Information on the Phase Boundary from Hadron Yields or the Unreasonable Success of the Thermal Model

EMMI Workshop Quarks, Hadrons, and the Phase Diagram of QCD
St. Goar, August 31 – September 3, 2009

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analysis of yields of produced hadronic species in statistical model – grand canonical

partition function: $\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln(1 \pm \exp(-(E_i - \mu_i)/T))$

particle densities: $n_i = N/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2\, \mathrm{d}p}{\exp((E_i - \mu_i)/T) \pm 1}$

for every conserved quantum number there is a chemical potential:

$$\mu_i = \mu_B B_i + \mu_S S_i + \mu_{I_3} I_i^3$$

but can use conservation laws to constrain V, μ_S, μ_{I_3} → **Fit at each energy provides values for T and μ_b**

technical details:

van der Waals type interaction via excluded volum correction following Rischke, Gorenstein, Stoecker, Greiner, 1991

finite volume correction a la Balian and Bloch

width of all resonances included by integrating over Breit-Wigner distributions

For a review see: Braun-Munzinger, Redlich, Stachel, QGP3, R. Hwa ed. (Singapore 2004) 491-599; nucl-th/0304013

successfully applied to AGS Si+Au data in 1994

J.Stachel, P.Braun-Munzinger, workshop in honor of the 75th birthday of R. Hagedorn, Divonne June 1994

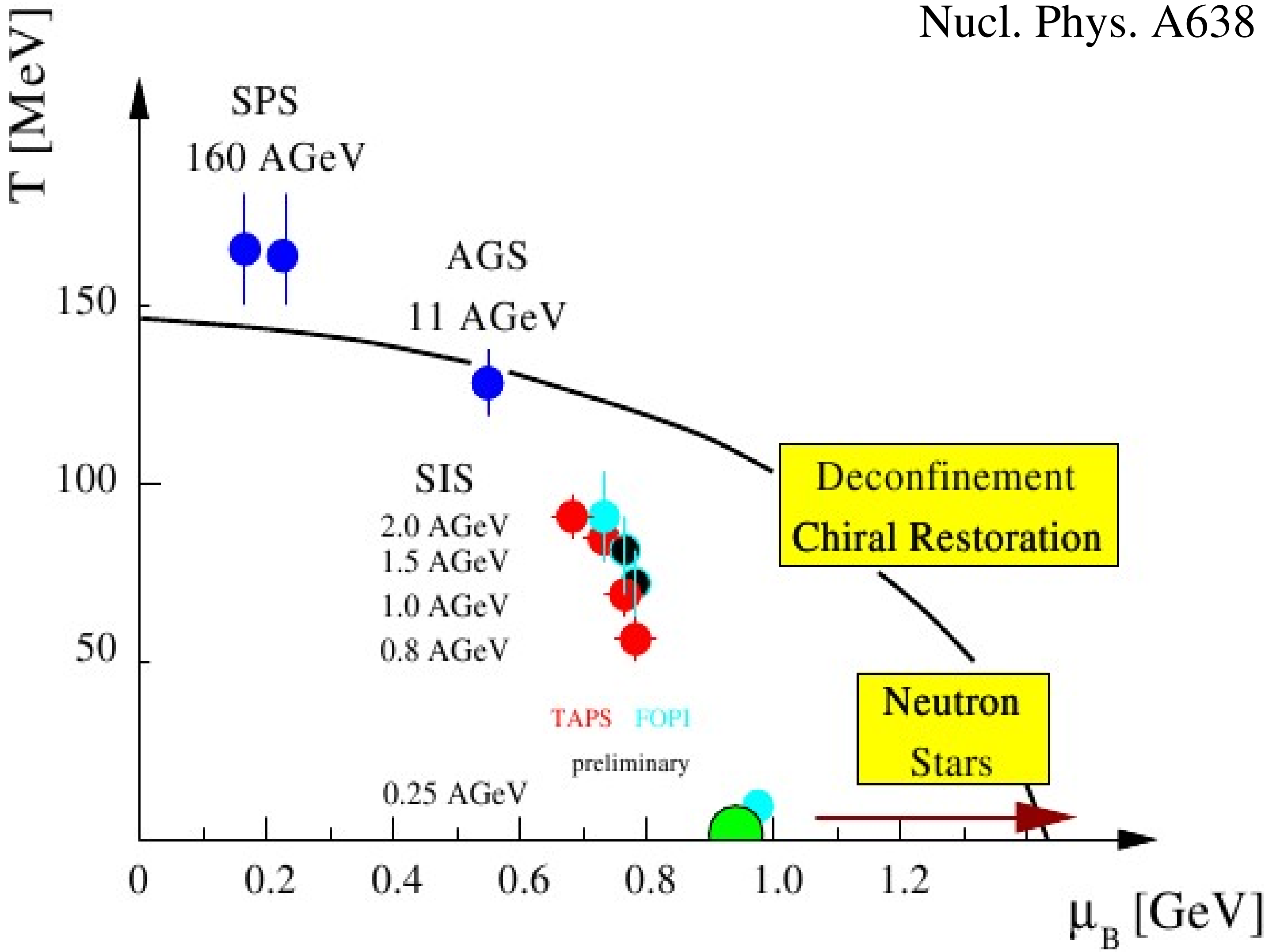
TABLE I. Particle ratios calculated in a thermal model for two different temperatures, baryon chemical potential μ_b= 0.54 GeV and strangeness chemical potential μ_s such that overall strangeness is conserved, in comparison to experimental data (with statistical errors in parentheses) for central collisions of 14.6 A GeV/c Si + Au(Pb).

Particles	Thermal Model		Data		
	T=.120 GeV	T=.140 GeV	exp. ratio	rapidity	ref.
π/(p+n)	1.29	1.34	1.05(5)	0.6 - 2.8	[4,3]
d/(p+n)	$4.3 \cdot 10^{-2}$	$5.8 \cdot 10^{-2}$	$3.0(3) \cdot 10^{-2}$	0.4 - 1.6	[4]
$\bar{p}$/p	$1.47 \cdot 10^{-4}$	$5.8 \cdot 10^{-4}$	$4.5(5) \cdot 10^{-4}$	0.8 - 2.2	[15]
K^+/π^+	0.23	0.27	0.19(2)	0.6 - 2.2	[4]
K^-/π^-	$5.0 \cdot 10^{-2}$	$6.2 \cdot 10^{-2}$	$3.5(5) \cdot 10^{-2}$	0.6 - 2.3	[4]
K^0_s/π^+	0.14	0.16	$9.7(15) \cdot 10^{-2}$	2.0 - 3.5	[16,4,21]
K^+/K^-	4.6	4.3	4.4(4)	0.7 - 2.3	[4]
Λ/(p+n)	$9.5 \cdot 10^{-2}$	0.11	$8.0(16) \cdot 10^{-2}$	1.4 - 2.9	[16,4,3]
$\bar{\Lambda}/\Lambda$	$8.8 \cdot 10^{-4}$	$3.7 \cdot 10^{-3}$	$2.0(8) \cdot 10^{-3}$	1.2 - 1.7	[15]
$\phi/(K^++K^-)$	$2.4 \cdot 10^{-2}$	$3.6 \cdot 10^{-2}$	$1.34(36) \cdot 10^{-2}$	1.2 - 2.0	[15]
Ξ^-/Λ	$6.4 \cdot 10^{-2}$	$7.2 \cdot 10^{-2}$	0.12(2)	1.4 - 2.9	[17]
$\bar{d}/\bar{p}$	$1.1 \cdot 10^{-5}$	$4.7 \cdot 10^{-5}$	$1.0(5) \cdot 10^{-5}$	2.0	[18]

works over 9 oom!

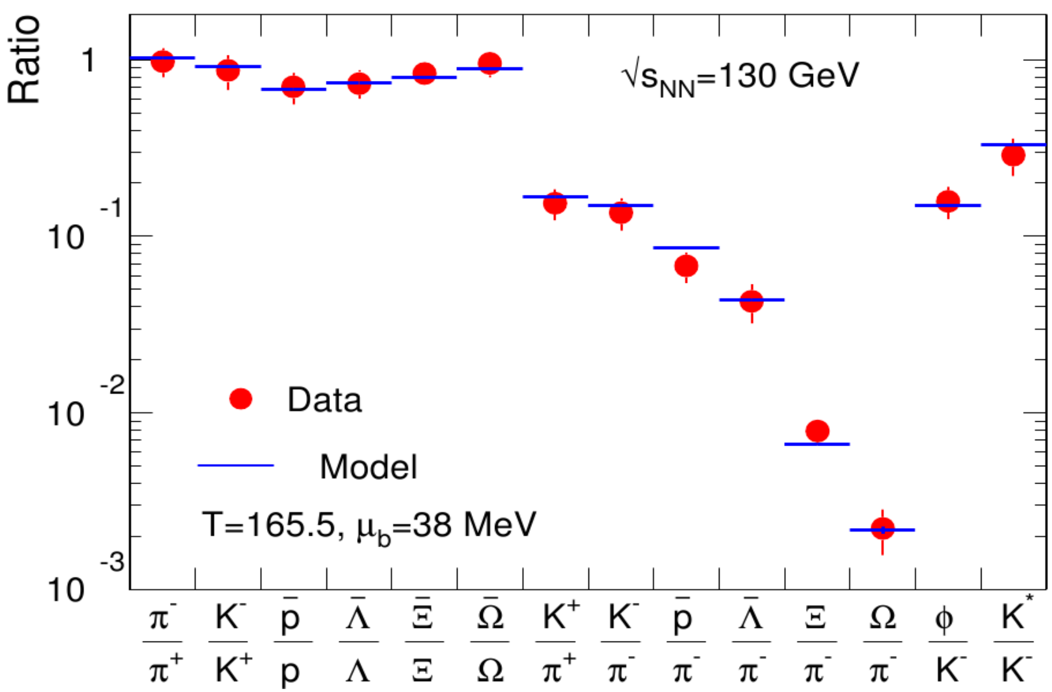
leading to the first phase diagram with experimental points

P.Braun-Munzinger and J. Stachel, nucl-th/9803015,
Nucl. Phys. A638 (1998) 3

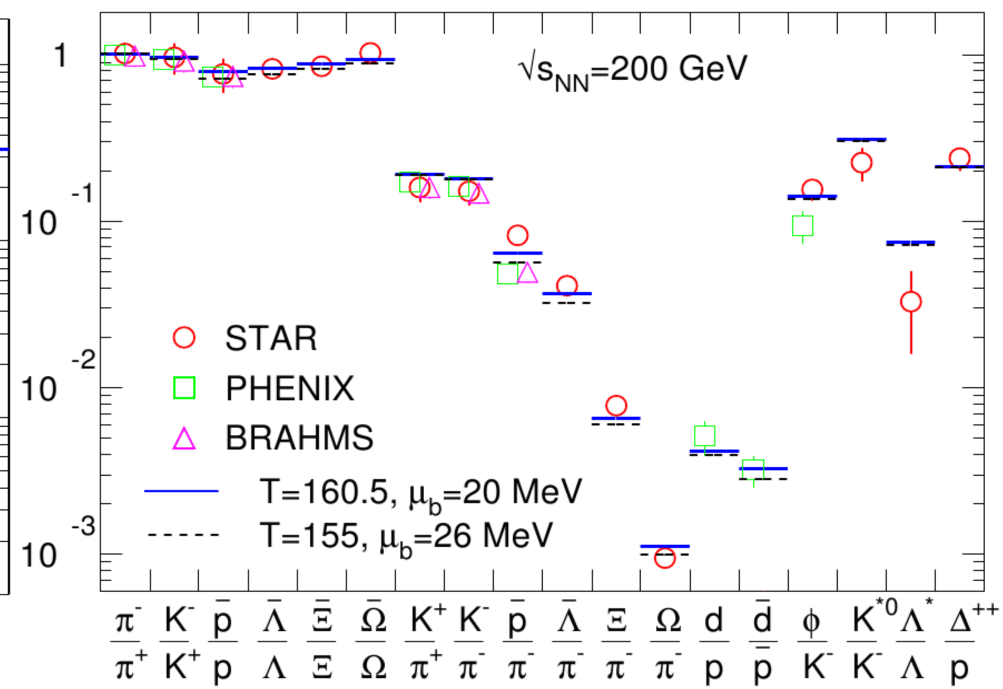


hadron yields at RHIC compared to statistical model (GC)

