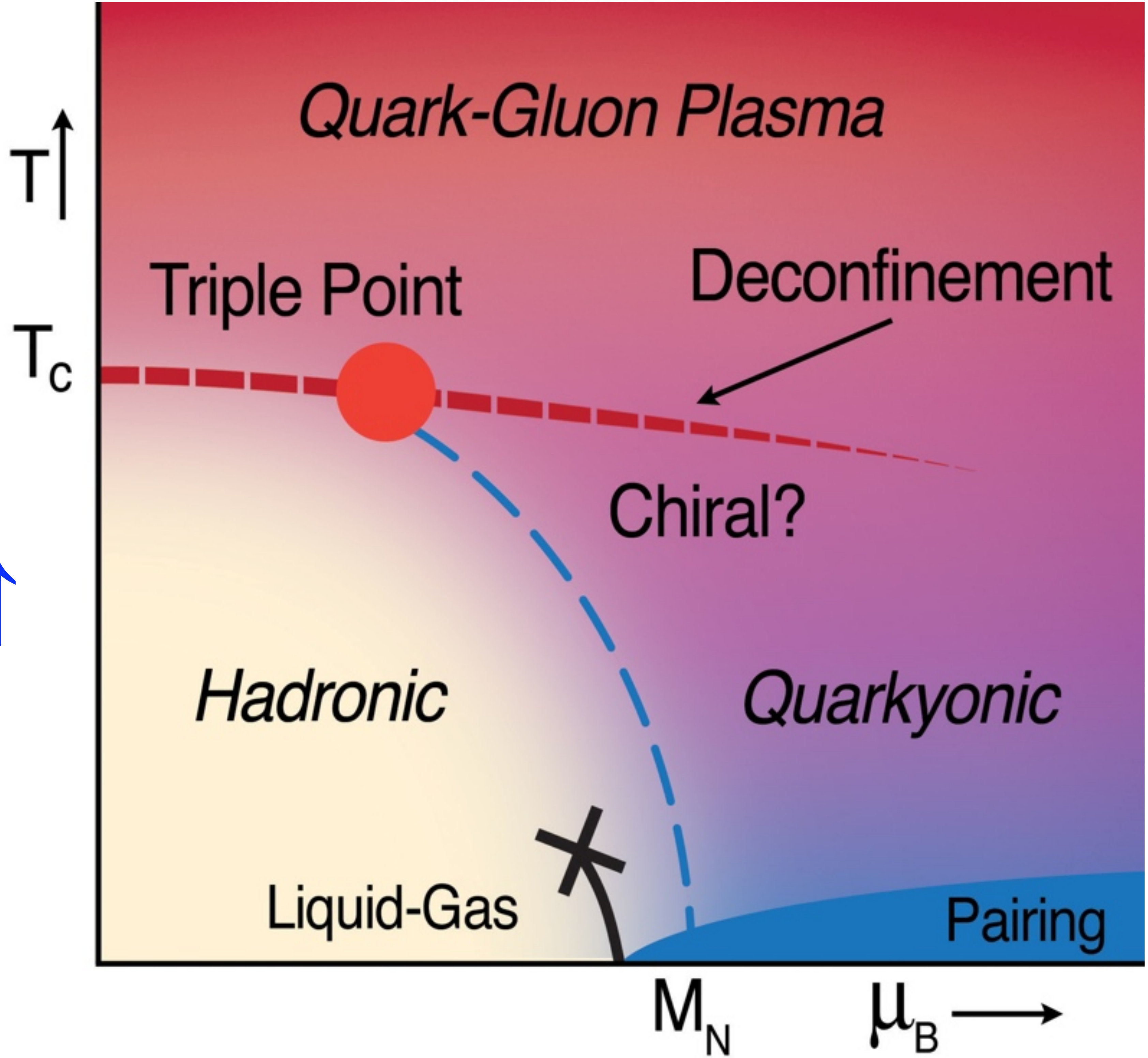

Quark-Gluon Plasma
T
Triple Point
Deconfinement
Tc
Chiral?
Hadronic
Quarkyonic
Liquid-Gas
Pairing
MN
μB

T↑

μB→

Quarkyonic Matter, a Triple Point, and Chiral Spirals in QCD

1. Large N_c, small N_f:

    Quark-yonic matter: a quark Fermi sea with a *confined* Fermi surface

    Triple point. Deconfining critical end point at *large* $\mu_{qk} \sim N_c^{1/2}$

2. A (different) phase diagram for QCD

3. Chiral spirals in Quarkyonic matter

4. "Purely pionic" effective Lagrangians and nuclear matter:

    The unbearable lightness of being (nuclear matter)?

L. McLerran & RDP, 0706.2191.
Y. Hidaka, L. McLerran, & RDP 0803.0279
L. McLerran, K. Redlich & C. Sasaki 0812.3585
Blaschke, Braun-Munzinger, Cleymans, Fukushima, Oeschler,
RDP, McLerran, Redlich, Sasaki, & Stachel (BBMCFOPMRSS) 0909...
T. Kojo, L. McLerran, & RDP 0909....
J. P. Blaizot, M. Nowak, L. McLerran & RDP 09......

So what *is* Quarkyonic matter?

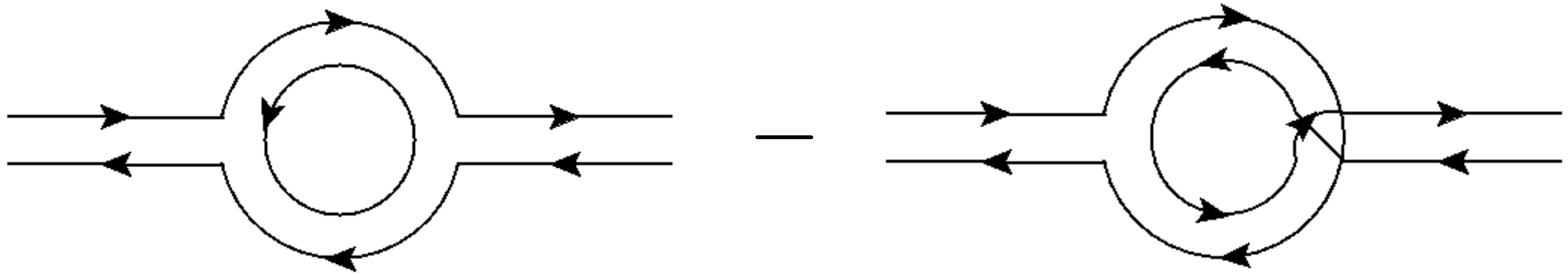
Dense nuclear matter

QCD at large N_c (small N_f)

In SU(N_c), gluons matrices, N_c x N_c, quarks column vectors.
Denote fund. rep. by a line: quarks have one line, gluons have two.

't Hooft '74: let N_c = # colors $\rightarrow \infty$, $\lambda = g^2 N_c$ *fixed*. Keep N_f = # flavors *finite*.

Consider gluon self energy at 1 loop order. For *any* N_c, color structure in all diagrams (3 gluon & 4 gluon vertices) reduces to (Hidaka & RDP 0906.1751)



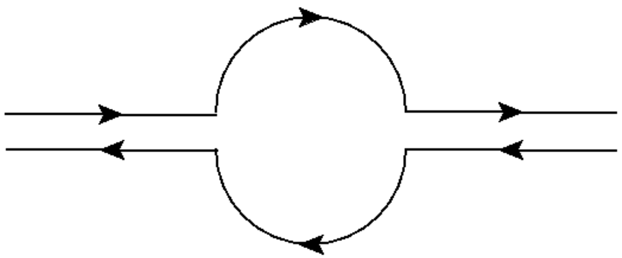
$$\sim g^2 N_c \sim \lambda \qquad\qquad \sim g^2 \sim \frac{\lambda}{N_c}$$

First diagram is "planar". Second, involving trace, is not, is down by $1/N_c$.

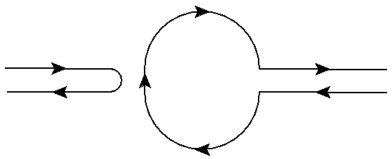
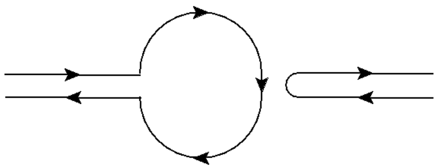
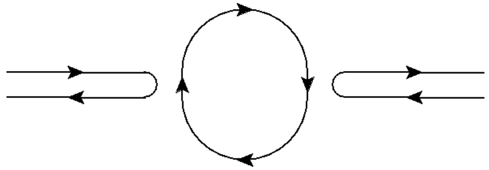
At large N_c and small N_f, planar diagrams dominate.

Large N_c and small N_f: *glue* dominates

Contribution of the quarks to the gluon self energy at 1 loop order, any N_c:



$$\sim g^2 N_f \sim \frac{1}{N_c} N_f \lambda$$

$- \frac{1}{N}$  $- \frac{1}{N}$  $+ \frac{1}{N^2}$ 

If $N_f/N_c \to 0$ as $N_c \to \infty$, loops *dominated* by gluons, *blind* to quarks.

Quarks act *something* like external sources, not quite.
N.B.: limit of large N_c, small N_f is *free* of the pathologies of $N_f = 0$ (quenched)

No problems considering nonzero quark density, μ_{qk}:
quarks do *not* affect gluons when $\mu_{qk} \sim 1$!

Phases at large N_c : *pressure* as an order parameter

$T = \mu_{qk} = 0$: confined, only color singlets. Glueballs, meson masses ~ 1.
Baryons *very* heavy, masses ~ N_c, so no virtual baryon anti-baryon pairs.

$T \neq 0$, $\mu_{qk} = 0$:

$T < T_c$: Hadrons. T_c ~ mass ~ 1. # hadrons ~ 1, so pressure = $p \sim 1$: *small*.

$T > T_c$: Quark-Gluon Plasma. Deconfined gluons & quarks.
gluons ~ N_c^2, so $p \sim N_c^2$: *big*. Dominated by gluons.

$T \neq 0$, $\mu_{qk} \neq 0$: usual mass threshold, baryons only when $\mu_{qk} > M_N/N_c = m_{qk} \sim 1$.

$T < T_c$, $\mu_{qk} < m_{qk}$: Hadronic "box" in T-μ_{qk} plane: *no* baryons.

$T > T_c$ any μ_{qk} : Quark-Gluon Plasma. Some quarks, so what, $p_{qk} \sim N_c$.

