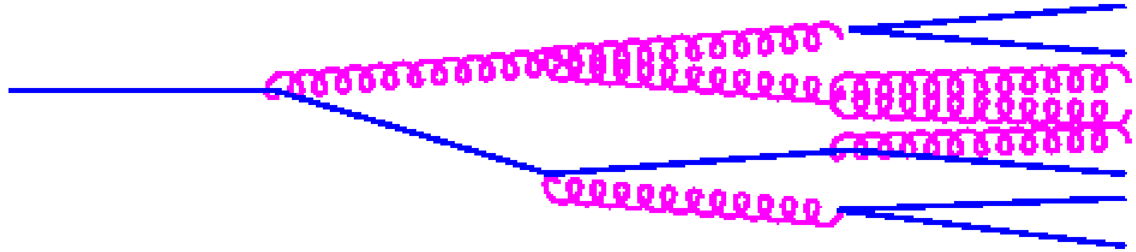
Peter Arnold

Gluon bremsstrahlung in QCD plasmas at very high energy

Why interesting?

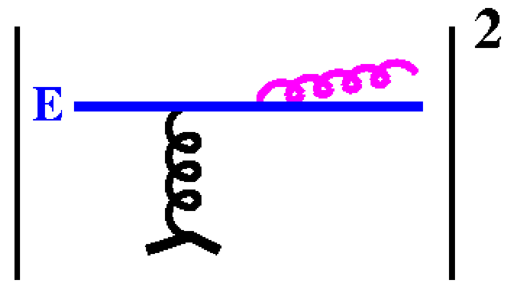
Perturbatively, gluon bremsstrahlung (and related process of pair production) dominates energy loss of high energy particles ($E >> T$) traversing a quark-gluon plasma.



Calculations complicated by the **Landau-Pomeranchuk-Migdal (LPM)** effect.

The LPM Effect

Naively

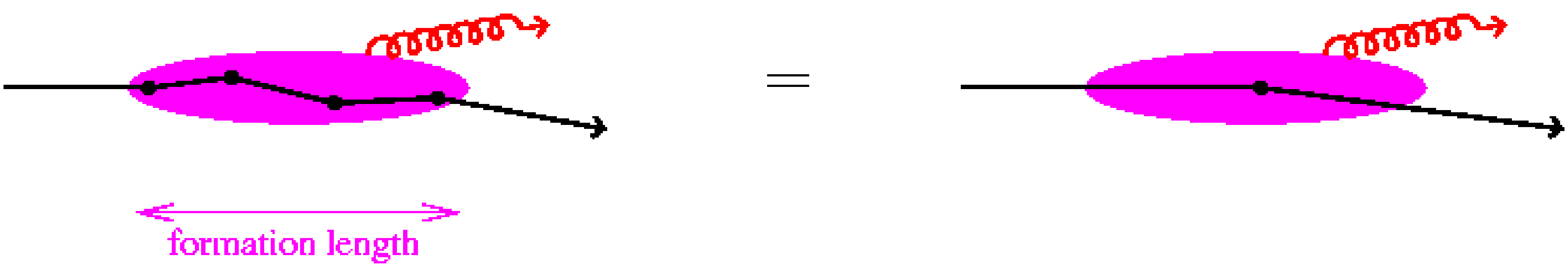
brem rate ~ $n\sigma v$ ~ (density of scatterers) ×  × 1

Problem

At very high energy,

probabilities of brem from successive scatterings no longer independent;

brem from several successive (small angle) collisions not very different from brem from one collision.



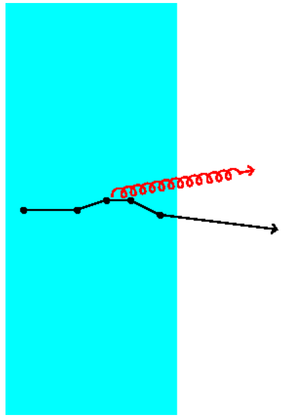
$$\text{formation length} \propto \sqrt{E}$$

Result: a reduction of the naive brem rate.

I. Review of the LPM effect

QED 1953-56, QCD 1996-98

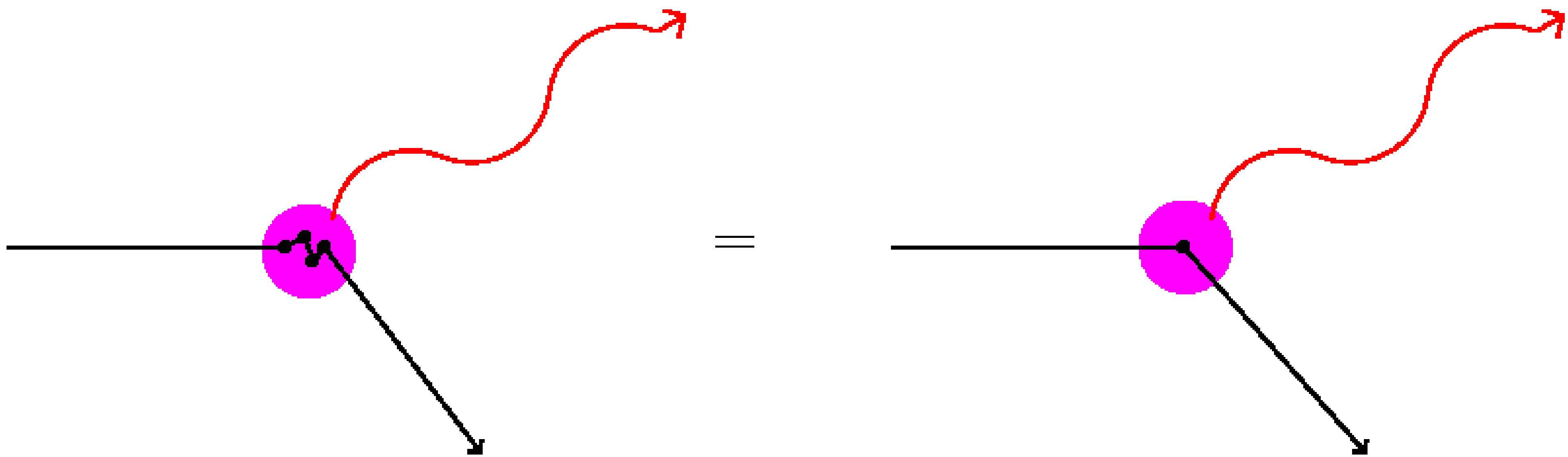
II. A theoretical puzzle



III. Its resolution

The LPM Effect (QED)

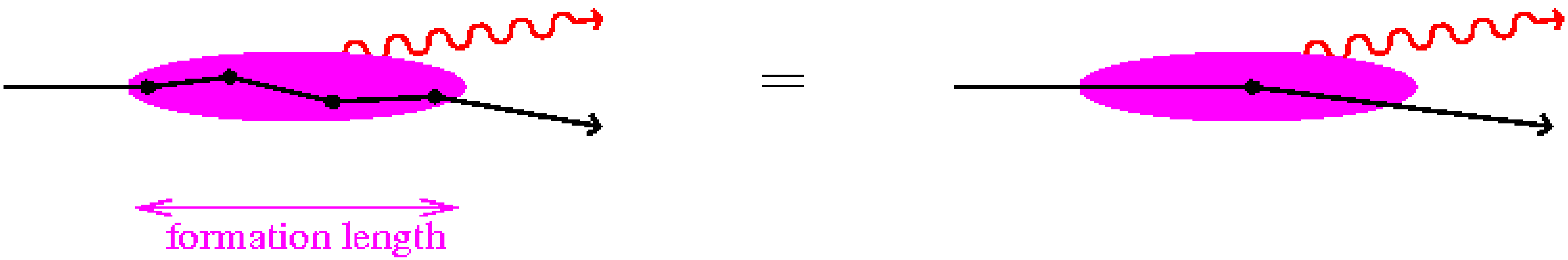
Warm-up: Recall that light cannot resolve details smaller than its wavelength.