130 GeV data in excellent agreement
with thermal model predictions



prel. 200 GeV data fully in line
still some experimental discrepancies



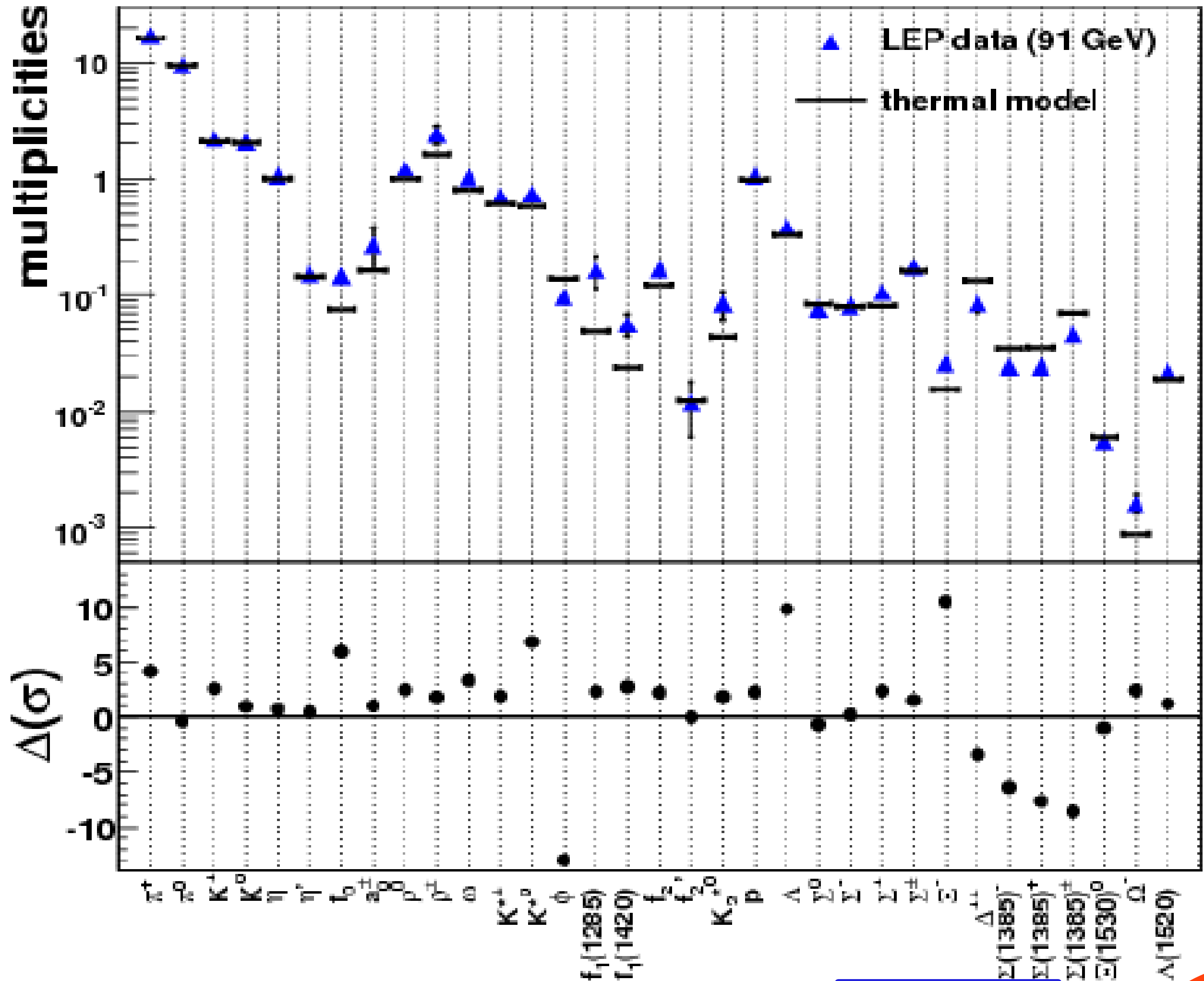
where yields are final, agreement in HI collisions much better than for e+e-

chemical freeze-out at: T = 165 ± 5 MeV

P. Braun-Munzinger, D. Magestro, K. Redlich, J. Stachel, Phys. Lett. B518 (2001) 41
A. Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys. A772 (2006) 167

e+e- collisions: initialize thermal model with u,d,s,c,b – jets according to measurement (weak isospin)

A. Andronic, P. Braun-Munzinger, F. Beutler, K. Redlich, J. Stachel, arXiv 0804.4132

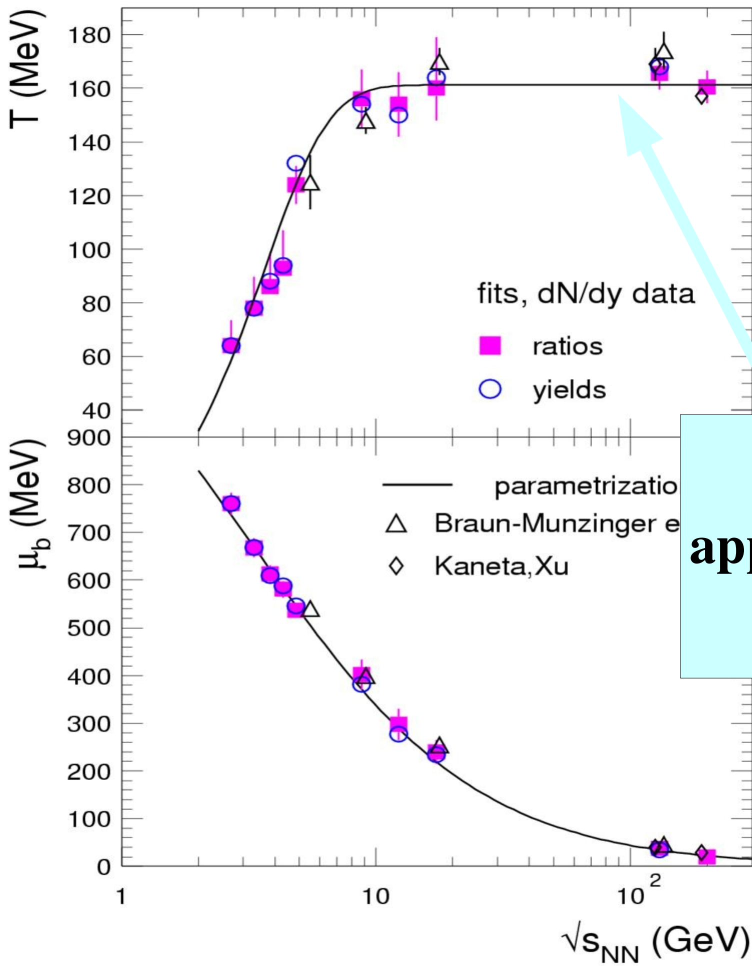


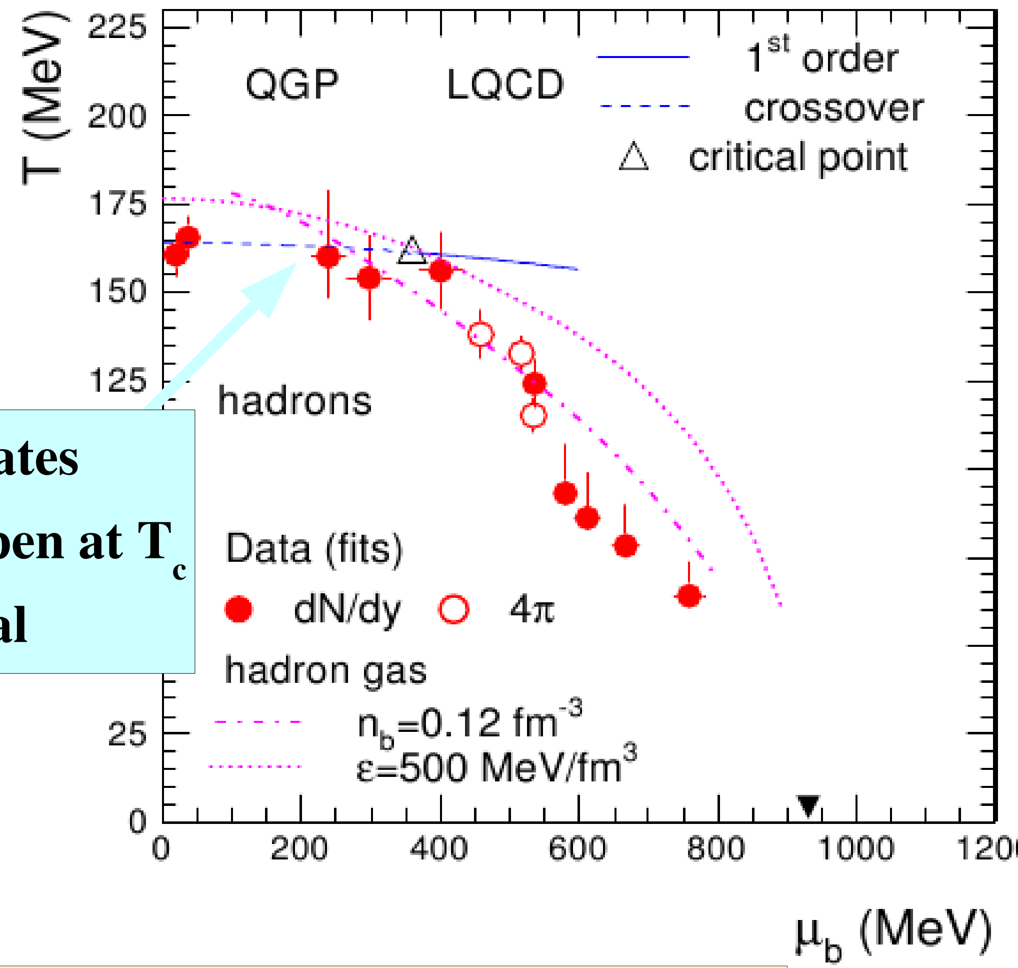
strangeness supressed – fit still not good!

parameter set: $T=164$ MeV, $V=20\ fm^3$, $\gamma_s=0.72$ with $\chi^2=718/30$

hadrochemical freeze-out points and the phase diagram

A. Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys. A772 (2006) 167





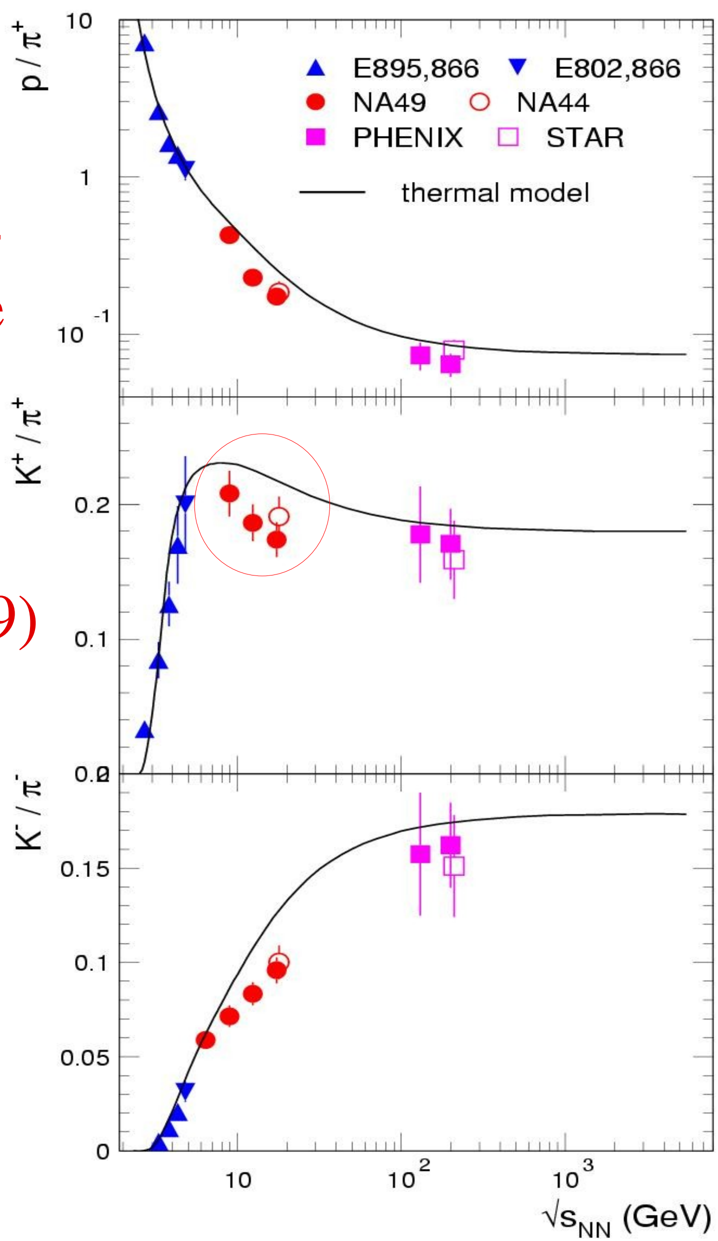
T_{chem} saturates appears to happen at T_c not trivial

