On the chiral phase transition and the relation to quark confinement

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in collaboration with:
Jens Braun, Florian Marhauser, Jan M. Pawlowski

hep-ph/0908.0008 & diploma thesis (LMH) 2008

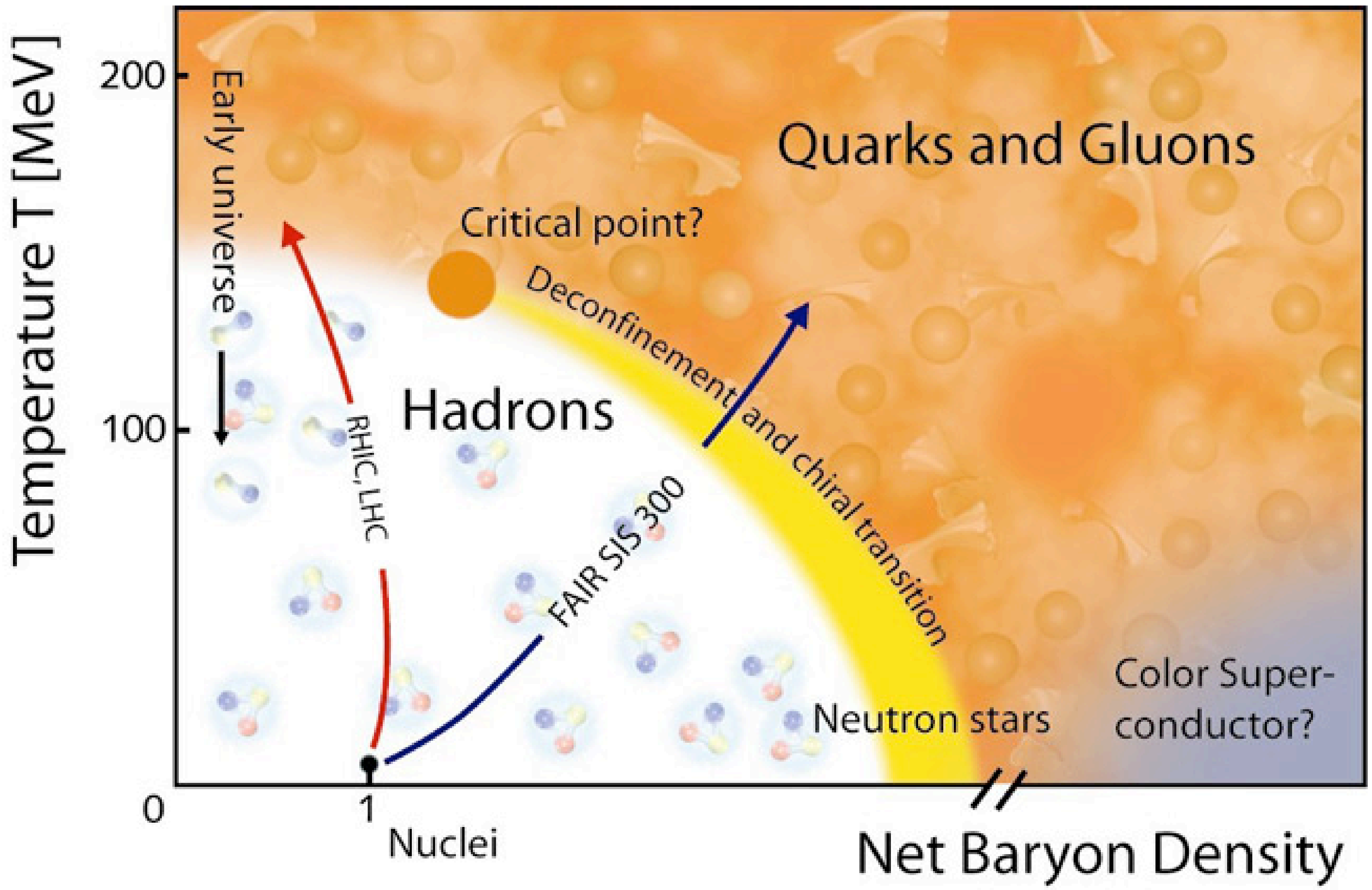
Quarks, Hadrons, and the Phase Diagram of QCD
St. Goar 2009

Outline

1. Motivation & Method
2. On the chiral phase transition
   at finite μ & $T = 0$
3. On the relation of quark confinement &
   chiral symmetry breaking
4. Summary & Outlook

