{3/2}$
strong-coupling to order $\lambda^{-3/2}$
$c_\Lambda = 0$
λ^1
NLA
Padé
$\lambda \to \infty$
$c_\Lambda = 2$
1
$\frac{1}{2}$
1
0.95
0.9
0.85
0.8
0.75
0.7
0
2
4
6
8
10
12
14
$\lambda \equiv g^2 N$

Insights from the functional renormalization group (scalar theory)

J.-P. B, A. Ipp, R. Mendez-Galain, N. Wschebor

(Nucl.Phys.A784:376-406,2007)

and work in progress

Scales, fluctuations and degrees of freedom in the quark-gluon plasma

$$\langle(\partial A)^2\rangle \qquad g^2\langle A^4\rangle \sim g^2\langle A^2\rangle^2$$

- **Hard degrees of freedom: the plasma particles**

$$k \sim T \qquad \langle A^2\rangle_T \sim T^2 \qquad \langle(\partial A)^2\rangle_T \sim T^4 \qquad g^2\langle A^2\rangle_T^2 \sim g^2T^4$$

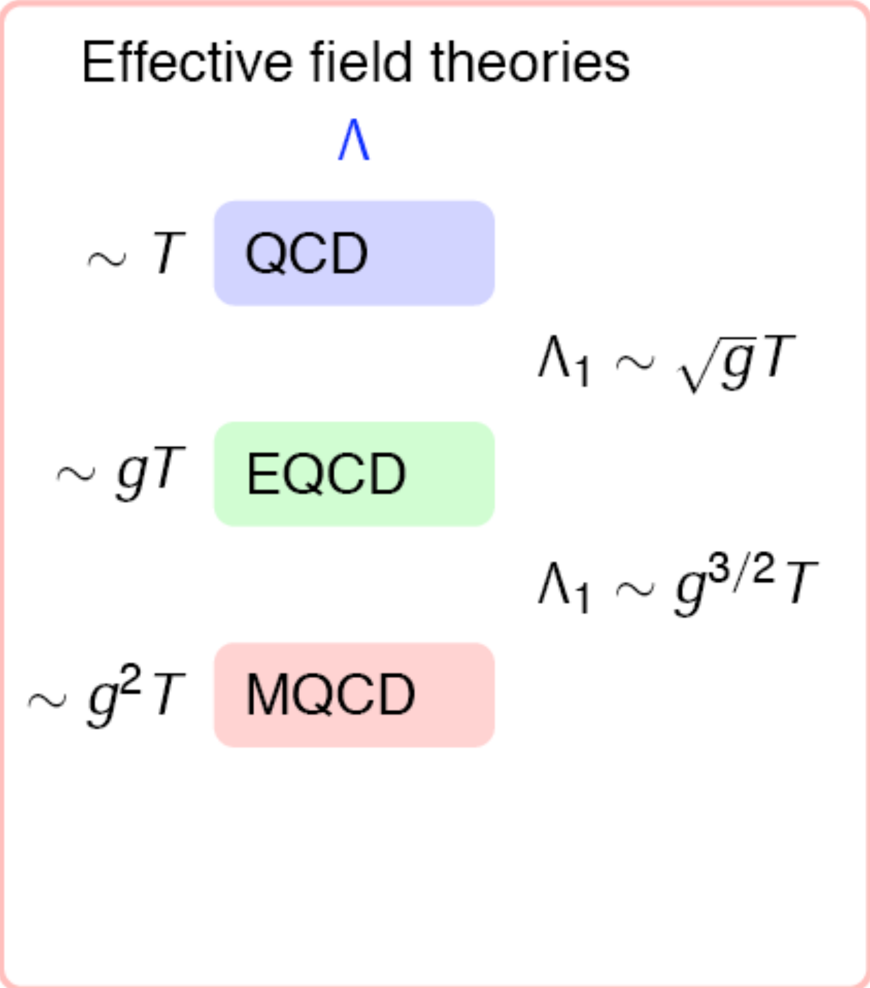
- **Soft degrees of freedom, collective modes**