$T < T_c$, $\mu_{qk} > m_{qk}$: # quarks ~ N_c , so $p \sim N_c$: *dense* nuclear matter (*not* dilute)
Confined phase! But Fermi sea of *quarks*? "*Quark-yonic*"

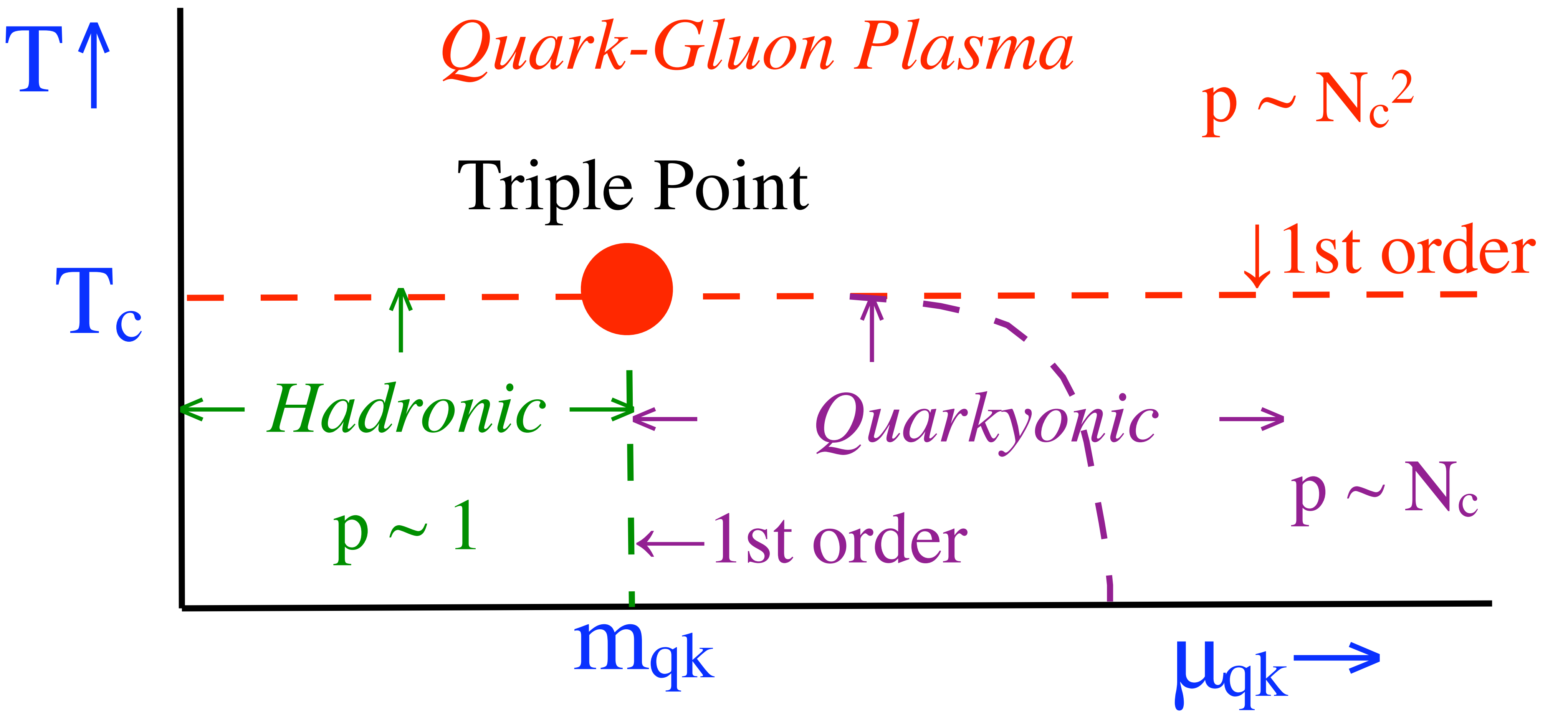
Phase diagram at large N_c and small N_f

Lattice (Teper, 0812.0085): deconfining transition 1st order at $T \neq 0$, $\mu_{qk} = 0$.
must remain so when $\mu_{qk} \neq 0$. *Straight* line in T - μ_{qk} plane.

Hadronic/Quarkyonic transition: energy density jumps by N_c, 1st order?

Chiral transition: in Quarkyonic phase?

True triple point!



Lattice: (pure glue) SU(3) close to SU(∞)

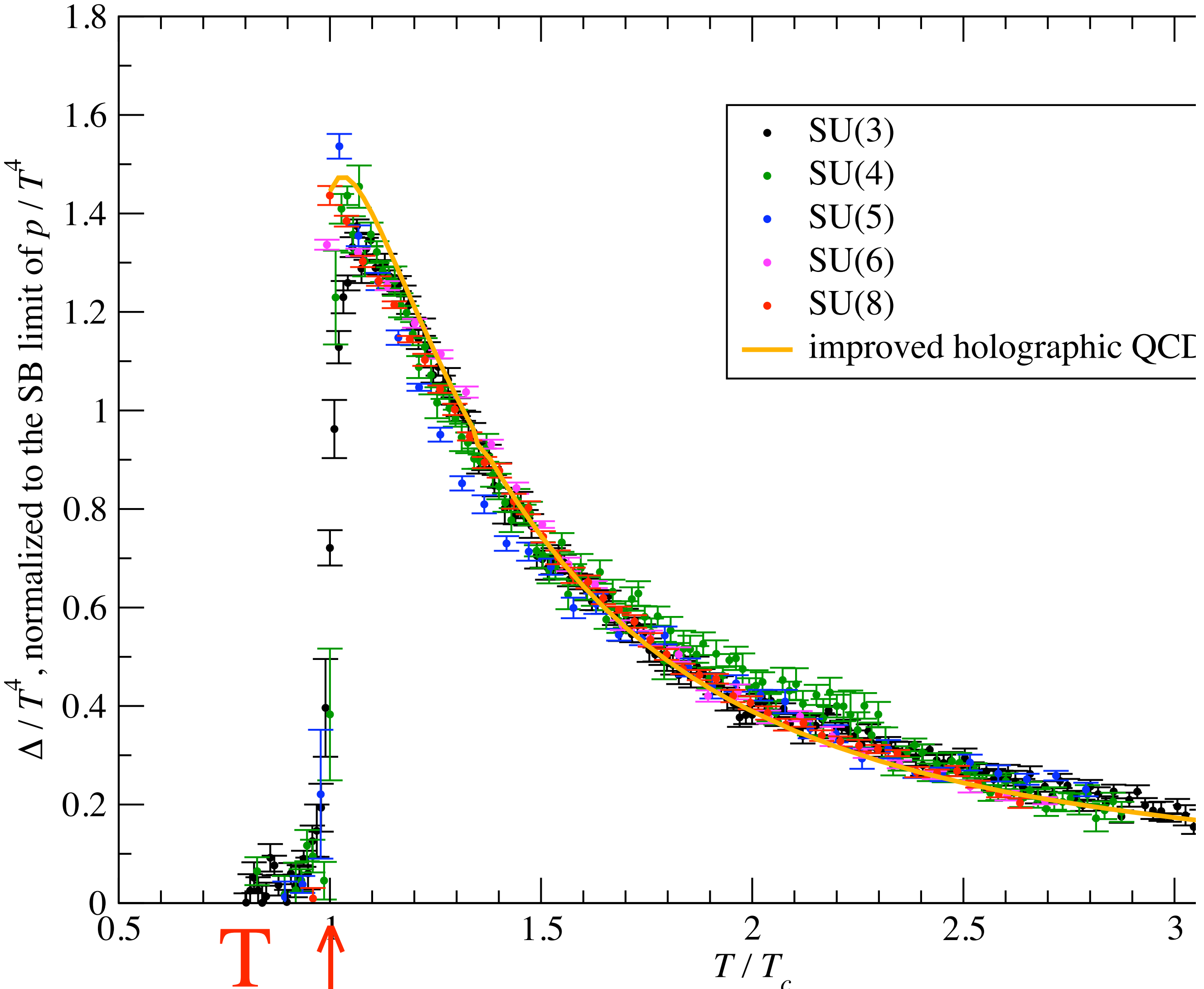
Panero, 0907.3719: $SU(N_c)$, *no quarks*, $N_c = 3, 4, 5, 6, 8$.

Deconfining transition first order, $N_c = 3$ close to $N_c = \infty$

Improved holographic: *fit* of scalar potential

$$\frac{e-3p}{N^2\,T^4} \sim \text{const.}$$

$$\frac{e-3p}{N^2\,T^4} \uparrow$$



$T_c \uparrow$

$T/T_c \rightarrow$

Triple point for water

Triple point where three lines of first order transitions meet.

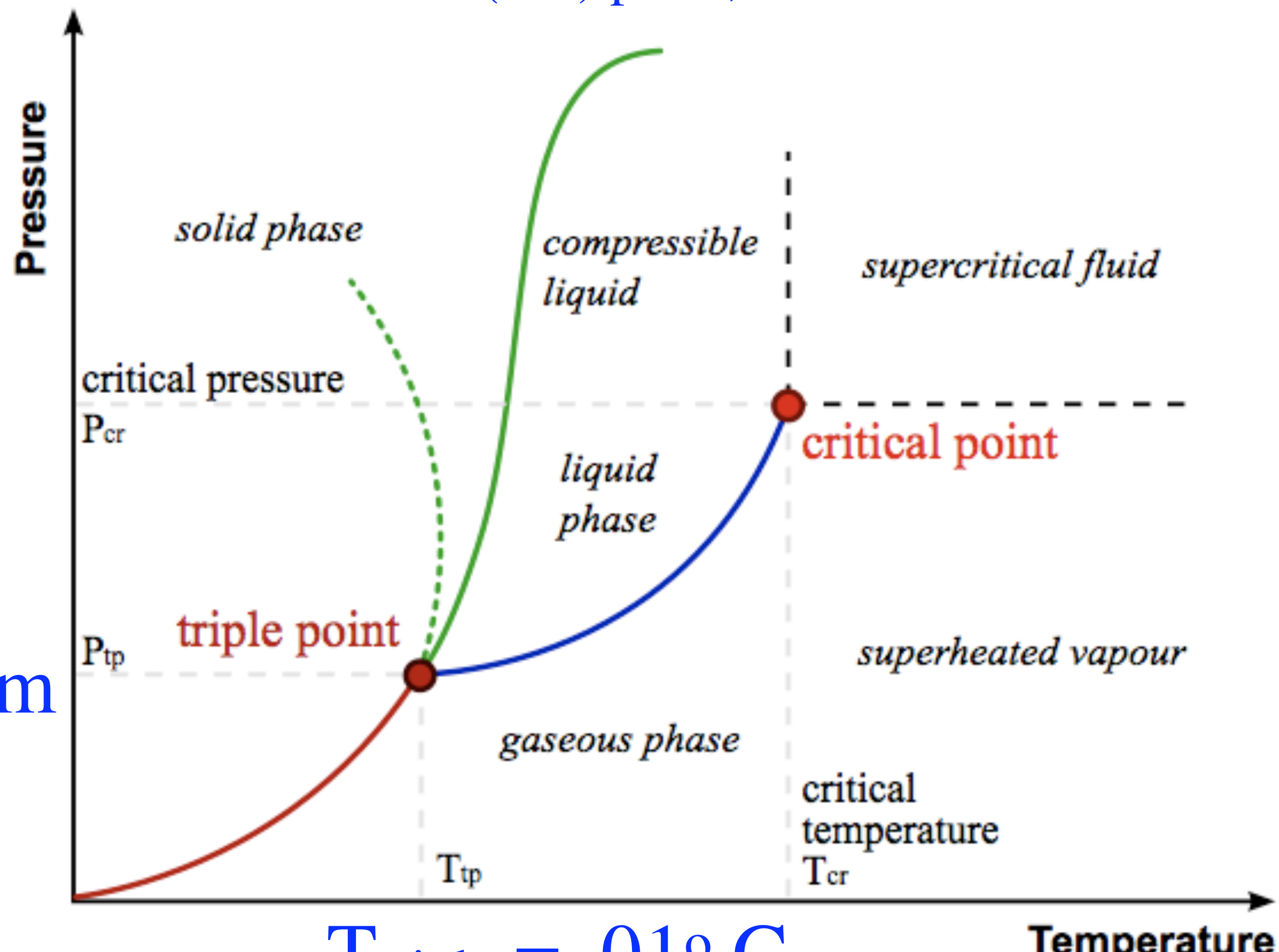
E.g., for ice/water/steam, in plane of temperature and pressure.

