From Quarks and Gluons to Hadrons: Functional RG studies of QCD at finite Temperature and chemical potential

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Quarks, Hadrons & the Phase Diagram of QCD, St. Goar

03/09/2009

From Microscopic Degrees to Macroscopic DoF

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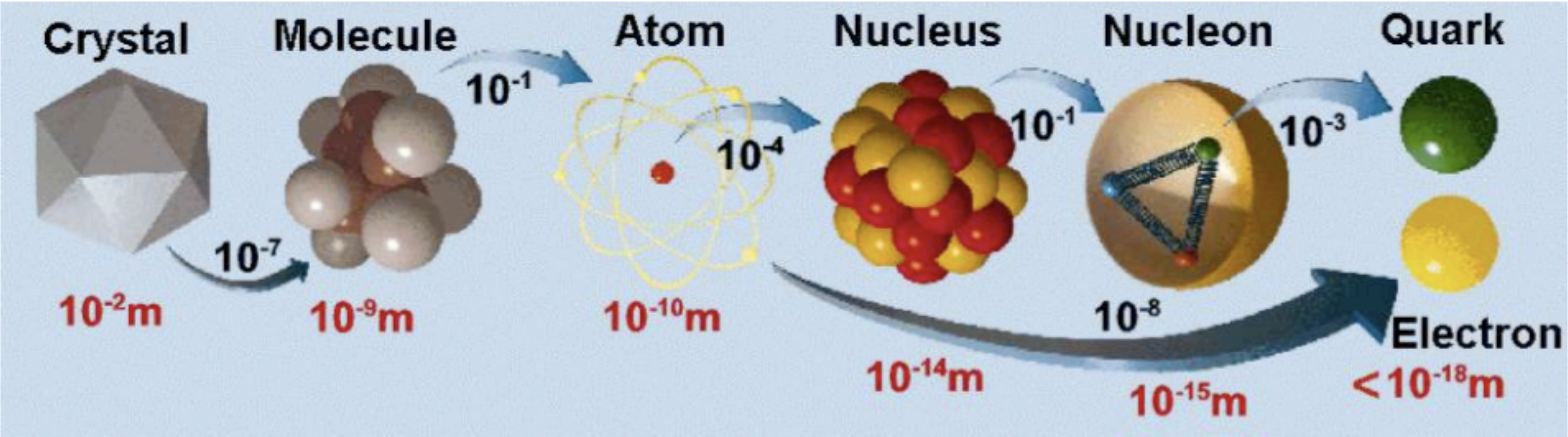

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large length scales

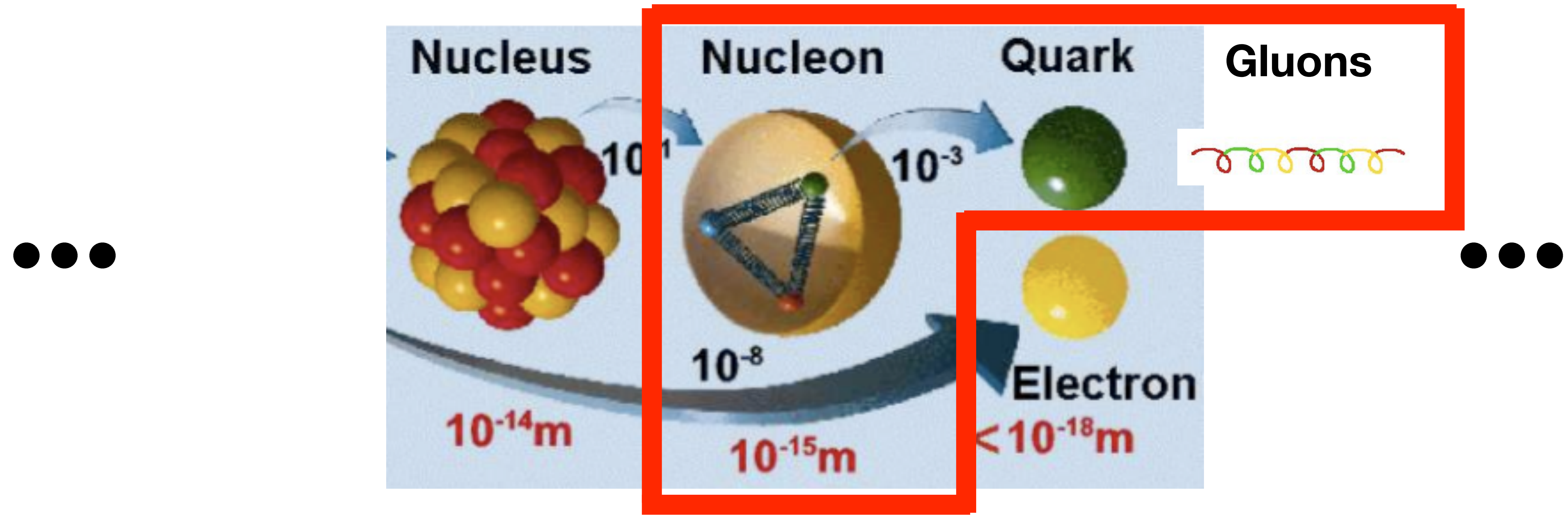
small length scales

small momentum scales

Renormalization Group

large momentum scales

From Microscopic Degrees to Macroscopic DoF



large length scales

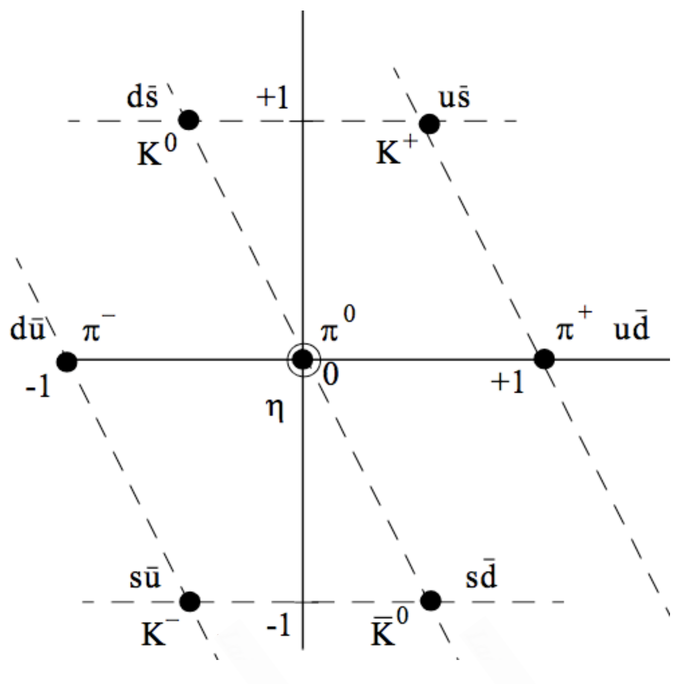
small length scales

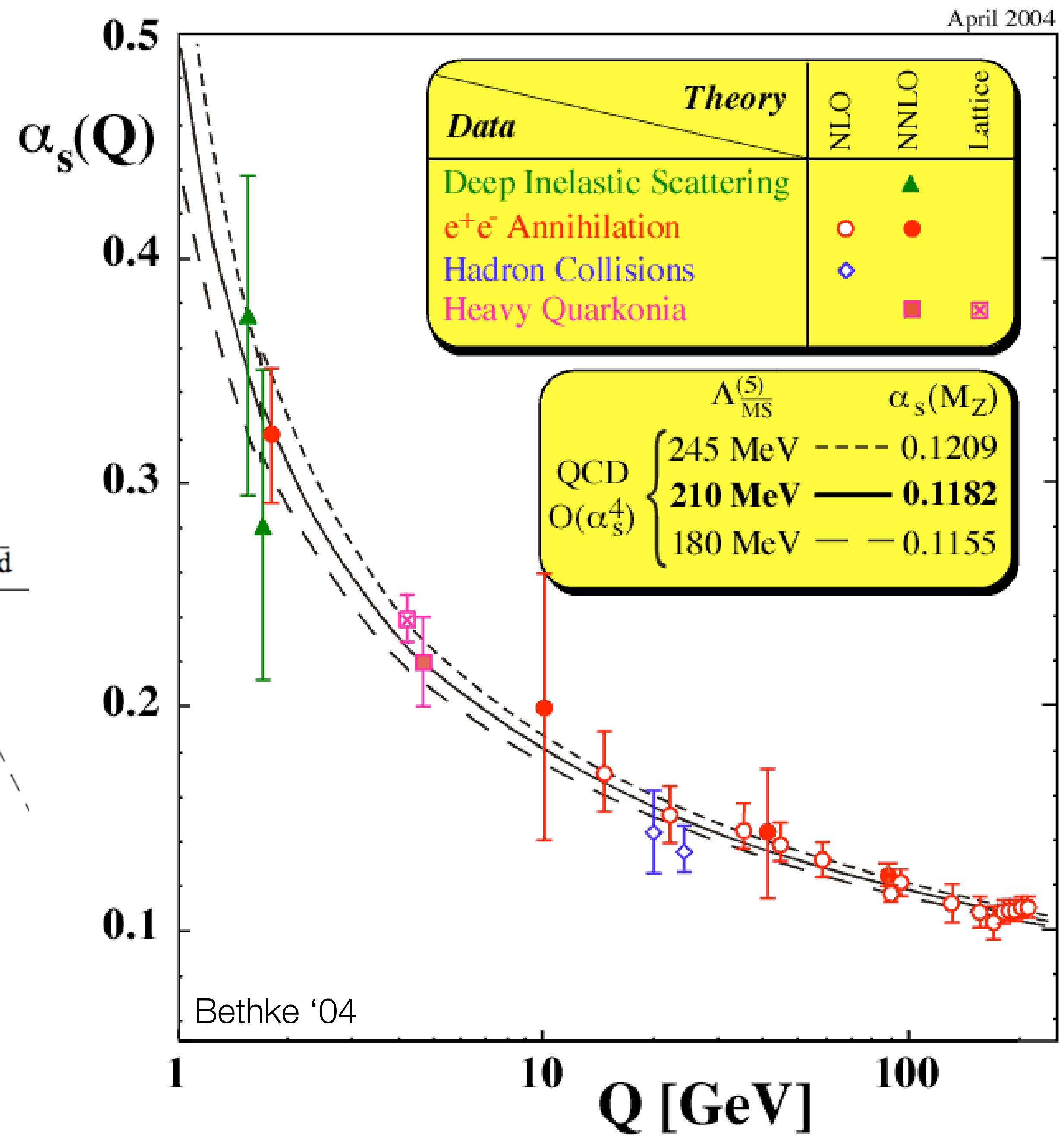
small momentum scales

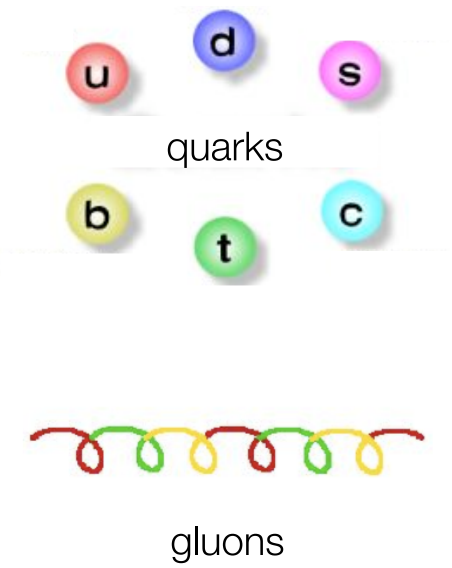
Functional
Renormalization Group flows

large momentum scales

Challenges in QCD

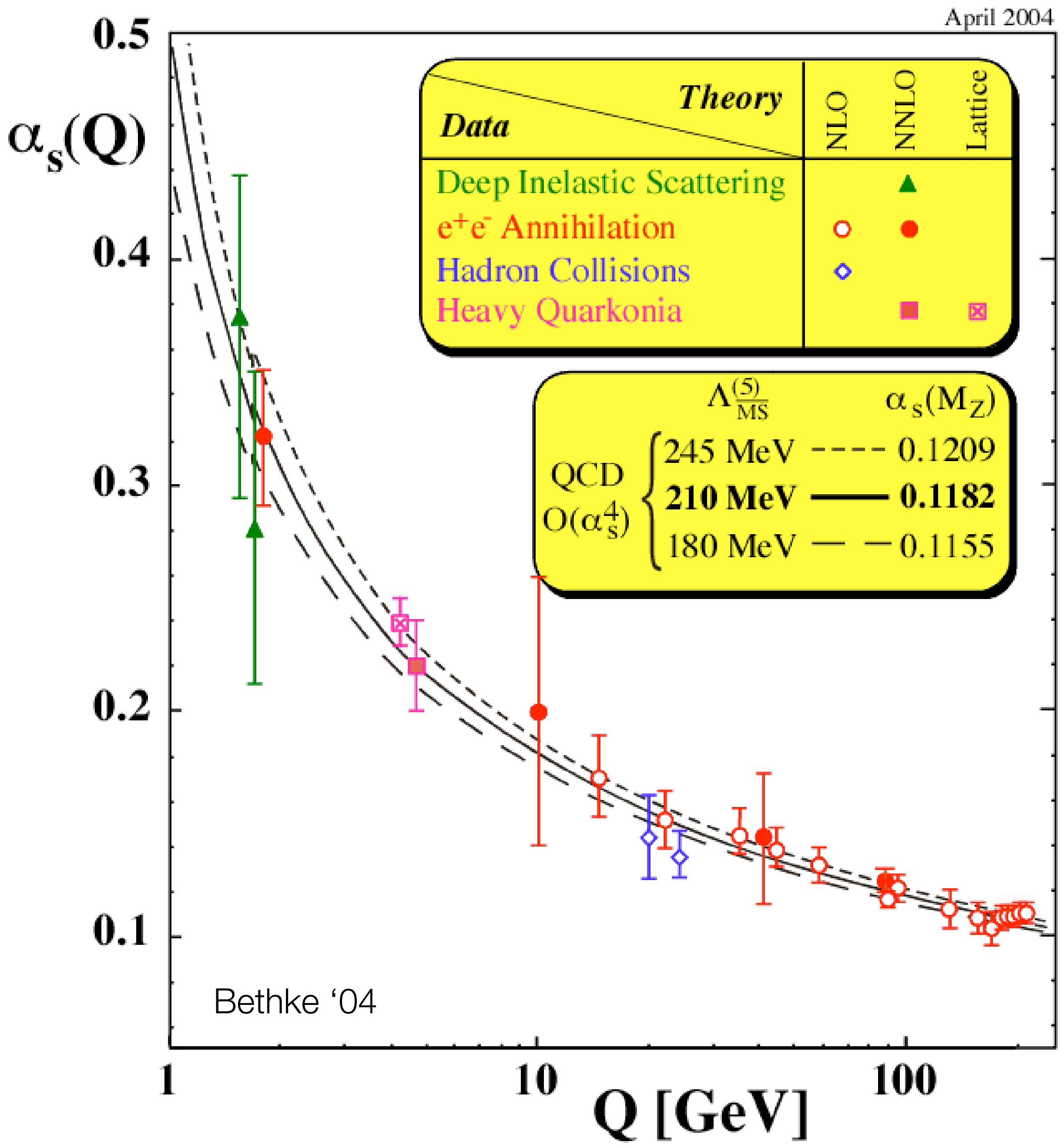






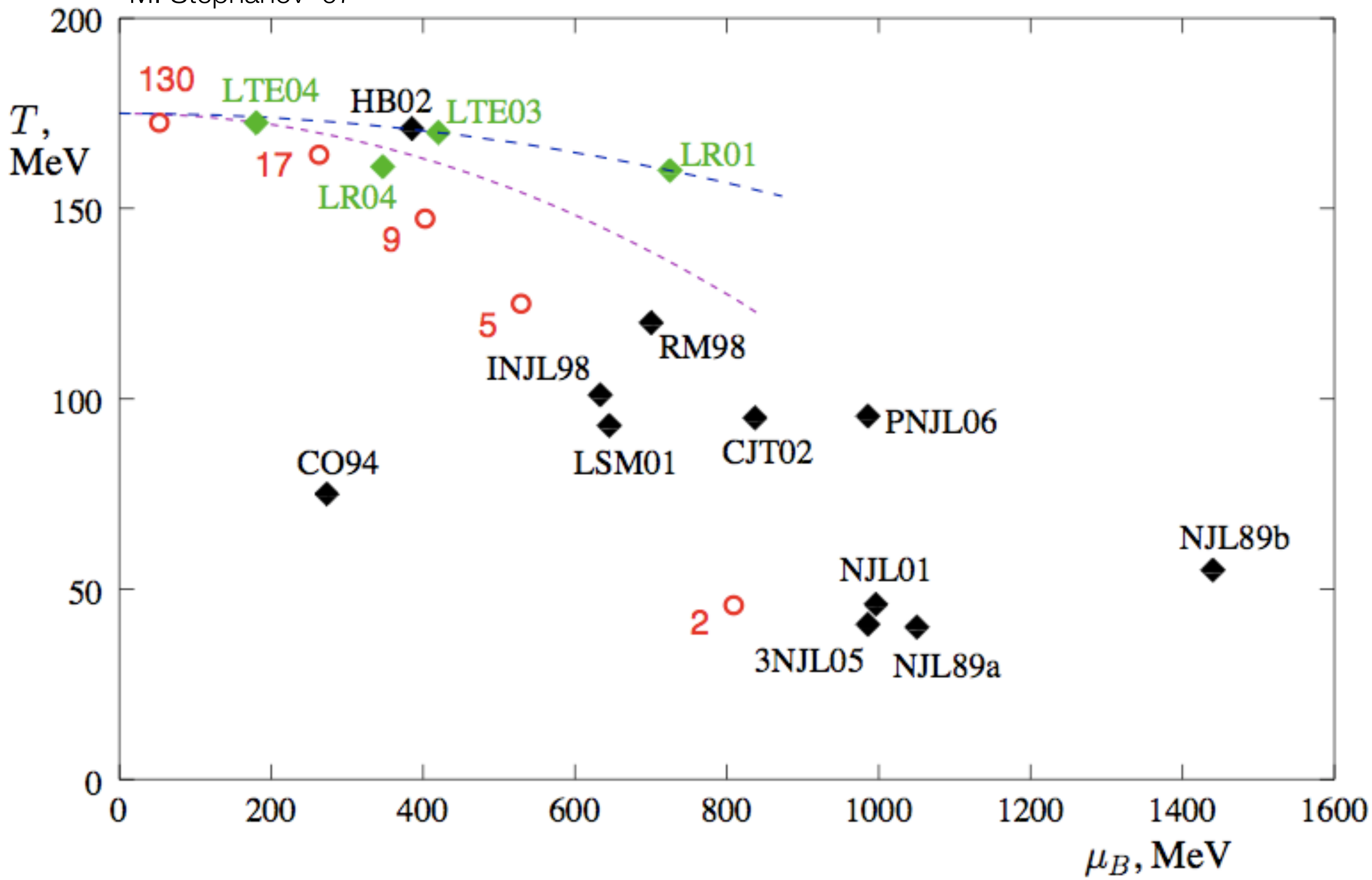
Challenges in QCD

- Asymptotic freedom at high momenta (Gross & Wilczek ’73, Politzer ’73)
- running coupling exhibits Landau pole at small momenta
  $\longrightarrow$ pQCD fails
- Understanding of QCD in the mid-momentum regime is needed to study confinement & chiral symmetry breaking



QCD phase diagram? Puzzles ...