Motivation & Method

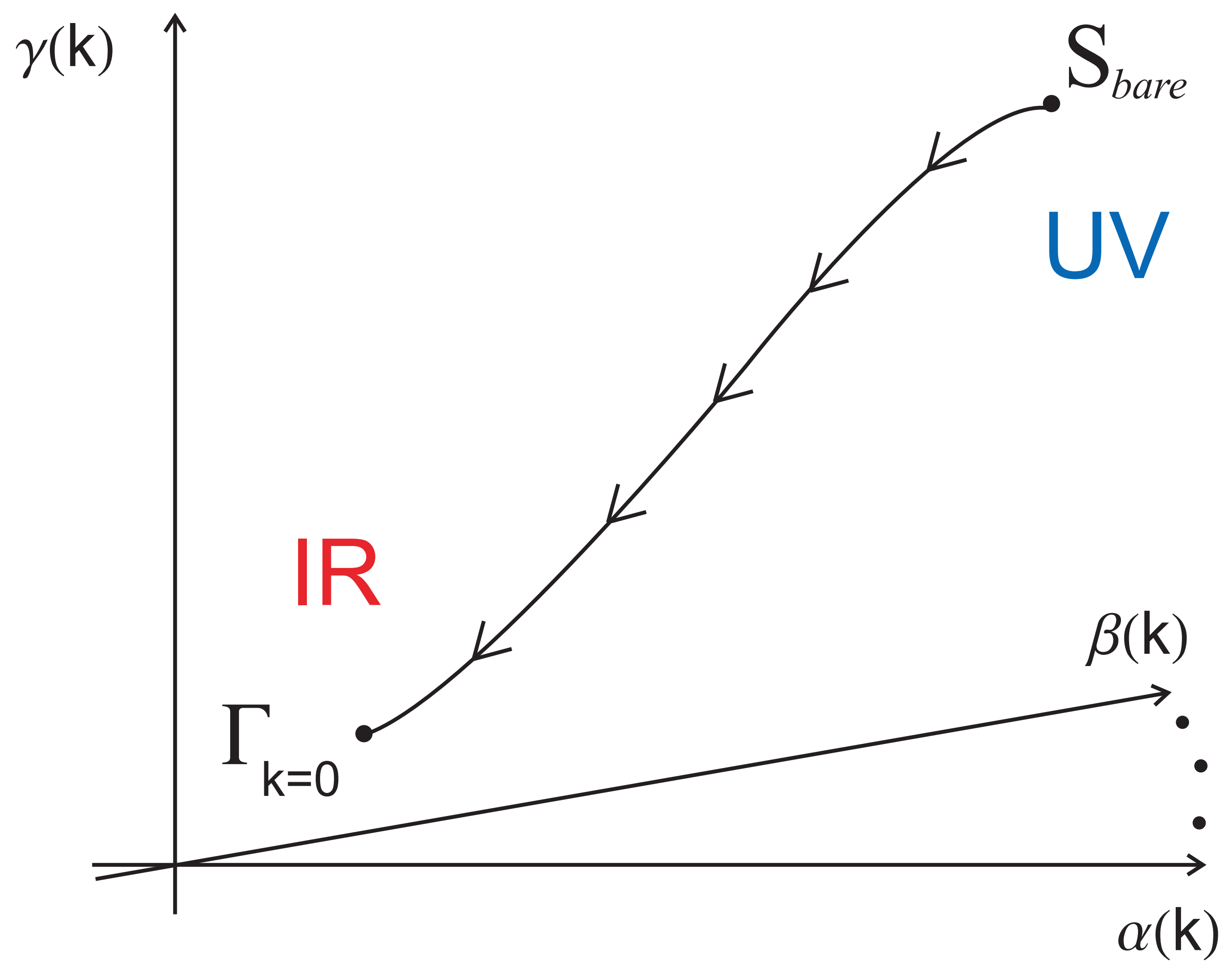
The QCD phase diagram

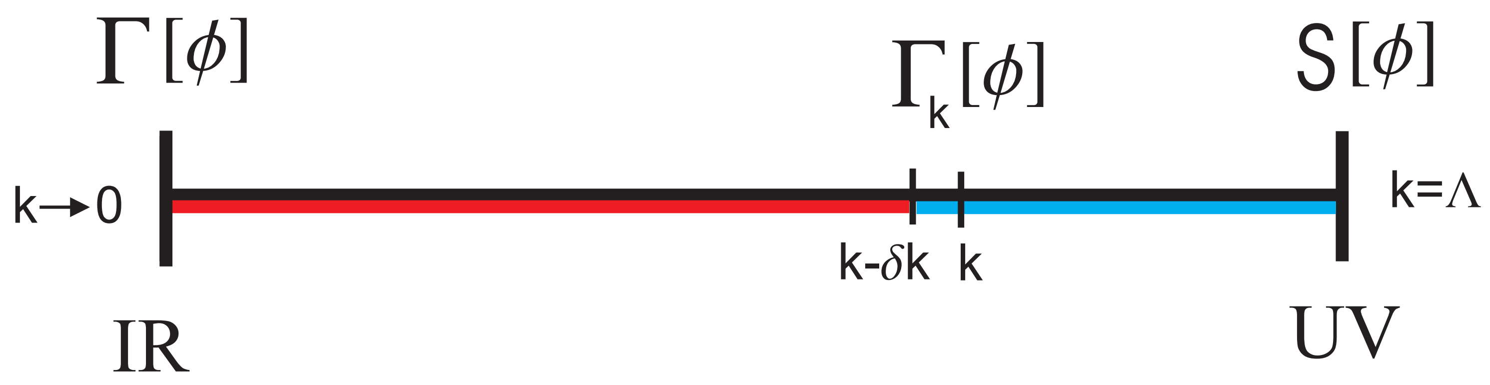


credits: GSI Darmstadt

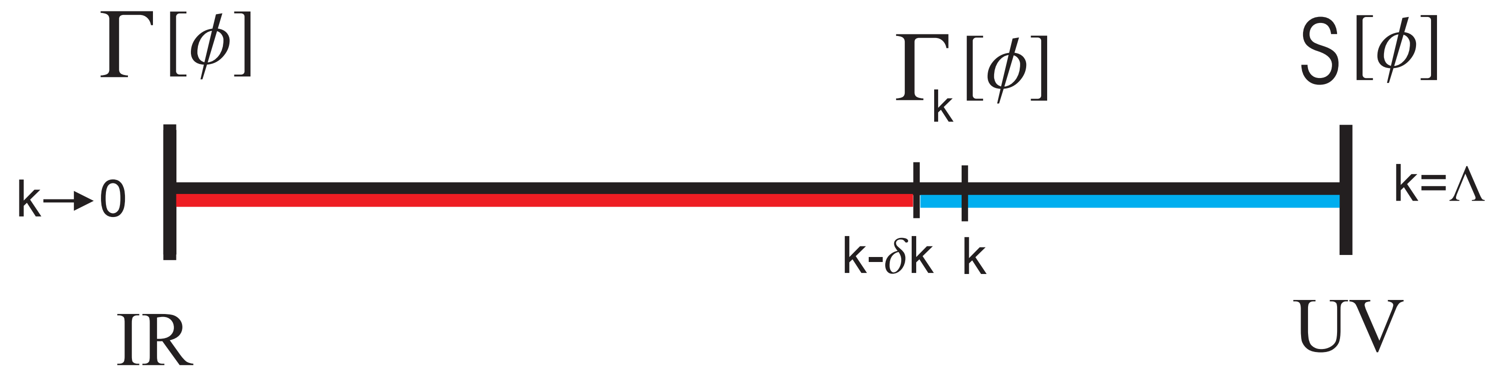
The flow of the effective action

Theory space:



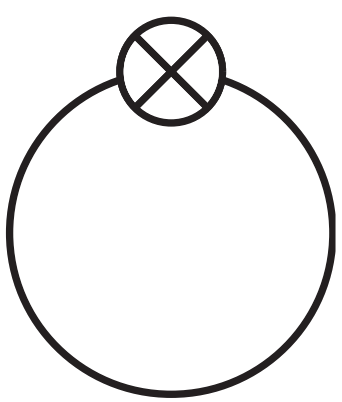


The flow equation



Flow equation for the effective action:

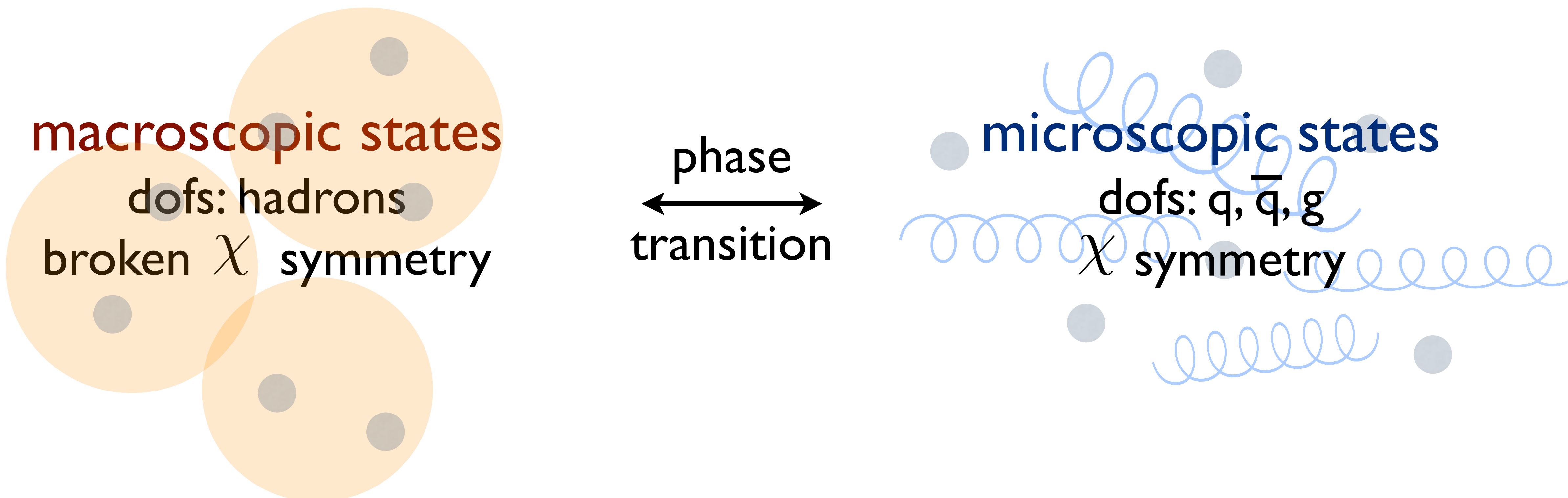
$$k\frac{\mathrm{d}}{\mathrm{d}k}\Gamma_k[\phi] = \tfrac{1}{2}\mathrm{Tr}$$



Wetterich's equation, '93

On the chiral phase transition: finite μ & $T = 0$

Chiral phase transition



symmetry of matter sector of QCD for $m_q = 0$

order parameter: chiral condensate $\langle \bar{\psi}\psi \rangle$

$$\langle \bar{\psi}\psi \rangle = \begin{cases} 0 & T > T_{c,\chi} \\ > 0 & T < T_{c,\chi}. \end{cases}$$

Approximation

finite μ and $T = 0$, $N_f = 1$, Landau gauge

$$\Gamma_{QCD}^{N_f=1} = \int d^4x \Big\{ \underbrace{\bar{\psi}\,(\mathrm{i}\,\not{D}[A] + \mathrm{i}\gamma_0\mu)\,\psi}_{\text{Dirac}} + \underbrace{\frac{\bar{\lambda}_\psi}{2}[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2]}_{\text{4-fermion}}$$

$$+ \underbrace{\frac{1}{2}Z_\phi\,(\partial_\mu\Phi)^2 + U(\Phi^2)}_{\text{scalar}} + \underbrace{\frac{\bar{h}}{\sqrt{2}}(\bar{\psi}(\vec{\tau}\cdot\Phi)\psi)}_{\text{Yukawa}}$$

$$+ \underbrace{\frac{Z_{A_{QCD}}}{4}F^a_{\mu\nu}F^a_{\mu\nu}}_{\text{Yang-Mills}} + \Gamma_{\text{gauge}} \Big\}$$

Approximation

finite μ and $T=0,\; N_f=1$

dynamical hadronisation

$$\Gamma_{QCD}^{N_f=1} = \int d^4x \left\{ \underbrace{\bar\psi\,(\mathrm{i}\,\not{D}[A] + \mathrm{i}\gamma_0\mu)\,\psi}_{\text{Dirac}} + \underbrace{\frac{\bar\lambda_\psi}{2}[(\bar\psi\psi)^2 - (\bar\psi\gamma_5\psi)^2]}_{\text{4-fermion}} \right.$$

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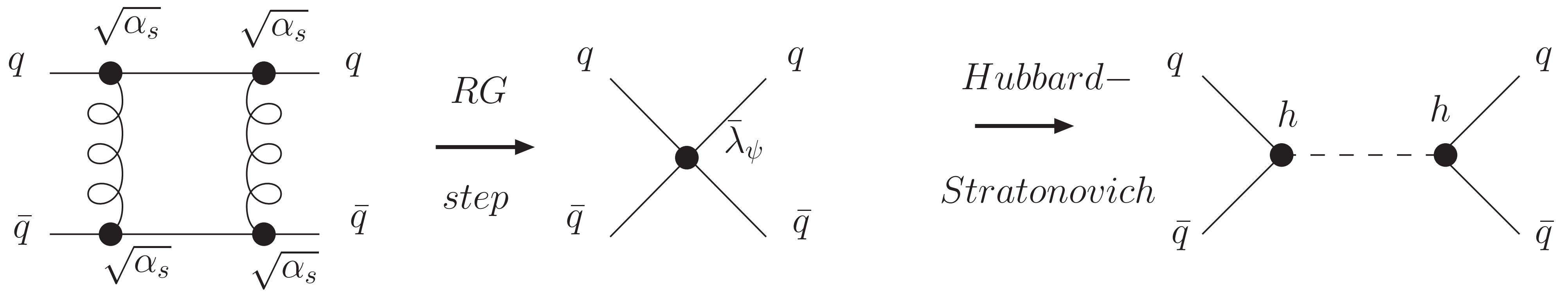
Dynamical hadronisation



