Quark and gluon properties in dense 2-colour QCD

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Quarks, hadrons and the phase diagram of QCD
St. Goar, 1 September 2009

Outline

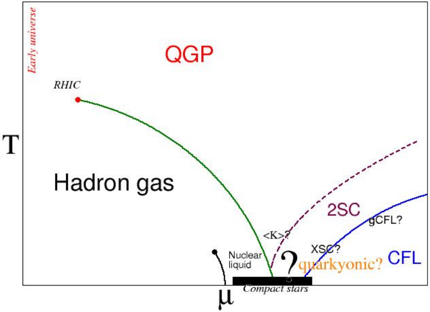
Background
- QC_2D vs QCD

Formalism
- Tensor structures
- Lattice formulation

Results
- Bulk thermodynamics
- Gluon propagator results
- Quark propagator results

Summary

Background



- A plethora of phases at high μ, low T
- Based on models and perturbation theory
- Details depend on diquark gaps and strange quark mass
- Diquark condensation a generic feature

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Indirect approach

Study QCD-like theories without a sign problem

- Generic features of strongly interacting systems at $\mu \neq 0$
- Check on model calculations

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$\rightarrow$ superfluidity rather than colour superconductivity

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Condensation of tightly bound diquarks (Goldstone baryons)

$\leftrightarrow$ Chiral perturbation theory

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$$\langle\psi\psi\rangle \propto \sqrt{1-(\mu/\mu_o)^4}$$

Bardeen–Cooper–Schrieffer:
Pairing of quarks near the Fermi surface

$$\langle\psi\psi\rangle \propto \Delta\mu^2$$

QC_2D vs QCD— Issues of interest

Gluodynamics — SU(2) and SU(3) very similar?

- Deconfinement at high density — effects on gluon propagator?
- Gap equation with effective or one-gluon interaction used to determine superconducting gap → more realistic input?
- Static magnetic gluon: unscreened at all orders in perturbation theory!

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Quark propagator

- Details of phase diagram depend critically on the effective quark mass in the medium.
- Location of Fermi surface?
- Direct determination of diquark gap, size of Cooper pairs?

Tensor structure in medium

The medium breaks Lorentz (Euclidean) symmetry to O(3)
$\Longrightarrow$ 1 $\rightarrow$ 2 scalar functions in gluon, 2 $\rightarrow$ 4 in quark:

$$D_{\mu\nu}(\vec{q}, q_t) = P^T_{\mu\nu} D_M(\vec{q}^{\,2}, q_t^2) + P^E_{\mu\nu} D_E(\vec{q}^{\,2}, q_t^2) + \xi \frac{q_\mu q_\nu}{q^4}$$

$$S^{-1}(\vec{p}, \tilde{\omega}) = i\vec{\not{p}} A(\vec{p}^{\,2}, \tilde{\omega}^2) + i\gamma_4 \tilde{\omega} C(\vec{p}^{\,2}, \tilde{\omega}^2) + B(\vec{p}^{\,2}, \tilde{\omega}^2) + i\gamma_4 \vec{\not{p}} D(\vec{p}^{\,2}, \tilde{\omega}^2)$$

$$S(\vec{p}, \tilde{\omega}) = i\vec{\not{p}} S_a + i\gamma_4 \tilde{\omega} S_c + S_b + i\gamma_4 \vec{\not{p}} S_d$$

where $\tilde{\omega} \equiv p_4 - i\mu$.

Gor'kov formalism

Quarks and antiquarks are in the same representation.
Construct Gor'kov spinor

$$\Psi = \begin{pmatrix} \psi \\ \overline{\psi}^T \end{pmatrix} \quad \Longrightarrow \quad \langle \Psi(x) \bar{\Psi}(y) \rangle \equiv \mathcal{G}(x, y) = \begin{pmatrix} S_N & -S_A \\ \bar{S}_A & \bar{S}_N \end{pmatrix}$$

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Symmetries

From isospin and charge conjugation symmetry it follows that

$$\bar{S}_N(x,y) = -S_N(y,x)^T , \qquad S_A(x,y) = S_A(y,x)^T$$

Fermi surface and Cooper pairs

Fermi surface

In a Fermi liquid the Fermi surface is given by

$$\det S^{-1}(\overrightarrow{p_F}, p_4 = 0) = 0 \quad \Longleftrightarrow \quad \overrightarrow{p}^2 A^2 + \tilde{\omega}^2 C^2 + B^2 = 0$$