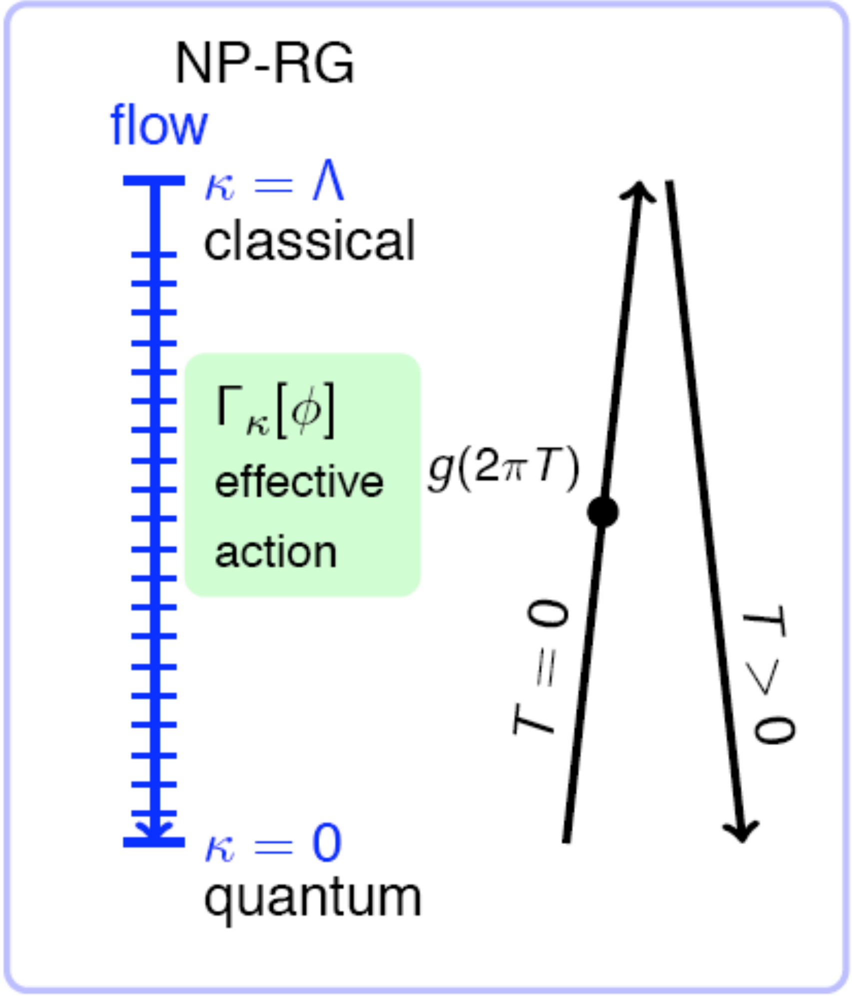
$$k \sim gT \qquad \langle A^2\rangle_{gT} \sim gT^2 \qquad \langle(\partial A)^2\rangle_{gT} \sim g^3T^4 \qquad g^2\langle A^2\rangle_{gT}^2 \sim g^4T^4$$

- **Ultrasoft degrees of freedom, unscreened magnetic fluctuations**

$$k \sim g^2T \qquad \langle A^2\rangle_{g^2T} \sim g^2T^2 \qquad \langle(\partial A)^2\rangle_{g^2T} \sim g^6T^4 \qquad g^2\langle A^2\rangle_{g^2T}^2 \sim g^6T^4$$

Non perturbative renormalization group





Courtesy, A. Ipp

Non perturbative renormalization group

effective action

$$\partial_t \Gamma_\kappa[\phi] = \frac{1}{2} T \sum_n \int \frac{d^3\mathbf{q}}{(2\pi)^3} \partial_t R_\kappa(Q) G_\kappa(Q;\rho)$$

=

(C. Wetterich 1993)

propagator $G_\kappa^{-1}(Q;\rho) \equiv \Gamma_\kappa^{(2)}(\mathbf{Q},-\mathbf{Q};\rho) + R_\kappa(Q)$

regulator

four-momentum vectors:

$\mathbf{Q} = (\omega_n, \mathbf{q})$

$Q = |\mathbf{Q}|$

Matsubara frequency $\omega_n = 2\pi n T$

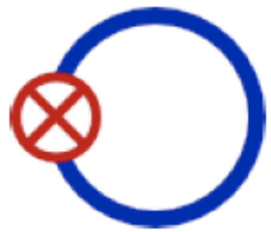
momentum derivative: $\partial_t \equiv \kappa\, \partial_\kappa$

scalar field: $\rho \equiv \frac{1}{2}\phi^2$

Local potential approximation

$$\Gamma_k^{LPA}[\phi] = \int d^d x \left\{ \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + V_k(\phi) \right\}$$

$$\partial_t V_\kappa(\rho) = \frac{1}{2} T \sum_n \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \partial_t R_\kappa(Q) G_\kappa(Q;\rho) \quad =$$



propagator $G_\kappa^{-1}(Q;\rho) \equiv \Gamma_\kappa^{(2)}(\mathbf{Q}, -\mathbf{Q}; \rho) + R_\kappa(Q)$