(Generalizes: four lines of first order transitions meeting is a quadruple point.)

Generically, *distinct* from critical (end) point, where one first order line ends.

$P \uparrow$

$P_{triple} = .006$ atm



$T \rightarrow$

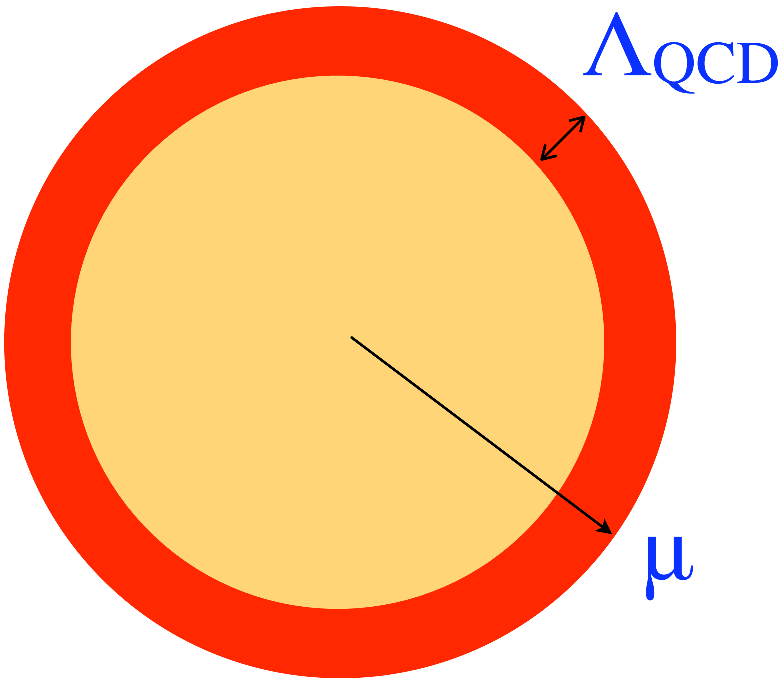
$T_{triple} = .01^{o}$ C

Quarkyonic phase at large N_c, large μ?

Let $\mu \gg \Lambda_{QCD}$ but $\sim N_c^0$. Coupling runs with μ, so pressure $\sim N_c$ is close to perturbative! How can the pressure be (nearly) perturbative in a confined theory?

Pressure: dominated by quarks far from Fermi surf.: perturbative,

$p_{qk} \sim N_c\, \mu^4 \,(1 + g^2(\mu) + g^4(\mu) \log(\mu) + ...)$



Within Λ_{QCD} of Fermi surface: *confined* states.

$p_{qk} \sim N_c\, \mu^4 \,(\Lambda_{QCD}/\mu)^2$, *non*-perturbative.

Within skin, only confined states contribute.

Fermi sea of quarks + Fermi surface of bar-yons
= "quark-yonic". *N=3?*

Pressure dominated by quarks.

But transport properties *dominated* by confined states near Fermi surface!

For QCD: what is (cold) nuclear matter like at high density?
Just a quark NJL model?

Deconfining critical end point at (large) $\mu_{qk} \sim N_c^{1/2}$

Semi-QGP theory of deconfinement: Hidaka & RDP 0803.0453

$$A_0 = \frac{T}{g}\, Q$$

For large μ: compute one loop determinant in background field.

Korthals-Altes, Sinkovics, & RDP hep-ph/9904305

$$S_{qk} = \mathrm{tr}\,(\mu + i\,T\,Q)^4\ ,\ T^2\ \mathrm{tr}\,(\mu + i\,T\,Q)^2\ ,\ N_c^2\,T^4\,V(Q)$$

RDP ‘09: for large μ, expand:

$$S^{qk}_{\mu\sim\sqrt{N_c},T\sim 1} \sim N_c\,\mu^4 - 6\,\mu^2\,T^2\,tr\,Q^2 + \ldots \sim N_c^3\ ,\ N_c^2\ (tr\,Q^2/N_c)$$

Consider $\mu \sim N_c^{1/2}$, $T \sim 1$: gluons *do* feel quarks.

Term $\mu^4 \sim N_c^3$ dominates, but *independent* of Q *and* temperature.

Term $\mu^2 \sim N_c^2$ Q-dependent. Breaks $Z(N_c)$ symmetry, so washes out 1st order deconfining transition: Deconfining Critical End Point (CEP)

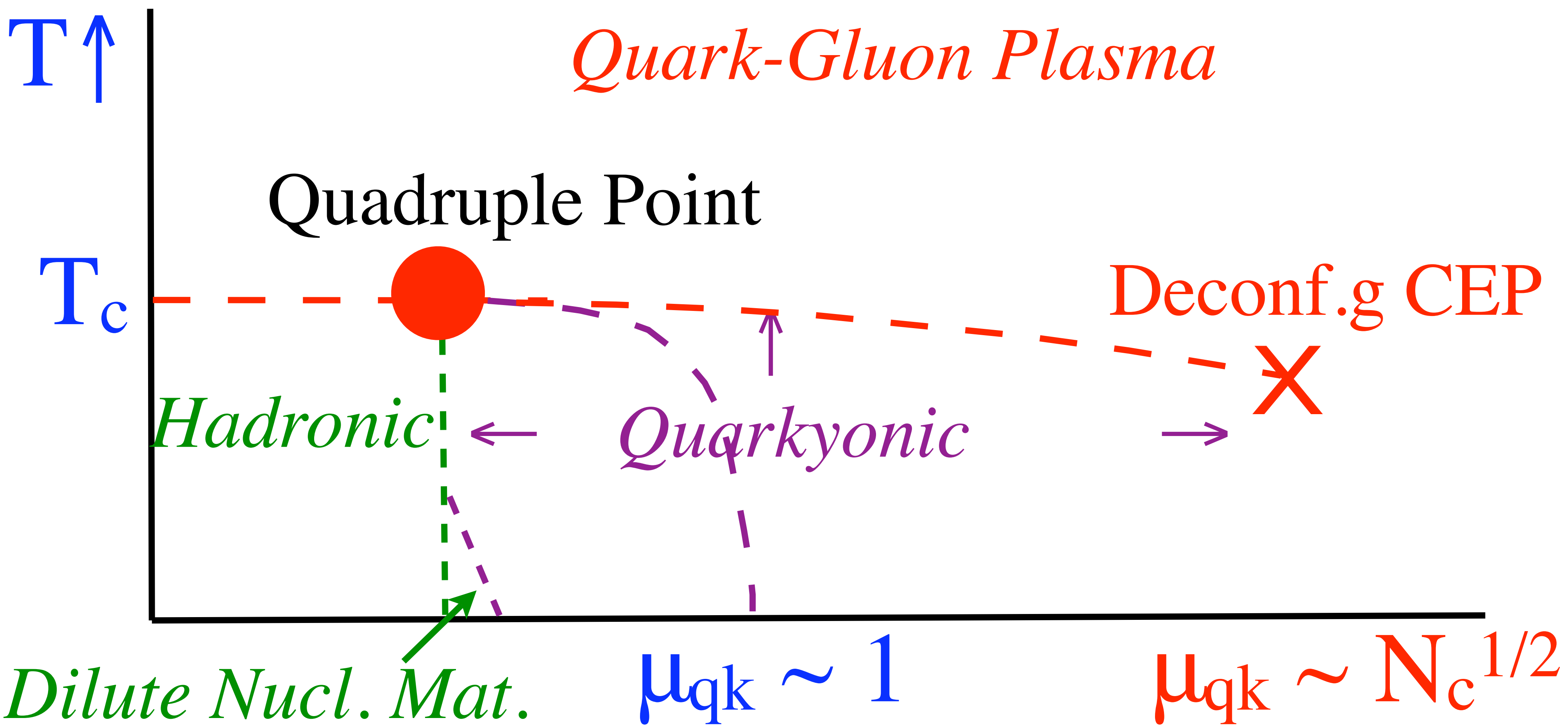
Phase diagram at large N_c and small N_f, II

About deconfining Critical End Point (CEP), smooth transition between deconfined and quarkyonic phases.

Since gluons are sensitive to quarks for such large μ, expect curvature in line.

Triple point still well defined, as coincidence of three 1st order lines.

Chiral transition?



So what does this have to do with experiment?