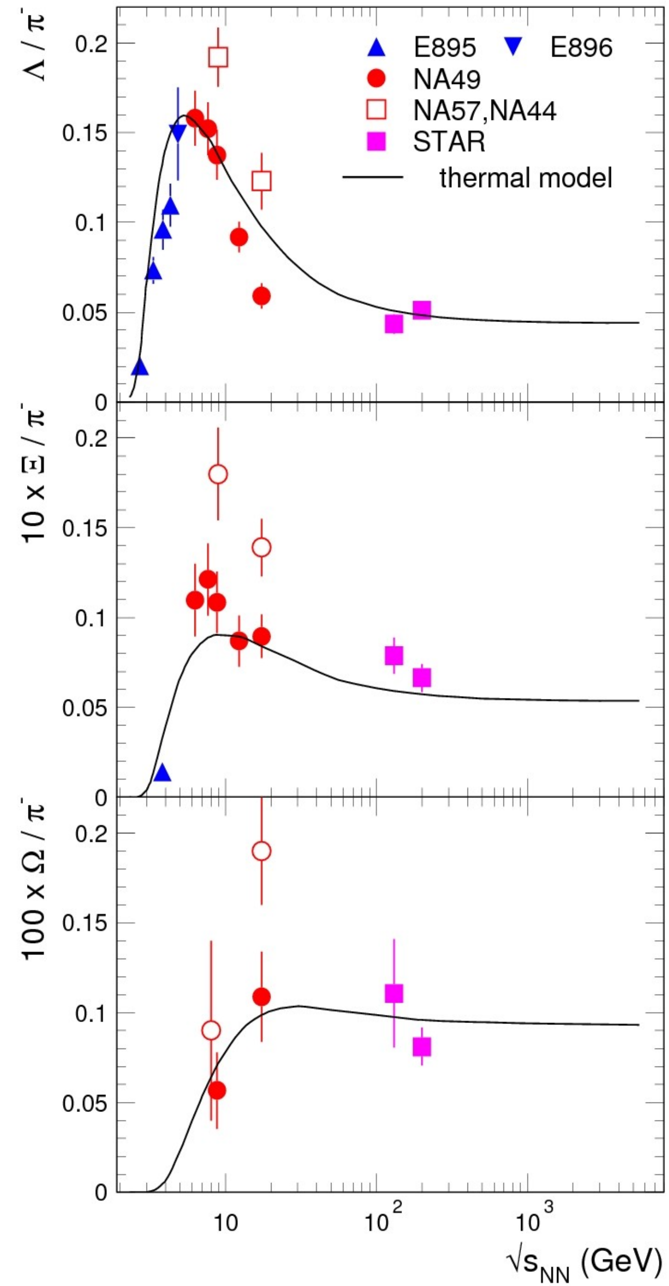
in general good fits to all central heavy ion collision data
for non-central collisions additional corona effect plus
thermalized core with same parameters good (Aichelin et al.)

detailed √s dependences: predicted features as peak in Λ/K appear, but K^+/π^+ cannot be really well reproduced

A. Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys. A772 (2006) 167

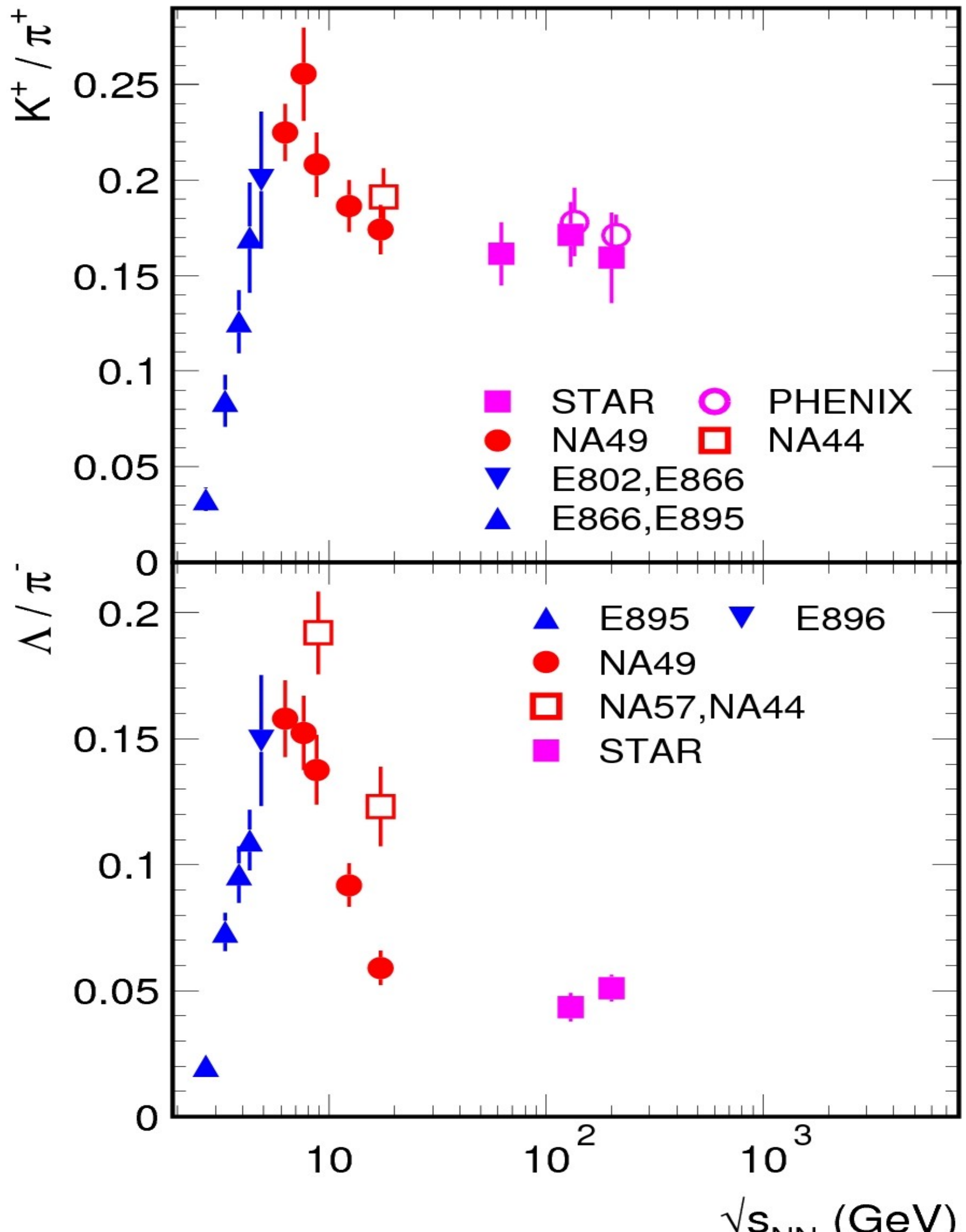
instead: suggestion of evidence for the onset of deconfinement (Gazdzicki, Gorenstein 1999)





why address the issue again now?

- extended and finalized data by NA49 and low energy point from STAR



why address the issue again now?

- study effect of consequences of extended hadronic mass spectrum using all information as of PDG2008 (was required for precision e+e- data)

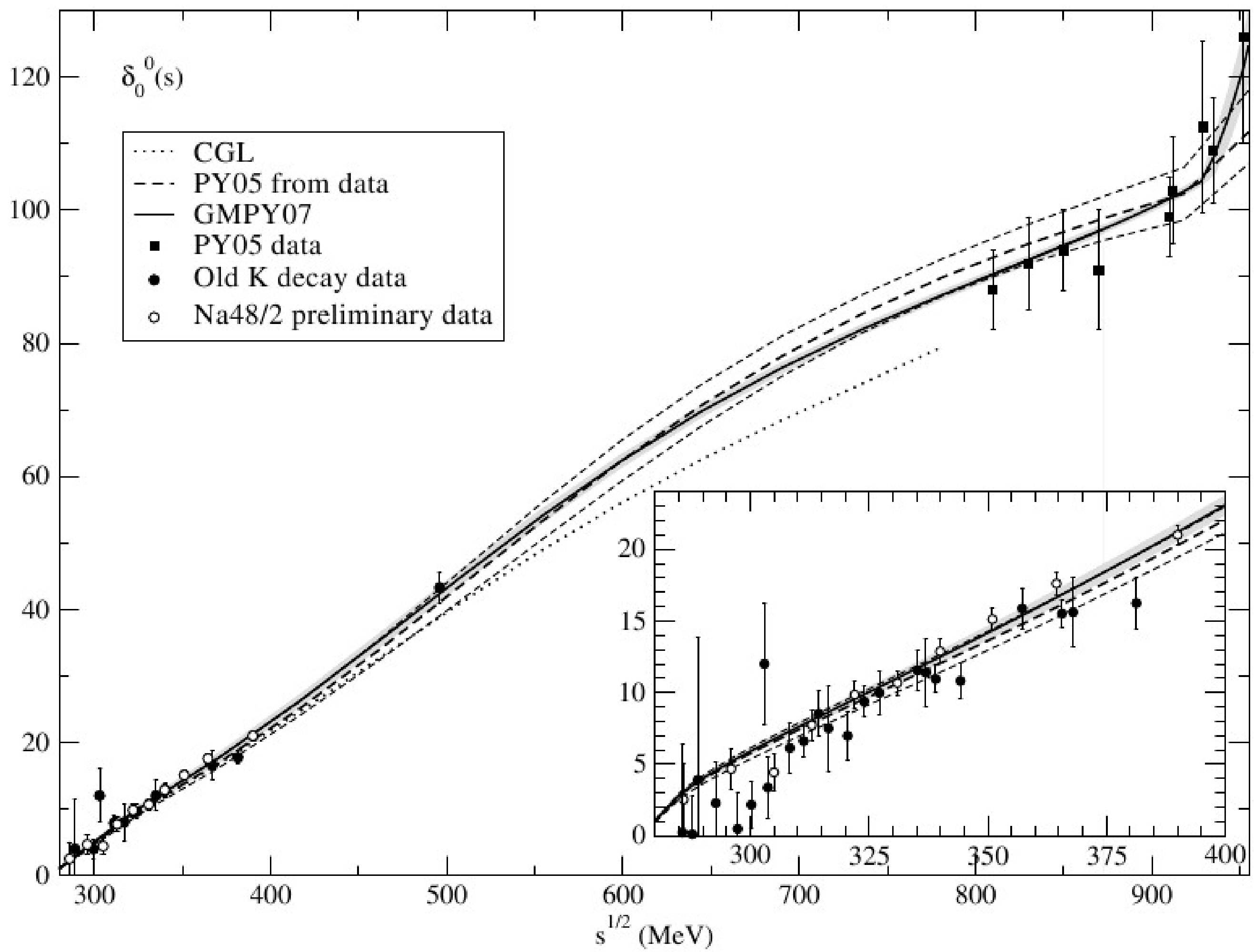
		2005 paper	now	
mesons	nonstrange	37	+ 86	= 123
	strange	28	+ 4	32
	charmed	15	+ 25	40
	beauty	16	+ 12	28
baryons	nonstrange	30	+ 36	66
	strange	33	+ 30	63
	charmed	10	+ 22	32
	beauty	0	+ 14	14
composites		28		28
total		197	+ 229	426

why address the issue again now?