Hubbard-Stratonovich transformation:

$$\bar{\lambda}_\psi(\bar{\psi}\psi)^2 = h\bar{\psi}\psi\sigma - \frac{1}{2}m^2\sigma^2 \qquad \rightarrow \;\bar{\lambda}_\psi = -\frac{h^2}{2m^2}$$

α_s at finite μ & T=0

$$\alpha_s(k) = \frac{g^2}{4\pi} \frac{1}{Z_{A_{QCD}}(k^2) Z_C^2(k^2)}$$

α_s at finite μ & T=0

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with $$Z_{A_{QCD}}(k^2) = Z_{A_{YM}}|_{p^2=k^2} + \Delta Z_{A_Q}(m, \mu, k)$$

α_s at finite μ & T=0

C.S. Fischer, A. Maas, J. M. Pawlowski, '08
J. M. Pawlowski '09

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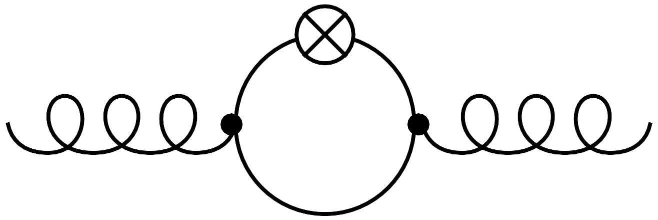

J. Braun '08
LMH diploma thesis '08

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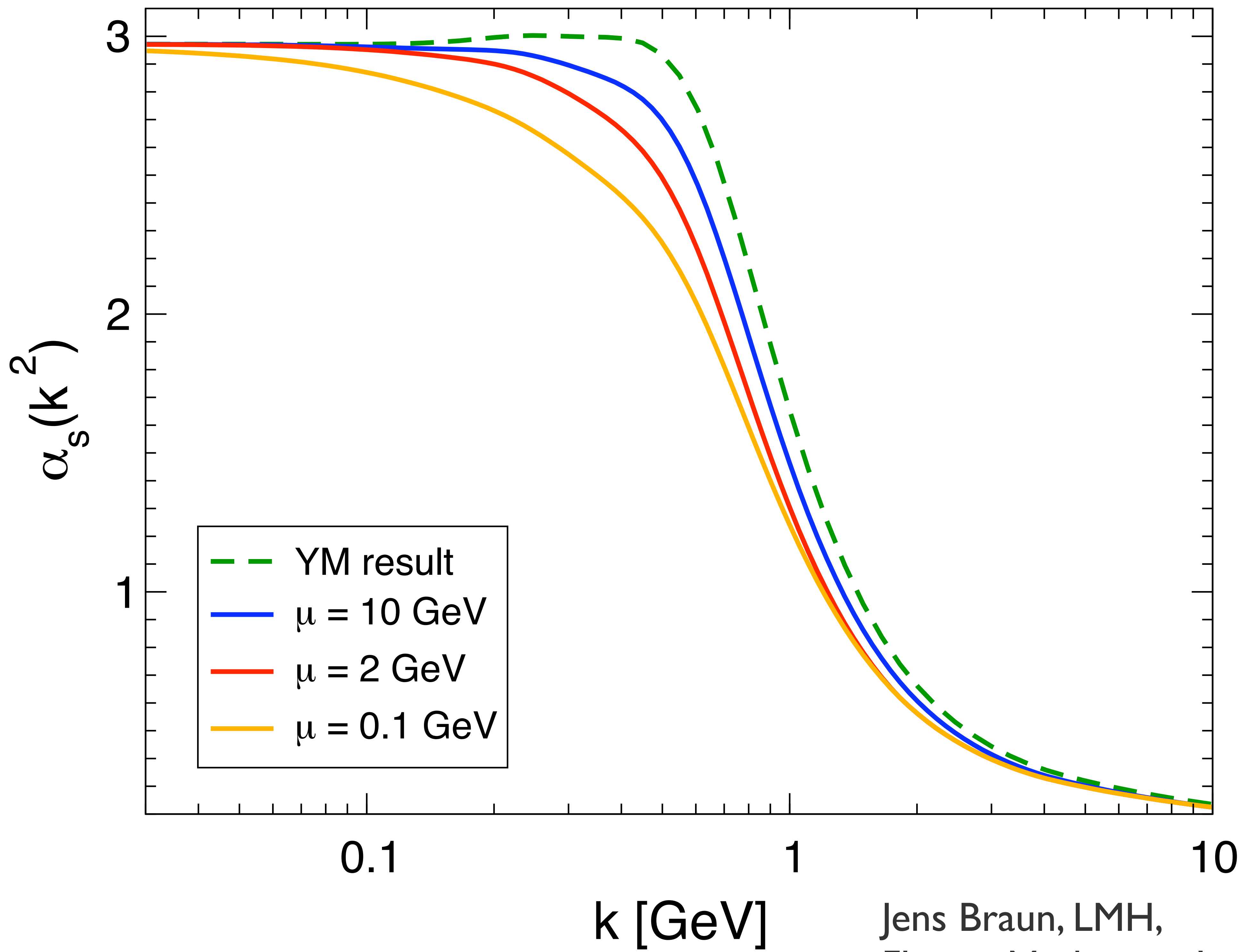
Initial conditions set at Z boson mass (90 GeV):

$$\Gamma_{QCD}^{N_f=1} = \int d^4x \left\{ \bar{\psi} \left(\mathrm{i}\, \not{D}[A] + \mathrm{i}\gamma_0 \mu \right) \psi + \frac{Z_{A_{QCD}}}{4} F^a_{\mu\nu} F^a_{\mu\nu} \right\}$$