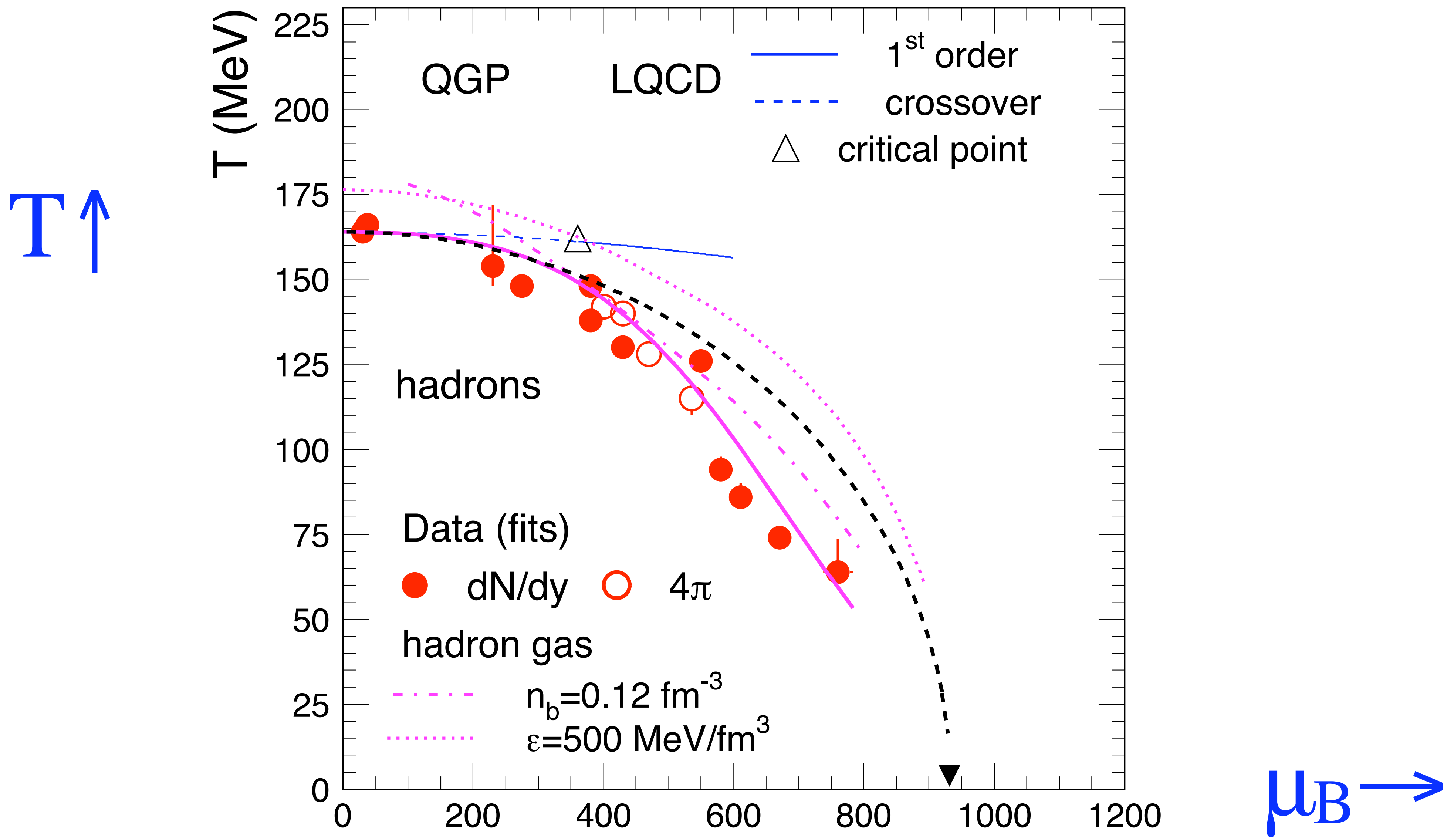
Strange "MatterHorn" ≈ Triple Point?

Wonderous utility of statistical/hadron resonance gas models

Chemical equilibriation at SIS, AGS, SPS, RHIC, and onto NICA and FAIR:

Braun-Munzinger, Cleymans, Oeschler, Redlich, Stachel

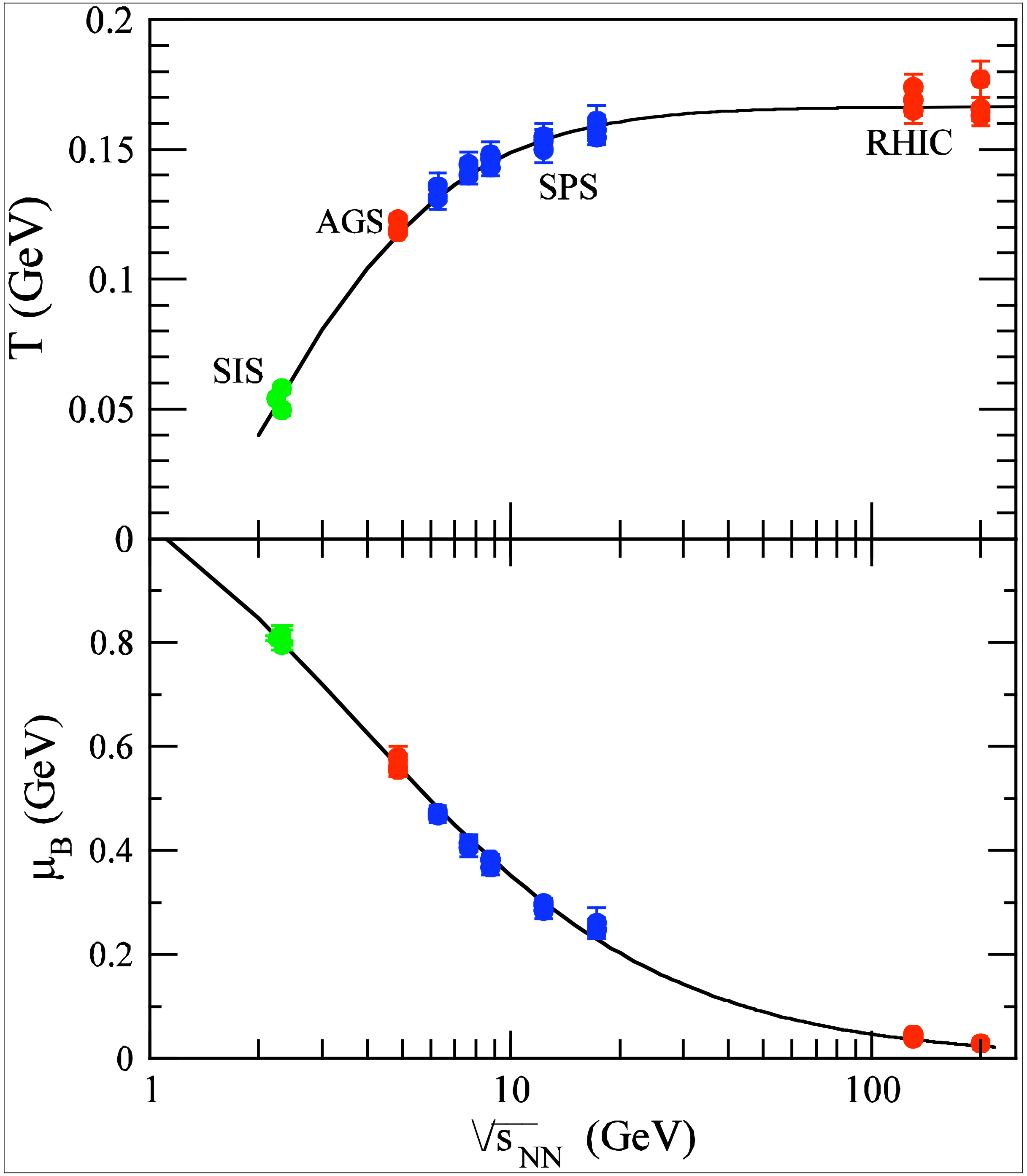
plus: Bialas, Biro, Broniowski, Florkowski, Levai, Ko, Satz + ...