M. Stephanov '07



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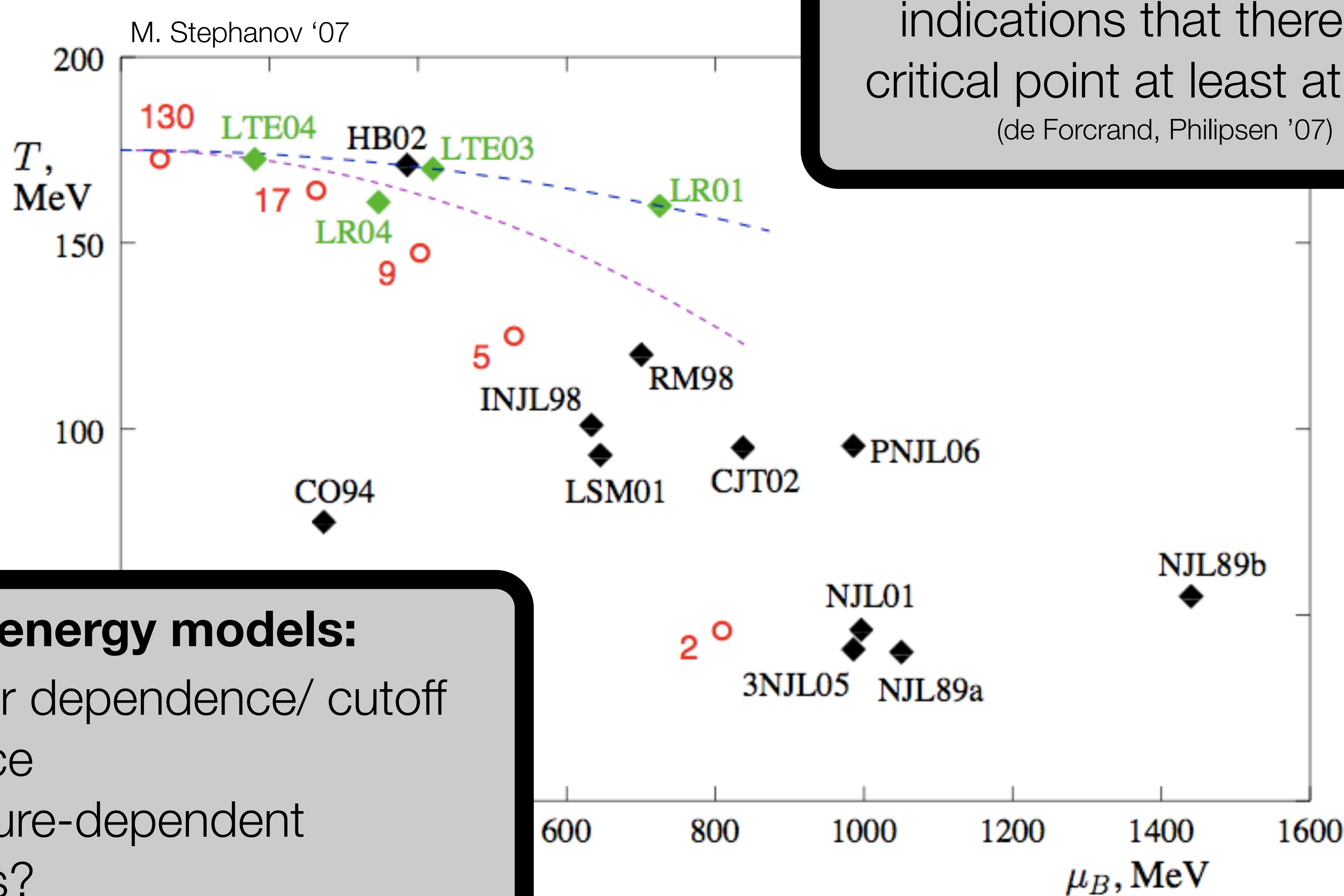


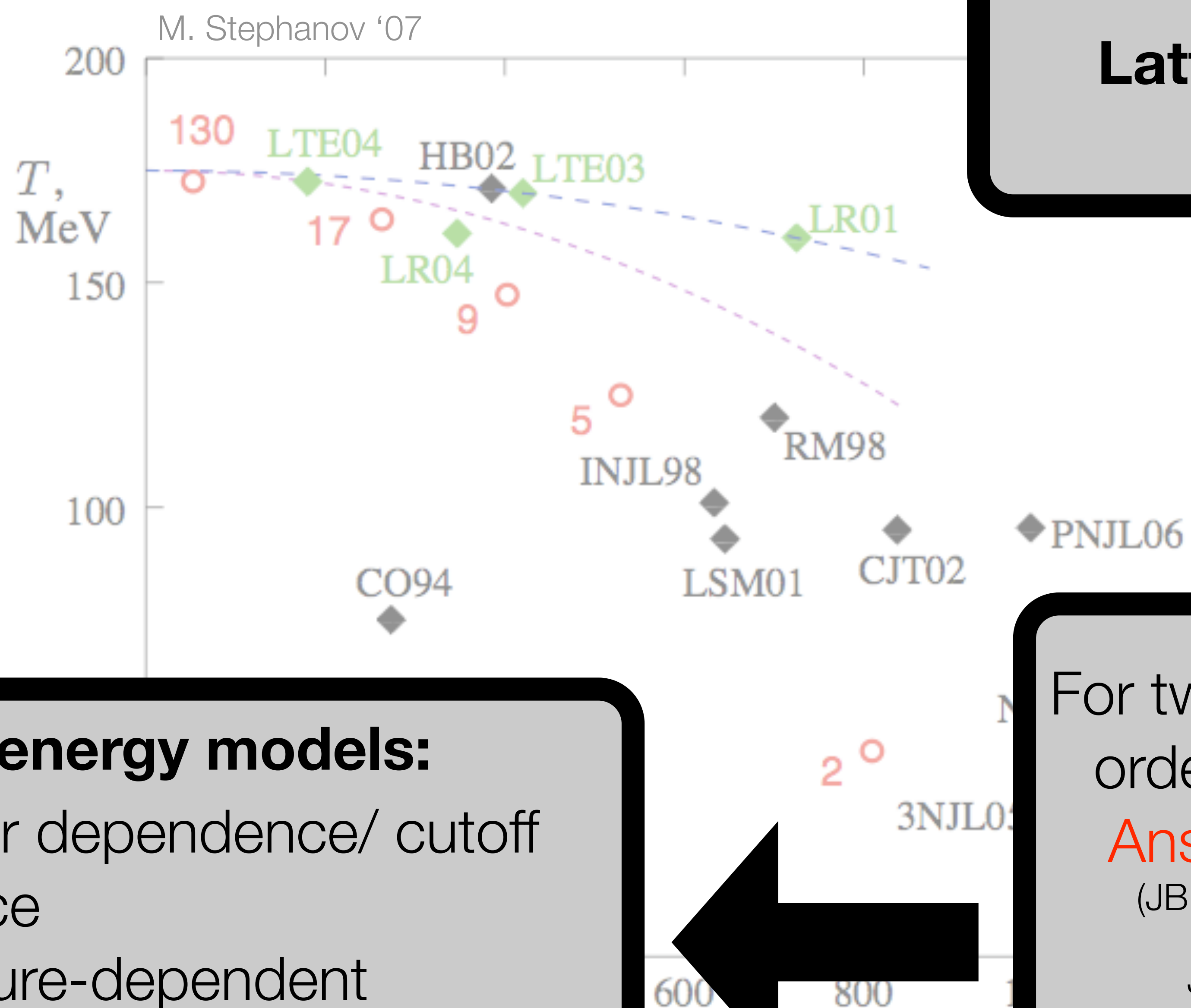
indications that there is no critical point at least at small μ

(de Forcrand, Philipsen '07)

low-energy models:

- parameter dependence/ cutoff dependence
- temperature-dependent parameters?
- truncated action

QCD phase diagram? Puzzles ...



Lattice QCD simulations

Analysis

For two (massless) quark flavors: order of the phase transition?

Answers from FV scaling (!?)

(JB, B. Klein, Phys. Rev. D '07, EPJ C '08; JB, B. Klein, P. Piasecki, in prep.; JB, B. Klein, B.-J. Schaefer, in prep)

→ **important for low-energy models**

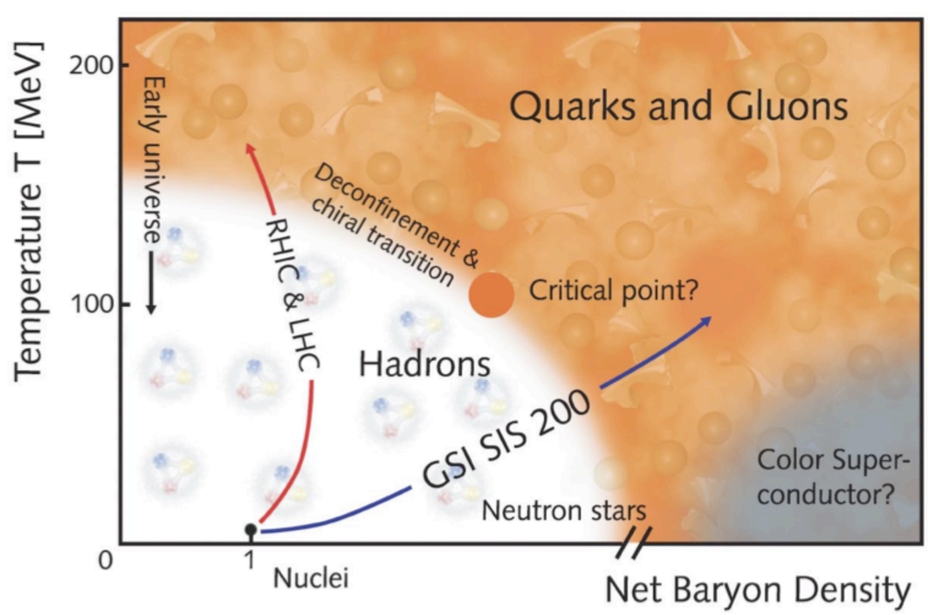
low-energy models:

- parameter dependence/ cutoff dependence
- temperature-dependent parameters?
- truncated action

How to tackle QCD at finite temperature?

Lattice QCD

Functional approaches
(Functional RG methods, Dyson-Schwinger Eqs., ...)



How to tackle QCD at finite temperature?

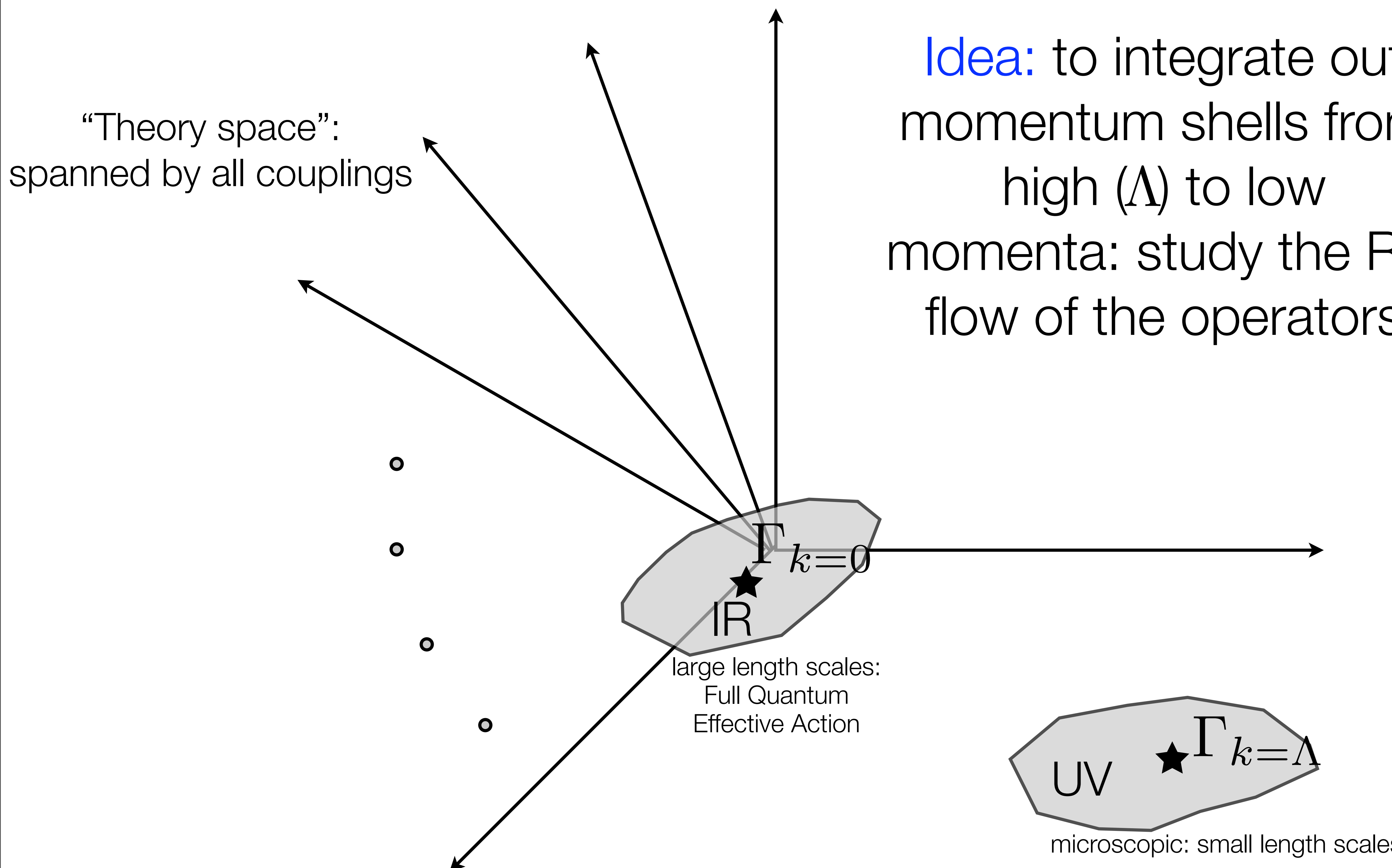
Lattice QCD

Functional approaches
(Functional RG methods, Dyson-Schwinger Eqs., ...)

pros	cons
• continuum formulation • chiral fermions • no sign-problem • computationally efficient	• search for controlled expansion point

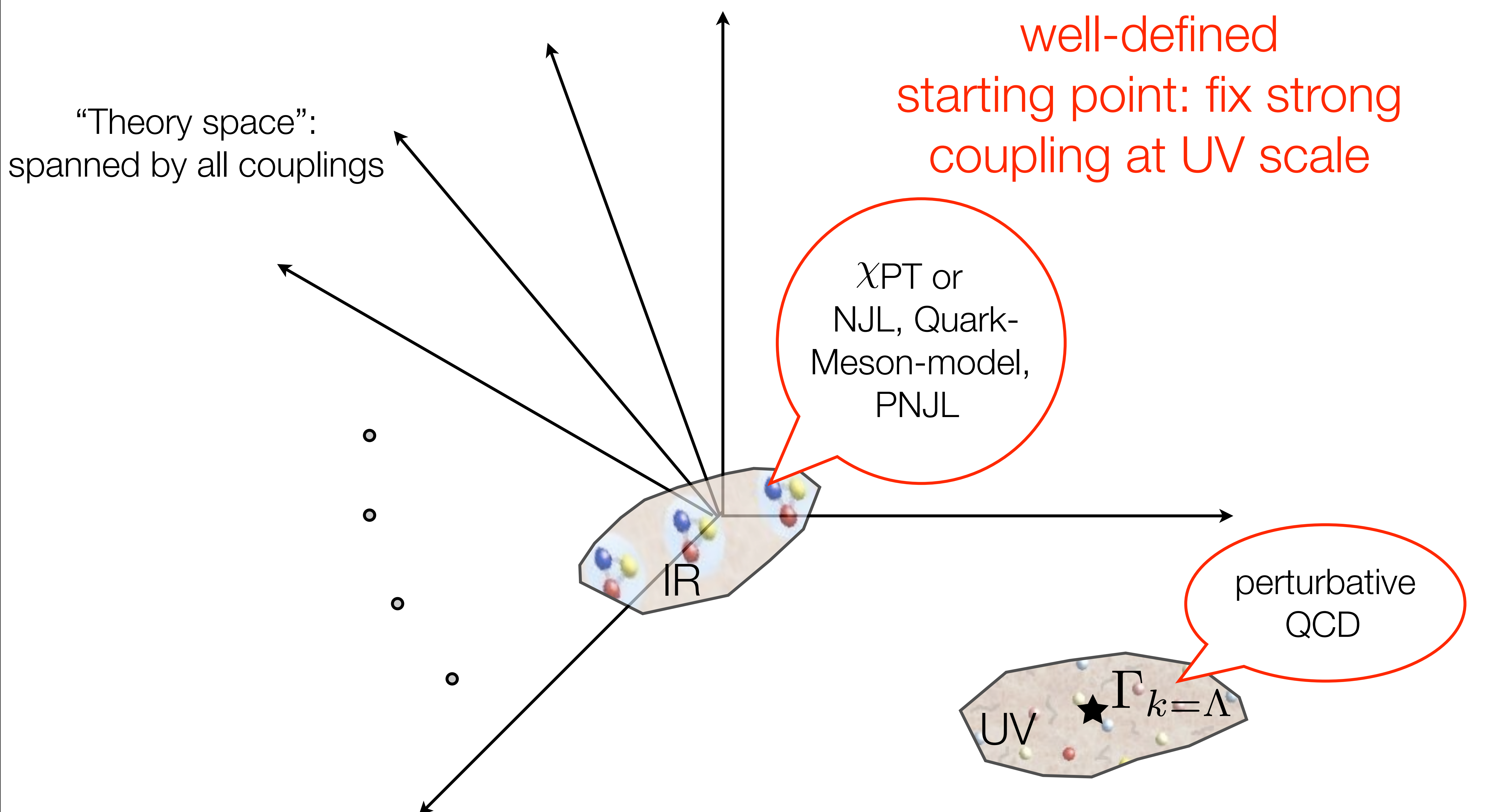
Functional Renormalization Group

Idea: to integrate out momentum shells from high (Λ) to low momenta: study the RG flow of the operators



Functional Renormalization Group - QCD

(C. Wetterich '92)



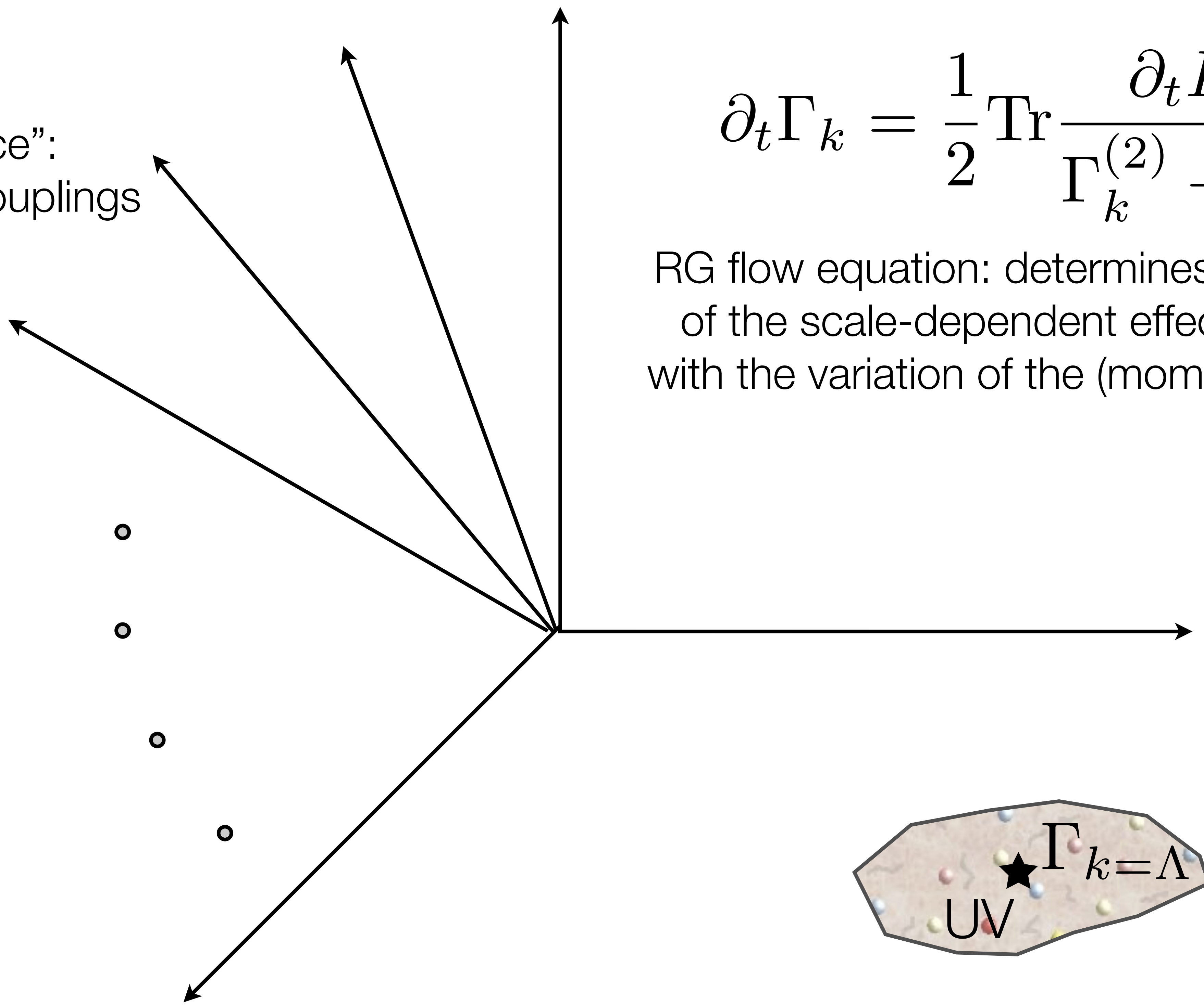
Functional Renormalization Group - QCD

(C. Wetterich '92)

"Theory space":
spanned by all couplings

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}$$

RG flow equation: determines the change of the scale-dependent effective action with the variation of the (momentum) scale



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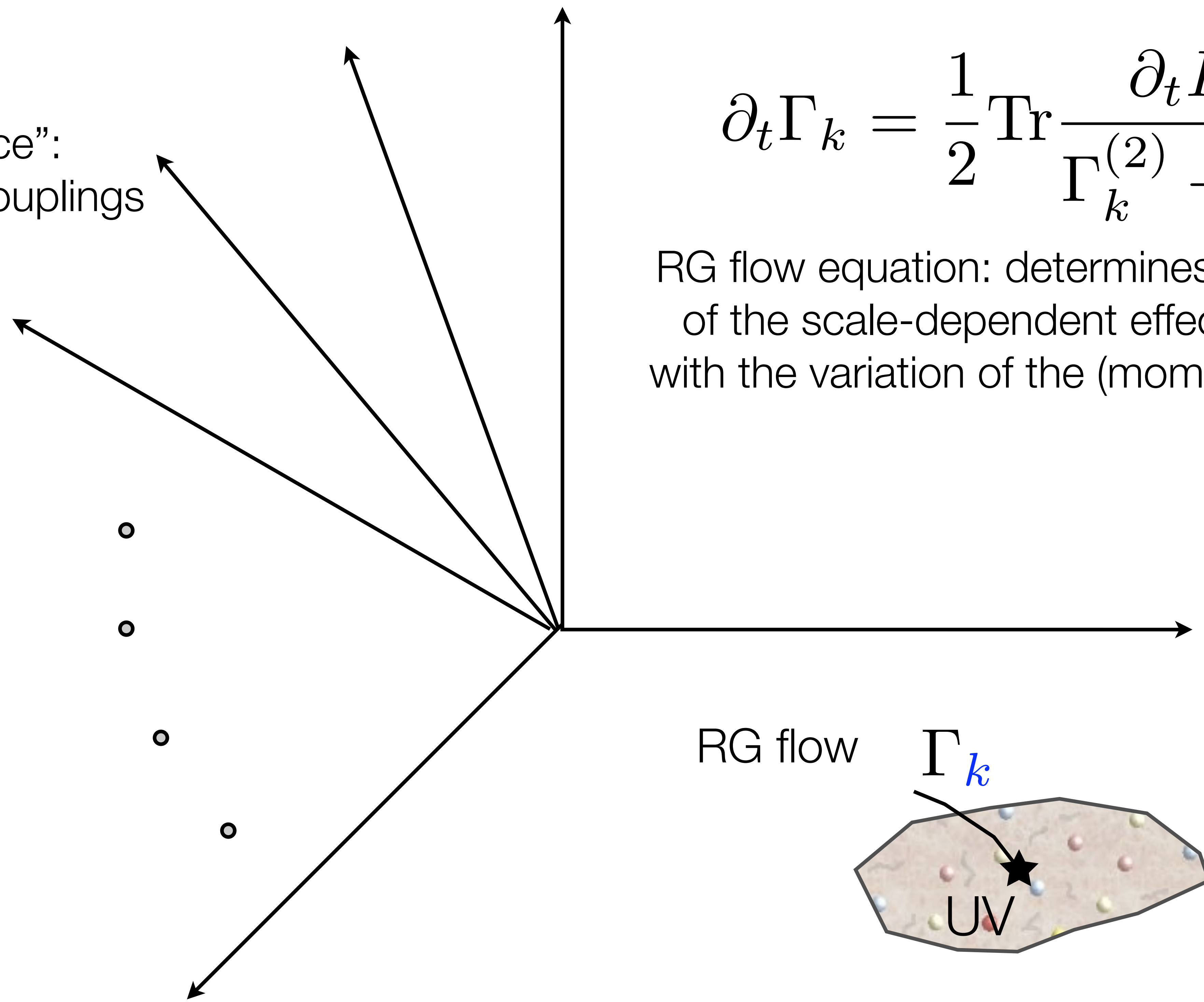