- solidified evidence for the σ meson

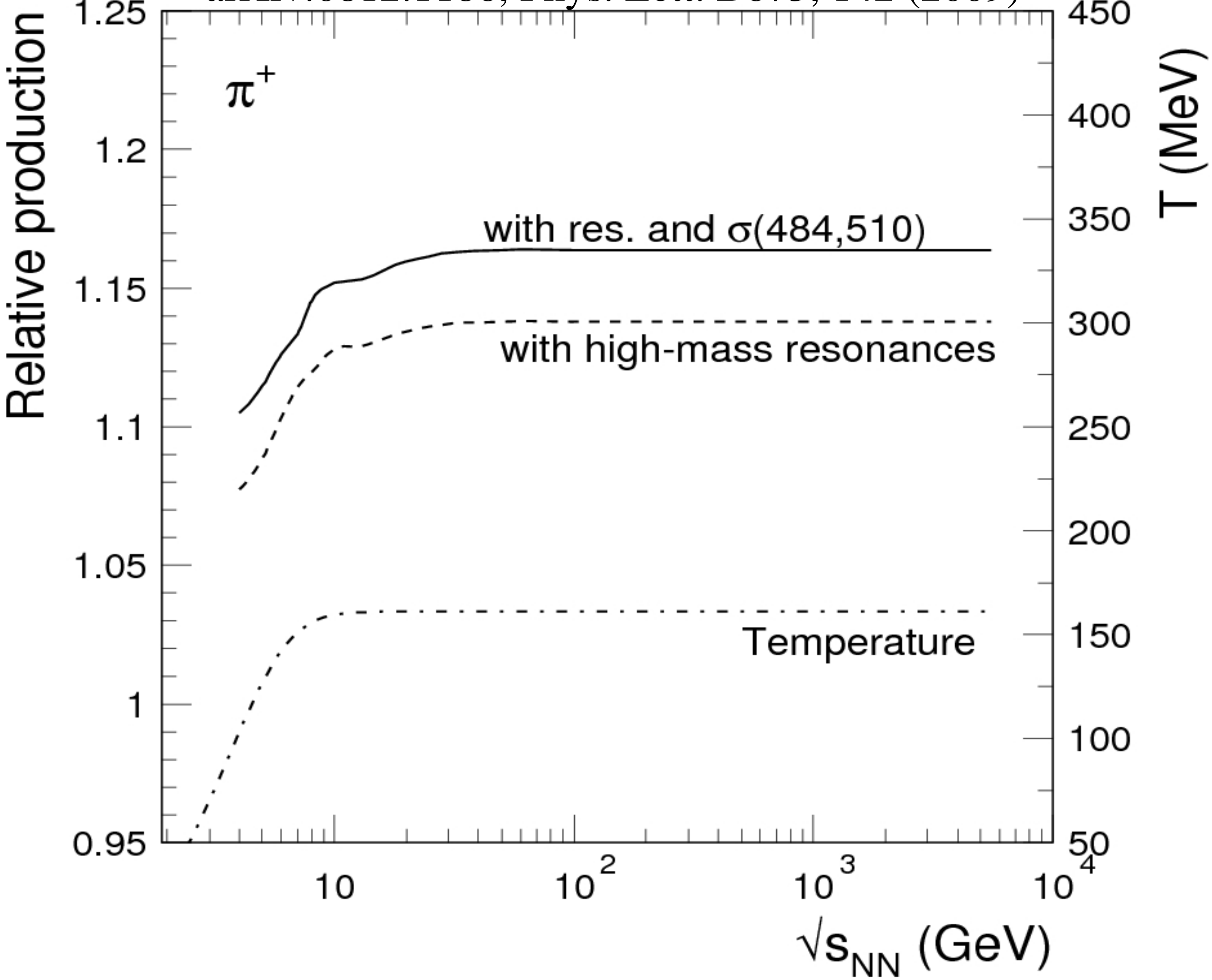
analysis by Garcia-Martin, Pelaez, Yndurain Phys. Rev. D76 (2007) 074034, hep-ph/0701025

$$M_\sigma = 484 \pm 17 \ \text{MeV}, \quad \Gamma_\sigma/2 = 255 \pm 10 \ \text{MeV}$$



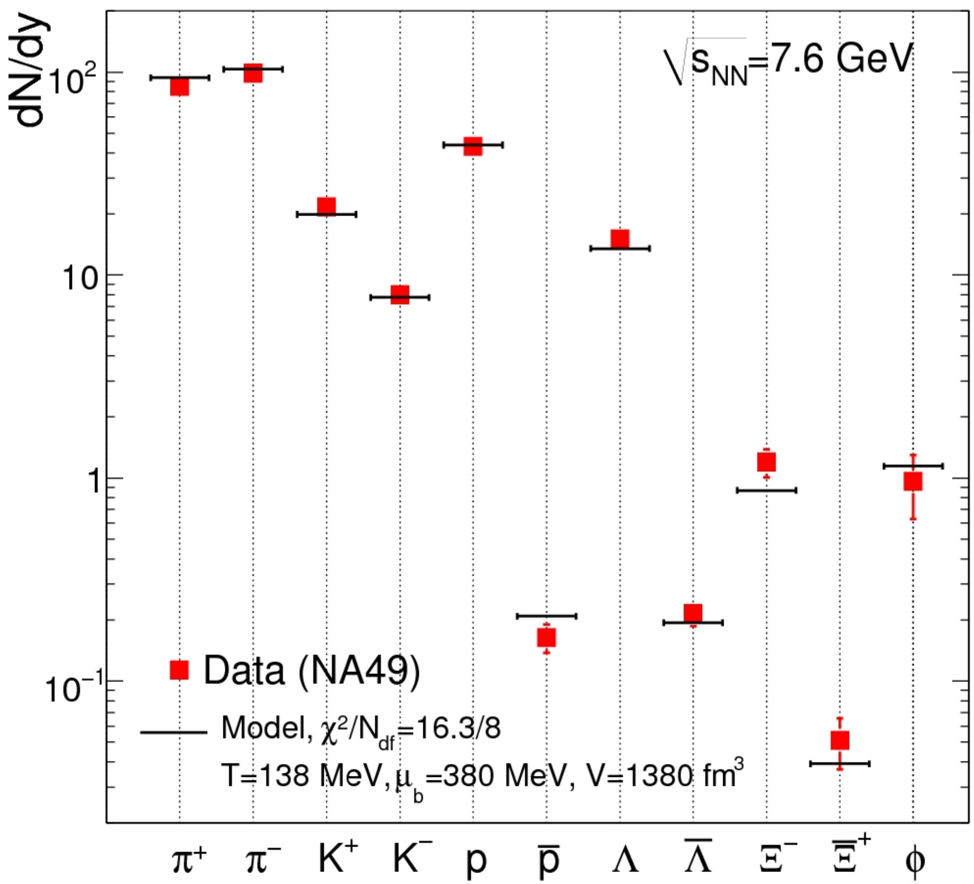
relative change in pion yield with more high mass resonances and the σ

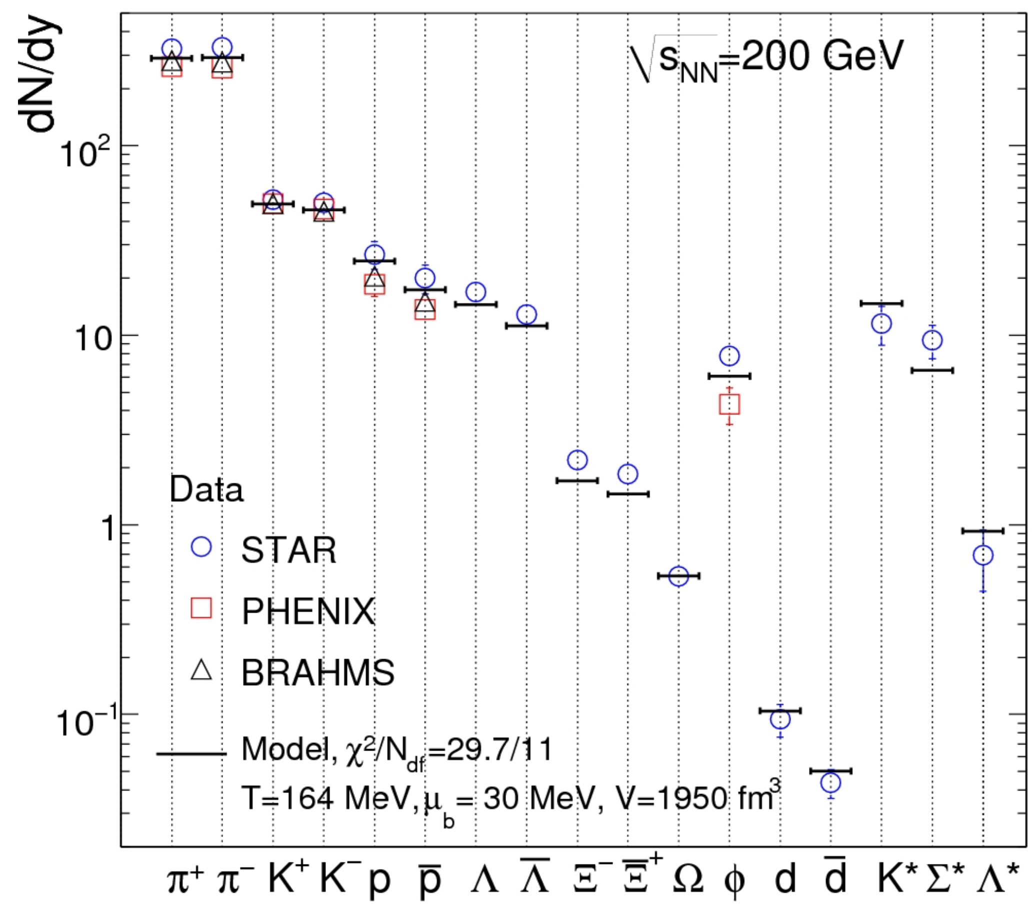
A. Andronic, P. Braun-Munzinger, J. Stachel, arXiv:0812.1186, Phys. Lett. B673, 142 (2009)



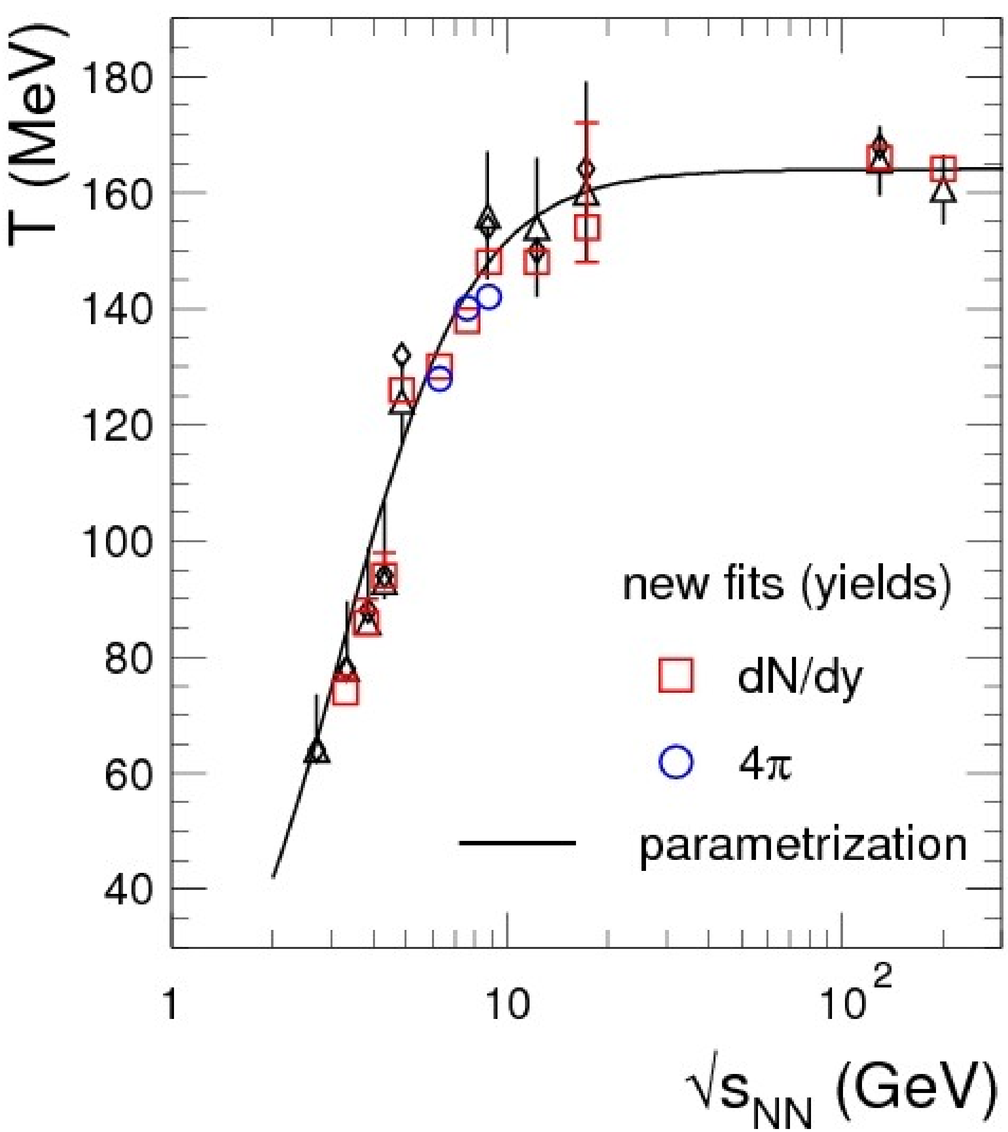
as T levels off,
so does the increase in pion yield