Pole in propagator

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Size of Cooper pair

If we know the anomalous propagator $S_A(x)$ we can compute the size of the Cooper pairs:

$$\xi^2 = \frac{\int d^3x \, \overrightarrow{x}^2 |\frac{1}{2} \operatorname{Tr}(S_A(x)\Lambda^+)|^2}{\int d^3x \, |\frac{1}{2} \operatorname{Tr}(S_A(x)\Lambda^+)|^2}$$

Lattice formulation

We use Wilson fermions:

- Correct symmetry breaking pattern, Goldstone spectrum
- $N_f < 4$ needed to guarantee continuum limit
- No problems with locality, fourth root trick
- Chiral symmetry buried at bottom of Fermi sea

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$$\gamma_5 M(\mu)\gamma_5 = M^\dagger(-\mu), \quad C\gamma_5\tau_2 M(\mu) C\gamma_5\tau_2 = -M^*(\mu)$$

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Diquark source J introduced to

- lift low-lying eigenmodes in the superfluid phase
- study diquark condensation without uncontrolled approximations

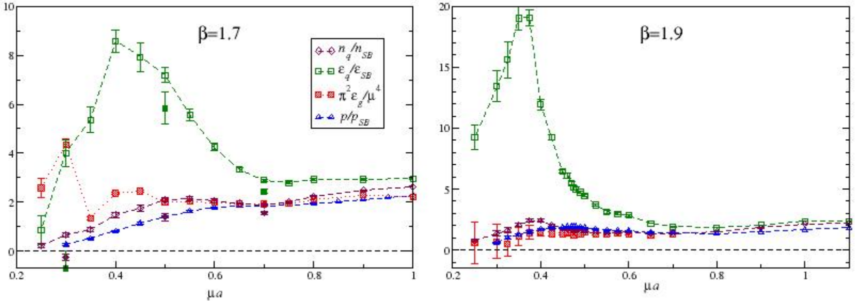
Simulation Parameters

We work on two lattices, 'coarse' and 'fine'.
Two 'finer' lattices are used for $\mu = 0$ simulations only

Name	β	κ	Volume	a	am_π	m_π/m_ρ
coarse	1.7	0.178	$8^3 \times 16$	0.23fm	0.79	0.80
fine	1.9	0.168	$12^3 \times 24$	0.18fm	0.65	0.80
finer, h	2.0	0.162	$12^3 \times 24$		0.64	0.83
finer, l	2.0	0.163	$12^3 \times 24$		0.52	0.76

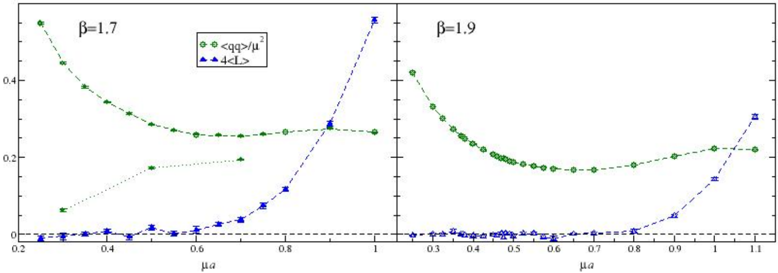
- Simulations performed with $j = J/\kappa = 0.04$ for $\mu = 0.3 - 1.0$
- 300–500 trajectories for each μ.
- Simulations with $j = 0.02, 0.06$ for $\mu = 0.3, 0.5, 0.7, 0.9$ (coarse lattice) $\rightarrow$ enable extrapolation to $j = 0$.

Thermodynamics results



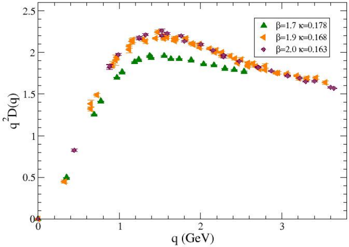
- Close to SB scaling for $\mu > \mu_d$
- $\varepsilon_q \sim 2\varepsilon_{SB} \rightarrow k_F > E_F \implies$ binding energy?
- 30–40% of total energy from gluons!?
- Renormalisation of energy densities in progress [with Joyce Myers, Simon Hands]

Phase transitions



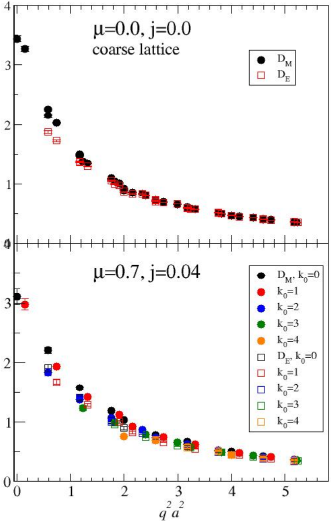
- Deconfining transition at $a\mu_d \sim 0.65$ on both lattices?!
- Still very far from saturation at μ_d
- BEC $\rightarrow$ BCS crossover becoming softer?
- Quarkyonic superfluid?