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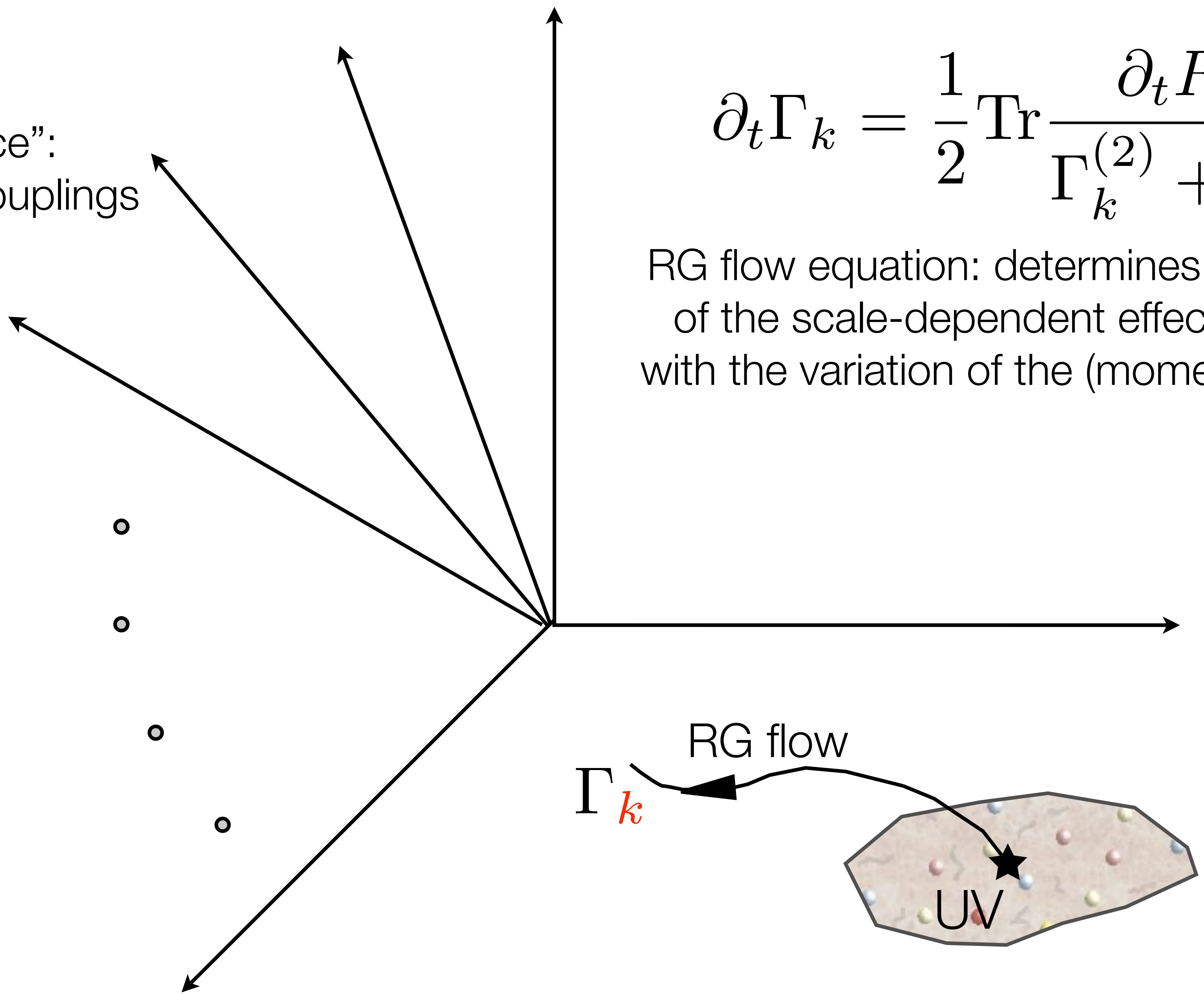

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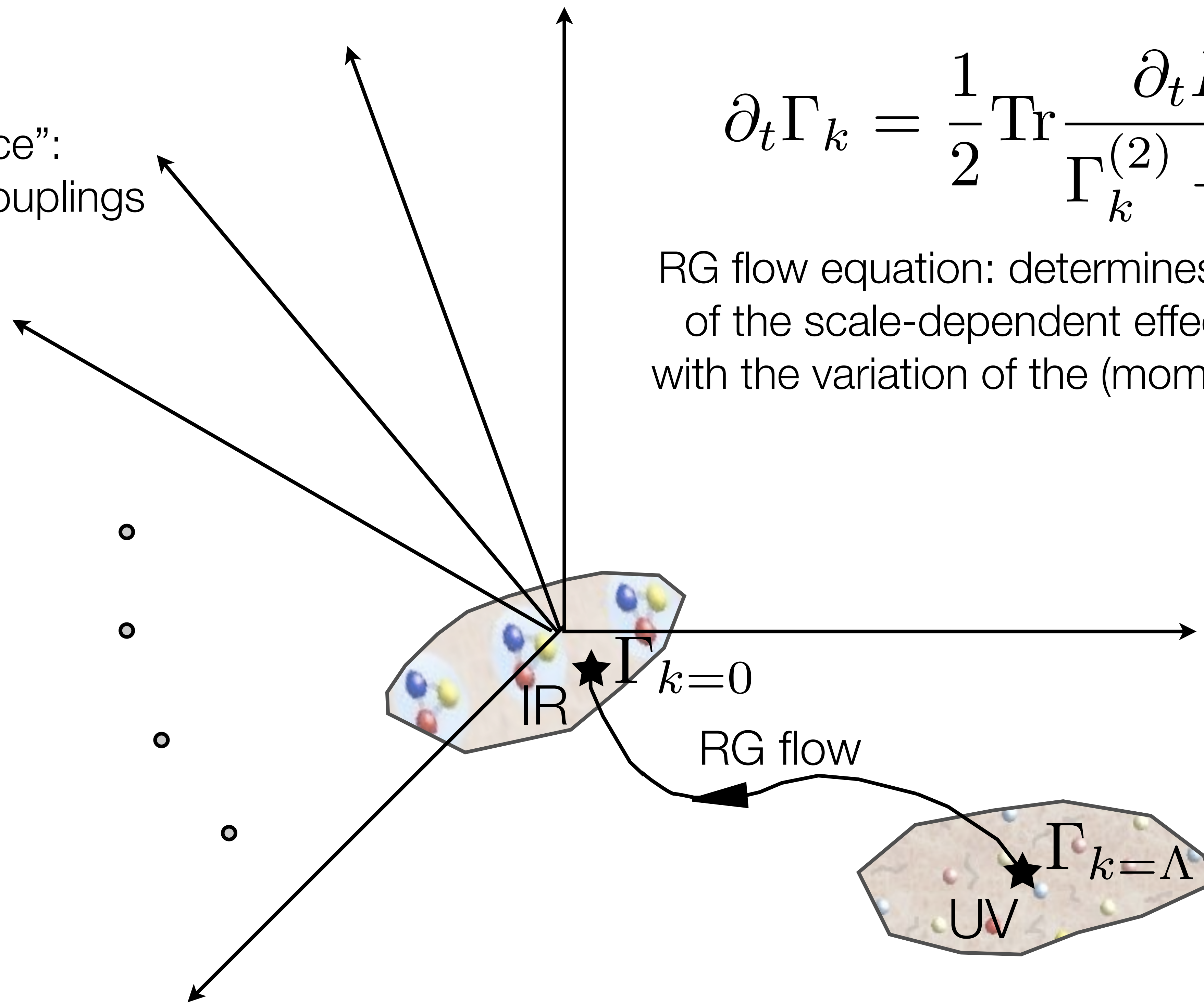
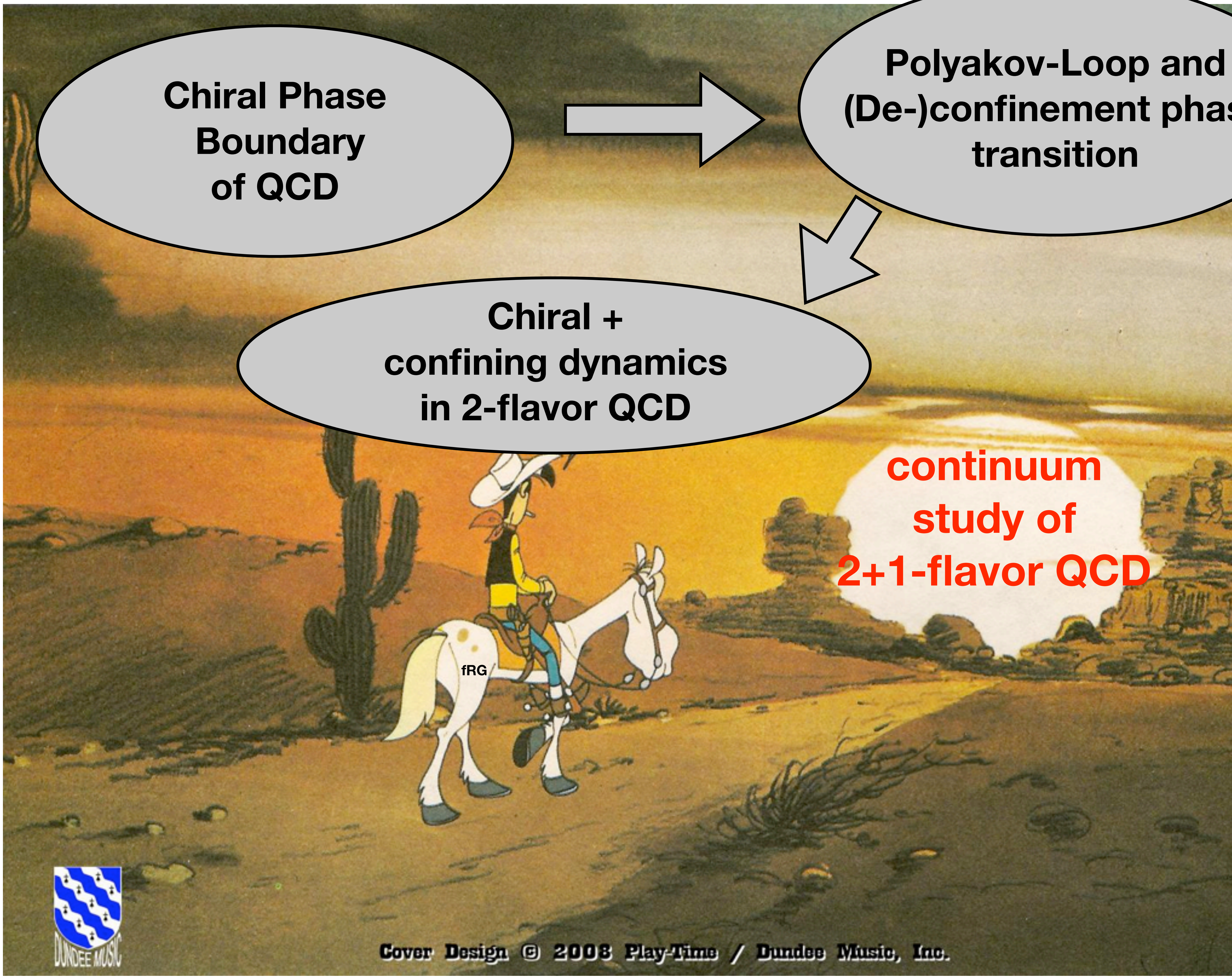


Outline


Chiral Phase Boundary of QCD
Polyakov-Loop and (De-)confinement phase transition
Chiral + confining dynamics in 2-flavor QCD
continuum study of 2+1-flavor QCD
fRG
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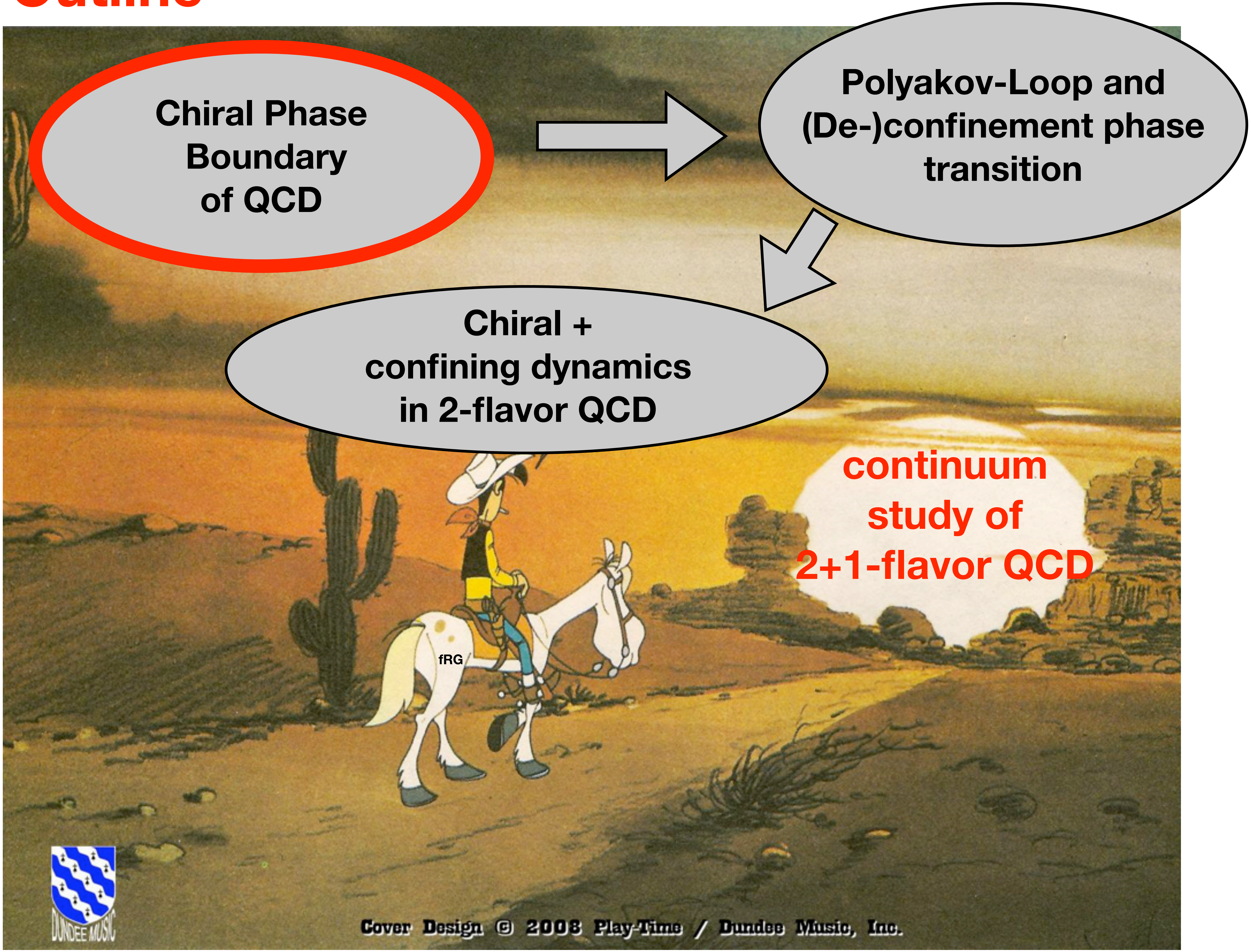
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Aspects of the NJL model

(Y. Nambu, G. Jona-Lasinio '61)

- classical action of the NJL model:

$$S = \int_x \{\bar{\psi} \mathrm{i} \partial\!\!\!/ \psi + \bar{\lambda}_\sigma [(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2]\}$$

- spontaneous symmetry breaking if quark condensate is non-vanishing: $\langle \bar{\psi}\psi \rangle \neq 0$

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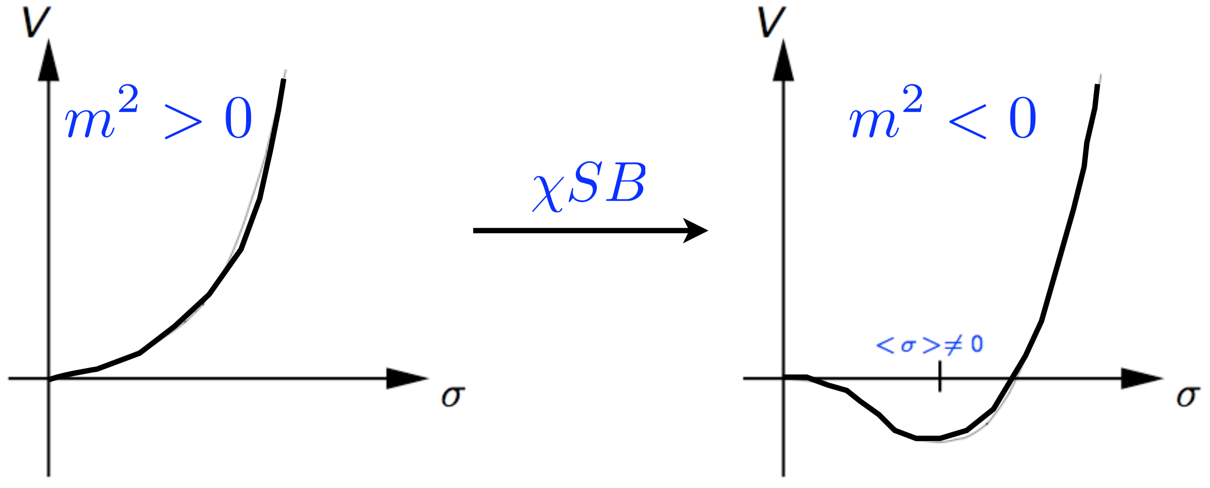
- bosonization of the NJL model yields $(\sigma = -2\bar\lambda_\sigma\bar\psi\psi\,,\pi = -2\bar\lambda_\sigma\bar\psi\gamma_5\psi)$

$$S = \int_x \left\{\bar\psi \mathrm{i}\partial\!\!\!/ \psi + \bar\psi(\sigma + \mathrm{i}\gamma_5\pi)\psi - \frac{1}{\bar\lambda_\sigma}(\sigma^2+\pi^2)\right\}$$ ("Quark-Meson model")

⟹ $\bar\lambda_\sigma$ is inverse proportional to the scalar mass parameter, $m^2 \propto \frac{1}{\bar\lambda_\sigma}$



Four-Fermion Interactions in QCD

- at the UV scale $(k = \Lambda \gg \Lambda_{\mathrm{QCD}})$:

$$\Gamma_{\Lambda} = \int_x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \bar{g} A\!\!\!/)\psi \right\}$$

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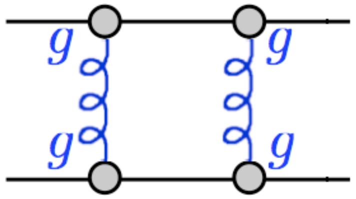
$$k = \Lambda - \delta k$$



$$\Gamma_{\Lambda-\delta k} = \int_x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \bar{g} A\!\!\!/)\psi + \frac{\lambda_\sigma}{2k^2}[(\bar{\psi}^a \psi^b)^2 - (\bar{\psi}^a \gamma_5 \psi^b)^2] + \ldots \right\}$$