[Photon emission from different scatterings have same phase → coherent.]

Now: Just Lorentz boost above picture by a lot!

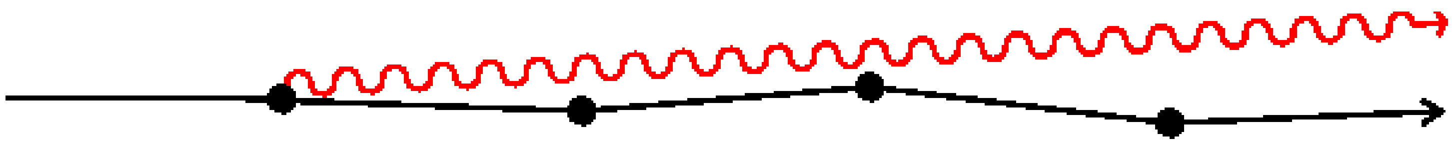
The LPM Effect (QED)



Note: (1) **bigger *E*** requires bigger boost → more time dialation → **longer formation length**

(2) big boost → this process is **very collinear.**

An alternative picture



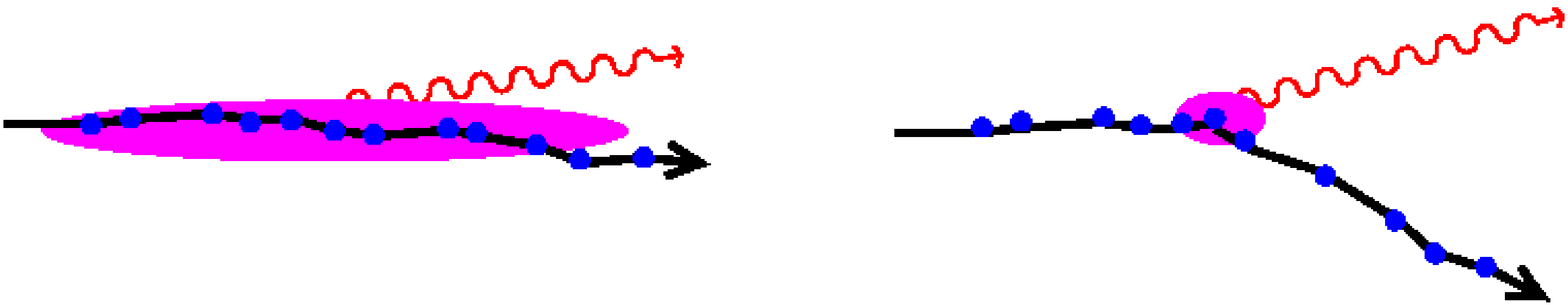
versus

Are these two possibilities in phase? Do they interfere coherently?

YES if (i) everything is nearly collinear ✓
(ii) particle and photon have nearly same velocity ✓ (*speed of light*)

The important point:

The more collinear the underlying scattering, the longer the formation time.



Note: the formation length
depends on the net angular deflection during the formation length, which
depends on the formation length

[Self-consistency → standard parametric formulas for formation length.]

The LPM Effect (QCD)

There is a qualitative difference for ***soft*** bremsstrahlung.:

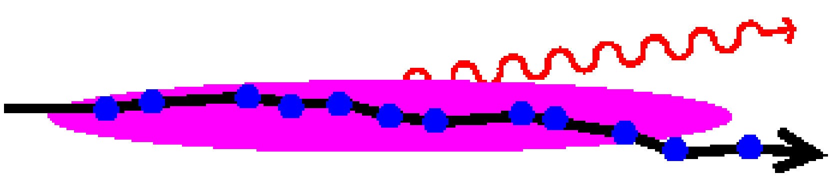
<u>QED</u>

Softer brem photon $\rightarrow$ longer wavelength
$\rightarrow$ less resolution
$\rightarrow$ more LPM suppression

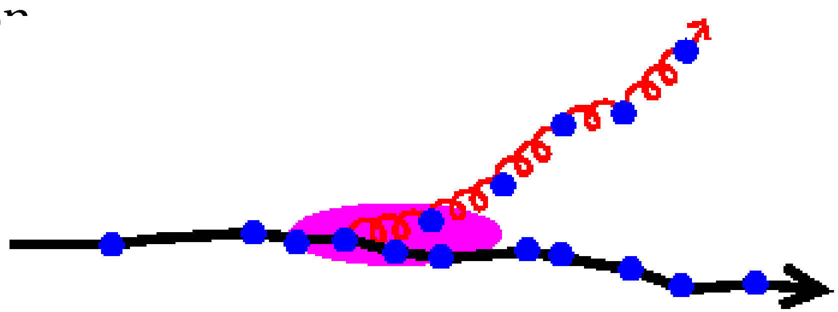
<u>QCD</u>

Unlike a brem photon, a brem gluon can easily scatter from the medium.

Softer brem gluon $\rightarrow$ easier for brem gluon to scatter
$\rightarrow$ less collinearity
$\rightarrow$ less LPM suppression