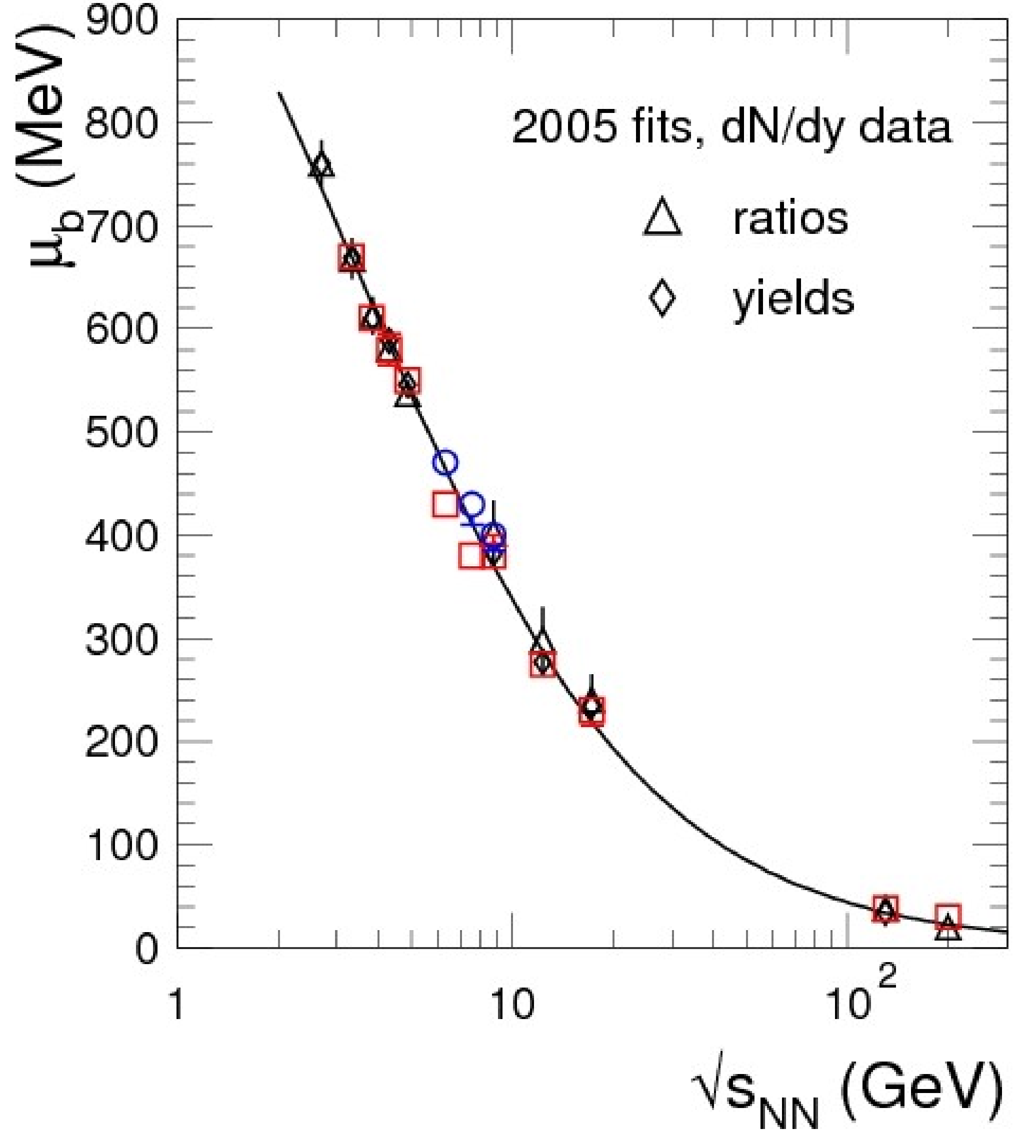
overall, good fit of hadron yields





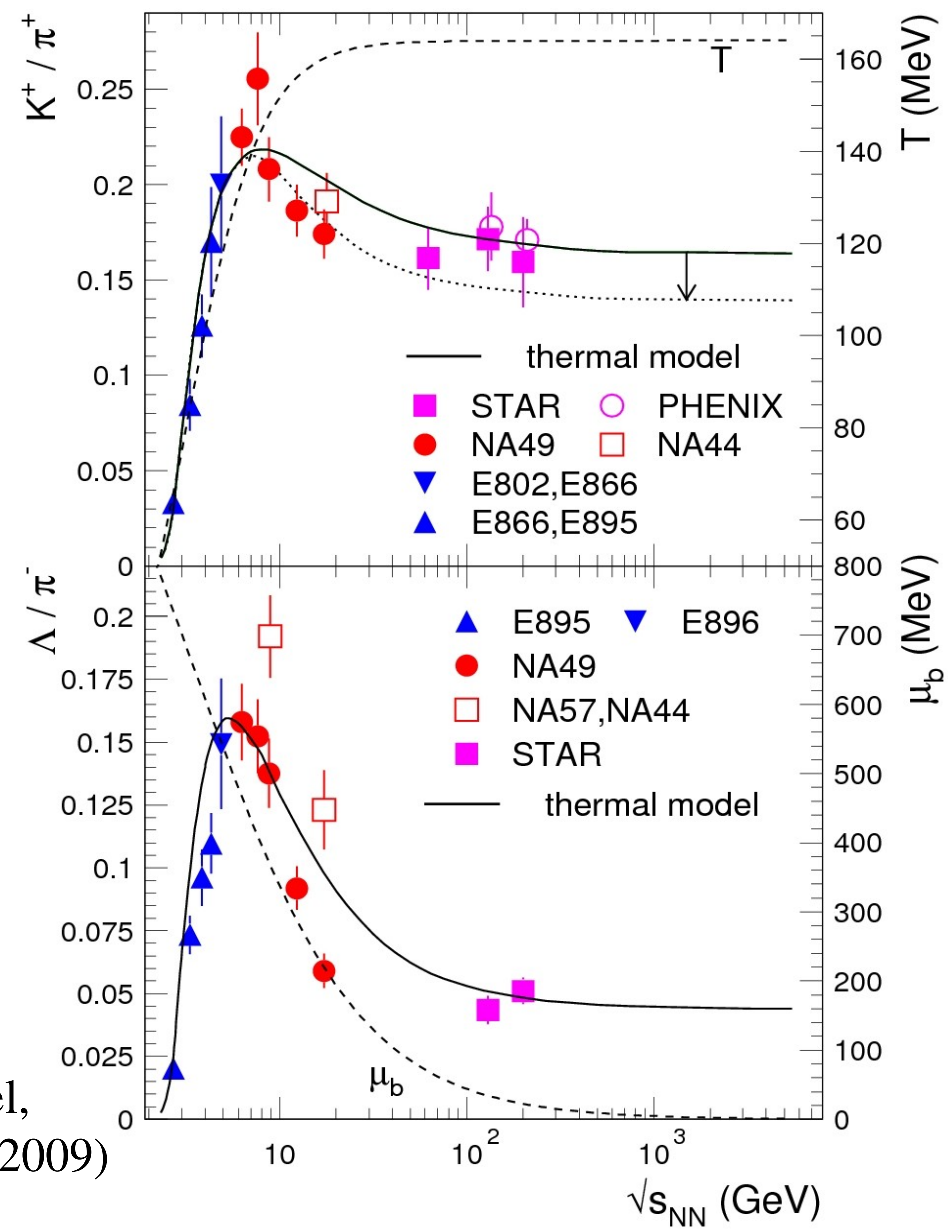
little change in systematic beam energy dependence of T and μ





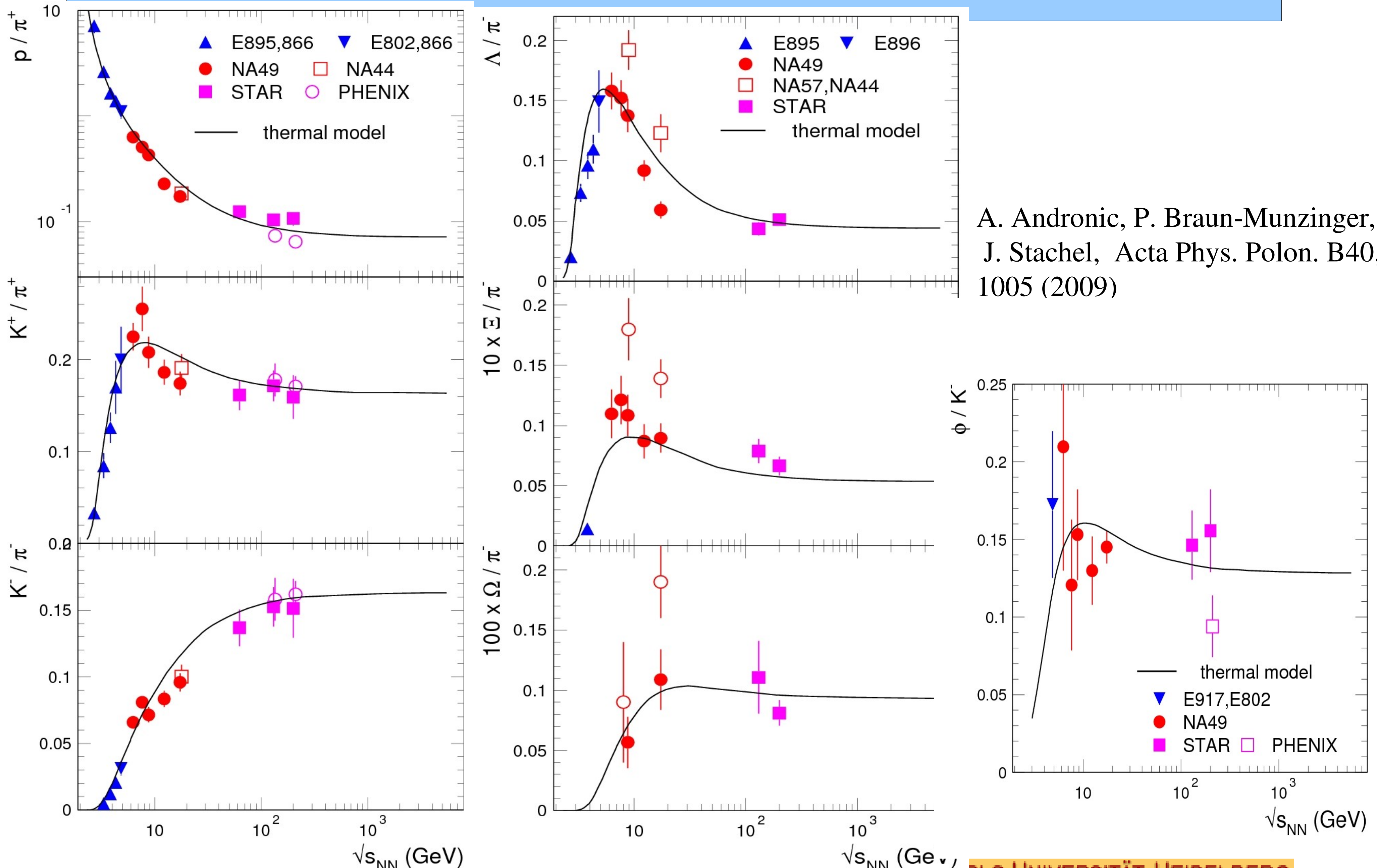
the horn becomes more pronounced in statistical model

levelling off of T and concomittant levelling off of increase in pion yield sharpens the maximum



A. Andronic, P. Braun-Munzinger, J. Stachel, arXiv:0812.1186, Phys. Lett. B673, 142 (2009)

beam energy dependence of hadron production well reproduced



A. Andronic, P. Braun-Munzinger, J. Stachel, Acta Phys. Polon. B40, 1005 (2009)

what is consequence of potentially still incomplete hadron spectrum?