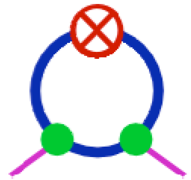
$$\Gamma_\kappa^{(2)}(\mathbf{Q}, -\mathbf{Q}; \rho) = Q^2 + m_\kappa^2(\rho) \quad \text{with } m_\kappa^2(\rho) \equiv \frac{\partial^2 V_\kappa}{\partial \phi^2}$$

Beyond the local potential approximation

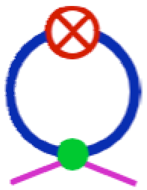
(J.-P. B, R. Mendez-Galain, N. Wschebor, Phys.Lett.B632:571-578,2006)

$$\partial_t \Gamma_\kappa^{(2)}(\mathbf{P},-\mathbf{P};\rho) = T\sum_n \int \frac{d^3\mathbf{q}}{(2\pi)^3}\, \partial_t R_\kappa(Q)\, G_\kappa^2(Q;\rho)$$

$$\times\Big\{ \Gamma_\kappa^{(3)}(\mathbf{P},\mathbf{0},-\mathbf{P}-\mathbf{0};\phi)\, G_\kappa(\mathbf{Q}+\mathbf{P};\rho)\, \Gamma_\kappa^{(3)}(-\mathbf{P},\mathbf{P}+\mathbf{0},-\mathbf{0};\phi)$$



$$-\frac{1}{2}\Gamma_\kappa^{(4)}(\mathbf{P},-\mathbf{P},\mathbf{0},-\mathbf{0};\phi)\Big\}$$



close equations with:

$$\Gamma_\kappa^{(3)}(\mathbf{P},-\mathbf{P},\mathbf{0};\phi) = \frac{\partial \Gamma_\kappa^{(2)}(P,\rho)}{\partial \phi}, \qquad \Gamma_\kappa^{(4)}(\mathbf{P},-\mathbf{P},\mathbf{0},\mathbf{0};\phi) = \frac{\partial^2 \Gamma_\kappa^{(2)}(P,\rho)}{\partial \phi^2}$$

Application to critical O(N) model

(from F. Benitez, J.-P. Blaizot, H. Chate, B. Delamotte, R. Mendez-Galain, N. Wschebor, arXiv:0901.0128 [cond-mat.stat-mech])

TABLE I: Coefficient c and critical exponents for the $O(N)$ models for $d = 3$.

N	BMW				6-loop						Monte-Carlo			
	η	ν	ω	c	η	ν	ω	c	Ref.[a]		η	ν	c	Ref.[a]
0	0.034	0.588	0.80		0.0285(6)	0.5881(8)	0.803(3)		[11]		0.030(3)	0.5872(5)		[12]
1	0.039	0.630	0.78	1.15	0.0335(6)	0.6303(8)	0.792(3)	1.07(10)	[11][15]		0.0368(2)	0.63020(12)	1.09(9)	[20][14]
2	0.041	0.672	0.78	1.37	0.0349(8)	0.6704(7)	0.784(3)	1.27(10)	[11][15]		0.0381(2)	0.6717(1)	1.32(2)	[21][13]
3	0.040	0.709	0.75	1.50	0.0350(8)	0.7062(7)	0.783(3)	1.43(11)	[11][15]	non	0.0375(5)	0.7112(5)		[22]
4	0.038	0.738	0.74	1.63	0.0350(45)	0.741(6)	0.774(20)	1.54(11)	[23][15]		0.0365(10)	0.749(2)	1.60(10)	[24][14]
10	0.022	0.860	0.81		0.024	0.859			[17]					
100[b]	0.0023	0.985	0.99	2.36	0.0027	0.989			[18]					

- **LPA**: 3D regulator possible.

$$R_\kappa(\omega_n, \mathbf{q}) = (\kappa^2 - \mathbf{q}^2)\theta(\kappa^2 - \mathbf{q}^2)$$