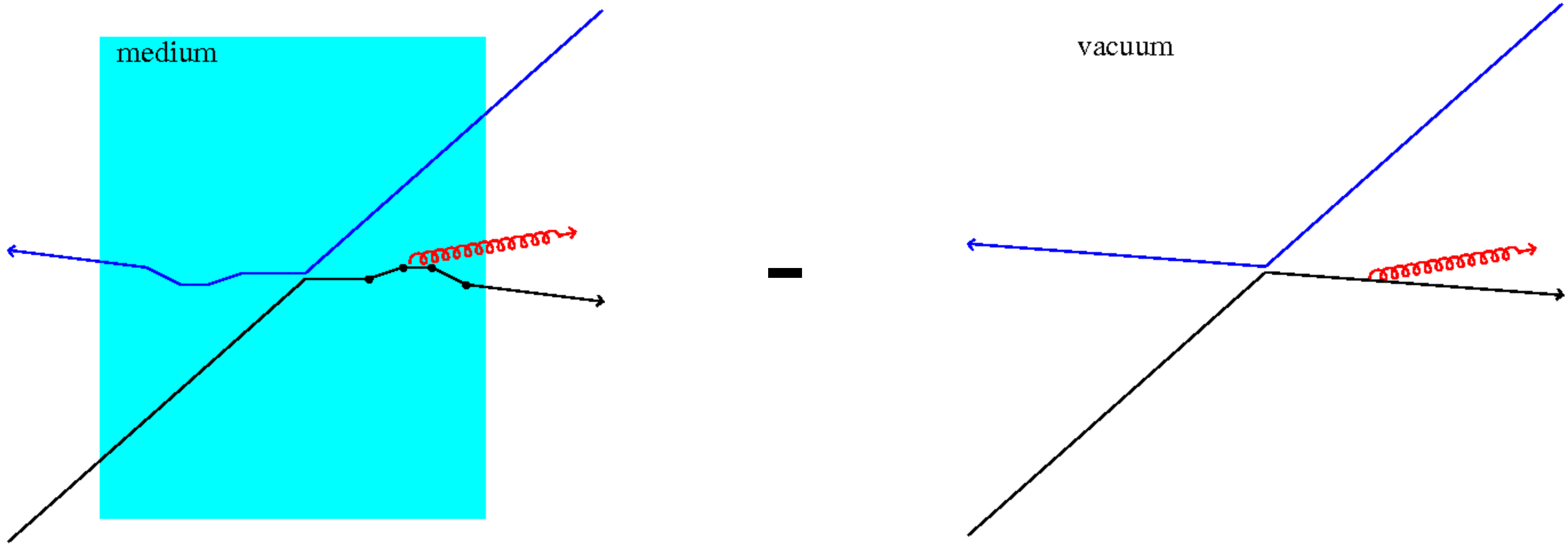


vs.



Upshot: Soft brem more important in QCD than in QED (for high-E particles in a medium)

A theoretical puzzle (background)



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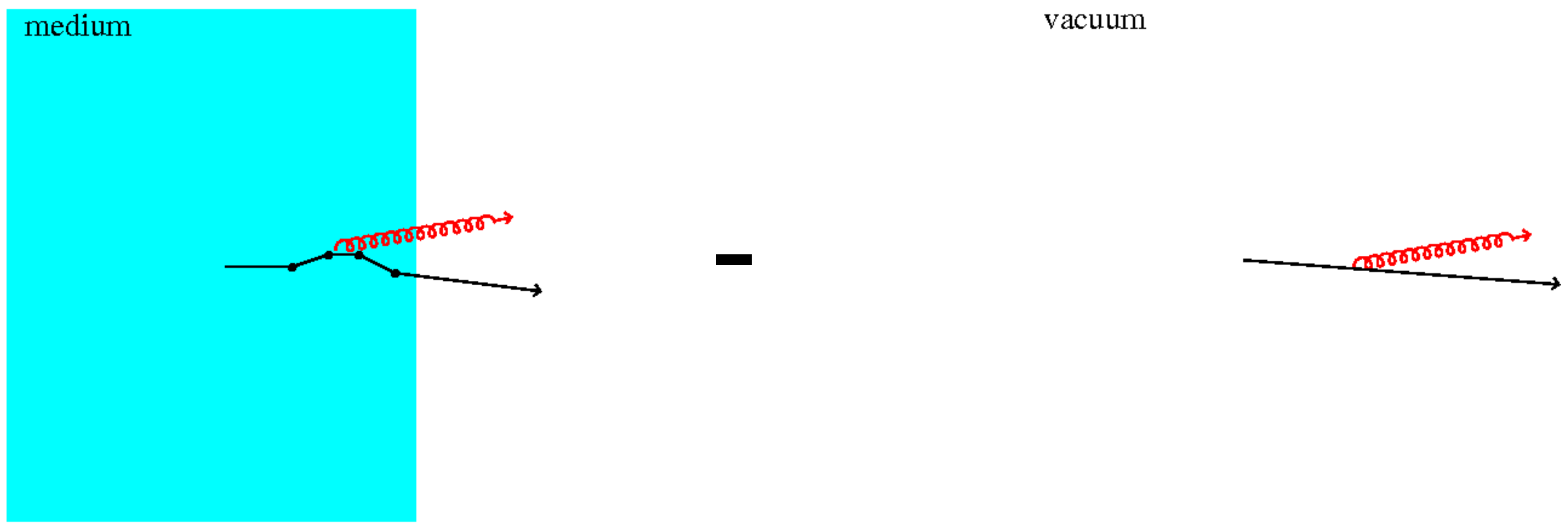