R. Hagedorn's statistical bootstrap model Nuovo Cimento 3 (1965) 147;
also CERN-TH.4100/85

strongly interacting particles form resonances (3,4,5,...n) and those may combine to form new resonances

only low lying ones experimentally known

all particles form mass spectrum $\rho(m)$ of which lower part is known

infer asymptotic behaviour from some new principle - $\rho(m)$ should generate itself from a self-consistency condition

density of states $$\sigma(E,V) = \sum_{n=1}^{\infty} \frac{V^n}{n!} \int \delta\left(E - \sum_{i=1}^{n} E_i\right) \prod_{i=1}^{n} \rho(m_i)\, dm_i\, d^3p_i$$

both $\rho(m)$ and $\sigma(E,V)$ should be the same if E=m and V is resonance volume V_0

single out one 'elementary' input state m_0 and identity of ρ and σ yields the bootstrap equation

$$\rho(m) = \delta(m - m_0) + \sum_{n=2}^{\infty} \frac{V_0^n}{n!} \int \delta\left(m - \sum_{i=1}^{n} E_i\right) \prod_{i=1}^{n} \rho(m_i)\, dm_i\, d^3p_i$$

'clusters consist of clusters'

physical solution $$\rho(m) = f(m)\, e^{m/T_0}$$

partition function has singularity at T_0 and possibly exhibits a phase transition

Hagedorn Spectrum

Note: an exponential mass spectrum is expected also in QCD

at least in the large Nc limit (T.L Cohen, arXiv:0901.0494)

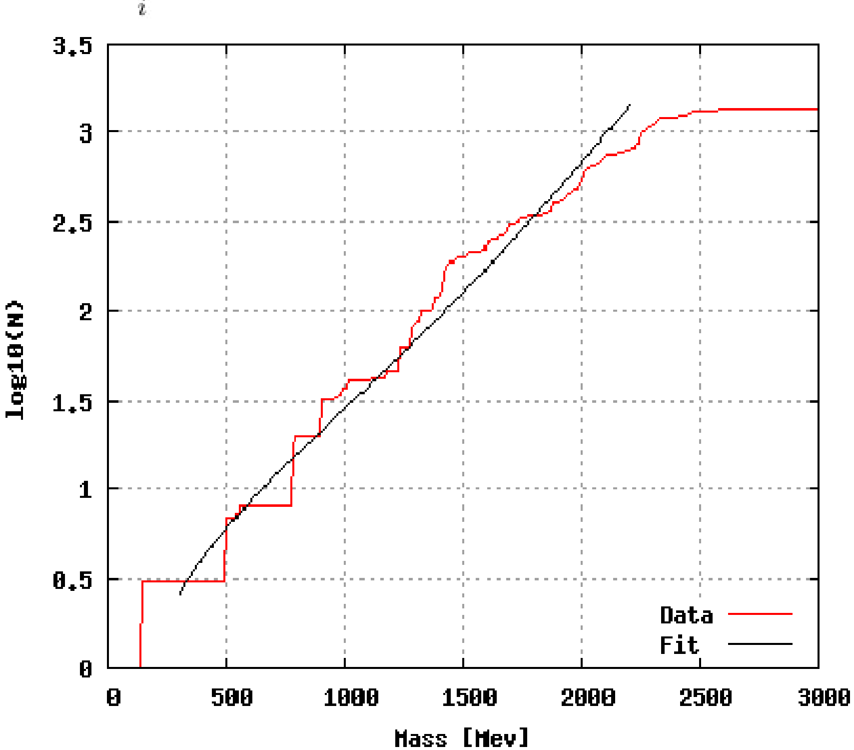
what do we know today about Hagedorn spectrum?

Maciej Sobczak – analysis of states listed in PDG2008 compilation

$$f_{FIT}(m) = log_{10}\left(\int_0^m \frac{c}{(x^2+m_0^2)^{5/4}} exp(x/T_H)\right)$$

$$\rho(m) = \frac{c}{(m^2+m_0^2)^{5/4}} exp(m/T_H)$$

$$N_{exp}(m) = \sum_i g_i \Theta\left(m - m_i\right)$$



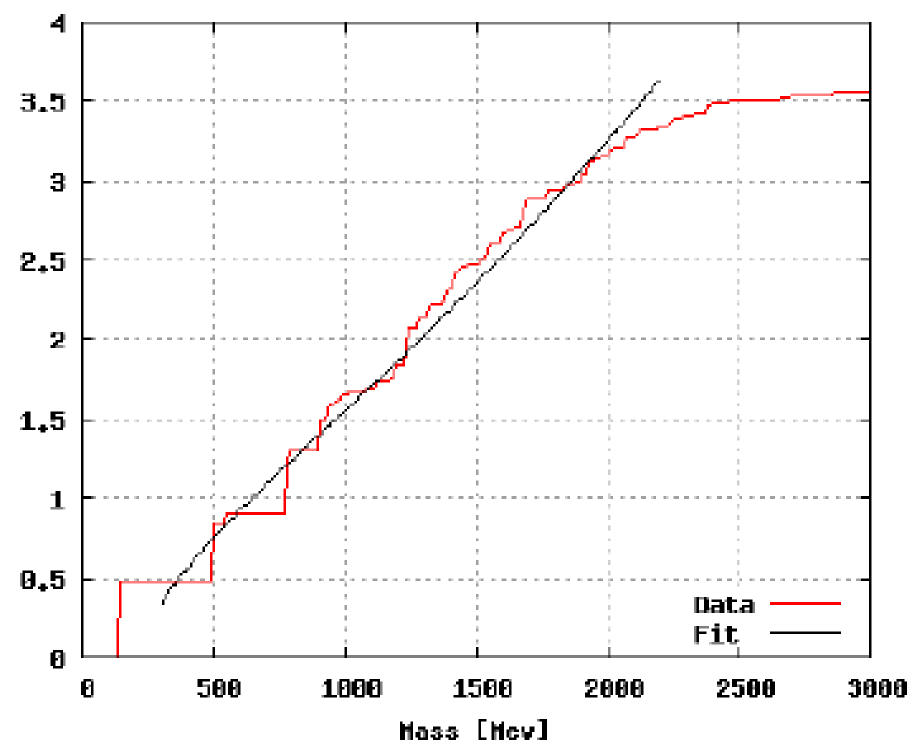


Figure 2: All mesons $T_H = 203.315$, $c = 25132.674$, range: $300 - 2200$ MeV All hadrons $T_H = 177.086$, $c = 18726.494$, range: $300 - 2200$ MeV

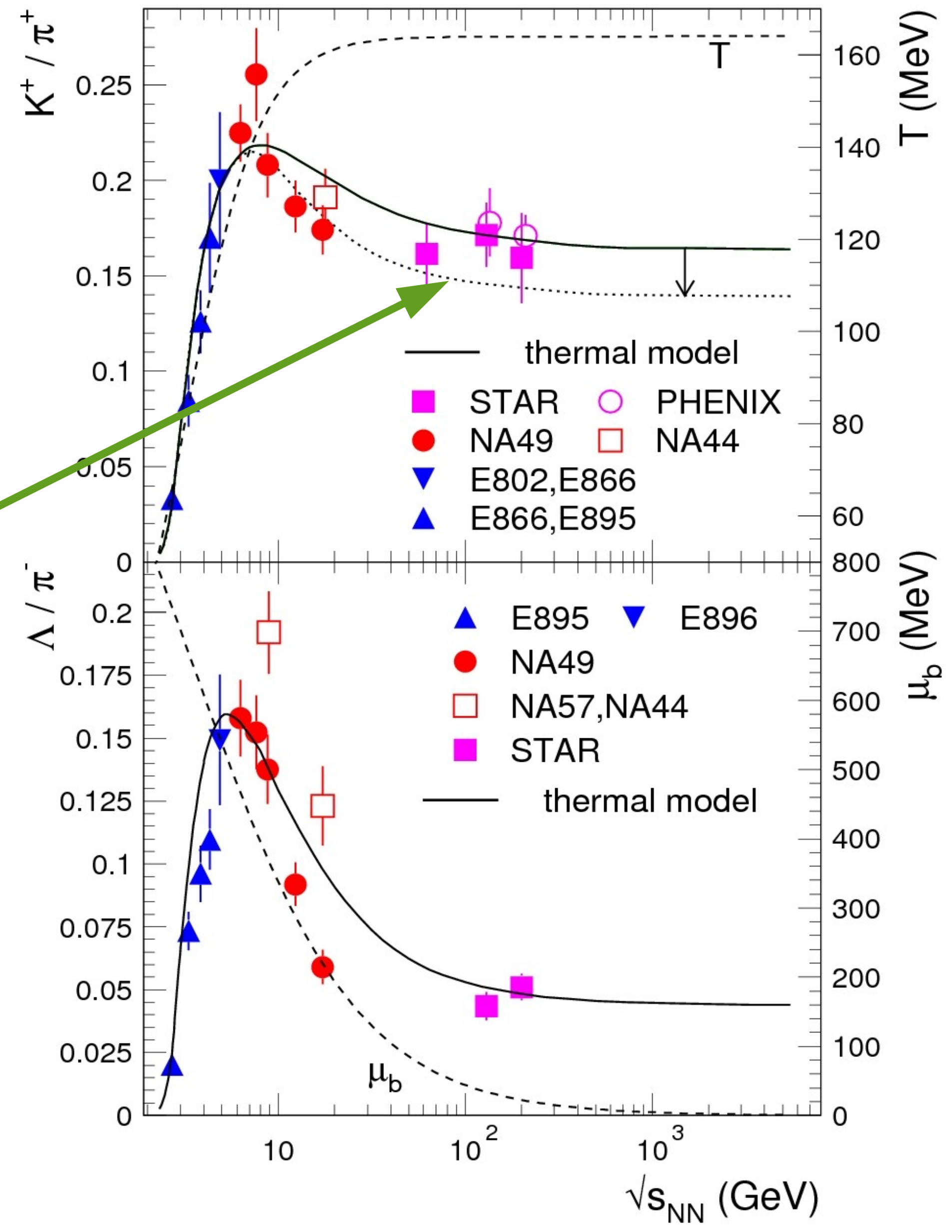
How would this affect K^+/π^+?

estimate effect by extending mass spectrum beyond 3 GeV based on T_H = 200 MeV and assumption how states decay