- quark-gluon dynamics generate four-fermion interactions

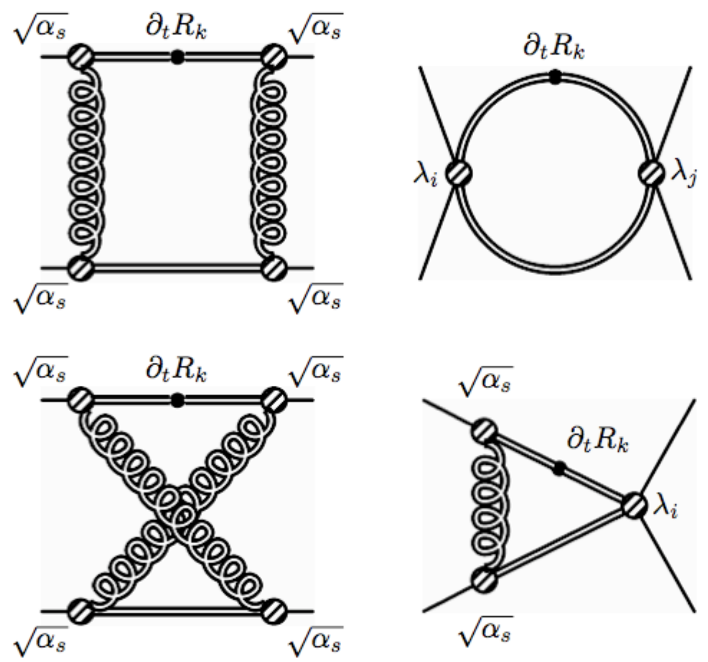
RG flow for the chiral QCD sector

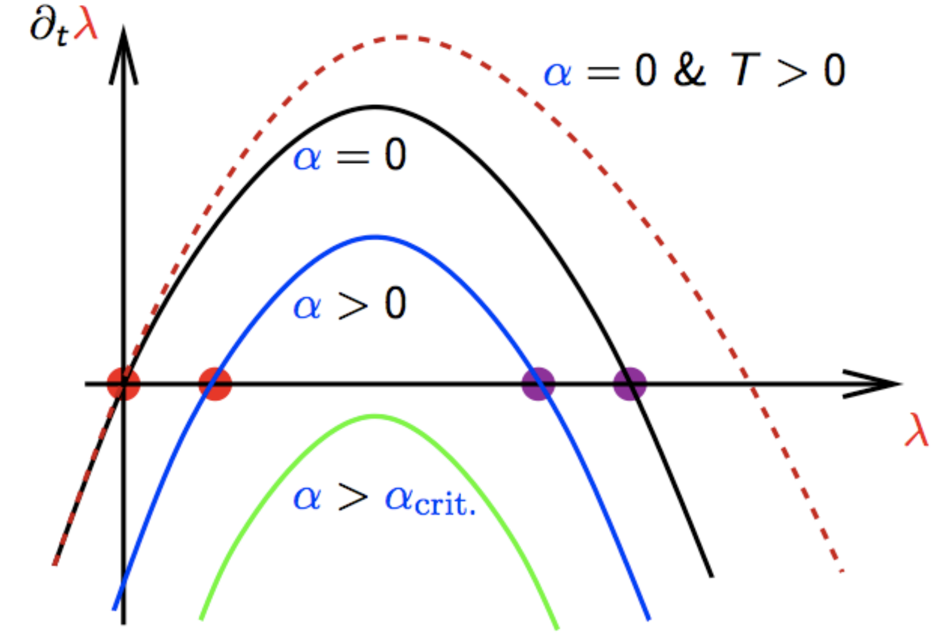
- effective action:

$$\begin{aligned}\Gamma_k \quad &= \int_x \Big\{ \frac{\bar{g}^2}{g^2} F^a_{\mu\nu} F^a_{\mu\nu} + w_2 (F^a_{\mu\nu} F^a_{\mu\nu})^2 + w_3 (F^a_{\mu\nu} F^a_{\mu\nu})^3 + \dots \Big\} \\ &+ \int_x \Big\{ \bar{\psi}(\mathrm{i} Z_\psi \partial\!\!\!/ + Z_1 \bar{g} A\!\!\!/)\psi + \frac{1}{2}\Big[\frac{\lambda_-}{k^2}(\mathrm{V}-\mathrm{A}) + \frac{\lambda_+}{k^2}(\mathrm{V}+\mathrm{A}) \\ &\quad + \frac{\lambda_\sigma}{k^2}(\mathrm{S}-\mathrm{P}) + \frac{\lambda_{\mathrm{VA}}}{k^2}[2(\mathrm{V}-\mathrm{A})^{\mathrm{adj}} + (1/N_c)(\mathrm{V}-\mathrm{A})]\Big] \Big\}\end{aligned}$$

- no Fierz-ambiguity
- four-fermion interactions ($\lim_{\Lambda\to\infty} \lambda_i = 0$)
- truncation checks: momentum dependencies, regulator dependencies, higher order interactions (H. Gies, J. Jaeckel, C. Wetterich '04, H. Gies, C. Wetterich '02, JB '08)

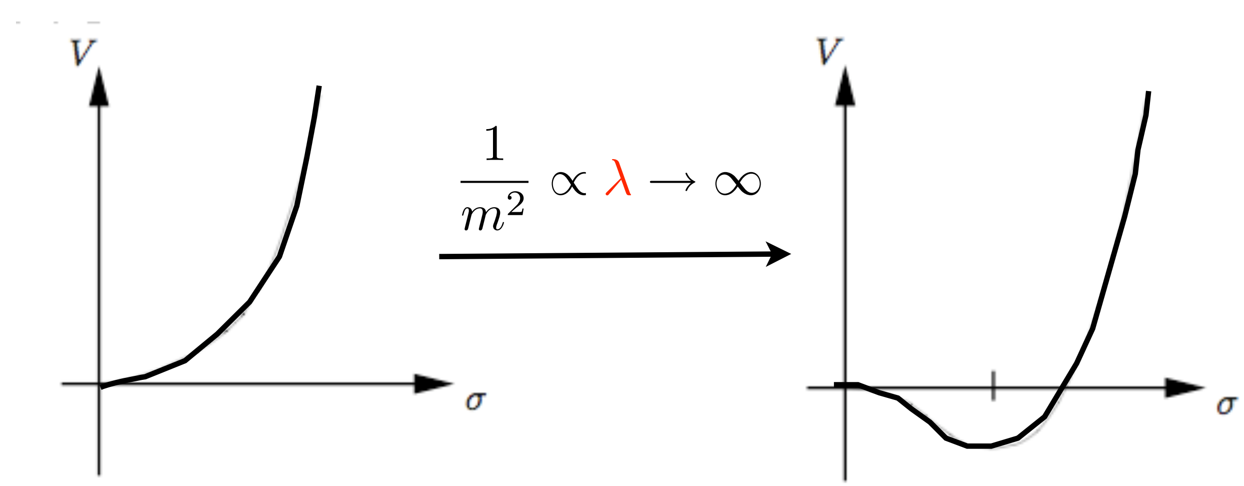
"Criticality" at zero and finite temperature





- flow of four-fermion couplings:

$$\partial_t \lambda = 2\lambda - \lambda A(\tfrac{T}{k})\lambda - b(\tfrac{T}{k})\lambda\alpha_s - c(\tfrac{T}{k}){\alpha_s}^2$$



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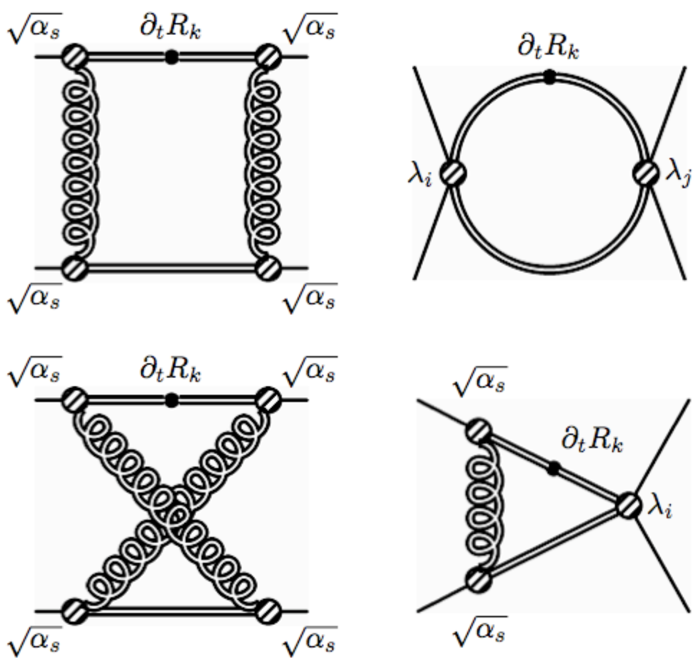
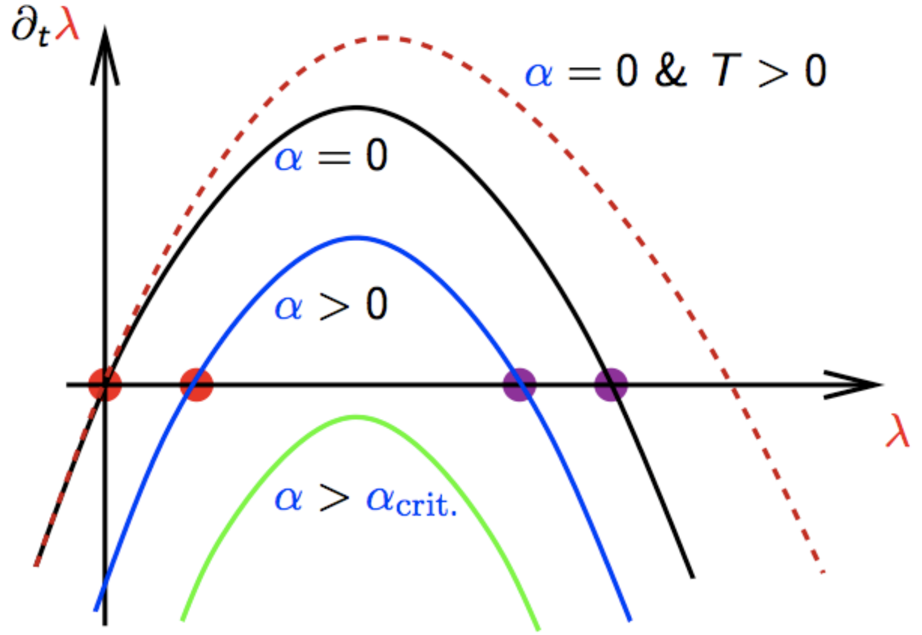


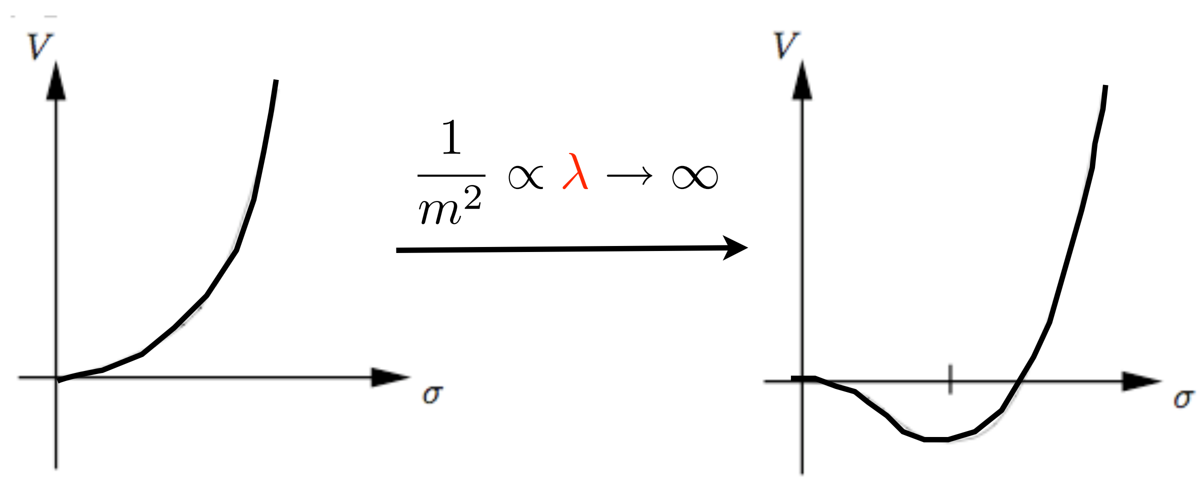


- critical gauge coupling α_{cr} :

if $\alpha_s > \alpha_{cr}$ ⟹ no fixed points ⟹ χSB

- at zero temperature: (H. Gies, J. Jaeckel ’05)

$$\alpha_{cr} \overset{N_c=N_f=3}{\approx} 0.85$$



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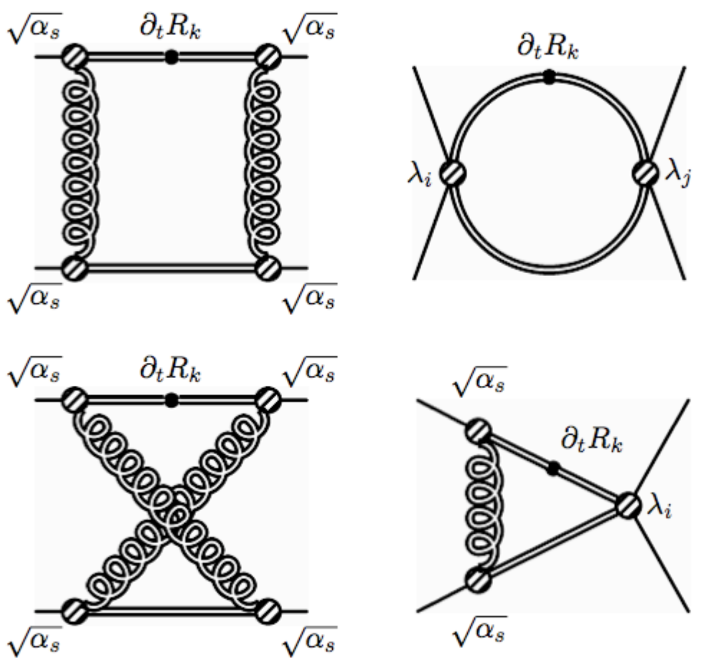
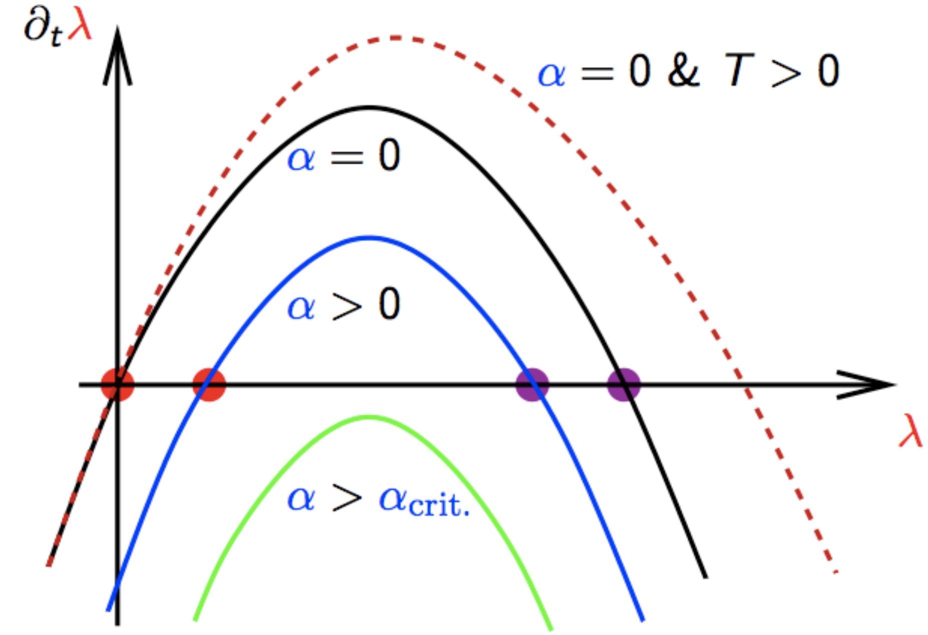




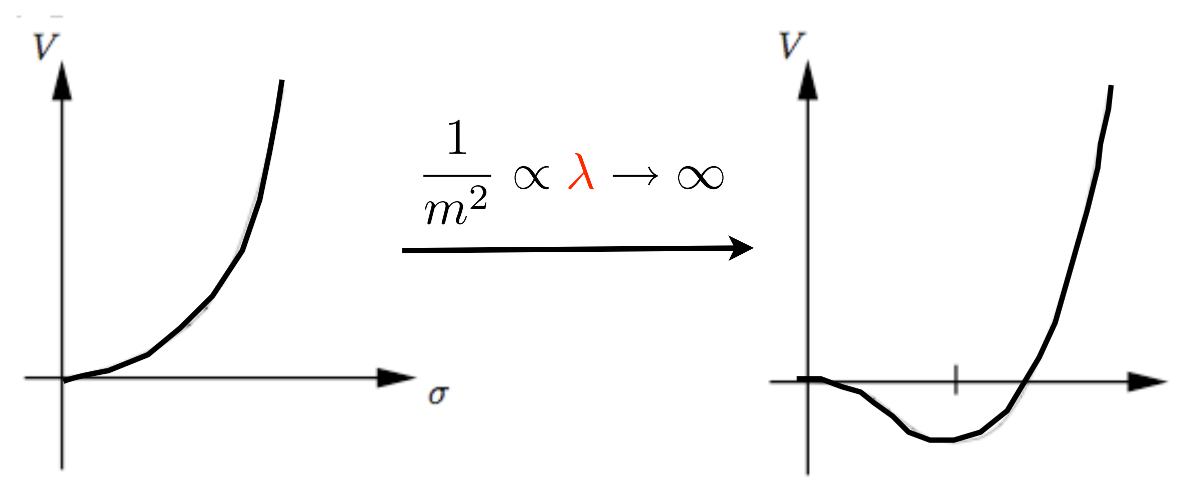
- critical gauge coupling α_{cr} :

  if $\alpha_s > \alpha_{cr}$ ⟹ no fixed points ⟹ χSB

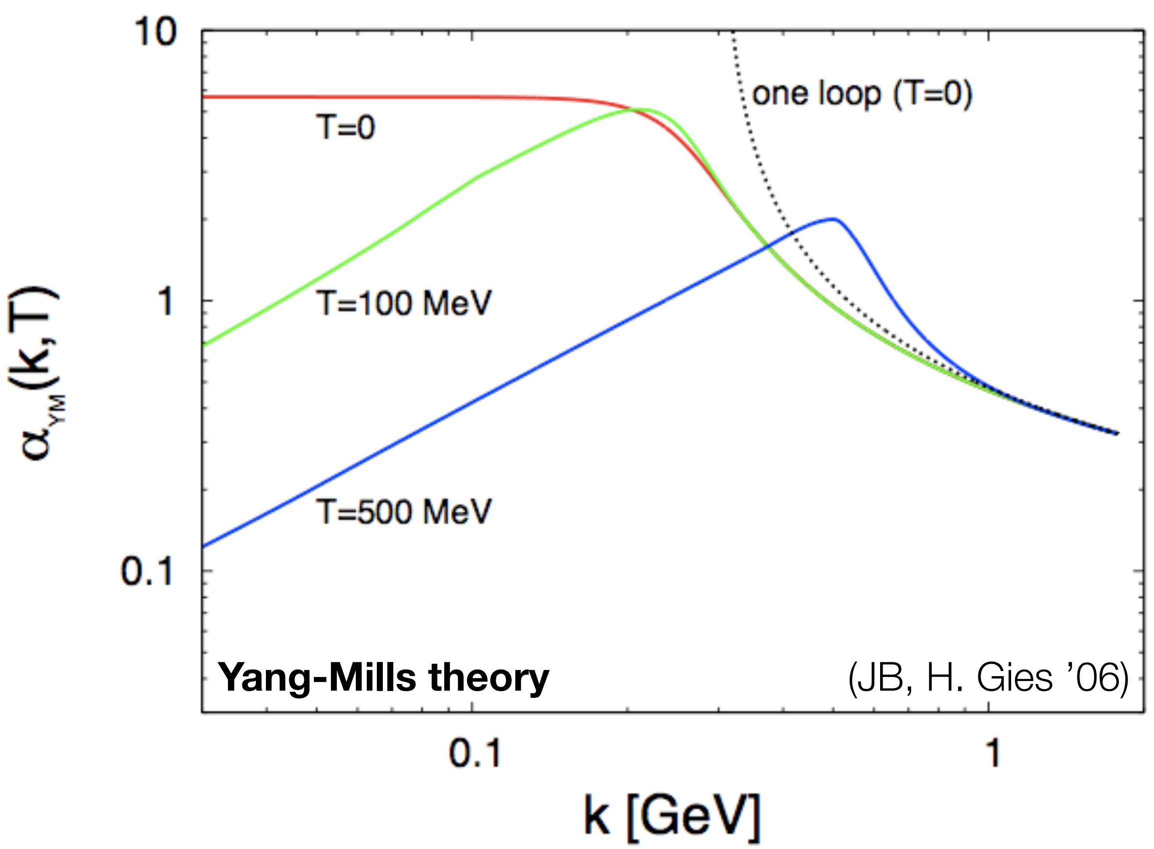
- at finite temperature: (JB, H. Gies ’05)

$$\alpha_{cr}(T/k) > \alpha_{cr}(T=0)$$

quarks acquire a thermal mass



RG flow of gluodynamics



cf. vertex expansion in **Landau-gauge QCD**:

SDE: v. Smekal et al. '97, Fischer et al. '02;
RG: Pawlowski et al. '03, Fischer&Gies '04;
Gies '02; Gies&Braun '05/'06
Lattice: e. g. Sternbeck et al. '05; ...