Litim regulator

- analytical treatment of Matsubara sums possible

$$T\sum_n \frac{1}{-(i\,\omega_n)^2 + \omega_\kappa^2} = \frac{1 + 2\mathrm{n}(\omega_\kappa)}{2\,\omega_\kappa}$$

- But: does not respect Euclidean invariance; frequency is not regulated

- **BMW**: Euclidean method.
- 4D regulator more appropriate

$$R_\kappa(Q) = Z_\kappa \kappa^2 r(Q/\kappa)$$

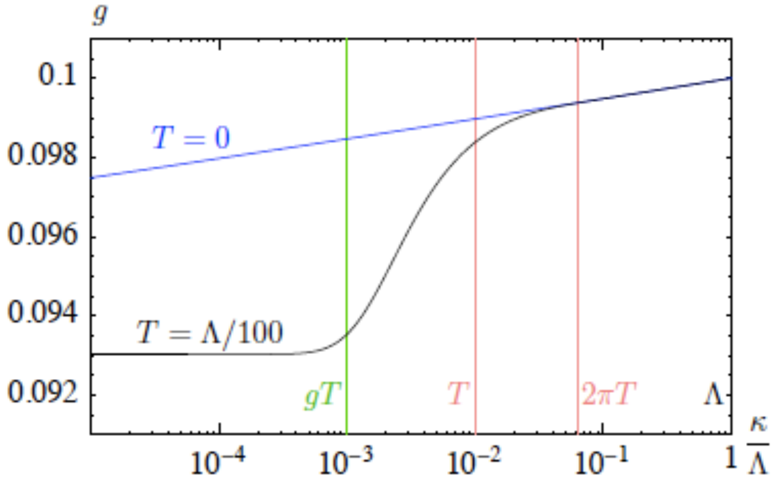
$$r(q) = \frac{\alpha q^2}{e^{q^2} - 1}$$

Exponential regulator

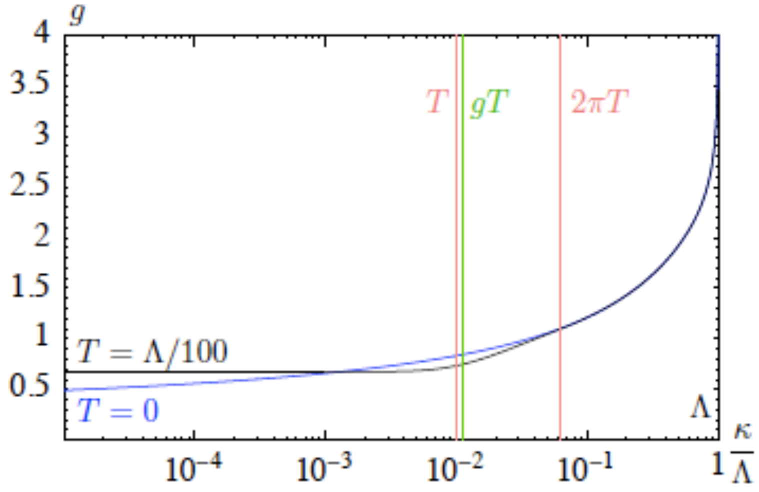
- Need to sum over Matsubara frequencies.

Flow of coupling constant (LPA)