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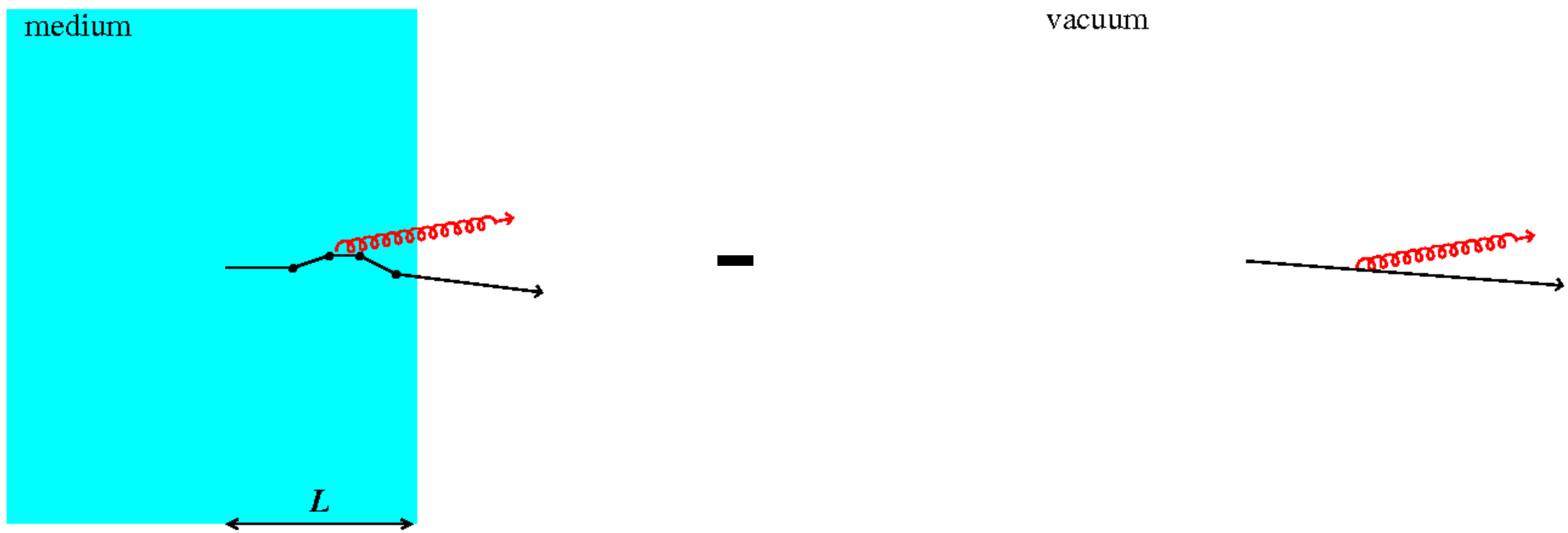
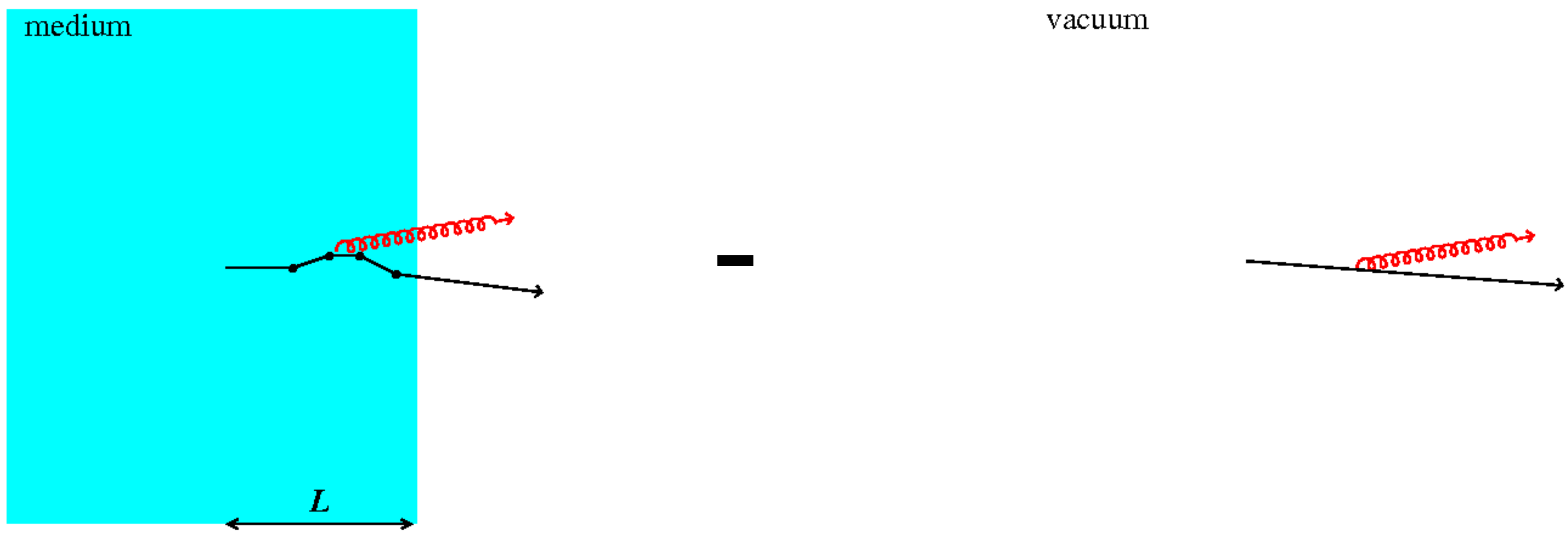


Naively: medium effect grows linearly with *L*..

For small enough *L*, instead grows like $L^2 \ln L$ because of the LPM effect. [BDMPS 1996]

A theoretical puzzle (background)



Naively: medium effect grows linearly with *L*..

For small enough *L*, instead grows like $L^2 \ln L$ because of the LPM effect. [BDMPS 1996]

<u>Assumptions I will make in this talk:</u>

$E \gg T$ and moreover $\ln(E/T) \gg 1$

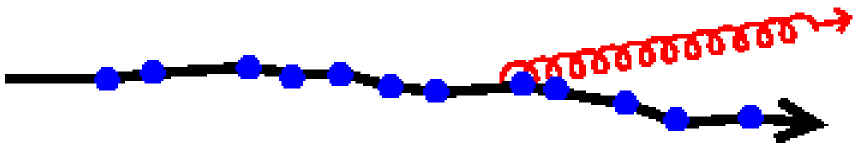
$\alpha_s \ll 1$ and moreover $\alpha_s \ln(E/T) \ll 1$

mean free path for elastic collisions $\ll L \ll$ formation length

The puzzle

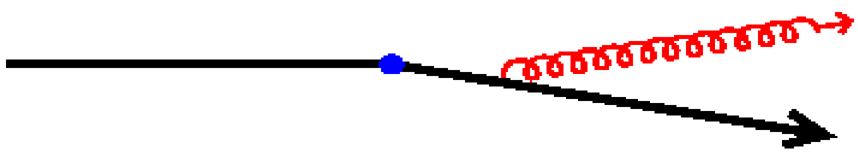
Treating $\ln(E/T) >> 1$, and trying to analyze the problem to leading order in inverse powers of this logarithm:

Harmonic oscillator (HO) approximation [BDMPS]



Consider only *typical* scattering events (no rare, large-than-usual scatterings)

single scattering (N=1) approximation [GLV, Salgado & Wiedemann]



Consider only *one* scattering from medium (both typical and rare deflection angles)

Naively, this might seem weird given my assumption that

$L \gg$ mean free path for elastic collisions

The puzzle: energy loss

Treating $\ln(E/T) >> 1$, and trying to analyze the problem to leading order in inverse powers of this logarithm:

Harmonic oscillator (HO) approximation [BDMPS]

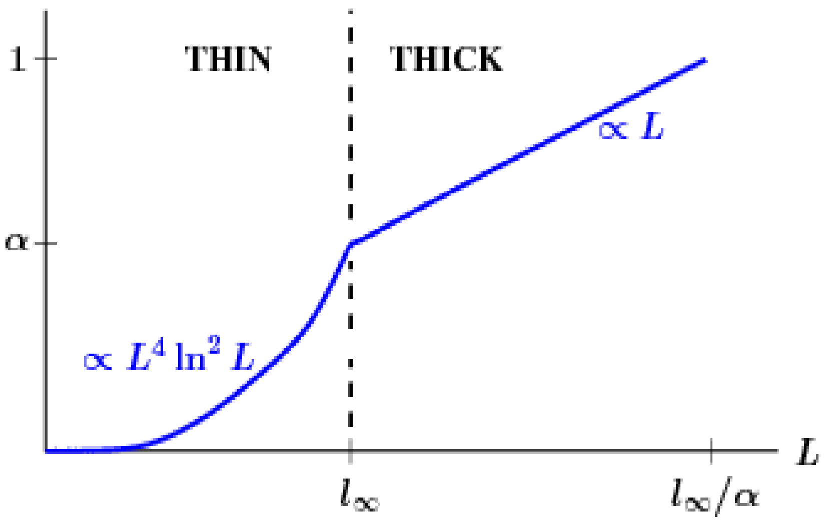
$$\langle \Delta E \rangle \simeq \# \alpha^3 n L^2 \ln\left(\frac{\hat{q} L}{m_{\mathrm{D}}^2}\right)$$

single scattering (N=1) approximation [GLV, Salgado & Wiedemann]