- $k_{max} \propto T$ decoupling of hard gluonic modes ⇒ "finite-size" effect:

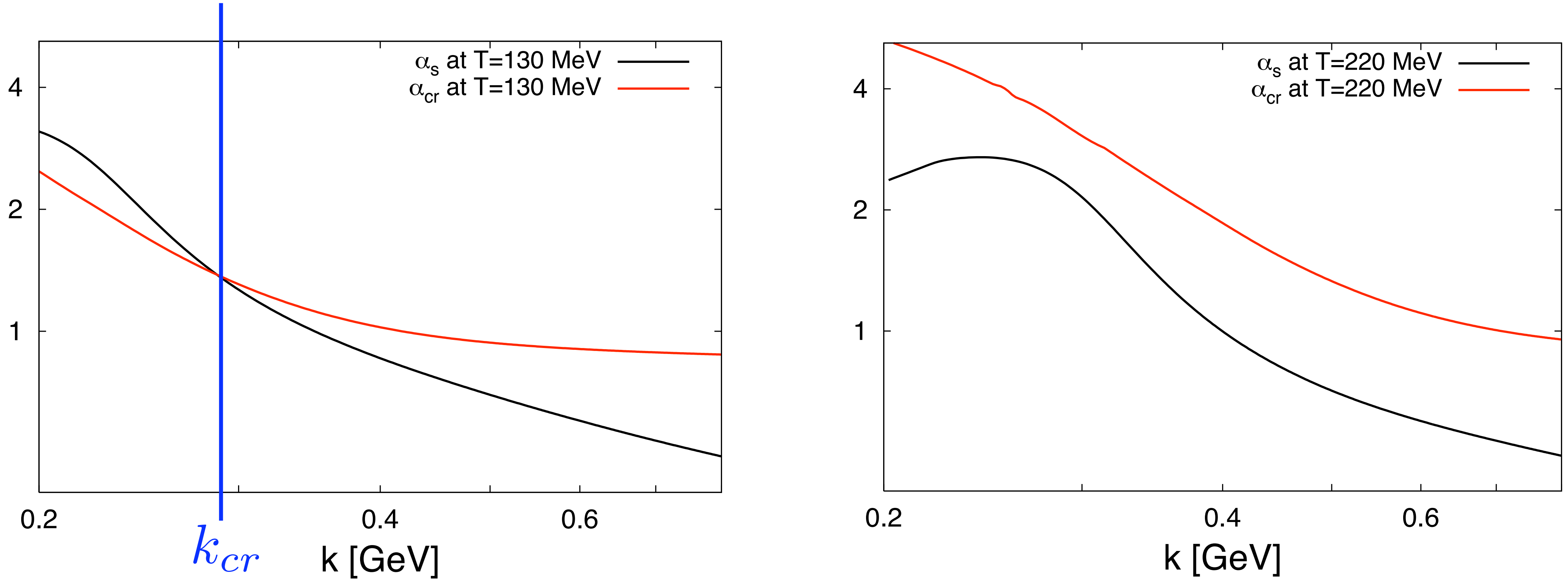
$$p_{g,0}^2 \equiv \omega_n^2 = 4n^2\pi^2T^2 \quad \rightarrow \quad \omega_0^2 = 0$$

- decrease for $T \gtrsim k$ due to existence of a non-trivial IR fixed point in 3d Yang-Mills theory: strong interactions at high temperatures (JB, H. Gies '06; Lattice: Cucchieri et al. '07)

$$\alpha_{4D} \approx \alpha_{3D}^* \frac{k}{T} + \mathcal{O}\left((k/T)^2\right) \quad \text{with} \quad \alpha_{3D}^* \approx 2.7\,;\ \eta_{3d} \rightarrow 1$$

Chiral Phase Transition in QCD

- study: $\alpha_{cr}(T/k)$ vs. $\alpha_s(T/k)$
- intersection point of α_{cr} and α_s indicates onset of χSB



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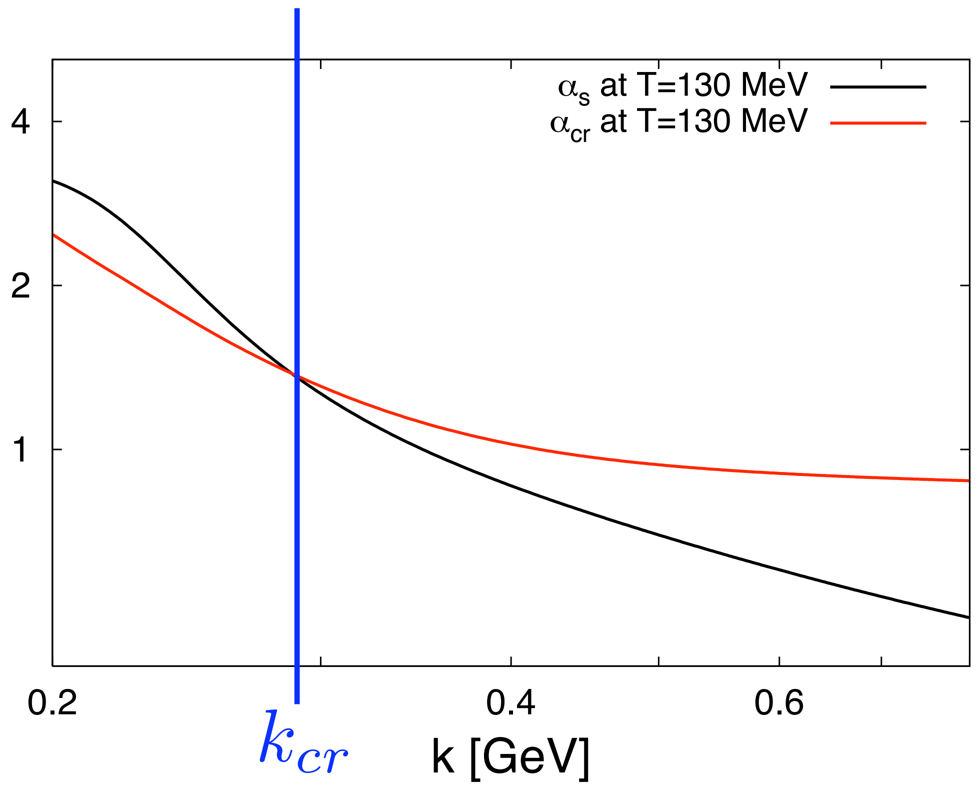
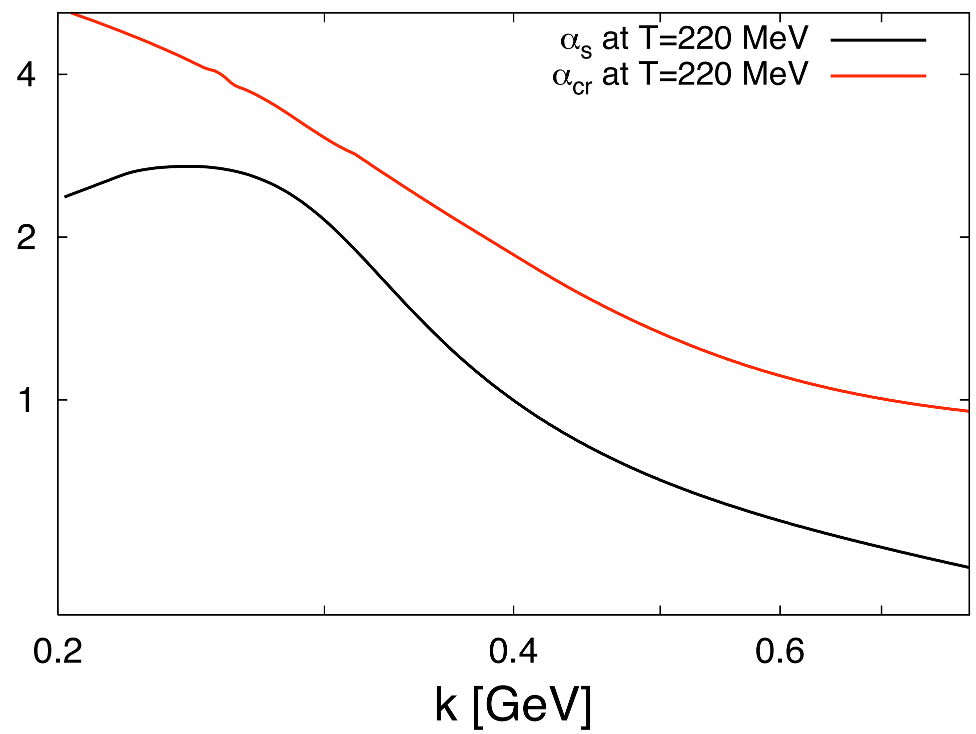




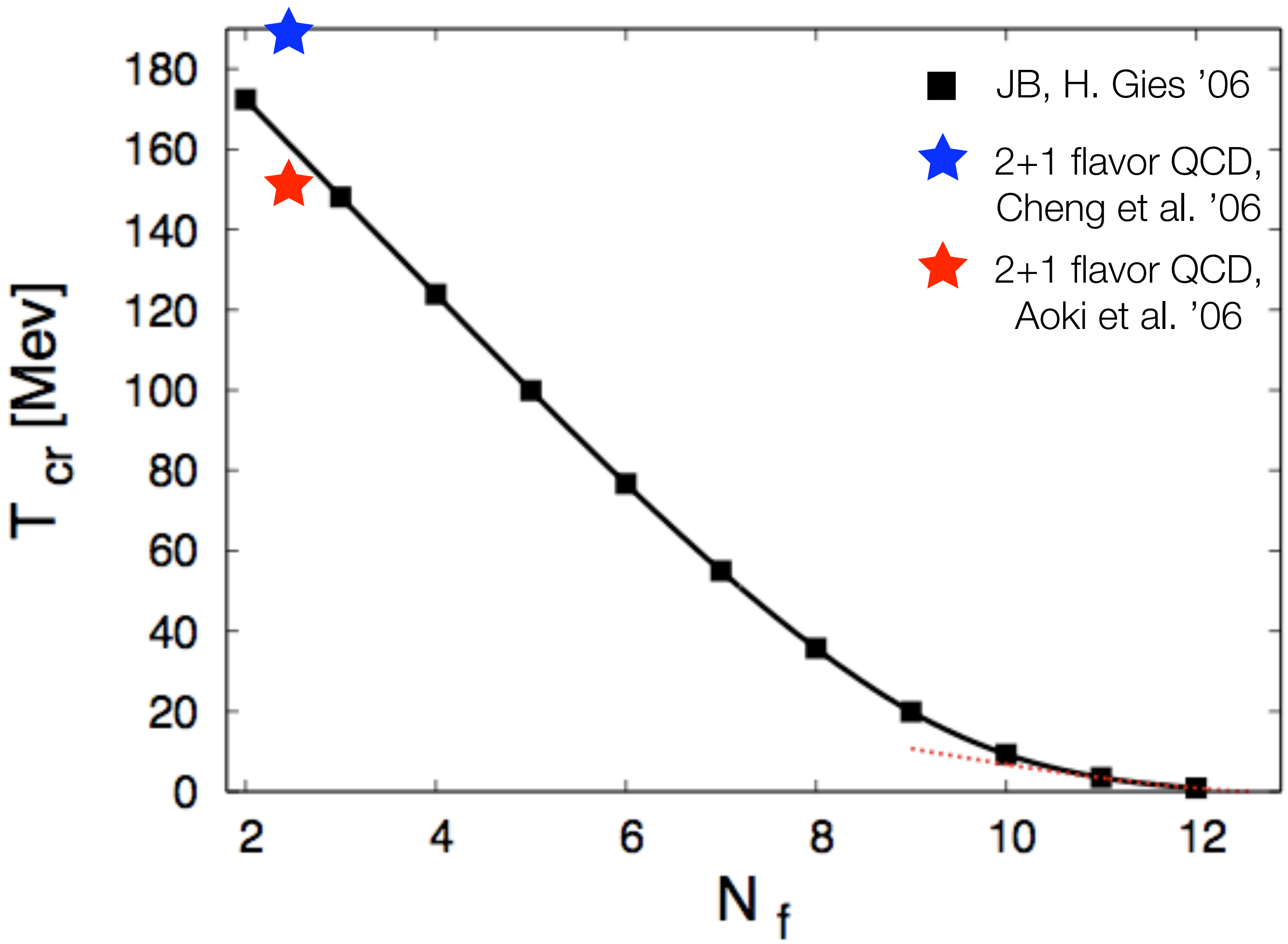
- single input parameter: $\alpha_s(m_\tau) = 0.322$

N_f	T_{cr}
2	$172\,\mathrm{MeV}$
3	$148\,\mathrm{MeV}$

(JB, H. Gies '06)

$N_f = 2+1$	T_{cr}
Lattice (Cheng et al. '06)	$192\,\mathrm{MeV}$
Lattice (Aoki et al. '06)	$151\,\mathrm{MeV}$

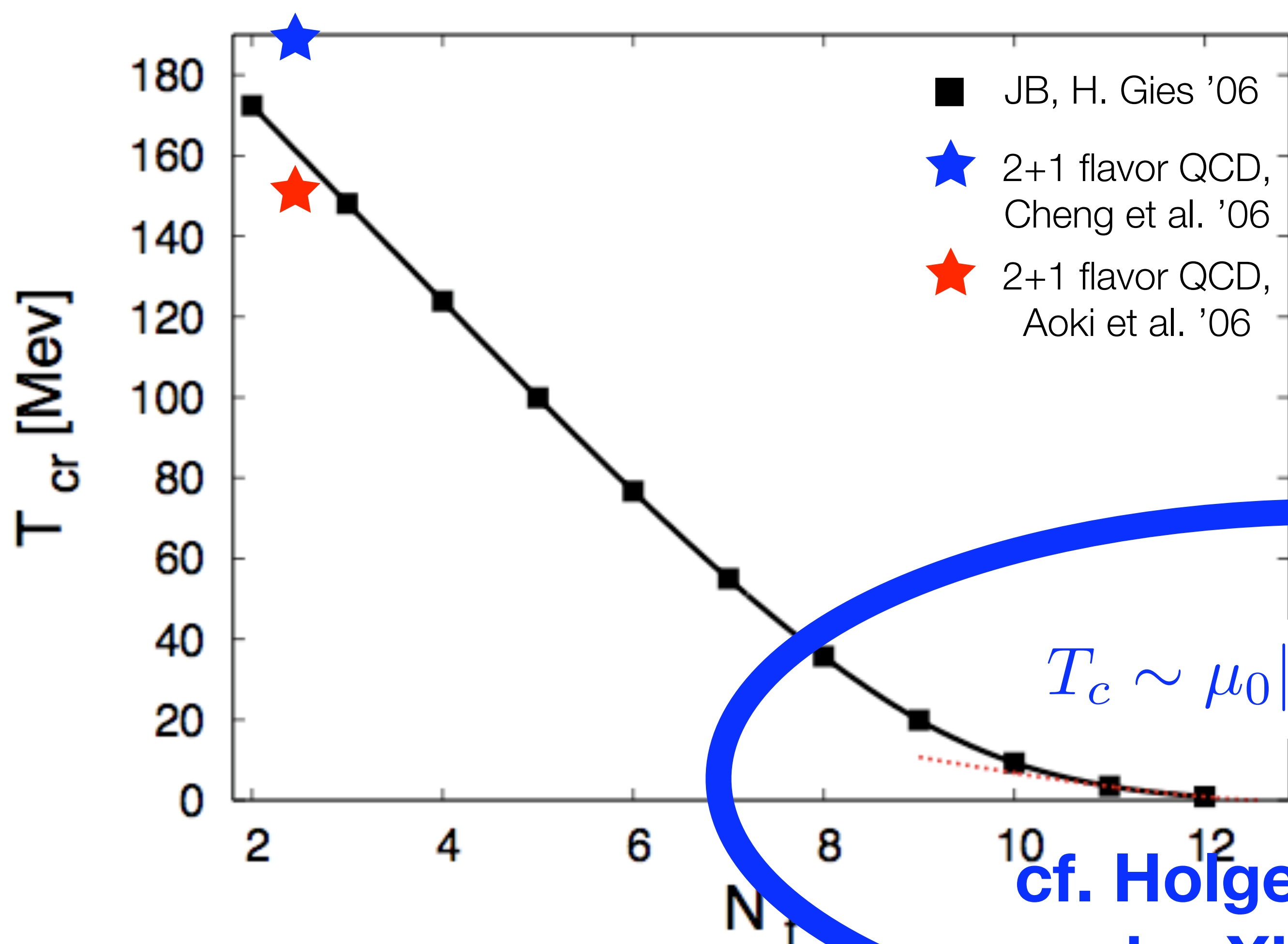
Many-flavor QCD

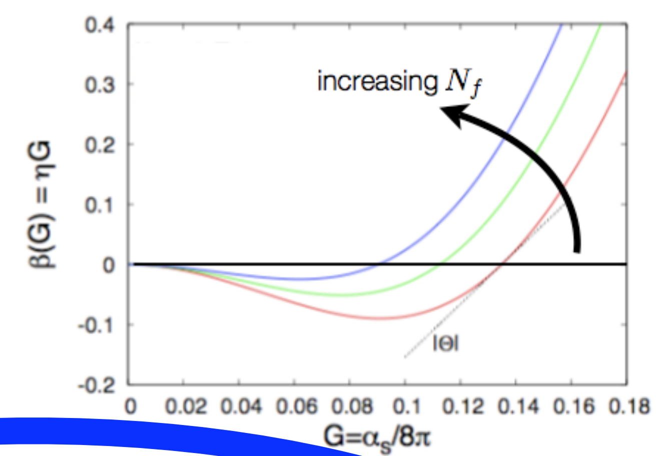


- small N_f : fermionic screening
- critical number of quark flavors: $N_{f,cr} \approx 12$ (cf. e. g. Appelquist et al. '07 & '96)
- "conformal phase" for $N_{f,cr} < N_f < 16.5$: asymptotic freedom but no χSB

(Appelquist et al. '07; Deuzeman, Lombardo, Pallante '08; Fodor et al. '09)

Many-flavor QCD





$$T_c \sim \mu_0 |N_f - N_f^{\mathrm{cr}}|^{\frac{1}{\Theta}}$$

cf. Holger Gies' talk, and arXiv:0909.xxxx

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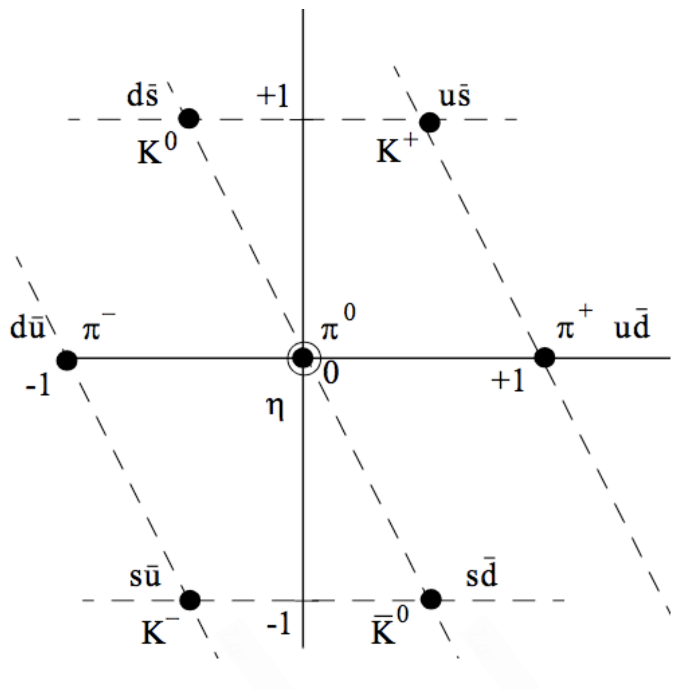
(Appelquist et al. '07; Deuzeman, Lombardo, Pallante '08; Fodor et al. '09)

Challenge:

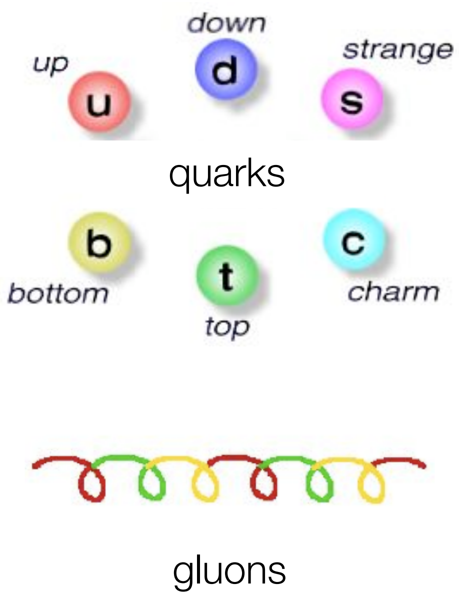
How to penetrate the phase boundary in
order to get access to
the low-energy observables
from first principles?