weak coupling

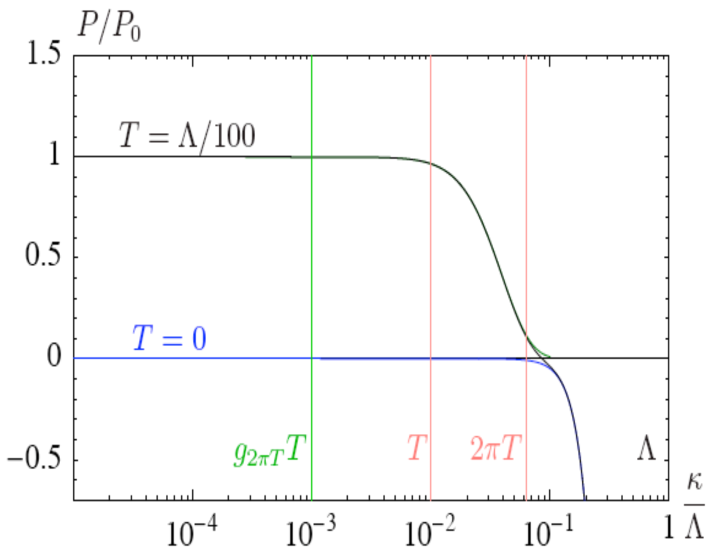


strong coupling

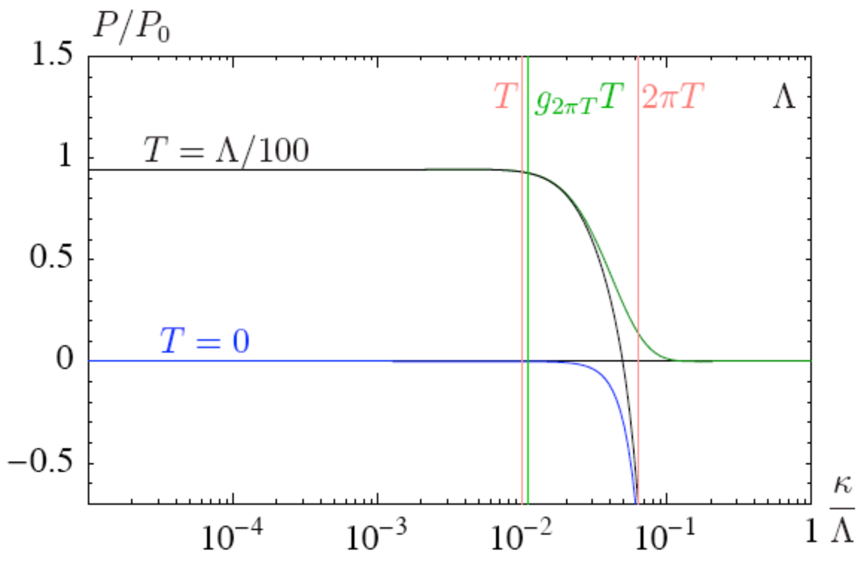


Flow of pressure (LPA)