Smooth evolution in T, μ_{Baryon} with $\sqrt{s_{NN}}$

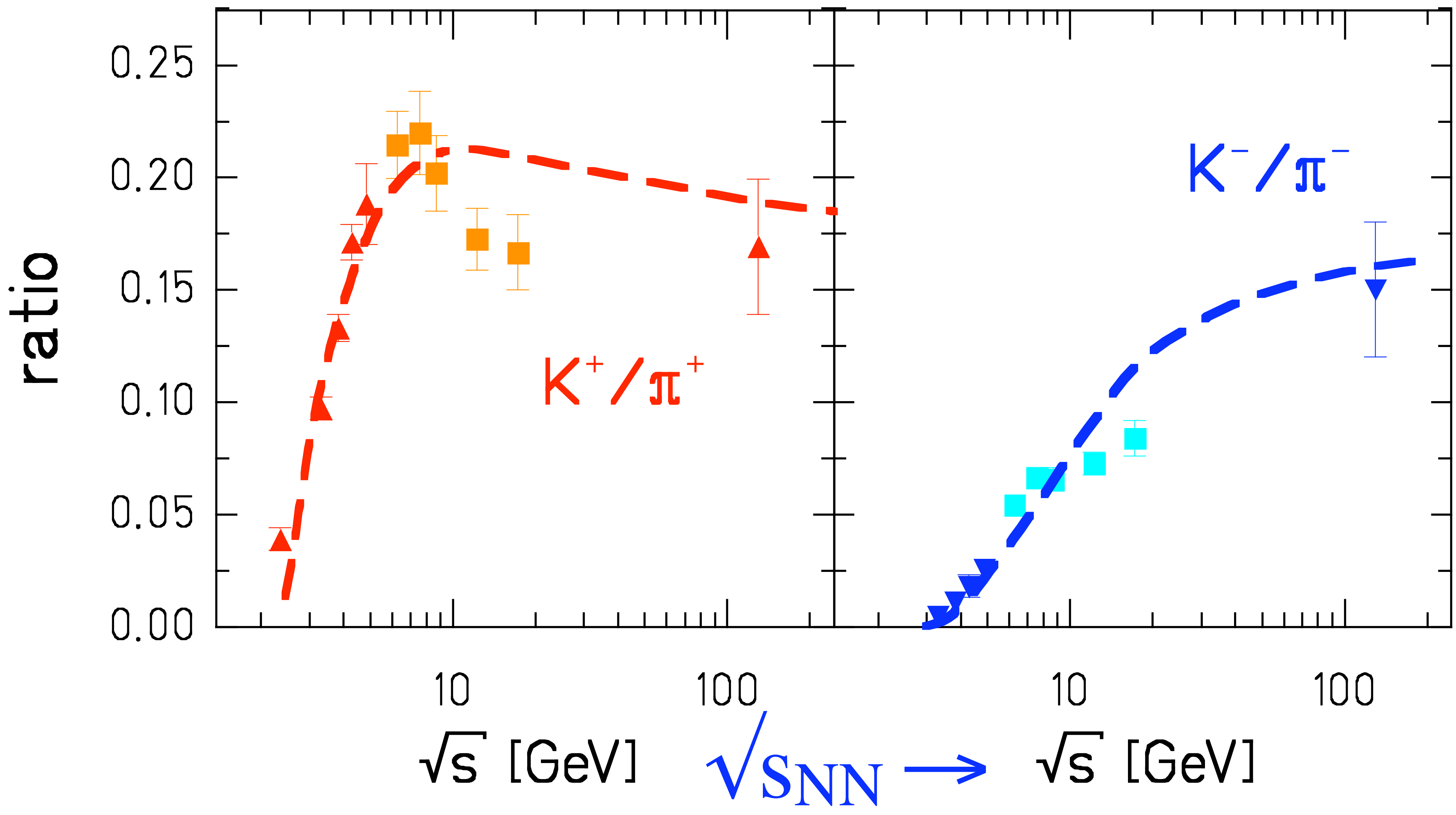
$T\uparrow$

$\mu_B\uparrow$



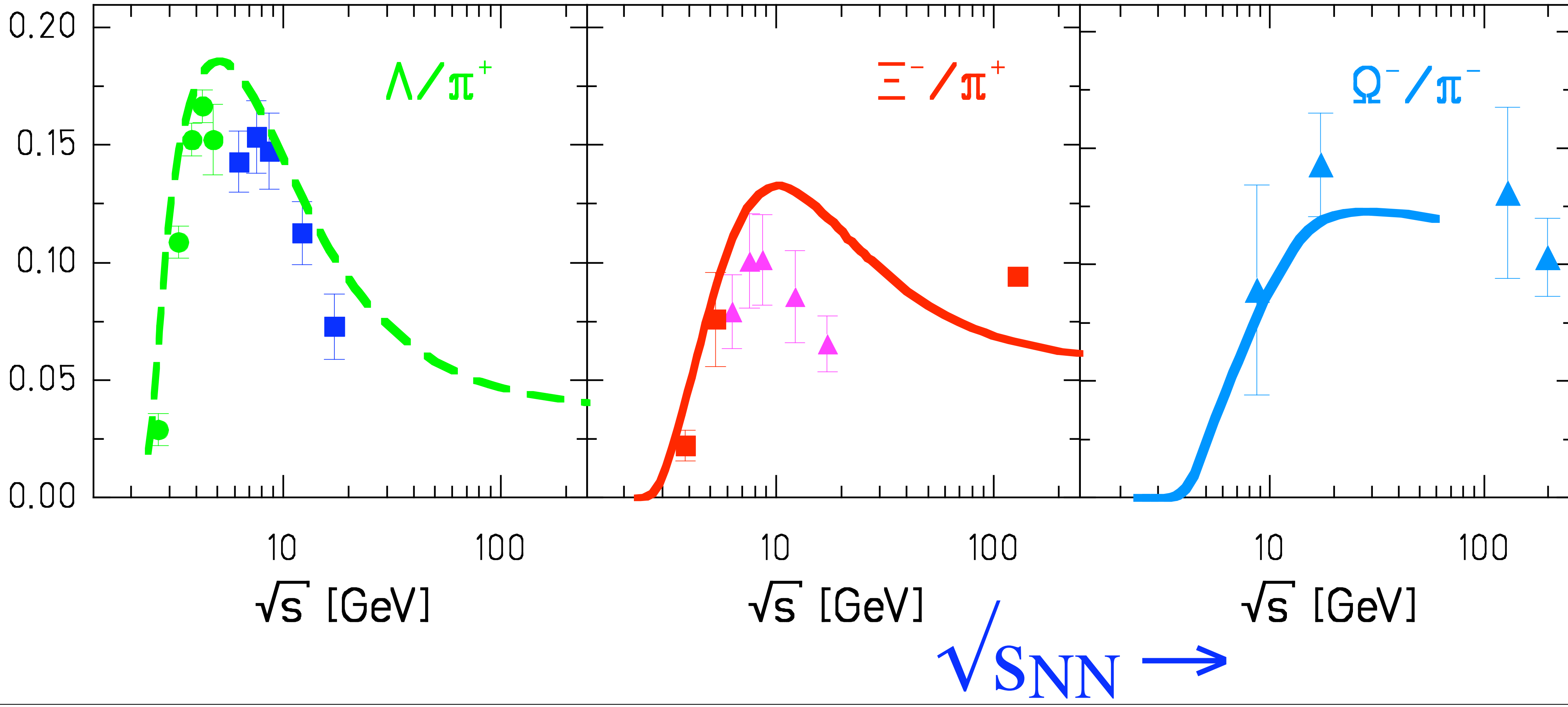
$\sqrt{s_{NN}} \rightarrow$

Strange MatterHorn: peak in K^+/π^+, *not* K^-/π^-



Strange MatterHorn: also in baryons

Natural to have peaks in K^+/π^+, strange baryons: start with (s s-bar) pairs.
At $\mu \neq 0$, strange quarks combine into baryons, anti-strange into pions.
For different baryons, peaks do not occur at same energy, but nearby, so not true phase transition, but approximate.

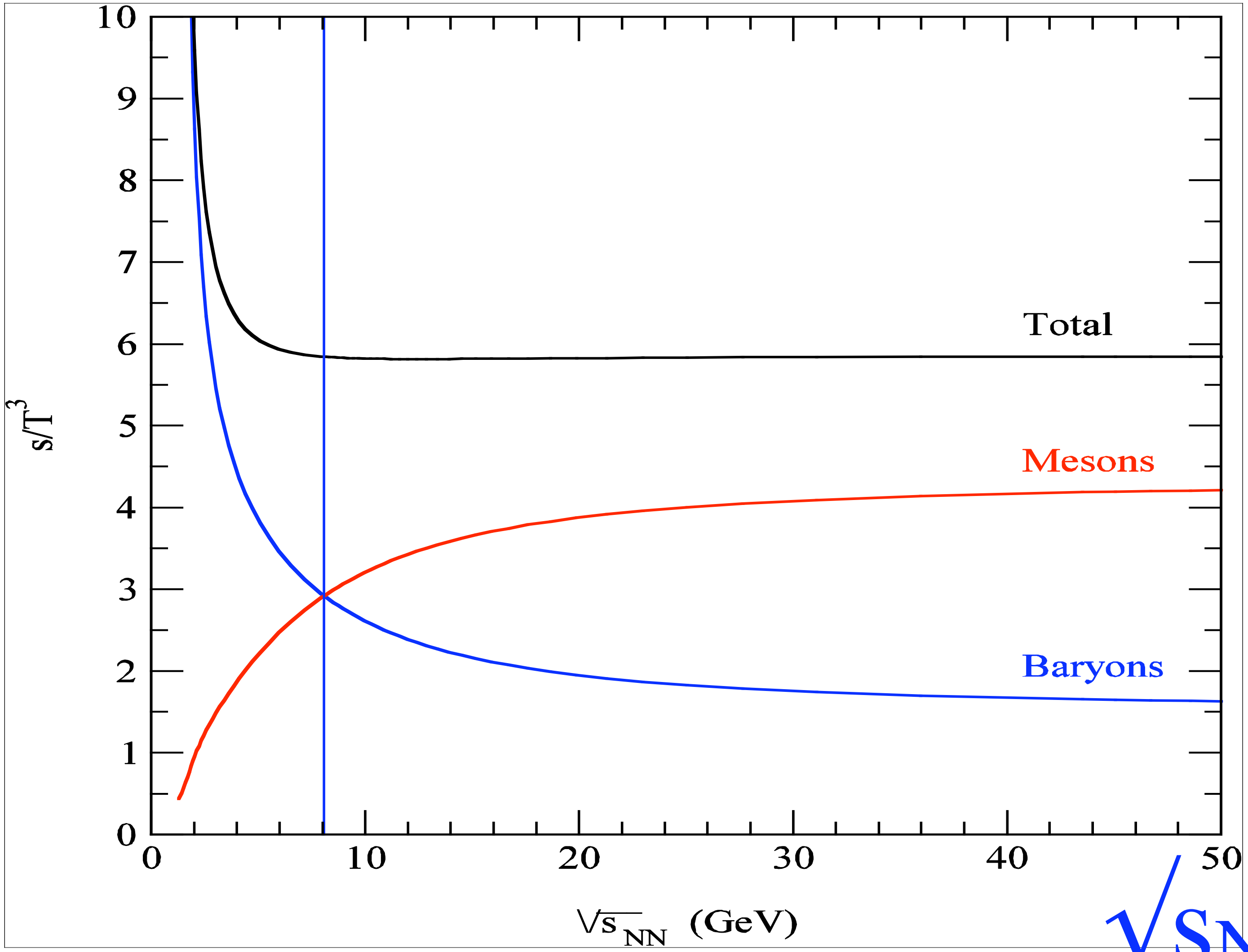


Strange MatterHorn and the triple point?

Usual explanation of MatterHorn: transition from baryons to mesons at freezeout.

Or: changing from Hadronic/Quarkyonic boundary to Hadronic/QGP boundary: i.e., (approximate) triple point.