From microscopic to macroscopic DoFs

macroscopic DoF



microscopic DoF



RG flow

$k \to 0$

large length scales

k

$k \to \Lambda$

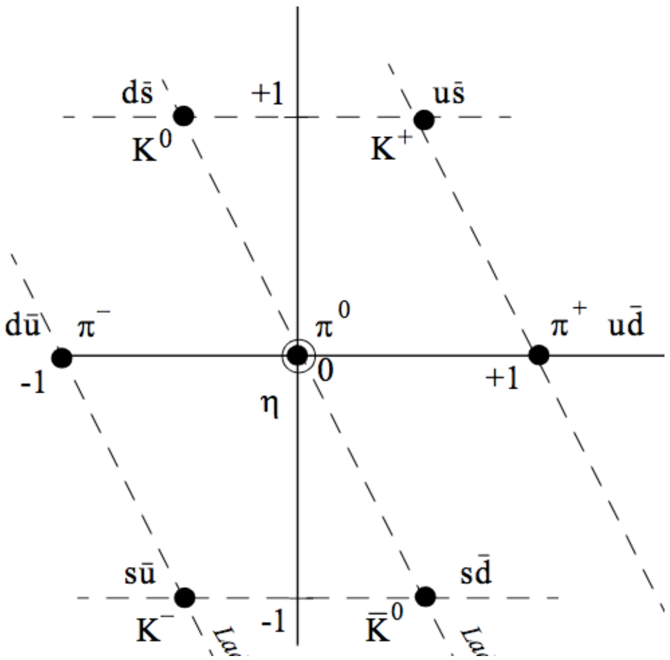
small length scales

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \frac{\partial_t R_k}{\Gamma_k^{(2)}[\phi] + R_k}$$

From microscopic to macroscopic DoFs: Do it by hand ...

macroscopic DoF

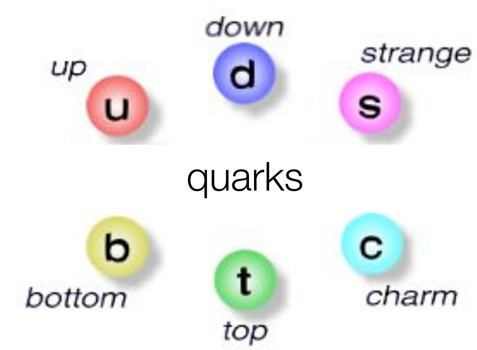
For example:
(constituent-quark-) meson-model



RG flow | RG flow

microscopic DoF

quark-gluon dynamics

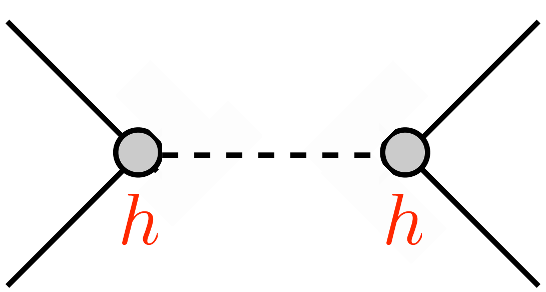


gluons

e. g.: $k \approx k_{cr}$
"set" by diverging
$\bar{\lambda}_\sigma(\mu, T)$

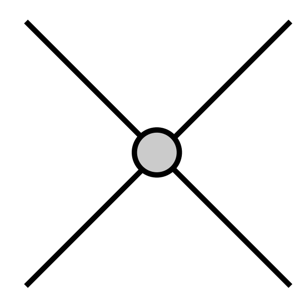
$$h\bar{\psi}(\sigma + \mathrm{i}\gamma_5\pi)\psi + m^2(\sigma^2 + \pi^2)$$

$$\bar{\lambda}_\sigma[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2]$$



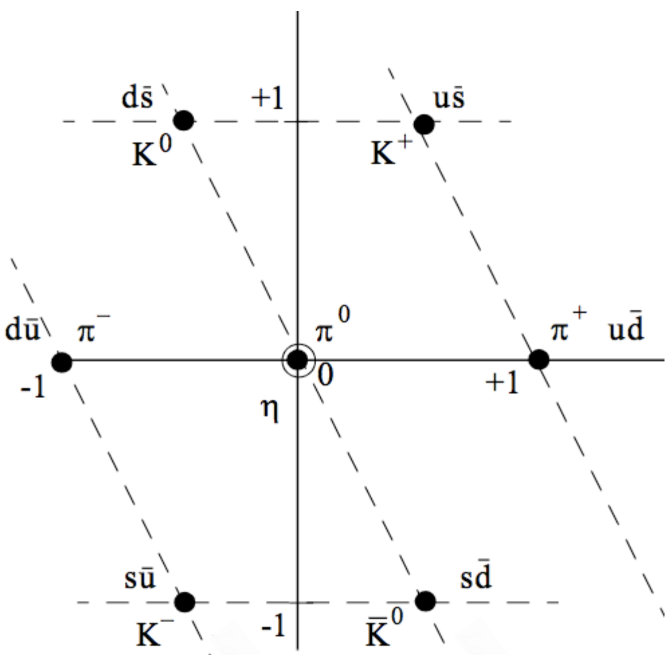
bosonization at fixed scale

Hubbard-Stratonovich transformation



From microscopic to macroscopic DoFs

macroscopic DoF

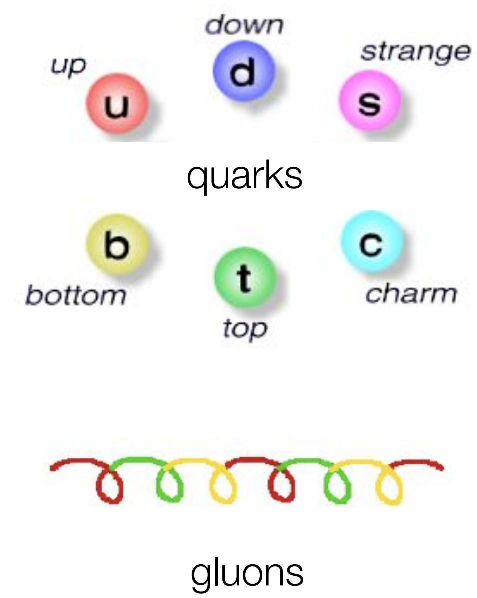


solution:

scale-dependent degrees of freedom

$\partial_t \phi_k \sim \bar{\psi}_L \psi_R$

microscopic DoF



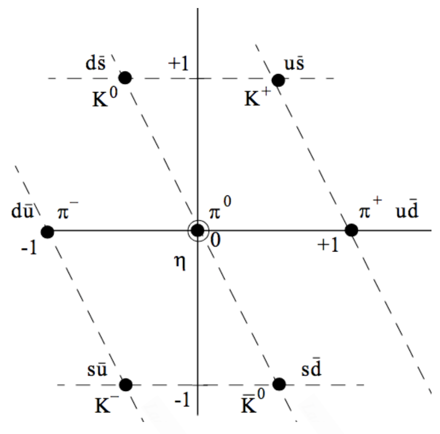
$k \to 0$ $\qquad k \qquad$ $k \to \Lambda$

$$\partial_t \Gamma_k[\phi_k] = \frac{1}{2}\mathrm{Tr}\frac{\partial_t R_k}{\Gamma_k^{(2)}[\phi_k] + R_k} - \int_x \frac{\delta \Gamma_k[\phi_k]}{\delta \phi_k}\partial_t \phi_k$$

(H. Gies, C. Wetterich '01, '02; J. M. Pawlowski '05)

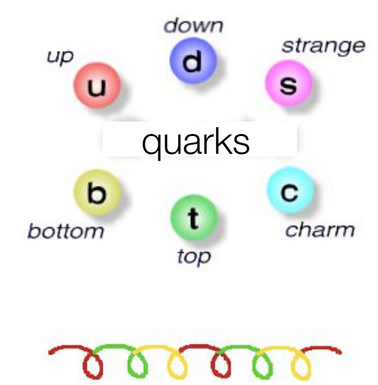
From microscopic to macroscopic DoFs

macroscopic DoF

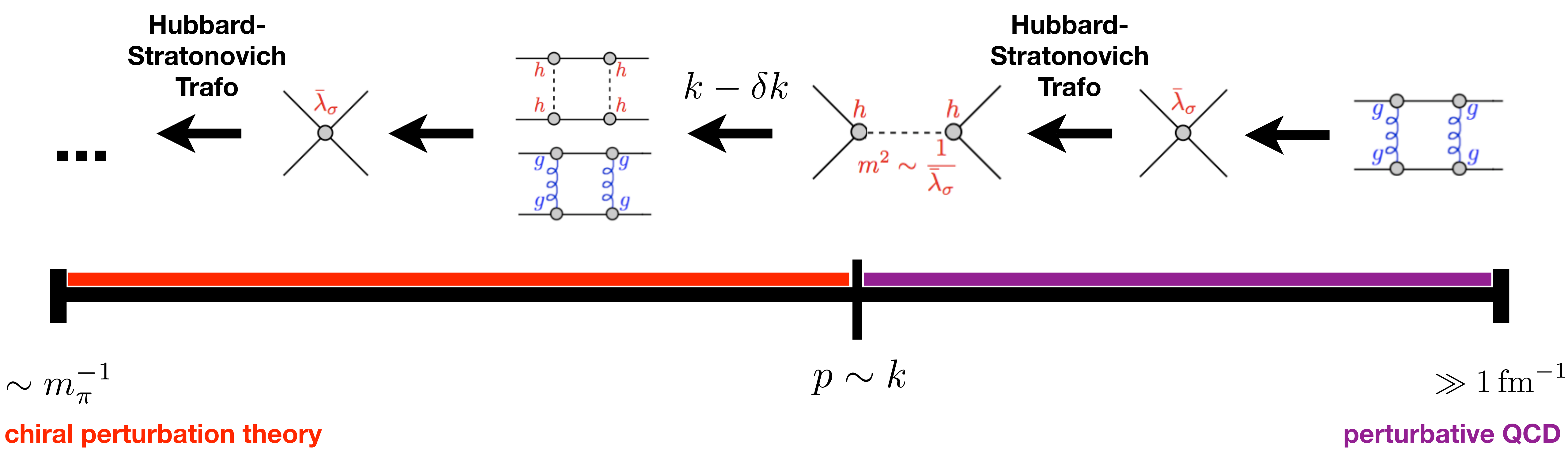


RG flow

microscopic DoF



- **solution:** cont. Hubbard-Stratonovich transformations



QCD with one quark flavor: setting up the stage

- (re-)bosonizaton technique allows to include (momentum-dependent) four-fermion interactions to arbitrary order

- ansatz: (mean-field)

$$\Gamma_k = \int_x \left\{ \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \mathrm{i}\gamma_0 \mu_q)\psi + \frac{m^2}{2}\phi^2 + \left[(\bar{\psi}_R \psi_L)\phi - (\bar{\psi}_L \psi_R)\phi^* \right] \right\}$$

QCD with one quark flavor: setting up the stage

- dynamical hadronization allows to include (momentum-dependent) four-fermion interactions to arbitrary order

- ansatz: (mean-field + bosonic fluctuations)

$$\Gamma_k = \int_x \left\{ Z_\psi \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \mathrm{i}\gamma_0 \mu_q)\psi + Z_\phi \partial_\mu \phi^* \partial_\mu \phi + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + h\left[(\bar{\psi}_R \psi_L)\phi - (\bar{\psi}_L \psi_R)\phi^*\right] \right\}$$

QCD with one quark flavor: setting up the stage

- dynamical hadronization allows to include (momentum-dependent) four-fermion interactions to arbitrary order

- ansatz: (mean-field + bosonic fluctuations + gauge field fluctuations) (JB '08,'09)

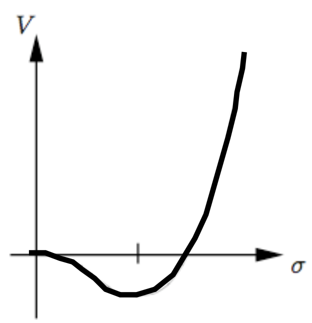
$$\Gamma_k = \int_x \Big\{ \bar{\psi}(\mathrm{i}\not{D} + \mathrm{i}\gamma_0 \mu_q)\psi + \frac{\bar{\lambda}_\sigma}{2}\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2\right] + Z_\phi \partial_\mu \phi^* \partial_\mu \phi + U(\phi^2)$$

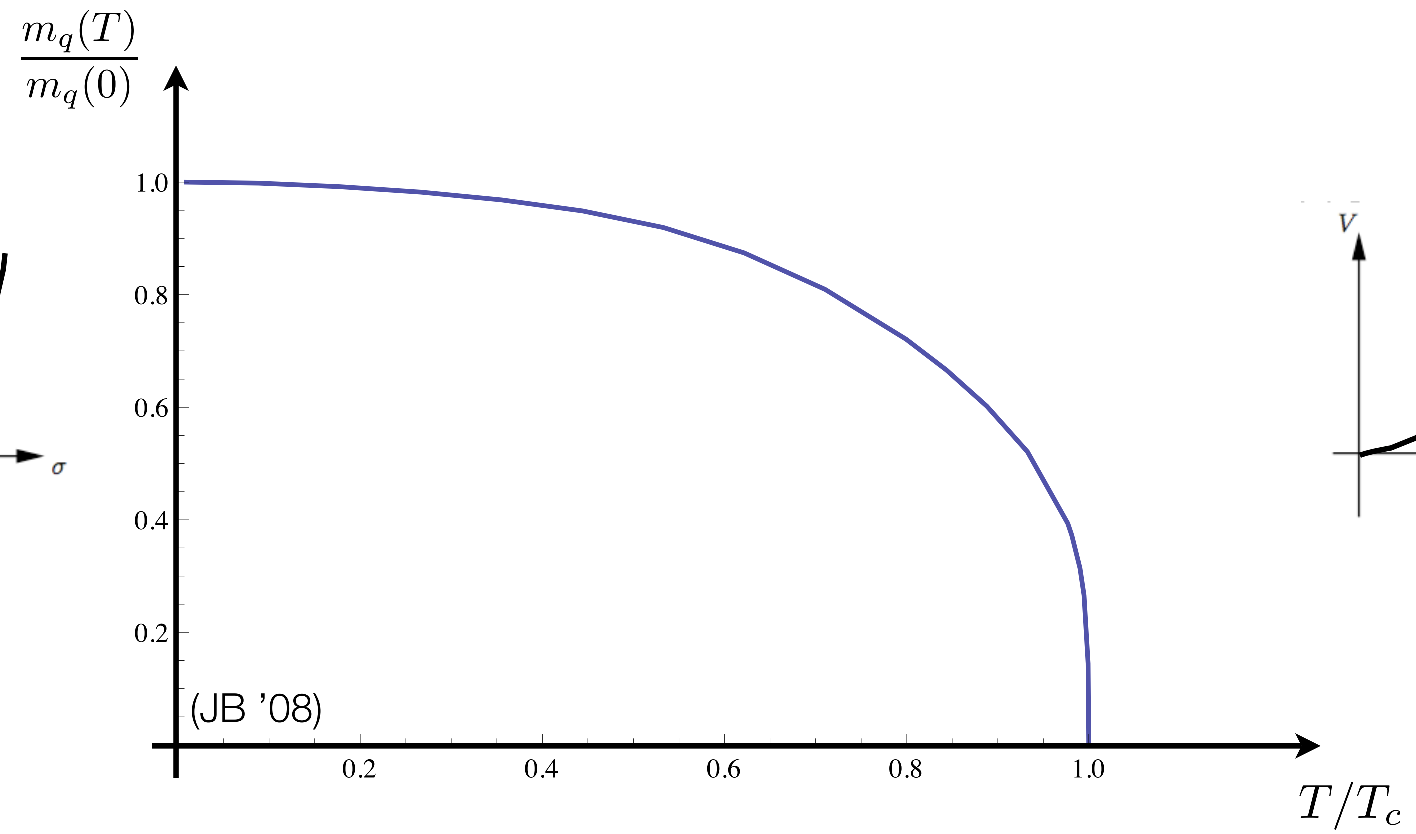
$$+\bar{h}\left[(\bar{\psi}_R\psi_L)\phi - (\bar{\psi}_L\psi_R)\phi^*\right] \Big\} + \Gamma_{gauge}$$

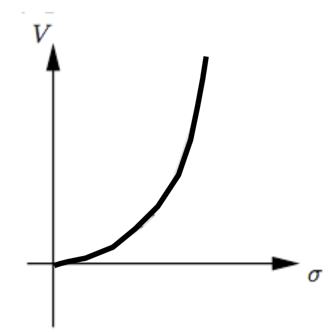
- initial conditions: $\bar{\lambda}_\sigma|_\Lambda = 0,\ \bar{\lambda}_\phi|_\Lambda = 0,\ \bar{h}|_\Lambda = 0,\ Z_\phi|_\Lambda = 0,\ \alpha_s(M_Z) = 0.118$

$$\Gamma_\Lambda = \int_x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}(\mathrm{i}\not{D} + \mathrm{i}\gamma_0 \mu_q)\psi \right\}$$

QCD with one quark flavor







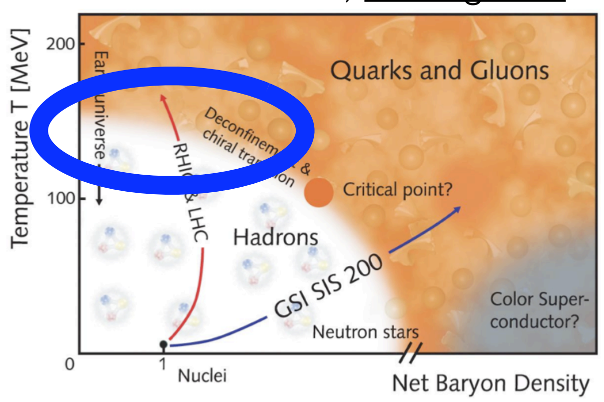
- with global $U_A(1)$ symmetry: 2nd order phase transition
- anomalously broken $U_A(1)$: crossover

QCD phase boundary at small chemical potentials

(J. Braun, accepted for publication in EPJ C)

$$\frac{T_c(\mu_q)}{T_c(0)} = 1 - t_2 \left(\frac{\mu_q}{\pi T_c(0)}\right)^2 + \ldots$$

FAIR, www.gsi.de



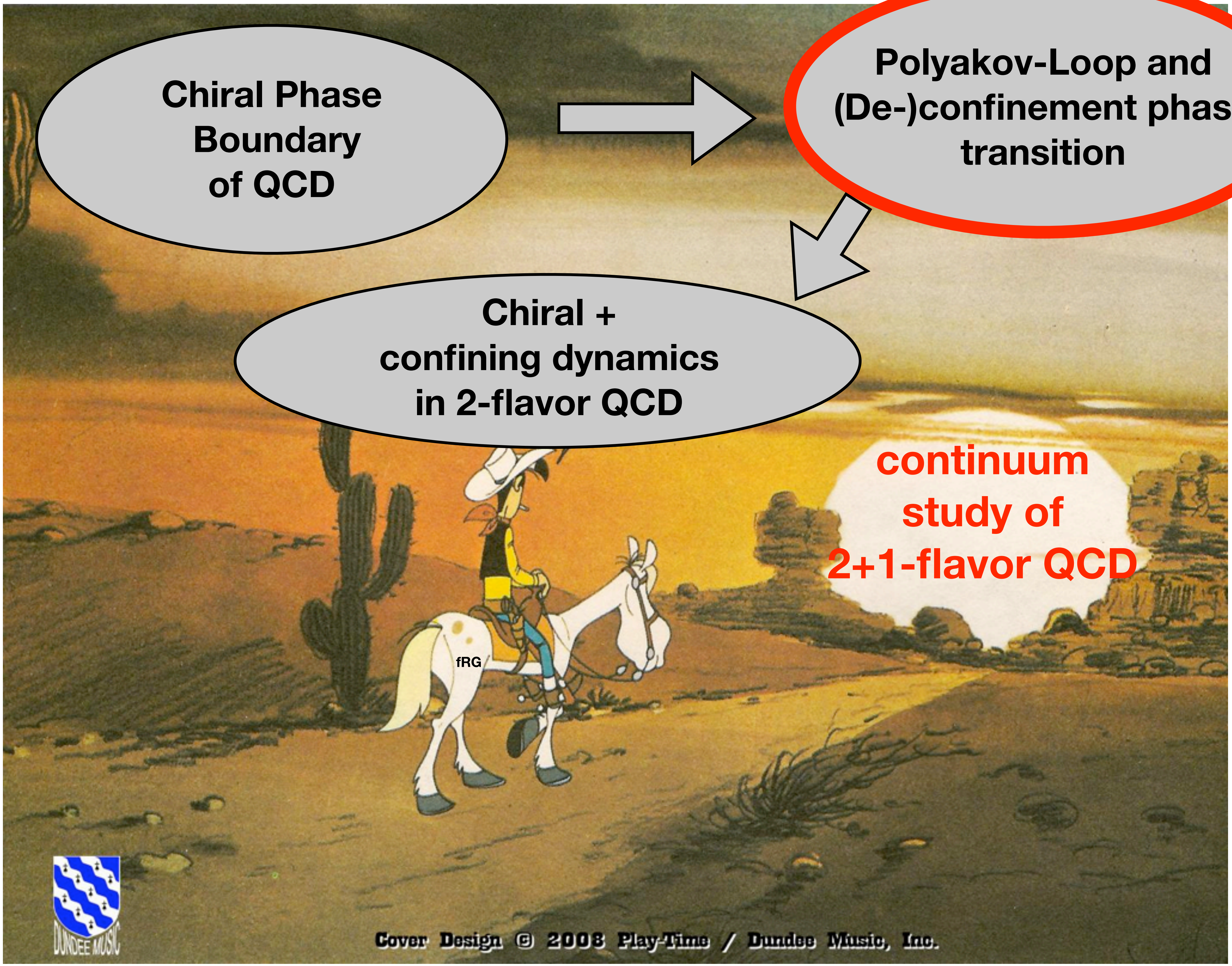
- large N_c expansion: $t_2 \sim \frac{N_f}{N_c}$ (D. Toublan '05, J. Braun '08)
- results from different approaches:

Method	$N_f = 1$	$N_f = 2$	$N_f = 3$	
FRG: QCD flow	0.97* / 0.4**	---	---	(JB '08)
Lattice: imag. μ	---	0.500(54)	0.667(6)	(de Forcrand et al. '02, '06)
Lattice: Taylor+Rew.	---	---	1.13(45)	(Karsch et al. '03)

*with global $U_A(1)$ symmetry
** anomalously broken $U_A(1)$ symmetry (lower bound)

- no parameters, relies solely on physical coupling: $\alpha_s(M_Z)$

Outline


Chiral Phase Boundary of QCD
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fRG
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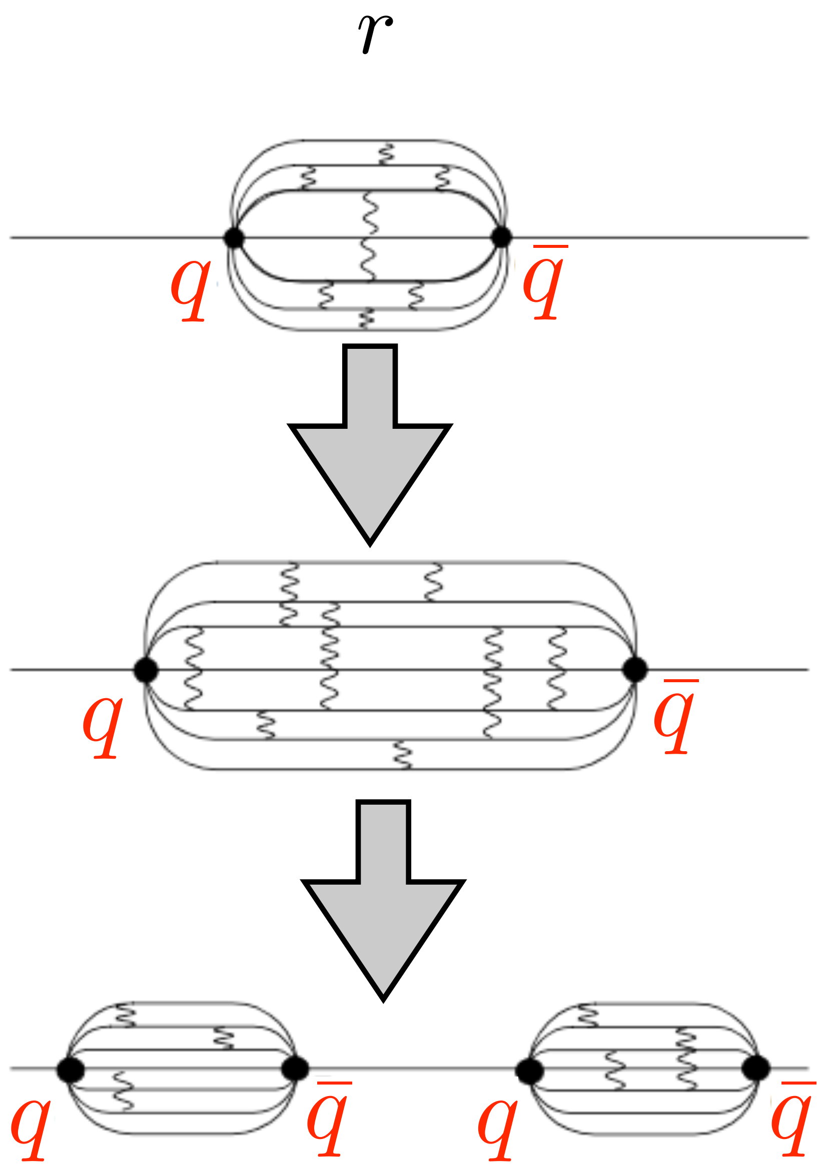
Confinement at zero temperature

potential of a quark-antiquark pair: $\mathcal{F}_{q\bar{q}}(r) \propto \sigma r$