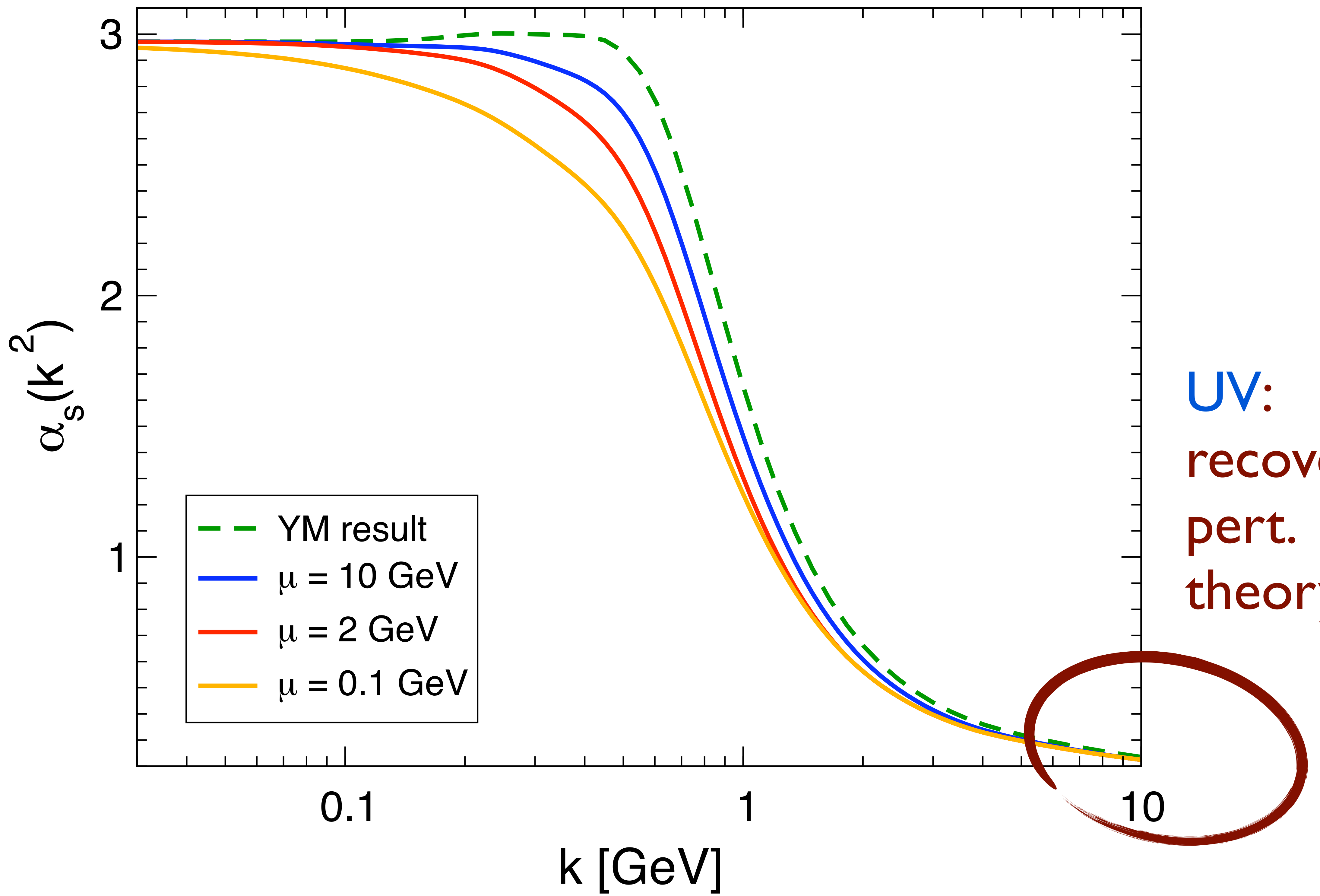
$$\alpha_s(m_Z) \approx 0.118$$

α_s at finite μ & T=0



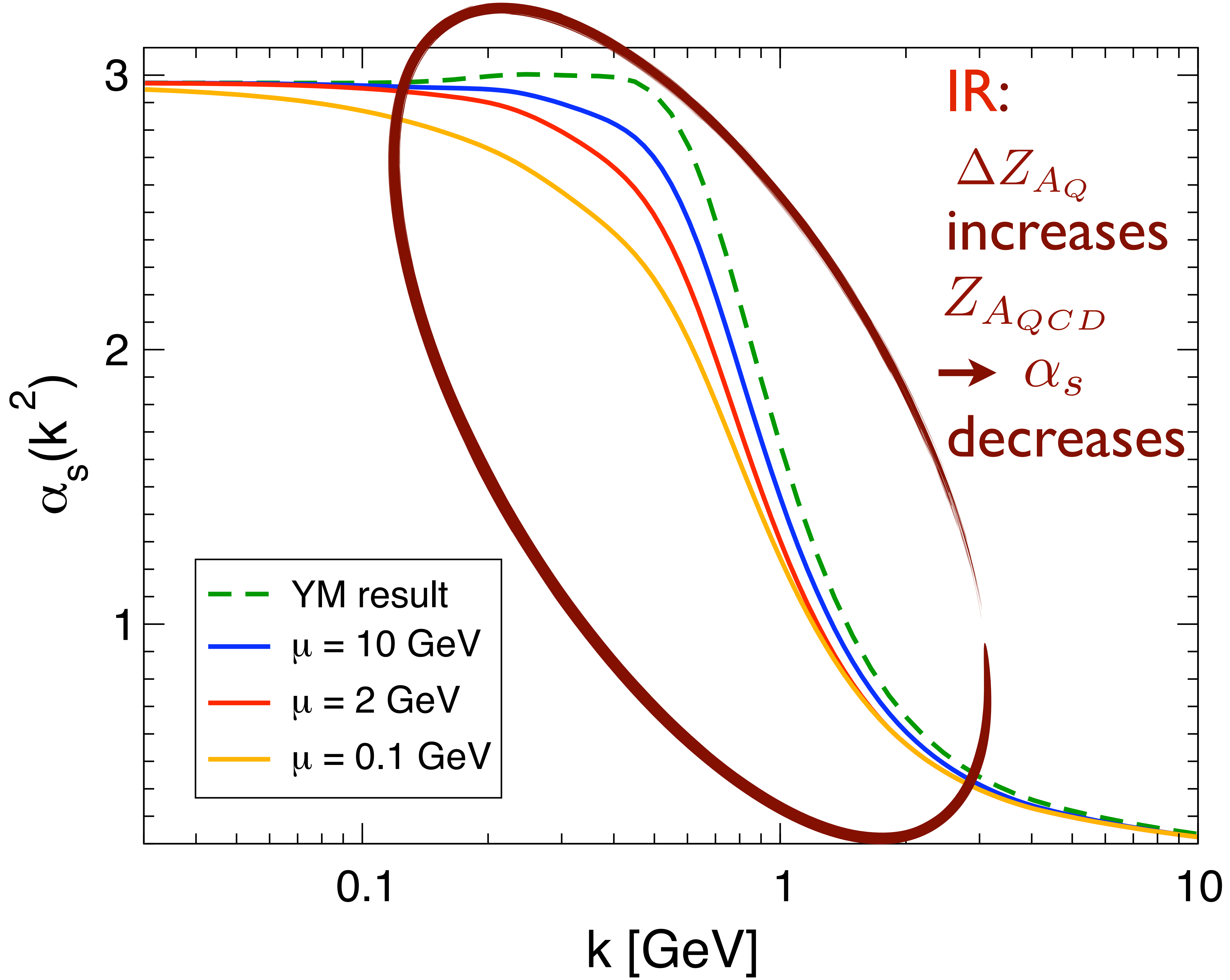
Jens Braun, LMH,
Florian Marhauser, Jan M. Pawlowski

α_s at finite μ & T=0



UV:
recover
pert.
theory

α_s at finite μ & T=0

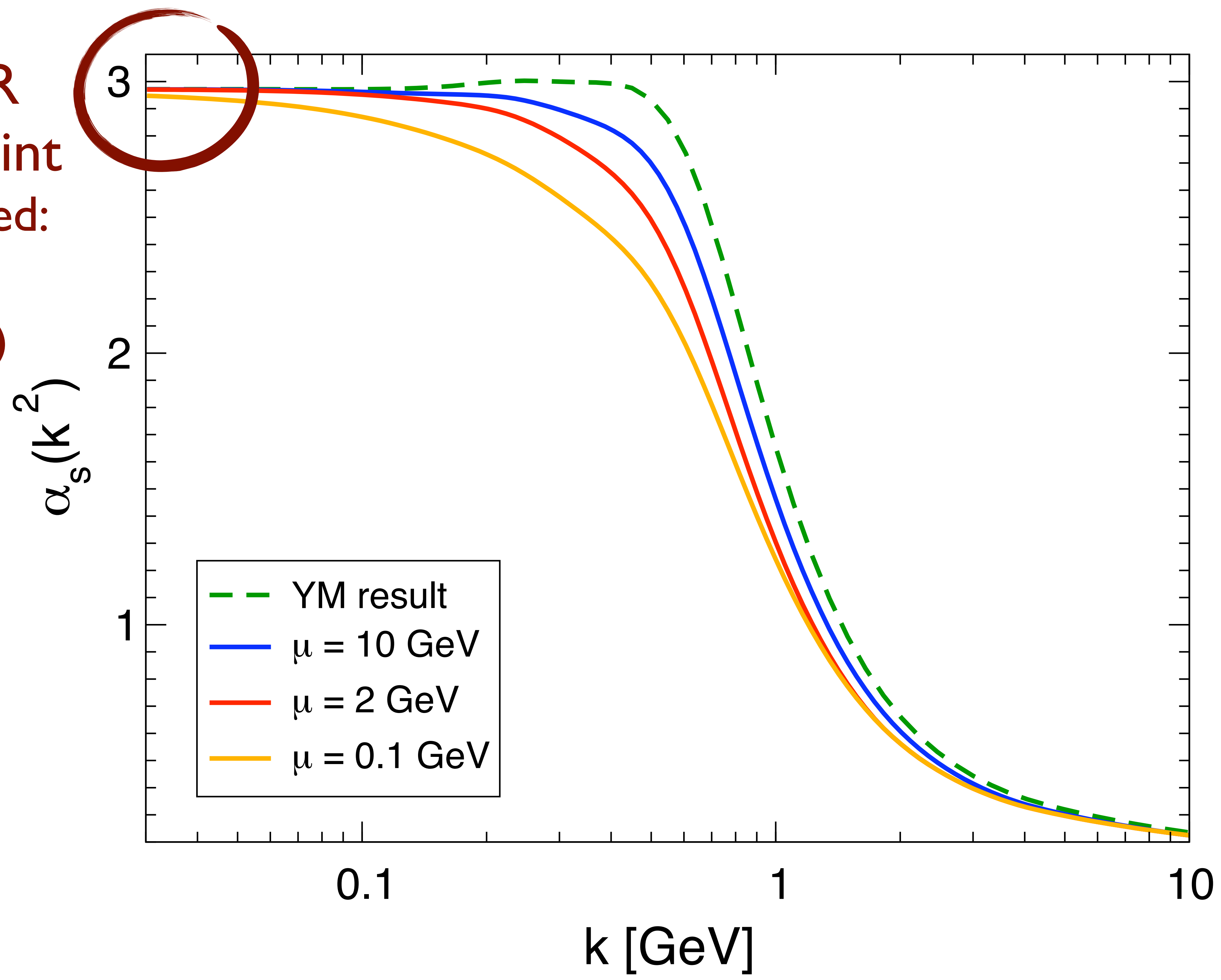


IR:

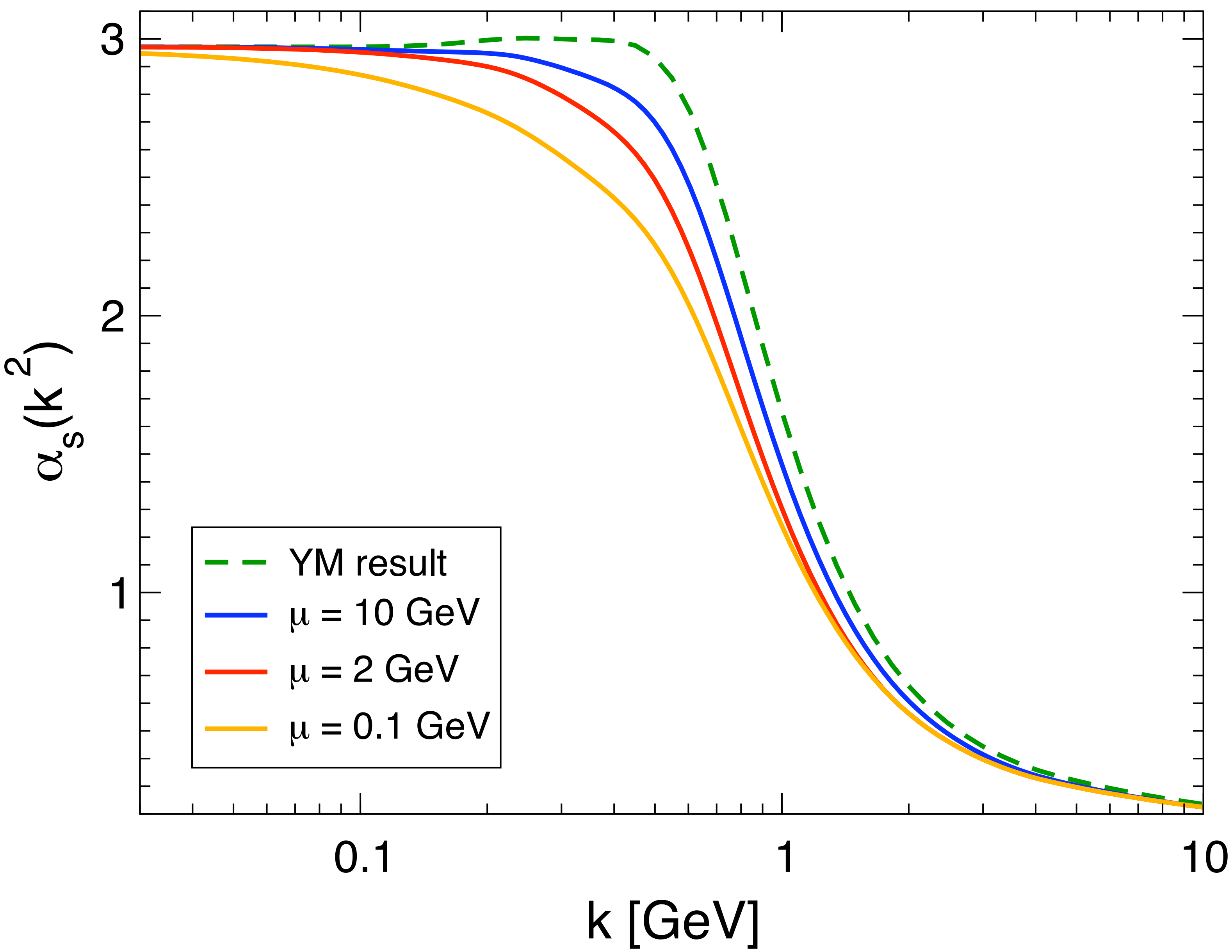
ΔZ_{A_Q} increases $Z_{A_{QCD}}$ → α_s decreases

α_s at finite μ & T=0

$k \to 0$
join in IR fixed point
(unchanged: fermions decouple)



α_s at finite μ & T=0



- $\mu = 0$: phase transition at $k_{\rm cr} \approx 440$ MeV, $k < k_{\rm cr}$: dynamical generation of quark mass, at $k \simeq m_\psi$ freeze-out point (quark decouples)
- $\mu > 0$: strong μ-dependence, above $\mu_{\rm cr} \approx 350$ MeV no χSB

Jens Braun, LMH, Florian Marhauser, Jan M. Pawlowski;
work in progress

On the relation of quark confinement & chiral symmetry breaking

Approximation

$$\Gamma_{QCD}^{N_f=2} = \int d^4x \Big\{ \underbrace{\bar{\psi}\,(\mathrm{i}\,\not{D}[A] \;+ 2\pi\theta\, T\gamma_0)\,\psi}_{\text{Dirac}} + \underbrace{\frac{\bar{h}}{\sqrt{2}}(\bar{\psi}(\vec{\tau}\cdot\Phi)\psi)}_{\text{Yukawa}} + \underbrace{\frac{1}{2}\,(\partial_\mu\Phi)^2 + U(\Phi^2)}_{\text{scalar}}$$

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Jens Braun, LMH, Florian Marhauser, Jan M. Pawlowski ’09

Approximation

imaginary μ and finite T, $N_f = 2$, chiral limit

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imaginary chemical potential

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imaginary chemical potential

$\mu = 2\pi\mathrm{i}\theta/\beta$

$\beta = 1/T$

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imaginary chemical potential

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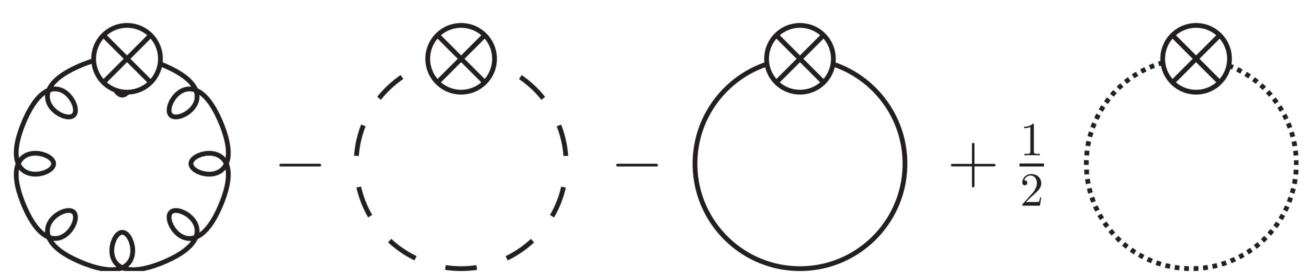
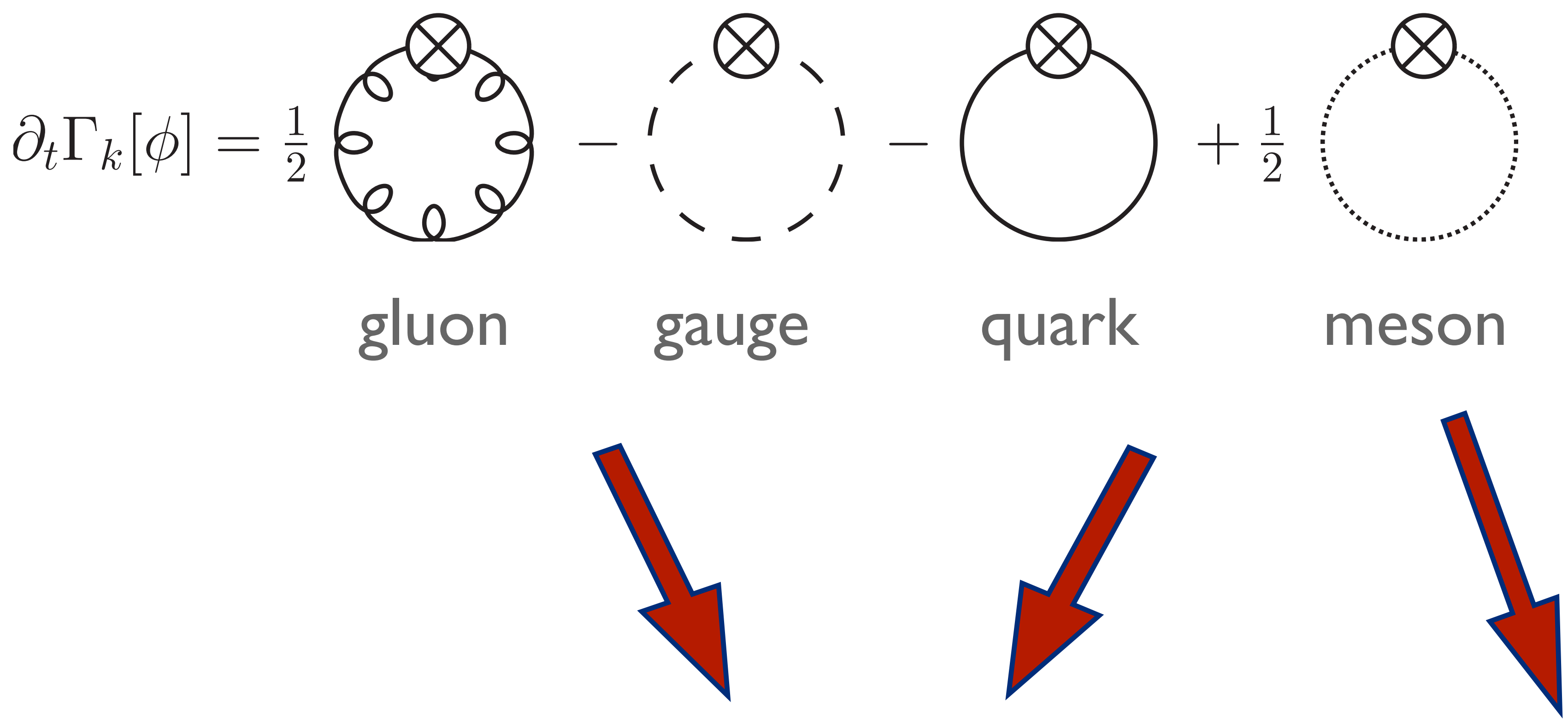
$\partial_t \Gamma_k[\phi] = \frac{1}{2}$ 