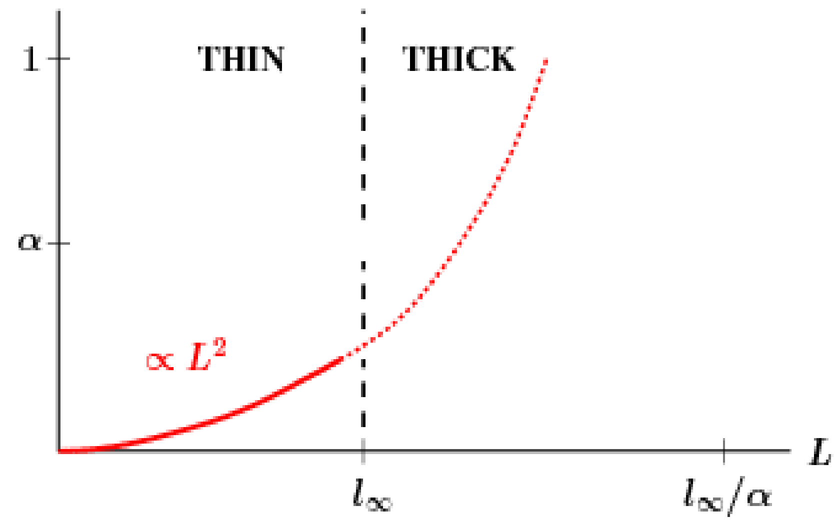
$$\langle \Delta E \rangle \simeq \# \alpha^3 n L^2 \ln\left(\frac{E}{m_{\mathrm{D}}^2 L}\right)$$

The puzzle: spectrum

$$\langle \Delta\,\text{spectrum} \rangle = \frac{dI}{d\omega} - \left[\frac{dI}{d\omega}\right]_{\text{vac}} \quad \text{vs. } L \text{ for fixed } \omega$$



harmonic oscillator (HO) approximation [BDMPS]



single scattering (N=1) approximation

Which approximation, if either, is right (at leading log order)?

[Zakharov 2001, BDMS 2001, Peigne & Smilga 2008, Arnold 2009]

The puzzle: spectrum

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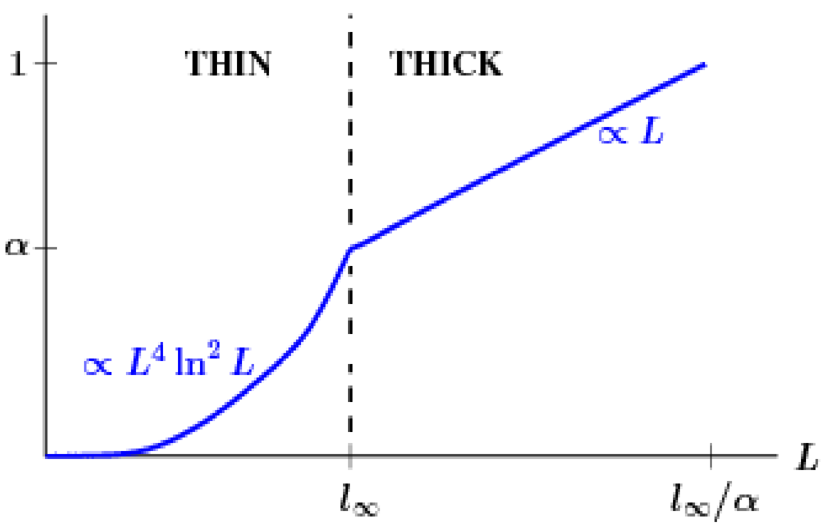

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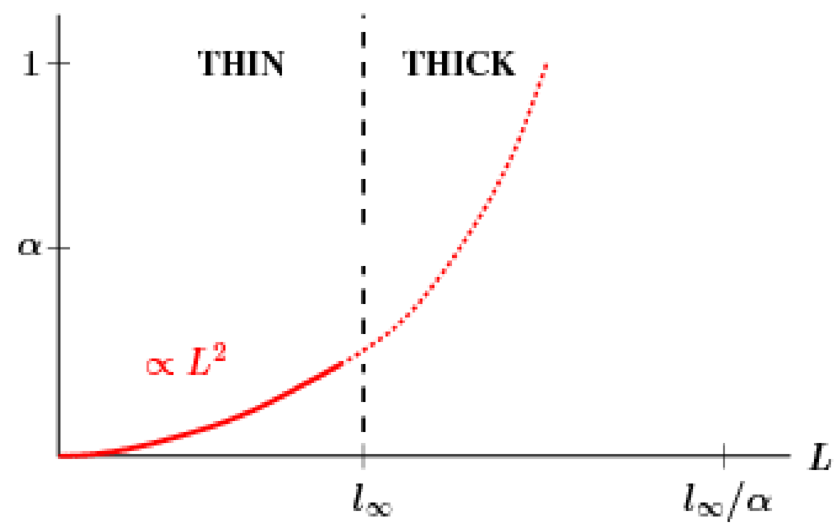


single scattering (N=1) approximation

Which approximation, if either, is right (at leading log order)?

Answer: They're both important.

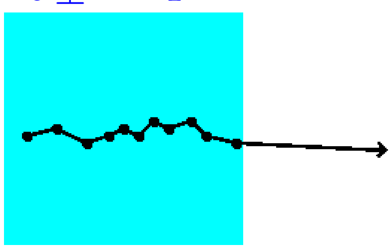
[Zakharov 2001, BDMS 2001, Peigne & Smilga 2008, Arnold 2009]