Gluon propagator results

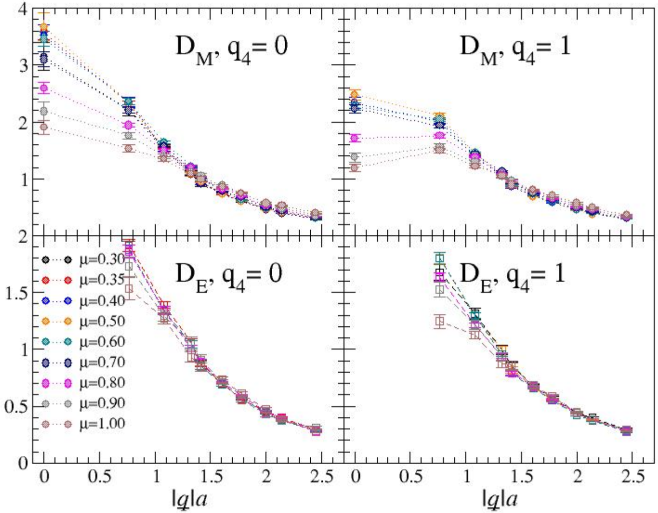


Some finite volume and lattice spacing effects at $\mu = 0$

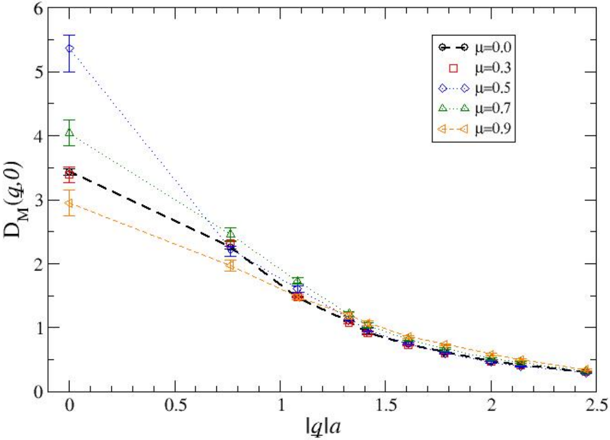
In-medium modifications, incl. violations of Lorentz symmetry, visible in magnetic gluon at $\mu = 0.7$



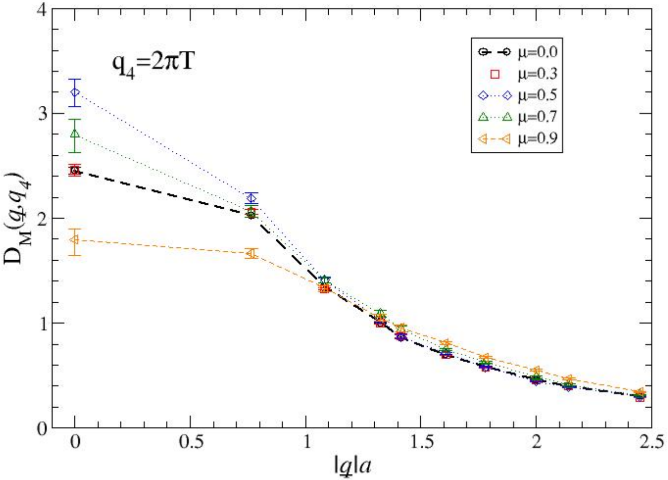
Coarse lattice results



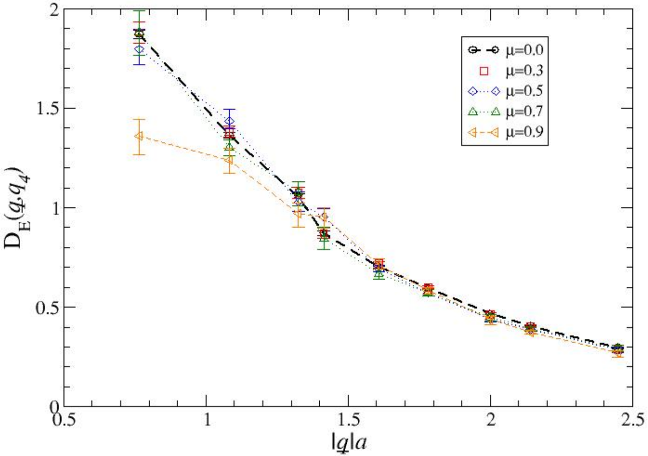
Static magnetic gluon extrapolated to j=0