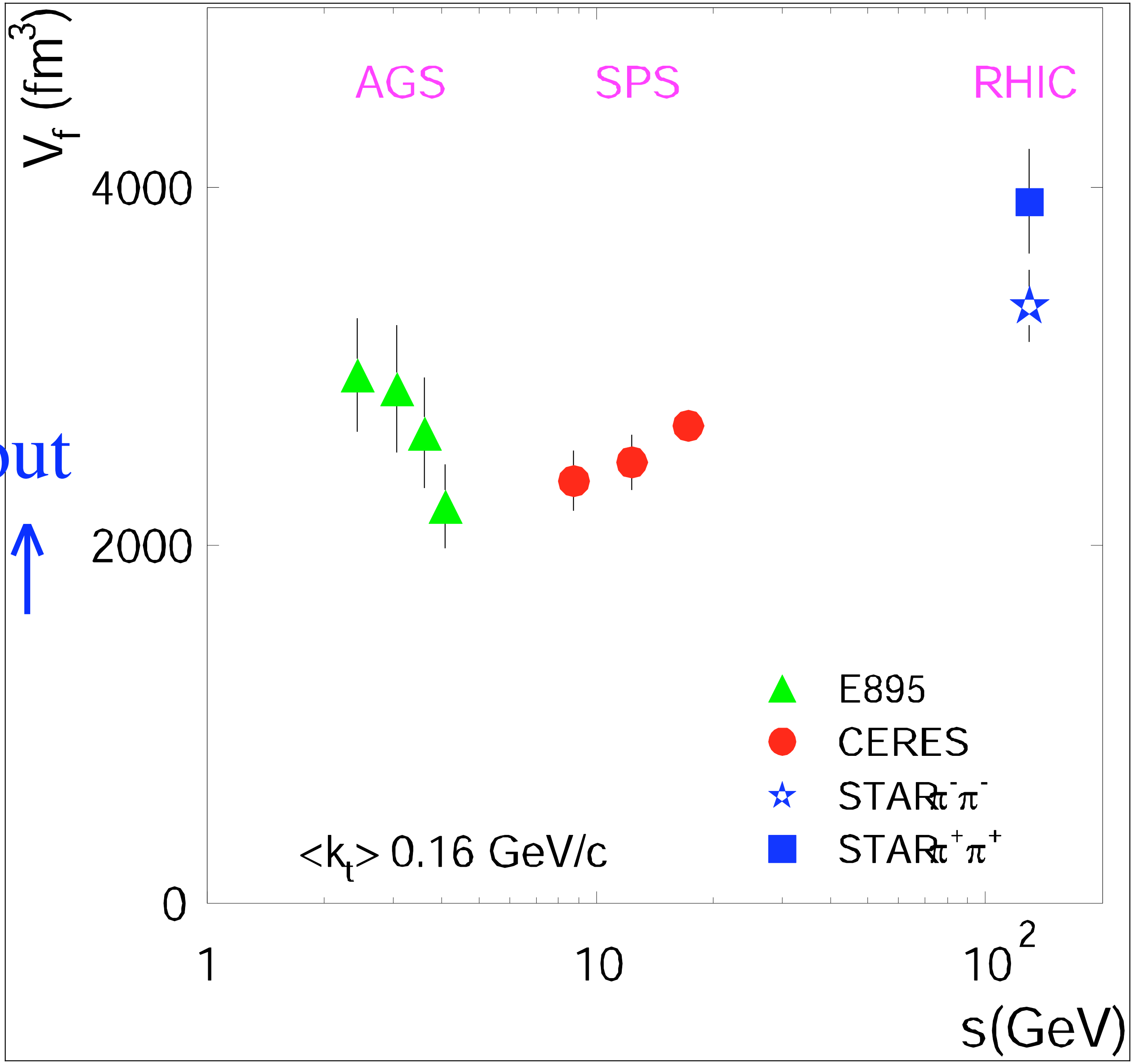
entropy density/T^3 ↑



$\sqrt{s_{NN}}$ →

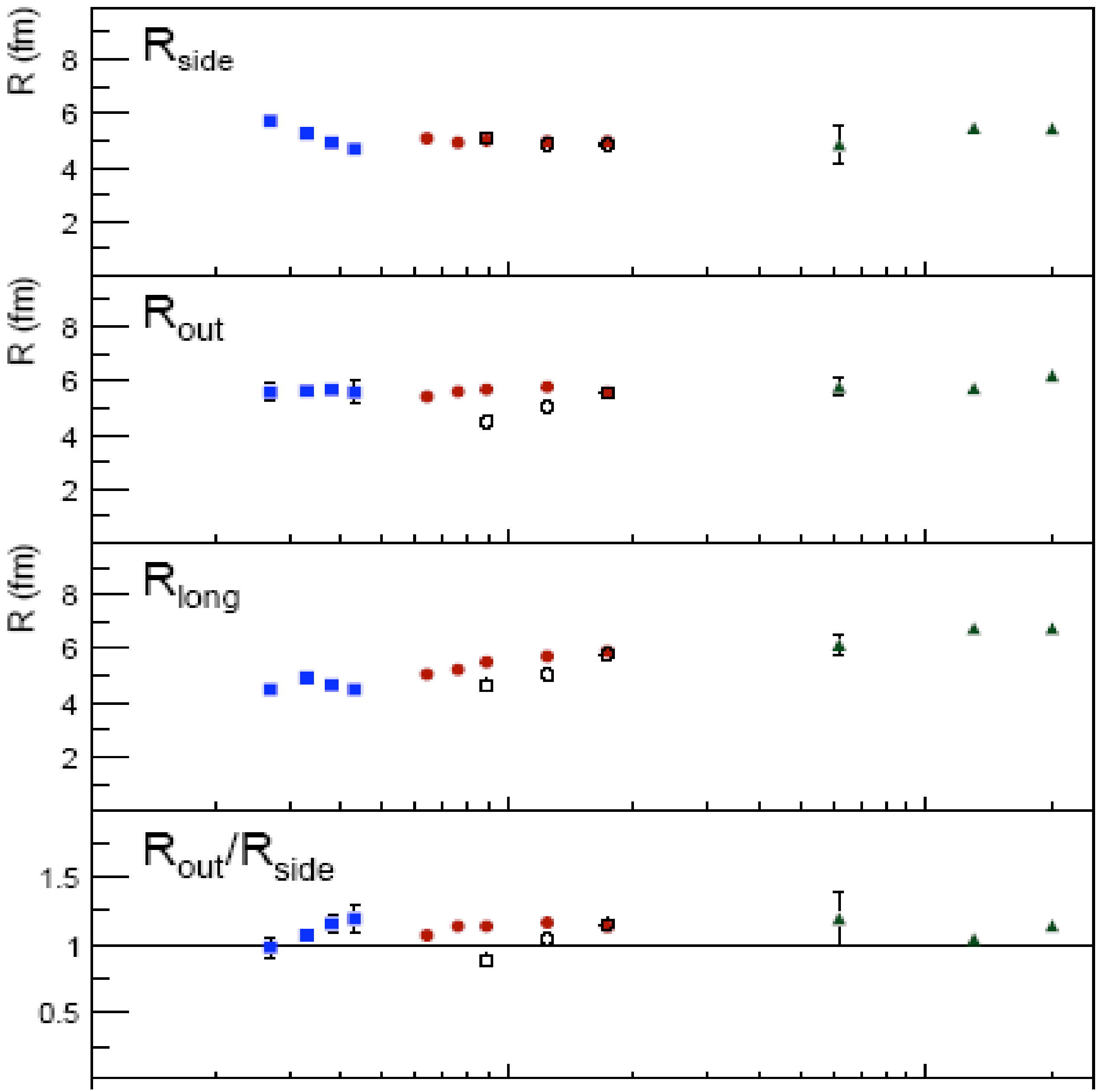
HBT radii: minimum near strange MatterHorn?

freeze out volume ↑



$\sqrt{s_{NN}}$ →

HBT radii: flat from NA49.



$\sqrt{s_{NN}} \rightarrow$

Triple point versus critical end point

Critical endpoint: correlation lengths *diverge*.

Hence: HBT radii should *increase*.

Effects should be greatest on the lightest particles, *not* the heaviest:

K^+/π^+ should *de*crease, not *in*crease. *Neither* is seen in the data.

Assume that at triple point, chiral transition splits from deconfining.

Leading operator which couples the two transitions is

Mocsy, Sannino, & Tuominen, hep-ph/0301229, 0306069, 0308135, 0403160:

$$c_1\,\ell\;\mathrm{tr}\,\Phi^\dagger\Phi \sim c_1\,\ell\;(\pi^2 + K^2 \ldots)$$

If this coupling c_1 flips sign, transitions diverge. Hence $c_1 = 0$ at triple point?

If so, leading coupling then becomes

$$c_2\,\ell\;\mathrm{tr} M\,\Phi \sim c_2\,\ell\;(m_\pi^2\pi^2 + m_K^2 K^2 + \ldots)$$

This coupling is proportional to mass squared: *bigger* for kaons than pions!

Enchancement of K^+/π^+, strange baryons due to dense environment.

Implicitly: line for chiral transition crossover, not 1st order.

Quarkyonic Chiral Spirals

"Chiral spirals" in the chiral Gross-Neveu model

Schon & Thies, hep-th/0003195; 0008175; Thies, 06010243: coined term
Basar & Dunne, 0806.2659, Basar, Dunne, & Thies, 0903.1868

Chiral Gross-Neveu model in 1+1 dimensions:

$$\mathcal{L} = \overline{\psi}\, i \not\partial\, \psi + G\left((\overline{\psi}\psi)^2 + (\overline{\psi}\gamma_5\psi)^2 \right)$$

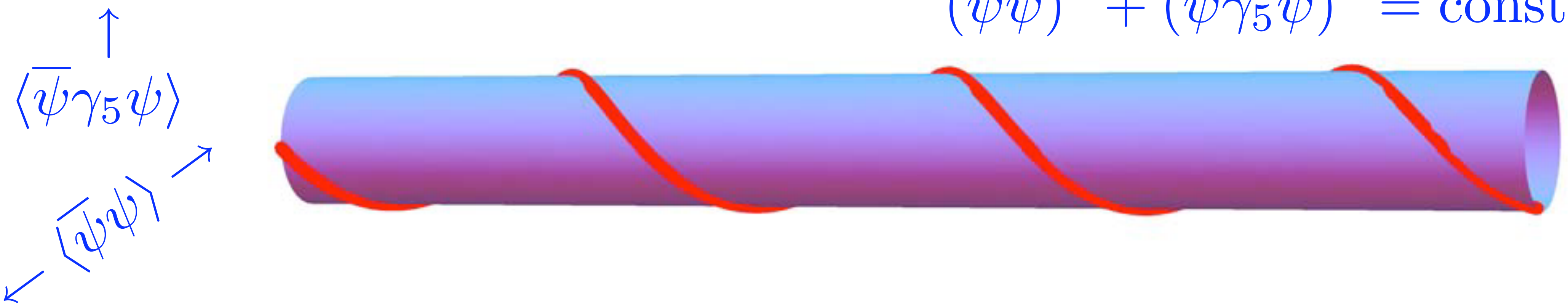
Continuous chiral symmetry:

$$\psi \rightarrow e^{i\theta\gamma_5}\, \psi$$

Model *exactly* soluble as # flavors → ∞.

At $\mu \neq 0$: periodic structure (crystal) which *oscillates* in space: "*chiral spiral*"
In chiral limit, oscillations symmetric about zero.

$$(\overline{\psi}\psi)^2 + (\overline{\psi}\gamma_5\psi)^2 = \text{const.}$$



Why lattice? Bosonization in 1+1 dim.'s:
Fermion current gives spatially varying scalar field