gluon gauge quark meson

Approximation

$$\Gamma_{QCD}^{N_f=2} = \int d^4x \left\{ \bar\psi \left(\mathrm{i}\, \not{D}[A] + 2\pi\theta\, T\gamma_0 \right) \psi + \frac{\bar h}{\sqrt{2}} (\bar\psi(\vec\tau \cdot \Phi)\psi) + \frac{1}{2} \left(\partial_\mu \Phi\right)^2 + U(\Phi^2) + \frac{Z_{A_{QCD}}}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \Gamma_{\text{gauge}} \right\}$$



coupled via dynamical quark-gluon interaction

dynamically included

Approximation

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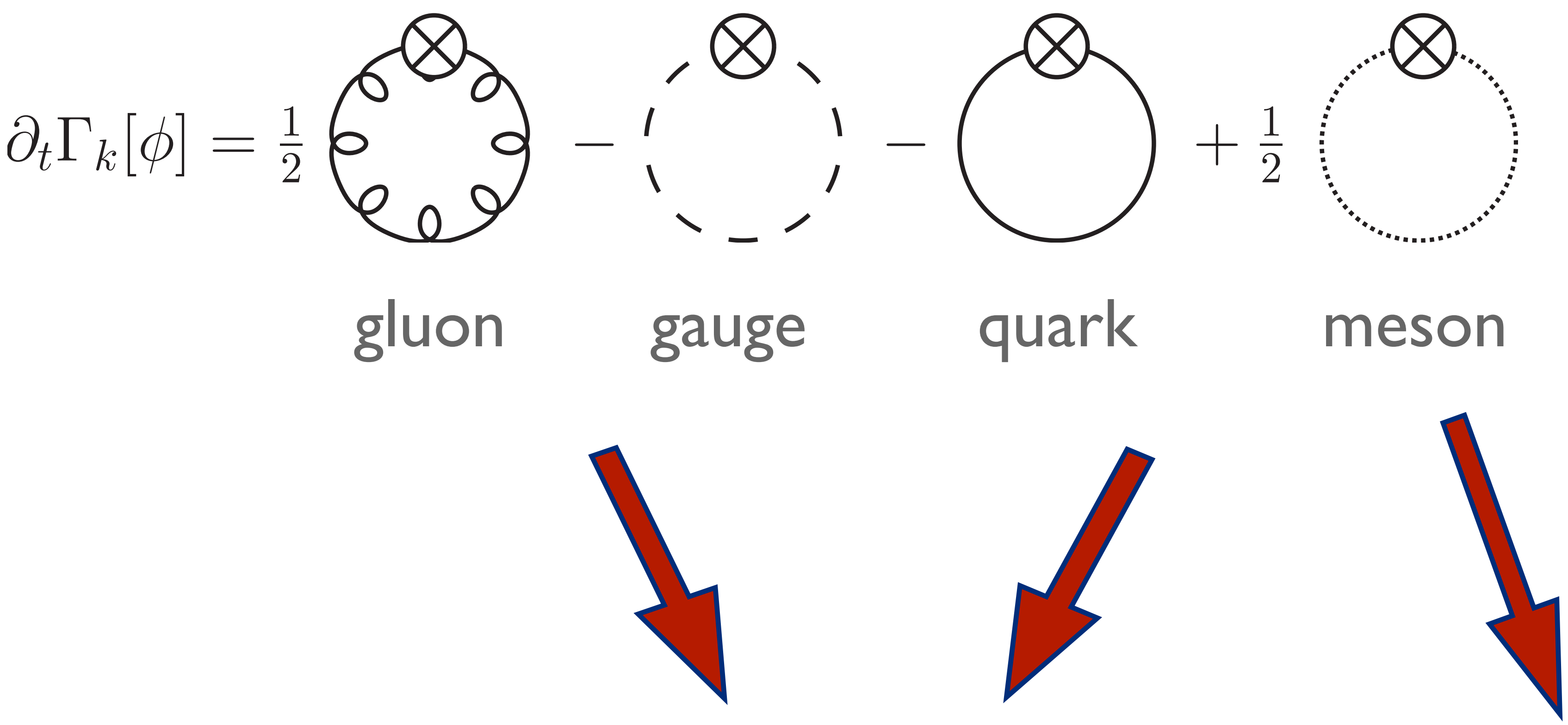


coupled via dynamical
quark-gluon interaction

dynamically included

confining properties: full momentum dependence
of ghost & gluon propagator

Confinement phase transition

Symmetry of gauge sector of QCD:

center symmetry Z_3 for SU(3)

Confinement phase transition

Symmetry of gauge sector of QCD:

center symmetry Z_3 for SU(3)

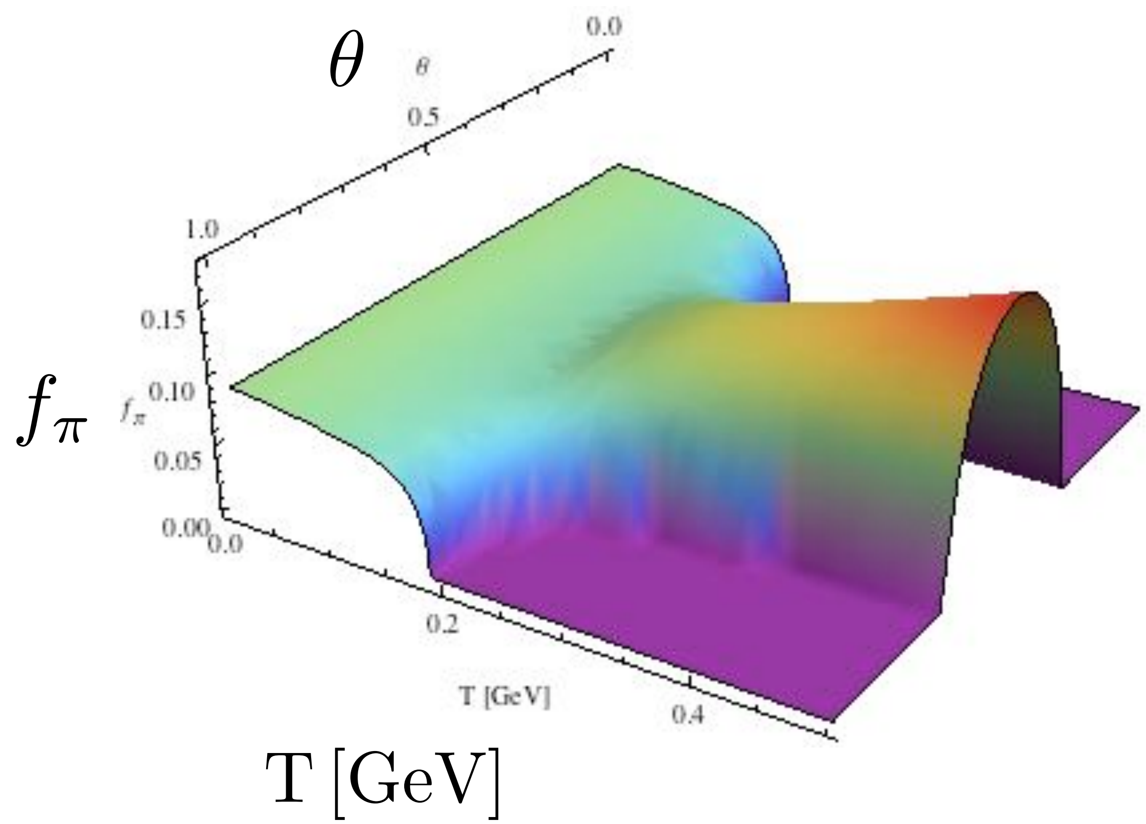
symmetry present in the limit of static quarks ($m_q \to \infty$)