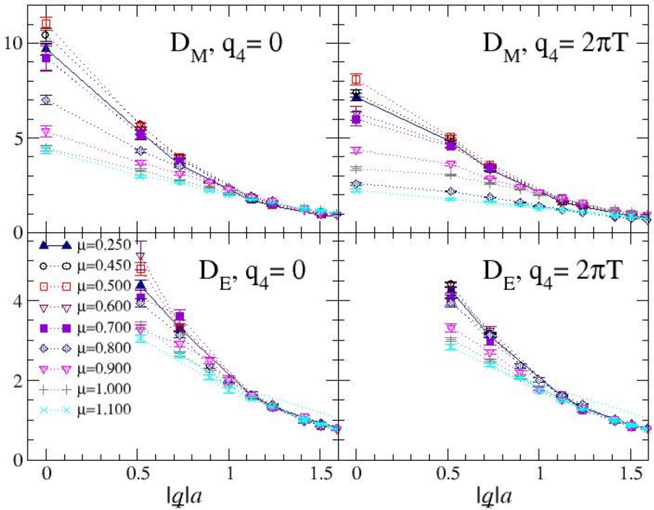
Magnetic gluon ($q_4 = 2\pi T$) extrapolated to j=0



Electric gluon extrapolated to j=0



Fine lattice results



In-medium gluon mass

Crude fit to 'massive' form

$$D_{E,M}(\overrightarrow{q}, q_4) = \frac{Z}{\overrightarrow{q}^2 + q_4^2 + m_{e,m}^2}$$

not a good fit!

Improvement:
Try HDL-inspired form?

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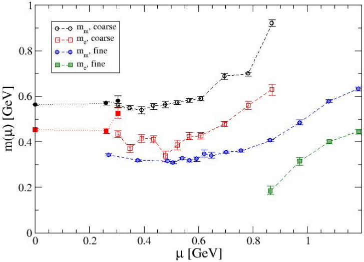


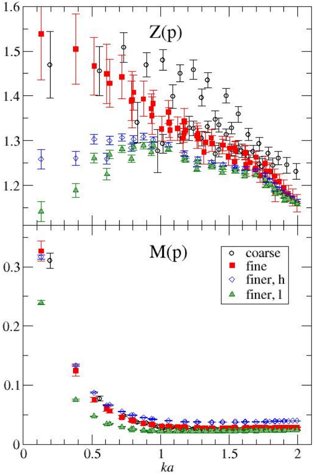
Fit gives $m_e = 0$ for $a\mu < 0.7$ on fine lattice

Quark propagator results

Quark propagator in vacuum

Raw data — not in physical units!

- Large lattice spacing dependence
- Substantial quark mass dependence for $Z(p)$
- Unusual momentum behaviour in $Z(p)$
- infrared suppression recovered in low-mass and continuum limit?



Tensor structure

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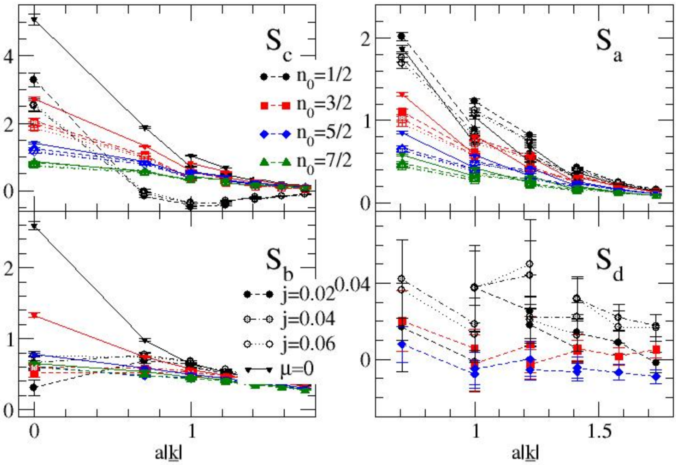
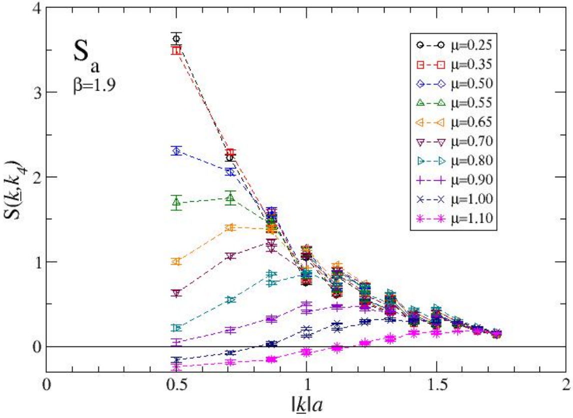
We would like to reduce the number of components to consider
We find that