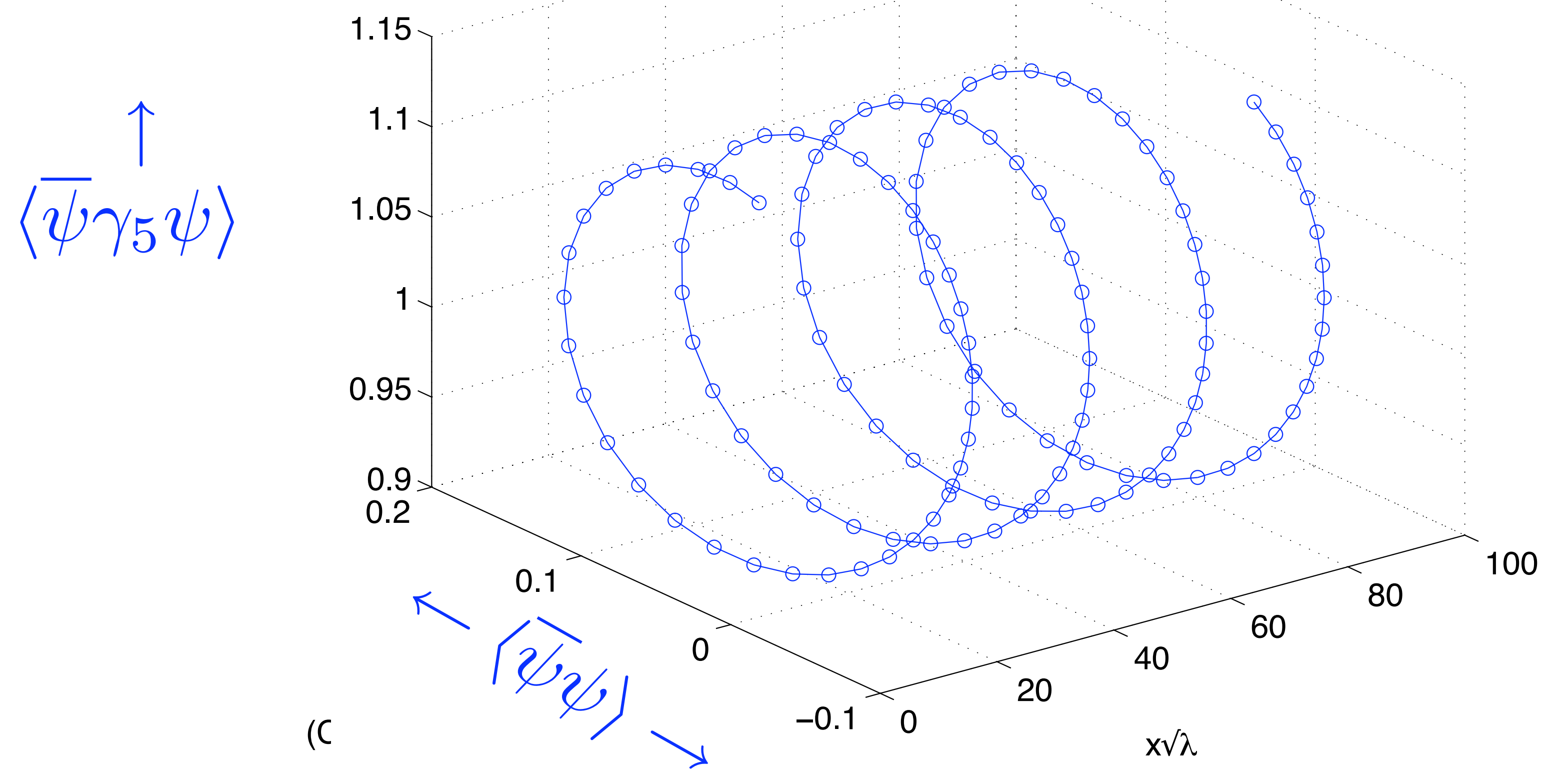
$$\overline{\psi}\,\gamma^0\psi = \partial_1\phi$$

Chiral spiral for QCD in 1+1 dimensions

Bringoltz, 0901.4035: ‘t Hooft model, QCD in 1+1 D, with *massive* quarks.

Works in Coulomb gauge, in *canonical* ensemble: fixed baryon number.
Solves numerically equations of motion under constaint of nonzero baryon #

Finds “chiral spiral”, with oscillations about *non*zero value:
magnitude of the oscillations decreases as g^2N_c/m^2 increases.



Quarkyonic Matter: from 4D to 2D

Simplest model of confinement: $$\Delta^{\mu\nu}_{\text{gluon}} = \frac{\sigma}{(P^2)^2} \left(\delta^{\mu\nu} - \frac{P^\mu P^\nu}{P^2} \right)$$

Bethe-Saltpeter kernel for a meson:

For quarks near the Fermi surface, can neglect transverse momenta, $p_\perp$:

$$ip_0 - \mu + \sqrt{(p_F + p_1)^2 + p_\perp^2} \approx ip_0 + p_1 + p_\perp^2/2\mu + \ldots$$

So can integrate over $p_\perp$
in the gluon propagator:

$$\int d^2 p_\perp \, \frac{1}{(p_0^2 + p_1^2 + p_\perp^2)^2} = \frac{1}{p_0^2 + p_1^2}$$

End up with an effective model for QCD (at nonzero density) in 1+1 dimensions.

SU(2 N_f) symmetry for N_f flavors of 2D quarks

Start with a chiral basis: $$\psi_{R,L} = (1 \pm \gamma_5)\,\psi/2$$

and introduce helicity projections: $$\psi_{R\pm,L\pm} = (1 \pm \gamma_0\gamma_1)\,\psi/2$$

Effective quark Lagrangian in 1+1 D: Shuster & Son, hep-ph/9905448

$$\mathcal{L}_{\rm qk} = \sum_{R,L}\sum_{\pm} \overline{\psi}_{R\pm,L\pm}(\gamma^0(\partial_0 - i\mu) + \gamma^1\partial_1)\psi_{R\pm,L\pm}$$

For N_f flavors, have SU($2N_f$) symmetry in 1+1 D. Like heavy quark symmetry.

1+1 D Dirac matrices = Γ^μ : $\Gamma_5 = \Gamma^0\,\Gamma^1$.

For one flavor: $$\Psi = (\,\psi_{R+}, \psi_{R-}, \psi_{L-}, \psi_{L+})$$

QCD Lagrangian maps directly: $$(\overline{\psi}\,\gamma^0\,\psi, \overline{\psi}\,\gamma^1\,\psi) \rightarrow (\overline{\Psi}\,\Gamma^0\,\Psi, \overline{\Psi}\,\Gamma^1\,\Psi)$$

Condensates map as: $$(\overline{\psi}\psi, \overline{\psi}\,\gamma^0\gamma^1\,\psi) \rightarrow (\overline{\Psi}\Psi, \overline{\Psi}\,\Gamma_5\,\Psi)$$

Many terms break SU($2N_f$) symmetry: $$\overline{\psi}\,\gamma_5\tau_3\,\psi \rightarrow \overline{\Psi}\,\Gamma_5\tau_3\,\Psi$$

Chiral Spirals in 1+1 D and 3+1 D

Effective quark Lagrangian in 1+1 D: 2D gluons plus 2D quarks:

$$\mathcal{L}_{\rm qk} = \overline{\Psi}\left(i\not{D}_{2D} + \mu\Gamma^0\right)\Psi \; ; \; g^2_{2D} = g^2\,\sigma$$

Perform anomalous chiral rotation *linear* in x: $\Psi \rightarrow \exp(i\,\mu\,\Gamma_5\,x)\,\Psi$

Find: anomalous chiral rotation *removes* μ from quark Lagrangian.

Fischler, Kogut & Susskind '79 : $\Gamma^1\,\Gamma_5 = \Gamma^0$

In 1+1 dimensions, Fermi sea => vacuum.

Where is the Fermi density? From the anomaly. Chiral condensate:

$$< \overline{\Psi}\,\Psi >_{\mu=0} \rightarrow \cos(2\mu x)\; < \overline{\Psi}\Psi >_\mu + i\sin(2\mu x)\; < \overline{\Psi}\,\Gamma_5\,\Psi >_\mu$$

$< \overline{\Psi}\,\Psi >_{\mu=0} \neq 0$ => Chiral Spiral (= Chiral Density Wave) in Fermi sea.

For 4D quarks, chiral spiral = condensate in helicity:

$$< \overline{\psi}\psi >= \cos(2\mu x)\,c \; ; \; < \overline{\psi}\,\gamma^0\gamma^1\,\psi >= i\sin(2\mu x)\,c$$

(Many) Massless Modes about Chiral Spirals

Excitations near the Fermi surface:

Witten '84: non-Abelian bosonization for QCD. a, b= 1...N_c. i,j = 1... N_f.

$$J_+^{ij} = \overline{\psi}^{a,i} \psi^{a,j} \sim g^{-1} \partial_+ g \; ; \; J_+^{ab} = \overline{\psi}^{a,i} \psi^{b,i} \sim h^{-1} \partial_+ h \; .$$

Steinhardt '80. Affleck '86. Frishman & Sonnenschein, hep-th/920717...

Armoni, Frishman, & Sonnenschein , hep-th/0011043..

QCD in 1+1 D: "fractionalization" of color and flavor.

Flavor currents: Wess-Zumino Witten model. *Many* massless excitations!

Color currents: gauged WZW model. Massive excitations of 't Hooft model

1+1 D: only quasi long range order. At large N_c, disorders at scales exp(-N_c).

Effective 1+1D model embedded in 3+1D.

Transverse dimensions: break SU(2 N_f) to SU(N_f), produce true long range order.

Quarkyonic Chiral Spirals versus ...

Chiral Density Waves (CDW) in *perturbative* regime:
Deryagin, Grigoriev, & Rubakov '92. Shuster & Son, hep-ph/9905448.
Rapp, Shuryak, and Zahed, hep-ph/0008207.
Shuster & Son: in perturbative regime, CDW only wins for $N_c > 1000\, N_f$!
Large N_c: Quarkyonic Chiral Spirals until $\mu \sim \sqrt{N_c}$.

QCD: *certainly* color superconductivity for asymptotically high density.
Quarkyonic Chiral Spirals (QCS) for intermediate density? Both pionic & kaonic

QCS analogous to pion condensation: (Migdal '71, Sawyer & Scalapino '72...)

$$< \overline{\psi}\psi >= \cos(2\mu x) < \overline{\psi}\psi >_0 \; ; \; < \overline{\psi}\, \gamma_5\, \psi >= i \sin(2\mu x) < \overline{\psi}\psi >_0$$

Pion condensation *like* chiral spiral in 1+1 D, *differs* from QCS in 3+1 D.

3+1 D NJL models (Nickel 0906.5295): flavor sym in 3+1 D ≠ 1+1 D so *no* QCS

Kaon condensation (Kaplan & Nelson '86) constant $<K^->$, *not* QCS.
Do expect kaonic QCS in QCD (if pionic QCS exist)!