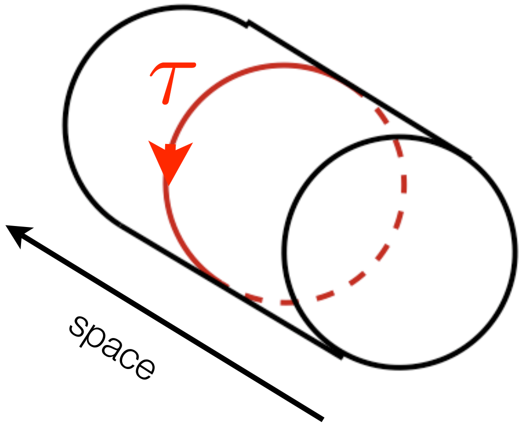


confinement at finite temperature

- infinitely heavy quark moving in Euclidean time direction:

$$\frac{\partial \Psi_q}{\partial \tau} = \mathrm{i}\bar{g} A_0 \Psi_q \quad \Longrightarrow \quad \Psi_q(\vec{x},\tau) = \left[\mathrm{P} \exp\left(i\bar{g} \int_0^\tau dt A_0 \right) \right] \Psi_q(\vec{x},0)$$

infinitely heavy quark propagating in (Euclidean) time direction



- Polyakov-Loop: $\tau = \beta = 1/T$ (Polyakov '78, Susskind '79)

$$\mathcal{P}(\vec{x}) = \frac{1}{N_c} \mathrm{P} \exp\left(i\bar{g} \int_0^\beta dt A_0(t,\vec{x}) \right)$$

confinement at finite temperature

- expectation value of Polyakov-loop is related to the quark free energy $\mathcal{F}_q$:

$$\langle \mathrm{Tr}_F \mathcal{P}(\vec{x}) \rangle \sim \int \mathcal{D}A \, \mathrm{Tr}_F \mathcal{P}(\vec{x}) \, e^{-S} \sim \mathrm{e}^{-\beta \mathcal{F}_q}$$

⇒ deconfinement:

$$\mathcal{F}_q \ \text{finite} \quad \Longleftrightarrow \quad \langle \mathrm{Tr}_F \mathcal{P}(\vec{x}) \rangle \neq 0$$

⇒ confinement:

$$\mathcal{F}_q \to \infty \quad \Longleftrightarrow \quad \langle \mathrm{Tr}_F \mathcal{P}(\vec{x}) \rangle = 0$$

confinement at finite temperature

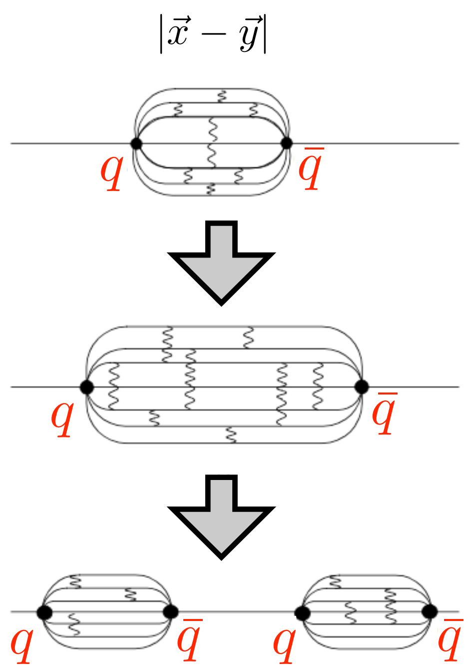
- quark-antiquark correlator:

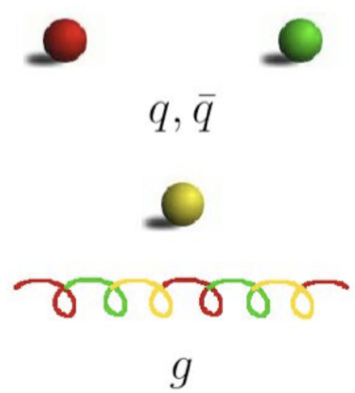
$$\lim_{|\vec{x}-\vec{y}|\to\infty} \mathrm{e}^{-\beta\mathcal{F}_{q\bar{q}}} \sim \lim_{|\vec{x}-\vec{y}|\to\infty} \langle \mathrm{Tr}\,\mathcal{P}(\vec{x}) \cdot \mathrm{Tr}\,\mathcal{P}^\dagger(\vec{y})\rangle \leq |\langle \mathrm{Tr}\,\mathcal{P}\rangle|^2 \sim |\mathrm{e}^{-\beta\mathcal{F}_q}|^2$$

$T < T_c$

$T \geq T_c$

$$\mathcal{F}_{q\bar{q}} \sim \sigma|\vec{x}-\vec{y}|$$





deconfinement

$$\mathcal{F}_q \to \infty \iff \langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle = 0$$

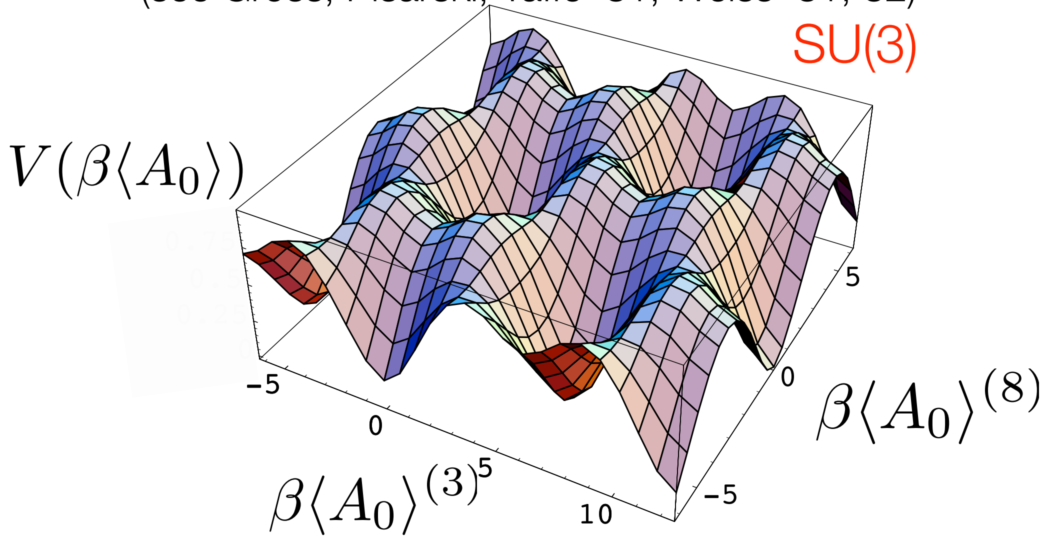
$$\mathcal{F}_q \text{ finite} \iff \langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle \neq 0$$

perturbative Polyakov-Loop potential

(JB, H. Gies, J. M. Pawlowski '07)

- perturbative Polyakov-loop potential in background-field gauge, $A_\mu = \delta_{\mu 0}\langle A_0\rangle$

(see Gross, Pisarski, Yaffe '81; Weiss '81,'82)



minimum at $\beta\langle A_0\rangle = 0$:
deconfinement (broken Z_3-symmetry)

$$\mathcal{F}_q \text{ finite} \quad \Longleftrightarrow \quad \langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle \neq 0$$

Landau-gauge propagators & color confinement

(JB, H. Gies, J. M. Pawlowski '07)

- Polyakov-loop potential:

$$V(\beta\langle A_0\rangle) = \frac{1}{\Omega T^4}\left(\frac{1}{2}\mathrm{Tr}\ln\Gamma^{(2)}_{\mathrm{A}}[\beta\langle A_0\rangle] - \mathrm{Tr}\ln\Gamma^{(2)}_{\mathrm{gh}}[\beta\langle A_0\rangle]\right)$$

- low-temperature:

$$(\Gamma^{(2)}_{A}) \sim (D^2[\langle A_0\rangle])^{1+\kappa_A}\,, \qquad (\Gamma^{(2)}_{\mathrm{gh}}) \sim (D^2[\langle A_0\rangle])^{1+\kappa_{gh}}$$

- **what if ...**

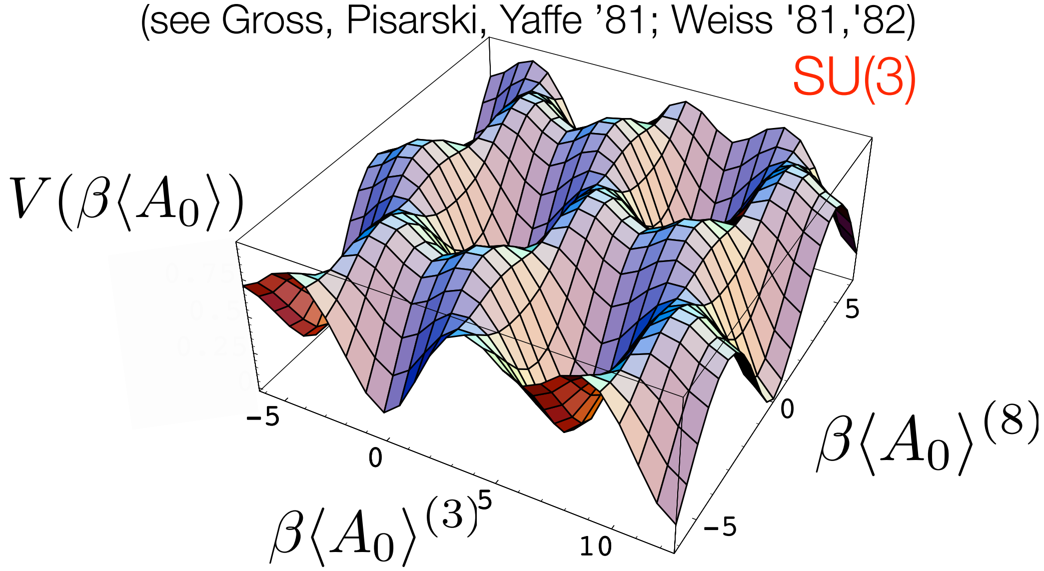
$$3\kappa_A - 2\kappa_{gh} < -2 \quad \Rightarrow \quad \kappa_{gh} > \frac{d-3}{4}$$

perturbative Polyakov-Loop potential

(JB, H. Gies, J. M. Pawlowski '07)

- perturbative Polyakov-loop potential in background-field gauge, $A_\mu = \delta_{\mu 0}\langle A_0\rangle$

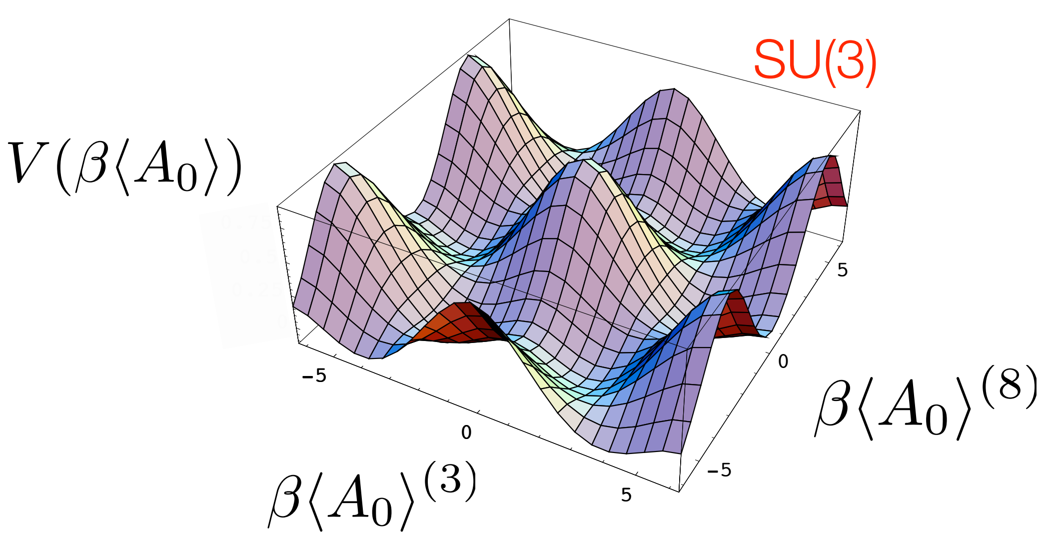
(see Gross, Pisarski, Yaffe '81; Weiss '81,'82)



minimum at $\beta\langle A_0\rangle = 0$:
deconfinement (broken Z_3 -symmetry)

$$\mathcal{F}_q \ \text{finite} \quad \Longleftrightarrow \quad \langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle \neq 0$$

- for $T < T_c$



minimum at $\beta\langle A_0\rangle = (2/3)2\pi$:
deconfinement (broken Z_3 -symmetry)

$$\mathcal{F}_q \to \infty \quad \Longleftrightarrow \quad \langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle = 0$$

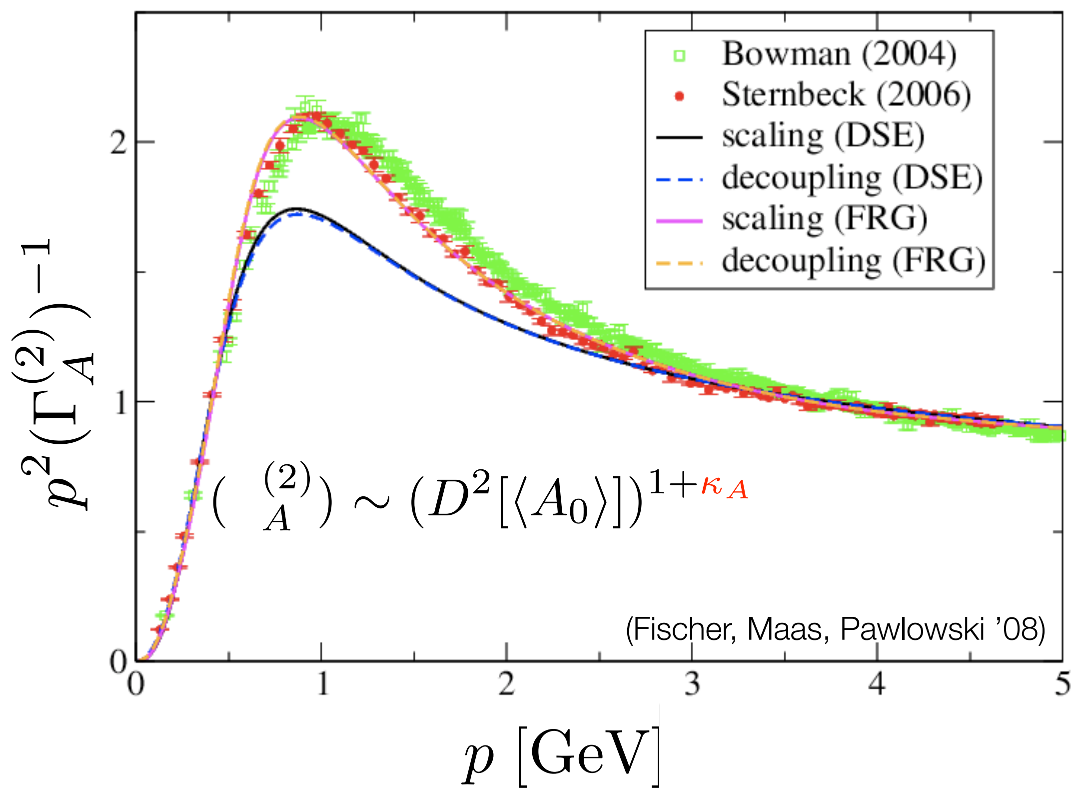
Landau-gauge propagators & color confinement

(JB, H. Gies, J. M. Pawlowski '07)

- Polyakov-loop potential:

RG improvement terms

$$V(\beta\langle A_0\rangle) = \frac{1}{\Omega T^4}\left(\frac{1}{2}\mathrm{Tr}\ln\Gamma^{(2)}_{\mathrm{A}}[\beta\langle A_0\rangle] - \mathrm{Tr}\ln\Gamma^{(2)}_{\mathrm{gh}}[\beta\langle A_0\rangle]\right) + \mathcal{O}(\partial_t\Gamma^{(2)}) + \mathcal{O}(V'')$$



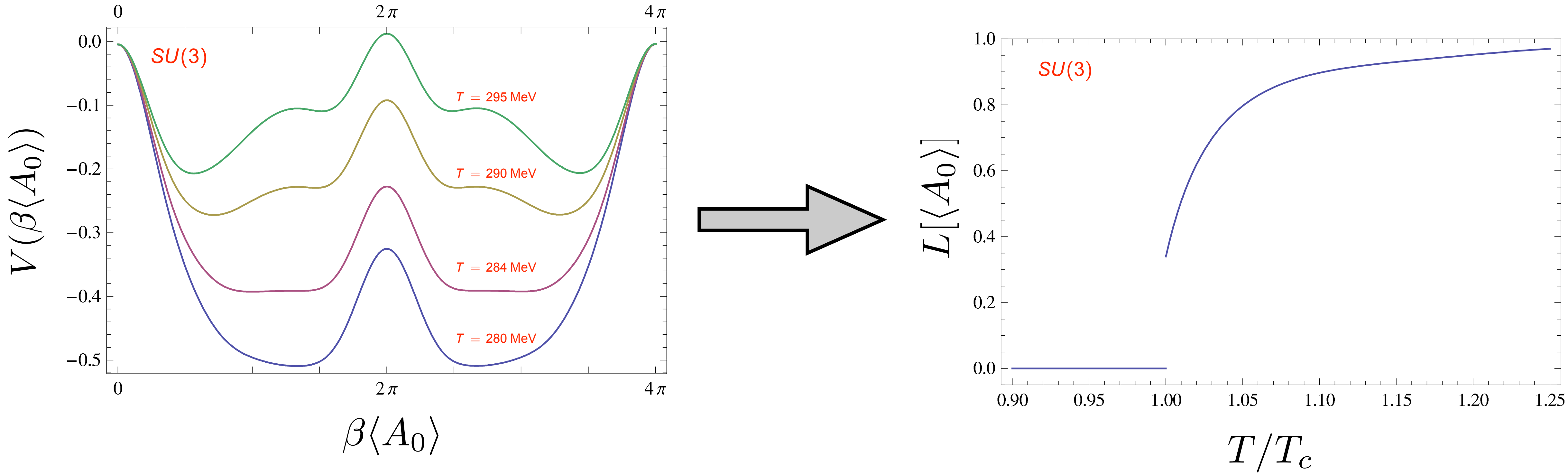
- **quark confinement criterion (Landau gauge)**:

$$\langle \mathrm{Tr}_F \mathcal{P}(\vec{x})\rangle = 0 \ : \ 3\kappa_A - 2\kappa_{gh} < -2$$

Polyakov-Loop Potential in Landau-gauge

(JB, H. Gies, J. M. Pawlowski '07)

- order parameter $L[\langle A_0\rangle] = \frac{1}{N_c}\mathrm{Tr}_F \exp\left(i\int_0^\beta dt\langle A_0\rangle\right)$



- first order phase transition for SU(3) (and second order for SU(2))
- SU(3): $T_c = 284\,\mathrm{MeV} (= 0.646\sqrt{\sigma})$ Lattice QCD: $T_c = 0.646\sqrt{\sigma}$ (Kaczmarek et al.)
- more gauge groups: (JB, A. Eichhorn, H. Gies, J. Pawlowski, in prep.)
  - SU(N) for N=4, 5, 6, 7, 8: first order
  - symplectic group Sp(2): first order (in agreement with Holland, Pepe, Wiese '04)
  - exceptional group G(2) and others are under investigation ...