Scattering probabilities

$Q_\perp \equiv$ net transverse momentum transfer in distance L

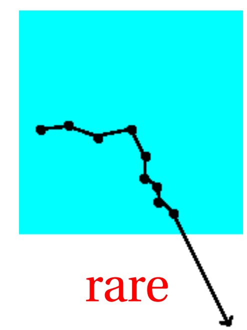
net deflection angle $\sim Q_\perp / E$

$Q_\perp^2 \sim \hat{q}L$

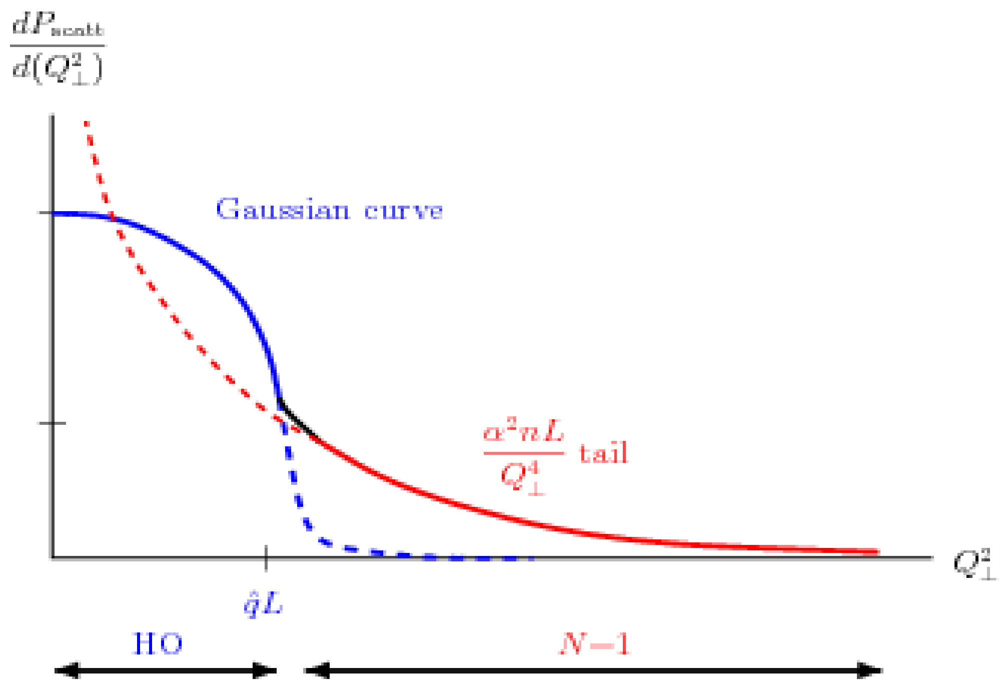


typical

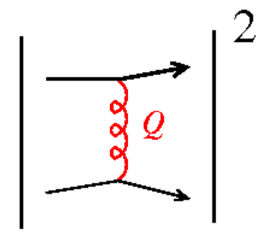
$Q_\perp^2 \gg \hat{q}L$



rare



$\hat{q} \equiv$ typical $Q_\perp^2 / L$



$\sim \frac{1}{Q^4} \sim \frac{1}{q_\perp^4}$

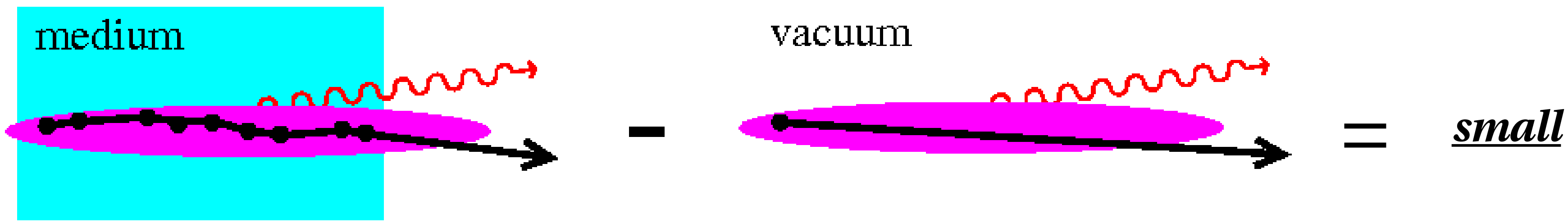
Return to thin media puzzle

<u>Typical scatterings</u>

Probability of underlying scattering event ***<u>large</u>*** but

relatively small deflection angle → large formation time

→ small medium effect on brem

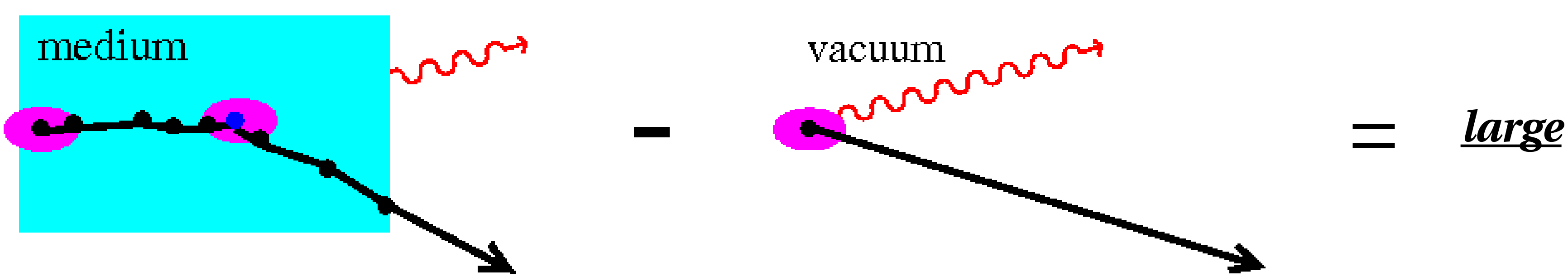


<u>Rare scatterings</u>

Probability of underlying scattering event ***<u>small</u>*** but

relatively large deflection angle → small formation time

→ significant medium effect on brem



Return to thin media puzzle

Typical scatterings

Probability of underlying scattering event ***large*** but

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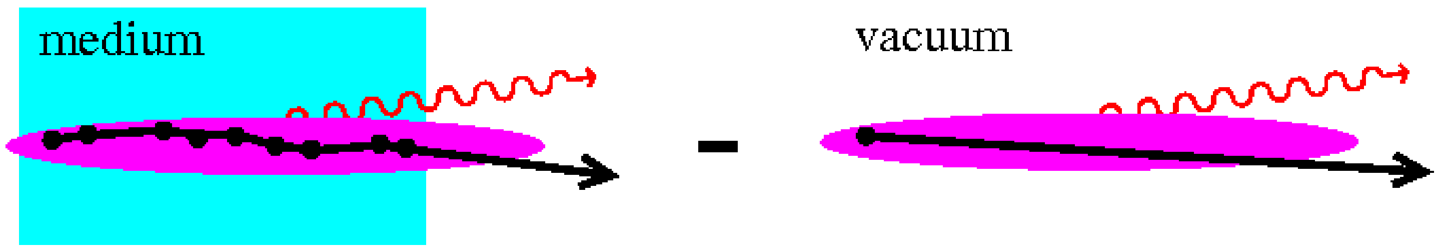