- the vector, scalar and temporal components of the normal propagator S_a, S_b, S_c and
- the scalar and tensor components A_b, A_d of the anomalous propagator are nonzero
- all other components are zero

Quark propagator on coarse lattice, $\mu = 0.5$



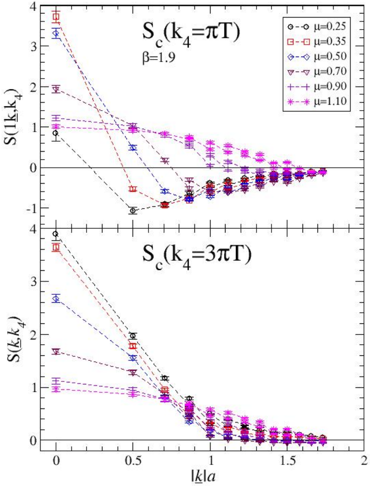
Normal propagator: spatial vector part



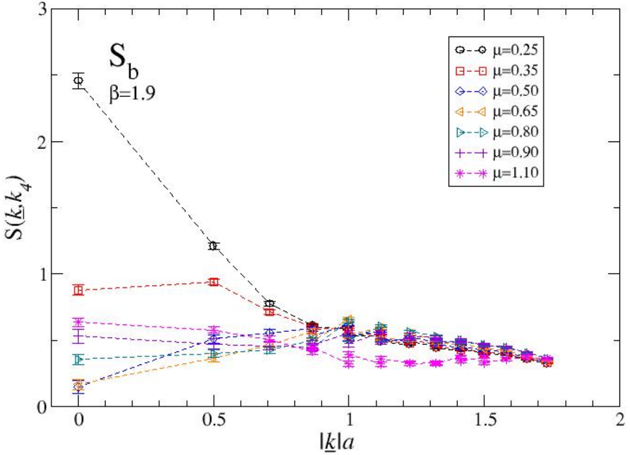
Normal propagator: temporal vector part

All data are for $aj = 0.04$!

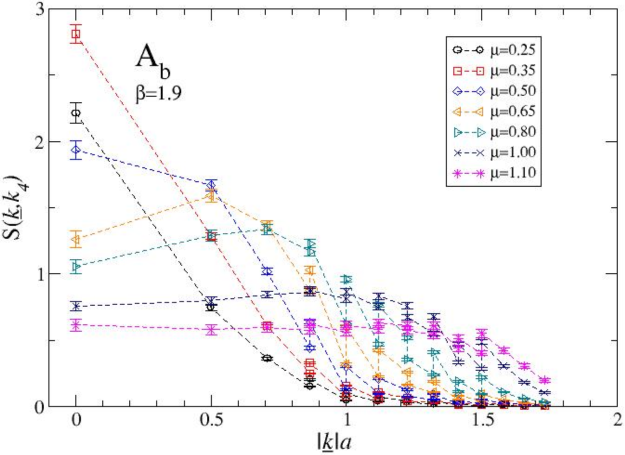
Fermi momentum may be found by extrapolating zero crossing to $k_4 = 0$?



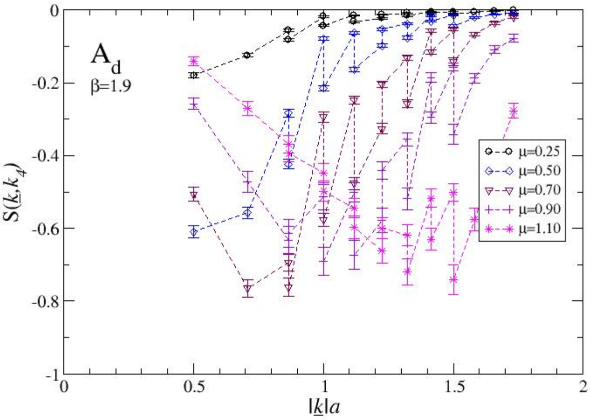
Normal propagator: scalar part



Anomalous propagator: scalar part



Anomalous propagator: tensor part



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- Clear signal for anomalous propagation
  - Scalar anomalous propagator becomes $\approx$ constant at large μ
  - Need to understand tensor component!

Outlook

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- Invert Gorkov propagator to obtain form factors and gaps
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- Study Gribov copy effects / gauge dependence using stereographic Landau gauge [with Dhagash Mehta]
- Continuum extrapolation?

Volume dependence

$[\mu = 0.9, j = 0.04]$