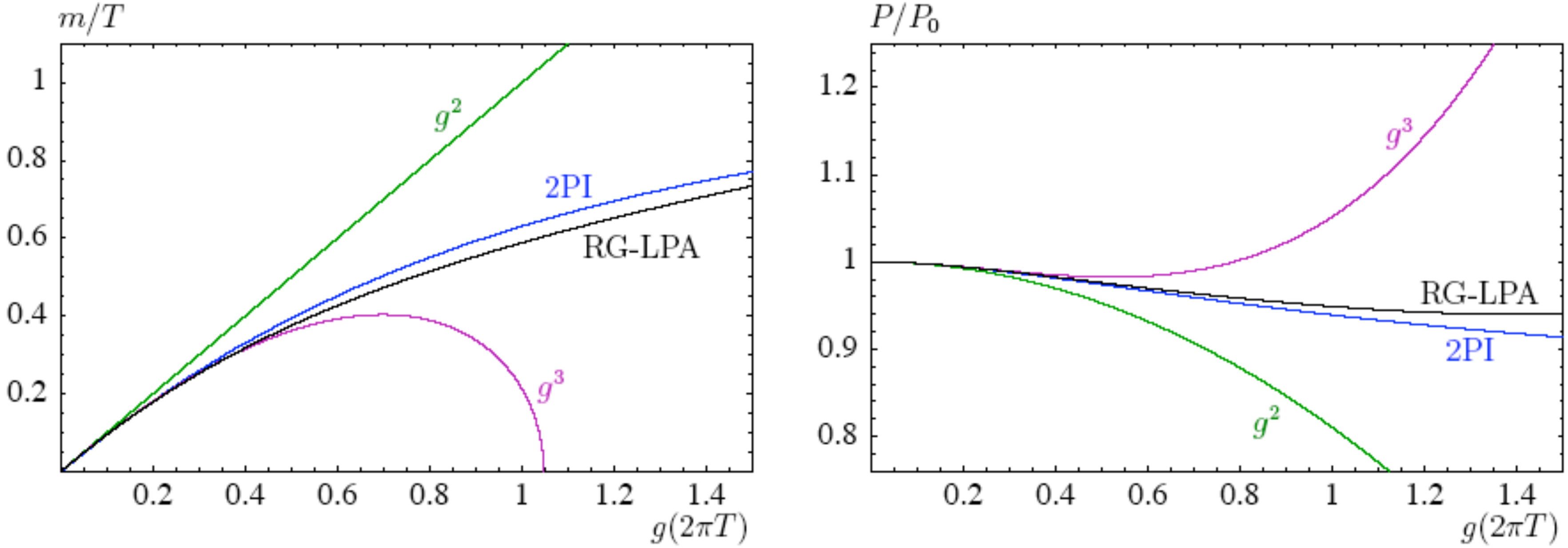
weak coupling



strong coupling

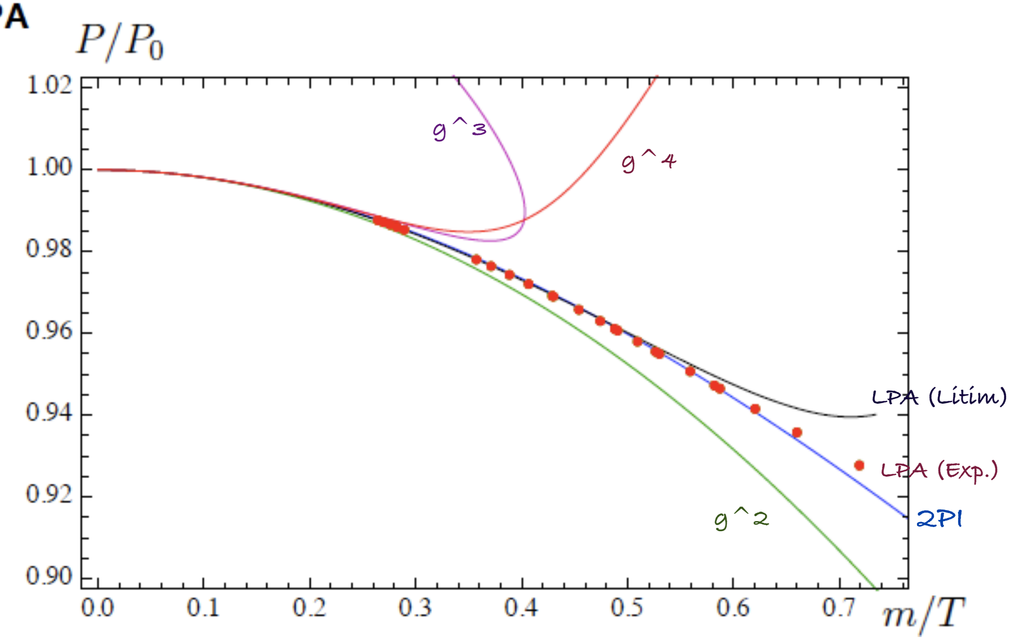


Mass and pressure as function of $g(\kappa = 2\pi T)$

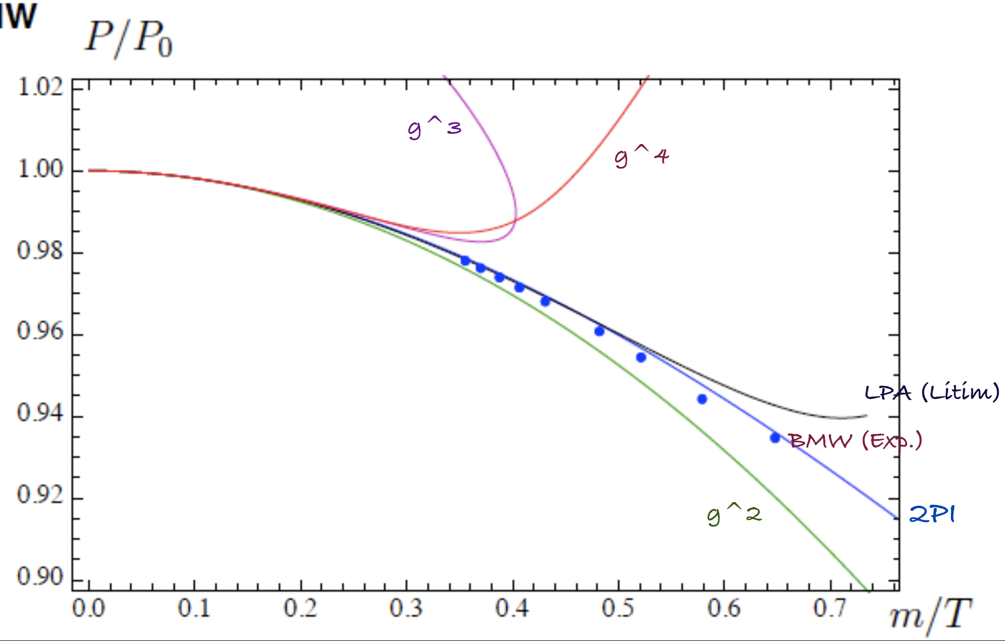


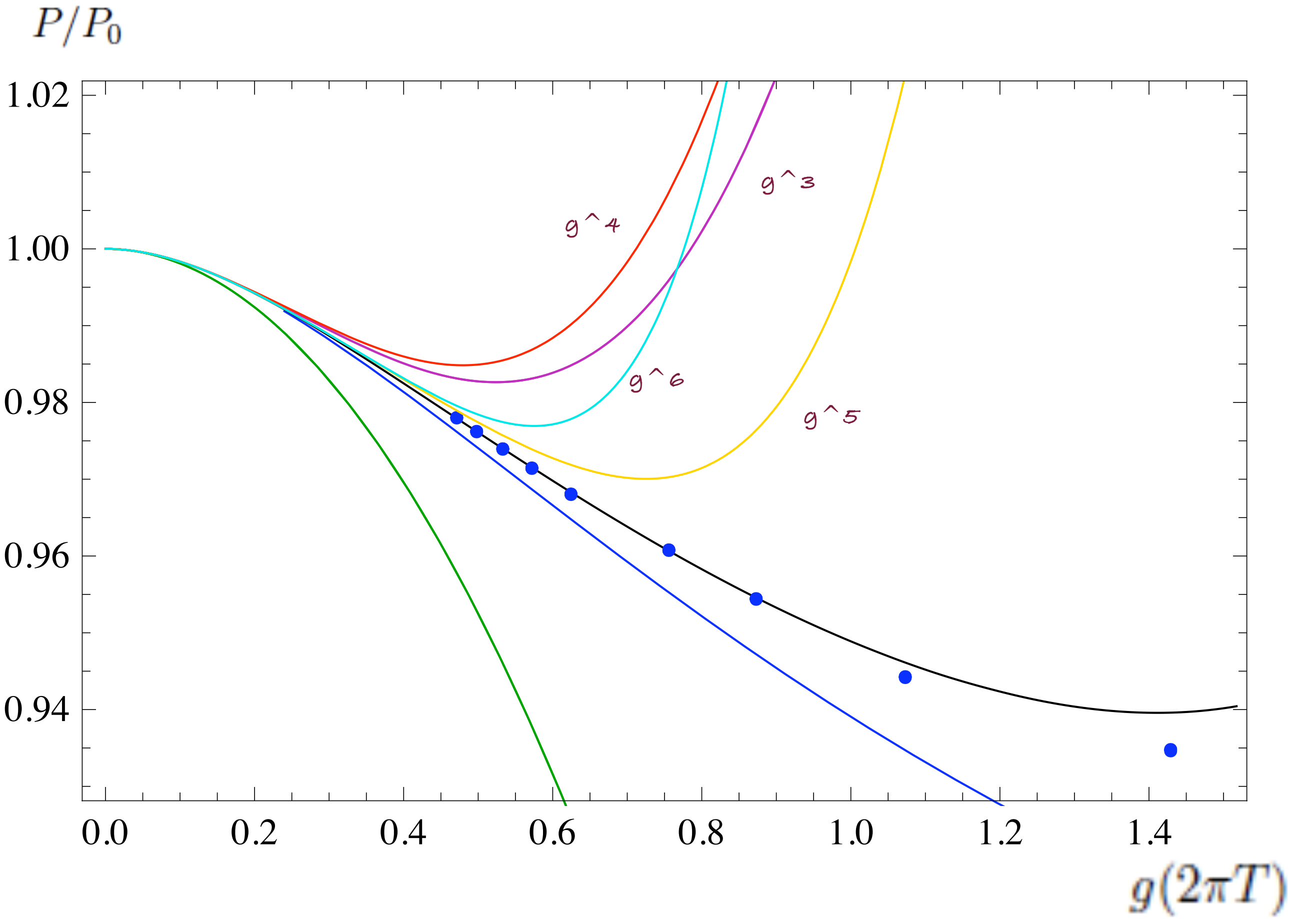
Pressure versus m/T

- **LPA**



- **BMW**





(g^6 from A. Gynther, et al ,hep-ph/0703307)

Summary

- While strict perturbation theory is meaningless, weak coupling methods (involving resummations) provide an accurate description of the QGP for

$$T \geq 3T_c$$

- The functional renormalization group confirms results obtained with various resummations (at least in scalar case)