= ***small***

Rare scatterings

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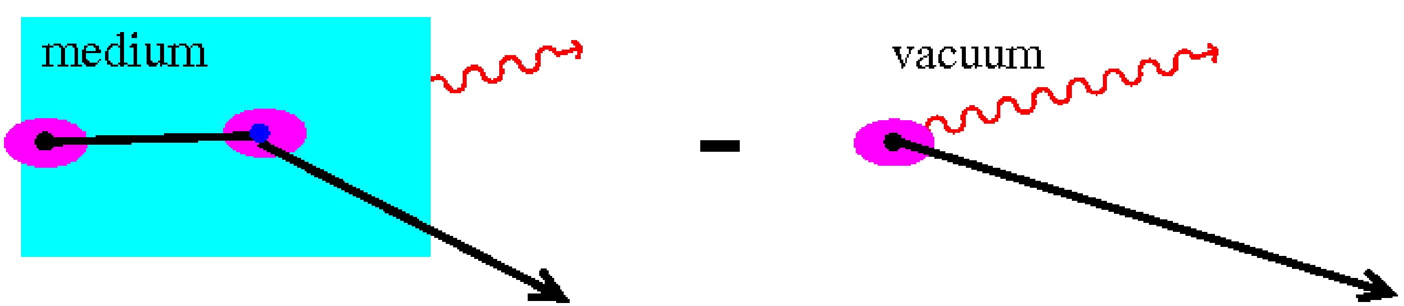
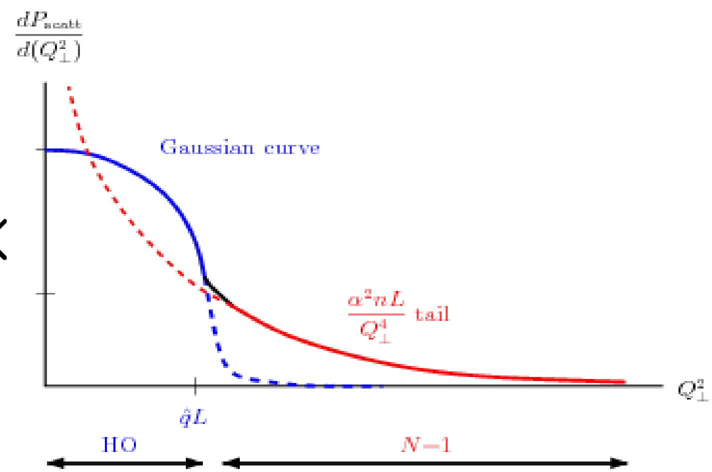


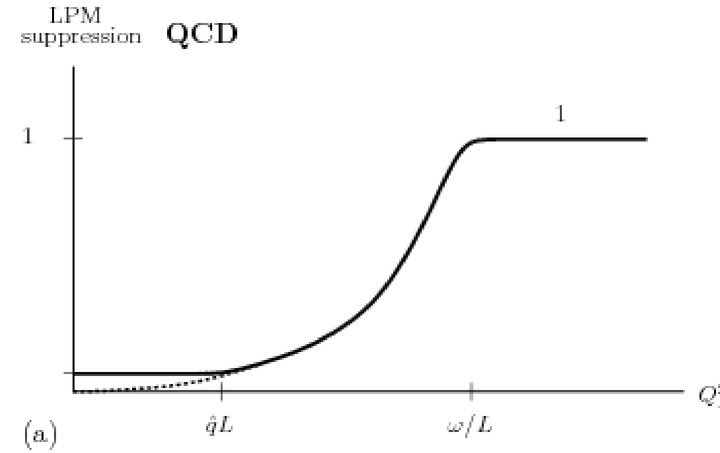
= ***large***

$$Q_\perp^2 \times \alpha \times$$



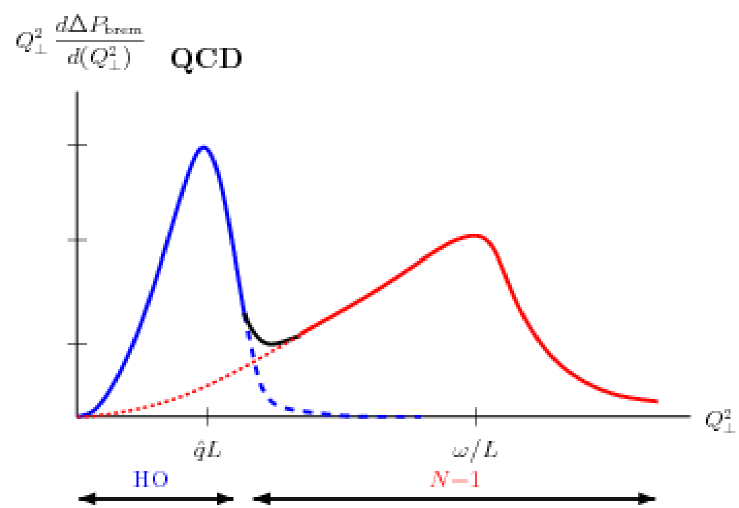
underlying scattering prob.

$\times$



LPM suppression

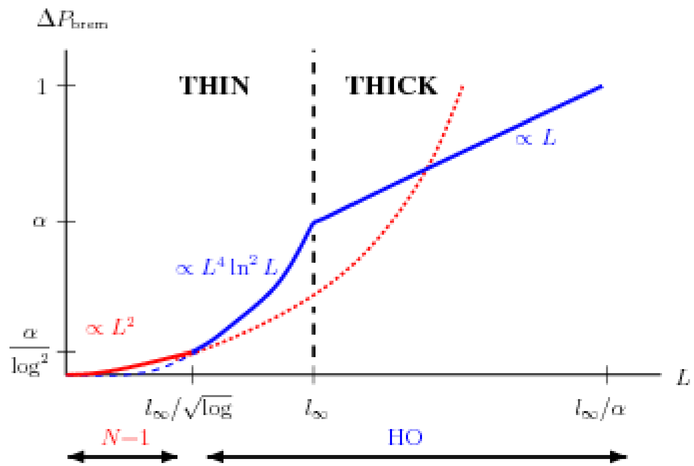
$=$



Brem probability

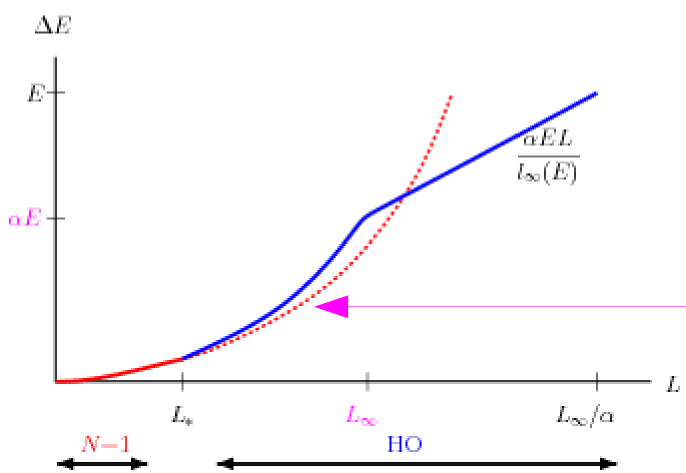
Which peak wins depends on frequency ω of gluon.

Δ (spectrum) vs. *L* for fixed ω



Total ΔE as function medium size *L*

$$L_\infty \equiv \sqrt{\frac{E}{\hat{q}}}$$ = formation time in infinite medium



$$\alpha^3 n L^2 \left[\ln\left(\frac{\hat{q}L}{m_{\rm D}^2}\right) + \ln\left(\frac{E}{\hat{q}L^2}\right) \right]$$

$$= \alpha^3 n L^2 \left[\ln\left(\frac{\sqrt{\hat{q}E}}{m_{\rm D}^2}\frac{L}{L_\infty}\right) + \ln\left(\frac{L_\infty^2}{L^2}\right) \right]$$

Lessons

The LPM effect is easy to understand qualitatively.

When computing average quantities like $\langle \Delta E \rangle$, the average is sometimes dominated by ***extremely rare*** events and so is not characteristic of what happens in most events.