order parameter: Polyakov loop ϕ

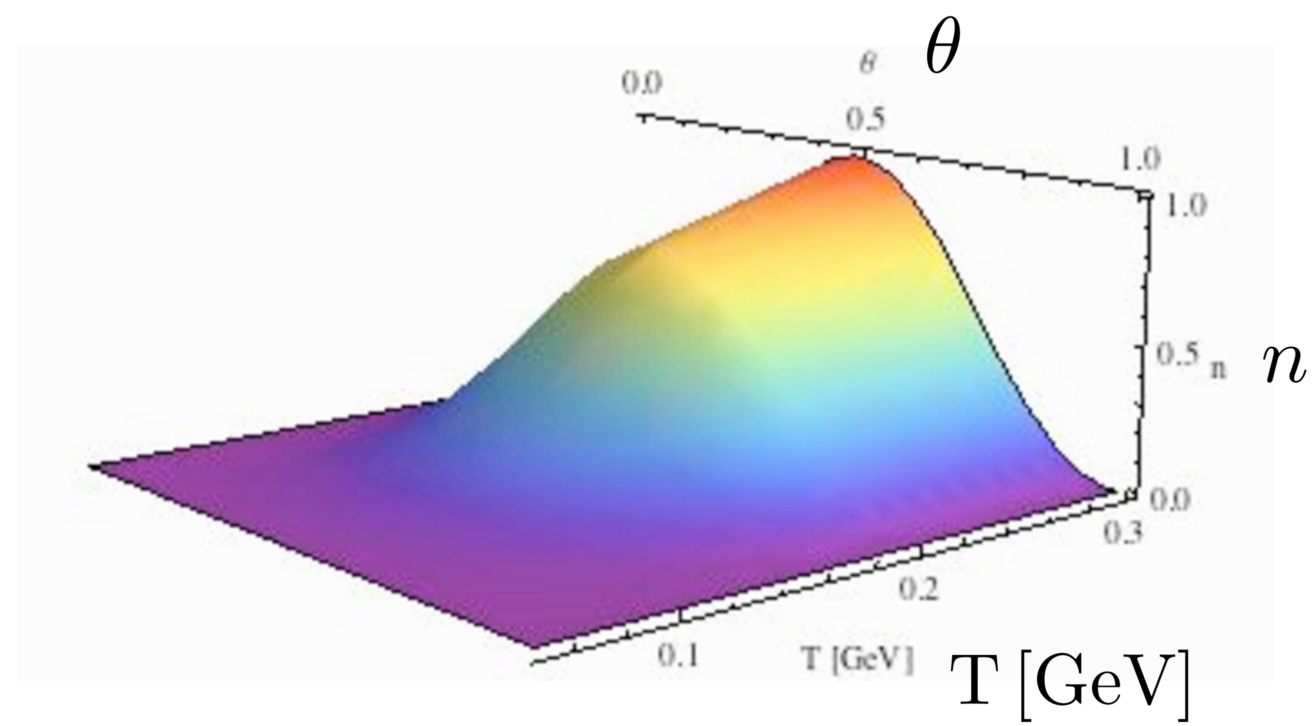
$$\phi = \frac{1}{N_c} \mathrm{Tr}\, \mathcal{P}\, e^{i \int_0^{1/T} dt\, \langle A_0 \rangle} = \begin{cases} > 0 & T > T_{c,\mathrm{conf}} \\ 0 & T < T_{c,\mathrm{conf}}. \end{cases}$$

Pion decay constant & dual density

C. Gattringer ’06;
F. Synatschke, A. Wipf, C. Wozar ’07;
F. Bruckmann, C. Hagen, C. Gattringer ’08;
C. S. Fischer ’09

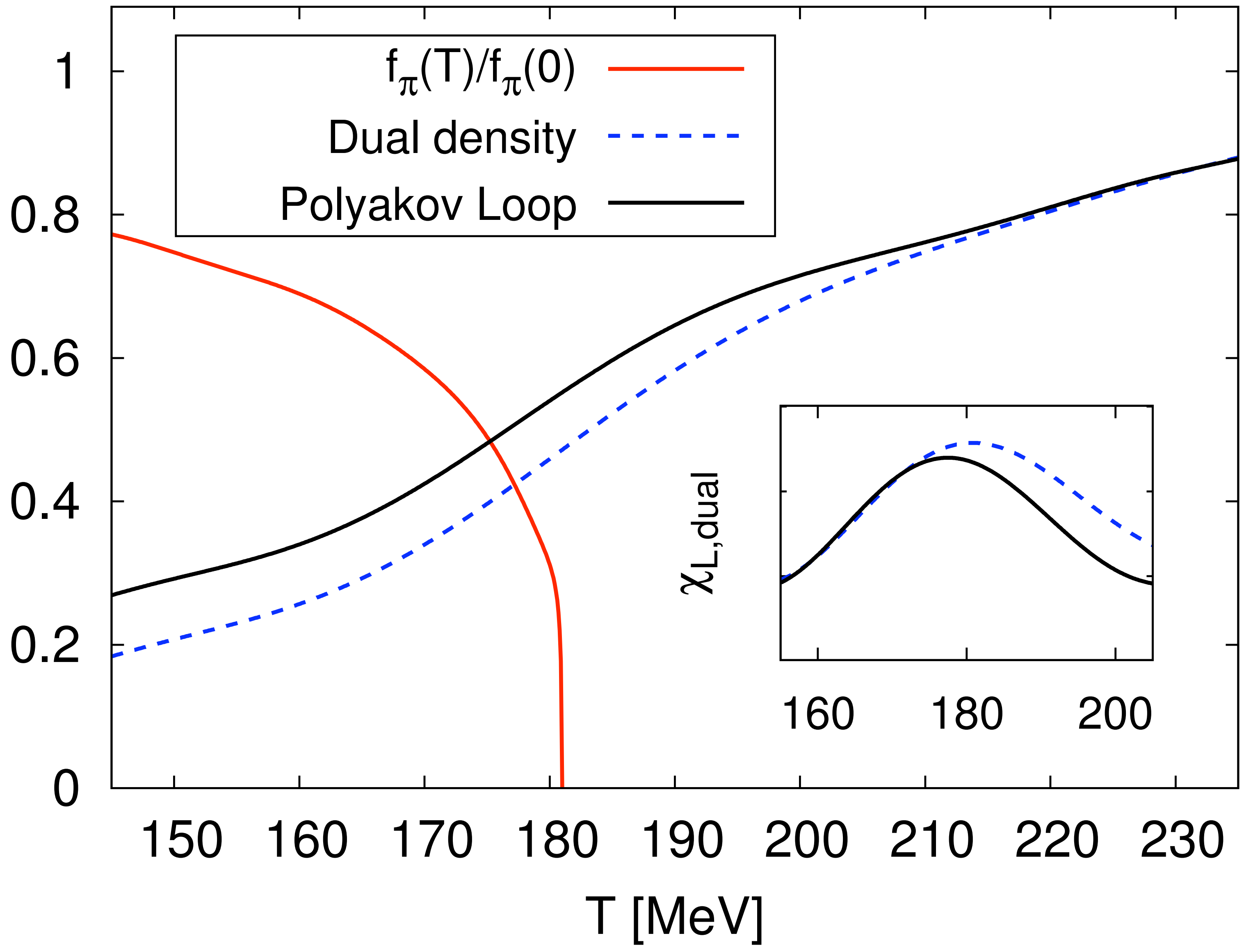


pion decay constant



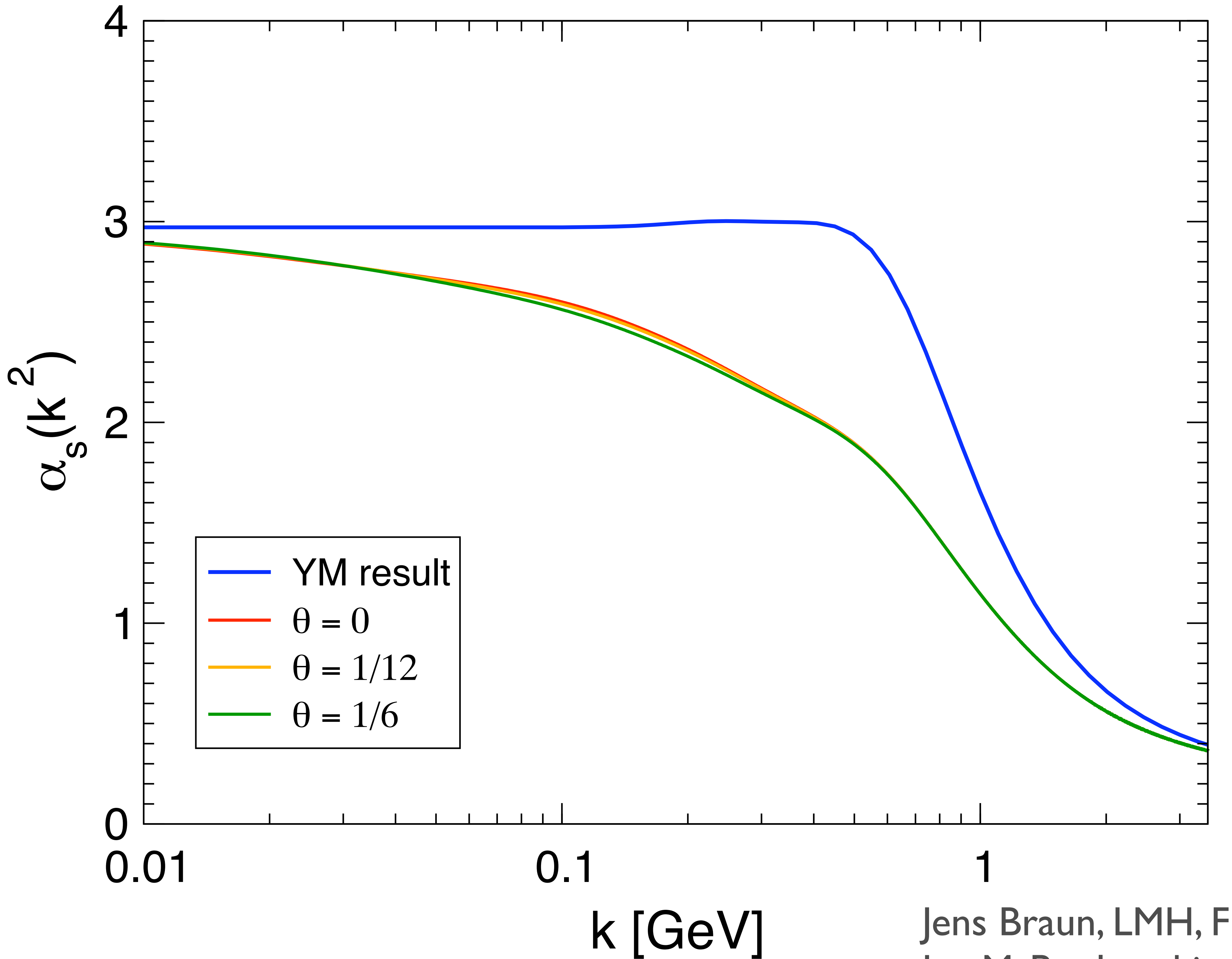
dual density n $\sim \mathrm{Log}\, Z(\theta)$