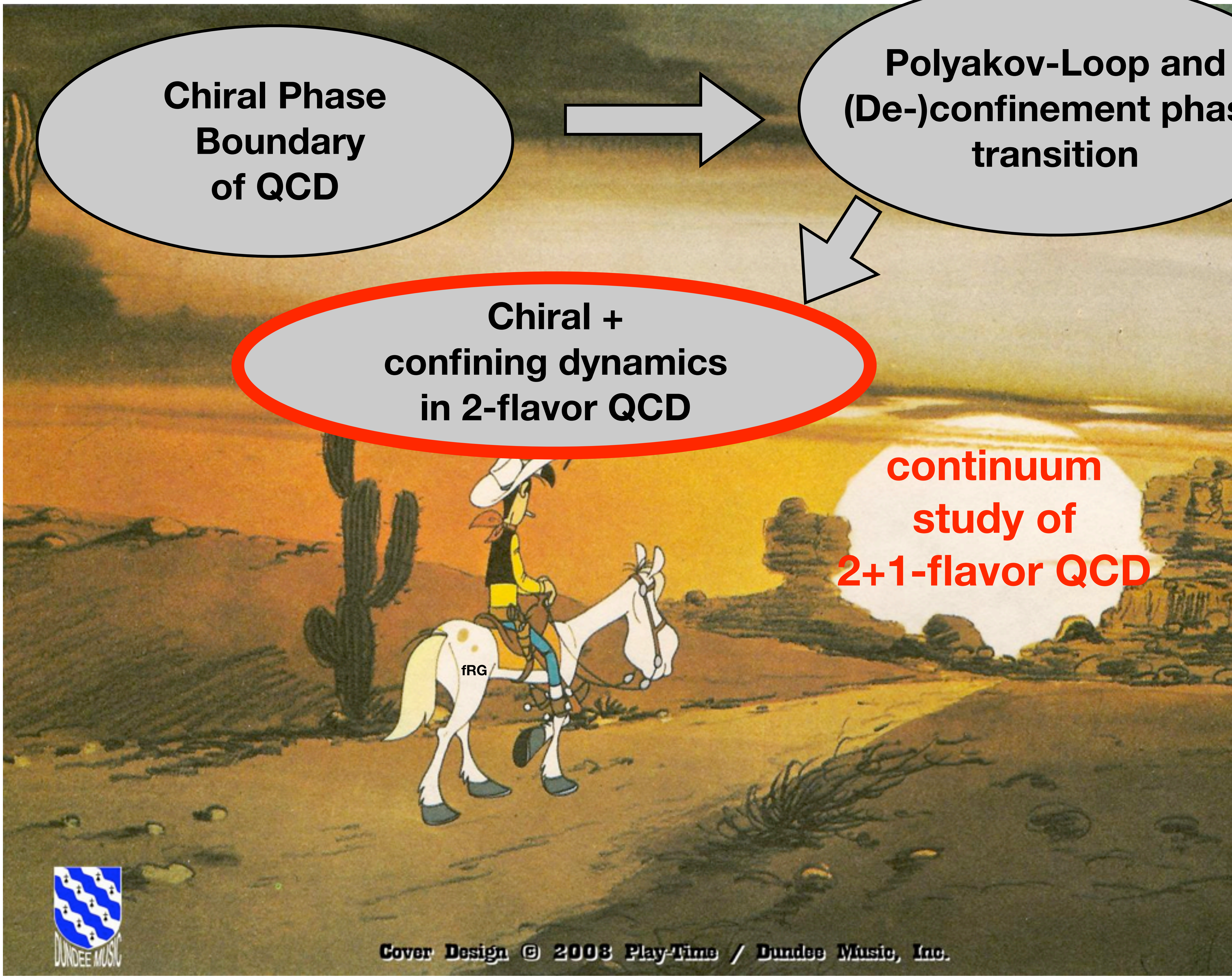
Outline

Chiral Phase Boundary of QCD

Polyakov-Loop and (De-)confinement phase transition

Chiral + confining dynamics in 2-flavor QCD

continuum study of 2+1-flavor QCD

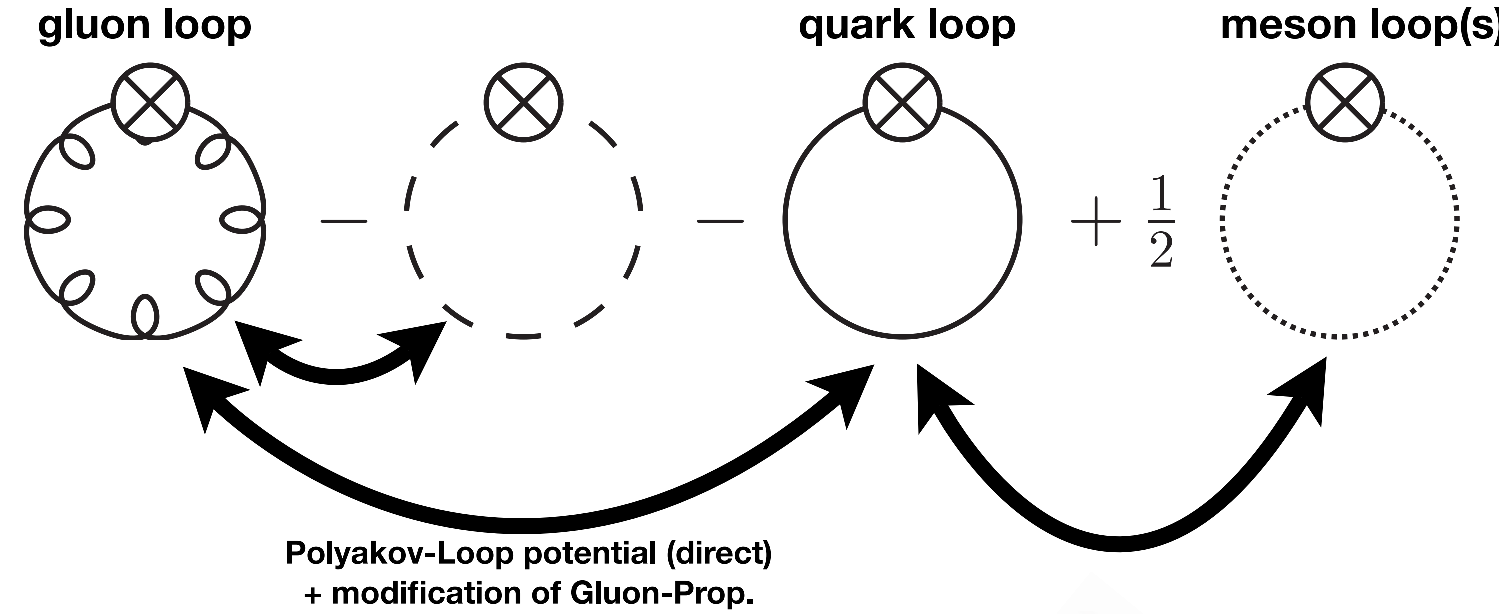


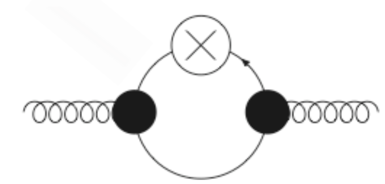
2-flavor QCD study in the chiral limit

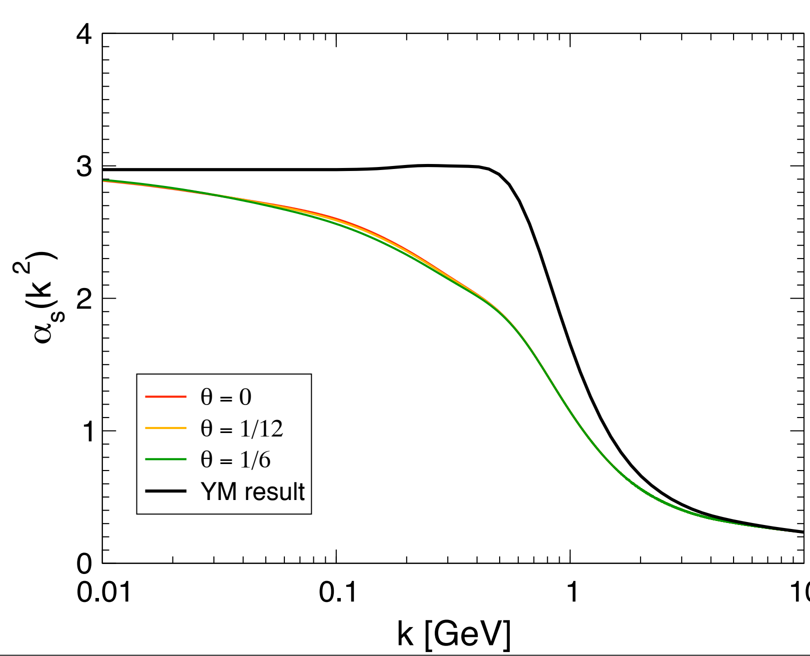
(JB, L. Haas, F. Marhauser, J. M. Pawlowski ’09)

- RG flow of the effective action

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \ldots$$



- quark contribution to the flow of the gluon propagator:  **(cf. L. Haas’ talk)**
- running coupling at zero temperature:



(see also JB, Gies ’05, ’06)

2-flavor QCD study in the chiral limit

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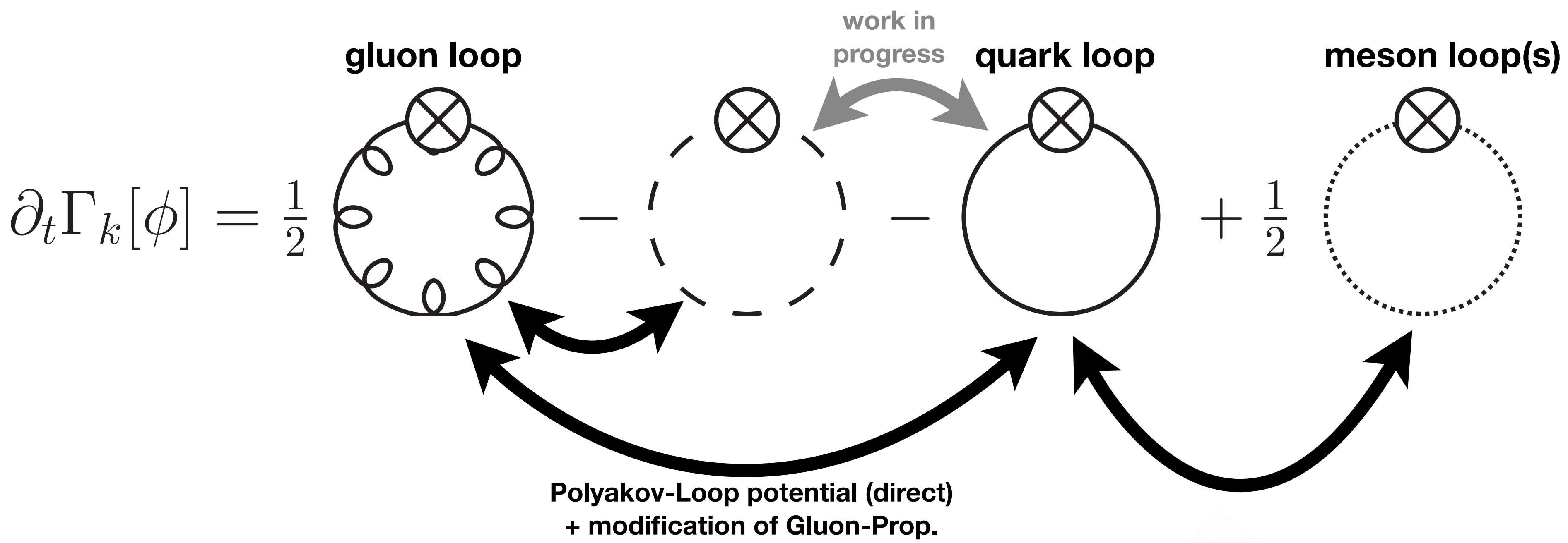

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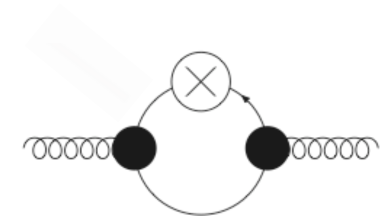
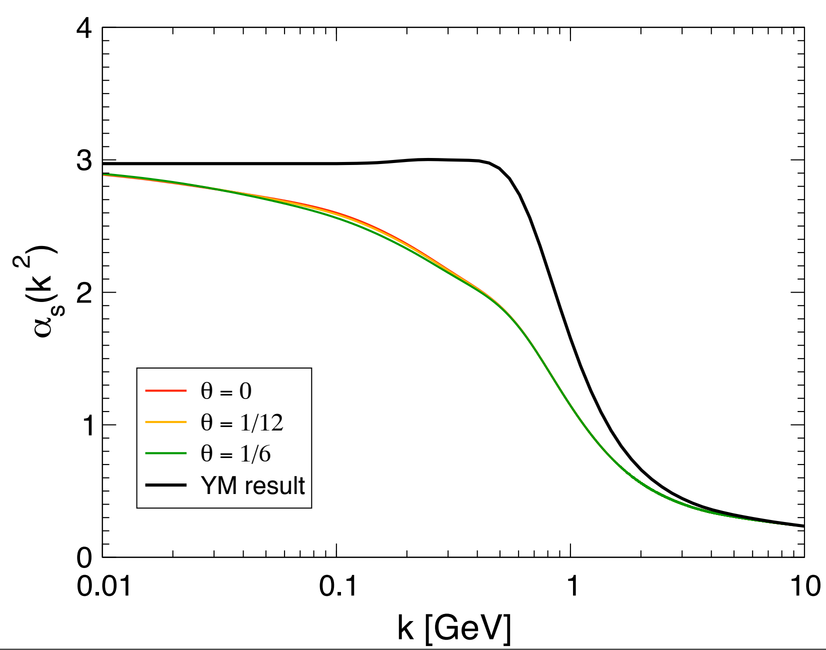


(cf. L. Haas’ talk)

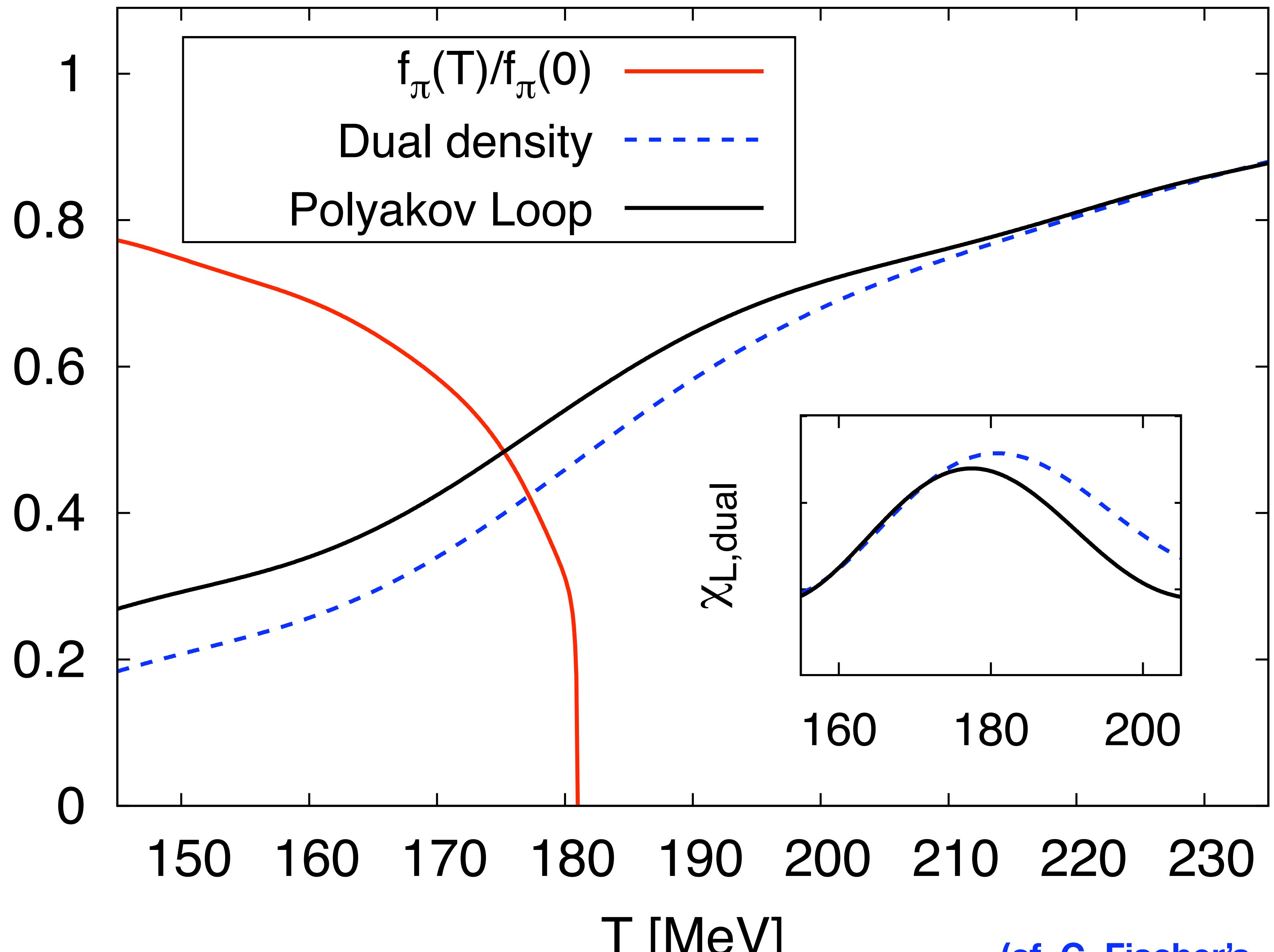
- running coupling at zero temperature:



(see also JB, Gies ’05, ’06)

Chiral and Confinement order parameters for vanishing chemical potential

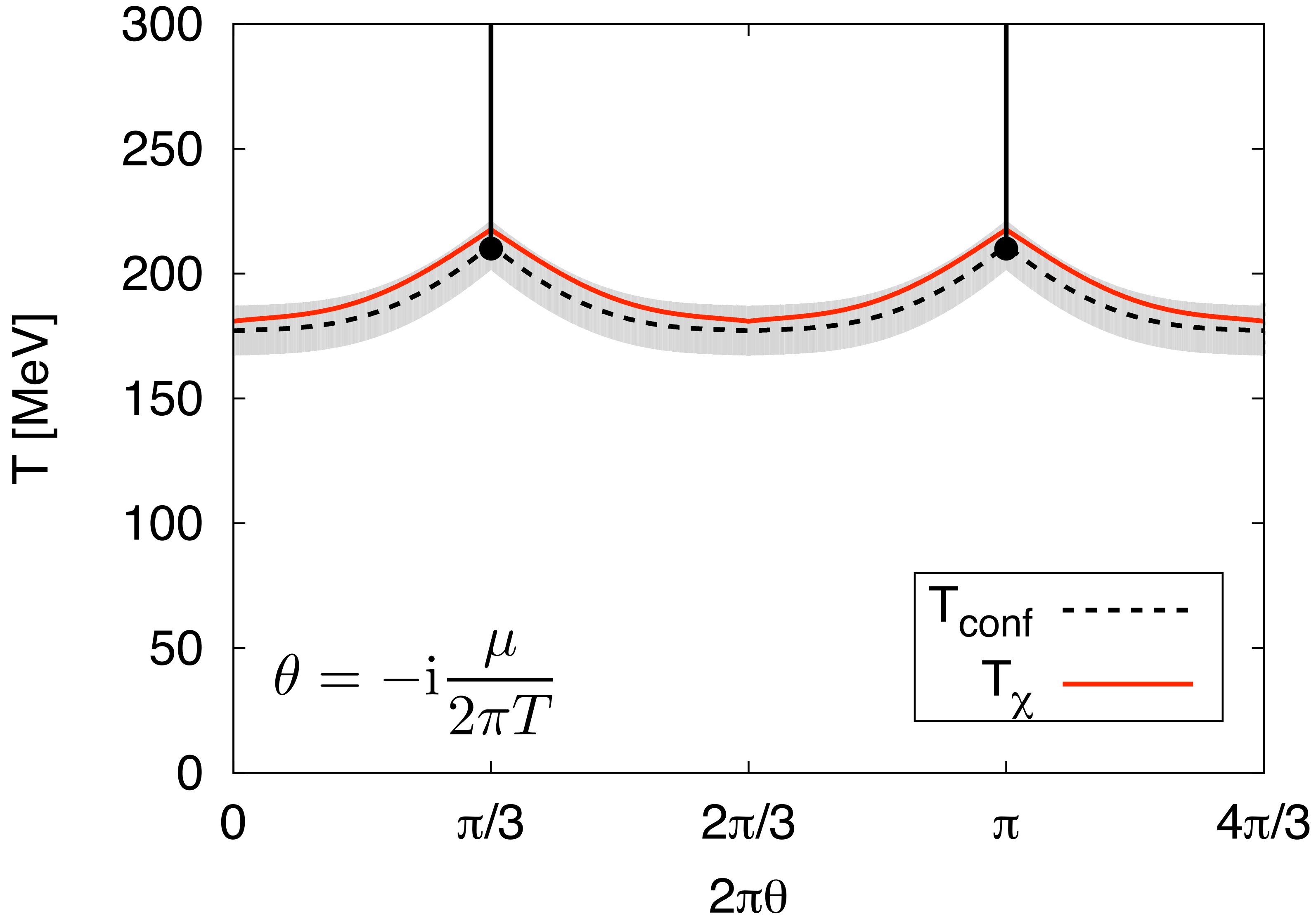
(JB, L. Haas, F. Marhauser, J. M. Pawlowski '09)



(cf. C. Fischer's, J. M. Pawlowski's and & A. Wipf's talk for a discussion of dual observables)

Roberge-Weiss phase diagram

(JB, L. Haas, F. Marhauser, J. M. Pawlowski '09)



- chiral phase transition and deconfinement crossover temperature follow each other closely (in accordance with Lattice QCD, see Kratochvila et al. '06 & Wu et al. '06)
- PNJL model studies agree with lattice QCD if an additional $\phi^4 \sim \psi^8$-coupling is introduced in the PNJL model (Sakai et al. '09)

Conclusions

- FRG allows to bridge the gap between regimes with different DoF
- good agreement with Lattice QCD studies for chiral as well as deconfinement phase transition
- criterion for quark confinement & studies of the Polyakov loop dynamics in various gauge groups
- critical number of quark flavors for SU(3): $N_{f,cr} \approx 12$
- shape of the phase boundary near $N_{f,cr}$ is determined by the underlying IR fixed point scenario (testable prediction!)
- promising results for finite chemical potential, including a relation of QCD at imaginary chemical potential and dual observables

Outlook

- quantitative study of the effect of anomalously broken $U_A(1)$ on the phase boundary (together with L. Haas, F. Marhauser and J. M. Pawlowski)

- RG analysis of lattice data and the nature of the chiral transition?
  (insights from finite-volume scaling (?), together with B. Klein and P. Piasecki)

- study finite (real) chemical potential for $N_f = 2$ and $N_f = 3$:
  radius of convergence?

- study of thermodynamic observables

- successive improvement of truncations (include dynamical hadronization in 2-flavor QCD study)