Scattering probabilities

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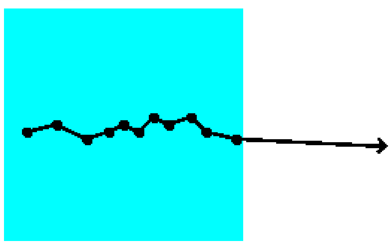
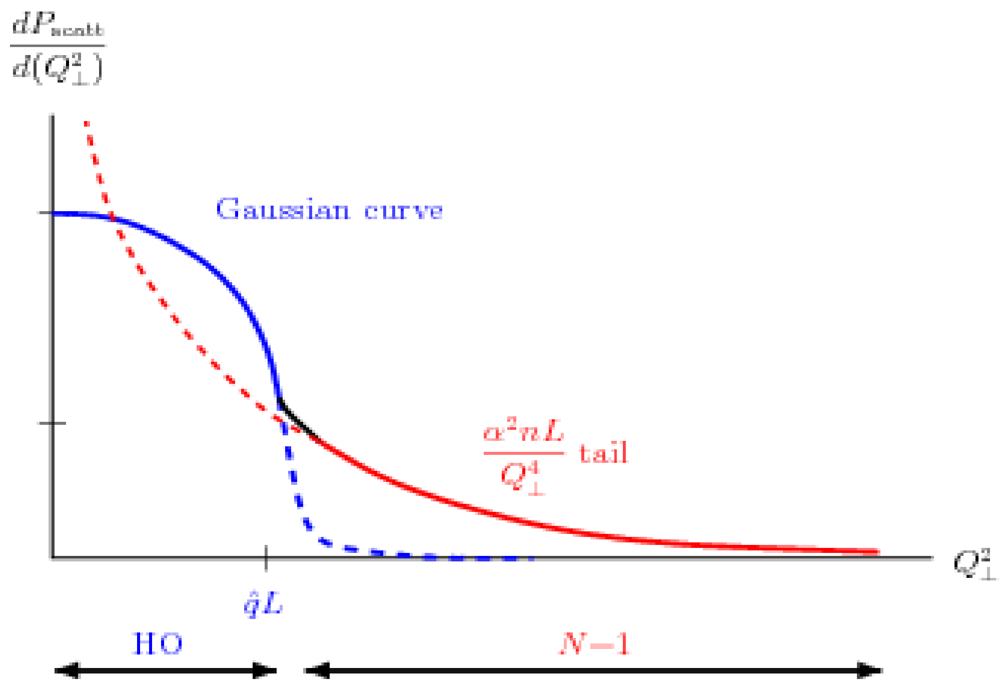


typical



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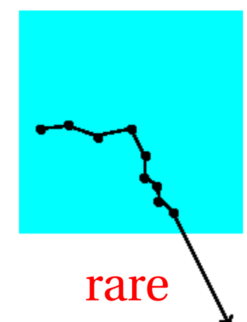

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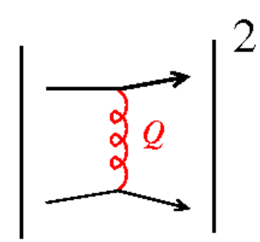
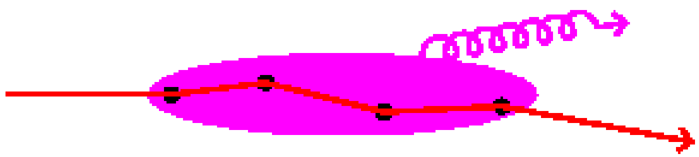


$\sim \frac{1}{Q^4} \sim \frac{1}{q_\perp^4}$

$\hat{q}$ in weakly-coupled plasmas



formation time
depends on collinearity of brem
depends on transverse momentum transfer $Q_\perp$

$$Q_\perp^2 = \hat{q} L$$

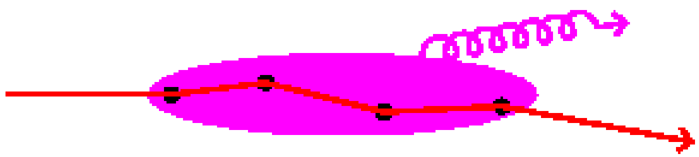
$\propto q_\perp^{-4}$ for large $q_\perp$

$$\hat{q} = \int d^2 q_\perp \, \frac{d\Gamma_{\rm el}}{d^2 q_\perp} \, q_\perp^2 =$$ squared transverse momentum transfer per unit length

$=$ UV log divergent (leading order)

$$\hat{q}_{\rm typical} = \hat{q}\big(\text{UV cutoff}^2 = \text{typical } Q_\perp^2 = \hat{q}_{\rm typical} L\big)$$

$\hat{q}$ in weakly-coupled plasmas



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squared transverse momentum transfer per unit length

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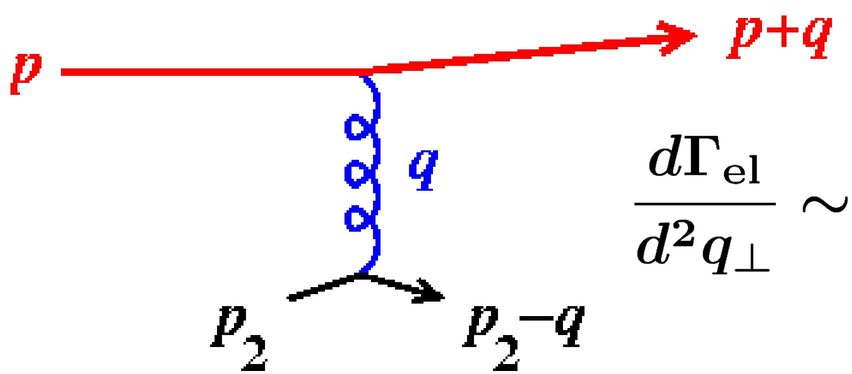

$$\frac{d\Gamma_{\rm el}}{d^2 q_\perp} \sim \int dq_z \int d^3 p_2 \frac{d\sigma_{\rm el}}{d^3 q} f(\vec{p}_2)[1 \pm f(\vec{p}_2 - \vec{q})]$$

Leading-order-in-α_s result for UV-regulated qhat

Pure gluon gas, for example:

$$\hat{q}(\Lambda) = \left[\zeta(3)\ \ln\frac{\Lambda}{\mu} + \zeta(2)\ \ln\frac{\mu}{m_{\rm d}} - \sigma_+\right]\frac{9g^4T^3}{\pi^3}$$

$$\mu \equiv 2Te^{\frac{1}{2}-\gamma_{\rm E}}$$

Λ = UV cut-off on $q_\perp$

$$\sigma_+ \equiv \sum_{k=1}^{\infty}\frac{\ln[(k-1)!]}{k^3}$$

[Arnold & Xiao(2008)]

WARNING: Corrections which are formally higher-order in coupling, of order m_d/T = O(g). are of order 100% for realistic couplings. [Caron-Huot (2008)]