Order parameters



$T_{\mathrm{c,conf}}$ **agree within a few MeV with** $T_{\mathrm{c},\chi}$

α_s at imaginary μ & finite T



Jens Braun, LMH, Florian Marhauser,
Jan M. Pawlowski, work in progress

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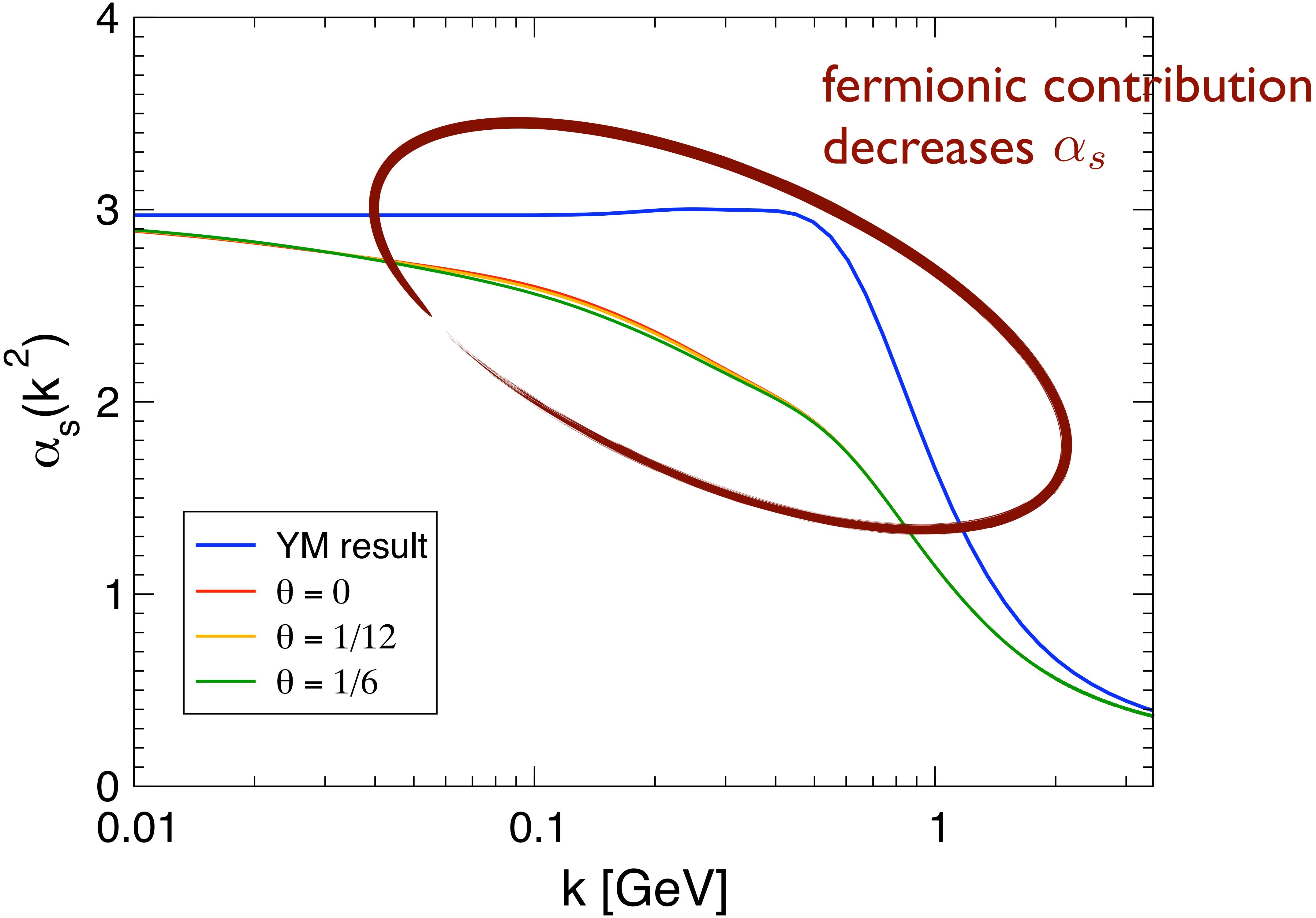

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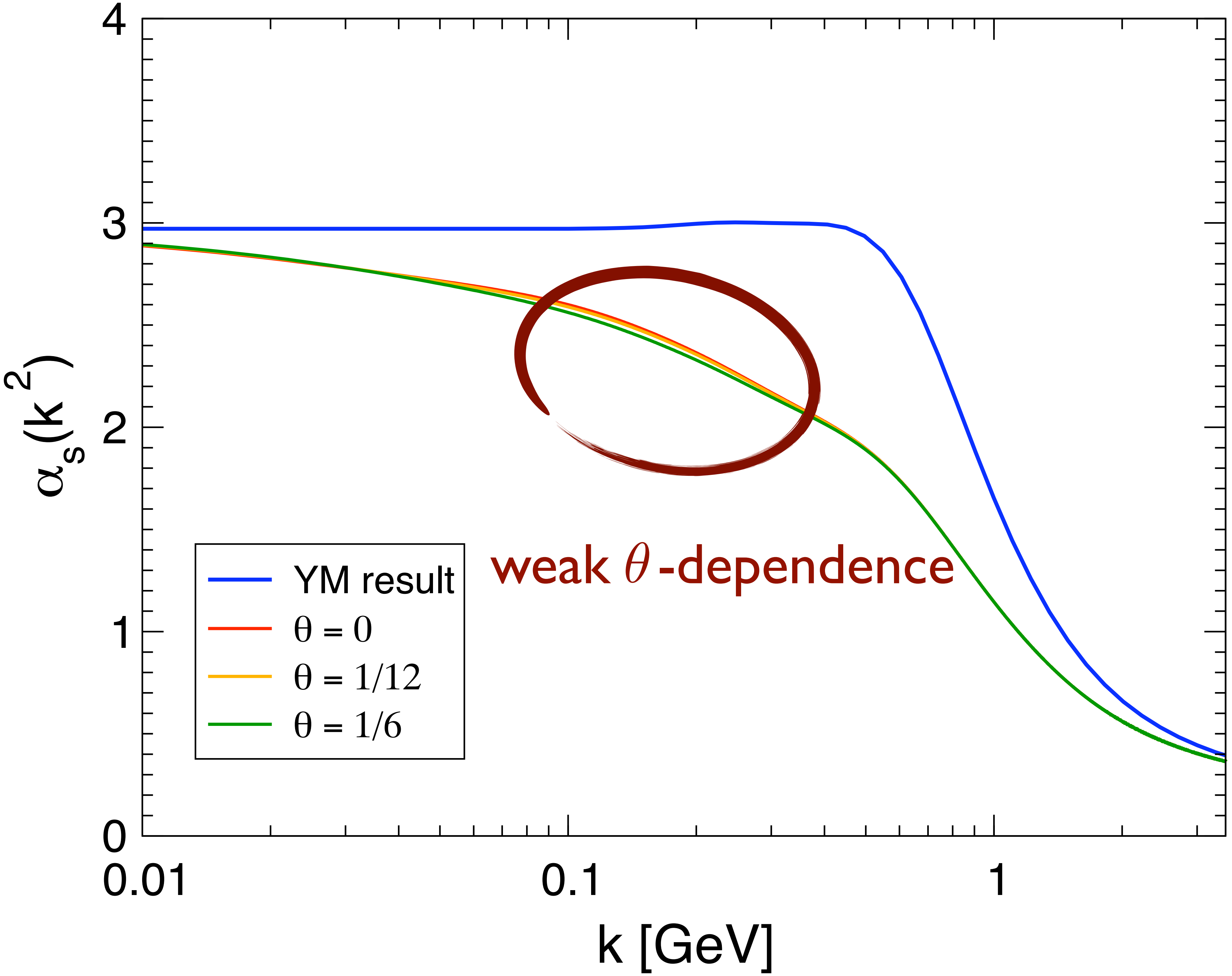

Summary & Outlook

Summary:

- $N_f = 1$: significant μ-dependence of α_s
- $N_f = 2$: weak θ-dependence of α_s
- $T_{c,\mathrm{conf}}$ agrees with $T_{c,\chi}$

Summary & Outlook

Summary:

- $N_f = 1$: significant μ-dependence of α_s
- $N_f = 2$: weak θ-dependence of α_s
- $T_{c,\mathrm{conf}}$ agrees with $T_{c,\chi}$

Outlook:

- $N_f = 1$: include the $U_A(1)$ anomaly
- $N_f = 2$ & $N_f = 2 + 1$: dynamical hadronisation