strongest contribution to kaon from K* producing one K

all high mass resonances produce multiple pions

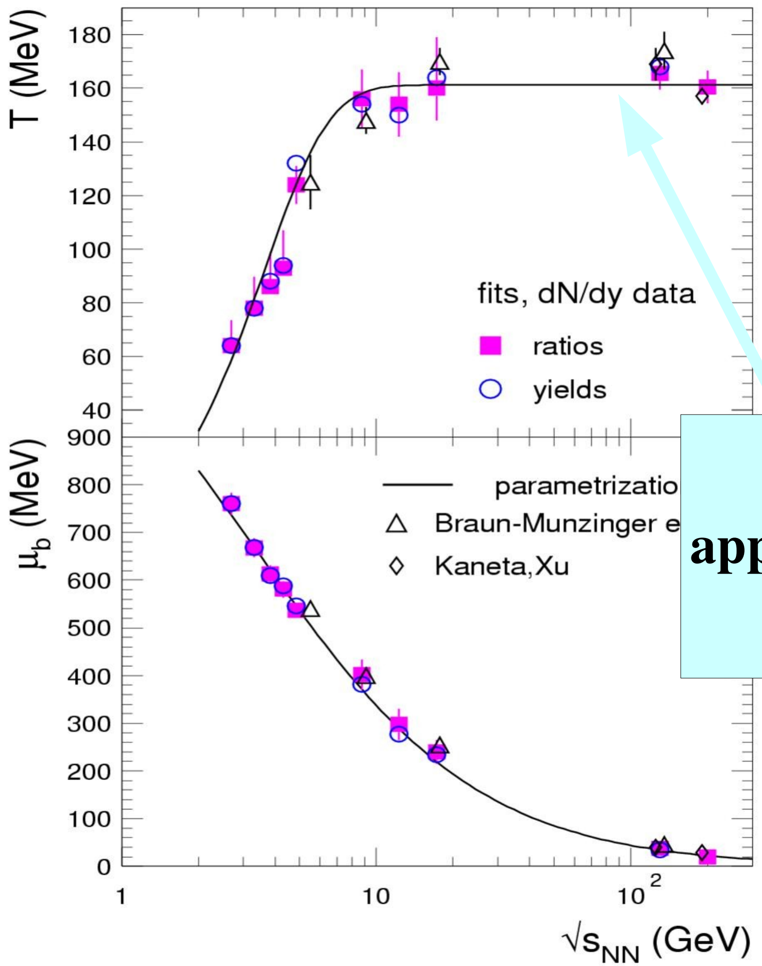
-> further reduction of K^+/π^+

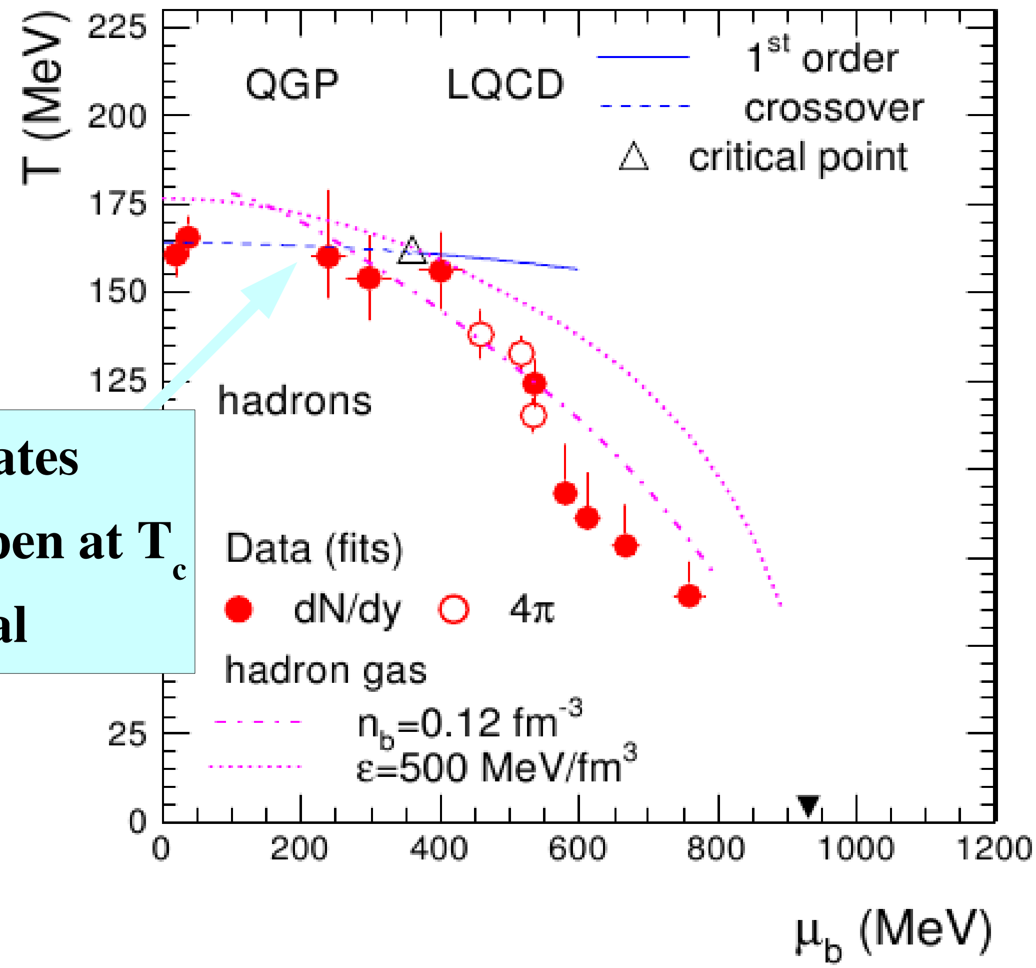


A. Andronic, P. Braun-Munzinger, J. Stachel, arXiv:0812.1186, Phys. Lett. B673, 142 (2009)

hadrochemical freeze-out points and the phase diagram

A. Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys. A772 (2006) 167





T_{chem} saturates appears to happen at T_c not trivial

how is equilibrium achieved?
why freeze-out at phase boundary?

where does chemical equilibration happen?

- Hadron yields determined by Boltzmann factors with 'free' vacuum masses.
- Particle distribution in QGP phase has no 'memory' of vacuum hadron masses .
- Relative yields are not determined by the strange quark mass but by individual strange hadron masses (at fixed T and m).

Hadron yields get into equilibrium in hadronic or mixed phase

- But: the number of strange quarks is determined in the QGP phase! (modulo modifications during hadronization) Equilibrium then implies redistribution of strange quarks.

How is chemical equilibration achieved?

P. Braun-Munzinger, J. Stachel, C. Wetterich, Phys. Lett. B596 (2004) 61

scenario:

- Strangeness saturation takes place in the QGP phase and/or during hadronzation
- Phase transition is crossed from above.
- Near T_c new dynamics associated with collective excitations will take place and trigger the transition.
- Propagation and scattering of these collective excitations is expressed in the form of multi-hadron scattering. Near T_c multi-hadron processes will therefore be dominant. Chemical equilibrium is reached via these multi-hadron scattering events.

2-body collisions are not enough

typical densities at $T_{ch}: \rho_\pi = 0.174/\mathrm{fm}^3(\mathrm{incl.res.}), \rho_K = 0.030/\mathrm{fm}^3 \rho_\Omega = 0.0003/\mathrm{fm}^3$

to maintain equilibrium even for 5 MeV below T_{ch} need relative rate change

$$\left|\frac{\bar{r}_\Omega}{n_\Omega} - \frac{\bar{r}_K}{n_K}\right| = \tau_\Omega^{-1} - \tau_K^{-1} = 1.10 - 0.55/\mathrm{fm} = 0.55/\mathrm{fm}$$

so, Ω density needs to change by 100 % withing 1 fm/c!
typical reactions with large cross sections (order 10 mb) and relative velocity of 0.6 give

$$\Omega + \bar{K} \rightarrow \Xi + \pi \quad \rightarrow \quad \bar{r}_\Omega / n_\Omega = n_{\bar{K}} \langle v\sigma \rangle = 0.018/\mathrm{fm}$$

$$\pi + \pi \rightarrow K + \bar{K} \quad (\sigma = 3\mathrm{mb}) \quad \bar{r}_K / n_K = 0.18/\mathrm{fm}$$

i.e. much too slow to maintain equilibrium even over $\Delta T = 5$ MeV
even much more difficult to produce large Ω abundancy
assume hadronization like in pp, factor 8 too few Ωs, to produce them within 1 fm/c need reactions that provide $\bar{r}_\Omega / n_\Omega = 1.0$ ⇨ not with 2-body reactions
<u>Consensus in the literature:</u> P. Koch, B. Müller, J. Rafelski, Phys. Rep. 142 (1986)
C. Greiner, S. Leupold, J.Phys. G27 (2001) L95; P. Huovinen, J. Kapusta, nucl-th/0310051

Evaluation of multi-strange baryon yield as most challenging test case

consider situation at T_{ch} = 176 MeV first

- rate of change of density for n_{in} ingoing and n_{out} outgoing particles

$$r(n_{in}, n_{out}) = \bar{n}(\mathrm{T})^{n_{in}} |\mathcal{M}|^2 \phi$$

with

$$\phi = \prod_{k=1}^{n_{out}} \left(\int \frac{d^3 p_k}{(2\pi)^3 (2E_k)} \right) (2\pi)^4 \delta^4 \left(\sum_k p_k^\mu \right)$$

- the phase space factor φ depends on √s
  needs to be weighted by the probability *f*(s) that multiparticle scattering occurs at a given value of √s
  evaluate numerically in Monte-Carlo using thermal momentum distribution

- typical reaction $\Omega + \bar{N} \rightarrow 2\pi + 3K$
  assume cross section equal to the measured one for $p + \bar{p} \rightarrow 5\pi$
  at proper energy above threshold, i.e. √s = 3.25 GeV ➡ 6.4 mb

- compute matrix element and use for rate of $2\pi + 3K \rightarrow \bar{N} + \Omega$

$$r_\Omega = n_\pi^5 (n_K / n_\pi)^3 |\mathcal{M}|^2 \phi$$

Evaluation of multi-strange baryon yield

reaction $2\pi + 3K \rightarrow \bar{N} + \Omega$ leads to

$$r_\Omega = 0.00014 \text{fm}^{-4} \quad \text{or} \quad r_\Omega / n_\Omega = 1/\tau_\Omega = 0.46/\text{fm}$$

can achieve final density starting from only pions and kaons at t=0 in 2.2 fm/c

similarly one obtains

for $3\pi + 2K \rightarrow \Xi + \bar{N}$ or $\tau_\Xi = 0.71\,\text{fm}$

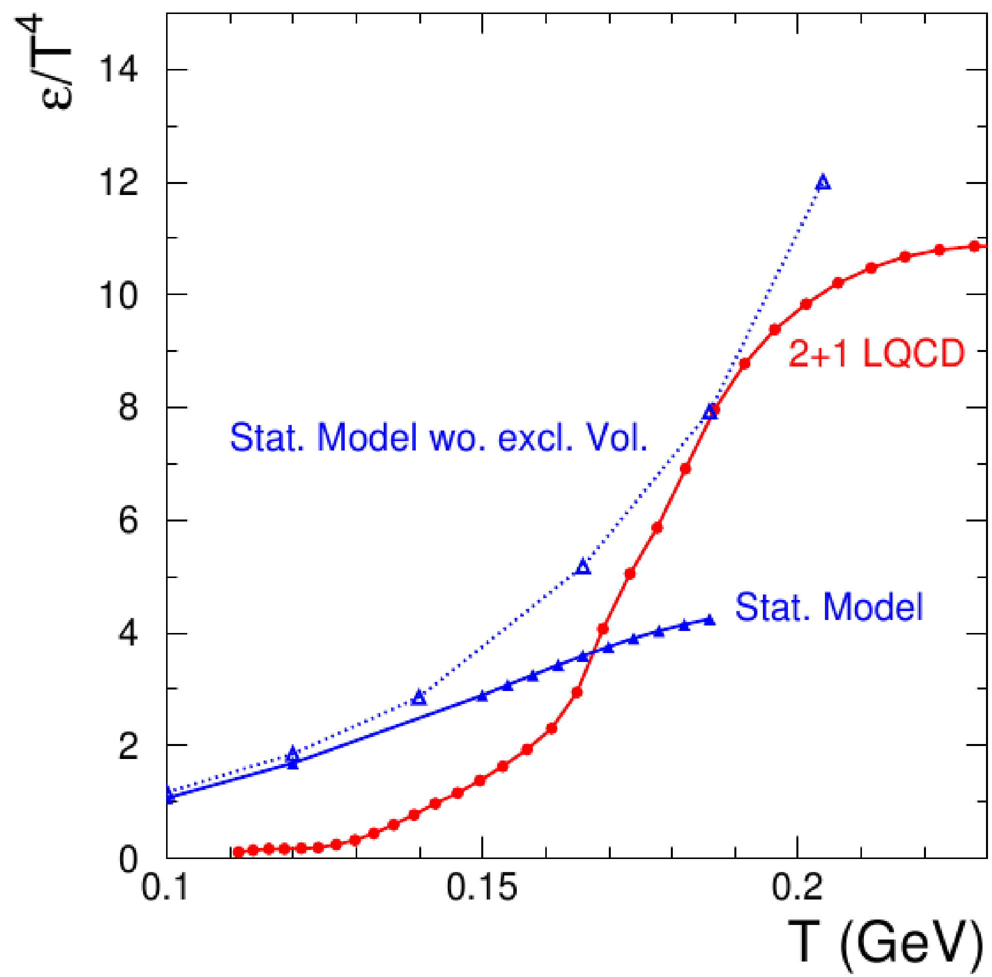
and

for $4\pi + K \rightarrow \Lambda + \bar{N}$ or $\tau_\Lambda = 0.66\,\text{fm}$

why do all particle yields show one common freeze-out T?