The unbearable lightness of being (nuclear matter)

Nucleon-nucleon potentials from the lattice

Ishii, Aoki & Hatsuda, PACS-CS, 0903.5497

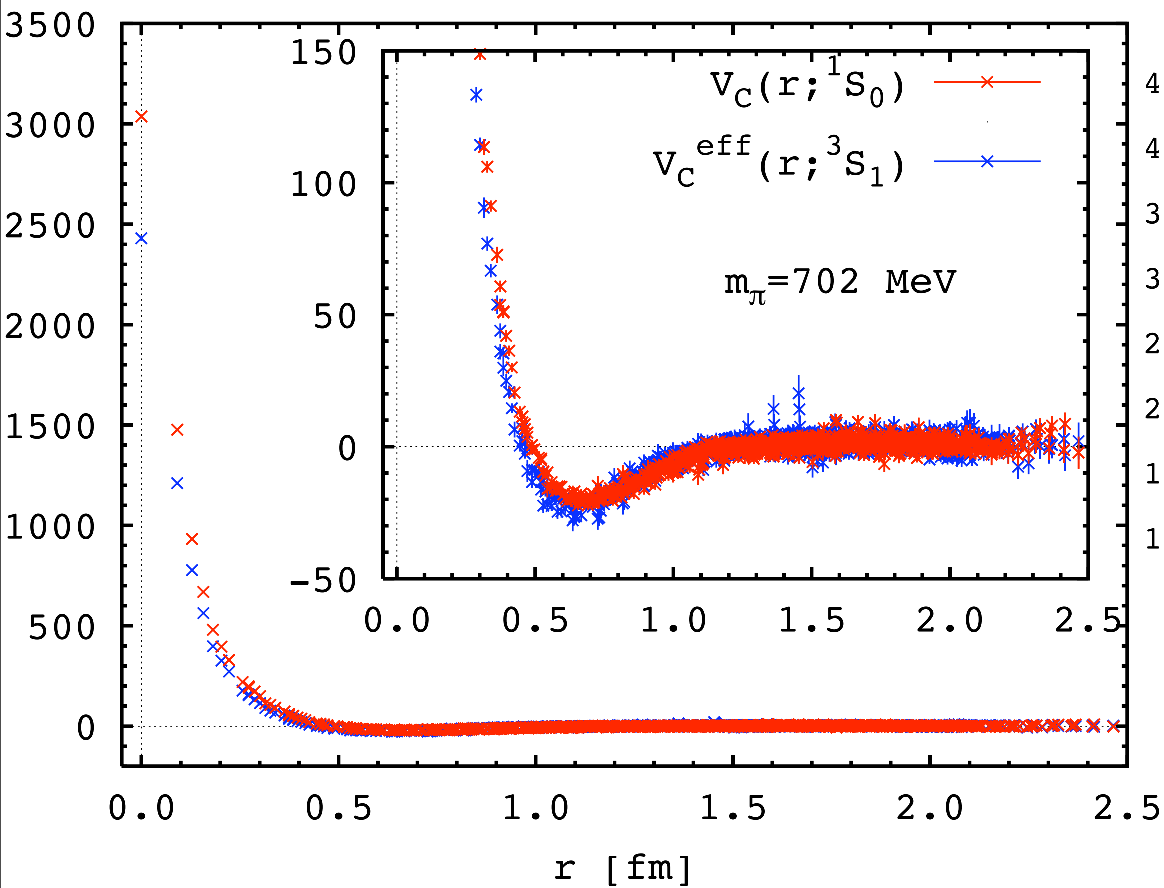
Nucleon-nucleon potentials from quenched and 2+1 flavors.

Pions heavy: 700 MeV (left) and 300 MeV (right)

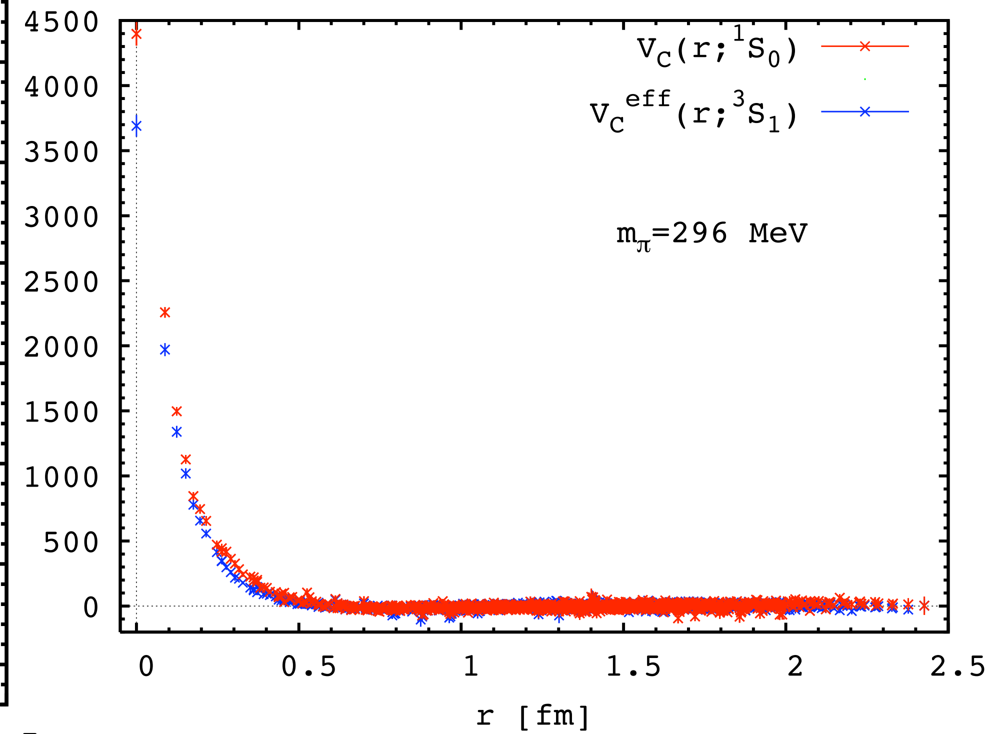
Standard lore: delicate cancellation. *So why independent of pion mass?*

Essentially *zero* potential plus strong hard core repulsion

$m_\pi = 702$ MeV



$m_\pi = 296$ MeV



Purely pionic nuclear matter

At infinite N_c, integrate out *all* degrees of freedom *except* pions:

Lagrangian power series in $U = e^{i\pi/f_\pi}$, $V_\mu = U^\dagger \partial_\mu U$

Infinite # couplings: Skyrme *plus* complete Gasser-Leutwyler expansion,

$$\mathcal{L}_\pi = f_\pi^2 \, V_\mu^2 + \kappa \, [V_\mu, V_\nu]^2 + c_1 \, (V_\mu^2)^2 + c_2 \, (V_\mu^2)^3 + \ldots$$

All couplings ~ N_c , every mass scale ~ typical hadronic.

Need *infinite* series, but nothing (special) depends upon exact values

Valid for momenta < f_π, masses of sigma, omega, rho...

Large N_c version of in-medium chiral perturbation theory:

W. Weise & ...: 0808.0856, 0802.2212,0801.1467, 0712.1613, 0707.3154 +...

Higher time derivatives, but no acausality at low momenta.

Purely pionic nuclear matter: *free* baryons

From purely pionic Lagrangian, take baryon as stationary point.

Find baryon mass ~ N_c, some function of couplings.

Couplings of baryon dictated by chiral symmetry:

$$\overline{\psi}\left(i\partial\!\!\!/ + M_B\, \mathrm{e}^{i\tau\cdot\pi\gamma_5/f_\pi}\right)\psi$$

By chiral rotation, $W = \exp(-i\pi\gamma_5/2f_\pi)$

$$\mathcal{L}_B = \overline{\psi}\left(iW^\dagger \partial\!\!\!/ W + M_B\right)\psi \sim \frac{1}{f_\pi}\overline{\psi}\gamma_5 \partial\!\!\!/ \pi\psi + \ldots$$

At large ~ N_c , $f_\pi \sim N_c^{1/2}$ is *big*. Thus for momenta k < hadronic, interactions are *small*, ~ $1/f_\pi^2 \sim 1/N_c$.

Thus: baryons from chiral Lag. free at large N_c , down to distances $1/f_\pi$.

Manifestly special to chiral baryons. True for u, d, s, but *not* charm?

The Unbearable Lightness of Being (Nuclear Matter)

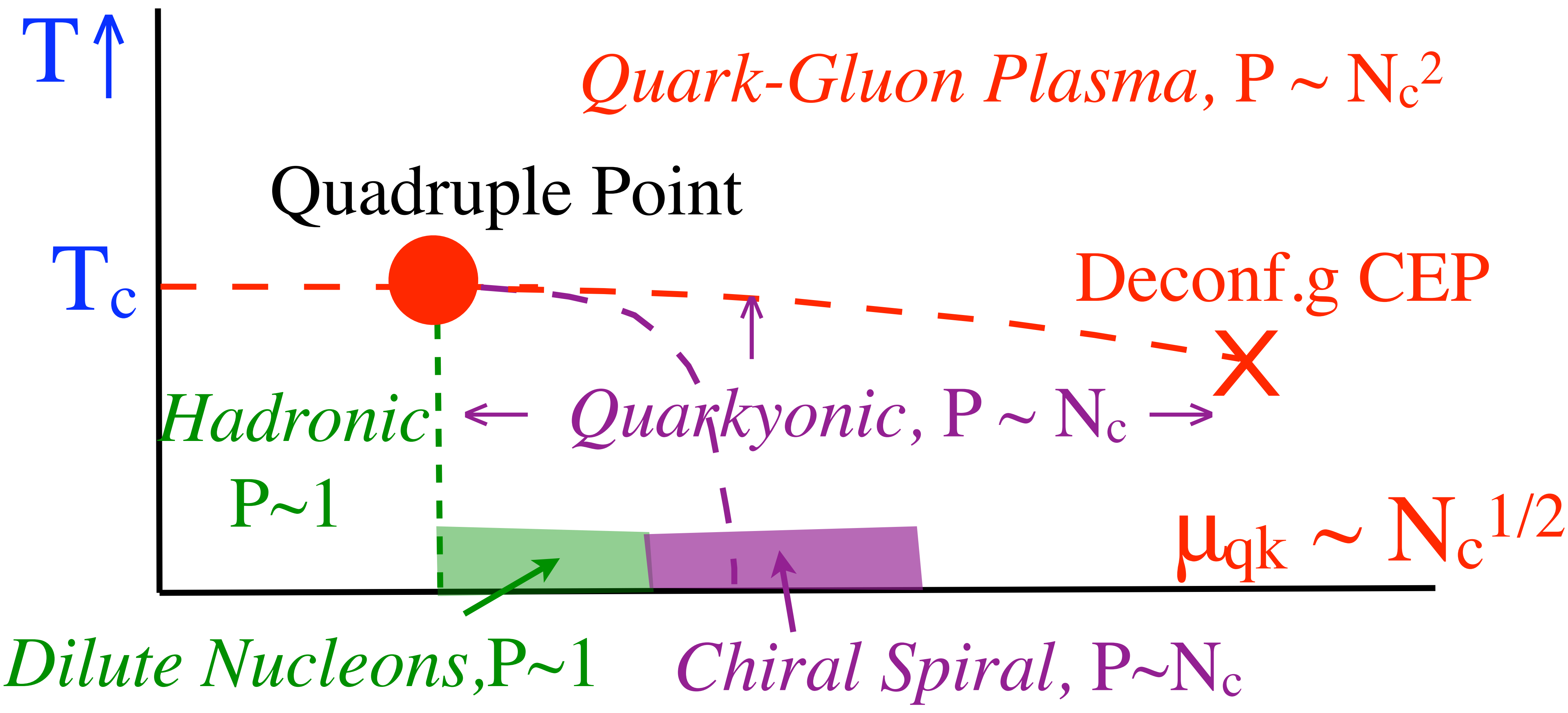
Use purely pionic Lagrangian for all of nuclear matter?

Then pressure ~ 1, and *not* N_c. Like hadronic phase, *not* quarkyonic.

*Un*like standard lore, where pressure(nucl mat) grows quickly, ~ N_c

Red line: 1st order. Green line: Baryons condense. Purple: chiral trans.

IF chiral transition 1st order, etc, Quadruple Point where *four* phases coexist.



Today's phase diagram for QCD