- **The density of particles varies rapidly (factor 2 within 8 MeV) with T near the phase transition due to increase in degrees of freedom.**
- **also: system spends time at Tc -> volume has to triple (entropy cons.)**
- **Multi-particle collisions are strongly enhanced at high density and lead to chem. equilibrium very near to T_c**
- **independently of cross section all particles can freeze out within narrow temperature interval**

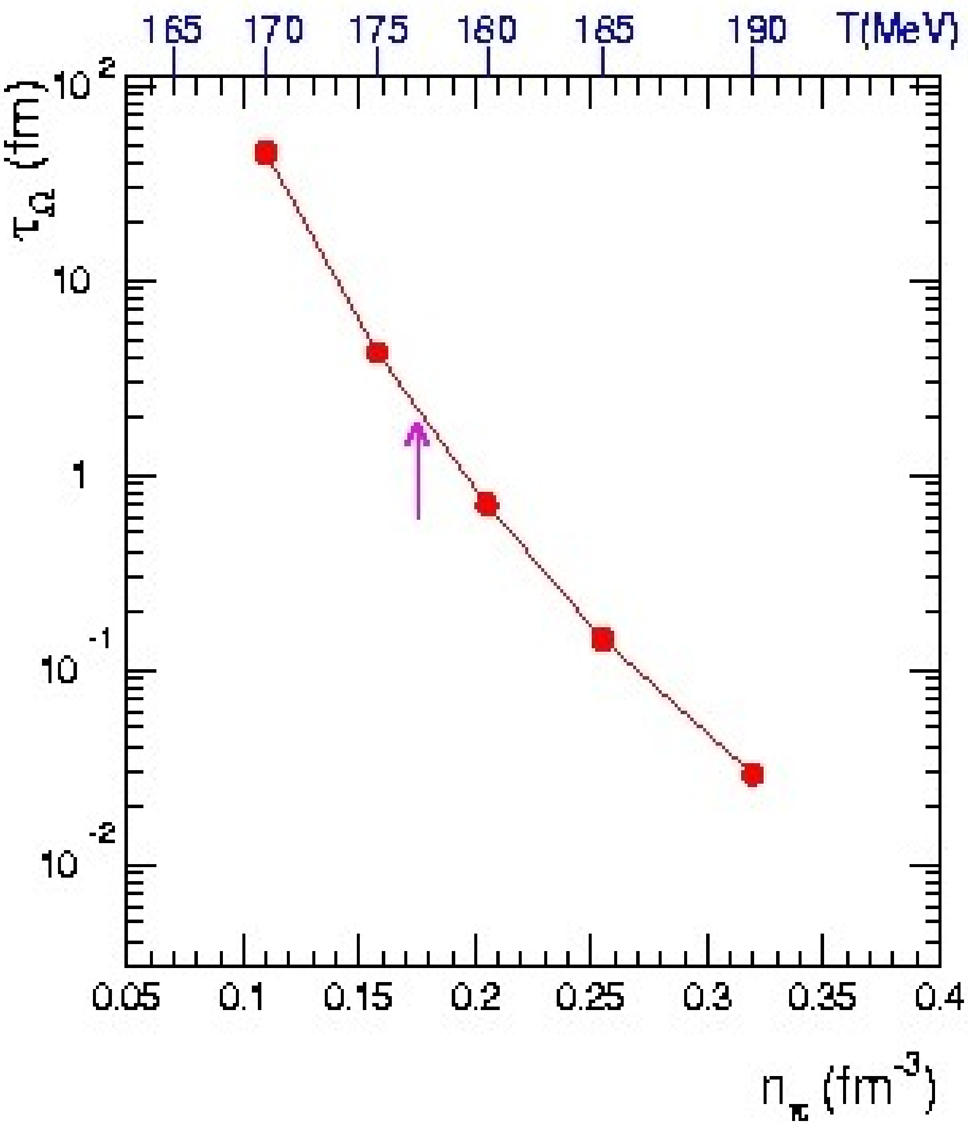
natural consequence that chemical freeze-out takes place at T_c!



Lattice QCD by F. Karsch et al.

P. Braun-Munzinger, J. Stachel, C. Wetterich, Phys. Lett. B596 (2004)61

Density dependence of characteristic time for strange baryon production



- Near phase transition particle density varies rapidly with T
- For small μ_b, reactions such as 2π+KKK$\rightarrow$ Ω Nbar bring multi-strange baryons close to equilibrium.
- in region around T_c equilibration time $\tau_\Omega \propto T^{-60}$!
- increase ρ_π by 1/3 or 8 MeV: τ = 0.2 fm/c

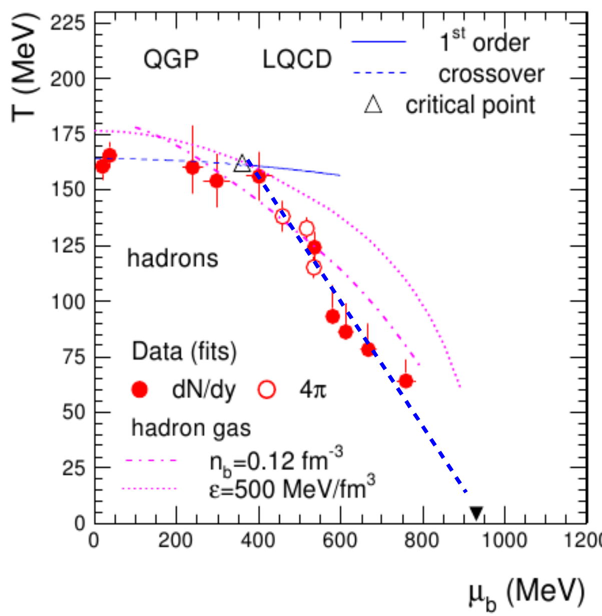
  decrease ρ_π by 1/3: τ = 27 fm/c
- All particles freeze out within a very narrow temperature window.

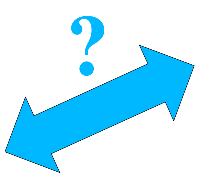
P. Braun-Munzinger, J. Stachel, C. Wetterich, Phys. Lett. B596 (2004)61

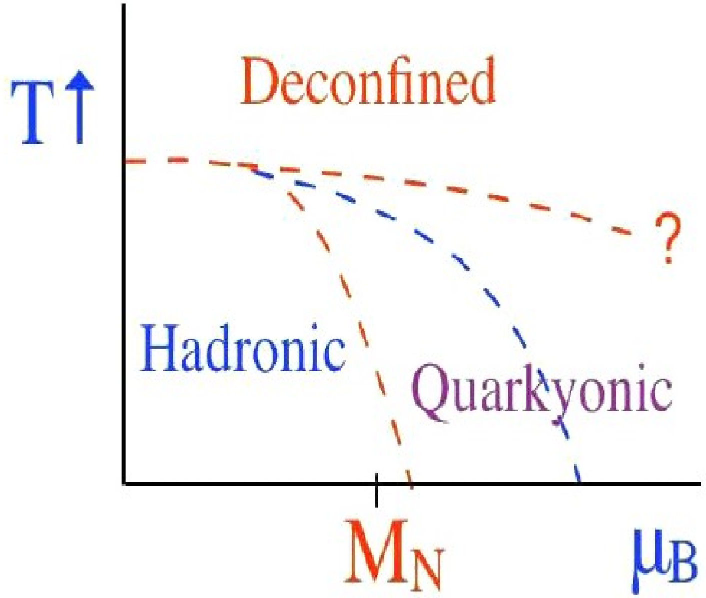
Conclusions

- hadron yields and their √s dependence - including the horn - produced by the statistical hadronization and the specific variation of the thermal parameters with beam energy
- in nature, rapid equilibration and freeze-out at a common T is achieved at the phase boundary due to steep density dependence on T and μ

  (shown for top SPS energy and above; P. Braun-Munzinger, J. Stachel, C. Wetterich, Phys. Lett. B596 (2004) 61)
- the thermal parameters delineate the phase boundary and thus the horn is due to the location of the phase boundary in the T-μ landscape







From Small $N_c = 2$ to

Large N_c : Proposed Phase Diagram

McLerran, Pisarski, 2007

and Sasaki, Redlich, McLerran 2008

Conclusions

the freeze-out points delineate the location of the phase boundary in the T-μ landscape

which phase boundary?

at high T/ moderate μ consistent with deconfinement phase transition (lattice QCD)

but in region where T is dropping?

could deconfinement transition come so low??

data follow path of approximately constant baryon density

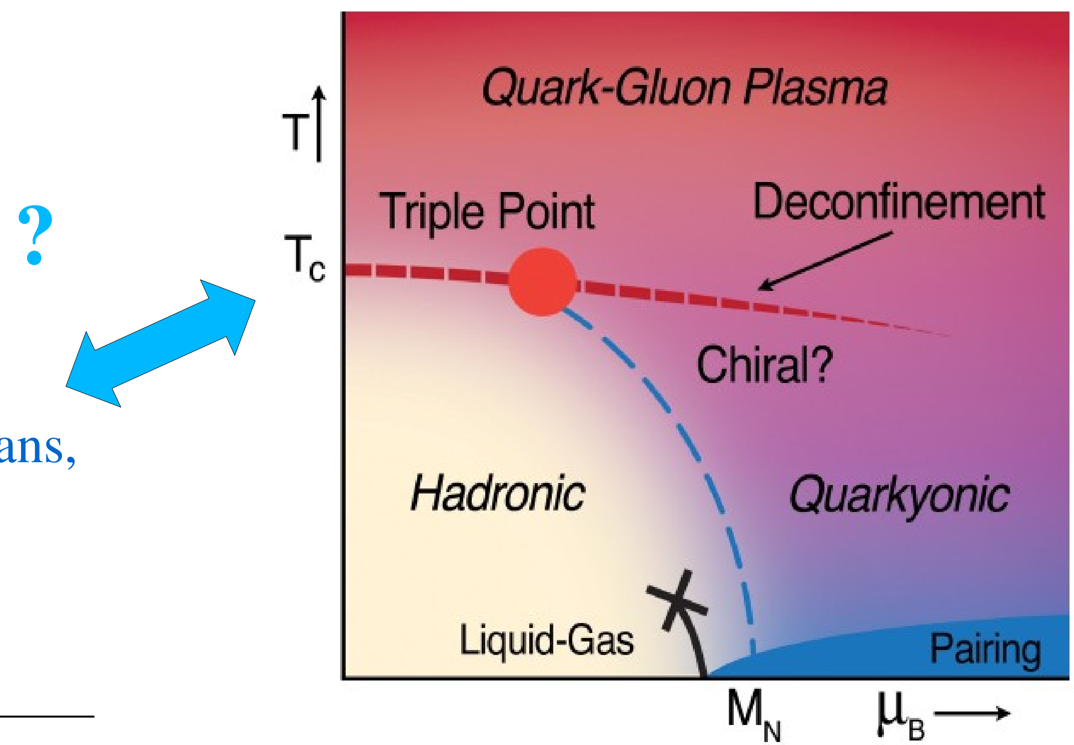
what transition is it?

quarkyonic? see talk Rob Pisarski

nuclear?

...?

subject of a publication in preparation by D. Blaschke, P. Braun-Munzinger, J. Cleymans, K. Fukushima, H. Oeschler, R.D. Pisarski, L. McLerran, K. Redlich, C. Sasaki and J.S.



Mass Changes close to T_c?

repeat fit of RHIC data with several hypotheses:

- change all masses by constant factor → similar fit quality if variation ≤ 20 %
  (see also Michalec, Florkowski, Broniowski, nucl-th/0103029)
  but don't expect π mass to scale with the rest (Goldstone boson)
- reduce m_ϕ by 5% → 3 σ discrepancy with data
- reduce $m_{K^{0*}}$ by 10% → 2.5 σ discrepancy with data

no room for very significant changes

Scenario of hadronic expansion between T_{ch} and T_f

choose values appropriate for RHIC Au-Au collisions

assume $T_{ch} = 176\,\mathrm{MeV}$

density decrease between chemical and thermal freeze-out: 30%

two pion correlation data: $R_{side} = 5.75\,\mathrm{fm}, R_{long} = 7.0\,\mathrm{fm}, \langle\beta_t\rangle = 0.5, \beta_{long} = 1$

isentropic expansion $\rightarrow \tau_f = 0.9 - 2.3\,\mathrm{fm}, T_f = 158 - 132\,\mathrm{MeV}$

(uncertainty due to variation in density profile)

near Tc: rate of cooling $|\dot{T}/T| = \tau_T^{-1} = (13 \pm 1)\%/\mathrm{fm}$

Check numerics via detailed balance

- Initially manifestly nonequilibrium situation - start with practially zero Ω density
- As equilibrium is approached
  rates $3K + 2\pi \to \Omega + \bar{N}$ and $\Omega + \bar{N} \to 3K + 2\pi$ have to become equal
- back and forth reactions scale very differently with pion density
  $\to$ only at one density can they be equal
- to explicitly check these rates now use pion, kaon, nucleon densities before strong decays, i.e. without resonance feeding
  (for all resonances corresponding rates have to be calculated accordingly)
- find: creation of Ω with $r_\Omega / n_\Omega = 3.4\ 10^{-3}$/fm
  and annihilation of Ω with $r_\Omega / n_\Omega = 1.4\ 10^{-3}$/fm

for equal rates reduce density by 25 %
reduce T by 2-3 MeV or excluded volume a bit larger